Emiratization Policies in the UAE: An Intrafirm Bargaining and Matching Approach

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Abstract

This paper examines government policies in a labor market where expatriates constitute a sizeable majority of the labor force. It develops a large firm version of the matching model with intrafirm bargaining and constant returns in production. In the model, the firm's hiring decisions are driven by the increasing costs of employing expatriates relative to nationals rather than workers' productivities. This approach contrasts with the prior literature which has captured the employment effect on wages based on assumptions of increasing or decreasing labor productivity rather than the firm's cost structure. The model is calibrated to the data from the United Arab Emirates (UAE) where approximately 90% of the workforce is imported labor. It finds that beyond a certain cost level of employing expatriates, the firm would decrease overall employment and unemployment for UAE nationals rises.

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Keywords: Private Sector Employment; Expatriates; Wage Bargaining; Labor Market Policies

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1 Introduction

This paper uses the Stole and Zwiebel (1996) intrafirm bargaining methodology to study the United Arab Emirates ("UAE") government policies aimed at reducing unemployment among UAE nationals. Stole and Zwiebel presented a novel bargaining methodology to analyze the equilibrium process within the firm. This paper applies the bargaining process to the large firm version of the search and matching model and incorporates government imposed job quotas for UAE nationals in the private sector. The job quotas form part of the UAE government's initiative to increase the employment of nationals in the private sector and are commonly referred to as "Emiratization" policies.

The "Emiratization" policies cause a firm's costs to increase as the number of expatriates increases in proportion to the number of nationals in a firm. These costs are perceived by the firms to constitute an implicit form of taxation on the hiring of expatriates. As a result, firms are incentivized to attract more nationals which in turn increases the bargaining power of nationals and thus impacts their wages. However, as the total number of employed nationals in a firm increases, the bargaining power of each additional national decreases. As such, there is an employment effect on the wages of nationals based on their proportion within the firm, even if the productivity of labor does not change. Therefore, we can relax the assumption of Stole and Zwiebel which requires that employment effects are solely contingent on changes in productivity and focus on the costs of hiring expatriate employees to model the firm's employment policies.

Although our model analyzes the UAE labor market, it has many characteristics common to the remaining Gulf Cooperation Council (GCC) countries.[1] In both the UAE and the remaining GCC countries, expatriates constitute a large proportion of the workforce. This is mainly due to government efforts to diversify their national economies away from natural resources following the 1970's oil boom (Winckler 1997). High-skilled and low-skilled labor were actively recruited from abroad as the required labor force could not be supplied locally. In 2005, 92% of the United Arab Emirates ("UAE") workforce was expatriate and constituted more than 98% of those employed in the private sector.[2] Similarly, in Qatar, the proportion of expatriates participating in the labor force was 88% in 2004.[3]

As the size of the expatriate labor force expanded, so did the national labor force. The private sector perceived expatriates to be cheaper and more productive than nationals. In the UAE, for example, this is underscored by the difference in means between private sector wages of UAE nationals and expatriates, which according to the household budget survey of 2008,[4] was highly significant and amounted to 8400 AED or 2283 USD per month for all skill levels.[5] In Saudi Arabia,

[1] The GCC countries are Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and the United Arab Emirates.

[2] UAE Census of 2005.

[3] Qatar Census of 2004.

[4] The survey contains information on 13, 992 national and expatriate households. The sample is restricted to heads of households who are employees in order to obtain independent and identically distributed data. The sample size is 10,513 individuals.

[5] The skill level classification is the International Standard Classification of Occupations "ISCO-88". The difference

the difference between average monthly wages of Saudi nationals and expatriates in 2000 amounted to 3566 SAR or 951 USD.[6] The public sector, which had acted as an employer of first and last resort, could no longer absorb all the new national labor market entrants (Fasano and Goyal 2004).[7] This resulted in the emergence of significant unemployment among GCC nationals.[8]

GCC countries were faced with a dilemma between alleviating unemployment among GCC nationals or importing foreign workers essential for economic growth (Winckler 1997). In response to the unemployment problem, GCC governments embarked on a process of nationalizing the workforce.[9] A quota system was imposed on private sector companies to increase their intake of national labor despite its potential impact on growth.

The literature is very limited in measuring the impact and effectiveness of the nationalization policies in the GCC countries. Chemingu and Roes (2008) assess these policies and their impact on employment in Kuwait using a dynamic computable general equilibrium ("CGE") model. They showed that a reduction in the supply of skilled expatriates in Kuwait would not increase the employment of Kuwaiti nationals significantly but instead increases the labor costs of the firm and adversely impacts the economy. They also examined other policies to stimulate private sector employment such as increasing the investment share in key sectors with high national labor and imposing a production subsidy of 20% for these sectors. For the period of 2001 to 2015, they concluded that these policies are not sufficient as they will not absorb all the nationals looking for employment.

Toledo (2006) extends the Ramsey theorem[10] to explain the problems associated with the "Emiratization" policies. Assuming the same productivity levels between UAE nationals and migrants, he concluded that if "Emiratization" were to succeed, then it must be implemented in the imperfectly competitive sector. Firms would have to be ready to give up their rents in exchange for providing jobs to nationals. In a perfectly competitive input and output market, firms would not want to hire UAE nationals. Toledo also concludes that allowing greater mobility of expatriates would increase their productivity, thereby raising their wages and tightening the wage gap between migrants and nationals. However, neither Toledo (2006) nor Chemingu and Roes (2008) model the labor market decisions of national workers, but assume that all nationals are employable. Toledo (2006) only analyzed the firm's demand for employment, while Chemingu and Roes's (2008) projections are based on the performance of the general macroeconomic factors of Kuwait.

On the other hand, Fasano and Goyal (2004) analyze both the firms and GCC workers' labor market decisions in the presence of the nationalization policies. They use the standard search and

in means between monthly private sector wages of nationals and expatriates is UAE Dirhams (AED) 2665, 7009, 3790, and 13,636 for skill levels 1, 2, 3 and 4 respectively, 4 being the highest skilled workers.

[6] Saudi Arabia Employment and Wages Survey of 2000.

[7] In the UAE, for example, employment of nationals in the public sector grew at an average annual rate of 5% during the period 1995 to 2005, while the UAE labor force grew at a rate of 6% (UAE 1995 and 2005 census).

[8] Unemployment rates in the UAE reached 19% in 2005, and 9.8% in Saudi Arabia in 2008.

[9] The nationalization programs are referred to as "Emiratization" , "Qatarization", "Saudization", "Omanization", "Kuwaitization", "Bahrainization" in the UAE, Qatar, Saudi Arabia, Oman, Kuwait and Bahrain respectively.

