Uncertain Fiscal Consolidations

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Abstract

In a non-linear New Keynesian economy, we explore the macro-economic consequences of undertaking state-dependent fiscal consolidations of uncertain timing and composition. Following the empirical evidence in Alesina and Ardagna (2010), we place particular emphasis on whether or not the fiscal consolidation is driven by tax rises or expenditure cuts. We find that the composition of the fiscal consolidation, its duration, the monetary policy stance, the level of government debt, the degree of price stickiness, and expectations over the likelihood and composition of fiscal consolidations all matter in determining the extent to which a given consolidation is expansionary and/or successful.

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1 Introduction

The financial crisis of 2007-9 has left the advanced economies with average levels of gross government debt relative to GDP breaching the 100% level for the first time since the aftermath of World War II (IMF (2011)). As a result, the IMF expects most governments of such economies, with the notable exception of Japan and the United States, to begin consolidation efforts by 2012. The expected pace of consolidation is particularly rapid in those economies subject to pressures in the financial markets due to worries over fiscal sustainability, see IMF (2011). Moreover, politicians in some economies, such as the UK, cite the need to avoid the possibility of rising debt costs as a key motivation in undertaking fiscal consolidations even in the absence of any current significant risk premia on government debt. It therefore appears to be the case that the dominant medium-term fiscal trend in such economies is the need to return to a position of fiscal sustainability, particularly when prompted to do so through rising costs of servicing government debt possibly due to fears of default/restructuring.

Conventional Keynesian analysis suggests that fiscal consolidations inevitably lead to a contraction in aggregate demand, and thereby lost output. However, following Giavazzi and Pagano (1990)'s analysis of fiscal consolidations in Denmark and Ireland in the 1980s, it appeared that such fiscal actions could be expansionary, since output growth actually accelerated following the fiscal tightening. A large volume of subsequent empirical work\(^1\) has considered a wider set of countries over a wider time period and has also found some evidence that fiscal consolidations can potentially be expansionary. The literature does not always fully agree on the relative importance of different factors in determining whether or not a consolidation will be either expansionary and/or successful (in the sense of achieving a sustained reduction in the government debt to GDP ratio). Nevertheless, it appears that the persistence and composition of the consolidation matters, with cuts to government spending being thought to be pro-growth relative to tax increases (see, for example, Alesina and Perotti (1995), Perotti (1996), Alesina and Ardagna (1998, 2010), and Ardagna (2004)). Other factors, which are considered by the literature, include the size of consolidation (Giavazzi and Pagano (1996) and Strauch and Von Hagen (2001)), the state of the public finances at the time of the consolidation (see, for example, Giavazzi, Jappelli, and Pagano (2000)) and, the state of the macroeconomy at the time of the consolidation (Perotti (1999), Alesina, Ardagna, and Trebbi (2006), Guichard, Kennedy, Wurzel, and Andre (2007)), whether or not monetary and/or exchange rate policy is accommodative at the time of the consolidation (Strauch and Von Hagen (2001), Ardagna (2004), Lamberti and Tavares (2005)), the nature of the political institutions undertaking the consolidation (Alesina (2000), Alesina, Ardagna, and Tavares (2005)), and the state of the macroeconomy at the time of the consolidation (Perotti (1999), Alesina, Ardagna, and Trebbi (2006), Guichard, Kennedy, Wurzel, and Andre (2007)), whether or not monetary and/or exchange rate policy is accommodative at the time of the consolidation (Strauch and Von Hagen (2001), Ardagna (2004), Lamberti and Tavares (2005)), the nature of the political institutions undertaking the consolidation (Alesina (2000), Alesina, Ardagna, and Trebbi (2006), Guichard, Kennedy, Wurzel, and Andre (2007)), whether or not monetary and/or exchange rate policy is accommodative at the time of the consolidation (Strauch and Von Hagen (2001), Ardagna (2004), Lamberti and Tavares (2005)), the nature of the political institutions undertaking the consolidation (Alesina (2000), Alesina, Ardagna, and Tavares (2005)) for a survey of the literature, see Briotti (2005).
and Trebbi (2006)). However, it is important to note that in many cases where one study finds a conditioning variable to be significant, another study will not - for example, while Lambertini and Tavares (2005) find that accompanying exchange rate devaluations help ensure fiscal consolidations are successful, Ardagna (2004) does not, and where Alesina and Ardagna (2010) find that the composition of consolidations affects both how expansionary and successful a consolidation is, Ardagna (2004) argues that composition does not matter for success. Our non-linear model can help explain these conflicting results in that we find significant non-linear interactions between debt levels, the degree of price stickiness, the monetary policy stance and the compositions of consolidations which are unlikely to be controlled for by adding individual variables to linear regressions or sorting samples according to a single variable.

While much of the literature on fiscal consolidations is empirical, the implicit theoretical mechanisms underpinning the results are discussed. Briotti (2005) identifies three conditions under which we could achieve an expansionary fiscal consolidation in a simple neo-classical setting. Firstly, households must undertake intertemporal consumption smoothing, such that their current consumption depends not just on current income, but expected future income. Secondly, taxes must be distortionary such that a timely consolidation implies that households no longer fear a larger and more costly consolidation in the future. This raises expected income and fuels the expansion. Thirdly, for such a consolidation to be expansionary, it must be unexpected/uncertain otherwise there would be no sense of relief that the consolidation was actually underway. However in order to motivate the need to condition regressions on the state of the economy, the size of the consolidation and the state of the public finances, the literature also often suggests that such variables provide additional information about future policies. For example, Bertola and Drazen (1993) develop a model where government spending is inherently unsustainable, but the government will satisfy its intertemporal budget constraint by periodically undertaking fiscal consolidations through spending cuts. These consolidations may occur at a low threshold, but if not, they will definitely occur at a second higher threshold. The current state, therefore, provides information about the likelihood of consolidations, and we may observe an expansion following a worsening fiscal position as this raises the probability that we shall shortly enter a period of beneficial fiscal correction. Similarly, Sutherland (1997) suggests that there will be non-linearities in the economic impact of fiscal policy associated with the level of government debt. At high debt levels fiscal consolidation is imminent, and in an economy populated with finitely-lived Blanchard-Yaari consumers this means that current generations will bear the tax costs of the consolidation, while at lower debt levels consolidation is delayed and future generations are more likely to bear the costs of future fiscal corrections. Therefore a given deficit financed tax cut may
have quite different consequences at different levels of debt.\footnote{Alesina and Perotti (1997) also argue that the response to changes in tax rates may be quite different depending on the extent and nature of union wage bargaining in an economy. Although our economy does not contain such distortions, there are non-linearities associated with Laffer curve effects arising from the distortionary taxation of labor income.}

In this paper we develop a non-linear DSGE model where fiscal consolidations occur with increasing probability as government debt levels rise. While the probability of consolidation is rising in debt levels, the exact timing of the consolidation is uncertain. This is consistent with the empirical observation that sizeable consolidations can take place at low as well as high debt/gdp ratios. We further introduce uncertainty over the composition of the fiscal consolidation. To do so we analyse the dataset of Alesina and Ardagna (2010) who define fiscal consolidations within OECD economies between 1970 and 2007 as either being expansionary or contractionary. Within each type of consolidation we compute the mean of the changes in government spending, fiscal transfers and tax revenues (all relative to GDP) during the course of a fiscal consolidation. In other words we have an empirical measure of two typical types of consolidation, which reveals that ‘expansionary’ fiscal consolidations are typically based on cuts in government spending, while ‘contractionary’ consolidations usually rely on tax increases. Using the relative frequency of ‘expansionary’ and ‘contractionary’ consolidations observed in the data, we assume that when a fiscal consolidation is undertaken it is either tax or expenditure based. This allows us to assess which type of consolidation turns out to be expansionary and which contractionary, and whether it is the resolution of the uncertainty associated with the timing or composition of fiscal consolidations that matters most in determining whether or not it is expansionary and/or successful.

