Strategic Vertical Separation under Diseconomy of Scale*†

Igor Sloev‡

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Abstract

The paper analyzes vertical separation in a duopoly with decreasing returns to scale. It demonstrates, in contrast to classical results, the existence of asymmetric equilibria, where one firm separates and another integrates, whenever diseconomy of scale is high for at least one firm. Moreover, if the diseconomy of scale is low for one firm and moderate for another, a more efficient firm separates in a unique equilibrium, while its rival integrates. Also, the research herein shows that asymmetric equilibria exist even in a completely symmetric game. This provides an explanation for a widely observed difference in firm sales strategies.

JEL classification: L11, L21, L22, L25

Keywords: vertical oligopoly, vertical separation, vertical integration

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‡Humboldt University at Berlin, Institute of Economic Theory I, Spandauer str. 1, Berlin, Germany; sloevigo@staff.hu-berlin.de.

1 Introduction

Commonly, *strategic separation* refers to a manufacturer's decision to sell goods through an exclusive retailer, who makes the final decision on quantity or price. *Strategic delegation* means that an owner delegates a decision on final output or price to a manager within the firm. If the agent (the retailer or the manager) has no bargaining power, then a principal (the manufacturer or the owner) completely controls his/her objective through contract terms. The strategic role of both separation and delegation remains the same: by manipulating the agent's objective, the principal alters its agent's as well as its rivals' behaviors to achieve a more preferable outcome. In contrast to separation, *integration* implies that the profit maximizing principal makes the decision on final output or price directly.

Classical works of Vickers (1985), Sklivas (1987), Fershtman and Judd (1987), and Bonanno and Vickers (1988) demonstrate that delegating a decission to an agebt is in the interest of each principal.[1] Hence, in a unique equilibrium, each principal takes advantage of delegation, and coexistence of vertical separation and vertical integration never occurs as an equilibrium outcome. This result proves robust in respect to the nature of a competition (price or quantity competition), number of firms, and a demand function specification. Moreover, the result holds under different assumptions of agents' objectives: linear combinations of profit and quantity (Vickers, 1985), profit and revenue (Sklivas 1987; Fershtman and Judd, 1987), or retailer profit (Bonanno and Vickers, 1988). More recently, Jansen et al. (2007) and Ritz (2008) assume that agents maximize a linear combination of the principal's profit and market share and also maintain that in a unique equilibrium each owner delegates the decision to a manager.

Basu (1995) and Jansen (2003) show one possibility of the coexistence of vertical separation and vertical integration. To obtain the result, the authors assume that separated firms bear an additional fixed cost. This effectively implies that separation alters a firm's technology. Thus, in their models, separation relates to both a choice of technology and a choice of the agent's incentive scheme.

A common feature of all the models mentioned above is the assumption of linearity of cost functions. This paper extends the existing analysis by allowing for decreasing return to scales. We analyze a case of duopolists competing in quantities and, following a standard approach, assume that each separated firm uses a two-part tariff in a trade with its retailer.

Under mild assumptions on demand and firm costs, we demonstrate that vertical separation and vertical integration coexist whenever a cost function of at least one firm exhibits a sufficient degree of scale diseconomy (or, in other words, when a marginal cost curve is steep enough). Furthermore, we show the existence of asymmetric equilibria in a perfectly symmetric game. The intuition for our results follows. If only one firm separates, then, effectively, it has the advantage of first mover and makes a Stackelberg leader's profit. Thus, each firm prefers to separate given that its rival integrates. If both firms separate, it may create strong competition

[1] If firms compete in prices, then delegation leads to a Pareto efficient outcome; while if firms compete in quantities, it lowers the profit of at least one firm.

between their retailers and, under a decreasing economy of scale, it may lead to profits lower than a Stackelberg follower's profit. In this case, each firm prefers to integrate given its rival separates, and, therefore, there exist two equilibria, where one firm separates and another integrates.

Moreover, we provide a closed form solution for the case of linear demand and quadratic cost. We show that, in addition to the previous result, if diseconomy of scale is low for one firm and moderate for another, then there exists a unique equilibrium where the more efficient firm separates and the less efficient integrates. Welfare analysis shows that both consumer surplus and total welfare are higher if both firms separate; thus, asymmetric equilibria are not optimal.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes subgame outcomes. Section 3 provides equilibrium analysis and the main results for the general case. Section 4 examines the case of linear demand and quadratic costs, and, finally, Section 5 concludes. The Appendix contains all proofs.

2 Model

We now consider two firms, $i = 1, 2$, producing homogeneous goods and competing in quantities. A demand function, $P(Q)$, with $Q = q_1 + q_2$, where q_i is an output of firm i, and cost functions, $C_i(q_i)$, $i = 1, 2$, are such that:

A1. $\exists \overline{Q} > 0$: $P(Q) > 0$ for $Q \in [0, \overline{Q})$ and $P(Q) = 0$ for $Q \geq \overline{Q}$; $P''(Q)$ is continuous; $P(0) = \overline{P} > 0$, $-P'(Q) > \delta > 0$, $P'(Q) + P''(Q)q_i < 0$ for $Q \in [0, \overline{Q})$.

A2. $C_i(q)$ is a twice continuously differentiable increasing convex function, $C_i(0) = 0$, $C_i'(0) = 0$, $0 < C_i''(Q) < b$ for all $q_i \in (0, \overline{Q}]$ and some $b > 0$.

A3. $P^{(3)}(Q) \geq 0$ for all $Q \in [0, \overline{Q})$.

Assumptions A1-A2 offer sufficient conditions for the existence of a unique equilibrium in a Cournot game[2] and together with Assumption 3 ensure existence of a solution in a whole game.

