Strategic Vertical Separation under Diseconomy of Scale\footnote{Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.} \footnote{Chapter 3 of the authors’ PhD thesis provided the basis for this paper. I thank to my supervisors, Emmanuel Petrakis and Chrysovalantou Milliou, for their valuable comments.}

Igor Sloev\footnote{Humboldt University at Berlin, Institute of Economic Theory I, Spandauer str. 1, Berlin, Germany; sloevigo@staff.hu-berlin.de.}

December 2010

Abstract

The paper analyzes vertical separation in a duopoly with decreasing returns to scale. It demonstrates, in contrast to classical results, the existence of asymmetric equilibria, where one firm separates and another integrates, whenever diseconomy of scale is high for at least one firm. Moreover, if the diseconomy of scale is low for one firm and moderate for another, a more efficient firm separates in a unique equilibrium, while its rival integrates. Also, the research herein shows that asymmetric equilibria exist even in a completely symmetric game. This provides an explanation for a widely observed difference in firm sales strategies.

JEL classification: L11, L21, L22, L25

Keywords: vertical oligopoly, vertical separation, vertical integration
1 Introduction

Commonly, *strategic separation* refers to a manufacturer’s decision to sell goods through an exclusive retailer, who makes the final decision on quantity or price. *Strategic delegation* means that an owner delegates a decision on final output or price to a manager within the firm. If the agent (the retailer or the manager) has no bargaining power, then a principal (the manufacturer or the owner) completely controls his/her objective through contract terms. The strategic role of both separation and delegation remains the same: by manipulating the agent’s objective, the principal alters its agent’s as well as its rivals’ behaviors to achieve a more preferable outcome. In contrast to separation, *integration* implies that the profit maximizing principal makes the decision on final output or price directly.

Classical works of Vickers (1985), Sklivas (1987), Fershtman and Judd (1987), and Bonanno and Vickers (1988) demonstrate that delegating a decision to an agent is in the interest of each principal.\(^1\) Hence, in a unique equilibrium, each principal takes advantage of delegation, and coexistence of vertical separation and vertical integration never occurs as an equilibrium outcome. This result proves robust in respect to the nature of a competition (price or quantity competition), number of firms, and a demand function specification. Moreover, the result holds under different assumptions of agents’ objectives: linear combinations of profit and quantity (Vickers, 1985), profit and revenue (Sklivas 1987; Fershtman and Judd, 1987), or retailer profit (Bonanno and Vickers, 1988). More recently, Jansen et al. (2007) and Ritz (2008) assume that agents maximize a linear combination of the principal’s profit and market share and also maintain that in a unique equilibrium each owner delegates the decision to a manager.

Basu (1995) and Jansen (2003) show one possibility of the coexistence of vertical separation and vertical integration. To obtain the result, the authors assume that separated firms bear an additional fixed cost. This effectively implies that separation alters a firm’s technology. Thus, in their models, separation relates to both a choice of technology and a choice of the agent’s incentive scheme.

A common feature of all the models mentioned above is the assumption of linearity of cost functions. This paper extends the existing analysis by allowing for decreasing return to scales. We analyze a case of duopolists competing in quantities and, following a standard approach, assume that each separated firm uses a two-part tariff in a trade with its retailer.

Under mild assumptions on demand and firm costs, we demonstrate that vertical separation and vertical integration coexist whenever a cost function of at least one firm exhibits a sufficient degree of scale diseconomy (or, in other words, when a marginal cost curve is steep enough). Furthermore, we show the existence of asymmetric equilibria in a perfectly symmetric game. The intuition for our results follows. If only one firm separates, then, effectively, it has the advantage of first mover and makes a Stackelberg leader’s profit. Thus, each firm prefers to separate given that its rival integrates. If both firms separate, it may create strong competition

\(^1\)If firms compete in prices, then delegation leads to a Pareto efficient outcome; while if firms compete in quantities, it lowers the profit of at least one firm.
between their retailers and, under a decreasing economy of scale, it may lead to profits lower than a Stackelberg follower’s profit. In this case, each firm prefers to integrate given its rival separates, and, therefore, there exist two equilibria, where one firm separates and another integrates.