[10] The Ramsey theorem provides a condition with regards to the price that a monopolist should set so as to maximize social welfare. The primary source of inefficiency in this case is monopoly.

matching model to account for labor market frictions. The "single-worker" matching model used in Fasano and Goyal (2004) assumes each vacancy represents one firm, without modeling the strategic interactions within a firm, especially as it relates to the hiring of expatriates and nationals. As a result, their paper does not explicitly model the hiring of expatriates, but simply acknowledges that their presence affects the bargaining power of GCC nationals. A model which allows for multiple vacancies within a firm would be required to explicitly account for the interaction between nationals and expatriates in the firm.

This paper applies the large firm version of the search and matching model to allow for multiple vacancies within one firm. It also incorporates the Stole and Zwiebel (1996) intrafirm bargaining methodology to enable the analysis of strategic interactions within a firm. Stole and Zwiebel (1996) model the firm's internal wage bargaining process by assuming that workers are irreplaceable, thereby providing them with bargaining power vis-à-vis the firm. Moreover, wages are renegotiated continuously such that neither party can commit to future wages and employment decisions which may have resulted otherwise due to contract incompleteness. Stole and Zwiebel (1996) show that, due to the diminishing returns of labor, firms can depress wages by expanding employment. Thus, bargained wages are driven down to workers' reservation values.

De Fontenay and Gans (2003) extend Stole and Zwiebel's theory by assuming that workers are not irreplaceable, and as long as there is a substitutable exogenously fixed finite pool of labor outside the firm, firms tend to under-hire because wages will exceed the reservation wages. Employees exert power over the firm; thereby, allowing them to capture some rents. However, if there is a cost to hiring outsiders, or if the pool of outsiders is endogenously determined by the firms' employment decisions, then there will be over-hiring. Stole and Zwiebel (2003) responded by stating that only low skilled can make up a perfectly substitutable pool of labor for which wage bargaining does not apply, and as such their results still hold.

Both Stole and Zwiebel (1996) and De Fontenay and Gans (2003) are partial equilibrium models. To extend the analysis from a partial equilibrium framework to a general equilibrium framework, the literature combined the Stole and Zwiebel intrafirm bargaining approach with the dynamic approach of Pissarides (2000).[11]

Cahuc and Wasmer (2001) combine both the Pissarides and the Stole and Zwiebel approaches. They then compare the combined model with the Pissarides standard "single-worker" firm model. Cahuc and Wasmer find that the two models are equivalent, if there are constant returns to scale in all factors and zero adjustment cost for capital. In such circumstances, they find that firms are no longer able to exploit the diminishing marginal productivity of workers and manipulate wages by over-employing workers. Cahuc and Wasmer also compare the combined model to that of Stole and Zwiebel and show that due to the existence of labor market frictions, employees are likely to receive rents even if the firms over-employ.

Cahuc and Wasmer (2008) extend their model to analyze wage and employment decisions when

[11] Pissarides developed a dynamic model which explained unemployment and the flows between workers and jobs using a matching function that allowed for labor market frictions.

workers are heterogeneous and have different bargaining powers. Cahuc and Wasmer not only model the interactions between a firm and its workers, but also among workers themselves. They show that introducing multi-labor inputs to the model alters the results such that firms may under-employ workers when some of the labor inputs complement one another. They find that workers with low bargaining power can be under-employed whilst those with high bargaining power can be over-employed.

Bertola and Garibaldi (2001) also combines the Pissarides "large firm" model with the Stole and Zwiebel intrafirm bargaining process in order to study the relationship between wages and firm size. This relationship cannot be captured in a world of constant returns or under the "single worker" model. Bertola and Garibaldi assume that there are hiring costs and decreasing returns to scale in the recruitment technology. Moreover, in their model, they assume that productivity is not constant across firms, but depends on a stochastic idiosyncratic labor demand shock and employment levels. Bertola and Garibaldi's results verify the empirical evidence of a positive relationship between wages and firm size. They find two opposing effects. On one hand, wages decrease with employment levels as in Stole and Zwiebel (1996), and on the other hand, wages increase with an increase in the labor demand shifter. Bertola and Garibaldi find that high wages may be associated with high employment levels because higher productivity levels, driven by increases in the idiosyncratic labor demand shock, cause firms to post more vacancies.

Mortensen (2009) uses the combined model to study the relationship between wages and productivity. He concludes that a second equilibrium of wage dispersion exists in addition to the single wage equilibrium if on-the-job search is allowed and firms are heterogeneous with respect to productivity. Beugnot and Tidball (2010) also proves the existence of multiple equilibria, but unlike Mortensen (2009), this is generated by assuming increasing returns to scale in the aggregate production function. They show through numerical simulations that there will always be two equilibria: a Pareto inferior equilibrium with a low employment rate and wage, and a Pareto superior equilibrium with a high employment rate and wage. They conclude that the "low" equilibrium may be reached only if the bargaining power of workers is lower than the matching elasticity. Therefore, government intervention may be required to achieve the Pareto superior equilibrium.

All the papers which use the combined model assume either decreasing or increasing marginal returns to labor to capture the impact of a firm's employment choices on wages. In effect, the results vary based on the productivity of workers. This paper is innovative because it modifies the combined model such that a firm's employment choices are driven by its cost structure rather than its workers' productivity levels. This enables us to measure the impact of the increasing marginal costs of hiring expatriates in the UAE caused by government intervention on the firm's internal bargaining process. Consequently, this paper also takes a first step in explicitly modeling the impact of the labor market nationalization policies on the bargaining process within the firm and ultimately on private sector employment.

The paper segments the labor market into two types of workers, nationals and expatriates, and two sectors, public and private. It models the labor market decisions of nationals and the queuing

effect for public sector jobs brought about by the many financial and cultural advantages that sector offers. The model also allows for on the job ("OTJ") search for nationals employed in the private sector. In addition, it models the private sector hiring decisions of both expatriates and nationals in the context of "Emiratization" policies.

In the model, the private sector is represented by one firm which maximizes profits by hiring both nationals and expatriates. The internal bargaining process as described by Stole and Zwiebel (1996) is conducted solely between nationals and the firm. We assume that the labor market for expatriates is frictionless because the private sector firm has access to an unlimited supply of foreign labor (Fasano and Goyal 2004) and that expatriate workers have negligible bargaining power. Unlike Cahuc and Wasmer (2001), the model assumes constant returns to labor and captures the impact of hiring an additional national worker on the wages through the "Emiratization" policies. We model these policies as a form of taxation for hiring expatriates, with no benefits to the firm (Forstenlechner 2008). Thus, the cost incurred by the firm increases with the proportion of expatriates hired within that firm.