We consider the following two stage game. At the first stage, each firm i chooses $m_i \in \{Separate, Integrate\} \equiv \{S, I\}$. If $m_i = I$, then firm i becomes the retailer of its goods. If $m_i = S$, then firm i sets the terms of a two-part tariff contact $\{\omega_i, A_i\}$ where ω_i is a per unit price, and A_i is a franchise fee.

At the second stage, all decisions are observed[3] and retailers choose quantities simultaneously and independently to maximize their own profits. The profit of the integrated firm i is $\pi_i = P(Q)q_i - C_i(q_i)$. If firm i separates, its own and its retailer's profits are $\pi_i^F = \omega_i q_i(\omega_i) + A_i - C_i(q_i(\omega_i))$ and $\pi_i^R = P(Q)q_i - \omega_i q_i - A_i$, respectively. Thus, $\pi_i(q_1, q_2) = \pi_i^F(q_1, q_2) + \pi_i^R(q_1, q_2)$, i.e., separation does not alter production technology, it only changes the retailer's objective.

[2] See Van Long and Soubeyranb (2000) for details.

[3] We assume that decisions are observable and irreversible and thus there is no commitment problem. For a discussion of observability and commitment in a delegation game see Katz (1991), Bagwell (1995), Corts and Neher (2003).

Let $\Gamma(P, C_1, C_2)$ denote the game described above for a given demand function $P(Q)$ and cost functions $C_1(q_1)$, $C_2(q_2)$. We use is the subgame perfect Nash Equilibrium in pure strategies as the solution concept.

Subgame outcomes

There are four subgames depending on the choice $m_i \in \{S, I\}, i = 1, 2$ of each firm at the first stage. If both firms integrate, they play the Cournot game. A unique equilibrium is determined by first order conditions

$$\begin{cases} P'q_i + P - C_i' = 0 \\ i = 1, 2. \end{cases} . \tag{1}$$

Let $\{q_1^{*C}, q_2^{*C}\}$ and $\{\pi_1^{*C}, \pi_1^{*C}\}$ denote equilibrium values and let q_i^{*M} be an output of firm i if it were a monopolist.

If firm 1 separates and firm 2 integrates, then the retailers' game[4]

$$\begin{cases} \max\limits_{q_1} \pi_1^R = Pq_1 - \omega_1 q_1 \\ \max\limits_{q_2} \pi_2^R = Pq_2 - C_2 \end{cases} \tag{2}$$

has first-order conditions

$$\begin{cases} P'q_1 + P - \omega_1 = 0 \\ P'q_2 + P - C_2' = 0 \end{cases} .$$

Retailer 2 has the Cournot reaction curve, while the position of retailer 1's reaction curve depends on firm 1's choice of ω_1. Thus, by choosing ω_1, firm 1 determines a point of intersection of reaction curves. Clearly, the optimal ω_1 is such that an equilibrium outcome replicates the Stackelberg outcome of the $[I, I]$-subgame. Therefore, we may characterize the solution of the $[S, I]$-subgame as the following: $q_2^F(q_1)$ solves

$$P'q_2 + P - C_2' = 0$$

and q_1^L is such that

$$P'q_1 + P - C_1' + P'q_1 \frac{\partial q_2^F(q_1)}{\partial q_1} = 0. \tag{3}$$

[4] Clearly, in equilibrium each separated firm entirely extracts its retailer's profit using a franchise fee, and the retailer gets zero profit. Thus, to keep notation brief, we skip A_i henceforth.

By the implicit function theorem, we have that

$$\frac{\partial q_2^F}{\partial q_1} = -\frac{P'' q_2^F + P'}{P'' q_2^F + 2P' - C_2''}.$$

Let ω_1^{*L}, $\{q_1^{*L}, q_2^{*F},\}$ and $\{\pi_1^{*L}, \pi_2^{*F}\}$ be equilibrium values of the $[S, I]$-subgame. Applying similar arguments and notation to the $[I, S]$-subgame, let ω_2^{*L}, $\{q_1^{*F}, q_2^{*L},\}$ and $\{\pi_1^{*F}, \pi_2^{*L}\}$ be equilibrium values of the $[I, S]$-subgame.

If both firms separate, then retailers' maximization problems are

$$\max_{qi} \pi_i^R = Pq_i - \omega_i q_i, i = 1, 2$$

where ω_1, ω_2 are set by firms at the previous stage. As the feasibility constraint, $q_i \geq 0$, implies that $q_i = 0$ for all $\omega_i \geq \overline{P}$, we assume without loss of generality that $\omega_i \leq \overline{P}$. Then, a solution of the retailers' problem is determined by the system of first order conditions:

$$\begin{cases} P' q_1 + P - \omega_1 = 0 \\ P' q_2 + P - \omega_2 = 0 \end{cases}. \tag{4}$$

The Jacobian matrix of (4) is

$$\mathbf{J} = \begin{pmatrix} P'' q_1 + 2P' & P'' q_1 + P' \\ P'' q_2 + P' & P'' q_2 + 2P' \end{pmatrix},$$

with $\det(\mathbf{J}) > 0$ for any (q_1, q_2) and by the implicit function theorem we have that

$$\frac{\partial \mathbf{q}}{\partial \boldsymbol{\omega}} = \frac{1}{\det(\mathbf{J})} \begin{pmatrix} P'' q_2 + 2P' & -(P'' q_1 + P') \\ -(P'' q_2 + P') & P'' q_1 + 2P' \end{pmatrix}, \tag{5}$$

where $\mathbf{q} = (q_1, q_2)$ and $\omega = (\omega_1, \omega_2)$.

