Mexican Immigration to the US: Selection, Sorting and Matching*

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Abstract
We propose a micro-founded theory of international migration, in which heterogeneous individuals endogenously sort into domestic and foreign labour markets and match with productivity-differentiated firms. Our approach is a fusion of the selection model by [Borjas (1987), the matching model by [Sattinger (1979) and the general equilibrium trade model by [Melitz (2003). We quantify various economic consequences of migration policies, including wage effects, entry/exit of firms, market size and fiscal implications. In a numerical experiment focused on Mexican immigration in the US, we find that imposing an infinite migration cost increases the wages of 44.4% of US residents – the less skilled ones, 47% of them are worse off, and 8.6% are indifferent. The main force that drives this result hinges on the pattern of selection of Mexican workers and their limited supply of American-specific skills.

Keywords: Migration; Matching; Selection; Sorting; Welfare; Inequality.


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1 Introduction

How does a cross-border movement of workers affect the economic environment in the sending and the destination country? This question becomes one of the major concerns in the current political debate. Topics related to immigration have proved to be central in many high-income countries, attracting significant interest during major elections.\footnote{To mention only few most influential: the UK vote for Brexit, German, French, Italian and Dutch parliamentary elections as well as the American vote for Republicans in 2016 presidential election.}

In this paper, by applying an original theoretical framework, we provide new, detailed, quantitative arguments for the academic and political debate about the economics of international migration. The proposed model includes four fundamental implications of workers’ movements: the wage effects, firms’ entry and exit processes, the market size effects as well as fiscal redistributions. With an endogenous treatment of continuous distributions of wages, we quantify the impact of the four-dimensional consequences on the welfare of individuals in the sending and the destination country. In numerical experiments, we focus on investigating the economic effects of Mexican immigration for the resident population in the United States and Mexico.

Our quantitative approach extends the theory of Gola (2016) by including migration costs, consumption, imperfect competition on the product market and endogenous trade. Thus, it is a combination of the selection model by Roy (1951) (formalized by Borjas (1987) in the context of international migration), the matching model inspired by Becker (1973) and Sattinger (1979) and the new trade theory with heterogeneous firms developed by Melitz (2003). We construct a framework which micro-found individual migration decisions, endogenizes migrants’ selection within the cohort of peers, their individual consumption decisions, and workers’ matching with firms. Economic agents are characterized by heterogeneous skills required to produce in distinct countries (modelled as separate labour markets). Individuals decide to migrate after comparing endogenously determined real wages and migration costs. This process determines whether they supply the source-specific skill in the sending country, or they decide to work in the host country using the destination-specific skill. To produce, workers have to match with heterogeneous firms. Each worker-firm pair generates a surplus, which is a function of employee’s skill and firm’s productivity. Matching is, though, not random: firms decide who they want to hire by maximizing their profit. Optimal assortment reveals the positive and assortative property, which means that in both economies the best workers are employed by the most productive firms. Finally, the supply of enterprises is endogenously determined through a market process, following Hopenhayn (1992) and Melitz (2003). From a pool of entrants,
only selected firms remain active; those with a productivity level below a certain threshold
decide to exit the market. Enterprises can also locate their products on foreign markets,
as in Melitz (2003). In the equilibrium, individuals endogenously sort into two countries,
firms hire workers to maximize their profits, the supply of entrepreneurs and exporters is
governed by zero profit conditions, and wages clear in both labour markets.

The proposed model allows for investigating rich economic effects of migration policies.
As a consequence of the endogenous matching process, an any mass of any type of workers
influences the whole distribution of wages, similarly to Costrell and Loury (2004). Con-
gruent employees are close substitutes, while individuals located in opposite tails of the
wage (and skill) distribution act as complements. The exact, local value of the elasticity
of substitution between any two workers is determined endogenously through the tech-
nology of production, optimal matching and the actual density of talents in the economy.

The wage effect of migration is a fundamental phenomenon investigated by Card (2001),
Borjas (2003), Manacorda et al. (2012) and Ottaviano and Peri (2012). The distributive
consequences of migration, reported by these papers, depend heavily on the elasticity of
substitution between natives and immigrants, and among worker types. The first two
papers find almost perfect substitution between native and foreign workforce, while the
two latter (for the US and the UK respectively) give evidence on a certain degree of
complementarity between the two groups. The above problem has been circumvented by
Dustmann et al. (2012), the closest approach in migration literature to ours. The authors
estimate the wage effects of international migrants for UK citizens along natives’ wage
distribution. Using a stylized CES model, and a detailed dataset from the Labour Force
Survey, their main finding is that the actual wage effect is counter-proportional to the
change in the density of native wages after an inflow of immigrants. Only 25% of the lowest
skilled British were worse-off due to immigration between 1997 and 2005. In this way,
the paper stresses the differentiation between “nominal” skill levels of immigrants, and
their “real” allocation in destination country’s wage distribution – immigrants arriving in
the UK tend to be strongly downgraded in terms of their actual occupation.

An important ingredient of the qualitative features of our results is brought about
by the endogenous firm entry and exit process. Even though neglected in the literature,
the change in the extensive margin of enterprises active on the market is instrumental
in our approach for shaping the welfare impact of migration in both sending and desti-
nation country. An increase in the number of immigrants imposes a downward pressure
on wages of the most substitutive native workers, some of whom may exit the labour
market. Entrepreneurs improve their bargaining position and collect higher profits, which
motivates new firms to enter. A greater number of firms benefits all workers, especially
those at the right tail of the skill distribution, whose wage is exceptionally sensitive to firm’s productivity, as in Costrell and Loury (2004).

Embedding a two-sided heterogeneous labour market into the general equilibrium structure in the vein of Melitz (2003) induces a third implication of migration: the market size effect highlighted in the recent quantitative trade and migration literature.\(^2\) Assuming that all individuals reveal love-for-variety and consume horizontally differentiated basket of goods, the change in the mass of firms on the market, has an influence on real wages through the ideal price index. Papers by Iranzo and Peri (2009), Biavaschi and Elsner (2013), Di Giovanni et al. (2015), Aubry et al. (2016), Biavaschi et al. (2016) develop multi-country models in a monopolistically competitive world in the vein of Krugman (1980) and Melitz (2003), giving evidence that the market size effect of migration is expected to be a crucial element of the general picture.\(^3\)

Finally, we experiment with fiscal consequences of immigration. Taking the tax schedule as given, and assuming a combination of lump-sum and proportional benefit system, we perform an accounting exercise that results in the estimate of the first order effect that migrants might impose on the governmental redistributive scheme in both countries. Auerbach and Oreopoulous (1999); Storesletten (2000); Rowthorn (2008); Dustmann and Frattini (2014) find the effects for the US and the UK to be positive, but small. Similarly, the reports published by the OECD, e.g. OECD (2013), draw a general picture of fiscally beneficial immigrants in almost all OECD member states.

In the majority of the above cited papers, except for Dustmann et al. (2012), the quantification is done for a few discrete worker types, assuming that all individuals are identical within a particular group, hence: perfect substitutes.\(^4\) These papers are thus susceptible to the critic about finite number of employee types. In particular, the multi-country general equilibrium models assume a nested CES production function, and differentiate between low-skilled (up to secondary education) and high-skilled (equipped with at least

\(^2\) The magnitude of the market size effect depends on the elasticity of substitution between the varieties of consumption good. Its high value imposes similarity of different vintages of products, which makes the actual market size effect moderate. Assuming consensual parameter values (as was done in the cited papers) implies a strong market size externality of migration. A conservative evaluation of the market size effect is presented in Burzynski et al. (2017), showing that nevertheless, it is still quantitatively significant.

\(^3\) The market size effect predicted by these models is, by construction, homogeneous across all types of workers. It gives further insights into global efficiency gains from migration, without altering redistribution across heterogeneous individuals.

\(^4\) In this sense a person with hardly any education and a worker who finished a secondary school, but has no tertiary education are treated uniformly. Furthermore, these modelling techniques are silent about unobservable qualities which differentiate individuals – and their remunerations – within a particular education group. Borjas (2003); Ottaviano and Peri (2012); Manacorda et al. (2012) control for individual-specific characteristics such as: age, time spent abroad or experience.
some tertiary, college-education) workers only. As stated by Dustmann et al. (2012) such a distinction is insufficient, especially when it comes to analysing the economic effects of (highly downgraded) international migrants.5

A second line of criticism related to the recent body of literature includes the assumption of a constant elasticity of substitution across worker types. Low-skilled and high-skilled individuals (grouped in two homogeneous sets) affect each on the labour market other through a single number that governs their degree of substitutability or complementarity. In this way, the wage effects of immigration shocks are proportionate to the changes in sizes of broad employment groups, without any differentiation across education levels and skill types. Having introduced a full distribution of skills and wages in our approach, we are totally agnostic about the exact value of the substitution elasticity, and leave it as an immanent, and endogenous outcome of the matching process with heterogeneous firms.6 This feature has important consequences for analysing the distributional effects of migration policies across the universe of skills.

A further problem is related to the (imperfect) mapping of migrants’ skills before and after moving. Recent macroeconomic quantifications of the impact of migration (except for Docquier et al. (2014) and Biavaschi et al. (2016) who perform a migrants’ downgrade exercise assuming multiple discrete education groups) assume that skills (represented by individual’s education level) are uniformly transmitted across borders and labour markets.7 A Polish holder of a university degree who decides to emigrate to the UK will earn a wage rate that is significantly different from the one of a highly skilled UK native (possibly a graduate from one of the top English universities). A low-skilled Polish labourer might be only slightly disadvantaged relative to his English colleague. As shown by Dustmann et al. (2012) this is a principle rather than an exception (especially considering the inflow of migrants from new EU member states to the UK). A non-linear skill-downgrading is indeed an immanent factor that shapes the wage distributions of native and foreign workers.

The labour market in the CES literature assumes that all firms (even when being heterogeneous in terms of their productivity) employ the same bundle of skills, a given

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5 The discretization of labour force makes these models hardly useful in a political debate on migration policies, because the median voter will always be a member of a uniform group of individuals.

6 Note, however, that our model exhibits close substitution between similar workers, and weak substitution between more distant ones, as in Costrell and Loury (2004). This is also in contrast to Dustmann et al. (2012), who define a constant elasticity of substitution between all continuous worker-types.

7 In a nested CES framework, it is easy to produce a mechanical skill downgrade of immigrants. The only assumption required is a lower relative productivity of immigrants versus native workers. Note, however, that this averages out the differential, and in particular, imposes that the ratio of wages for low-skilled natives/migrants is very close (but generically not identical) to the one of the high-skilled natives/migrants.
mass of “skill composite”, which is a nested CES combination of low/high skilled natives/migrants. Wages are equal to marginal productivities of group-specific workers. Some recent findings by Bartolucci and Devicienti (2012); Håkanson et al. (2015); Eeckhout and Kircher (2016), though, suggest that employees sort across employers and this process reveals positive and assortative matching (PAM) pattern. Sorting plays a critical role in the propagation of wage effects along distribution of skills. With PAM, workers in the right tail of wage distribution are much more vulnerable to changes in firms’ productivity (brought about by endogenous entry and exit decisions of entrepreneurs, induced by an inflow of migrants), as neatly quantified by Costrell and Loury (2004). In contrast, in a standard CES world this entry and exit process of firms uniformly affects all workers and does not produce additional dimension of redistribution.

In what follows, we propose a theoretical framework that deals with all the issues mentioned above. By modelling the whole distribution of wages in sending and destination countries, we explicitly include a continuous domain of individual skill levels. The model endogenously produces elasticities of substitution between any of these worker types (including education, and origin specific ones). By calibrating dependencies in our two-dimensional skill distribution, the model is capable of reproducing the actual skill downgrade of migrants. Finally, a micro-founded sorting of workers and firms is the core element of the theory.

In our quantitative results, we find that the calculation of ultimate benefits and losses from migration is strongly dependent on the actual selection of immigrants and their sorting between labour markets. The exercises we perform depend on a specific case of Mexican emigration to the US, and according to our data (and in line with recent findings by Moraga (2011)) Mexicans are negatively selected with respect to the Mexican-relevant skill. Simultaneously, they are strongly positively selected in terms of their US-specific skill. Nonetheless, the mass of Mexicans who are proficient in US skills is small. Overall, immigrants are characterized by a worse distribution of skills than native American workers, which is decisive in terms of the economic effect in the host country. In our main counterfactual scenario, in which we impose prohibitive migration costs for Mexicans, we find that in the US economy without Mexican immigrants 44.4% of American native population is strictly better-off. 47% of working-age Americans are losing due to this policy, while 8.6% are unaffected (this group includes the unemployed individuals who remain inactive on the labour market). Closing the Mexico-US border reduces inequalities in the US, increases them in Mexico, while global inequality (taking both countries together) is raised as well. Even though the no-migration scenario improves or keeps constant remunerations of 53% of the least skilled American natives, it dampens or keeps constant
wages of 40% least skilled Mexicans. The overall effects in both countries are, however, limited: they range from -0.3% (+1.5%) for the American (Mexican) high earners to +4.0% (-4.5%) for those who are indifferent between employment and inactivity in the US (Mexico). More importantly, while the unemployment rate in the US decreases only slightly, the extensive margin of inactive Mexican natives raises by 2.5%.

In another exercise we investigate alternative migration policies through the lens of our quantitative model. We find that there exist combinations of additive and multiplicative migration costs that would fulfill a conjunction of three requirements in the destination: increase (or keep constant) the number of migrants, augment the average wage of American workers and be acceptable for the absolute majority of US residents. These undoubtedly beneficial policies impose an improvement of the selection of Mexican immigrants in the US, without changing their aggregate number. We conclude that such policies should be strictly preferred to those that aim to reduce the number of Mexicans in the US. We find the latter to be detrimental for the majority of the US resident population and to reduce the average wage of US based workers. Finally, we conclude that such a policy evaluation depends crucially on the economic effects taken into consideration. Without internalizing the fiscal and the market size consequences of immigration, the general picture turns opposite: policies that limit the number of Mexican immigrants end up being acceptable by the majority of US residents.

The rest of the paper is organized as follows. Sections 2 and 3 discuss the theoretical model and its numerical calibration. In Section 4 we analyse the economic consequences of experimenting with migration costs between Mexico and the US. Additional robustness checks of our main results are provided in Section 5. Section 6 concludes.

2 The Model

The model consists of four building blocks: populations of workers and firms in the sending (Mexico) and the destination country (United States). Mexican workers decide about the preferred destination, by maximising their real wages net of migration cost. Therefore, Mexico is populated with non-migrant workers, while the US host Mexican immigrants. Firms, while operating on the market, choose which workers to hire given

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8 The effect in the US is due to the exit of firms, and the fact that Mexican immigrants and currently inactive American workers (who are very low skilled) are not close substitutes. Contrary to this, Mexico experiences a positive supply shock which invites more firms to enter, reduces their average productivity, and forces the marginal Mexican worker to resign from working due to a loss in her wage.

9 The population of the US consists of Mexican migrants and the pooled group of US native citizens and immigrants from other countries. Thus, while talking about US workers we think about all US
the surplus function and labour costs. They also sort into exporting to the partner country and the Rest of the World (ROW). In both economies wages are set to clear the labour market for every level of skill. Along with the optimal consumption patterns of each individual, this provides the general equilibrium in the economy.