The plan of the paper is as follows. Below we discuss the empirical evidence in Alesina and Ardagna (2010) describing the nature of large-scale fiscal consolidations. Section 3 then outlines our New Keynesian model, monetary and fiscal policies and the range of state-dependent fiscal consolidations that may occur. Section 4 describes the fiscal limit which determines the state-dependent probability of observing a fiscal consolidation. In Section 5 we describe the calibration and solution for our non-linear model, before considering a wide range of fiscal consolidations in Sections 6 and 7. We reach our conclusions in Section 8.

## 2 Fiscal Consolidations Data

In this section, we outline the features of the dataset on fiscal consolidations constructed by Alesina and Ardagna (2010) (henceforth AA).

AA undertake an empirical analysis of episodes of fiscal stimulas (rise in deficit/fall in surplus) and fiscal adjustment (fall in deficit/rise in surplus) of more than 1.5% of GDP, where
the data is cyclically adjusted. Each such episode is then classified as being ‘expansionary’ if GDP growth in the two years following the stimulus/adjustment is greater than the 75th percentile of the same variable empirical density in all episodes of fiscal stimulus/adjustment. They also define a ‘successful’ fiscal adjustment as being where the debt/GDP ratio falls by 4.5% three years later. Using data for a sample of developed OECD economies between 1970 and 2007, there are 107 episodes of fiscal adjustment amounting to 15.1% of the observations in their sample. Of these 107 episodes, 65 last for one period/year, 13 last two years, 4 last three years and 1 fiscal adjustment episode lasts for four years. While 26 years of fiscal consolidation fulfill AA’s definition of expansionary fiscal consolidations, such that one in four fiscal consolidations are deemed expansionary. We use these latter two observations to calibrate both the typical length of time spent in an episode of consolidation and the relative frequency of consolidations that AA would label ‘expansionary’ and ‘contractionary’, respectively.

We then follow AA to compute the average change in key fiscal variables in the two years following a fiscal consolidation relative to the two years prior to the fiscal adjustment. Our numbers differ slightly from those in AA as we exclude consolidations where we do not have observations either prior to, or following the episode. We do this as we wish to assess the statistical significance of the changes in fiscal variables over the course of a consolidation episode. Table 1 details the average change in fiscal variables under both types of consolidation, where all variables are measured relative to output.

Table 1 reveals some quite striking differences between episodes which meet AA’s definitions of expansionary and contractionary, respectively. Expansionary consolidations feature a statistically significant fall in government spending of 2.19% of GDP and subsidies of 0.32%, and a statistically insignificant rise in tax revenues of 0.35% and fall in transfers of 0.58% of GDP. There are also statistically significant falls in public investment and public-sector wages, of 0.76% and 0.4%, respectively, although these tend to be similar across both types of consolidation. In contrast, contractionary consolidations have a far smaller fall in government spending of only 0.8%, as well as a statistically significant rise in tax revenues transfers of 1.11% and 0.47%, respectively. In other words the fiscal consolidations which meet the definition of expansionary in AA’s dataset are driven by spending cuts with no significant increases in aggregate tax revenues, while the contractionary episodes are far more heavily dependent on increases in taxation, particularly business and indirect taxes.

Given the significant differences across consolidation regimes apply to aggregate spending and taxation, we focus on these fiscal variables in our theoretical model below.
3 Model

Our aim is to explore the macroeconomic consequences of uncertain fiscal consolidations. Since debt service costs are particularly important in determining debt dynamics at high debt levels, we consciously use a conventional new Keynesian model of the kind typically used to explore monetary and fiscal policy interactions, modified by allowing explosive lump-sum transfers and occasional fiscal consolidations which are uncertain in both their timing and composition.

Households in our economy supply labor to imperfectly competitive intermediate goods producing firms who do not completely adjust prices in the face of shocks since they face costly Rotemberg-style price adjustment. Moreover, rather than rendering fiscal policy redundant by balancing the budget through lump-sum taxes, we assume that households’ labor and profit income is taxed. This influences their labor supply decisions, which in turn affects firms’ marginal costs and pricing decisions. Taken together, this implies a relatively rich set of monetary and fiscal policy interactions: monetary policy has real effects due to the assumption of price stickiness, which in turn affects both the size of the tax base and real debt service costs. While fiscal policy, in the form of tax or government spending changes have the obvious fiscal consequences, but also influence inflation either through the labor supply response to distortionary taxation or the aggregate demand effect of changes in government spending.

In our model fiscal consolidations are triggered after debt rises to a level which breaches a ‘fiscal limit’. The upper bound of the fiscal limit is the maximum level of debt the government is able to support, where the government’s ability to sustain a given level of debt depends upon the nature of the tax Laffer curve it faces and the shocks it may experience, such that it is stochastic. However, it is anticipated that governments will attempt to stabilise debt through fiscal consolidations in advance of reaching this upper bound, although due to political factors such as a war of attrition over who bears the costs of a particular consolidation they may leave such a consolidation to the last minute (see the empirical evidence in Alesina, Ardagna, and Trebbi (2006) who find that political factors do appear to play a significant role in determining when a consolidation is instigated in a manner consistent with war of attrition type effects). Consistent with this evidence, although we do not formally model the political decision making process, the probability of a fiscal consolidation is rising in the level of government debt.
3.1 HOUSEHOLDS  Our cashless economy is populated by a large number of identical households of size 1, who have preferences given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

where $$\beta \in (0, 1)$$ is the households’ subjective discount factor, $$c_t$$ is consumption and $$n_t$$ the households’ labor supply. The household receives nominal wages $$W_t$$ and monopoly profits $$\Upsilon_t$$ from the firm, both of which are taxed at the rate, $$\tau_t$$, and lump-sum transfers $$z_t$$ from the government. The household chooses consumption, $$c_t$$, hours worked, $$n_t$$, and nominal bond holdings, $$B_t$$, to maximize utility subject to their budget constraint,

$$P_t c_t + \frac{B_t}{R_t} = B_{t-1} + (1 - \tau_t) (W_t n_t + P_t \Upsilon_t) + P_t z_t$$ (1)

The budget constraint can be written as in real term as

$$c_t + \frac{b_t}{R_t} = \frac{b_{t-1}}{\pi_t} + (1 - \tau_t) (w_t n_t + \Upsilon_t) + z_t$$ (2)

where $$w_t \equiv W_t/P_t$$ is the real wage. The first condition describes the household’s optimal allocation of consumption over time, and the second, their optimal labor supply decision. Notice in the case of the latter, labor income is taxed so that changes in the tax rate will influence households’ desire to work.

$$\frac{1}{R_t} = \beta E_t \frac{u_c(t+1)}{u_c(t)} \frac{1}{\pi_{t+1}}$$ (3)

$$-\frac{u_n(t)}{u_c(t)} = w_t (1 - \tau_t)$$ (4)

3.2 FINAL GOOD PRODUCTION  Final goods production is for the purposes of private and public consumption and competitive final goods firms buy the differentiated products produced by intermediate goods producers in order to construct consumption aggregates, which have the usual CES form,

$$Y_t = \left( \int_0^1 y_t(i) \frac{2}{\sigma} di \right)^{1/\theta}$$ (5)

where $$Y_t$$ is aggregate output, $$y_t(i)$$ the output of intermediate good firm i, and $$\theta > 1$$ is the elasticity of demand for each firm’s product. Cost minimization on the part of final goods
producers results in the following demand curve for intermediate good $i$,

$$ y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t $$

and an associated price index for final goods,

$$ P_t = \left( \int_0^1 p_t(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}} $$

### 3.3 Intermediate Goods Production

The imperfectly competitive intermediate goods firms enjoy some monopoly power in producing a differentiated product such that they face a downward sloping demand curve, but are also subject to Rotemberg quadratic-adjustment costs such that large price changes in excess of steady-state inflation rates are particularly costly. The quadratic price adjustment costs renders the firm’s problem dynamic,

$$ \max_{\pi_t} \sum_{t=0}^{\infty} R_{0,t} \left( p_t(i)y_t(i) - mc_tP_t y_t(i) - \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \pi_t - 1 \right)^2 P_t Y_t \right) $$

$$ s.t. \quad y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t $$

where $mc_t = w_t/A_t$ is the real marginal cost implied by a linear production function, $y_t(i) = A_t n_t(i)$. Productivity, $A_t$ is common to all firms and follows an $AR(1)$ process:

$$ A_t - A = \rho^A(A_{t-1} - A) + \varepsilon_t^A \quad \varepsilon_t^A \sim i.i.d. \mathcal{N}(0, \sigma^2_A) $$

The first-order condition, after imposing symmetry across firms, is,

$$ (1 - \theta) + \theta mc_t - \phi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} + \beta \phi E_t \left( \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1} Y_{t+1}}{Y_t} \right) = 0. $$

which represents the non-linear New Keynesian Phillips curve (NKPC) under Rotemberg pricing and which would, upon linearization, correspond to the standard NKPC under Calvo (1983) pricing.