The paper then studies the impact of the "Emiratization" policies on private sector employment of nationals. It finds that increasing the cost of hiring expatriates may in fact decrease the private sector employment of nationals and subsequently increase unemployment.

The remainder of the paper is organized as follows. The next section provides an overview of the "Emiratization" policies in the UAE. Section 3 describes the model and framework. Section 4 determines the intrafirm wage bargaining between nationals and the firm and proves the existence of an equilibrium. Section 5 explores the comparative statics. Section 6 calibrates the model to UAE data. Finally, section 6 concludes.

2 Overview of the "Emiratization" Policies

The Emiratization policies imposed job quotas on organizations engaged in certain economic activities such as banking, insurance, and trade which required certain levels of UAE national employment. The first resolution was passed in 1998 and targeted the banking system[12]. It obliged all the banks operating in the UAE to increase their intake of national employees at the rate of 4% annually. In 2003, another resolution[13] was passed requiring all insurance companies operating in the UAE to raise their intake of national employees at an annual rate of 5%.In 2004, a resolution[14] was approved obliging all trading firms employing 50 or more workers to raise their intake of national employees at the annual rate of 2%. Finally, in 2006, secretarial and human resources occupations were also nationalized[15]. The National Human Resource Development & Employment Authority TANMIA was assigned the task of monitoring the compliance of firms to the Emiratization policies.

Subsequently, firms were classified according to three categories: A, B, or C depending on

[12] Ministerial Resolution No. 10 for 1998.
[13] Ministerial Resolution No. 202/2 for 2003.
[14] Ministerial Resolution No. 259/1 for 2004.
[15] Ministerial Resolutions No. 442 and 443 for 2006.

whether they fulfilled the Emiratization requirements. Non-compliance demoted the firms to categories B or C. The costs and fees associated with recruiting expatriates depended on the category a firm is classified in[16]. For instance, the renewal of a labor card[17] costs category A firms approximately 136 USD; whereas, it costs category B and C firms 408 USD and 680 USD respectively.

3 The Model

3.1 Population

We consider a continuous time economy with two types of workers, nationals and expatriates, and two sectors, public and private. Nationals can be: a) unemployed and searching for either a public or private sector job; b) employed in the private sector whilst searching for a public sector job; or c) employed in the public sector. Nationals when changing jobs across sectors can only move from the private sector to the public sector. The index $i \in \{g, p\}$ denotes the public and private sector respectively. Expatriates, on the other hand, are always employed in the private sector and cannot be both unemployed and residing in the UAE due to immigration policies. The population is thus distributed as follows:

N: measure of nationals employed in the private sector
U: measure of unemployed nationals
E_g: measure of nationals employed in the public sector
X: measure of expatriates

where the national population is normalized to 1: $N + U + E_g = 1$. All workers are risk neutral, infinitely-lived and discount the future at the common rate r.

3.2 Search and Matching

The model assumes that only nationals engage in random search. The labor market for expatriates is frictionless, as in Fasano and Goyal (2004), since the private sector has an unlimited supply of foreign labor.

National workers match with either sector through a constant returns to scale matching function which depends on the number of vacancies posted and the number of individuals looking for jobs. The matching functions for each sector are denoted as $M^g(U + \lambda N, V^g)$ and $M^p(U, V^p)$ where V^g and V^p each represent the total vacancies in the public and private sector respectively, and $\lambda \in (0, 1)$ is an exogenous search intensity of individuals working in the private sector and searching for a job in the public sector.

The probability that the public sector fills its vacancy with a national is:

[16] Ministerial Resolution No. 19 for 2005.
[17] A card issued by the Ministry of Labor for all foreign employees working in the private sector.

$$\frac{M^g(U+\lambda N, V^g)}{V^g} = \frac{m_g(\theta_g)}{\theta_g}$$

The probability that the private sector fills its vacancy with a national is:

$$\frac{M^p(U, V^p)}{V^p} = \frac{m_p(\theta_p)}{\theta_p}$$

In the model, $\theta_g = \dfrac{V^g}{(\lambda N + U)}$ and $\theta_p = \dfrac{V^p}{U}$ denote the labor market tightness in the public and private sector respectively. Similarly, unemployed nationals find jobs at the rate of $m_i(\theta_i)$, $i \in \{g, p\}$ and employed private sector nationals obtain work in the public sector at the rate of $\lambda m_g(\theta_g)$. The matching functions have the following properties: $\dfrac{d(m_i(\theta_i))}{d\theta_i} > 0$ and $\frac{d(\frac{m_i(\theta_i)}{\theta_i})}{d\theta_i} < 0, i \in \{g, p\}$. Matches break up at an exogenous rate δ.

3.3 Labor Demand

3.3.1 Public Sector

The model assumes, as in Alfonso and Gomes (2008), that the government's objective is to preserve a particular level of employment E_g given that it is not our intention to determine the optimal level of employment in the public sector. Therefore, in the steady state, the public sector posts vacancies to maintain E_g such that:

$$\delta E_g = m_g(\theta_g)(\lambda N + U)$$

3.3.2 Private Sector

The private sector consists of a large representative firm which hires both nationals and expatriates. Production from both types of labor input is constant and denoted by Y^N and Y^X. The firm incurs a search cost, γ, for each vacancy posted to nationals, but can hire expatriates instantaneously without incurring any search costs. However, due to labor market policies, the firm is penalized for hiring expatriates at the expense of nationals, and thus incurs a cost $cf(N, X)$ which is dependent on the employment levels of nationals and expatriates in the firm and on an exogenous cost parameter c, where $c > 0$. The cost function is characterized by the following properties:

$$f(N,0) = 0 \quad f_N(N,X) < 0 \quad f_{NN}(N,X) > 0 \quad f_X(N,X) > 0$$

The cost decreases with the employment of nationals and increases with the employment of expatriates. The firm also pays wages w^X and $w^N(N)$ when it hires an expatriate and a national respectively. The expatriate's wage is assumed to be exogenous since it depends mainly on the labor market of foreign countries, which is considered fixed in this paper. The national's wage is endogenous and determined by intrafirm bargaining. The intrafirm bargaining mechanism was first introduced by Stole and Zwiebel (1996) and later incorporated into the search and matching

model. This mechanism assumes that wages are a function of the employment level because they are continuously renegotiated. Therefore, in this paper, the national's bargaining position is affected by the number of national workers already employed in the firm. Finally, national workers leave the firm either because they find a job in the public sector or because a shock occurs.