2.1 Basic Concepts

**Workers** There is a unit measure of Mexican workers, each of whom is endowed with a vector of skills \((x_U, x_M) \in [0,1]^2\). The skill \(x_U\) determines the worker’s productivity in the US and the skill \(x_M\) specifies her productivity in Mexico. The joint distribution of \(X_U, X_M\) conditional on the workers being Mexican is denoted as \(C(\cdot)\), which is a twice continuously differentiable copula function with strictly positive density on its support. Without loss of generality, we assume that the marginal distributions of \(X_U\) and \(X_M\) in the population of Mexicans are standard uniform.

There is also a measure \(R_{WU} > 0\) of US-resident workers. For simplicity, we assume that they cannot move to Mexico and are fully described by their skill \(x_U \in [0,1]\). The distribution of \(X_U\) conditional on the workers being American is labelled as \(F(\cdot| i = U)\). \(F\) is twice continuously differentiable and strictly increasing, hence, it is not necessarily a uniform distribution.

**Firms** In each country \(i \in \{U, M\}\), there is a measure \(R_{Fi} > 0\) of firms. Enterprises are ranked according to their productivity level on a unit segment. By paying the fixed cost of entry (thus, entering the market), each firm discovers its actual type \(h_i \in [0,1]\). Then, knowing its productivity, firms optimally choose a worker to hire. If an American residents, rather than only US-born native citizens.

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10The descriptive focus of this paper is restricted to modelling explicitly the Mexican and the US economy. The Rest of the World, as a trade partner for both of the mentioned countries, serves as an exogenously defined entity, included in the model only for the purpose of allowing trade imbalance between Mexico and the US.

11This does not mean that wages are uniformly distributed, rather: we normalize Mexican skills to be equal to the inverse CDF of wages. Consequently, this allows to treat \(X_U\) and \(X_M\) as orderings of individuals with respect to their observed wage level. In fact, these individual characteristics, called “skills” throughout the paper, are nothing else but quantiles of wage distributions in the total population of Mexicans. For a formal proof of this fact see Gola (2016).

12Note that if \(F\) first order stochastically dominates the standard uniform distribution, then we can meaningfully say that the population of US residents is more proficient than the Mexican population in the skill used in the US. \(F\) is identified conditional on the normalization of the distribution of US skills among Mexicans, as argued above, this is done without loss of generality.

13Parallelly to the distribution of Mexicans’ skills, we normalize the distribution of firms’ productivities in each country, so that the type of the firm corresponds to the quantile in which a particular enterprise is located. This assumption is made without loss of generality, as shown in Gola (2016).
entrepreneur $h_U$ hires a worker with skill $x_U$, then such a match produces a surplus of $\pi_U(x_U, h_U)$, whereas if a Mexican firm $h_M$ hires an agent with skill $x_M$, then such a match yields $\pi_M(x_M, h_M)$. Note that the surplus functions $\pi_i : [0, 1]^2 \to \mathbb{R}$ are strictly increasing in worker’s skill and firm’s productivity, and (weakly) supermodular. If an entrepreneur decides not to hire any worker, she receives a reservation profit normalised to zero. All the entrants, whose productivity is inferior to the zero-profit threshold, decide to exit the market, as in Hopenhayn (1992) and Melitz (2003). The remaining group of successful entrepreneurs stays active, hires workers and produces a match-specific surplus. Furthermore, in line with Melitz (2003), we assume that each firm produces a differentiated variety of the consumption good, which implies that the measure of all active firms in a particular market is equivalent to the measure of all varieties available for consumption. Varieties can be imported, as firms can serve both domestic and foreign markets. Whether or not the firm sells abroad, is decided by comparing the expected profit from exporting with the fixed cost of opening to exports. However, trade is costly and requires to pay $\tau_{ij}$ for every unit of consumption good shipped from country $i$ to $j$.

**Individual consumption and welfare** Each individual in country $i \in \{U, M\}$ has homothetic preferences over the set of all available varieties (goods produced in the home country and those imported). The utility of consumption of a $i$-based resident writes:

$$U_i(x_i) = \left[ R^F_1 \int_{h_i^*}^1 q_i(x_i, h_i) \frac{\varepsilon-1}{\varepsilon} dh_i + R^F_1 \int_{h_j^*}^1 q_i(x_i, h_j) \frac{\varepsilon-1}{\varepsilon} dh_j + R^F_{iW} q_{iW}(x_i) \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

subject to a standard budget constraint:

$$R^F_1 \int_{h_i^*}^1 p(h_i)q_i(x_i, h_i)dh_i + R^F_1 \int_{h_j^*}^1 p(h_j)q_i(x_i, h_j)dh_j + R^F_{iW} p_{iW} q_{iW}(x_i) = w_i(x_i),$$

where $q_i(x_i, h_j)$ denotes the quantity of variety $h_j$ (domestic or imported) consumed by a worker of skill $x_i$ in country $i$. This amount depends not only on individual characteristics (nominal wage earned by $x_i$), but also on the actual type of consumed good: $h_j$. Varieties, likewise firms, are heterogeneous with respect to market prices, which affects consumers’ demand decisions. A standard solution of this individual utility maximization problem

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To ensure that there are no corner solutions to firms’ exit decision, we assume that $\pi_i(x_i, 0) < 0 < \pi_i(x_i, 1)$ for all $x_i \in [0, 1]$, which implies that the most productive enterprise always stays on the market, while the worst one never does.
yields:

\[ q_i(x_i, h_j) = \frac{p(h_j)^{-\varepsilon}}{P_i^{1-\varepsilon}} w_i(x_i), \]  

(3)

where \( \varepsilon \) is the elasticity of substitution between any two varieties. Utility in each country \( i \) is also gained from consuming goods from the Rest of the World (indexed by a subscript \( W \)). We assume that firms in ROW are homogeneous, thus the goods originating from ROW are not indexed by a firm type. What matters for individual welfare, the mass of all varieties available for consumption in country \( i \), is determined by the total supplies of firms that are active on the internal and international markets. The domestic varieties are restricted to the quantiles \([h_i^c, 1]\) of the whole distribution of domestic firms of mass \( R^F_i \). Simultaneously, only \([h_j^*, 1]\), firms from the whole distribution of foreign firms of mass \( R^F_j \) are present on the domestic market, with: \( h_j^* \geq h_i^* \). ROW firms are characterized by an exogenously given mass \( R^F_W \). Ultimately, the welfare of any \( i \) resident boils down to her real wage (nominal wage divided by the ideal price index): \( U_i(x_i) = \bar{w}_i(x_i) \equiv w_i(x_i)/P_i \), where:

\[ P_i = \left[ R^F_i \int_{h_i^c}^{1} p(h_i)^{1-\varepsilon} dh_i + R^F_j \int_{h_j^*}^{1} p(h_j)^{1-\varepsilon} dh_j + R^F_W (p_i W)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \]  

(4)

**Firms’ decisions and trade**  
Firms perfectly foresee the demand for their products. Their hiring and pricing decisions are taken parallelly, as a solution to the profit maximization problem. Here, let us focus on the goods market dimension, and leave its the labour market part for the next subsections. The aggregated demand for any variety \( h_i < h_i^* \), denoted by: \( q_i(h_i) = R^W_i \int_0^1 q_i(x_i, h_i) dx_i \), comes from the domestic consumers only, and is equal to the product of a unit price and the quantity sold in country \( i \):

\[ p(h_i)q_i(h_i) = \frac{p(h_i)^{1-\varepsilon}}{P_i^{1-\varepsilon}} Y_i \equiv \pi^{NT}_i(h_i), \quad h_i < h_i^*, \]  

(5)

where: \( Y_i = R^W_i \int_0^1 w_i(x_i) dx_i \) is the total wage bill in country \( i \). This revenue from selling variety \( h_i \) on domestic market is by definition equivalent to the surplus generated by firm \( h_i \), labelled by \( \pi^{NT}_i(h_i) \). The superscript \( NT \) indicates a non-tradable variety \( h_i \). As for the exported varieties, \( h_i > h_i^* \), the value of demand is a sum of domestic and foreign
sales:

\[ p(h_i)q_i(h_i) + \tau_{ij}p(h_i)q_j(h_i) + \tau_{iW}p(h_i)q_W(h_i) = \]

\[ p(h_i) \left( \frac{Y_i}{P_i^{1-\varepsilon}} + \frac{Y_j^{1-\varepsilon}}{P_j^{1-\varepsilon}} + \frac{Y_W^{1-\varepsilon}}{P_W^{1-\varepsilon}} \right) \equiv \pi^T_i(h_i), \quad h_i > h_i^{\ast}. \tag{6} \]

The collection of the two above mentioned equations defines the actual price schedule for all the goods produced in country \( i \), as a function of trade costs, aggregated incomes and exogenously given prices in ROW\(^{15}\).

Knowing the price levels, it is straightforward to determine the aggregated value of trade between \( i \) and \( j \):

\[ Y_{ij} = R_i^F \int_{h_i}^{h_i^{1-\varepsilon}} \pi_i(h_i)dh_i \left( 1 + \frac{Y_i}{P_i^{1-\varepsilon}} \frac{P_j^{1-\varepsilon}}{Y_j^{1-\varepsilon}} + \frac{Y_W^{1-\varepsilon}}{P_W^{1-\varepsilon}} \frac{P_j^{1-\varepsilon}}{Y_j^{1-\varepsilon}} \right)^{-1}, \tag{7} \]

as well as domestic absorption:

\[ Y_{ii} = R_i^F \int_{h_i}^{h_i^{1-\varepsilon}} \pi_i(h_i)dh_i + R_i^F \int_{h_i}^{h_i^{1-\varepsilon}} \pi_i(h_i)dh_i \left( \frac{Y_i}{P_i^{1-\varepsilon}} + \frac{Y_j^{1-\varepsilon}}{P_j^{1-\varepsilon}} + \frac{Y_W^{1-\varepsilon}}{P_W^{1-\varepsilon}} \right)^{-1}, \tag{8} \]

By definition total income equals the value of production: \( Y_i = Y_{ii} + Y_{ij} + Y_{iW} \), and trade is balanced: \( Y_{ij} + Y_{iW} = Y_{ji} + Y_{Wi} \). However, we allow for: \( Y_{ij} \neq Y_{ji} \) which enables us to endogenously model the Mexico-US trade balance.

\[^{15}\text{Note that the price levels set by producers are equal to constant markups over marginal cost, as in Melitz (2003). To show this key property of monopolistically competitive economy, let us write firms’ first order conditions from the supply perspective: } \partial \pi_i(x_i, h_i)/\partial x_i = \partial \omega_i(x_i)/\partial x_i = \partial \omega_j(x_i)/\partial h_i(h_i(x_i)) \cdot \partial q_i(h_i(x_i))/\partial x_i. \text{ Assume, that there is a unique mapping that relates } x_i \text{ and } h_i, \text{ so that we can write: } h_i = h_i(x_i) \text{ (necessary and sufficient condition for this to hold are provided in the following sections). Since employing a different skill level } x_i \text{ influences not only the wage rate paid by the firm, but also the quantity of manufactured product, the marginal cost writes as: } MC_i(x_i) = \partial \omega_i(x_i)/\partial q_i(h_i(x_i)). \text{ Simultaneously, assuming that firms know the exact form of consumers’ demand function, we have: } \partial \pi_i(x_i, h_i)/\partial x_i = \partial q_i(h_i(x_i))/\partial x_i = \varepsilon - 1 \frac{\partial q_i(h_i(x_i))}{\partial x_i} = \varepsilon - 1 \frac{\partial (h_i(x_i))}{\partial x_i}. \text{ Equalizing both of the above mentioned expressions, we arrive at: } MC_i(x_i) = (\varepsilon - 1)/\varepsilon \cdot p(h_i(x_i)), \text{ which, after a simple rearrangement, proves that price is equal to a constant markup over the marginal cost of production.}\]
2.2 Supply of Skills

The overall supply of skills in each country is determined by two factors: the Mexican workers’ migration decisions and all workers’ declaration to participate on the labour market. Both of these choices are made rationally, and are governed by wage maximizing behaviour. In reaching their labour market entry and migration decisions, individuals take wages \( w_i(x_i) \) as given. They have also the option of not working at all, in which case their pay-off is normalised to \( w^c_i \).\(^{16}\)

Let an individual of skill \( x^c_i \) be the least skilled person active on the labour market in country \( i \), that is the one who possesses the critical skill. This means that everyone, who is characterized by \( x_i < x^c_i \) stays unemployed, while those who have a higher skill \( x_i \geq x^c_i \) work and earn a nominal wage: \( w_i(x_i) \).\(^{17}\)

A Mexican worker \((x_U, x_M)\) migrates to the US if and only if:

\[
(1 - \delta_1)\bar{w}_U(x_U) - \delta_0 \geq \bar{w}_M(x_M),
\]

where: \( \delta_1 \) is a multiplicative cost of moving from Mexico to the US, while \( \delta_0 \) stands for an additive migration cost. \( \bar{w}_i(x_i) \) represents the real wage in country \( i \) for skill \( x_i \), which is the nominal wage \( w_i(x_i) \) divided by the ideal price index \( P_i \). The inequality above defines the no-arbitrage condition in Mexicans’ sorting into both of the labour markets. When a Mexican worker finds it profitable to emigrate, so that (9) is strict, she chooses to work in the US using her \( x_U \) skill instead of supplying skill \( x_M \) in Mexico. If this condition is not met, \((x_U, x_M)\) works in Mexico whenever \( x_M \geq x^c_M \) or remains unemployed otherwise. Assume that there exists a Mexican worker with a Mexican critical skill \( x^c_M \), who is indifferent between emigrating, working at home, or being unemployed is in possession of a migration critical American skill: \( x^m_U \).\(^{18}\) Such an individual earns: \( \bar{w}_M = (1 - \delta_1)\bar{w}_U - \delta_0 \) in real terms, where \( \bar{w}_U \) might be greater (smaller) than \( \bar{w}_U \). The least skilled Mexican immigrant earns more (less) than the least skilled native American, only if \( x^m_U \) is above (below) \( x^c_U \).

The cumulative supply of Mexican workers’ skill \( x \) in country \( i \) – \( S^M_i(x) \) – is defined as a measure of Mexican workers living in country \( i \) with a skill greater than \( x \). According to

---

\(^{16}\)We label to \( w^c_i \) as an \( i \)-specific outside option, that is the reservation wage of an \( i \)-born, marginal person that decides to enter the labour market in country \( i \). The wage of the least skilled employee is equal to the outside option of the unemployed individuals if and only if the mass of unemployed in positive (which we assume to be the generic situation).

\(^{17}\)In our model, the unemployment rate has to be treated as a long-term voluntary unemployment. In this sense, this macroeconomic variable is close to the definition of inactivity rate. People’s insufficient levels of relevant skills make it impossible to earn more than the reservation wage, and pushes them into inactivity on the labour market.