The associated monopoly profit, which is taxed by the government when received by households, is,

$$ \Upsilon_t = Y_t - mc_t Y_t - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t. $$
The aggregate resource constraint is,

\[ c_t + g_t = A_t n_t \left( 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 \right). \]  

\[ 13 \]

3.4 Monetary and Fiscal Policy Combining the households’ budget constraints and noting the equivalence between factor incomes and national output allows us to derive the government budget constraint:

\[ \frac{B_t}{R_t} + \tau_t (W_t n_t + P_t \Upsilon_t) = B_{t-1} + P_t g_t + P_t \zeta_t \]  

where we see that while fiscal policy in the form of tax, transfers and government spending changes will obviously affect debt dynamics, monetary policy will also have a role to play, especially when debt stocks are large. The government’s budget constraint can be rewritten as:

\[ \frac{b_{t-1}}{\pi_t} = \frac{b_t}{R_t} + T_t - g_t - \zeta_t \]

Monetary policy: We assume that monetary policy follows a simple rule of the form,

\[ R_t - R = \alpha(\pi_t - \pi) \]  

(15)

Fiscal policy outside of fiscal consolidations: Before considering what happens to fiscal policy variables during consolidation episodes, we describe what happens to those variables outside of consolidations. We allow fiscal transfers to depend on a regime-switching index \( r_s^z_t \),

\[ z(r_s^z_t) = \begin{cases} 
(1 - \rho^z)z + \rho^z z_{t-1} + \epsilon^z_t & \text{if } r_s^z_t = 1 (\rho^z < 1) \\
\zeta^z z_{t-1} + \epsilon^z_t & \text{if } r_s^z_t = 2 (\zeta^z > 1)
\end{cases} \]

with \( \epsilon^z_t \sim i.i.d. \mathcal{N}(0, \sigma^2) \) and \( r_s^z_t \) following a transition matrix of \( \begin{pmatrix} p_1^z & 1 - p_1^z \\ 1 - p_2^z & p_2^z \end{pmatrix} \).

Within the Markov regime-switching process we move from a stationary process for transfers with \( \rho^z < 1 \) to one where transfers explode with \( \zeta^z > 1 \). Therefore, although transfers will ultimately be stabilized, there can be prolonged periods during which transfers increase leading to sustained increases in government debt which can prompt attempts at fiscal consolidation. Such localized instability in transfers is common to many advanced economies. Figure 2 and 3 illustrate that the transfers-GDP ratios during the past 40 years are stable in some countries but not in others.
Outside of consolidations, the government spending, $g_t$, follows an AR(1) process:

$$g_t - g = \rho^g(g_{t-1} - g) + \varepsilon^g_t \quad \varepsilon^g_t \sim \text{i.i.d.} \mathcal{N}(0, \sigma^2_g)$$

(16)

and tax rates are adjusted to stabilize government debt,

$$\tau_t - \tau = \gamma^\tau(b_{t-1} - b)$$

**Fiscal policy during fiscal consolidations:** Fiscal consolidations are then considered in the form of non-zero values for $m^g_t$ and $m^\tau_t$, implying reductions in government spending and increases in taxation relative to the values implied by the exogenous processes for spending and the tax rule.

$$g_t - g = m^g_t + \rho^g(g_{t-1} - g) \quad (17)$$

$$\tau_t - \tau = m^\tau_t + \gamma^\tau(b_{t-1} - b) \quad (18)$$

There are two layers of uncertainty about the fiscal consolidations: when will it occur and what form will it take? The probability of beginning a consolidation depends on the current level of government debt $b_{t-1}$ and a stochastic fiscal limit $b^*_t$. Households know the distribution of fiscal limit, but the timing of consolidations is uncertain due to the stochastic realized fiscal limit. At period $t$, if the existing government liability is lower than the realized fiscal limit, the government doesn’t take consolidations; otherwise, consolidations, either through tax increases or spending cuts, occur and last for one year, in line with the majority of consolidations observed in AA data. Whether consolidations are taken through tax or spending is random, governed by the parameter $\omega$.

$$\begin{cases} 
\text{if } b_{t-1} < b^*_t: & \text{no FC} \\
\text{otherwise, with prob of } \omega: & \text{tax-type FC (} m^\tau > 0) \\
\text{with prob of } 1 - \omega: & \text{spending-type FC (} m^g > 0) 
\end{cases}$$

As shown in Figure 5, a state variable $rs_t$ tracks the path of consolidations. The probability that $rs_t > 1$ such that we begin a consolidation depends on the distribution of the fiscal limit and the current level of government debt $\Phi(b_{t-1})$ ($\Phi' > 0$).

### 4 Fiscal Limit

Laffer curves provide a natural starting point for quantifying the fiscal limit from the tax revenue side of the government’s budget constraint. At the peak of the Laffer curve tax
revenues reach their maximum and, given some level of total government expenditures, the expected present value of primary surpluses and, therefore, the value of government debt, are maximized. Revenues, expenditures, and discount rates, of course, vary with the shocks hitting the economy, generating a distribution for the maximum debt-GDP level that can be supported. We refer to this as the distribution of the fiscal limit. This section describes more precisely how we derive that distribution.

4.1 Laffer Curve

Assume the utility function is \( u(c_t, n_t) = \log c_t + \chi_N \log(1 - n_t) \). Labor supply can be solved analytically as a function of \((\tau_t, \pi_t, A_t, g_t)\) using the first-order conditions. Work effort is given by

\[
n_t = \frac{w_t X_{1,t} + \chi n g_t}{w_t X_{1,t} + \chi n X_{2,t}} \tag{19}
\]

with

\[
X_{1,t} = 1 - \tau_t \tag{20}
\]

\[
X_{2,t} = A_t \left(1 - \frac{\phi}{2} \left(\frac{\pi_t}{\pi} - 1\right)^2\right) \tag{21}
\]

Total tax revenue is

\[
T_t = (w_t n_t + \Upsilon_t) \tau_t = A_t n_t \tau_t \left(1 - \frac{\phi}{2} \left(\frac{\pi_t}{\pi} - 1\right)^2\right). \tag{22}
\]

When the monetary authority keeps the inflation rate at its target \((\pi_t = \pi)\), the peak of the Laffer curve is a function only of the exogenous state of the economy \((A_t, g_t)\).

\[
\tau_t^{\text{max}} = \tau^{\text{max}}(A_t, g_t) \tag{23}
\]

\[
T_t^{\text{max}} = T^{\text{max}}(A_t, g_t) \tag{24}
\]

Evidently, the stochastic processes governing the exogenous states induce stochastic processes for both the tax rate that maximizes revenues and the level of revenues.

4.2 Distribution of the Fiscal Limit

The fiscal limit is defined, following Bi (2011), as the maximum expected present value of future primary surpluses.

\[
B^* = E \sum_{t=0}^{\infty} \beta^t \beta_p \frac{u^\text{max}_c(A_t, g_t)}{u^\text{max}_c(A_0, g_0)} \left(T^{\text{max}}(A_t, g_t) - g_t - z_t\right) \tag{25}
\]
Calculation of the fiscal limit uses the stochastic discount factor that obtains when tax rates are at the peak of the Laffer curve, $\beta^t u^c_{\max}(A_t, g_t)/u^c_{\max}(A_0, g_0)$, but modified to allow for a political risk parameter $\beta_p$.