Let $\{q_1^s(\omega_1, \omega_2), q_2^s(\omega_1, \omega_2)\}$ denote the solution of (4). Then optimal values of ω_1, ω_2 satisfy:

$$\frac{\partial \pi_i^S}{\partial \omega_i} = P' \left(\frac{\partial q_i^s}{\partial \omega_i} + \frac{\partial q_j^s}{\partial \omega_i} \right) q_i^s + P \frac{\partial q_i^s}{\partial \omega_i} - C_i' \frac{\partial q_i^s}{\partial \omega_i} = 0, i \neq j. \tag{6}$$

Directly differentiating (6) in respect to ω_i and using (5) one can obtain that $\partial^2 \pi_i / \partial \omega_i^2 < 0$ under Assumptions 1 and 2 and provided $P^{(3)}(Q) \geq 0$. Therefore, Assumptions 1-3 ensure the existence of a pure strategy equilibrium in the $[S, S]$-subgame. It is convenient to rewrite (6) in the form

$$P' q_i^s + P - C_i' + P' q_i^s \left(\frac{\partial q_j^s}{\partial \omega_i} / \frac{\partial q_i^s}{\partial \omega_i} \right) = 0,$$

where

$$\frac{\partial q_j^s}{\partial \omega_i} / \frac{\partial q_i^s}{\partial \omega_i} = -\frac{P'' q_j^s + P'}{P'' q_j^s + 2P'}. \tag{7}$$

Let $\{q_1^{*S}, q_2^{*S}\}$ and $\{\pi_1^{*S}, \pi_1^{*S}\}$ be the equilibrium values in the $[S, S]$-subgame. Now we may represent the game as the following table.

		Firm 2	
		Separate	*Integrate*
	Separate	π_1^{*S}, π_2^{*S}	π_1^{*L}, π_2^{*F}
Firm 1	*Integrate*	π_1^{*F}, π_2^{*L}	π_1^{*C}, π_2^{*C}

3 Equilibrium

First, we note that the Stackelberg leader's profit always exceeds the Cournot profit, $\pi_i^{*L} > \pi_i^{*C}$, and therefore we have the following result.

Proposition 1 *The $[I, I]$-subgame is never played in equilibrium.*

The equilibrium is determined by a relation of manufacturers' profits π_i^{*L}, π_i^{*F}, and π_i^{*S}. More specifically, in the equilibrium, $[S, I]$ is played if $\pi_2^{*F} \geq \pi_2^{*S}$; $[I, S]$ if $\pi_1^{*F} \geq \pi_1^{*S}$; $[S, S]$ if both $\pi_1^{*F} \leq \pi_1^{*S}$ and $\pi_2^{*F} \leq \pi_2^{*S}$ hold.

Now, we consider a family of games $\{\Gamma(P, C_1, \alpha C_2)\}|_{\alpha>0}$.

Lemma 1 *As $\alpha \to \infty$, both $q_2^{*F}(q_1, \alpha)$ and $\partial q_2^F / \partial q_1(q_1, \alpha)$ uniformly converges to zero.*

Lemma's statement is quite intuitive: the steeper is firm 2's marginal curve (the greater is α), the more negligibly are both its output, $q_2^{*F}(q_1, \alpha)$, and its response to changes in q_1, $\partial q_2^F / \partial q_1(q_1, \alpha)$, for any $q_1 \in [0, \overline{Q})$.

In general, separation has two effects. First, it allows a firm to manipulate its retailer's reaction curve and thus gives a strategic advantage to a separated firm. Second, it increases competition between retailers. This occurs because retailers' reaction curves (determined by (4)) are steeper than firms' reaction curves (determined by (1)). While the first always increases the firm's profit, the second harms it. Thus, a decision on separation depends on which effect dominates. The following proposition states that if a firm's cost function exhibits a high enough diseconomy of scale, then the firm prefers to integrate given that its rival separates.

Proposition 2 *$\exists \overline{\alpha} > 0$ such that for any $\alpha \geq \overline{\alpha}$ the game $\Gamma(P, C_1, \alpha C_2)$ has equilibrium with $[S, I]$ played at the first stage.*

The intuition for the result follows. As firm 2's cost function exhibits high diseconomy of scale, an advantage of manipulating the retailer's objective is small, while more aggressive behavior by retailer 1 lowers firm 2's profits below the Stackelberg follower's profits. Thus, firm 2 prefers to integrate given that firm 1 separates.

In contrast to Proposition 2, Proposition 3 states that if firm 2 being inefficient separates, then firm 1 prefers to integrate.

Proposition 3 $\exists\widehat{\alpha} > 0$ *such that for any* $\alpha \geq \widehat{\alpha}$ *the game* $\Gamma\left(P, C_1, \alpha C_2\right)$ *has equilibrium with* $[I, S]$ *played at the first stage.*

This occurs for the following reason. As firm 2 is inefficient, firm 1, being a Stackelberg follower, achieves almost monopolistic profits. If it separates, its own and firm 2's retailers compete more aggressively, which leads to an excessive output and significantly lowers firm 1's profits. Thus, firm 1, operating even more efficiently, prefers to integrate given that firm 2 separates.

Combining Propositions 2 and 3, we arrive at a Corollary that states that *two equilibria* exist whenever at least *one firm* has a sufficiently steep marginal curve.

Corollary 1 *For any* $\alpha \geq \max\{\widehat{\alpha}, \overline{\alpha}\}$ *the game* $\Gamma\left(P, C_1, \alpha C_2\right)$ *has two equilibria where one firm separates and another integrates.*

Note that Corollary 1 does not imply existence of asymmetric equilibria in a symmetric game. To cover this case, we formulate the following proposition.

Proposition 4 $\exists\widehat{\alpha} > 0$ *such that for any* $\alpha > \widehat{\alpha}$ *the symmetric game* $\Gamma\left(P, \alpha C, \alpha C\right)$ *has two asymmetric equilibria with* $[S, I]$ *and* $[I, S]$ *played at the first stage.*

Proposition 4 says that in the symmetric game, whenever the slop of marginal cost curve for each firm is steep enough, a loss from more aggressive retailers' behavior dominates a gain from separation. Thus, each firm prefers to integrate given that its rival separates, and hence there exist two asymmetric equilibria in a symmetric game.