\(^{18}\)Note that we could write: \( x^m_M \equiv x^c_M \), since there is no immigration in Mexico.
what has already been stated, we can aggregate the mass of Mexican workers who decided to emigrate to the US, as a conditional, reversed cumulative density function:

\[ S_M^U(x) = \Pr [X_U \geq x, (1 - \delta_1)\bar{w}_U(X_U) \geq \max(\bar{w}_M(X_M), \bar{w}_M^c) + \delta_0]. \]

An analogous supply function for Mexican stayers, can be derived as:

\[ S_M^M(x) = \Pr [X_M \geq x, (1 - \delta_1)\bar{w}_U(X_U) < \max(\bar{w}_M(X_M), \bar{w}_M^c) + \delta_0]. \]

Hence, \( S_M^U(0) \) gives us the total measure of Mexicans who emigrated to the US.

Finally, the cumulative supply of skill \( x \) in country \( i \) -- \( S_i(x) \) -- is defined as the measure of workers of either nationality living in country \( i \) with skill greater than \( x \). In the case of the US, the resident workers (natives plus non-Mexican immigrants) constitute the major group. They are complemented by Mexican immigrants, whose mass equals \( S_M^U(x) \), for any skill level \( x \)\(^{19}\).

\[
S_U(x) = R_U^M \Pr [X_U \geq x, w_U(X_U) \geq \bar{w}_U] + S_M^M(x). \tag{10}
\]

We assume prohibitive migration costs of moving from the US to Mexico, thus the only group of workers active on Mexican labour market, are Mexican natives:

\[ S_M(x) = S_M^M(x). \tag{11} \]

### 2.3 Demand for Skills

Each firm in country \( i \in \{U, M\} \) has to reach four decisions: whether to enter the market, whether to stay active, whom to hire on a competitive labour market and whether to produce for domestic market or realise the option to export. Denote the profit of enterprise \( h_i \) as: \( r_i(h_i) \) and the skill of the agent it employs as: \( x_i^*(h_i) \). Firm’s operating profits equal to the remaining surplus after paying wages to employees:

\[
r_i(h_i) = \max_{x_i \in [0,1]} \pi_i(x_i, h_i) - w_i(x_i), \tag{12}
\]

\(^{19}\)The comparison of wages within a particular country can be simplified to considering nominal wages only, because: \( w_i(x_i) \geq w_i(x_i') \leftrightarrow \bar{w}_i(x_i) \geq \bar{w}_i(x_i') \).
while the skill level of an employed worker fulfils:

\[ x_i^*(h_i) \in \arg \max_{x \in [0,1]} \pi_i(x, h_i) - w_i(x). \] (13)

The entry decision of every entrepreneur is motivated by an ex-ante analysis of the market. Being not aware of their actual type (productivity ranking \( h_i \)), the firm has to formulate expectations about its future prospects. However, discovering the productivity level (equivalent to entering the market) is costly, and requires to pay a sunk expenditure of \( \phi_i \) dollars. Entrepreneurs continue to enter the market only if their expected profits cover the fixed cost of entry:

\[ \mathbb{E}[r_i] = \int_0^1 r_i(h_i) dh_i \geq \phi_i. \] (14)

The left hand side summarizes the expected (average) profits of firms in country \( i \), assuming that those who enter, but immediately exit the market receive a zero profit (instead of entering and encountering a loss). When \( \mathbb{E}[r_i] > \phi_i \), positive profits can be collected by new entrants, so the mass of enterprises, \( R^F_i \), increases. In contrast, a cost of entry higher than expected profits discourages entrepreneurship, and implies a fall in \( R^F_i \).

A similar argument pins down the mass of country \( i \) firms that decide to export to \( j \) and to ROW. The equilibrium condition requires that the expected profit from exporting equalizes the fixed cost of entering the foreign market:

\[ \mathbb{E}[r_i^{exp}] = \int_{h_i^*}^1 r_i(h_i) dh_i = \phi_i^{exp}. \] (15)

To simplify notation, we assume without loss of generality, that \( x_i^*(\cdot) \) is a function, so that all firms of type \( h_i \) hire the same agent.\(^{20}\) Then the cumulative demand for skill \( x \) in country \( i \) is defined as the measure of country \( i \) firms who hire workers with skill greater than \( x \):

\[ D_i(x) = R^F_i \Pr \{ x_i^*(H_i) \geq x, \mathbb{E}[r_i(H_i)] \geq \phi_i \}. \] (16)

---

\(^{20}\)This does not have to be the case in equilibrium if \( \pi_i(\cdot) \) is weakly supermodular. However, even then there exists an equilibrium with a \( x_i^*(\cdot) \) that not only is a function, but also a differentiable bijection. Furthermore, as it follows from the results in \( \text{Chiappori et al. (2010)} \) that the wage functions are unique in this model, the supply of talent will be the same for any \( x_i^*(\cdot) \) that meets \( (12) \), including those that are correspondences, rather than functions. Hence, restricting attention to functions does not change the set of possible equilibria and is without loss of generality (see the proof of Proposition 1 in \( \text{Gola (2016)} \) for details and Appendix C therein for a definition of skill demand that does not assume that \( x_i^*(\cdot) \) is a function).
In this simple two-country labour market a general definition of equilibrium sorting writes as follows:

**Definition 1.** An equilibrium is characterised by:

1. the supply of skills $S_i : [0, 1] \rightarrow [0, 1]$ in each country, which is determined by workers sorting decisions and given by Equations (10) and (11);
2. the demand for skills $D_i[0, 1] \rightarrow [0, 1]$ in each country, which is determined by firms’ profit maximisation and given by Equation (16);
3. firms’ measure $R^F_i$, determined by the zero profit condition (14);
4. the measure of exporting firms’, $R^F_i(1 - h^*_i)$, defined by the zero profit condition from exporting (15);
5. wages $w_i : [0, 1] \rightarrow \mathbb{R}$ in each country, which are set to clear the markets: $S_i(x) = D_i(x)$ for $i \in \{U, M\}$ and all $x \in [0, 1]$.

### 2.4 Equilibrium and its Characteristics

In this subsection we provide a general description of the solution procedure of the model. The derivation of equilibrium makes use of a two-step procedure proposed in [Gola (2016)](http://example.com). First, the wages are computed treating equilibrium supply of skills as given. Secondly, the supply of skills is aggregated treating equilibrium wages as given. These two combined result in a fixed-point problem that fully characterises the equilibrium.\(^{21}\)

**Wages** Fixing the supply of skills, and assuming that: 1) matching between workers and firms is positive and assortative (PAM), 2) the wage function is twice differentiable, and 3) the hiring function $x^*_i(\cdot)$ (and its inverse, the matching function $h^*_i(\cdot)$) are differentiable bijections, allows us to derive the explicit forms of wage equations. First, note that the profit function is strictly increasing in $h_i$, thus firms’ demand for skill can be simplified to\(^{22}\)

$$D_i(x) = R^F_i \Pr(H_i \geq h^*_i(x), H_i \geq h^*_i) = \begin{cases} R^F_i (1 - h^*_i(x)) & \text{for } x \geq x^*_i(h^*_i) \\ R^F_i (1 - h^*_i) & \text{for } x < x^*_i(h^*_i). \end{cases}$$

---

\(^{21}\)For a detailed proof of existence and uniqueness of the equilibrium, consult Appendix A.

\(^{22}\)The profit of a firm $h_i$ is: $r_i(h_i) = \pi_i(x^*_i(h_i), h_i) - w_i(x^*_i(h_i))$. For any $h'_i > h_i$ we have, by profit maximisation: $r_i(h'_i) \geq \pi_i(x^*_i(h_i), h'_i) - w_i(x^*_i(h_i)) > r_i(h_i)$, as $\pi_i(x_i, \cdot)$ is strictly increasing. Additionally, labelling the critical (minimal) skill and productivity levels with superscript $c$ gives us: $\pi_i(x^*_i, h^*_i) = 0$. 15
Labour markets must clear both internationally and within each country\footnote{Thus, from the definition of demand and market clearing we get that: $D_i(0) = R_i^F \Pr(H_i \geq h_i^c) = (1 - h_i^c)R_i^W = S_i(0)$.} The latter implies that $S_i(x) = D_i(x)$, which pins down both $h_i^*(\cdot)$ and $h_i^c$:

$$h_i^*(x) = 1 - S_i(x)/R_i^F, \quad h_i^c = 1 - S_i(0)/R_i^F.$$ 

The matching function derived above defines the structure of production through the surplus generated by the optimally chosen matches. Any individual $x_i$ meets the firm $h_i^*(x_i)$ and they jointly generate $\pi_i(x_i, h_i^*(x_i))$. In this setting, a marginal increase in worker’s wage is brought about by the marginal increase in surplus:

$$\frac{\partial w_i(x_i)}{\partial x_i} = \frac{\partial \pi_i(r, 1 - S_i(r)/R_i^F)}{\partial x_i} = \frac{\partial \pi_i(x_i, h_i^*(x_i))/\partial x_i}{\partial x_i}. $$

Solving this differential equation with an initial condition: $w_i(x_i^c) = w_i^c$ gives us the general form of the wage equation in country $i$:

$$w_i(x_i) = \int_{x_i^c}^{x_i} \frac{\partial \pi_i(r, 1 - S_i(r)/R_i^F)}{\partial r} \, dr + w_i(x_i^c) \quad \text{for } x_i \geq x_i^c. \quad (17)$$

**Sorting** This step establishes the equilibrium supply of skills taking the wage structure as given. We start with rewriting the definition of supply of skills in the US stated in (10):

$$S_U(x) = R_U^W (1 - F(x)) + S_M^U(x). \quad (18)$$

While US residents supply their skills according to the cumulative density function $F$, which is given ex ante, Mexican immigrants sort into both of the labour markets according to real wages and migration costs. Consequently, $S_M^U(x)$ is the endogenously determined supply of Mexican labour in the US.

In line with our notation, the only unemployed Mexican workers are those with both skills below the critical thresholds: $(x_{U}, x_{M}) < (x_{U}^m, x_{M}^m)$. Their mass can be computed using the two-dimensional distribution of skills modelled with a copula function:

$$C(x_{U}^m, x_{M}^m) = C(x_{U}^m, x_{M}^m) = 1 - S_M^U(0) - S_M(0). \quad (19)$$

The rest of the Mexican population is active either on the Mexican or the US labour market – the endogenous sorting takes place in a two-dimensional space of Mexican and American-specific skills. To reveal it, let us define $x_i^s$, which play a similar role to $x_i^m$, but on the opposite end of the skill distribution. The skill $x_i^s$ is the lowest level of country $i$ skill such that any worker with $x \geq x_i^s$ works in country $i$ with probability 1, irrespective
of her skill in the other country. Formally, this can be defined as:

\[
x_U^* = \min\{x_U \in [0, 1] : (1 - \delta_1)\bar{w}_U(x_U) - \delta_0 \geq \bar{w}_M(1)\}, \quad (20)
\]

\[
x_M^* = \min\{x_M \in [0, 1] : \bar{w}_M(x_M) \geq (1 - \delta_1)\bar{w}_U(1) - \delta_0\}. \quad (21)
\]

It follows that \(x_M^m < x_U^*\) and \(\max\{x_U^*, x_M^*\} = 1\). This leads us to a definition of the separation function \(\psi : [x_M^m, x_M^*] \times [x_U^m, x_U^*]\), which differentiates Mexican stayers from Mexican migrants to the US. Formally, it provides us with an American skill level \(\psi(x_M) \in [x_U^m, x_U^*]\), such that a Mexican worker equipped with a bundle of: \((\psi(x_M), x_M), \forall x_M \in [x_M^m, x_M^*]\) receives an identical remuneration in both countries:

\[
(1 - \delta_1)\bar{w}_U(\psi(x_M)) - \delta_0 = \bar{w}_M(x_M). \quad (22)
\]

Hence, any Mexican worker \((\psi(x_M), x_M), \forall x_M \in [x_M^m, x_M^*]\) is indifferent between migrating to the US and remaining in her home country. In this sense, \(\psi\) separates the population of Mexicans migrants from the group of stayers.

Knowing the critical skills of Mexican workers, the star skills and the separation function we can easily determine the supply of Mexican workers’ skill in each country. Mexicans with \(x_U < x_U^*\) (\(x_U > x_U^*\)) never (always) work in the US, while those characterized by any \(x_U \in [x_U^m, x_U^*]\) migrate if \(x_U > \psi(x_M)\), which happens with probability \(\partial C(x_U, \psi^{-1}(x_U))/\partial x_U\).\(^{24}\)

Analogously, Mexicans with \(x_M < x_M^m\) (\(x_M > x_M^*\)) never (always) work in Mexico, while those having any \(x_M \in [x_M^m, x_M^*]\) stay in Mexico if \(x_U < \psi(x_M)\), which occurs with probability \(\partial C(\psi(x_M), x_M)/\partial x_M\). Therefore:

\[
S_U^M(x_U) = \begin{cases} 
\int_{x_U^m}^{x_U^*} \frac{\partial}{\partial x_U} C(r, \psi^{-1}(r))dr + 1 - x_U^*, & x_U < x_U^m, \\
1 - x_U, & x_U \in (x_U^m; 1], 
\end{cases} \quad (23)
\]

\[
S_M(x_M) = \begin{cases} 
\int_{x_M^m}^{x_M^*} \frac{\partial}{\partial x_M} C(\psi(r), r)dr + 1 - x_M^*, & x_M < x_M^m, \\
1 - x_M, & x_M \in (x_M^m; 1]. 
\end{cases} \quad (24)
\]

The endogenous supply of skills is the last condition necessary to prove the following:

**Theorem 1.** The equilibrium defined in Definition \([\underline{1}]\) exists and is unique.
Proof. Apply the proof of Theorem 1 in Gola (2016).

3 Calibration

In this section we proceed with a numerical calibration of the model. After specifying and motivating the chosen forms of a few critical functions (Section 3.1), we provide a detailed description of the dataset used to calibrate the model (Section 3.2). Then, we perform a validation of our benchmark calibration (Section 3.3).

3.1 Functional forms

Copula The general version of the model presented in Section 2 allows for any relationship between the marginal distributions of skills carried by Mexican citizens. In our quantitative exercise, we will impose a positive connection between their rankings with respect to both types of skills. However, the strength of this relation might not be identical across rankings. For our baseline calibration we propose the Clayton copula, which is a member of Archimedean copula family. There are several reasons that motivate our choice. First, this copula is described by a fairly simple formula:

\[ C(u, v) = \left( u^{-\theta} + v^{-\theta} - 1 \right)^{-1/\theta}, \quad \theta > 0, \]

which brings no computational difficulties in calculating its CDF as well as marginal PDF. Second, it allows for very strong relations in the bottom-left corner of the two-dimensional distribution (Mexicans ranked low in their domestic skill will almost surely be ranked low in the foreign skill). Moving up the rankings to the top-right corner, these relations become weaker (Mexicans ranked high in their domestic skill have a high probability to be ranked high).

25 Assuming a negative correlation between skills would imply that Mexicans that are highly ranked in the Mexican skill, would be characterized by a relatively low level of the American skill, and vice versa. We consider this case as being less probable than the positive relation, since the process of accumulation of both skills requires a similar set of individual traits (ability to learn, adapt and develop in a particular environment, manual capabilities or educational proficiency).