Moreover, we provide a closed form solution for the case of linear demand and quadratic cost. We show that, in addition to the previous result, if diseconomy of scale is low for one firm and moderate for another, then there exists a unique equilibrium where the more efficient firm separates and the less efficient integrates. Welfare analysis shows that both consumer surplus and total welfare are higher if both firms separate; thus, asymmetric equilibria are not optimal.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes subgame outcomes. Section 3 provides equilibrium analysis and the main results for the general case. Section 4 examines the case of linear demand and quadratic costs, and, finally, Section 5 concludes. The Appendix contains all proofs.

2 Model

We now consider two firms, \( i = 1, 2 \), producing homogeneous goods and competing in quantities. A demand function, \( P(Q) \), with \( Q = q_1 + q_2 \), where \( q_i \) is an output of firm \( i \), and cost functions, \( C_i(q_i), i = 1, 2 \), are such that:

A1. \( \exists Q > 0: P(Q) > 0 \) for \( Q \in [0, Q] \) and \( P(Q) = 0 \) for \( Q \geq Q \); \( P''(Q) \) is continuous; \( P(0) = P > 0 \), \( -P'(Q) > \delta > 0 \), \( P'(Q) + P''(Q)q_i < 0 \) for \( Q \in [0, Q] \).

A2. \( C_i(q_i) \) is a twice continuously differentiable increasing convex function, \( C_i(0) = 0, C_i'(0) = 0, 0 < C_i''(Q) < b \) for all \( q_i \in (0, Q] \) and some \( b > 0 \).

A3. \( P^{(3)}(Q) \geq 0 \) for all \( Q \in [0, Q] \).

Assumptions A1-A2 offer sufficient conditions for the existence of a unique equilibrium in a Cournot game\(^2\) and together with Assumption 3 ensure existence of a solution in a whole game.

We consider the following two stage game. At the first stage, each firm \( i \) chooses \( m_i \in \{ \text{Separate, Integrate} \} \equiv \{ S, I \} \). If \( m_i = I \), then firm \( i \) becomes the retailer of its goods. If \( m_i = S \), then firm \( i \) sets the terms of a two-part tariff contact \( \{ \omega_i, A_i \} \) where \( \omega_i \) is a per unit price, and \( A_i \) is a franchise fee.

At the second stage, all decisions are observed\(^3\) and retailers choose quantities simultaneously and independently to maximize their own profits. The profit of the integrated firm \( i \) is \( \pi_i = P(Q)q_i - C_i(q_i) \). If firm \( i \) separates, its own and its retailer’s profits are \( \pi_i^r = \omega_i q_i(\omega_i) + A_i - C_i(q_i(\omega_i)) \) and \( \pi_i^R = P(Q)q_i - \omega_i q_i - A_i \), respectively. Thus, \( \pi_i(q_1, q_2) = \pi_i^r(q_1, q_2) + \pi_i^R(q_1, q_2) \), i.e., separation does not alter production technology, it only changes the retailer’s objective.

\(^3\)We assume that decisions are observable and irreversible and thus there is no commitment problem. For a discussion of observability and commitment in a delegation game see Katz (1991), Bagwell (1995), Corts and Neher (2003).
Let $\Gamma(P, C_1, C_2)$ denote the game described above for a given demand function $P(Q)$ and cost functions $C_1(q_1), C_2(q_2)$. We use is the subgame perfect Nash Equilibrium in pure strategies as the solution concept.

### Subgame outcomes

There are four subgames depending on the choice $m_i \in \{S, I\}, i = 1, 2$ of each firm at the first stage. If both firms integrate, they play the Cournot game. A unique equilibrium is determined by first order conditions

$$\begin{align*}
P' q_i + P - C'_i &= 0 \\
i &= 1, 2.
\end{align*}$$

(1)

Let $\{q_1^*, q_2^C\}$ and $\{\pi_1^*, \pi_1^C\}$ denote equilibrium values and let $q_i^{*M}$ be an output of firm $i$ if it were a monopolist.