4 Linear demand and quadratic costs

This section provides analysis for the case of linear demand, $P(Q) = 1 - Q$, and quadratic cost functions, $C_i(q_i) = \gamma_i q_i^2/2$, where $\gamma_i \geq 0$. This form of demand and cost functions allows us, first, to capture the effects of scale diseconomies on the equilibrium structure and, second, to obtain a closed-form solution.

Using (1) we get the solution of the $[I, I]$-subgame:

$$q_i^{*C} = \frac{1+\gamma_j}{(3+2\gamma_i+\gamma_j\gamma_i+2\gamma_j)} \text{ and } \pi_i^{*C} = \frac{(2+\gamma_i)(1+\gamma_j)^2}{2(3+2\gamma_j+\gamma_j\gamma_i+2\gamma_i)^2}, i, j = 1, 2.$$

If firm 1 separates and firm 2 integrates, the retailers' profit maximization problems (3) have the following solution:

$$\begin{cases} q_1^L = \frac{1+\gamma_2-\omega_1(2+\gamma_1)}{3+2\gamma_2} \\ q_2^F = \frac{1+\omega_1}{3+2\gamma_2} \end{cases}. \tag{8}$$

Plugging (8) into firm 1's profits and optimizing with respect to ω_1, we obtain:

$$\omega_1^{L*} = \frac{(1+\gamma_2)(2\gamma_1 + \gamma_{1\gamma 2} - 1)}{(2+\gamma_2)(2+2\gamma_2+\gamma_1\gamma_2+2\gamma_1)}.$$

Equilibrium quantities and profits are given, respectively, by:

$$\begin{cases} q_1^{L*} = \frac{1+\gamma_2}{(2+2\gamma_2+\gamma_1\gamma_2+2\gamma_1)} \\ q_2^{F*} = \frac{1+\gamma_2+2\gamma_1+\gamma_1\gamma_2}{(2+\gamma_2)(2+2\gamma_2+\gamma_1\gamma_2+2\gamma_1)} \end{cases}$$

and

$$\begin{cases} \pi_1^L = \frac{(1+\gamma_2)^2}{2(2+\gamma_2)(2+2\gamma_2+\gamma_1\gamma_2+2\gamma_1)} \\ \pi_2^F = \frac{(1+\gamma_2+2\gamma_1+\gamma_1\gamma_2)^2}{2(2+\gamma_2)(2+2\gamma_2+\gamma_1\gamma_2+2\gamma_1)^2} \end{cases}.$$

If both firms vertically separate, first-order conditions of the retailers' profit maximization problems (4) give optimal outputs q_1, q_2 as functions of (ω_1, ω_2),

$$q_i = \frac{1-2\omega_i^s+\omega_j^s}{3}, i,j = 1,2,$$

and the total output and the final price, respectively:

$$Q = \frac{2-\omega_1^s-\omega_2^s}{3}, P = \frac{1+\omega_1^s+\omega_2^s}{3}.$$

First-order conditions of firms' maximization problem (6) determine firms' reaction curves in the space $\{\omega_1^s, \omega_2^s\}$:

$$\omega_i^s = \frac{(-1+2\gamma_i)(1+\omega_j^s)}{4(1+\gamma_i)}, i,j = 1,2. \tag{9}$$

Note that $\partial\omega_i^s/\partial\omega_j^s$ strictly increases in γ_i and does not depend on γ_j. Thus, the degree of substitution between ω_i^s and ω_j^s decreases in γ_i; if $\gamma_i < 1/2$, ω_i^s is a substitute for ω_j^s, and if $\gamma_i > 1/2$, ω_i^s is a complement to ω_j^s. Moreover, if $\gamma_i > 1/2$ and $\gamma_j < 1/2$, then ω_i^s is a complement to ω_j^s, whereas ω_j^s is a substitute for ω_i^s.

System (9) has the solution:

$$\omega_i^{s*} = \frac{2\gamma_i - 2\gamma_j + 4\gamma_i\gamma_j - 1}{5+6\gamma_i+6\gamma_j+4\gamma_i\gamma_j}, i,j = 1,2.$$

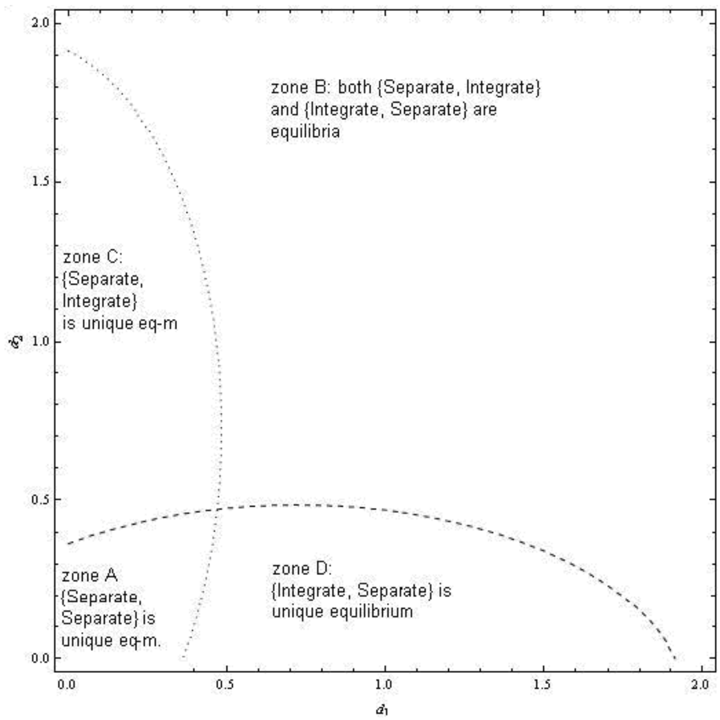
Respective equilibrium quantities and profits are given by:

$$q_i^S = \frac{2+4\gamma_j}{(5+6\gamma_i+4\gamma_i\gamma_j+6\gamma_j)},$$
$$\pi_i^S = \frac{2(1+\gamma_i)(1+2\gamma_j)^2}{(5+6\gamma_i+4\gamma_i\gamma_j+6\gamma_j)^2},$$
$$i,j=1,2.$$