26 One of the most apparent choices for a two-dimensional function that could link marginal distributions of individual skill rankings would be the Gaussian copula. This would generate, however, two issues. First is connected with the fact that a Gaussian copula does not have a closed form expression which would significantly increase the computation time. Second, this family of copula functions (assuming high and positive correlation coefficient between the marginal distributions) imposes strong relations in the extremes, and weak relations in the middle. This means that there is a very high probability that a person with a low (high) ranking in Mexican skill will have a low (high) ranking in the American skill. Gaussian copulas with low and positive correlation would imply that people with low (high) rankings in Mexican skills can easily be classified as high (low) ranked individuals with American skills.

27 Please consult Appendix B for a graphical comparison of Clayton, Gaussian and Gumbel copulas.
Finally, the Clayton copula is characterized by only one parameter, $\theta$, that defines the strength of the relationship between the two marginal distributions. In particular, this parameter can be easily mapped into a well known rank correlation measure, the Kendall’s $\tau$. One can show that: $\tau = \theta/(\theta + 2)$.

**Surplus functions** The specification of the model introduces a surplus function which maps the rankings of individuals in both countries into value added produced by a specific match. This general formulation of functions $\pi_i(x_i, h_i)$ fulfils two conditions. It has to be strictly increasing in both arguments, and weakly supermodular. Having a large margin of flexibility in defining these functions, we aspire to provide the closest possible fit to the distribution of wages in all the groups of workers we consider, without inflating the set of parameters. Since the total surplus is divided into worker’s income and firm’s profit, we propose a simple, multiplicatively separable non-normalized function that meets some additional requirements. Assume that the normalized surplus equals:

$$
\pi_i(x_i, h_i) = k_i \Phi_i^{-1}(x_i, t_1, s_i)(1 - t_2 h_i)^{-\gamma_i} + k_i^0, \quad i \in \{1, 2\}, \tag{26}
$$

where: $k_i > 0$ is a multiplicative constant (which serves as a location modifier of the surplus), $\Phi_i^{-1}(\cdot)$ is an inverse CDF which maps skills into surplus and $k_i^0$ is a normalization constant (to ensure that in the equilibrium $\pi_i(x_i^c, h_i^c) = 0$). The second factor in Equation (26) maps firm’s ranking in terms of productivity into surplus. It also exhausts the assumption that firms’ performance is distributed according to a truncated Pareto CDF with a given shape parameter, and cut-off at quantile $t_2 \rightarrow 1$.

Many studies argue that the wage distribution in a population is close to log-normal. We verify, however that the model by Borjas (1987) struggles to fit the right tails of wage distribution.
distributions, which motivates us to introduce PAM (and consequently: $\gamma_i > 0$). Since our approach acts as an extension of Borjas (1987), we choose $\Phi_i$ to be the CDF of a truncated log-normal distribution with a location parameter equal to 0, a scale parameter $s_i$ and a truncation parameter $t_1$. Therefore, while the first factor in the surplus function maps individual skill ranking in country $i$ into real numbers, the second one is related to the matching process with heterogeneous firms. The calibrated value of $\gamma_i$ is a product of Pareto shape parameter and the exponent that governs the relative importance of firms in producing the surplus (which is reflected in their bargaining power).

### 3.2 Data Description

Our calibration represents a static state of the Mexican and American economies in year 2015. There are few model objects that are easily represented in the data. Starting with the least data-demanding ones, we can pin down several scalar demographic and macroeconomic variables. First, we normalize the working age population of Mexicans (sum of those residing in Mexico and in the US) to unity. This gives us the relative size of working age population of native Americans equal to $R_W = 2.643$. We can then identify the number of legal Mexican workers in the US in year 2015, using the Database on Immigrants in OECD and non-OECD Countries (DIOC) by the OECD. There, we find that their mass is equal to: $S_{MU}(0) = 0.137$. Finally, according to the Labour Force Surveys, unemployment in the population of American natives aggregates to: $u_U = 8.62\%$, and the same statistic in Mexico equals: $u_M = 4.0\%$. All these values are taken as given in the calibration procedure and are matched perfectly in the reference scenario.

Then, we feed our model with the data related to international trade. The actual trade flows across the US, Mexico and the ROW are taken from the Trade in Value Added database by the OECD. Then, using the data from the US Department of Trade, we find

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30 Note that $\gamma_i = 0$ imposes both: no matching and a degenerated distribution of firms’ productivity, which brings us back to the selection model by Roy (or Borjas (1987), if the copula function was Gaussian) with truncated log-normal wages.

31 An important notice has to be made in terms of this modelling choice. The existence of equilibrium in the model requires all function to be Lipschitz continuous, as in Gola (2016). This puts an additional constraint on the inverse CDF we use in the definition of surplus. We assume $\Phi^{-1}(\cdot)$ is a truncated log-normal, and we choose to cut it at $t_1$ sigmas on both sides of the distribution. The value of $t_1$ (as well as $t_2$ for Pareto) is calibrated, is driven by wage distribution data, but is assumed to be identical in Mexico and the US.

32 In terms of total numbers, according to the DIOC, we find that there are 51.3 million working-age Mexicans (natives and migrants), 42.2 million of whom being employed in Mexico. In the US we identify 149 million working-age residents (natives and non-Mexican immigrants, 136 employed. The database published by the OECD reveals 7 million legal Mexican immigrants in the US, who are employed.

33 The value added exported from US to Mexico amounts to 1% of US GDP (7.5% to ROW), while Mexican value added in exports to the US constitutes almost 10% of Mexican GDP (11% to ROW).
that 6% of all employed workers contribute to exports in the American economy, while in Mexico this share is set to 18%. The latter data points pin down the extensive margin of exporting firms, $h_i^e$. Finally, we assume the price index in Mexico to serve as a numeraire (set to 1), while $P_U$ is computed to match the US/Mexico differential in minimal wages, yielding a value of 2.1. The price index in ROW is determined by the trade-weighted PPP differentials with the US and the relative number of exporting/importing firms from/to the US, resulting in $P_W = 0.58$.

The second set of objects we fit are the distributions of wages in the analysed three groups of individuals: residents in the US, natives in Mexico and Mexican migrants in the US. We calibrate the model on publicly available intercensal data from Mexico and the US. For the US, we use the 2015 1% sample provided by IPUMS, and we compute yearly wage data for 1.23 million US native workers (excluding managers) and 52 thousand Mexican immigrants in the US. We similarly dispose of Mexican 2015 intercensal survey, from which we extract a single variable: earnings per month (Ingresos por trabajo mensualizado) in Mexican peso, for 5.78 million Mexican natives (excluding managers). To make it comparable with the US, we multiply it by 12, divide it by the average exchange rate USDMEX equal to 17.81 for 2015 according to the OECD, and modify it using the PPP adjustments, also taken from the OECD.

For the fiscal part, we collect the actual rates and thresholds for American (as well as Mexican) income and corporate tax rates (the source is OECD). We match the (reduced) actual income structure quite well, by obtaining that corporate taxes constitute around 20.5% of tax revenues, while the summary by National Priorities Project (provided using The Office of Management and Budget data) reveals it to be close to 18.8%. The rest

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34 In the census wages are presented as “Wages or salary income last year” and are quoted in USD. However, we would like to control for the fact that people serve a different intensive margin of labour supply. There is a significant heterogeneity of individuals with respect to actual hours worked per week. Since in our model all workers supply an identical amount of labour per unit of time, we have to take this fact into consideration. Then, people work part-time, or not supply labour for the entire year. Thus, we store information about individual-specific usual hours worked and the number of weeks worked in the analysed year. From this, we calculate a wage rate that is earned in a standard, 40-hour per week shift, assuming a person is employed for all 52 weeks: $w = \frac{w_{\text{monthly}} \cdot 40}{\text{hours per week}} \cdot \frac{52}{\text{weeks per year}}$.

35 For all wage distributions we delete 0.5% of lowest and 0.5% of highest values to remove outliers. Then, we smooth them out by interpolating locally the missing values (in case of sudden jumps in the empirical distribution). Finally, with these upgraded data, we compute kernel densities that allow us to generate $K = 100,000$ density points for 100,000 quantiles of each distribution. Since we have to discretize the whole model, we choose a grid of $K = 100,000$ points on which all the functions and distributions are going to be computed. This enables us to retain considerable accuracy of our computations, and keep our calibration and simulation algorithms relatively fast.

36 For more information on the structure of inflows and outflows to/from the US governmental budget, please consult: https://www.nationalpriorities.org/budget-basics/federal-budget-101/spending/ Note that we are concerned only with the two types of taxes: individual income and corporate, which constitute
is financed by personal income taxes (in particular, our calibration gives that 2% of total revenues originate from Mexican immigrants). 30% of all US governmental spendings are related to social security, while 16% - to military. Thus, we broadly assume that 50% of all transfers are lump-sum, identical for all workers in the US. The rest, that is around 50% of governmental expenditures (including medicare, education and science) are treated as proportional to incomes. We further assume that the margin of adjustment is incorporated in the lump-sum transfers, taking the tax schedule and wage-proportional transfers as constants in the counterfactual scenario. In Mexico, we assume a similar structure of benefits.

3.3 The reference calibration

The chosen vector of parameters allows us to compute wage distributions that fit the actual data roughly well. According to Figure 1, in which the empirical distributions are depicted in black, while model distributions are in colour (blue for Mexicans in Mexico, red for Mexicans in the US and green for US residents), in almost all instances the quantiles are well matched, apart from left tails of the distribution of Mexican native wages. We overestimate the minimal wage rate in Mexico, and slightly underestimate the minimal remuneration of Mexican immigrants. Since we solve the model from the right hand side (starting from \( x_U = 1 \)), the maximal wages in all distributions are accurately fitted.

The model matches the mass of immigrants from Mexico to the US almost perfectly. The difference between model and data is at the sixth decimal place, which is less than 0.005% of the total mass of migrants. Similarly, the unemployment rates in both countries are matched precisely with very small errors. We also estimate the equilibrium fixed costs of entry into both production sectors, which in the calibration equal the average profit per worker earned by the operating firms. For the US this threshold equals: 12,855 USD, while for Mexico it is: 483 USD. When it comes to exporting, the fixed costs amount to 76,253 USD in the US and 2,338 USD in Mexico.

The final element to discuss is the selection pattern generated by our calibration, depicted in Figure 2. Figure 2a plots the inverse separation function \( \psi^{-1} \) in red, in the \((x_U, x_M)\) space, and shows who decides to migrate (surface to the right from the red function), and who decides to stay in Mexico (left from the red function). We find that emigrants from Mexico to the US are strongly positively selected among their peers with 89% of all taxes collected and 49% of total budget revenues.

\[37\] This set of assumptions ignores the fact that migrants might have a different net fiscal impact through lower probability of collecting welfare benefits. In this sense, our estimates of fiscal effects would be the lower bound of the real ones.
Figure 1: Evaluation of model fit: distributions of wages

Note: Figure 1 depicts the closeness of fit of our model to the data on wage distributions. The blue line represents model wage distribution in Mexico, the red line is the immigrants’ wage distribution, while the green line is the American natives’ wage distribution. Black lines depict the respective distributions in the data.

respect to the American skill $x_U$. This rather expected result is illustrated in Figure 2b, where we compare the American skill distribution across three populations of interest: Mexican natives (black), migrants (red) and American residents (blue). Even though Mexican emigrants possess a significantly higher level of American skill than Mexicans in total (they are positively selected with regards to American skill), they underperform in comparison with American residents. We see that Mexican immigrants in the US provide a generally lower supply of American skill than US workers (apart from the low skill levels, where Americans are more numerous). This feature is preserved mainly due to the fact that Mexicans as a whole population are strongly downgraded in terms of US-relevant skill, comparing to US natives. In particular, it means that the mass of Mexicans which are proficient in US-specific tasks is very limited. At the same time, Mexican emigrants are significantly negatively selected in terms of the Mexican skill, a phenomenon stressed by recent empirical literature (Moraga, 2011). Had those migrants returned to Mexico, they would have earned lower wages than stayers, as it is apparent from Figure 2c. A confirmation of a huge positive selection of Mexicans with respect to their American skill can be found in Figure 2d, where we compare the wage distributions of current migrants and Mexican natives, had they moved to the US. More than 50% of them would not even be skilled enough to participate on the US labour market.
Figure 2: Selection of Mexican emigrants to the US

Note: Figure 2a depicts the patterns of Mexican migrants’ selection. Figure 2a plots the separation function (in red) compared to the 45 degree line (black) in the space of Mexicans’ skill levels. Figure 2b presents the distributions of American skills among American natives (blue), Mexican natives (black) and Mexican immigrants (red). Figure 2c illustrates the distribution of Mexican natives’ wages (black) and the hypothetical distribution of immigrants’ wages after their return to Mexico (red). Figure 2d compares the wage distribution of immigrants’ wages in the US (red) with the hypothetical distribution of Mexican natives’ wages had all of them moved to the US (black).

4 Main results

This section provides quantitative economic arguments regarding the impact that Mexican immigration has on the welfare of US residents. In what follows, we investigate the consequences of setting prohibitive immigration costs for Mexican workers. We do this by analysing a counterfactual state of the world in which all legal Mexican immigrants are removed from the US and inserted into the Mexican economy. In Subsection 4.2 we decompose the aggregate effects into four channels enumerated in the Introduction. Further on, in Subsection 4.3 we conduct a search through the space of immigration...
policies and report their implications for US resident.\footnote{All of the counterfactual experiments are done using a single simulation algorithm, whose short description can be found in Appendix C.}

4.1 No Mexican migration in the US

We commence with a description of the economic effects of increasing migration costs from Mexico to the US to an infinite value. No Mexican citizen is no longer authorized to stay in the US; thus the overall number of Mexican workers in the US is reduced to zero and all the former Mexican immigrants in the US are returned to their homeland.

Mexican immigration to the US is less skilled than the population of the receiving country. This fact shapes the results in the no-migration counterfactual. Since the majority of Mexicans is located below the median skill level of American workers, low-skilled Americans are the closest substitutes for Mexican immigrants and gain most after removing them. American low-skilled workers can now match with more productive firms, which boosts their wages, while at the same time deteriorates firms’ profits. This triggers two effects: on the one hand, American workers who were close to the margin, but inactive on the labour market become employed, because the wage they can earn in the new economic environment strictly surpasses the outside option. On the other hand, firms at the margin have to form matches with less productive workers, become less profitable and are forced to exit the market (note that every American who changes her/his labour market status from inactive to employed, necessarily posses a strictly lower skill than any Mexican immigrant). Moreover, the expected profits decline, so less entrepreneurs decide to enter the market. The latter causes a rise in entrants’ average productivity, through a reduction in their mass. This Melitz-type effect impacts all the workers along two channels. First, all the individuals residing in the US suffer an increase in prices: as the mass of domestic varieties decreases. This effect is uniform across the whole distribution of wages and amounts to an increase in the US price index by 0.4%. Second, all workers match with less productive firms, which harms their wage level, since the surplus function is increasing in both arguments. Furthermore, the high-earners located in the right part of the distribution, are over-proportionally affected by this deterioration of match quality due to the assumption of supermodularity of surplus functions.

A part of the losses induced by the Melitz-type externalities is mitigated by the fiscal effect of immigration. In our simulations we assume a rather pessimistic (playing against immigration) case of equal participation of US residents and Mexican immigrant in the US redistributive scheme. This, by construction, implies that Mexicans are net receivers of
benefits since they earn significantly less than the rest of the US workforce. Controlling for the first-order changes in inflows and outflows from the US governmental budget, results in an augmentation in the country-wide lump-sum transfer by 88 USD per year.