If firm 1 separates and firm 2 integrates, then the retailers’ game\textsuperscript{4}

$$\begin{align*}
\max_{q_i} \pi_1^R &= P q_1 - \omega_1 q_1 \\
\max_{q_2} \pi_2^R &= P q_2 - C_2
\end{align*}$$

(2)

has first-order conditions

$$\begin{align*}
P' q_1 + P - \omega_1 &= 0 \\
P' q_2 + P - C'_2 &= 0
\end{align*}$$

Retailer 2 has the Cournot reaction curve, while the position of retailer 1’s reaction curve depends on firm 1’s choice of $\omega_1$. Thus, by choosing $\omega_1$, firm 1 determines a point of intersection of reaction curves. Clearly, the optimal $\omega_1$ is such that an equilibrium outcome replicates the Stackelberg outcome of the $[I, I]$-subgame. Therefore, we may characterize the solution of the $[S, I]$-subgame as the following: $q_2^F(q_1)$ solves

$$P' q_2 + P - C'_2 = 0$$

and $q_1^F$ is such that

$$P' q_1 + P - C'_1 + P' q_1 \frac{\partial q_2^F(q_1)}{\partial q_1} = 0.$$

(3)

\textsuperscript{4}Clearly, in equilibrium each separated firm entirely extracts its retailer’s profit using a franchise fee, and the retailer gets zero profit. Thus, to keep notation brief, we skip $A_i$ henceforth.
By the implicit function theorem, we have that

$$\frac{\partial q_i^F}{\partial q_1} = \frac{P''q_i^F + P'}{P''q_2^F + 2P'' - C_{2}''}.$$  

Let $\omega^*_L$, $\{q_1^L, q_2^F,\}$ and $\{\pi_1^L, \pi_2^F\}$ be equilibrium values of the $[S, I]$-subgame. Applying similar arguments and notation to the $[I, S]$-subgame, let $\omega^*_S$, $\{q_1^F, q_2^S,\}$ and $\{\pi_1^F, \pi_2^S\}$ be equilibrium values of the $[I, S]$-subgame.

If both firms separate, then retailers’ maximization problems are

$$\max_{q_i} \pi_i^R = Pq_i - \omega_i q_i, i = 1, 2$$

where $\omega_1, \omega_2$ are set by firms at the previous stage. As the feasibility constraint, $q_i \geq 0$, implies that $q_i = 0$ for all $\omega_i \geq P$, we assume without loss of generality that $\omega_i \leq P$. Then, a solution of the retailers’ problem is determined by the system of first order conditions:

$$\begin{cases} P'q_1 + P - \omega_1 = 0 \\ P'q_2 + P - \omega_2 = 0 \end{cases}.$$  

(4)

The Jacobian matrix of (4) is

$$J = \begin{pmatrix} P''q_1 + 2P' & P''q_1 + P' \\ P''q_2 + P' & P''q_2 + 2P' \end{pmatrix},$$

with $\det(J) > 0$ for any $(q_1, q_2)$ and by the implicit function theorem we have that

$$\frac{\partial \bf q}{\partial \bf \omega} = \frac{1}{\det(J)} \begin{pmatrix} P''q_2 + 2P' & -(P''q_1 + P') \\ -(P''q_2 + P') & P''q_1 + 2P' \end{pmatrix},$$  

(5)

where $\bf q = (q_1, q_2)$ and $\bf \omega = (\omega_1, \omega_2)$.

Let $\{q_i^*(\omega_1, \omega_2), q_j^*(\omega_1, \omega_2)\}$ denote the solution of (4). Then optimal values of $\omega_1, \omega_2$ satisfy:

$$\frac{\partial \pi_i^S}{\partial \omega_i} = P' \left( \frac{\partial q_i^*}{\partial \omega_i} + \frac{\partial q_j^*}{\partial \omega_i} \right) q_i^* + P \frac{\partial q_i^*}{\partial \omega_i} C_i^* \frac{\partial q_i^*}{\partial \omega_i} = 0, i \neq j.$$  

(6)