We can easy see that $\pi_i^L > \pi_i^C > \pi_i^F$ and, as prescribed by Proposition 1, the outcome of the subgame $[I,I]$ is never played in SPNE. To determine the equilibrium outcome, we compare π_i^F and π_i^S. Consider the set (γ_1,γ_2) such that firm i is indifferent to separating and integrating, given that firm j separates:

$$\pi_i^F = \frac{(1+\gamma_i+2\gamma_j+\gamma_i\gamma_j)^2}{2(2+\gamma_i)(2+2\gamma_j+\gamma_i\gamma_j+2\gamma_i)^2} = \frac{2(1+\gamma_i)(1+2\gamma_j)^2}{(5+6\gamma_i+4\gamma_i\gamma_j+6\gamma_j)^2} = \pi_i^S. \quad (10)$$

It can be shown that $\gamma_i = \theta_i(\gamma_j)$, $i,j=1,2$ determined by (10) are such that: (i) $\gamma_i = \theta_i(\gamma_j), i,j=1,2$ are strictly concave and have a unique maximum, (ii) $\theta_i(0) > 0$ and (iii) $\exists\overline{\gamma_j} < +\infty : \theta_i(\overline{\gamma_j}) = 0$. Figure 1 gives a graphical representation of $\theta_i(\gamma_j)$ and $\theta_j(\gamma_i)$.



$\pi_2^F > \pi_2^S$ above the dashed line, $\pi_1^F > \pi_1^S$ right of the dotted line.

In zone A (low γ_1 and low γ_2), both firms have relatively flat marginal cost curves. Each firm prefers to separate given that its rival separates; hence, in the unique equilibrium, firms play $[S,S]$ at the first stage. The equilibrium profit of each firm is lower than in the Cournot equilibrium, yet higher than the Stackelberg follower's profit: $\pi_i^F < \pi_i^S < \pi_i^C$. Although inside zone A, firms may differ in efficiency, this difference ranks as sufficiently small. In zone C (low γ_1 and moderate γ_2), firm 1 is more efficient than the firm 2, but the difference in efficiency is not too high. Then the strategy *Separate* is dominant for firm 1, while firm 2 chooses *Integrate*, $\pi_1^F < \pi_1^S, \pi_2^F > \pi_2^S$. In zone D (low γ_1 and moderate γ_2), the situation is the opposite of that in zone C: firm 2 is more efficient than firm 1, and the difference in efficiency is not too high. The strategy *Separate* is dominant for firm 1, while firm 2 chooses *Integrate*: $\pi_1^F < \pi_1^S$, $\pi_2^F > \pi_2^S$. Finally, zone B is such that (either γ_1 or γ_2 or both are sufficiently high), either both firms are sufficiently inefficient, or the asymmetry in costs is very high. In this case, each firm integrates if its rival separates. Therefore, two asymmetric equilibria, with strategies $[I,S]$ and $[S,I]$, exist.

The following Proposition summarizes the above results:

Proposition 5 *At the first stage firms play*
(i) $[S,S]$ *if* $(\gamma_1,\gamma_2) \in A \equiv \{(\gamma_1,\gamma_2)|\, \gamma_1 \le \theta_1(\gamma_2), \gamma_2 \le \theta_2(\gamma_1)\}$*;*
(ii) either $[S,I]$ *or* $[I,S]$ *if* $(\gamma_1,\gamma_2) \in B \equiv \{(\gamma_1,\gamma_2)|\, \gamma_1 \ge \theta_1(\gamma_2), \gamma_2 \ge \theta_2(\gamma_1)\}$*;*
(iii) $[S,I]$ *if* $(\gamma_1,\gamma_2) \in C \equiv \{(\gamma_1,\gamma_2)|\, \gamma_1 \le \theta_1(\gamma_2), \gamma_2 \ge \theta_2(\gamma_1)\}$*;*
(iv) $[I,S]$ *if* $(\gamma_1,\gamma_2) \in D \equiv \{(\gamma_1,\gamma_2)|\, \gamma_1 \ge \theta_1(\gamma_2), \gamma_2 \ge \theta_2(\gamma_1)\}$*.*

Intuition for this result is the following. If firm 1 separates and firm 2 integrates, then they get Stackelberg leader and follower profits, respectively. Let's consider $\widetilde{\omega}_1$ and $\widetilde{\omega}_2$ which replicates this Stackelberg outcome in the $[S,S]$-subgame, that is $q_1^S(\widetilde{\omega}_1,\widetilde{\omega}_2) = q_1^{L*}$ and $q_2^S(\widetilde{\omega}_1,\widetilde{\omega}_2) = q_2^{F*}$. Now, a deviation of firm 2 from strategy *Integrate* to strategy *Separate* is equivalent to switching from the outcome determined by $\{\widetilde{\omega}_1,\widetilde{\omega}_2\}$ to the outcome determined by $\{\omega_1^{s*},\omega_2^{s*}\}$ in the $[S,S]$-subgame. The deviation implies that $\widetilde{\omega}_2 > \omega_2^{s*}$, while either $\widetilde{\omega}_1 < \omega_1^{s*}$ or $\widetilde{\omega}_1 > \omega_1^{s*}$ depends on values of γ_1 and γ_2.