Subject to the law of motion of jobs, the firm maximizes the discounted value of profit as follows:

$$\max \pi(N,X) = \int_0^\infty e^{-rt}[Y^N N + Y^X X - w^X X - N w^N(N) - cf(N,X) - \gamma V^p]dt$$

$$s.t. \quad \frac{dN}{dt} = \frac{m_p(\theta_p)}{\theta_p} V^p - (\delta + \lambda m_g(\theta_g))N$$

All the analysis in this paper will be conducted at the steady state, i.e., where $\frac{dN}{dt} = 0$. The first order conditions in the steady state for an optimal level of N and X are given by:

$$J^p = \frac{\theta_p \gamma}{m_p(\theta_p)} \tag{1}$$

$$Y^X - w^X = cf_X(N,X) + r \tag{2}$$

where J^P, the marginal profit of employing a national, depends on the productivity of nationals and their wages, on the cost of hiring expatriates, and on the impact of hiring an additional national on the wages of all other nationals.

$$J^p = \frac{Y^N - N\frac{dw^N(N)}{dN} - w^N(N) - cf_N(N,X)}{(\delta + r + \lambda m_g(\theta_g))} \tag{3}$$

The first order condition for nationals (1) equates the marginal profit of hiring nationals to the expected cost of searching for them. On the other hand, the first order condition of expatriates (2) equates the marginal profit of employing expatriates to the discount factor and the marginal cost of hiring expatriates which in turn depends on the employment level of nationals in the firm. Expression (2) does not account for the effect of expatriates on national private sector wages since the employment of expatriates can be instantaneously changed[18].

3.4 Workers

This paper only solves for the value functions of national workers since the expatriate labor market is frictionless. The discounted value of an unemployed national worker, denoted by U^N, solves:

[18] We assume that the employment level of expatriates is not a predetermined variable. Cahuc and Wasmer (2001) imposed the same assumption for capital.

$$rU^N = b + m_g(\theta_g)Max[(N^g - U^N), 0] + m_p(\theta_p)Max[(N^p - U^N), 0] \tag{4}$$

All national unemployed workers receive an unemployment benefit (or the value for leisure) b, but face a sector dependent probability $m_i(\theta_i)$, $i \in \{g, p\}$ of receiving a job offer from the public and private sector. Similarly, the discounted value functions of an employed national in the public and private sectors are:

$$rN^g = w_g + \delta(U^N - N^g) \tag{5}$$

$$rN^p = w^N(N) + \delta(U^N - N^p) + \lambda m_g(\theta_g)Max[(N^g - N^p), 0] \tag{6}$$

Employed workers in the public sector receive the exogenous wage w_g until they lose their jobs and become unemployed at the rate of δ. The public sector wage w_g is assumed to be exogenous and determined by government policies. In the private sector, workers receive the bargained wage $w^N(N)$ until the match is dissolved because either the worker finds a public sector job or a shock occurs at the rate δ.

4 Labor Market Equilibrium

4.1 Intrafirm Wage Bargaining

As in Stole and Zwiebel (1996), this paper assumes that instantaneous and continuous wage renegotiation occurs between nationals and the firm, which means that all wages are renegotiated if an employee leaves the firm. Moreover, government jobs are so attractive that it is assumed that the firm will not be able to offer a higher wage to keep workers from searching in the public sector. Private sector wages for UAE nationals are determined according to the surplus sharing rule based on the amount of nationals already employed in the firm:

$$\beta J^p = (1 - \beta)(N^p - U^N) \tag{7}$$

where $\beta \in (0, 1)$ is an exogenous parameter representing the bargaining power of workers. Expressions (3), (5) and (6) yield the following differential equation for the steady state wage of UAE nationals in the private sector:

$$w^N(N) = \beta[Y^N - N\frac{dw^N(N)}{dN} - cf_N(N, X)] + (1 - \beta)rU^N - (1 - \beta)\frac{\lambda m_g(\theta_g)}{(r + \delta)}(w_g - rU^N) \tag{8}$$

Therefore, contrary to Cahuc and Wasmer (2001), the wage function stays different from that

obtained in the Pissarides standard matching model even with the assumption of constant productivity. The $N\frac{dw^N(N)}{dN}$ term remains because of the firm's cost structure which depends of the employment levels of expatriates and nationals within the firm. As the number of nationals employed in the firm increases, the cost savings derived from employing an additional national on the hiring of expatriates diminishes[19]. As a result, the firm exploits the diminishing marginal benefit to depress the wages of nationals. This effect is referred to as the Stole and Zwiebel effect since it results from the intrafirm bargaining mechanism of wages developed in Stole and Zwiebel (1996).

The solution to the differential equation yields[20]:

$$w^N(N) = \beta Y^N + (1-\beta) r U^N - (1-\beta)\frac{\lambda m_g(\theta_g)}{(r+\delta)}(w_g - rU^N) - cN^{-\frac{1}{\beta}} \int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du \quad (9)$$

The first two terms in the wage function are similar to the Pissarides standard search and matching model, the third term reflects the willingness of workers to agree to a reduction in their wages when OTJ search is allowed, and finally, the last term denotes the employment effect on wages. The employment effect is positive because the cost function decreases with the employment of nationals ($f_N(N,X) < 0$). Moreover, the wage function decreases with the employment level of nationals, $\frac{\partial w^N(N)}{\partial N} < 0$ (Stole and Zwiebel effect).

The model focuses only on interior solutions, where there are no negative surpluses resulting from filling private sector job vacancies with nationals. Condition (10) can be expressed in terms of the exogenous parameters of the model (see Appendix A).

$$N^p + J^p \geq U^N$$

$$\beta[(r+\delta)Y^N + \lambda m_g(\theta_g) w_g] - (r+\delta)cN^{-\frac{1}{\beta}} \int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du \geq \beta r U^N (r+\delta+\lambda m_g(\theta_g)) \quad (10)$$

4.2 Existence

Definition 1 *A Steady State Equilibrium with on the job search is defined by* $\{N, X, U, \theta_g, \theta_p\}$ *such that the following conditions hold:*

1- The first order condition for an optimal choice of UAE nationals within the firm:

$$\frac{m_p(\theta_p)}{\theta_p}\frac{(1-\beta)}{\beta}\frac{[\beta(Y^N(r+\delta+m_g(\theta_g)) - \beta(1-\lambda)m_g(\theta_g)w_g - \beta b(r+\delta+\lambda m_g(\theta_g)) - (r+\delta+m_g(\theta_g))cN^{-\frac{1}{\beta}}\int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du]}{(\delta+r+\lambda m_g(\theta_g))(r+\delta+m_g(\theta_g)+\frac{1}{2}m_p(\theta_p))} = \gamma \quad (11)$$

[19] This is due to the cost function being convex in N, i.e., $f_{NN}(N,X) < 0$.