Finally, the endogenous reaction in international trade between Mexico and the US partly counterweights the reduction in peoples’ mobility. According to Table 1, the bilateral trade flows between the US and Mexico increase by 4.5%. Since the data reveal that the US observes a trade deficit vis-à-vis Mexico, the trade balance deteriorates by further 1.3% after imposing the no migration policy. In addition to reducing domestic absorption by 4%, the US reduces exports to the ROW by 1.9%, while imports from ROW drop by 2%. The trade balance with the ROW improves by 0.08% and counterbalances the Mexican net flow.

Table 1: Percent changes in trade matrix

<table>
<thead>
<tr>
<th>from:</th>
<th>ROW</th>
<th>MEX</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW</td>
<td>0.00%</td>
<td>6.27%</td>
<td>-2.02%</td>
</tr>
<tr>
<td>MEX</td>
<td>6.54%</td>
<td>13.73%</td>
<td>4.65%</td>
</tr>
<tr>
<td>USA</td>
<td>-1.92%</td>
<td>4.47%</td>
<td>-4.05%</td>
</tr>
</tbody>
</table>

In sum, according to the results in Figures 3a and 3c, 44.4% of American working-age population gains from repatriating Mexican workers back home. This includes employed, low-earning workers, as well as previously unemployed Americans who entered the labour market. 47% of American population loses in the no-migration scenario. High-earners encounter losses in their real wages mainly due to a negative firms’ entry and exit effect (the extensive margin of entering firms drops by 4%). Finally, 8.6% of American population remains unaffected by the change in Mexican immigration – these are the unemployed in both states of the world. The real wage effects range from 4% (approximately 115 USD of annual remuneration) for the 9th percentile to -0.3% (-1150 USD) for the 99th percentile. The employment effect is, however, small: the unemployment rate in the US reduces by only 0.03 percentage points (from 8.62% to 8.59%). Even though inequalities among US residents reduce, the average wage in the US decreases slightly by 0.2%, which indicates that the mean efficiency of the remaining American residents reduces marginally.

Returning Mexican migrants induce visible economic effects in Mexico. They tend to be less proficient in Mexico-specific skill, even though, they are strongly positively

39Exports to the US increase due to the fact that consumers in the US substitute domestic varieties with more abundant and cheaper foreign goods. Exports to Mexico increases because of the size effect: an increase in overall demand (larger population of Mexican residents), even though the prices of US goods rise.
selected with respect to their American-specific skill. Therefore, the location of former Mexican migrants in the home country wage distribution is biased towards the left tail. In consequence, the low-earning Mexican natives lose in the no-migration counterfactual, as the closest substitutes of the returning population. The losses reach up to -4.5% (43 USD) for the 5th percentile of wage distribution, while the 40th percentile is indifferent, as Figures 3b and 3d depict. The biggest winners in Mexico are the most skilled individuals who receive 1.5% of their real wage (780 USD). The macroeconomic implications on Mexican labour market are precisely opposite to the ones we observed in the US. More Mexicans below median wage increases competition among less proficient Mexican natives, and makes some of them turn to unemployment (which goes up from 4% to 4.1%). This, however, raises the average productivity of Mexican employees, and encourages more
firms to enter the market (the mass of entrepreneurs surges by 13.5%, roughly the size of migration wave). More firms means more domestic varieties, which significantly reduces the price index by -1.4%. Finally, the budgetary consequences in Mexico are detrimental for an average worker – the net lump-sum benefit drop by 49 USD.

The biggest winners from current migration patterns are the Mexican migrants. Figure 4 illustrates the magnitudes of their welfare effects. Note that the extent to which an immigrant is better off from moving abroad is dependent on her combination of skills, more precisely: on how far is she located away from the separation function (see Figure 2a). People characterized by a skill pair exactly on this line are indifferent between staying and moving. This means that their net gain (the difference between US and Mexican wage corrected with migration costs, recall Equation (9)) equals zero. However, the majority of emigrants are strictly better off, which can be verified in both Figures 4a and 4b. In the former, we plot two “gross” wages distributions of Mexican workers in their home country (blue line) and in the US (black line). On top of that, we add the “net” wage distribution which includes migrants’ gross wage corrected for additive and multiplicative migration costs (red line). The latter graph summarizes the difference between the red and the blue line and presents the net gain of migrants along their distribution. Note that the gains are sizeable, their median is close to 2,000 USD, while the maximum is reached by the 99th percentile of immigrants, and amounts at 6,000 USD.

![Figure 4: Net gains of Mexican migrants](image)

(a) CDFs of Mexican wages  
(b) Distribution of net gains in USD

Note: Figure 4 presents the welfare gains for current Mexican migrants. Figure 4a plots three distributions: the actual distribution of migrants’ wages in the US (black), the actual distribution of Mexican natives’ wages (blue), and the distribution of Mexican migrants’ wages net of migration costs (red). Figure 4b plots the absolute gains (in USD) of Mexican migrants along their cumulative density.

A question of great social and political importance relates to the changes in inequalities in both countries, as well as in the global population of Mexico and the US. As we learned
from our previous analysis, expatriation of Mexican immigrants decreases inequalities in the US and amplifies them in Mexico. The global picture is analysed in Figure 5. Figure 5a compares the “with-migration” distribution of global wages (jointly for all Mexicans and Americans, black line) to the “without-migration” one (red line). The changes concentrate in the middle-left segment of the distribution, between the 10\textsuperscript{th} and the 30\textsuperscript{th} percentiles. These are mainly low-skilled Mexican workers, and migrants who are now back in their home country. A better overview on the distributional effects is given in Figure 5b, where we compare the changes in wages earned by workers in particular quantiles of the global distribution. More than 50\% of the pooled US and Mexican population loses from closing the border. This includes 40\% of the poorest people (mainly Mexicans), and more than 10\% of the richest - highly skilled American workers. Gains are observed for 40-90\textsuperscript{th} percentiles, which are the high-skilled Mexicans and the low and middle-skilled Americans. In quantitative terms, the Gini coefficient of income inequality goes up from 0.4779 in the status quo, to 0.4813 in the counterfactual. Current Mexico-US migration scheme reduces global inequalities as a consequence of a negatively selected flow of low-skilled workers into the US.

Firms also respond to changing labour structure after imposing the migration shock. Due to the fact that they are strongly heterogeneous in terms of their productivity (distributed according to a Pareto CDF) and as a consequence of PAM, firms’ profits are significantly differentiated (as depicted in Figures 6a and 6b for the US and Mexico, respectively). Due to supermodularity of surplus functions the effects for the right tail of
firms’ profit distribution are expected to be quantitatively stronger. Closing the border reduces the supply of low and medium-skilled workers in the US. This improves their bargaining position, forces entrepreneurs to increase the offered wages (as we saw in Figure 3) and to match with cheaper, marginally less productive workers (close substitutes to the ones they matched with). In consequence, firms located between 10th and 75th percentile lose, as presented in Figure 6c. The effect that dominates from 75th percentile onwards relates to the number of entrants. Recall, that the lower number of workers in the US decreases expected profits and diminishes the mass of firms entering the market. This symmetrically reduces the supply of firms along the whole distribution. Accordingly, there are less highly productive firms, and those who stay on the market after removing migrants, face lower competition for talents and experience a boost in their profits. A reverse situation takes place in Mexico (Figure 6d), where an inflow of low-skilled returnees invites new entrants, benefits low-productivity firms, and depresses profits of the most productive entities. Comparing the results in Figure 6 to the ones presented in Figure 3 we see a clear pattern of redistribution between capital owners (shareholders of firms) and workers due to closing the Mexican-US border. In the US, the gains are transferred from the best workers to the best firms, and from the less productive firms to the less productive workers. In Mexico the outcomes are exactly opposite.

4.2 Decomposition of the total effect

In this subsection we compare the magnitude of four main effects of international migration in our model: wage effects across the whole distribution of skills, firms’ entry and exit effect, the market size effect and the fiscal impact. We proceed with isolating the wage implications and then adding each of the remaining effects one by one.

To start with, we focus on the wage effects generated by our model, by neutralizing the three remaining sources of deviations in welfare. Nominal wages float as responses to changes in relative skill supplies, as well as due to altering the mapping between firms and employees in the labour market matching. According to Figure 7, red lines, in a world without firm entry, market size and fiscal implications, the wages of all US residents grow, while all Mexicans experience a sharp loss. The explanation of the model mechanics that yield such results is straightforward and in line with standard assignment models. A drop in the supply of workers in the US benefits all due to a crude supply-demand interaction. While the mass of firms stays constant, a decrease in labour offered on the US market boosts the wages for all of the workers, since different skills are imperfect substitutes. The heterogeneity in the effect is brought about by an unequal spread of Mexican immigrants.
along the whole distribution of wages. Thus, all American workers gain from a lower
competition on the market and a better bargaining position with respect to firms. Their
benefits rise up to nearly 7% (200 dollars) for the marginal American and almost 0.5%
(800 USD) for the best American worker. Conversely, in Mexico the return of former
emigrants depresses wages of all native citizens up to -4% for the marginal Mexican.
However, the absolute magnitudes are limited and range from -40 to -150 USD. Note that
in the economy characterized by wage effects only (which might be close to the short-run
interpretation of the results), the US vote over no Mexican immigration policy would be

40The gain for the worst American worker is by definition the highest in relative terms, because each
and every relocated migrant improves her wage rate. For a given American worker, positive impact is
brought by those ex-migrants, who were located to the right of her position in the distribution. Moving
further to the right hand side of the cumulative curve, a lower mass of Mexican migrants is more skilled
than a given American, thus the relative gains are lower.
Figure 7: decomposition of the total effects in the case of no migration from Mexico to the US

Note: Figure 7 depicts the welfare consequences of setting prohibitive migration barriers from Mexico to the US, broken into four economic effects. The red two-dashed line depicts the magnitudes of wage effects. The blue dashed line presents the wage and the firms’ entry and exit effect. The green long-dashed line incorporates wage, firms’ entry and exit and the market size effects. The black solid line summarizes the total effect of all four sub-effects (including the fiscal one). Figure 7a (7b) contains relative changes in American (Mexican) natives’ wages along the distribution, while Figure 7c (7d) gives absolute variations in American (Mexican) natives’ wages (in USD). Quantiles of respective distribution on the horizontal axis.

unanimously confirmatory.

The second step of our analysis consists of adding the firms’ entry and exit effect to the previous picture (blue lines in Figure 7). Note the crucial role of firms’ entry and exit not only in terms of quantitative measures, but more importantly in the directions of the overall effects of migration. We now observe a significant polarization of both populations into a well defined group of winners (low and medium-skilled in the US, high-skilled in Mexico) and losers (high-skilled in the US, low and medium-skilled in Mexico). In the US around 60% of residents are strictly better-off, while in Mexico around 30% are winning.
The absolute magnitude of the firms’ entry and exit effect (differences between the blue and the red curves) increases as we move towards the highly skilled Americans/Mexicans, as a consequence of production function supermodularity. Firms’ entry and exit attenuates wage effects of migration, the gains (and losses, respectively) are shared between workers and entrepreneurs through an endogenous bargaining scheme according to the changes in relative strengths of both groups along their distributions. A change in the number of firms (a rather medium-run phenomenon) has significant implications for the overall shape of our results in both countries. This process, possibly not internalized immediately by all the economic agents, appears to be crucial in the overall evaluation of migration policies, and should be taken into account in public debates on the economic implications of global labour movements.

The third element of our decomposition is adding the market size effect to the previous analysis (the green lines in Figure 7). The latter works through the ideal price indexes induced by the change in the number of available varieties of consumption good, as in Iranzo and Peri (2009), Di Giovanni et al. (2015) and Aubry et al. (2016). Assuming that individuals reveal love-for-variety results in an even more pessimistic evaluation of the no Mexican migration policy in the US. Lower supply of workers in the US means higher wages and a reduction in the mass of firms which translates into higher prices and a poorer array of consumption goods for all American residents. This squeezes the mass of winners in the US to only 10%, keeping their gains at negligible levels. In Mexico, in comparison, the market size effect significantly dominates and turns the overall results positive for all the residents. Similarly to the firms’ entry and exit effect: the change in the number of available varieties is a medium-run phenomenon, and might not be fully accounted for by all individuals.

The last piece of puzzle is adding the fiscal effect, which completes the total result depicted with a black solid line in Figure 7 being identical to what was presented in Figure 3. Despite being quantitatively small, the change in net benefits received by US residents sets the final measure of winners and losers almost at par. Note, however, that we assumed identical fiscal participation of Mexican immigrants and US residents, which might be considered as a pessimistic scenario. According to OECD (2013) all US based immigrants act as net payers to the governmental redistributive scheme. The interplay of all four effects is strictly negative for 47% of American residents, while 44.4% are better-off. This result corroborates that the dispute on economic effects of migration, along its social and political dimensions, is supposed to be on the knife-edge.
4.3 Search through migration policies

Apart from a positive evaluation of modifying migration barriers between Mexico and the US, this paper offers a normative analysis of a wide range of migration policies. In this subsection, we search for combinations of additive and multiplicative migration costs that provide most beneficial outcomes for US residents regarding three criteria. The first one is the change in the number of immigrants from Mexico to the US. The second one includes the average wage earned by American residents. The third one considers the proportion of American population that experiences a positive wage effect after implementing a given policy mix.

For the purpose of finding preferential migration policies, we conduct $11 \times 11$ counterfactual simulations with various values for both migration costs. In each case we record the main macroeconomic indicators that characterise the US economy, as well as keep track of the changes in the distribution of American residents’ wages. In Figure 8 we present a graphical analysis of our findings in division into the above discussed evaluation criteria. Two columns of contour graphs are displayed: the left hand side panel analyses the results in a benchmark model (we call this group of figures: reference case), while the right hand side panel quantifies the shocks in a model with wage effect and firms’ entry and exit effect only (named: restricted case). The first row (sub figures (a) and (b)) considers shares of Mexican immigrants in the US, in percent. The second row depicts the changes in average real wage received by US residents, in USD, while the last row presents the share of American resident population that is strictly better-off after applying a particular policy. The latter result can be interpreted as if all individuals voted for immigration policies after perfectly anticipating the overall change in real wage rates. Each graph is organized as follows. The horizontal axis represents the additive migration cost which is in the range of: $-\delta_0 \in [-3500, 1500]$. The vertical axis covers the multiplicative migration cost in a form: $1 - \delta_1 \in [0.1, 0.5]$. In this way, high values on both axis are equivalent to low migration costs, and imply a particular type of liberalization. Variables of interest are plotted using a coloured contour map with red shades depicting negative, while green shades - positive deviations. The black dot indicates the current structure of costs of migrating to the US for Mexicans, which refers to the point $(-102, 0.282)$.