A decrease in ω_2 raises firm 2's output and lowers firm 1's. If $\widetilde{\omega}_1 < \omega_1^{s*}$, an increase in ω_1 has the same qualitative effect on outputs: q_2 increases and q_1 decreases. Both changes raise firm 2's profit, and thus firm 2 strictly prefers to deviate from *Integration* to *Separation.* This holds on the subset of zone A with γ_1 and γ_2 close enough to zero.

In contrast, if $\widetilde{\omega}_1 > \omega_1^{s*}$, a decrease in ω_1 raises firm 1's output, lowers firm 2's output, and negatively affects firm 2's profit. In this case, a total effect of both changes depends on values of γ_1 and γ_2. If both γ_1 and γ_2 are sufficiently low (the rest of zone A and zone D), the positive effect of the change in ω_2 dominates the negative effect of the change in ω_1 and the deviation is profitable. Otherwise, firm 2 prefers to integrate, given that firm 1 separates (zones C and B).

Symmetric arguments apply for the case when firm 1 chooses between separation and integration given that firm 2 separates. Therefore, zone A is the set of $\{\gamma_1,\gamma_2\}$ such that each firm strictly prefers to separate, and thus, $[S,S]$ is played in a unique equilibrium. In zone C separation is a dominant strategy for firm 1 while firm 2's profit is higher if it integrates. Thus, we have a unique asymmetric equilibrium. Similarly, in zone D, there exists a unique asymmetric equilibrium where firm 2 separates and firm 1 integrates. Finally, each firm prefers to integrate, given that its rival separates in the zone B, and there exist two asymmetric equilibria.

Corollary 2 *In the symmetric game with $\gamma_1 = \gamma_2 = \gamma$, there exists a unique $\widehat{\gamma}$ such that, if $\gamma < \widehat{\gamma}$, then in the unique equilibrium $[S, S]$-played at the first stage, and if $\gamma > \widehat{\gamma}$, then there exist two asymmetric equilibria with $[I, S]$ and $[S, I]$ played at the first stage.*

In particular, for given demand and cost specifications, we have that $\widehat{\gamma} \approx 0.47$ and in equilibrium $(\partial\omega_i/\partial\omega_j)|_{\omega_1=\omega_2=\omega^*} \approx -0.01 < 0, i, j = 1, 2$. Thus, asymmetric equilibria in the symmetric game arise even if the wholesale prices are not strategic complements, but given that the degree of substitution between them is sufficiently low.

4.1 Welfare analysis

Using explicit solutions for outputs in every subgame we have that for any γ_1, γ_2 the consumer surplus is maximized if both firms separate, that is

$$q_1^C + q_2^C < q_1^L + q_2^F < q_1^S + q_2^S.$$

Thus, if $(\gamma_1, \gamma_2) \in A$, then firms' actions $\{Separate, Separate\}$ are optimal from the point of view of the consumers surplus.

In respect to profits, we have reverse relationship

$$\pi_1^C + \pi_2^C > \pi_1^L + \pi_2^F > \pi_1^S + \pi_2^S,$$

which implies that total industry profit maximized in the Cournot setting. Calculating total welfare as the sum of consumers surplus and firm profits for each subgame, arrive at the result that total welfare in the $[S, S]$-subgame (W^{SS}) is always greater than the one in both the $[S, I]$-subgame (W^{SI}) and the $[I, I]$-subgame (W^{II}):

$$W^{SS}(\gamma_1, \gamma_2) > W^{SI}(\gamma_1, \gamma_2); W^{SS}(\gamma_1, \gamma_2) > W^{II}(\gamma_1, \gamma_2) \text{ for any } \gamma_1, \gamma_2.$$

Thus, if $(\gamma_1, \gamma_2) \in A$, then the firm's choice of separation at the first stage also maximizes total welfare.

5 Conclusion

This study investigates firms' incentives to vertically separate (i.e., to sell their products through independent exclusive retailers) and to vertically integrate (i.e., to retail their own products) in a framework of Cournot duopoly with decreasing returns to scale. In contrast to existing results, our analysis shows that the equilibrium market structure critically depends on a firms' cost structure.

Under very mild assumptions of demand and cost functions, we demonstrate that vertical integration and separation coexist whenever diseconomy of scale is high enough for at least one firm. Moreover, two asymmetric equilibria may exist in a completely symmetric game.

The intuition for the results follows. If only one firm separates, then it makes a Stackelberg leader's profit while its rival gets a Stackelberg follower's profit. As the Stackelberg leader's profit is always greater than a Cournot profit, each firm prefers to separate given that its rival integrates.

If both firms separate, then they compete at per unit prices. Retailers' reaction curves in this case are steeper than those of integrated firms' and, therefore, an increase in their own retailer' output leads to a greater reduction in the output of the rival's retailer as compared to the Cournot game. This leads to tough competition in per unit prices between separated firms and may lead to excessive production and low final profits.

If the separated firm, say firm 2, is inefficient enough (that is, it has a steep enough marginal cost curve), then firm 1 - the Stackelberg follower - makes an almost monopolistic profit. In this case, separation is not profitable for firm 1 as it would raise its retailer's output beyond the monopolistic output and, thus, would lower firm 1's profits. If, in contrast, firm 1, being more efficient, separates, then firm 2 prefers to integrate as a tough competition in per unit prices would lower its profit below the Stackelberg follower's profit.

In the symmetric case, the higher the diseconomy of scale, the lower the gain each firm gets by manipulating its own retailer's objectives; moreover, the higher the losses from tough competition on per-unit prices. Hence, if the diseconomy of scale is sufficiently high, each firm prefers to integrate given that its rival separates.

Furthermore, we provide a closed form solution for the case of quadratic costs and a linear demand. Under these assumptions, we show that in addition to previous results, one additional type of equilibrium arises. In particular, if diseconomy of scale is low for one firm and moderate for another, then there exists a unique equilibrium where the more efficient firm separates and the less efficient one integrates. We show also that both consumer surplus and total welfare are maximized if both firms separate, while the firms' profits are higher under Cournot competition.