[20] The method for solving the differential equation follows that of Bertola and Garibaldi (2001).

2- The first order condition for an optimal choice of expatriates within the firm:

$$Y^X - w^X = cf_X(N, X) + r \tag{12}$$

3- The flow of UAE nationals into unemployment is equal to the flow out of unemployment:

$$(m_g(\theta_g) + m_p(\theta_p))U = \delta(N + E_g) \tag{13}$$

4- The steady state condition of the government's law of motion:

$$\delta E_g = m_g(\theta_g)(\lambda N + U) \tag{14}$$

5- The normalization of the UAE national population:

$$E_g + N + U = 1 \tag{15}$$

Proposition 1 *If condition (10) holds and the cost parameter, c, is sufficiently large, then there exists at least one Steady State Equilibrium where both types of workers are hired by the firm.*

The existence of at least one equilibrium is established from the continuity of the first order conditions of the firm. The values of N, U, X,and θ_g can all be expressed in terms of θ_p from equations (12), (13), (14), and (15). Therefore, the existence of the equilibrium ultimately depends on the manner in which the first order condition of nationals changes when θ_p varies (see Appendix A for the proof). Moreover, if c is sufficiently large, then the "Emiratization" policies bind leading to an interior solution.

Lemma 1 *If the cost function is concave in X, i.e., $f_{XX} < 0$, then the equilibrium in unique*[21].

Please refer to Appendix A for the proof.

5 Comparative Statics

This section explores the impact of labor market policies on the employment of nationals and expatriates in the private sector using comparative statics. The comparative statics depend on the curvature of the cost function $f(N, X)$. Therefore, four possibilities arise:

1. $f_{XX}(N, X) < 0$ and $f_{NX}(N, X) < 0$

[21] This condition is sufficient but not necessary. Even if the cost function were convex in X, i.e., $f_{XX} > 0$, then the equilibrium could be unique. This occurs if the impact of the search frictions on the first order condition of nationals is larger than that of the penalty. In this case, the uniqueness depends on the specification of the matching function. Moreover, the calibration of the model to the UAE labor market generates a unique equilibrium even by assuming $f_{XX} > 0$ (see section 5 of the paper).

2. $f_{XX}(N,X) < 0$ and $f_{NX}(N,X) > 0$

3. $f_{XX}(N,X) > 0$ and $f_{NX}(N,X) < 0$

4. $f_{XX}(N,X) > 0$ and $f_{NX}(N,X) > 0$

The first two cases generate a unique equilibrium, while the last two may produce multiple equilibria. Henceforth, all policy analysis will be conducted for the stable interior equilibria of the model where the first order condition of nationals is decreasing in θ_p. Moreover, the paper will focus on the third case where $f_{XX}(N,X) > 0$ and $f_{NX}(N,X) < 0$, as this specification of the cost function is in line with the "Emiratization" policies. These policies cause a firm's costs to rise as the number of expatriates increase in proportion of the number of nationals within the firm. An example of such a function is $f(N,X) = \frac{X^2}{N+1}$. In the next section, the model is calibrated to UAE data in order to determine the magnitude of the impact of the cost parameter, c, and public sector employment, E_g, on private sector employment and labor market tightness in the private and public sector. During the calibration, the cost function will be specified as $f(N,X) = \frac{X^2}{N+1}$.

Corollary 1 *An increase in the cost parameter, c, when* $f_{XX}(N,X) > 0$ *and* $f_{NX}(N,X) < 0$ *leads to*[22]*:*

1. *an ambiguous effect on the labor market tightness in the public sector*

2. *an ambiguous effect on the labor market tightness in the private sector*

3. *an ambiguous effect on the employment levels of UAE nationals in the private sector*

4. *an ambiguous effect on the employment levels of expatriates in the private sector*

An increase in the cost parameter c has two countervailing effects on the employment of nationals and expatriates. One one hand, an increase in the cost parameter c leads to an increase in the national wages because nationals will have greater bargaining power. This causes firms to overemploy nationals in order to moderate their wage increases (Stole and Zwiebel effect). The increase in the employment levels of nationals in turn has an effect on the first order condition of expatriates thus allowing the firm to increase its intake of expatriates. On the other hand, an increase in c also has a direct effect on the first order condition of expatriates. It increases the costs of a firm which in turn lowers the employment level of expatriates. As fewer expatriates are hired, there is less need to employ nationals, and as a result, the firm decreases its intake of nationals.

The specification of the cost function determines which effect dominates. The second effect of decreases in the employment levels of both nationals and expatriates is always dominant if the following two sufficient conditions hold:

[22] The proof is available upon request.

1. $f_X \int_0^N u^{\frac{1-\beta}{\beta}} f_{uX}(u,X)du < f_{XX} \int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du$

2. $f_{XN} \int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du < f_X[\frac{d}{dN}(\int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du) - \frac{1}{\beta}N^{-1} \int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du]$

The two sufficient conditions above pertain to the nature of the cost function which determines whether the change in the cost parameter c influences the first order condition of expatriates more than the first order condition of nationals. The two conditions are always satisfied in the example of the cost function illustrated above ($f(N,X) = \frac{X^2}{N+1}$). With such a function, the Stole and Zwiebel effect is weakened by the decrease in the employment levels of expatriates. Indeed, the Stole and Zwiebel effect exists because there are diminishing marginal benefits from employing each additional national. However, the firm relies less on exploiting this diminishing marginal benefit as fewer expatriates are employed in the firm. The overall effect of an increase in the cost parameter c is a decrease in the employment levels of nationals and expatriates. It follows that the labor market tightness in both sectors decreases as fewer nationals are hired in the private sector.

Corollary 2 *An increase in the government employment level when* $f_{XX}(N,X) > 0$ *and* $f_{NX}(N,X) < 0$ *leads to*[23]*:*

1. *an increase in the labor market tightness in the public sector:* $\frac{d\theta_g}{dE_g} > 0$
2. *a decrease in the labor market tightness in the private sector:* $\frac{d\theta_p}{dE_g} < 0$
3. *a decrease in the employment levels of UAE nationals in the private sector:* $\frac{dN}{dE_g} < 0$
4. *a decrease in the employment levels of expatriates in the private sector:* $\frac{dX}{dE_g} < 0$

The rise in government employment causes an increase in the labor market tightness in the public sector. As a result, national workers move from the private to the public sector thus decreasing private sector employment. The firm is also forced to decrease its intake of expatriates because the penalty for employing them increases when fewer nationals are hired in the firm.