Higher political risk—lower $\beta_p$—lends itself to multiple interpretations that reflect the private sector’s beliefs about policy. Most straightforward is the idea that policymakers are believed to have effectively shorter planning horizons than the private sector [see, for example, Acemoglu, Golosov, and Tsyvinski (2008)]. To see this, rewrite the discount factor in (25) as $(\beta_p^p \beta^t)/(\beta_p^p)^{t-1}$, so that a lower value of $\beta_p$ reduces the present value of maximum surpluses. An alternative interpretation is that a lower $\beta_p$ implies that private agents place probability mass on both the maximum surpluses, $s^\max$ reflected in (25), and on surpluses being zero. Rewrite the surpluses as $\beta_p s^\max + (1-\beta_p) \cdot 0$ for this interpretation. Nothing we do hinges on the precise interpretation attached to $\beta_p$. As a practical matter, setting $\beta_p < 1$ serves to shift down the distribution of the fiscal limit, which generates occurrences of fiscal consolidations at lower levels of debt like those observed in data.

Since there exists a unique mapping between the exogenous state space, $(A_t, g_t)$, to $\tau^\max_t$ and $T^\max_t$, the unconditional distribution of the fiscal limit, $f(B^*)$, can be derived from a Markov Chain Monte Carlo simulation following the steps that appendix A describes.

The choice of $b_t^*$, which we treat as random, is determined by political considerations that are driven by the policymakers’ assessments of the costs associated with implementing fiscal consolidations.

5 Calibration and Solution

Calibration: The model is calibrated at a quarterly frequency. The household discount rate is 0.99 and the net real interest rate is 4.04 percent at annual rate. The utility function is assumed to be $u(c, L) = \log c + \chi_n \log(1 - n)$. The leisure preference parameter, $\chi_n$, is calibrated in such a way that the household spends 25 percent of its time working and the Frisch elasticity of labor supply is 3. Time endowment and the productivity level at the steady state are normalized to 1.

Table 2 summarizes the calibration. Parameterizations of the shock processes for $A_t$ and $g_t$ follow the literature.\(^3\) The price elasticity of demand, $\theta$, is assumed to be 11 and the Rotemberg adjustment parameter, $\phi$, is 100, which is equivalent to Calvo-type overlapping contracts models where 26.7 percent of the firms reoptimize each quarter [see Keen and Wang (2007)]. The gross inflation rate is calibrated to 1.03 at annual rate and the Taylor rule parameter is assumed to be 1.5.

\(^3\)For instance, Schmitt-Grohe and Uribe (2007) assume $\rho_A$ to be 0.8556, $\sigma_A$ to be 0.0064, $\rho_g$ to be 0.87, and $\sigma_g$ to be 0.016g.
The International Country Risk Guide’s (ICRG) index of political risk offers one way to calibrate the political factor, \( \beta_p \) [see Arteta and Galina (2008)]. The ICRG index of political risk for Greece stayed at the level of 60 (out of 100) during the period between 1984 and 1993, and then rose above the level of 80 between 1994 and 1996 before coming down in the recent financial crisis. We calibrate \( \beta_p \) to the pre-EMU ICRG level in Greece.

The fiscal parameters are roughly calibrated to match Greek data from 1971 to 2007. In steady state, government purchases are 16.7% of GDP and lump-sum transfers are 13.34% of GDP. The tax rate is 0.315 at the steady state, and the resulting government debt is 35.26% of GDP at an annual rate. The tax adjustment parameter, \( \gamma \), is calibrated to 0.5 at annual rate, which is roughly consistent with estimates.

For the countries with unstable transfers, shown in Figure 2, the transfer growth varies from 1.2% in Italy to 3.2% in Japan at annual rate. \( \zeta_z \) is calibrated to the transfer growth in Greece since 1970, 2.6% at annual rate. The regime-switching parameters \( p_{z1} \) and \( p_{z2} \) are calibrated to 0.95, implying that the average length of each regime is 5 years. A higher \( p_z \) leads to a more dispersed distribution of fiscal limits.

Under the calibration in table 2, the simulated distribution of the fiscal limit is shown in the top plot of figure 1, with the middle plot being the estimated distribution. The fat tail is generated by the possible explosive transfers. If the political risk factor, \( \beta_p \), were constant and equal to unity, the distribution of the fiscal limit would be centered at about 300 percent of GDP.

As discussed in section 2, the length of consolidations \( h \) in our model is calibrated to one year, while the size of consolidations \( m^r \) and \( m^g \) are calibrated to 1% of the steady-state level of GDP.

Solution: We solve the full non-linear model laid out in section 3, coupled with the fiscal limit described in section 4, using the monotone map method, which discretizes the state space and finds fixed points in the space of decision rules.

The solution method, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the system to a set of expectation first-order difference equations. In this model, the decision rule maps the state at period \( t \) into the stock of government debt, the real wage, and the inflation rate in the same period. Given the state denoted as \( \psi_t = \{ b_{t-1}, g_t, z_t, r_s_t, r_s^z_t \} \), the mappings can be written as \( b_t = f^b(\psi_t), w_t = f^w(\psi_t), \pi_t = \frac{\pi_{t+1}}{\pi_t} \).

---

4Linear regression of the tax rate on the government debt-GDP ratio from 1971 to 1995 is 0.42, while the debt-GDP ratio is almost flat from 1995 to 2007.

5Calibrating \( m^r \) and \( m^g \) to match AA data doesn’t change the main results, but may complicate identifying the relative efficacy of the different types of fiscal consolidation if they have a different scale.
The conjectured rules can be substituted into the non-linear system, in which the expectation terms are evaluated using a numerical quadrature. The model is solved for each set of state variables defined over a discrete partition of the state space. The decision rules are updated at every node of the state space. The procedure is repeated until the iterations update the current decision rules by less than some \( \epsilon > 0 \). After finding the decision rules, we can solve the pricing rule \( (q_t = f^a(\psi_t)) \) using the government budget constraint. The interest rate on government bonds can also be solved using \( R_t = \frac{1}{q_t} \), denoted as \( f^R(\psi_t) \).

### 6 Fiscal consolidation with only time uncertainty

As described in section 2, fiscal consolidations can occur across a wide range of debt/gdp ratios, but we anticipate that the probability of observing a fiscal consolidation is rising in the debt/gdp ratio. Therefore, consolidations at low debt levels are more likely to be something of a surprise, than the consolidations that follow sustained increases in debt. We also know from the AA data that large scale fiscal consolidations typically last for one year. Accordingly, expectations over both the likelihood of a consolidation and its duration may affect the impact of a given consolidation.

We begin by exploring the importance of uncertainty over the timing and duration of fiscal consolidations. Tax-type FC \( (RS^\tau) \) and spending-type FC \( (RS^g) \) are specified as,

\[
RS^\tau : \quad \tau_t - \tau = m^\tau(rs_t) + \gamma(b_{t-1} - b) \\
RS^g : \quad g_t - g = -m^g(rs_t)
\]

As illustrated in Figure 4, the fiscal consolidation measurements, \( m^\tau \) and \( m^g \), depend on the state-dependent variable \( rs_t \), which in turn hinges on the government liability \( b_{t-1} \) and the stochastic fiscal limit \( b^*_t \).

\[
\begin{align*}
&\begin{cases}
\text{if } rs_t = 1: & \text{No FC, } m^\tau_t = 0, m^g_t = 0 \\
\text{if } h + 1 \geq rs_t \geq 1 & \text{either } RS^\tau \text{ or } RS^g \text{ FC}
\end{cases}
\end{align*}
\]

We therefore contrast the impact of a fiscal contraction which raises tax revenues or cuts government spending by 1% of GDP for one year, when economic agents anticipate that the consolidation will be sustained for one year relative to the case where we observe the same consolidation but as a result of a series of unanticipated iid shocks. The latter case, denoted...
as no-RS model, is specified as following,

\[ \tau_t - \tau = \gamma (b_{t-1} - b) + \varepsilon_t^\tau \]
\[ g_t - g = \varepsilon_t^g \]

6.1 Tax-type fiscal consolidation: flexible price \( \phi = 0 \)

We initially assume that prices are flexible, such that the real economy, including real debt service costs, are not affected by anticipated monetary policy although any inflation surprises will drive a wedge between ex ante and ex post real interest rates.