Figure 8a shows the contour of shares of Mexican immigrants in the US for all combinations of analysed policy variables. Clearly, a liberalization of migration in both dimensions (high values on both axes, top-right corners of graphs) invites more Mexicans to the US. This, however, has a negative impact on the average wages of US residents, as noted in Figure 8c. The latter is caused by significantly negative fiscal implications of
Figure 8: The analysis of alternative combinations of migration policies

Note: The figures present selected macroeconomic variables for alternative values of migration costs: $1 - \delta_1 \in [0.1, 0.5]$ and $-\delta_0 \in [-3500, 1500]$. Panels (a), (c), (e) consider the reference case (benchmark model), while panels (b), (d), (f) provide the reduced case (wage effect and firms’ entry and exit effect only). (a) and (b) depict shares of Mexican immigrants in the US in percent; (c) and (d) contain the changes in average wages of US residents in USD; (e) and (f) compare the results of democratic poll on migration policies in percent. The black dot indicates the status quo.
inviting many low-skilled Mexicans to the US. Interestingly, limiting our analysis to the wage and firms’ entry effects in Figure 8d results in a reverse picture: more immigration is equivalent to higher average gains for the domestic country population. In both cases, though, less than 50% of US population is benefiting from such liberal migration policies, as indicated in Figures 8e and 8f. While in the reference case this is brought about by adverse fiscal impact, in the restricted case the effect operates only through the negative selectivity of immigrants and adverse wage implications for those US residents who are closest substitutes to the inflowing workers.

A more restrictive migration policy in both dimensions (low values on both axes, bottom-left corners of graphs) deteriorates slightly the average remuneration of American residents in the reference case. Even though their selection is improved comparing to the status quo, a lower number of immigrants induces a strongly negative market size effect, which dominates in terms of the overall welfare impact. The restricted case provides a clear-cut indication that after ignoring the fiscal and market size implications, the firms’ entry and exit effect puts a downward pressure on average wages. Moving to the poll results, restrictive migration policies are expected to benefit almost a half of the US population (since these policies are close to the no-migration scenario analysed in previous subsections). What is striking, is that basing on the restricted case it would lead to a misleading conclusion that these policies are beneficial for the majority of (low and medium-skilled) American workers, as the market size effect (which plays an instrumental role in this environment) is disregarded.

Finally, let us focus on the diagonal line that links upper-left with bottom-right corners of each of the graphs. These are the policies which keep the number of Mexican immigrants in the US approximately constant (see Figures 8a and Figure 8b), but change the selectivity of immigrants. The bottom-right corners represent less selective waves of Mexican immigrants (low additive and high multiplicative costs), while the upper-left ones: a highly selective policy scheme (high additive and low multiplicative costs). From what we obtain in the reference case, we can conclude that a more selective immigration policy towards Mexican immigrants to the US is beneficial for the vast majority of US residents as well as for the mean remuneration. Particular combinations of additive and multiplicative migration costs are capable of improving wages of all US based workers, by slightly improving the selectivity of Mexican immigrants, but keeping their number roughly unchanged or slightly below the current level. In these cases, the fiscal effects are weakly positive (more high-earning immigrants leads to higher benefits for everyone), individuals experience a subtly positive market size effect, while a combinations of the wage and firms’ entry effects are most negative just above the median US worker, but beneficial
for both of the tails of the distribution. A comparison with the restricted case reveals a shocking finding that the very same policies would have been almost surely turned down in a democratic vote, if immigration induced only the wage and the firms’ entry effects. This suggests that an internalization of both fiscal and market size consequences of immigration policies can significantly change the overall economic evaluation of immigration policies. As a final remark, note that immigration policies that improve the selection of Mexican immigrants without strongly altering their number (situated on the ray that originates in the status quo and takes the North-East direction) can be considered as recommendations for the US authorities. Altering the skill selection of Mexicans immigrants to the US, rather than reducing their number, is expected to improve welfare across the whole distribution of US wages.

5 Robustness checks

We verify the robustness of our main results by performing several additional experiments. Detailed descriptions can be found below, while graphical presentations are gathered in Appendix C.

Alternative distributions of the skills of unemployed Any shock to the supply of skills in the US affects workers’ participation. More precisely: the removal of Mexicans invites some of the previously inactive Americans to join the labour market. Importantly, we do not observe the wages (nor the skills) of these unemployed individuals, thus we can only speculate about their distribution. In the benchmark, we assume that they are distributed uniformly in the population of inactive Americans. In what follows, we verify taking an exponential (strictly convex) and logarithmic (strictly concave) CDFs. The latter has a slight impact on the wage effect, as depicted in Figure C.1a. Our reference results (the solid black line) are close to the concave scenario (the dashed red line), where the mass of skills is concentrated at their low levels. Taking the convex version (the long-dashed blue line) in which mass is concentrated at skills close to the margin, reveals slightly more moderate welfare effect for the Americans residents (due to closer substitution of the former unemployed to the expatriated Mexican immigrants), while the high earners suffer a less pronounced loss (due to a less severe entry/exit effect of firms, since more Americans join the labour market). All in all, there is almost no qualitative difference between these alternative scenarios.
**Welfare effects for 40 best calibrations** Even though we strive at minimizing the goal function in the proposed calibration algorithm, we find some vectors of model parameters, which do fit the data well, but are dominated by the reference set of model parameters. In order to increase the credibility of our (random) Monte Carlo procedure, it is inevitable to verify what are the quantitative and qualitative effects of simulating the no-migration counterfactual in models with these alternative sets of parameters. The results are presented in Figure C.1b with solid grey lines. 40 alternative parametrisations are compared to the reference one (black line). All of them do not deviate significantly from our benchmark, and in qualitative terms they predict almost identical patterns in welfare changes.

**Modifying the market size effect** The literature provides numerous estimates of the elasticity of trade flows with respect to trade costs (equivalent to the elasticity of substitution between varieties, ε, in our approach). Various model specifications and datasets used allow, however, to formulate a convergent view on the magnitude of this particular variable. In the Melitz (2003) trade model with heterogeneous firms, Simonovska and Waugh (2014b) indicate that the 80% confidence interval is: [4.1, 6.2]. Melitz and Redding (2015) use ε = 4 in their simulations. In the framework developed by Eaton and Kortum (2002), this elasticity is found to be in the range of: [3.8, 5.2] according to Bernard et al. (2003); Donaldson (2010); Burstein and Vogel (2010); Eaton et al. (2011); Parro (2013); Simonovska and Waugh (2014a); Caliendo and Parro (2015), though Eaton and Kortum (2002) estimate it at the level of 8. Therefore, we verify the consequences of alternative estimates of ε for our main results. Figures C.1c and C.1d summarize the effects of removing Mexican immigrant for the US residents. The black line indicates the reference value of ε = 5, red and blue lines assume higher values (7 and 9 respectively), while green and orange lines: 4 and 3 respectively. Higher elasticities (lower market size effects) move the welfare effects slightly upwards, but change the extensive margin of winners to 60%. Stronger market size effect has a significantly negative impact on the gains from removing immigrants, which reduce the mass of winners up to 15%.

**Adding illegal Mexican immigrants** Illegal migration from Mexico to the US proves to be one of the key points in the overall discussion about American migration policy. Therefore, we assess the welfare effects of removing all Mexican immigrants with an inclusion of the undocumented Mexicans. Unfortunately, there exists no official dataset that would give us an idea about the wage distribution of illegals. We overcome this difficulty by taking the necessary numbers from estimates available in the literature.
First, the actual number of undocumented Mexicans in the US is unknown. A recent briefing by Pew Research Center provides some trustworthy estimates of this figure. They assess that out of 11.7 million Mexican immigrants in the US in 2014, there were about 5.8 million of illegals. Our data considers 7 million working age migrants (according to the crude estimates a third/fourth of illegals are included in the US census), thus we will increase their number to 10.5 million, which leaves us with $S_U(0) = 0.206$. Illegal migrants earn substantially lower wages than their legal peers. Thanks to the kindness of Caponi and Plesca (2014) we are able to compute the wage penalty for illegals along the wage distribution. We find that the penalty is around 15-20%, in line with the findings of Massey and Gentsch (2014). The new distribution of immigrants’ wages would now be an average of the distribution of legal migrants (taken with a weight of 2/3) and the illegals (weight of 1/3). While the former is the one used in the reference calibration, the latter includes a 20% reduction of wages across all worker types.

After running the model including illegal Mexican migrants, we find that the wage effect is higher for the low-skilled Americans, see Figures C.1e and C.1f. In line with our expectations: a larger number of less skilled Mexicans on the American market dampens the wage of substitutive, low-skilled American workers, thus closing the borders is more profitable for these individuals. Conversely, high earners in the US suffer a substantially higher loss after including undocumented Mexicans, due to a more severe entry/exit effect on firms caused by the increase in the size of the inflow (which is 50% larger than in the reference counterfactual). Ultimately, migration is positive for a slightly larger share of US native population: strictly positive effects are reported for 48.7% (instead of 44.4% in the reference).

6 Conclusions

In this paper we construct an alternative model for analysing the consequences of international migration flows across the distribution of wages. We develop a framework with two separate labour markets (Mexico and the US) that require two distinct types of skills. Both economies are populated with heterogeneous individuals, who are distributed in this two-dimensional skill space. Since migration is costly, and because the two skills are dif-

\footnote{For details please consult: http://www.pewresearch.org/fact-tank/2017/03/02/what-we-know-about-illegal-immigration-from-mexico/}

\footnote{Caponi and Plesca (2014) dispose of data describing illegal immigrants to the US from all origins, and compute the kernel distributions of gender-specific penalties, see their Figure 1. Using their estimates, we pool the sample and produce a single distribution of differences between legal and undocumented immigrants.}
ferently priced in both countries, Mexican workers sort into two labour markets seeking to maximize their expected real wage. This selection model is then enriched by a matching process between workers and firms as well as endogenous consumption decisions and trade. Finally, the extensive margins of both factors (skills and entrepreneurship) is endogenous and driven by economic incentives to enter or exit labour markets and production sectors.

By calibrating the model to Mexican and American data, we verify that the selection of Mexican immigrants in the US is the key factor that shapes the overall welfare implications. We confirm that Mexicans in general are strongly downgraded in terms of their aggregate supply of US-specific skill, compared to the American workforce. Moreover, even though Mexican movers are strongly positively selected with respect to US-based skill compared to Mexican stayers, they still exhibit a visibly lower proficiency with regards to American residents. On top of that, emigrants tend to be negatively selected in terms of Mexican skill, in relation to the stayers. All of these factors structure the nominal wage effect of Mexican immigration for American and Mexican natives. The latter is, to some extent, counterbalanced by firms’ entry and exit, being a result of endogenous matching process on the labour market. Further consequences are brought about by individuals’ love-for-variety and the change in net fiscal position of resident American workers. These four economic forces bring a non-trivial and non-linear impact along the two distributions of real wages under investigation. In total, removing Mexican immigrants from American economy benefits the 44.4% of the US-based population – predominantly, the low-skilled individuals.

Our hypothetical evaluation of alternative immigration policies between Mexico and the US reveals that there might exist a combination of additive and multiplicative costs that would increase the average wage among US residents and be beneficial for virtually all of them. Inviting a slightly more skilled selection of Mexican immigrants, without changing their overall number is the recipe for achieving a Pareto-improving distribution of real wages in the population of American workers. The latter signals that the current immigration policy versus Mexico of the US is suboptimal from the American society’s point of view.

The discussion about the welfare effects of international migration is far from being terminated, especially in the case of Mexican migration to the US. The results we report underline the huge importance of the distributional aspects in both destination and sending countries. Keeping this in mind, the fierce political and social debate on immigration will surely continue. Taking our results as a cautious recommendation, the future modification of the US immigration policy versus Mexico should converge towards inviting more skilled individuals rather than eliminating all Mexican immigrants from the US.
References


Appendix A - theoretical model details

Wages - detailed derivation  Following Sattinger (1979) we start the derivation of equilibrium wages by tentatively assuming that the equilibrium hiring function $x_i^*(\cdot)$, and hence also its inverse: the matching function $h_i^*(\cdot)$, are strictly increasing. This immediately results in a positive, assortative matching (PAM) between workers and firms. For the sake of exposition, we will further assume that: a) $w_i(\cdot)$ is twice differentiable and b) $x_i^*(\cdot)$ is a bijection and is differentiable. Assuming supermodularity of the surplus function suffices to obtain a strictly increasing matching function, regardless of the supply of skills.

Let us denote the type of the least productive entrant firm as $h_i^c$. It nevertheless still remains on the market and receives: $\pi_i(x_i^c, h_i^c) = 0$. Note that enterprises’ profits $r_i(\cdot)$ are strictly increasing in equilibrium, which follows immediately from the fact that the surplus function strictly increases in $h_i$. Therefore, for any $h_i > h_i^c$ we have: $r_i(h_i) > 0$. This allows us to significantly simplify Equation (16): 

$$D_i(x) = R_i^F \Pr(H_i \geq h_i^*(x), H_i \geq h_i^c) = \begin{cases} R_i^F (1 - h_i^*(x)) & \text{for } x \geq x_i^*(h_i^c) \\ R_i^F (1 - h_i^c) & \text{for } x < x_i^*(h_i^c). \end{cases}$$

Labour markets must clear both internationally and within each country. The latter implies that $S_i(x) = D_i(x)$, which pins down both $h_i^*(\cdot)$ and $h_i^c$:

$$h_i^*(x) = \frac{R_i^F - S_i(x)}{R_i^F}, \quad h_i^c = \frac{R_i^F - S_i(0)}{R_i^F}.$$

As $h^*(\cdot)$ is strictly increasing it follows that we can define the critical skill $x_i^c$, i.e. the skill of the least talented worker who finds employment in country $i$, in a formal way:

$$x_i^c = \sup\{x \in [0, 1] : S_i(x) = S_i(0)\}. \quad (A.1)$$

From the discussion so far, it follows that the equilibrium supply $S_i(\cdot)$ determines the skill of the least talented worker and the least productive firm in each country, as well as the matching functions. The next step is to establish wages and profits of $x_i^c$ and $h_i^c$, respectively. Starting with the latter, note that as the surplus of any match involving the worst enterprise is negative, so is its profit: $r_i(0) < 0$. This gives us $h_i^c > 0$, which in turn

---

43These assumptions are also not necessarily met for $\pi_i(\cdot)$, but are nevertheless without loss in generality, for the same reasons as those in footnote 20.

44The first order condition of firms’ profit maximisation is: $w_i'(x_i^*(h_i)) = \partial \pi_i(x_i^*(h_i), h_i)/\partial x$, while the second order condition is simply: $\partial^2 \pi_i(x_i^*(h_i), h_i)/\partial x^2 \leq w_i''(x_i^*(h_i))$. Hence: $w_i'(x) = \partial \pi_i(x, h_i^*(x))/\partial x$. The two imply that: $dh^*(x)/dx \cdot \partial^2 \pi_i(x, h^*(x))/\partial x = w_i''(x_i) = \partial^2 \pi_i(x_i^*(h_i), h_i)/\partial x^2 \geq 0$. That the RHS is weakly positive follows from the second order condition. This and supermodularity of the surplus function implies that a strictly increasing matching function can always be supported in an equilibrium, regardless of the supply of skills. Furthermore, if the surplus function is strictly supermodular, then only a strictly increasing $h_i^*(\cdot)$ can be supported in equilibrium. This is a well known result, first shown for a discrete version of the one-market model by Becker (1973).