6 Calibration

The model is calibrated using the UAE labor market data to determine the magnitude of the effect of an increase in the cost parameter, c, on the hiring choices of the private sector and ultimately on the the unemployment rate of nationals. This data has never been used to numerically evaluate the impact of "Emiratization" policies on the labor market outcomes of nationals and expatriates.

[23] The proof is available upon request.

In this section, the cost function will be specified as $f(N,X) = \frac{X^2}{N+1}$ consistent with the "Emiratization" policies which increase the cost of hiring expatriates in proportion to nationals. Even though the cost function is not concave in X (Lemma 1), the calibration still generates a unique interior steady state equilibrium. The calibration also yields a corner solution in which the firm does not hire any nationals. The level of the cost parameter, c, determines which solution generates a higher profit for the firm. If $c < 0.021$, then it is more profitable for the firm not to hire any nationals given the baseline parameters specified in Table 1. This section also analyzes numerically the consequences of a variation in the exogenous level of government employment on the private sector hiring of nationals and expatriates.

6.1 Baseline Parameters

The data is extracted from the UAE Household Expenditure survey of 2008, where 73.8% of nationals are employed in the public sector and only 7.4% are working in the private sector[24]. Expatriates working in the private sector constitute almost five times the population of nationals[25]. Average annual private sector wages for nationals and expatriates are AED 140,000 and AED 63,000 respectively. Nationals employed in the private sector earn on average more than twice as much as expatriates. Average annual public sector wages reach AED 191,000. The flow value of unemployment for nationals is taken to be the log of the lowest wage earned by private national workers[26]. The average separation rate for nationals working in either sector, δ, is calculated from the Dubai Labor Force Survey of 2008[27]. The interest rate, r, represents the interest rate on inter-bank deposits for 2008. The time period in the calibration is one year.

The bargaining power, β, and the search intensity, λ, are set respectively to 0.5 and 0.2[28]. Further, the private sector matching function for nationals is assumed to take a Cobb-Douglas form of $m_p(\theta_p) = \theta_p^{1/2}$, while the public sector matching function is set to be $m_g(\theta_g) = \theta_g$ (as in Quadrini and Trigari 2007). The choice of the public sector matching function is consistent with the queuing of nationals for government jobs because the number of individuals searching for such

[24] We have restricted the population of nationals to those who are working or unemployed and are of skill levels 1,2 or 3 according to the International Standard Classification of Occupations "ISCO-88". We exclude skill level 4 because it is not a credible assumption that the labor market for expatriates is frictionless for that particular skill level.

[25] The population of nationals is normalized to one.

[26] We take the log of wages when we calibrate the model. Therefore, the productivity and search cost which are inferred from the model will be in logs too.

[27] In reality, the separation rate is larger in the private sector; however, for mathematical simplicity, the separation rate is assumed to be the same for both sectors. This does not alter our results. Moreover, the Dubai labor force data provides elapsed durations as opposed to completed durations. However, in such models, we typically assume that there is no duration dependence.

We have used data on Dubai to calculate the separation rate. Dubai is one of the seven emirates which make up the UAE. It is reasonable to calculate the separation rates using data from Dubai because the seven emirates are fairly homogeneous in the regard.

[28] If the search intensity is higher than 0.2, then the only way that firms will hire 7.4% of the nationals is if nationals' productivity exceeds that of expatriates.

jobs is larger than the posted vacancies.

It now remains to identify the nationals' and expatriates' productivity, the cost parameter, c, and the search cost, γ. The cost parameter cannot be extracted easily from the government regulations; and therefore, the model will be used to infer both c and γ. Moreover, we assume the productivity of nationals to be the same as expatriates because we are interested mainly in analyzing the effect of the "Emiratization" policies on the hiring of nationals irrespective of their productivity[29]. Based on this assumption, productivity is also deduced from the model.

Table 1: Baseline Case

Observed	**Values**	**Source**
E_g	0.7381	UAE Household Expenditure Survey - 2008
N	0.0741	UAE Household Expenditure Survey - 2008
X	4.6809	UAE. Household Expenditure Survey - 2008
w_g	12.16	UAE. Household Expenditure Survey - 2008
w^N	11.85	UAE. Household Expenditure Survey -2008
w^X	11.05	UAE. Household Expenditure Survey -2008
b	9.86	UAE. Household Expenditure Survey - 2008
r	0.04	UAE. Central Bank
δ	0.1	Dubai Labor Force - 2008
Assumptions	**Values**	
λ	0.2	
β	0.5	
$m_p(\theta_p)$	$\theta_p^{1/2}$	
$m_g(\theta_g)$	θ_g	
Calibrated	**Values**	
$Y^N = Y^X$	11.42	Parameters
γ	31.31	Parameters
θ_p	0.0047	Endogenous
c	0.0374	Parameter
θ_g	0.3643	Endogenous
v_g	0.0738	Endogenous
v_p	0.0009	Endogenous

[29] The productivity of expatriates is in reality believed to be higher than that of nationals. However, by assuming similar productivity, we do not alter our results with respect to the effect of the cost parameter on the hiring decisions of the firm.

6.2 Results

We conduct policy experiments using the baseline specification above by varying: i) the cost parameter; and ii) public sector employment[30].

6.2.1 Variation in the cost parameter c

A 1% increase in c decreases national employment in the private sector by 0.71 percentage points (p.p.) relative to its baseline value, and subsequently, the unemployment rate increases by an equivalent amount since government employment level is fixed. Moreover, as a response to the increase in c, the firm also reduces expatriate employment by 21%. As fewer expatriates are employed, there is less need to hire nationals which affects their bargaining position vis-à-vis the private sector and weakens the Stole and Zwiebel effect. As a result, nationals' reservation value falls and they are willing to accept a 14% cut in their wages.

Alternatively, if the government decreases c by 1%, overall private sector employment rises and unemployment among nationals decreases by 1.1 p.p. However, if c becomes too low, then it becomes more profitable for the firm not to hire any nationals. Therefore, the level of c that maximizes national employment in the private sector is attained when the firm is indifferent between hiring both types of workers and employing only expatriates.

Given the parameters specified in Table 1, the optimal cost parameter, c, is calculated to be 2.077%. At this level, national employment in the private sector reaches its maximum value of 9.54%. Thus, even if the "Emiratization" policies are set in such a way as to achieve the optimal c, they are not sufficient in promoting national employment in the private sector.

[30] Dynamic effects are ignored in this paper.