Impulse responses when the initial debt is low (40% of GDP): Figure 6 compares the impulse responses from the no-RS and \( RS^\tau \) models when the initial level of debt is low, namely 40% of annualized GDP and prices are flexible. In the case of the \( RS^\tau \) model the tax based fiscal consolidation is known to last for four quarters should it occur. While in the no-RS variant of the model there is no expectation that the fiscal consolidation will continue, although to illustrate the properties of the model we assume that iid tax shocks are drawn which happen to mimic the duration of the \( RS^\tau \) consolidation.

In Figure 6 the probability of consolidation is the level of the variable, while all other variables compute the difference in the outcome under a fiscal consolidation relative to the outcome without consolidation, although economic agents will form rational expectations as to whether or not a consolidation will occur in the future in both cases. When prices are flexible, anticipated monetary policy has no real effects, and inflation is best thought of as jumping to the level required to achieve the nominal interest rate (via the monetary policy rule) consistent with the path for ex ante real interest rates that apply along the model’s saddlepath equilibrium. However, it should also be noted that although anticipated monetary policy does not have real effects, any inflationary surprises will affect the real value of nominal government debt by placing a wedge between the ex ante and ex post real interest rates which determine debt service costs. Such surprises take place due to uncertainties over the timing and duration of fiscal consolidations (and later, their composition) and are not the same as the price level determination seen under the Fiscal Theory of the Price Level (Leeper (1991), Sims (1994), Woodford (1994, 1995)), since monetary policy remains active, and fiscal policy passive throughout.

At such low levels of debt the probability of fiscal consolidation is very low, such that the observed consolidation comes as a surprise in both cases. In the case of a fiscal consolidation of known duration, the sustained tax rise discourages labor supply and reduces consumption. At the end of the consolidation, tax rates will return to normal levels, as will
consumption and labor supply. Anticipating the rise in consumption at the end of the fiscal consolidation, ex ante real interest rates will rise in the final period of the consolidation. During the consolidation, consumption levels are reduced but relatively flat, such that ex ante real interest rates are in line with households’ rate of time preference. The observed path for inflation ensures that the monetary policy rule is consistent with these equilibrium real interest rates. Accordingly, inflation jumps up and continues rising throughout the consolidation, before falling back to normal levels once the consolidation has passed. The initial increase in inflation reduces ex post interest rates relative to ex ante rates and helps reduce the real value of nominal debt. In the case of the $RS^\tau$ consolidation there are no further surprises and the rise in interest rates at the end of the consolidation is entirely anticipated.

In contrast, when economic agents do not anticipate the consolidation continuing beyond the initial period, they expect consumption to recover and as a result ex ante real interest rates rise. In order for this increase in the ex ante real interest rate to be consistent with the monetary policy rule and the anticipated return to normal levels of inflation next period, current inflation jumps up. However, we have imposed that the same tax shock is drawn in the next period, so that tax rates remain high, and labor supply and consumption do not recover. So in period two of the no-RS consolidation we again expect consumption to recover, so that inflation stays high to achieve the increased equilibrium ex ante real interest rate. Therefore, we have two opposing effects on debt service costs - the repeated surprise of inflation being higher than expected helps deflate the real value of debt. However, this effect is dominated by the higher ex ante real interest rates as economic agents expect the tax increase to be reversed and consumption to recover. As a result the $RS^\tau$ fiscal consolidation where economic agents realize that the fiscal consolidation is durable, is more effective in reducing the debt burden, which suggests that any uncertainty over the duration of fiscal consolidations may affect their likelihood of success.

**Impulse responses when the initial debt is high (170% of GDP):** Figure 7 does the same comparison except that the initial debt level is high. As a result in the $RS^\tau$ model economic agents now expect there to be a fiscal consolidation, although not with a probability of one. Therefore, when the consolidation is realized, the tax increase is still something of a shock and the surprise inflation reduces ex post real interest rates. However, from that point onwards, economic agents know that the consolidation will last for one year, and there will be no consolidation in the period immediately following the end of the consolidation. This explains the path in expected inflation. Although there are no inflation surprises once the consolidation has begun, the base against which we compare the consolidation implies that there are no consolidations even although economic agents expect there to be one, such that
there are negative inflation surprises in our no-consolidation benchmark. To put it another way, the fact that we know there will be no further consolidations immediately after the current consolidation means that inflationary expectations are lower than they would be relative to the case where the consolidation had not taken place, but is expected to.

However, the most striking aspect of this high debt case, is that the tax based fiscal consolidations (of either known or unknown duration) actually raise government debt relative to the case where no consolidation had taken place. The reason is that at these debt levels the higher ex ante interest rates which arise due to the anticipated recovery in consumption following a consolidation, actually raise debt service costs on the larger stock of debt so much that this dominates the increased tax revenues gained from the consolidation.

Inflation dynamics in $RS^T$ model: In order to understand the inflation dynamics and the nature of the surprises induced by fiscal consolidations, we plot the level of inflation and expected inflation in Figure 8 and 9.

In Figure 8 the top panel compares the inflation rate $\pi_t$ and the one-step ahead expected inflation $E_{t-1}\pi_t$ when the debt goes up to a low level (40% of GDP) at $t=5$ but no fiscal consolidation ever occurs. Following the debt shock, given the fiscal rule, tax rates rise with it. As the debt is successfully stabilized over time, tax rates fall and consumption rises, implying a higher ex ante real interest rate. Given the interest rate rule this implies that inflation (and its expectation) rise in the debt level. The bottom panel adds two more paths: $\pi_t$ and $E_{t-1}\pi_t$ when a fiscal consolidation occurs at $t=9$. Here when the consolidation hits, labor supply contracts and consumption falls. The anticipated recovery in consumption immediately after the consolidation, implies a rise in ex ante real interest rates, which is associated with a rise in inflation during the fiscal consolidation given the monetary policy rule. Since the consolidation was unexpected, there is an initial inflation surprise.

The dynamics are quite different if the initial debt level is high (170% of GDP at $t=5$), shown in the Figure 9. The top panel compares actual inflation, $\pi_t$ and expected inflation, $E_{t-1}\pi_t$ when no fiscal consolidation ever occurs. When the debt shock occurs, tax rates rise in line with the fiscal rule and, given debt levels are so high, economic agents are anticipating a consolidation (although no actual consolidation takes place). Since a fiscal consolidation is associated with higher inflation, inflationary expectations rise significantly following the debt shock. However when this consolidation doesn’t occur, taxes are not as high as expected and there is a negative inflation shock. When a consolidation actually does occur at $t=9$ in the lower panel, inflation rises to ensure the nominal interest rate is set appropriately in anticipation of the rise in ex ante real interest rates upon exiting the consolidation. It is also worth pointing out that upon exiting the fiscal consolidation economic agents know
that there will no further fiscal consolidation for at least one period. As a result there are no surprises either during or in the period immediately following the consolidation, while in all other periods there is a non-zero probability attached to consolidation which implies a surprise no matter whether a consolidation is realized or not.

6.2 Tax-type fiscal consolidation: Sticky price $\phi = 100$

Having explored the nature of the surprises hitting the economy when either the timing or duration of a tax-based fiscal consolidation was uncertain, but prices were flexible, we now turn to consider the sticky-price case. This is of interest as it now implies that monetary policy can have real effects, affecting both the size of the tax base and the cost of debt financing and that the inflationary consequences of future consolidations will affect forward-looking price setting behavior today.

**Impulse responses when the initial debt is low (40% of GDP):** Figure 10 compares the impulse responses from the no-RS and $RS^\tau$ models. In the latter case, once the fiscal consolidation begins economic agents know that taxes will remain high for four quarters. This discourages labor supply and raises real wages and therefore marginal costs. Sticky-price firms raise prices in anticipation of this sustained rise in marginal costs and inflation jumps up and gradually declines over the course of the consolidation. While the initial jump helps deflate the real value of government debt, the active monetary policy in a sticky-price environment raises real interest rates in response to the rise in inflation offsetting some of the debt reduction arising from the fiscal consolidation.

When the fiscal consolidation is only expected to last one period, but actually lasts for four, price-setters are surprised by the sustained increase in marginal costs and inflation does not rise by as much. As a result, the active monetary policy does not raise real interest rates by as much and the fiscal consolidation under the iid tax shocks is more effective. This is in contrast to the flex price case considered above.