Our analysis provides a possible explanation for a widely observed asymmetry in firm sales strategies based on decreasing economies of scales and cost asymmetry. Notably, in the model, separation neither implies a change in the production function nor relates to additional costs. In this sense, we have shown the existence of asymmetric equilibria in a "pure" separation game.

The literature has applied the separation game to analyze a variety of issues, including mergers (Krakel and Sliwka, 2006; Ziss, 2001) and collusions (Lambertini and Trombetta, 2002; Spagnolo, 2005). All these applications employ the assumption of constant returns to scale. As our analysis shows, the equilibrium structure of the separation game itself is robust to this assumption only to a certain extent. Thus, we expect that substantially different results in these applications may be obtained for cases when the diseconomies of scale are significant. We leave this for future research.

A Appendix

Proof of Lemma 1. $q_2^F(q_1,\alpha)$ solves $P'q_2 + P - \alpha C_2'(q_2) = 0$ for all $\alpha > 0$ and all $q_1 \in [0, \overline{Q})$. Both $P'(\cdot)$ and $P(\cdot)$ are bounded, therefore $\alpha C_2'(q_2^F(q_1,\alpha))$ is bounded also for all $q_1 \in [0,\overline{Q})$. With necessity this implies that $q_2^F(q_1,\alpha) \underset{\alpha\to\infty}{\to} 0$ uniformly. Moreover,

$$\lim_{\alpha\to\infty} \alpha C_2'(q_2^F(q_1,\alpha)) = \lim_{\alpha\to\infty} P(q_1) > 0$$

for all $q_1 \in [0,\overline{Q})$.

Now, we show that

$$\lim_{\alpha\to\infty} \alpha C_2'(q_2^F(q_1,\alpha)) \in (0,\infty)$$

implies that

$$\lim_{\alpha\to\infty} \alpha C_2^{''}(q_2^F(q_1,\alpha)) = \infty$$

for all $q_1 \in [0,\overline{Q})$. If $C''(0) = 0$, then by the Taylor theorem $\exists \eta \in [0,q]$: $C_i'(q) = C_i^{''}(\eta)q$. As $C_i''(q) > 0$ for any $q > 0$, we have that $C_i''(q)$ is increasing at $q = 0$ and, hence, $C_i^{''}(\eta) < C_i^{''}(q)$ for q small enough. Thus,

$$\lim_{q\to 0}\frac{C_i'(q)}{C_i^{''}(q)} \le \lim_{q\to 0}\frac{C_i^{''}(\eta)}{C_i^{''}(q)}q \le \lim_{q\to 0} q = 0.$$

If $C''(0) > 0$, then $\lim\limits_{q\to 0} C_i'(q)/C_i^{''}(0) = 0$.

Therefore, for any $q_1 \in [0,\overline{Q})$ we have that

$$\lim_{\alpha\to\infty} \alpha C_2^{''}(q_2^F(q_1,\alpha)) = \lim_{\alpha\to\infty}\left(\alpha C_2^{'}(q_2)\frac{C_2^{''}(q_2)}{C_2^{'}(q_2)}\right)\Bigg|_{q_2=q_2^F(q_1,\alpha)} =$$

$$= P(q_1)\left(\lim_{\alpha\to\infty}\frac{C_2^{'}(q_2)}{C_2^{''}(q_2)}\right)^{-1}\Bigg|_{q_2=q_2^F(q_1,\alpha)} = \infty.$$

Finally, this provides that $\frac{\partial q_2^F}{\partial q_1} = -\frac{P''q_2^F+P'}{P''q_2^F+2P'-\alpha C_2''}$ uniformly converges to zero as $\alpha\to\infty$. ■

Proof of Propositions 2 and 3. We consider $[S,I]$-subgame. By Lemma 1, $\partial q_2^F/\partial q_1 \to 0$ and $q_2^F(q_1,\alpha) \to 0$ uniformly as $\alpha\to\infty$ and, therefore, (3) uniformly converges to the monopolist's first order condition:

$$P'(Q)q_1 + P(Q) - C_1'(q_1) + P'(Q)q_1 \frac{\partial q_2^F}{\partial q_1} \underset{\alpha\to\infty}{\to} P'(q_1)q_1 + P(q_1) - C_1'(q_1). \tag{11}$$

Thus, $\{q_1^{*L}(\alpha), q_2^{*F}(\alpha)\} \to \{q^{*M}, 0\}$ as $\alpha \to \infty$.

Now, we consider $[S, S]$-subgame. As there exists an internal solution of $\partial\pi_2^S(\alpha)/\partial\omega_2 = 0$ for all $\alpha > 0$, we have that $\alpha C'(q_2^{*S})$ is bounded and thus $q_2^S(\alpha) \to 0$ as $\alpha \to \infty$. Therefore, we have that $-\frac{P''q_2^S+P'}{P''q_2^S+2P'} \to -\frac{1}{2}$ and, in the limit, firm 1's first order condition takes a form:

$$\begin{aligned} &P'(Q^S)q_1^S + P(Q^S) - C_1'(q_1^S) + P'(Q^S)q_1^S \left(\frac{\partial q_2^S}{\partial\omega_1^S}/\frac{\partial q_1^S}{\partial\omega_1^S}\right) \underset{\alpha\to\infty}{\to} \\ &\underset{\alpha\to\infty}{\to} P'(q_1^S)q_1^S/2 + P(q_1^S) - C_1'(q_1^S) = 0. \end{aligned} \tag{12}$$

Comparing (11) and (12), we get that $\lim_{\alpha\to\infty} q_1^{*S}(\alpha) > q^{*M} = \lim_{\alpha\to\infty} q_1^{*L}(\alpha)$. Thus, if firm 2 separates, then firm 1's output is higher than the Stackelberg leader's output. As the firm 2's profit, $\pi_2(q_1, q_2)$, decreases in q_1 we have that $\pi_2^{*F} = \max_{q_2}\pi_2(q_1^{*L}, q_2) > \max_{q_2}\pi_2(q_1^{*S}, q_2) = \pi_2^{*S}$ whenever $q_1^{*S} > q_1^{*L}$. Thus, for α big enough $\pi_2^{*F} \geq \pi_2^{*S}$, which proves Proposition 2.