45Thus, from the definition of demand and market clearing we get that: $D_i(0) = R_i^F \Pr(H_i \geq h_i^c) = (1 - h_i^c)R_i^W = S_i(0)$.
We have that \( x_i \) of skills. Denote the critical skill of Mexican workers in country \( i \).

\[ \text{Sorting - detailed derivation} \]

In this step, we will establish the equilibrium supply of skills. Denote \( S_i(x) \) the supply and the critical skill (Equations (10) and (A.1)) lead us to:

\[
\begin{align*}
   w_i(x) &= \begin{cases} 
   \int_{x_i}^{x} \frac{\partial}{\partial x} \pi_i(r, 1 - S_i(r)/R_i)dr + w_i(x_i) & \text{for } x_i \geq x_i^c \\
   \pi_i(x, h_i^c) - \pi_i(x_i^c, h_i^*) + w_i(x_i^c) & \text{for } x_i < x_i^c,
   \end{cases}
\end{align*}
\]  

(A.2)

where \( w_i(x_i^c) = \pi_i(x_i^c, h_i^*(x_i^c)) \) \[47\]

\[ \text{Sorting - detailed derivation} \]

In this step, we will establish the equilibrium supply of skills. Denote the critical skill of Mexican workers in country \( i \) as:

\[ x_i^m = \sup \{ x \in [0, 1] : S_i^M(x) = S_i^M(0) \}. \]  

(A.3)

We have that \( x_i^m = x_i^m \) as there is no immigration to Mexico, and: \( S_i^M(\cdot) = S_i(\cdot) \). We will focus on deriving the supply of Mexican workers’ skill in each country: once we know \( S_i^M(\cdot) \), finding the overall supply of skills in the US, as well as the critical skill \( x_i^U \), is straightforward. Recall that \( w_i(x_i^U) = \pi_i(x_i^U, h_i^*(x_i^U)) \). Therefore, the definitions of supply and the critical skill (Equations (10) and (A.1)) lead us to:

\[
S_i(x) = \begin{cases} 
   S_i^M(x) + R_i^W(1 - F(x)) & \text{for } x \geq x_i^U \\
   S_i^M(0) + R_i^W(1 - F(x_i^U)) & \text{for } x < x_i^U,
   \end{cases}
\]  

(A.4)

where:

\[ x_i^U = \min \{ x_U \in [0, 1] : \pi_i \left( x_U, \frac{R_i^F - S_i^M(x_U) - R_i^W(1 - F(x_U))}{R_i^F} \right) \geq 0 \}. \]  

(A.5)

Furthermore, as long as there are some Mexican workers in both countries, it has to be the case that workers with talent \( (x_i^m, x_i^m) \) need to earn the same wages in both countries, otherwise they would also join the market, which contradicts the definition of \( h_i^c \).

\[46\] This follows from the facts that \( r_i(\cdot) \) depends only on the surplus, wage and hiring functions and, therefore, is continuous. Hence, if \( r_i(h_i^c) > 0 \), then firms with productivity marginally lower than \( h_i^c \) would also join the market.

\[47\] Note that Equation (A.2) implies that the wage function is, indeed, differentiable, but differentiability of the matching function holds if and only if \( S_i(\cdot) \) is differentiable. This is not a problem, however, as e.g. Hopkins (2012) or Gola (2016) offer alternative derivations of the wage function in the differential wage matching model, which do not rely on differentiability of the matching function. Furthermore, the results in Chiappori et al. (2010) imply that the first derivative of the wage function – hence, here, also the wage function itself – is unique in our model (more specifically, this is implied by their Proposition 3). For details of why does this result apply here, see proof of Proposition 1 and footnote 42 in Gola (2016). Therefore, the wage function will be exactly the same as in (A.2) even if the hiring function is not differentiable, or if it is not a function at all.
as otherwise some Mexicans would want to relocate:

**Lemma 1.** If $x^m_U, x^c_M < 1$ then $(1 - \delta_1)w_U(x^m_U) - \delta_0 = w_M(x^c_M)$.

**Proof of Lemma 1.** Suppose that $x^m_2 = \min\{x^m_1, x^m_2\}$. Suppose further that $w_1(x^m_1) - \delta < w_2(x^m_2)$. There are two possibilities. Firstly, $S^2_1(0) + S_2(0) < 1$; then $w_2(x^m_2) = 0$ and any worker with $w_1(x_1) - \delta < w_2(x^m_2)$ will choose unemployment. Secondly, $S_1^2(0) + S_2(0) = 1$; then $x^m_2 = 0$ and any worker with $w_1(x_1) - \delta < w_2(x^m_1)$ will choose to work in Mexico. In either case, the continuity of $w_1(\cdot)$ implies that there exists some $x'_1 \in (x^m_1, 1]$ such that $w_1(x_1) - \delta < w_2(x^m_1)$ for any $x_1 \in [x^m_1, x'_1]$. Hence, $S^2_1(x'_1) = S^2_1(x^m_1)$, which contradicts the definition of $x^m_1$.

Now suppose that $w_1(x^m_1) - \delta > w_2(x^m_2)$. Again, there are two possibilities. The case of $x^m_1 = x^m_2 = 0$ is essentially equivalent to the case above (swap $x^m_1$ with $x^m_2$ and apply the same reasoning). This leaves us with the case of $x^m_1 > 0$. By continuity of $w_1(\cdot)$ there exists some $x''_1 \in [0, x^m_1)$, such that $w_1(x''_1) - \delta > w_2(x^m_2)$. Further, by continuity of $w_2(\cdot)$, there exists some $x'_2 \in (x^m_2, 1]$, such that $w_2(x'_2) < w_1(x''_1) - \delta$. Then for any worker with $(x_1, x_2) \in S = [x''_1, x^m_1] \times [x^m_2, x'_2)$ we have that $w_1(x_1) - \delta > w_2(x_2) = 0$ and hence they strictly prefer to move to the US. As $S$ is of a strictly positive Lebesgue measure and $C_{x_1x_2}$ is strictly positive on $[0, 1]^2$, the measure of workers with skill $< x^m_1$ willing to move to US is strictly positive as well, which implies $S^2_1(x'_1) \neq S^2_1(0)$ and contradicts the definition of $x^m_1$.

This raises the question: in which circumstances do Mexicans work in both countries? Or equivalently: is the assumption in Lemma (1) plausible? Intuitively, some Mexicans will move to US if the no-migration wages in the US (net of migration costs) are high enough compared to the no-migration Mexican wages. And *vice versa*, some Mexicans will want to remain in their home country, as long as the surplus produced by the best possible match in Mexico is higher than the surplus produced by the worst match in US under full migration.

**Definition 2.** The no-migration wage function $w^m_1(\cdot)$ in country $i$ is the wage function given by Equation (A.2) that holds a) if $i = 1$, for $S^2_1(x_2) = 0$, and b) if $i = 2$, for

$$S_2(x_2) = \begin{cases} (1 - x_2) & \text{for } x > x_b^2, \\ (1 - x_b^2) & \text{otherwise}, \end{cases}$$

where:

$$x_b^2 = \min\{x_2 \in [0, 1] : \pi^c_1(x_2, \frac{R^M_2 - x_2^2 + 1}{R^M_2})\}.$$

**Assumption 1.** The highest no-migration wage in US (net of migration cost) is higher than the lowest no-migration wage in Mexico: $w^m_1(1) - \delta > w^m_2(x^{m_2})$.

**Assumption 2.** Either the highest possible surplus in Mexico is greater than the lowest surplus in the US under full migration (net of migration cost) or the measure of managers in the US is smaller than the measure of all workers in both countries:

$$\pi_2(1, 1) > \pi_1(0, \frac{R^M_1 - 1 - \pi^W_1}{R^M_1}) - \delta \text{ or } R^M_1 < 1 + R^W_1.$$

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Lemma 2. In any equilibrium: a) $x_1^m < 1$ iff Assumption 1 holds; b) $x_2^m < 1$ iff Assumption 2 holds.

Proof of Lemma 2. $x_1^m = 1$ implies $S_1^2(\cdot) = 0$ and, hence, means equilibrium wages are given by $w_1^m(\cdot)$. But then, by Assumption 1 and continuity of wage functions, there exist some $x_2^b > x_2^b$ and $x_1' < 1$ such that any worker with $(x_1', x_2) \in [x_1', 1] \times [x_2^b, x_2^b]$ strictly prefer to migrate to the US, which implies $x_1^m < 1$.

Consider any supply function $S_1^2(\cdot)$ that is strictly greater than zero for at least some $x_1 \in [0, 1]$. By inspection of Equations (A.2) and (A.5) follows than $w_1(\cdot) \leq w_1^m(\cdot)$ and $w_2(\cdot) \geq w_2^m(\cdot)$. Now suppose that Assumption 1 is not met. This implies that $w_1(1) - \delta < w_2(x_2^b)$ and contradicts strictly positive $S_1^2(\cdot)$.

For b), $x_2^m = 1$ if and only if $S_2(\cdot) = 0$. As markets clear in equilibrium, this means that $w_2(x_2) \geq \pi_2(x_2, 1); \text{otherwise a positive mass of managers could earn positive profits and demand would, therefore, be positive. Any worker with } x_1 = 0 \text{ is receiving a payoff of max}\{\pi_1(0, \frac{R_1^M - 1 - R_1^W}{R_1^W}) - \delta, 0\}: \text{she earns the first element if she is employed in the US and the second if she is not. As } w_2(1) > \max\{\pi_1(0, \frac{R_1^M - 1 - R_1^W}{R_1^W}) - \delta, 0\} \text{ and wage and surplus functions are continuous, we can find } x_2'' > 0 \text{ and } x_2'' < 1, \text{ such that for any } (x_1, x_2) \in [0, x_1'] \times [x_2', 1] \text{ the wage she could earn in Mexico is greater than the highest pay-off she could receive otherwise. But this contradicts the fact that } x_2^m = 1.

Analogously to the proof of a), consider $S_2(\cdot) > 0$. Then $w_2(x_2) < \pi_2(1, 1)$. Suppose Assumption 2 is not met, this implies $\pi_1(0, \frac{R_1^M - 1 - R_1^W}{R_1^W}) - \delta > \pi_2(1, 1) > 0$. Hence, any Mexican worker will be better of in the US, which contradicts $S_2(\cdot) > 0$ and therefore implies $x_2^m = 1$.

Hence, if only Assumption 1 is met, then everyone migrates to the US and if just Assumption 2 is met, then there is no migration to the US. In the remainder of this section we will assume that both assumptions hold, which – by Lemma (1) – implies that:

$$
(1 - \delta_1) \left( \pi_1 \left( x_1', \frac{R_1^M - S_1(0)}{R_1^M} \right) + \int_{x_1'}^{x_1} \frac{\partial}{\partial x_1} \pi_1 \left( r, \frac{R_1^M - S_1(r)}{R_1^M} \right) dr \right) = \pi_2 \left( x_2^c, \frac{R_2^M - S_2(0)}{R_2^M} \right) + \delta_0.
$$

(A.6)

In what follows, we take this situation as our reference scenario\(^{48}\) We therefore continue with: $x_1^m, x_2^m < 1$.

Let us find what determines the sizes of the Mexican workforce in each country. Given that $w_i(x_i^c) = \pi_i(x_i^c, 1 - S_i(0)/R_i^c) \geq 0$, we can define the maximal possible supply of

\(^{48}\)For a detailed analysis of other cases: no migration to the US, and emigration of all Mexicans to the US, please consult Appendix 6.
Mexican workers in country $i$ as $M_i \geq S^M_i(0)$, such that:

\[
0 = \pi_U \left( x_U^i, \frac{R^F_U - M_U - R^W_U (1 - F(x_U^i))}{R^F_U} \right), \tag{A.7}
\]

\[
0 = \pi_M \left( x_M^i, \frac{R^F_M - M_M}{R^F_M} \right). \tag{A.8}
\]

Whether the actual supply of Mexican workers will be equal to $M_i$ depends on the sum $M_U + M_M$. In particular, as the total supply of Mexicans cannot be greater than 1, we have that:

\[
S^N_U(0) + S_M(0) = \min\{M_U + M_M, 1\}. \tag{A.9}
\]

Moreover, as the only unemployed Mexican workers are those with $(x_U, x_M) < (x_U^m, x_M^c)$ it has to be the case that:

\[
C(x_U^m, x_M^c) = C(x_U^m, x_M^c) = 1 - S^M_U(0) - S_M(0). \tag{A.10}
\]

The final elements of Mexicans’ sorting description are the star skills $x_U^i$, which play a similar role to $x_U^m$, but on the opposite end of the skill distribution. The star skill $x_U^i$ is the lowest level of country $i$ skill such that any worker with $x \geq x_U^i$ works in country $i$ with probability 1, irrespective of her skill in the other country. Formally, this can be defined as:

\[
x_U^i = \min\{x_U \in [0, 1] : (1 - \delta_1)w_U(x_U) - \delta_0 \geq w_M(1)\}, \tag{A.11}
\]

\[
x_M^i = \min\{x_M \in [0, 1] : w_M(x_M) \geq (1 - \delta_1)w_U(1) - \delta_0\}. \tag{A.12}
\]

Note that Lemma (1), the strict increasingness of wage functions and the fact that $x_U^m, x_M^c < 1$ imply immediately that $x_U^m < x_U^i$. This allows us to define a function of Mexicans’ skills that differentiates Mexican stayers from emigrants to the US. We call this the separation function $\psi : [x_U^m, x_M^c] \times [x_U^m, x_U^i]$, and define it as an American skill level $\psi(x_M) \in [x_U^m, x_U^i]$, such that a Mexican worker equipped with a bundle of: $(\psi(x_M), x_M), \forall x_M \in [x_M^m, x_M^c]$ receives an identical remuneration in both countries:

\[
(1 - \delta_1)w_U(\psi(x_M)) - \delta_0 = w_M(x_M). \tag{A.13}
\]

Hence, any Mexican worker $(\psi(x_M), x_M), \forall x_M \in [x_M^m, x_M^c]$ is indifferent between migrating to the US and remaining in her home country.