**Impulse responses when the initial debt is high (170% of GDP):** In Figure 11, we also consider the marginal impact of a fiscal consolidation in a sticky-price economy but where the debt to gdp ratio is 170%, making the probability of fiscal consolidation very high. Since the consolidation was already expected, inflation is already high and consumption and labor supply are already low with the deleterious consequences on debt service costs caused by an active monetary policy. As a result when the fiscal consolidation is realized, its relative impact is not as great as it would have been if the consolidation had been unanticipated.
Inflation dynamics in $RS^\tau$ model: Figures 12 and 13 compare the inflation and one-step-ahead expected inflation when the initial debt is low or high. The comparison is similar to the model with flexible prices (Figure 8 and 9), except that inflation rate increases gradually over the FC period in the flexible price model, but decreases gradually in the sticky price model. This reflects the fact the inflation dynamics under flexible prices are driven by the need to ensure the nominal interest rate, generated by the monetary policy rule, be consistent with the equilibrium real interest rate, implied by the steady consumption increases observed as tax rates fall with stabilized debt. In contrast, in the sticky-price economy current price setting anticipates future fiscal policy, so that inflation is highest at the start of a fiscal consolidation as price-setters anticipate a prolonged period of higher marginal costs.

Output multiplier: Figure 14 shows the output multiplier, which is defined as

$$\Gamma_{t+k}^y = \frac{\sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) (\tau_{t+j}^{\text{shock}} y_{t+j}^\tau - y_{t+j}^{\text{no}})}{\sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \tau_{t+j}^{\text{shock}} y} \quad (28)$$

In other words it measures the discounted percentage change in cumulative output for one discounted unit of fiscal consolidation. It confirms that at low debt levels the tax-based fiscal consolidation of known duration ($RS^\tau$) is less expansionary than an equivalent fiscal consolidation of unanticipated iid tax increases, but these relative rankings are reversed at higher debt levels. This reflects the fact that in the base case for the high debt levels, price-setting behavior is already factoring in the probability of a sustained fiscal consolidation which raises inflation, debt service costs and has a negative impact on debt dynamics. The relative size of the fiscal multipliers captures this effect of expectations on the no consolidation base case.

6.3 Spending-type fiscal consolidation: flexible price $\phi = 0$ We now turn to consider the outcomes under government-spending based consolidations, where again these maybe of uncertain duration. We explore their impact in low and high debt environments, and under flexible and sticky prices.

Impulse responses when the initial debt is low (40% of GDP): Figure 15 compares the impulse responses from no-RS and $RS^\tau$ models when the initial level of debt is low, where again spending based consolidations are iid in nature in the no-RS case, but are known to last for one year in the $RS^\tau$ case. Once again since anticipated monetary policy has no real effects in a flexible price economy, the path for inflation is driven by the need to ensure the monetary policy rule is consistent with the equilibrium path for ex ante real interest rates. In the case where debt levels are low, any fiscal consolidation is unexpected. However, in
In the case of the \( RS^g \)-type consolidation economic agents know how long the consolidation will last and that there is at least a one period gap before a new consolidation begins. In this flex price economy the government spending cut, initiated when the consolidation begins, crowds-in private consumption. Therefore, when the consolidation ends, government spending rises and private consumption is anticipated to fall. From the consumption Euler equation, we know that the anticipated fall in consumption is associated with a reduced ex ante real interest rate. In contrast to the tax-based consolidations, this implies that inflation is falling throughout the consolidation.

When the duration of the consolidation was only expected to be one period, consumption is expected to fall after one period and ex ante real interest rates are correspondingly reduced. However, by construction and in order to facilitate exposition, we have assumed that the reduction in government consumption unexpectedly occurs for four periods in a row. This means that inflation remains low, even although it was expected to recover to normal levels implying a negative inflation shock which raises ex post real interest rates relative to ex ante rates. Nevertheless, the lower ex ante real rates throughout the no-RS consolidation dominate such that government spending based consolidations of unexpectedly long duration are more effective in reducing debt than those whose duration was known in advance. Again, this is in contrast to the relative efficacy of the two types of tax-based consolidation in a reversal of the reasoning considered above.

**Impulse responses when the initial debt is high (170% of GDP):** Figure 16 does the same comparison except that the initial debt level is high. In this case economic agents are anticipating that government spending cuts are imminent. The outcome is qualitatively similar to the low debt case, although the reduced debt interest costs have a greater impact when operating on the higher debt stock. Moreover, the base case now contains positive inflation surprises as consolidations are expected but not realized, and higher ex ante interest rates as economic agents anticipate the rise in consumption that will occur should a consolidation be realized.

6.4 **Spending-type fiscal consolidation: Sticky price \( \phi = 100 \)** We now turn to consider the same experiments, but where prices are sticky. This allows monetary policy to have real effects and implies that current inflation is influenced by expectations of future consolidations since price-setting behavior is intertemporal in nature due to desire of price-setters to avoid costly price adjustments.

**Impulse responses when the initial debt is low (40% of GDP):** Figure 17 compares the impulse responses from no-RS and \( RS^g \) models, but with sticky-prices. The re-
introduction of sticky prices enables monetary policy to have real effects. It also implies that price setters will attempt to anticipate future fiscal consolidations. As a result, when a consolidation begins and price-setters anticipate it will last for a year, inflation falls immediately and then slowly returns to normal as the period of low government spending passes. Under the active monetary policy, this reduction in inflation, results in lower real interest rates which help reduce debt service costs and maintain the size of the tax base. In contrast, when the expected duration of the consolidation was only one period, price-setters fail to anticipate the subsequent decreases in government spending such that inflation doesn’t fall by as much on impact.

As a result, the uncertainty over the duration of the consolidation serves to reduce its deflationary consequences which reduces the offsetting monetary policy response. This is also in contrast to the tax-based consolidations in a sticky-price economy, where uncertainty over the duration of the consolidation was less inflationary and so raised debt service costs by less.

**Impulse responses when the initial debt is high (170% of GDP):** In Figure 18, we contrast the two types of government spending consolidation, but where the initial debt stock is very large, namely 170% of annualized GDP. However, the outcomes are quite similar to those under lower debt levels. The noticeable differences are that there is a smaller relative increase in consumption when the consolidation is realized as households were already expecting government spending to be cut at any moment due to the high level of the debt stock. Similarly, the initial deflation is smaller as it was already factored into inflationary expectations, and the spike in inflationary expectations as the end of the consolidation period reflects the fact that in the base economic agents are expectation a deflationary consolidation even although none has occurred, while in the period following a realized consolidation they know that no consolidation will take place.

6.5 **Key message of time uncertainty:** To sum up, Figure 19 compares the output multipliers for no-RS, $RS^\tau$ and $RS^g$ models with sticky prices. The multiplier under a expenditure based fiscal consolidation/shock is defined as,

$$
\Gamma^y_{t+k} = \frac{\sum_{j=0}^{k} \left( \prod_{i=0}^{j-1} r_{t+i}^{-1} \right) \left( y_{t+j}^{shock} - y_{t+j}^{no} \right)}{\sum_{j=0}^{k} \left( \prod_{i=0}^{j-1} r_{t+i}^{-1} \right) \left( -\gamma^g_{t+j} \right)}
$$

We can see that with a low initial debt level, the tax and government spending consolidations, which are not expected to last, provide upper and lower bounds, respectively, for the same consolidations of known duration. This is for the reasons discussed above: not
anticipating the duration of the consolidation limits the inflationary (deflationary) response to the tax (spending)-based fiscal consolidation which, in turn, affects the extent to which monetary policy raises (reduces) real interest rates during the consolidation. In contrast, at high initial debt levels, tax based consolidations of known duration outperform those of uncertain duration, while government spending-based consolidations performs in a similar way regardless of the extent to which the duration of the consolidation is uncertain. The expansionary effect from $RS^\tau$ is due to the fact that the tax increase today can reduce the probability of future consolidations which have negative effects on current debt service costs and tax base due to expectations effects.