Finally, applying a similar argument to the $[I, S]$-subgame, we get that $\{q_1^{*F}(\alpha), q_2^{*L}(\alpha)\} \to \{q^{*M}, 0\}$ as $\alpha \to \infty$. Therefore, $\lim_{\alpha\to\infty} q_1^{*S}(\alpha) > q^M = \lim_{\alpha\to\infty} q_1^{*F}(\alpha)$ implies that $\pi_1^{*F} \geq \pi_1^{*S}$ for α big enough, which proves Proposition 3. ■

Proof of Proposition 4. Suppose firm i separates and firm j integrates. As $\alpha \to \infty$, both $q_i^{*L}(\alpha)$ and $q_i^{*F}(\alpha)$ converge to zero as well as

$$\frac{\partial q_j^F}{\partial q_i} = -\frac{P''q_j^F + P'}{P''q_j^F + 2P' - \alpha C''}$$

does. Thus, the term $P'(Q)q_i\left(\partial q_j^F/\partial q_i\right)$ is of higher order of smallness than q_i. Therefore, first order conditions of the Stackelberg game

$$\begin{cases} P'(Q)q_i + P(Q) - \alpha C'(q_i) + P'(Q)q_i\frac{\partial q_j^F}{\partial q_i} = 0 \\ P'(Q)q_j + P(Q) - \alpha C'(q_j) = 0 \end{cases}$$

uniformly converge to first order conditions of the Cournot game:

$$\begin{cases} P'q_i + P - \alpha C' = 0 \\ i = 1, 2 \end{cases}. \tag{13}$$

This means that

$$\lim_{\alpha\to\infty}\frac{q_i^L(\alpha)}{q^C(\alpha)} = \lim_{\alpha\to\infty}\frac{q_j^F(\alpha)}{q^C(\alpha)} = 1.$$

Equilibrium values of $\{q_1^S(\alpha), q_2^S(\alpha)\}$ converge to $\{0, 0\}$ as $\alpha \to \infty$ and thus

$$\frac{\partial q_j^s}{\partial \omega_i} / \frac{\partial q_i^s}{\partial \omega_i} = -\frac{P'' q_j^s + P'}{P'' q_j^s + 2P'} \to -1/2.$$

Therefore, first order conditions of the $[S, S]$-subgame

$$\begin{cases} P' q_i^s + P - \alpha C_i' + P' q_i^s \left(\frac{\partial q_j^s}{\partial \omega_i^s} / \frac{\partial q_i^s}{\partial \omega_i^s} \right) = 0, \\ \qquad i, j = 1, 2, i \neq j \end{cases}$$

converge to the system

$$\begin{cases} P' q_i^s + P - \alpha C' - \frac{1}{2} P' q_i^s = 0, \\ \qquad i = 1, 2. \end{cases} \tag{14}$$

as $\alpha \to \infty$. Comparing (13) and (14) we obtain

$$\lim_{\alpha \to \infty} \frac{q_i^C(\alpha)}{q_i^L(\alpha)} = 1 > \lim_{\alpha \to \infty} \frac{q_i^C(\alpha)}{q_i^S(\alpha)}.$$

This implies $q_i^L(\alpha) < q_i^S(\alpha)$ for α big enough. Therefore $\pi_j^F > \pi_j^S, \pi_i^L > \pi_i^S$,which proves Proposition 4. ■

References

Bagwell, K. Commitment and observability in games. Games and Economic Behavior 1995; 8; 271-280.

Basu, K. Stackelberg equilibrium in oligopoly: An explanation based on managerial incentives. Economic Letters 1995; 49; 459-464.

Bonanno, G., Vickers, J. Vertical separation. The Journal of Industrial Economics 1988; 36; 257-265.

Corts, K. S., Neher, D.V. Credible delegation. European Economic Review 2003; 47; 395-407.

Fershtman, C., Judd, K.L. Equilibrium incentives in oligopoly. The American Economic Review 1987; 77; 927-940.

Jansen, J. Coexistence of strategic vertical separation and integration. International Journal of Industrial Organization 2003; 21; 699-716.

Jansen, T., van Lier, A., van Witteloostuijn, A. A note on strategic delegation: The market share case. International Journal of Industrial Organization 2007; 25; 531–539.

Katz, M.L. Game-playing agents: Unobservable contracts as precommitments. Rand Journal of Economics 1991; 22; 307–328.

Krakel, M., Sliwka, D. Strategic delegation and mergers in oligopolistic contests. Journal of Economics and Business 2006; 58; 119-136.

Lambertini, L., Trombetta, M. Delegation and firms' ability to collude. Journal of Economic Behavior & Organization 2002; 47; 359-373.

Ritz, R.A. Strategic incentives for market share. International Journal of Industrial Organization 2008; 26; 586– 597.

Sklivas, S. D. The strategic choice of managerial incentives. The RAND Journal of Economics 1987; 18; 452-458.

Spagnolo, G. Managerial incentives and collusive behavior. European Economic Review 2005; 49; 1501-1523.

Van Long, N., Soubeyranb, A. Existence and uniqueness of Cournot equilibrium: A contraction mapping approach. Economics Letters 2000; 67; 345–348.

Vickers, J. Delegation and the theory of the firm. The Economic Journal 1985; 95, Supplement: Conference Papers, 138-147.

Ziss, S. Horizontal mergers and delegation. International Journal of Industrial Organization 2001; 19; 471–492.