Knowing the critical skills of Mexican workers, the star skills and the separation function we can easily determine the supply of Mexican workers’ skill in each country. Mexicans with $x_U < x_U^m$ never work in the US; Mexicans with $x_U > x_U^i$ always work in the US; Mexicans with any $x_U \in [x_U^m, x_U^i]$ work in the US if $x_U > \psi(x_M)$, which happens with probability $\partial/\partial x_U C(x_U, \psi^{-1}(x_U))^{49}$ Analogously, Mexicans with $x_M < x_M^m = x_M^c$ never work in Mexico; Mexicans with $x_M > x_M^c$ always work in Mexico; finally, Mexicans

\[\text{have that:}\]

\[
\psi(\cdot) \text{ is a bijection, and } \psi^{-1}(\cdot) : [x_U^m, x_U^i] \times [x_M^m, x_M^c] \text{ is well defined.}\]
with any \( x_M \in [x_M^m, x_M^s] \) work in Mexico if \( x_U < \psi(x_M) \), which occurs with probability \( \partial / \partial x_M C(\psi(x_M), x_M) \). Therefore:

\[
S_U^M(x_U) = \begin{cases} 
\int_{x_U^m}^{x_U^s} \frac{\partial}{\partial x_U} C(r, \psi^{-1}(r)) dr + 1 - x_U^s, & x_U < x_U^m, \\
1 - x_U, & x_U \in (x_U^m; 1]; 
\end{cases} 
\tag{A.14}
\]

\[
S_M(x_M) = \begin{cases} 
\int_{x_M^m}^{x_M^s} \frac{\partial}{\partial x_M} C(\psi(r), r) dr + 1 - x_M^s, & x_M < x_M^m, \\
\int_{x_M^m}^{x_M^s} \frac{\partial}{\partial x_M} C(\psi(r), r) dr + 1 - x_M^s, & x_M \in [x_M^m, x_M^s], \\
1 - x_M, & x_M \in (x_M^m; 1]. 
\end{cases} 
\tag{A.15}
\]

By plugging this back into (A.13) we arrive at an integral equation, which holds for \( x_M \in [x_M^m, x_M^s] \):

\[
\int_{x_M^m}^{x_M^s} \frac{\partial}{\partial x_M} \pi_M \left( t, 1 - \frac{S_M(0) + \int_{x_M^m}^{t} \frac{\partial}{\partial x_M} C(\psi(r), r) dr}{R_M^F} \right) dt + w_M^c + \delta_0 = (1 - \delta). 
\tag{A.16}
\]

This identity, together with Equations (A.4), (A.5), (A.7), (A.8), (A.9), (A.10), (A.11), (A.12), (A.14), (A.15), and the fact that \( S_i^m \in (0, M_i] \) fully characterises the equilibrium.

**Theorem 2.** The equilibrium defined in Definition 1 exists and is unique.

**Proof.** Apply the proof of Theorem 1 in Gola (2016).

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Figure B.1: Two-dimensional distributions for Clayton, Gaussian and Gumbel copulas

Note: Figure B.1 presents the distributions of skills assuming different copula functions (row 1: Clayton, row 2: Gaussian, row 3: Gumbel), and low (column 1) and high (column 2) correlations between skills.
Appendix C - calibration details

Solution Algorithm

The equilibrium of the model requires a conjunction of several conditions: the three wage equations of form: [A.2] for all the individuals regardless their current location and skill level, and the arbitrage equation [9] for Mexican immigrants in the US. The model is closed with setting the unemployment rates, defining the reservation wages, normalizing firms’ profits and computing the costs of entry into production. Taking these elements as given, the model generates the equilibrium selection mechanism of Mexican migrants (represented by the separation function [A.13]) and the density of skills supplied by the actual migrants (summarized by [A.14]). In the calibration, however, we aim to extract this pattern of migrants’ selection from the available data on wage distributions.

For a given vector of parameters Ξ, the solution algorithm starts with exploiting the distribution of American natives’ wages – the only one that is not affected by the selection mechanism.\(^{50}\) Taking the first derivative of relation (A.2), we arrive at the following differential equation:

\[
\frac{\partial}{\partial x_U} w_U(x_U) = \frac{\partial}{\partial x_U} W(F(x_U)) \leftrightarrow \frac{\partial}{\partial x_U} \pi_U(x_U, h_U(x_U)) = W'(F(x_U))F'(x_U), \quad (C.1)
\]

where the left hand side function is the derivative of the surplus with respect to its first argument (skill ranking \(x_U\)), while the right hand side function is the observed inverse distribution of wages \(W'()\) being a function of the distribution of American skills \(F()\), multiplied by the density of skills supplied by Americans: \(F'(x_U)\). Expression [C.1] serves as the first element of the two-dimensional system of differential equations, and is solved with an initial condition: \(w_U(1) = W(1)\). The solution is discretized on the assumed grid, and computed using the Euler method.\(^{51}\)

The second step is to reveal the underlying selection mechanism induced by a tuple: \(\{\Xi, F()\}\). We therefore proceed with exhausting the arbitrage condition [9], and taking its first derivative:

\[
\frac{\partial}{\partial x_M} w_M(\psi^{-1}(x_U)) = (1 - \delta_1) \frac{\partial}{\partial x_U} w_U(x_U) \leftrightarrow \frac{\partial}{\partial \psi^{-1}(x_U)} \pi_M(\psi^{-1}(x_U), h_M(\psi^{-1}(x_U))) (\psi^{-1}(x_U))' = (1 - \delta_1) \frac{\partial}{\partial x_U} \pi_U(x_U, h_1(x_U))(C.2)
\]

The latter serves as the second equation in the two-dimensional system, solved simultaneously with Equation [C.1], using the Euler method on the assumed grid, and taking the initial condition: \(\psi^{-1}(x_U^*) = x_M^*\). For the identified selection pattern, the mass of Mexican immigrants in the US can be computed by using equation [A.14], discretized in

---

\(^{50}\)Recall that \(\Xi = \{k_U, \gamma_U, R_U^a, s_U, k_M, \gamma_M, R_M^a, s_M, \theta, \delta_0, \delta_1, x_U^0, t_1, t_2\}\), and for \(i \in \{U, M\}\): \(k_i, \gamma_i, R_i, s_i > 0; \theta, \delta_1, t_1, t_2 > 0; x_U^0 \in [0, 1]; \delta_0 \in \mathbb{R}\).

\(^{51}\)Euler method is the simplest numerical way to solve an ordinary differential equation with a given initial condition. For a given ODE: \(y'(x) = f(x), y(1) = f(1)\), and a given series of grid points: \(\{x(1), ..., x(K)\}\), one can compute the values of \(y\) by setting: \(y(x(t)) = y(x(t - 1)) + (x(t) - x(t - 1))f(x(t - 1))\).
the following way:

$$S^*_U(x_U - \Delta x_U) = S^*_U(x_U) + \Delta x_U \partial C(x_U, \psi^{-1}(x_U))/\partial x_U,$$  \hspace{1cm} (C.3)

for all rankings \( x_U \) ranging from \( x_U^r \) down to \( x_U^0 \), with step \( \Delta x_U = 1/K \). The starting point requires that: \( S^*_U(x_U^* U) = 0 \).

At this stage, it is straightforward to determine the wage distributions of Mexican workers in the US and in Mexico. To this end, we use the Euler-discretization of country-specific equations \( \{A.2\} \). The final result of the calibration for a given vector of parameters \( \Xi \) is a set of three wage distributions: American natives, \( \hat{W}^U \equiv (w_U(x_U), F^U(x_U)) \), Mexican immigrants in the US, \( \hat{W}^I \equiv (w_U(x_U), F^I(x_U)) \), and Mexican stayers, \( \hat{W}^M \equiv (w_M(x_M), F^M(x_M)) \).  \hspace{1cm} (52)

Our goal in the calibration procedure is to find such a vector of parameters \( \Xi \) that gives the best possible fit of \( W^U \), \( W^I \) and \( W^M \) to the observed distributions \( W^U \), \( W^I \) and \( W^M \). In doing so, we need to search through a 14-dimensional space, and each vector of parameters requires a full solution of the model on the defined grid (performed by using the above described algorithm). Therefore, to maximize the performance of such a computationally-intensive search, we propose a version of basing-hopping algorithm, enriched by a Monte Carlo search procedure, with quantile distribution fitting goal function.  \hspace{1cm} (53)

Each vector \( \Xi \) is evaluated using a subjective goal function:

$$\zeta(\Xi) = p_1 e(W^U) + p_2 e(W^I) + p_3 e(W^M) + p_4 e(F^U(x_U^r)) + p_5 e(u^M) + p_6 e(S^*_U(x_M^0)),$$  \hspace{1cm} (C.4)

where \( e(\cdot) \) is an error function that computes the squared difference between an object from the model and its empirical counterparty in the data, and \( p \)'s are subjective weights.  \hspace{1cm} (54)

The proposed Monte Carlo search method settled in the vector of parameters indicated in Table C.2. Location \( (k_i) \) and spread \( (s_i) \) of the skill-component in the US-based surplus function take higher values than their counterparts in Mexico. The former is driven by a significant first-order stochastic dominance of US wage distribution relative to Mexican, while the latter indicates a slightly larger dispersion in skills pricing on American market comparing to Mexico. Then, firms’ component in wages appears to be significantly more.

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52 The proposed notation includes the relative densities of the analysed groups of workers. \( F^I(x_U) = (S^*_U(x_M) - S^*_U(x_U))/S^*_U(x_M) \), while: \( F^M(x_M) = (S_M(x_M) - S_M(x_M))/S_M(x_M) \).

53 Standard, one-dimensional selection models can be identified using a Maximum Likelihood Estimation. In the case of our model this is not feasible due to two complexities. Firstly, the selection patterns are difficult to solve algebraically. This means that we are unable to obtain closed form solution for the distributions of wages, which makes it impossible to use a standard MLE algorithm. Secondly, we do not fit individual observations in our datasets, rather: we set the model parameters to match the full distributions of the three groups of workers that we observe. This method is computationally less demanding, but arrives at a similar outcome: a MLE of \( \Xi \) would aim at equalizing the model distribution of wages to the observed ones, so that the probability of selecting an individual from a given wage distribution (that is an ordered pair of wage rate and ranking) is maximized. In one of the robustness checks in Section \( 5 \) we discuss the results of a pseudo-MLE calibration, in which the goal function in our standard procedure is weighted by the density of observations in the data.

54 For the scalars: \( F(x_U^r) \), \( u^M \), \( S^*_U(x_M^0) \) the reference values are 0, the unemployment rate in Mexico and the number of Mexican immigrants in the US respectively. For distributions we compute Euclidean distances between quantiles of data and model distributions, including every grid point.
important in the US than in Mexico, since the Pareto parameter (the relative elasticity of surplus to firm’s input) $\gamma$ is visibly higher in the US. We find an almost identical relative number of potential firms in the US than in Mexico ($R^F_U/R^W_U \approx R^F_M$). Interestingly enough, our best calibration returns a rather low value of the copula parameter $\theta$. Its value close to 1 indicates that American and Mexican skills are weakly related with an average rank correlation of 0.32. A good fit of our model requires a far cut in the distribution of skill input into surplus. $t_1 = 3.59$ is significantly higher than $t_0 = 3.09$ used for the truncation of the empirical distributions of wages (equivalent to cutting 0.5% from both tails). The same can be said about firms’ productivity distribution with $t_2 = 0.98$. Migration costs take values in expected ranges: the multiplicative one equals $1 - \delta_1 = 71.8\%$ of migrant’s wage in the US, while the additive one is close to zero. A technical parameter $x^*_1$ shows that the skill threshold after which Mexicans always choose to emigrate is very high.

Table C.1: Calibrated values of parameters

<table>
<thead>
<tr>
<th>American market</th>
<th>Mexico market</th>
<th>Common parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1 = 19,247.09$</td>
<td>$k_2 = 6,546.10$</td>
<td>$\theta = 0.963$</td>
</tr>
<tr>
<td>$s_1 = 0.546$</td>
<td>$s_2 = 0.501$</td>
<td>$x^*_1 = 0.99994$</td>
</tr>
<tr>
<td>$\gamma_1 = 0.219$</td>
<td>$\gamma_2 = 0.092$</td>
<td>$\delta_0 = -103.73$</td>
</tr>
<tr>
<td>$R^F_U = 3.000$</td>
<td>$R^F_M = 1.216$</td>
<td>$\delta_1 = 0.282$</td>
</tr>
<tr>
<td>$t_1 = 3.591$</td>
<td></td>
<td>$t_2 = 0.980$</td>
</tr>
</tbody>
</table>

Note: Parameters of the benchmark calibration are rounded to two or three decimal points, though in the actual computations we use 13 decimal digits.

**Simulation algorithm** In counterfactual simulations we manipulate the values of additive and multiplicative migration costs. We solve for the new equilibrium, keeping the set of parameters: $\{k_i, \gamma_i, s_i\}$ for $i \in \{U, M\}$ and $\theta$, $t_1$, $t_2$, constant. $\delta_0$ and $\delta_1$ change by definition, while $x^*_i$, $X^M_U$ and $u_i, R^F_i$ are now endogenous, and calculated in the new equilibrium. Finally, the three skill distributions $S^i$ and $S^M_U$, as well as all wage schedules are updated.

The algorithm solves for the new equilibrium by iterating on all the unknowns mentioned above. Taking a first guess of the selection $S^M_U$, it recomputes all characteristics of skill and wage distributions, for the new migration costs. Then, separately, the procedure computes the mass of firms in Mexico and in the US by fixing the costs of entry, and solving for $R^F_i$. The algorithm stops when the summary deviation in all endogenous variables in consecutive steps is lower than $1/K$. 

53
Table C.2: Income tax schedules in the US and Mexico

<table>
<thead>
<tr>
<th>US tax rate</th>
<th>threshold</th>
<th>MEX tax rate</th>
<th>threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>9,225</td>
<td>1.92%</td>
<td>334.1</td>
</tr>
<tr>
<td>15%</td>
<td>37,450</td>
<td>6.4%</td>
<td>2,835</td>
</tr>
<tr>
<td>25%</td>
<td>90,750</td>
<td>10.88%</td>
<td>4,983</td>
</tr>
<tr>
<td>28%</td>
<td>189,300</td>
<td>16%</td>
<td>5,792</td>
</tr>
<tr>
<td>33%</td>
<td>411,500</td>
<td>17.92%</td>
<td>6,935</td>
</tr>
<tr>
<td>35%</td>
<td>413,200</td>
<td>21.36%</td>
<td>13,987</td>
</tr>
<tr>
<td>39.6%</td>
<td>-</td>
<td>23.55%</td>
<td>22,046</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>42,090</td>
<td></td>
</tr>
<tr>
<td></td>
<td>32%</td>
<td>56,121</td>
<td></td>
</tr>
<tr>
<td></td>
<td>34%</td>
<td>168,363</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35%</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Corporate tax rate (flat):

|            | 35%       | 30%       |

Note: Thresholds are in 2015 PPP USD. source: OECD.

Table C.3: Calibrated trade costs

<table>
<thead>
<tr>
<th>from:\to</th>
<th>ROW</th>
<th>MEX</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW</td>
<td>1.00</td>
<td>2.78</td>
<td>6.15</td>
</tr>
<tr>
<td>MEX</td>
<td>1.72</td>
<td>1.00</td>
<td>3.89</td>
</tr>
<tr>
<td>USA</td>
<td>0.35</td>
<td>0.41</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Appendix D - robustness checks

Graphical evaluation of all robustness checks investigated in Section 5.
Figure C.1: Robustness checks

Figure C.1a investigates the welfare effects of closing the Mexico-US border for American natives assuming different distributions of skills in the population of unemployed Americans. The reference scenario (black) assumes a linear CDF, the convex scenario (blue) assumes exponential CDF, while the concave scenario (red) assumes logarithmic CDF. Figure C.1b plots the welfare effects for 40 best parametrisations generated by the calibration algorithm. Figure C.1c (C.1d) analyses the results assuming alternative values for the elasticity of substitution between varieties (black benchmark: \( \varepsilon = 5 \), red: \( \varepsilon = 7 \), blue: \( \varepsilon = 9 \), green: \( \varepsilon = 4 \), orange: \( \varepsilon = 3 \) ) Figure C.1e (C.1f) presents the relative (absolute) welfare effects of closing the border in an upgraded model with illegal immigrants.