7 Fiscal consolidation with time and composition uncertainty

Now we consider the time and composition uncertainty jointly: in the $RS$ model,

$$\tau_t - \tau = m^\tau(rs_t) + \gamma(b_{t-1} - b)$$
$$g_t - g = -m^g(rs_t)$$

As illustrated in Figure 5, the fiscal consolidation measurements depend on the state-dependent variable $rs_t$, which in turn hinges on the government liability $b_{t-1}$ and the stochastic fiscal limit $b^*_t$.

$$\begin{cases} 
\text{if } b_{t-1} < b^*_t: & \text{no FC} \ (m^\tau_t = 0, m^g = 0) \\
\text{otherwise, with prob of } \omega: & \tau-\text{type FC} \ (m^\tau_t = m^\tau, m^g = 0) \\
\text{with prob of } 1 - \omega: & g-\text{type FC} \ (m^\tau_t = 0, m^g = m^g) 
\end{cases}$$

The parameterizations are as follows: $m^\tau = 0.01, m^g = 0.01y$, which implies that the magnitude of the tax and government spending based consolidations amount to 1% of GDP. As noted above we also calibrate the probability that the consolidation is tax based to be $\omega = 0.75$ which is in line with the AA data. However, we explore how variations in this probability can influence the impact of alternative types of consolidation.

7.1 Benchmark case: $\alpha = 1.5$ and $\omega = 0.75$ We first calibrate $\omega$ to the data, implying the probability of having a tax-based ($\tau$-type) fiscal consolidation is three-times as high as a government expenditure ($g$-type) fiscal consolidation.

Impulse responses when the initial debt is low (40% of GDP): Figure 20 compares the impulse responses for $\tau$-type vs. $g$-type FC when the initial debt to gdp ratio is low. When debt levels are this low there is little expectation of there being a fiscal consolidation
in the near future, and, as a result there are little expectations effects, beyond the fact that when a consolidation is actually initiated economic agents know it will last for one year. Accordingly, at low levels of debt, if the fiscal consolidation turns out to be of the tax-based kind, then the impulse responses are very similar to those observed when tax-based consolidations are the only possible kind of consolidation as in Figure 10. Similarly, when the fiscal consolidation is essentially unexpected, if the realized fiscal consolidation happens to be based on government spending cuts, then the impulse responses are very similar to the outcomes of fiscal consolidations where the only kind of consolidation is of the government spending type, Figure 17. In other words when debt levels are low, economic agents do not expect there to be a consolidation of any kind, such that uncertainty of which type of consolidation would emerge upon realization of such an unlikely event is not important.

**Impulse responses when the initial debt is high (170% of GDP):** In Figure 21, we have very high levels of debt such that economic agents believe that a fiscal consolidation is imminent. In such an environment uncertainty over which type of consolidation will be realized starts to matter. Given that economic agents expect consolidations to involve tax increases 75% of the time, their expectations are that there will be inflationary increases in distortionary taxation when the consolidation begins. Accordingly, if there is actually a government spending based consolidation this is something of a surprise, which reduces inflation relative to the no-consolidation base (which incorporates expectations of predominately tax-based consolidations even although they are not actually realized in the base.) The deflationary government spending-based consolidation then, via the active monetary policy rule, reduces real interest rates, which helps support the tax base and reduces debt service costs. Moreover, the consumption gain from realizing the government spending consolidation, when households were worried that the consolidation would be tax-based, is significantly higher. As a result, relative to the no-consolidation base case, real wages actually rise when the government spending consolidation is realized while they would have fallen if there was no composition uncertainty and all consolidations were spending-based.

When the realized consolidations are of the tax-based type, then since these were largely anticipated, the results are qualitatively similar to those seen when all consolidations are tax-based without any composition uncertainty. Therefore, we see that the higher distortionary tax rates observed during the fiscal consolidation, raise marginal costs which fuels inflation during the course of the consolidation. The active monetary policy then responds to this higher inflation by raising real interest rates, which diminishes the tax base and raises debt service costs. This accounts for the relatively poor performance of the tax-based consolidations in stabilizing debt when debt levels are high, even although both tax and spending
based consolidations have roughly the same impact on the primary deficit and are equally effective at stabilizing debt when debt levels are relatively low.

This intuition is consistent with the decision rules plotted in Figure 22.

**Output multiplier:** Figure 23 compares the output multiplier under $\tau-$type and $g-$type consolidations when the type of consolidation is uncertain, and tax increase and spending cuts in no-RS model where there is no composition uncertainty. At low levels of debt, the two types of fiscal consolidation (tax and spending) applied without composition uncertainty, provide upper and lower bounds for the regime-switching model, similar to the comparison with only time uncertainty in Figure 19. When debt levels are higher, $g-$type fiscal consolidations in the regime-switching model with composition uncertainty can outperform i.i.d spending cuts (no-RS $(g)$) by a substantial margin, while $\tau-$type fiscal consolidation in the model with composition uncertainty is very similar to i.i.d. tax increases (no-RS $(\tau)$). This is due to the expectation spill-over effect, as explained above. As we shall now see, this ranking could be different depending on the monetary policy stance ($\alpha$) and economic agents’ expectations about the likely composition of any fiscal consolidation, $\omega$.

7.2 **Discussion of MP stance and household’s expectation of FC**

**Less active MP:** The discussion of the relative efficacy of tax and government spending based consolidations when debt levels are high highlighted the importance of the repercussions on debt service costs of the monetary policy response to the consolidation. Accordingly, deflationary government spending cuts facilitate a relaxation of monetary policy which helps stabilize debt through its impact on the tax base and debt service costs in a sticky price economy. In contrast, when monetary policy responds to the higher inflation generated by distortionary tax based consolidations, this has the opposite effect of raise the interest rates payable on government debt, which is particularly destabilizing when debt levels are large. This would tend to suggest that the responsiveness of monetary policy to inflation is a key ingredient in defining the relative efficacy of the alternative types of fiscal consolidation. To explore this, for high debt levels of 170% of gdp, Figure 24 shows the impulse responses across the two types of fiscal consolidation where monetary policy is less active ($\alpha = 1.2$) relative to the benchmark of $\alpha = 1.5$ considered in Figure 21, such that nominal interest rates only rise by 1.2 percentage point for every 1 percentage point rise in inflation above target, rather than the more aggressive response of 1.5 percentage point.

Contrasting the two figures, we can see that reducing the responsiveness of interest rates to excess inflation deepens the recession under government-spending based consolidations, and reduces its ability to stabilize debt. While, tax based consolidations are not thwarted
by monetary policy to the same extent, such that there is a more pronounced decline in debt following the tax based consolidation when monetary policy is less active. Nevertheless, it remains the case that government spending based consolidations are relatively more effective in reducing the debt burden, and this relative efficacy at high debt levels is likely to exist as long as monetary policy is active.

Figure 25 compares the output multiplier under \( \tau \)-type and \( g \)-type of RS model, and tax increase and spending cut in no-RS model. Compared to the calibration with \( \alpha = 1.5 \), under the less active monetary policy, tax increases can be more expansionary (the output multiplier turns to positive upon the exit of fiscal consolidation), while the spending cut becomes much more contractionary.

**Lower probability of tax increases:** In our final experiment we return to our benchmark monetary policy of \( \alpha = 1.5 \), but reverse the relative likelihood of tax and government spending based consolidations by setting \( \omega = 0.25 \), such that the probability of spending cuts is three-times high as that of tax increases. As before, at low debt levels this reversal makes no difference since no consolidations of either kind are expected. However, if we consider high debt levels then fiscal consolidations are thought to be imminent and it matters which type of consolidation economic agents anticipate will occur. When we reverse the relative probabilities of tax and government spending-based consolidations, then economic agents believe the consolidations will lead to deflationary cuts in government spending. Accordingly, when the relatively low probability tax-based consolidation is realized inflation rises relative to base, monetary policy responds by raising real interest rates, reducing the size of the tax base when prices are sticky and fueling debt service costs. This is actually enough to imply that government debt rises relative to the no-consolidation base case.