Table 2: Variation in the cost parameter

Variation in c	$\mathbf{c = 0.02077^*}$	$\mathbf{c = 0.0274}$	$\mathbf{c = 0.0374}$	$\mathbf{c = 0.0474}$
U	**0.1665** ($-2.13 p.p.$)	**0.1771** ($-1.1 p.p.$)	**0.1878**	**0.1949** ($+0.71 p.p.$)
N	**0.0954** ($+2.13 p.p.$)	**0.0848** ($+1.1 p.p.$)	**0.0741**	**0.067** ($-0.71 p.p$)
X	**8.589** ($+83.49\%$)	**6.455** ($+37.9\%$)	**4.6809**	**3.668** (-21.6%)
$\mathbf{w}^N$	**12.43** ($+58\%$)	**12.1** ($+25\%$)	**11.85**	**11.71** (-14%)
rU	**12.11**	**11.9**	**11.75**	**11.67**
$\boldsymbol{\theta}_p$	**0.0106**	**0.0071**	**0.0047**	**0.0034**
$\boldsymbol{\theta}_g$	**0.3978**	**0.3804**	**0.3643**	**0.3543**
$\mathbf{v}_p$	**0.0018**	**0.0013**	**0.0009**	**0.00067**
$\mathbf{v}_g$	**0.0738**	**0.0738**	**0.0738**	**0.0738**
Pr **ofit**	**1.5904** ($+79\%$)	**1.2116** ($+36\%$)	**0.89**	**0.7** (-21%)

6.2.2 Variation in Public Sector Employment

Table 3 evaluates the impact of a change in public sector employment on the equilibrium values of private sector wages and employment composition. A 10% increase in E_g lowers private sector employment of nationals by 3.54 p.p. since nationals would move from the private to the public sector as more government jobs are made available. As a result, the firm has to reduce its intake of expatriates as fewer nationals are employed within the firm.

National private sector wages decrease by 2% despite the increase in the nationals' outside option. This is due to the reduction in expatriate employment which weakens the bargaining position of nationals.

Table 3: Variation in Public Sector Employment

Variation in E_g	**E_g= 0.6643**	**E_g= 0.7381**	**E_g= 0.8119**
c	**0.0374**	**0.0374**	**0.0374**
U	**0.2193** ($+3.15 p.p.$)	**0.1878**	**0.1494** ($-3.84 p.p.$)
N	**0.1164** ($+4.23 p.p.$)	**0.0741**	**0.0387** ($-3.54 p.p.$)
X	**4.8652** ($+3.94\%$)	**4.6809**	**4.5268** (-3.29%)
$\mathbf{w}^N$	**11.89** ($+4\%$)	**11.85**	**11.83** (-2%)
rU	**11.76**	**11.75**	**11.79**
$\boldsymbol{\theta}_p$	**0.0067**	**0.0047**	**0.0028**
$\boldsymbol{\theta}_g$	**0.2738**	**0.3643**	**0.5167**
$\mathbf{v}_p$	**0.0015**	**0.0009**	**0.0004**
$\mathbf{v}_g$	**0.0664**	**0.0738**	**0.0812**
Pr **ofit**	**0.8854** (-4%)	**0.8899**	**0.8893** (-0.03%)

7 Conclusion

In this paper, we develop a two-sector search model with intrafirm bargaining in the private sector to analyze "Emiratization" policies in the UAE. The "Emiratization" policies aim to encourage national employment in the private sector by penalizing the firm for hiring expatriates at the expense of nationals. Therefore, we model the policies as a form of cost incurred by the firm which depends on the employment levels of both nationals and expatriates and on an exogenous cost parameter set by the government.

The contribution of our paper is twofold. First, departing from prior literature, we base our assumptions on cost rather than productivity to analyze the employment effect on wages set forth by Stole and Zwiebel. Second, we explicitly model the impact of the "Emiratization" policies on the bargaining process within the firm and on private sector employment.

We first determine the private sector wage of nationals and find that the wage value is different from the one obtained in the standard matching model even with constant productivity. This is due to the "Emiratization" policies which generate an employment effect on wages.

In addition, if the "Emiratization" policies bind, i.e., so long as c is sufficiently large, we prove

the existence of an interior equilibrium where both nationals and expatriates are hired by the firm. Further, we show that the uniqueness of the equilibrium depends on the curvature of the cost function.

We also consider the implication of "Emiratization" policies on labor market outcomes. We find that under certain conditions pertaining to the specification of the cost function, an increase in the penalty for employing expatriates can decrease private sector employment of nationals and expatriates. Moreover, we find that an increase in public sector employment also lowers the employment levels of nationals and expatriates within the firm.

Finally, we deduce from the calibration the cost level for which private sector employment is maximized given the baseline parameters of the model. We find that an increase in the cost level lowers national employment in the private sector; however, if the cost level becomes too low, then it becomes more profitable for the firm not to hire any nationals.

Many extensions arise from our paper. We have assumed throughout our analysis that the firm has an unlimited access to expatriates. However, this assumption does not hold for the highest skilled workers. It would be therefore interesting to account for frictions in the expatriate labor market and apply the intrafirm bargaining process to a model of international migration. In addition, we have also assumed a continuous cost function to model the "Emiratization" policies and capture the employment effect on wages. Removing the continuity assumption would alter the results of the paper in that the pivotal national would now have the highest bargaining position vis-à-vis the firm thereby affecting all wages and subsequently the firm's employment choices.

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8 Appendix A: Existence of the Equilibrium

8.1 Proof of Proposition 1

Proposition 1 establishes the existence of at least one equilibrium where both types of workers are hired. The values of N, U, X,and θ_g can all be expressed in terms of θ_p from equations (12), (13), (14) and (15). Therefore, the existence of at least one equilibrium depends on examining the first order condition of nationals when θ_p changes. The first order condition can be expressed as $rV = \gamma$, where rV is expressed as:

$$rV = \frac{m_p(\theta_p)}{\theta_p}\frac{(1-\beta)}{\beta}\frac{[\beta(Y^N(r+\delta+m_g(\theta_g))-\beta(1-\lambda)m_g(\theta_g)w_g-\beta b(r+\delta+\lambda m_g(\theta_g))-(r+\delta+m_g(\theta_g))cN^{-\frac{1}{\beta}}\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du]}{(\delta+r+\lambda m_g(\theta_g))(r+\delta+m_g(\theta_g)+\frac{1}{2}m_p(\theta_p))}$$

It can be shown that as $\theta_p \to 0$ then $rV \to \infty$, and as $\theta_p \to \infty$ then $rV \to 0$. Therefore, rV crosses the search cost, γ, at least once.

8.2 Proof of Lemma 1

Lemma 1 determines the sufficient condition for which the equilibrium is unique. The equilibrium is unique if rV is decreasing in θ_p.

Using the Chain Rule:

$$\frac{\partial rV}{\partial \theta_p} = \frac{\partial rV}{\partial \theta_p} + \frac{\partial rV}{\partial \theta_g}\frac{d\theta_g}{d\theta_p} + \frac{\partial rV}{\partial N}\frac{dN}{d\theta_g}\frac{d\theta_g}{d\theta_p} + \frac{\partial rV}{\partial X}\frac{\partial X}{\partial N}\frac{dN}{d\theta_g}\frac{d\theta_g}{d\theta_p}$$

It can be shown that $\frac{drV}{d\theta_p} + \frac{drV}{d\theta_g}\frac{d\theta_g}{d\theta_p} < 0$ without imposing any assumptions on f_{XX}.

Proof:

Define $S_g = w_g - rU^N \geq 0$ as the surplus from filling a public sector job, and $S_p = N^p + J^p - U^N \geq 0$ as the surplus from filling a private sector job with a UAE national. S_g and S_p must be non-negative for a match to occur.

Substituting the appropriate values yields:

$S_g = w_g(r+\delta)(r+\delta+\lambda m_g(\theta_g)+\beta m_p(\theta_p)) - b(r+\delta)(r+\delta+\lambda m_g(\theta_g)) - (r+\delta)m_p(\theta_p)[\beta Y^N - cN^{-\frac{1}{\beta}}\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du] \geq 0$

$S_p = (r+\delta)(r+\delta+m_g(\theta_g))[\beta Y^N - cN^{-\frac{1}{\beta}}\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du] - \beta(r+\delta)(1-\lambda)m_g(\theta_g)w_g - \beta b(r+\delta)(r+\delta+\lambda m_g(\theta_g)) \geq 0$

After differentiation, substitution and some mathematical manipulations we get:

$$\frac{\partial rV}{\partial \theta_p} = \frac{(1-\beta)[\frac{d(\frac{m_p(\theta_p)}{\theta_p})}{d\theta_p}(r+\delta+m_g(\theta_g)+\beta m_p(\theta_p)\ ) - \beta\frac{m_p(\theta_p)}{\theta_p}\frac{d(m_p(\theta_p))}{d\theta_p}]S_p}{(\delta+r+\lambda m_g(\theta_g))(r+\delta+m_g(\theta_g)+\beta m_p(\theta_p))^2} < 0$$

$$\frac{\partial rV}{\partial \theta_g} = -\frac{(1-\beta)\frac{m_p(\theta_p)}{\theta_p}\frac{d(m_g(\theta_g))}{d\theta_g}[\lambda S_p((r+\delta+m_g(\theta_g)) + S_g(1-\lambda)]}{(\delta+r+\lambda m_g(\theta_g))(r+\delta+m_g(\theta_g)+\frac{1}{2}m_p(\theta_p)\ )^2} < 0$$

$$m_g(\theta_g^*) = \frac{-(\lambda m_p(\theta_p)(1-E_g)+\delta(1-E_g)-\lambda\delta E_g)+\sqrt{(\lambda m_p(\theta_p)(1-E_g)+\delta(1-E_g)-\lambda\delta E_g)^2+4\delta E_g\lambda(1-E_g)}(m_p(\theta_p)+\delta)}{2\lambda(1-E_g)}$$

$$\frac{d\theta_g}{d\theta_p} = [\frac{\frac{d(m_p(\theta_p))}{d\theta_p}}{\frac{d(m_g(\theta_g))}{d\theta_g}}][-\frac{1}{2}+\frac{1}{2}\frac{(\lambda m_p(\theta_p)(1-E_g)+\delta(1-E_g)-\lambda\delta E_g+2E_g)}{\sqrt{(\lambda m_p(\theta_p)(1-E_g)+\delta(1-E_g)-\lambda\delta E_g)^2+4\delta E_g\lambda(1-E_g)}(m_p(\theta_p)+\delta)}] > 0$$

For the equilibrium to be unique, it is sufficient to show that $\frac{\partial rV}{\partial N}\frac{dN}{d\theta_g}\frac{d\theta_g}{d\theta_p}+\frac{\partial rV}{\partial X}\frac{\partial X}{\partial N}\frac{dN}{d\theta_g}\frac{d\theta_g}{d\theta_p} < 0$

Proof:

$$\frac{\partial rV}{\partial N} = \frac{(1-\beta)}{\beta}\frac{m_p(\theta_p)}{\theta_p}\frac{(\delta+r+m_g(\theta_g))cN^{-\frac{1}{\beta}}(\frac{1}{\beta}N^{-1}\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du - \frac{d}{dN}(\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du))}{(\delta+r+\lambda m_g(\theta_g))(r+\delta+m_g(\theta_g)+\beta m_p(\theta_p)\ )}$$

$$\frac{\partial rV}{\partial X} = -\frac{(1-\beta)}{\beta}\frac{m_p(\theta_p)}{\theta_p}\frac{(\delta+r+m_g(\theta_g))cN^{-\frac{1}{\beta}}(\int_0^N u^{\frac{1-\beta}{\beta}}f_{uX}(u,X)du)}{(\delta+r+\lambda m_g(\theta_g))(r+\delta+m_g(\theta_g)+\frac{1}{2}m_p(\theta_p)\ )}$$

$\frac{\partial rV}{\partial N} < 0$ because $\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du < \beta N\frac{d}{dN}(\int_0^N u^{\frac{1-\beta}{\beta}}f_u(u,X)du)$ due to the convexity of $f(N,X)$ with respect to N (Stole and Zwiebel effect).

Moreover, the first order condition of expatriates $\frac{(Y^X-w^X-r)}{c} = f_X(N,X)$ is used to determine $\frac{\partial X}{\partial N}$.

$$\frac{\partial X}{\partial N} = -\frac{f_{XN}(N,X)}{f_{XX}(N,X)}$$

Replacing $(\frac{\partial rV}{\partial N}+\frac{\partial rV}{\partial X}\frac{dX}{dN})\frac{dN}{d\theta_g}\frac{d\theta_g}{d\theta_p}$ by the appropriate values yields the following sufficient

condition for the existence of a unique equilibrium.

$$\frac{f_{XN}(N,X)}{f_{XX}(N,X)} \int_0^N u^{\frac{1-\beta}{\beta}} f_{uX}(u,X)du - \frac{d}{dN}(\int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du + \frac{1}{\beta}N^{-1} \int_0^N u^{\frac{1-\beta}{\beta}} f_u(u,X)du \leq 0$$

A sufficient condition for the expression above to be negative is $f_{XX}(N,X) < 0$.