The government spending-based consolidations are still relatively effective in stabilizing debt, but the benefits of economic agents finding that the consolidation was not of the expected tax-based kind are not so apparent when they attach a smaller probability weight to tax-based consolidations. Effectively when debt levels are high, the no consolidation base case contains surprises since economic agents are expecting fiscal consolidations to occur, when they don’t. When these consolidations are expected to be tax based, they raise inflationary expectations which fuel current inflation in a sticky price economy. In the presence of an active monetary policy the monetary policy response to high inflation will both contract the economy (and thereby the tax base) and raise debt service costs. In contrast, when fiscal consolidations are likely to involve substantial government spending cuts, which are deflationary, this reduces inflationary expectations which results in price setters moderating their price increases, facilitating a relaxation in monetary policy. Accordingly, there is a
clear incentive for governments to clarify that any fiscal consolidations that emerge are likely to expenditure based since this will help moderate the destabilizing implications of the inflationary consequences of tax-based consolidations.

Figure 27 compares the output multiplier under tax and expenditure based consolidations, when there is uncertainty as to which will be realized ($\tau$—type and $g$—type regime switching consolidations), and tax increase and spending cut consolidations in the absence of any composition uncertainty. In line with the impulse responses reported above, if the household puts a higher probability weight on spending cuts as the likely form of consolidations, tax-based consolidations can be much more contractionary.

8 Summary

In this paper we have explored the non-linearities inherent in state-dependent fiscal consolidations, where the exact timing of these consolidations is uncertain, but the likelihood of observing a consolidation is rising as debt levels rise. Moreover, consistent with the different types of fiscal consolidation observed in the data, we have contrasted fiscal consolidations which raise taxes to stabilize government debt, with those which opt to cut government expenditure. These have been examined in a variety of contexts, including low and high debt levels, flexible and sticky prices, different degrees of monetary policy activism and different beliefs about the relative probability that any future consolidations will be tax or expenditure based. Our results show that there are significant interactions between all these factors in determining the marginal impact of a given fiscal consolidation.

For example, at low debt levels, tax or expenditure based consolidations, of the same magnitude in terms of their effect on the primary budget, are almost equally successful in stabilizing debt. However, at higher debt levels when economic agents start to expect fiscal consolidations to occur, expectational effects can have significant impacts on the relative efficacy of alternative types of fiscal consolidation. Cuts in government spending are particularly helpful in stabilizing government debt as their deflationary impact on aggregate demand allows for a relaxation in monetary policy which has particularly beneficial effects on debt dynamics when debt levels are high. In contrast, tax based consolidations which raise distortionary taxes during the consolidation episode, discourage labor supply, raise marginal costs and fuel inflation. This in turn prompts a monetary policy response from an inflation targeting central bank which is damaging for debt dynamics. The relative magnitude of these affects depends on the extent of price-stickiness, the strength of the monetary policy response to excess inflation and the extent to which economic agents were expecting the consolidation to be of one type or another. While some of these factors are considered in the
empirical literature on fiscal consolidations, particularly the notion that expenditure based consolidations are more likely to be associated with ‘successful’ consolidations, our analysis suggests that the non-linearities inherent in the interactions between these elements make empirical identification of these effects through standard regression analysis difficult.


A Simulating the Fiscal Limit

The fiscal limit $B^*$ can be obtained using Markov Chain Monte Carlo simulation:

- First, for each simulation, we randomly draw the shocks of productivity, government purchases, and transfers for 1500 periods. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables using the household first-order conditions and the budget constraints. According to equation 25, we compute the discounted sum of maximum fiscal surplus by discarding the first 500 draws as a burn-in period.

- Second, we repeat the simulation for 100,000 times and obtain the distribution of the fiscal limit, which is then approximated through kernel density estimation.

- At each period of time, the effective fiscal limit $b_t^*$ is a random draw from the distribution.
Figure 1: Distribution of Fiscal Limits
Figure 2: Transfer-GDP Ratios: Unstable Group

Figure 3: Transfer-GDP Ratios: Stable Group
Figure 4: Time Uncertainty

Figure 5: Composition Uncertainty
Figure 6: Impulse Response: no – RS vs. RS* model with flexible price under low initial debt level

Figure 7: Impulse Response: no – RS vs. RS* model with flexible price under high initial debt level
Figure 8: Inflation dynamics in the $RS^T$ model with flexible price under lower initial debt level

Figure 9: Inflation dynamics in the $RS^T$ model with flexible price under high initial debt level
Figure 10: Impulse Response: noRS vs. $RS^\tau$ model with sticky price under low initial debt level

Figure 11: Impulse Response: noRS vs. $RS^\tau$ model with sticky price under high initial debt level
Inflation and Expected Inflation

\[ \pi_t \] (no FC) \quad E_{t-1} \pi_t \] (no FC)

Figure 12: Inflation dynamics in the \( RS^\tau \) model with sticky price under lower initial debt level

Inflation and Expected Inflation

\[ \pi_t \] \quad \pi_t \] (no FC) \quad E_{t-1} \pi_t \] (no FC)

Figure 13: Inflation dynamics in the \( RS^\tau \) model with sticky price under high initial debt level
Figure 14: Output Multiplier: noRS vs. $RS^\tau$ model with sticky price
Figure 15: Impulse Response: noRS vs. $RS^g$ model with flexible price under low initial debt level

Figure 16: Impulse Response: noRS vs. $RS^g$ model with flexible price under high initial debt level
Figure 17: Impulse Response: noRS vs. $RS^g$ model with sticky price under low initial debt level

Figure 18: Impulse Response: noRS vs. $RS^g$ model with sticky price under high initial debt level
Figure 19: Output Multiplier: noRS vs. $RS^\tau$ vs. $RS^g$ model with sticky price
Figure 20: Impulse Response: RS model with sticky price under low initial debt level

Figure 21: Impulse Response: RS model with sticky price under high initial debt level
Figure 22: Decision rules: $RS$ vs. $RS^r$ vs. $RS^g$ model with sticky price
Figure 23: Output Multiplier: no-RS vs. RS model with sticky price
Figure 24: Impulse Response under less active MP: RS model with sticky price under high initial debt level
Figure 25: Output Multiplier: no-RS vs. RS model (with less active MP) with sticky price
Figure 26: Impulse Response under $\omega = 0.25$: RS model with sticky price under high initial debt level
Figure 27: Output Multiplier: no-RS vs. RS model (with $\omega = 0.25$) with sticky price
<table>
<thead>
<tr>
<th></th>
<th>Expansionary</th>
<th>Contractionary</th>
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<tbody>
<tr>
<td>Debt</td>
<td>−4.93*</td>
<td>5.42*</td>
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<tr>
<td></td>
<td>(1.69)</td>
<td>(1.41)</td>
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<tr>
<td>Change in Debt</td>
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<td>−2.22*</td>
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<td></td>
<td>(1.21)</td>
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<td>Total Deficit</td>
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<td>(0.33)</td>
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<td>Primary Deficit</td>
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<td>−1.91*</td>
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<td>(0.58)</td>
<td>(0.31)</td>
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<td>Primary Expenditures</td>
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<td>−0.80*</td>
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<td>(0.08)</td>
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<td>Subsidies</td>
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<td>−0.16*</td>
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<td>Total Rev</td>
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<td>(0.24)</td>
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<td>Business Tax</td>
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<td>Indirect Tax</td>
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<td>(0.22)</td>
<td>(0.13)</td>
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Table 1: Expansionary and Contractionary Fiscal Adjustments (Size and Composition): * denotes statistical significance at the 5% level, all variables are the average changes in the variable relative to GDP in the two years preceding and following a fiscal consolidation and the standard errors are in brackets.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibration</th>
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<tr>
<td>Discount factor</td>
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<td>Rotemberg adjustment parameter</td>
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<td>Technology</td>
<td>$A$</td>
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<td>Labor supply</td>
<td>$n$</td>
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<td>Government transfer-GDP</td>
<td>$z/y$</td>
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<td>Government debt-GDP</td>
<td>$b/y$</td>
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<td>Technology</td>
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<td>Government transfer-GDP</td>
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<tr>
<td>Government debt-GDP</td>
<td>0.3526 (annual)</td>
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<td>Taylor rule parameter</td>
<td>$\alpha$</td>
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<td>Political factor</td>
<td>$\beta_P$</td>
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<td>Technology shock persistence</td>
<td>$\rho_A$</td>
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<td>Technology shock variance</td>
<td>$\sigma^2_A$</td>
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<td>Length of consolidations</td>
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<td>Tax-type consolidation</td>
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<td>Probability of tax-type FC</td>
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Table 2: Model Calibration