Directly differentiating (6) in respect to $\omega_i$ and using (5) one can obtain that $\partial^2 \pi_i / \partial \omega_i^2 < 0$ under Assumptions 1 and 2 and provided $P^{(3)}(Q) \geq 0$. Therefore, Assumptions 1-3 ensure the existence of a pure strategy equilibrium in the $[S, S]$-subgame. It is convenient to rewrite (6) in the form

$$P'q_i^* + P - C_i^* + P'q_j^* \left( \frac{\partial q_i^*}{\partial \omega_i} / \frac{\partial q_j^*}{\partial \omega_i} \right) = 0,$$
where
\[
\frac{\partial q_i^s}{\partial q_i^s} / \frac{\partial q_i^s}{\partial q_i^s} = -\frac{P''q_i^s + P'}{P''q_i^s + 2P'}.
\] (7)

Let \(q_1^S, q_2^S\) and \(\pi_1^S, \pi_2^S\) be the equilibrium values in the \([S,S]\)-subgame. Now we may represent the game as the following table.

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Separate</th>
<th>Integrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separate</td>
<td>(\pi_1^S, \pi_2^S)</td>
<td>(\pi_1^L, \pi_2^F)</td>
</tr>
<tr>
<td>Firm 1</td>
<td>Integrate</td>
<td>(\pi_1^F, \pi_2^L)</td>
</tr>
</tbody>
</table>

### 3 Equilibrium

First, we note that the Stackelberg leader’s profit always exceeds the Cournot profit, \(\pi_i^S > \pi_i^C\), and therefore we have the following result.

**Proposition 1** The \([I, I]\)-subgame is never played in equilibrium.

The equilibrium is determined by a relation of manufacturers’ profits \(\pi_i^S, \pi_i^F, \pi_i^L, \pi_i^C\). More specifically, in the equilibrium, \([S, I]\) is played if \(\pi_2^S \geq \pi_2^S\); \([I, S]\) if \(\pi_1^F \geq \pi_1^S\); \([S, S]\) if both \(\pi_1^F \leq \pi_1^S\) and \(\pi_2^F \leq \pi_2^S\) hold.

Now, we consider a family of games \(\Gamma (P, C_1, \alpha C_2)\) with \(\alpha > 0\).

**Lemma 1** As \(\alpha \to \infty\), both \(q_2^F(q_1, \alpha)\) and \(\partial q_2^F / \partial q_1(q_1, \alpha)\) uniformly converges to zero.

Lemma’s statement is quite intuitive: the steeper is firm 2’s marginal curve (the greater is \(\alpha\)), the more negligibly are both its output, \(q_2^F(q_1, \alpha)\), and its response to changes in \(q_1\), \(\partial q_2^F / \partial q_1(q_1, \alpha)\), for any \(q_1 \in [0, Q]\).

In general, separation has two effects. First, it allows a firm to manipulate its retailer’s reaction curve and thus gives a strategic advantage to a separated firm. Second, it increases competition between retailers. This occurs because retailers’ reaction curves (determined by (4)) are steeper than firms’ reaction curves (determined by (1)). While the first always increases the firm’s profit, the second harms it. Thus, a decision on separation depends on which effect dominates. The following proposition states that if a firm’s cost function exhibits a high enough diseconomy of scale, then the firm prefers to integrate given that its rival separates.

**Proposition 2** \(\exists \alpha > 0\) such that for any \(\alpha \geq \alpha\) the game \(\Gamma (P, C_1, \alpha C_2)\) has equilibrium with \([S, I]\) played at the first stage.
The intuition for the result follows. As firm 2’s cost function exhibits high diseconomy of scale, an advantage of manipulating the retailer’s objective is small, while more aggressive behavior by retailer 1 lowers firm 2’s profits below the Stackelberg follower’s profits. Thus, firm 2 prefers to integrate given that firm 1 separates.

In contrast to Proposition 2, Proposition 3 states that if firm 2 being inefficient separates, then firm 1 prefers to integrate.

**Proposition 3** \( \exists \bar{\alpha} > 0 \) such that for any \( \alpha \geq \bar{\alpha} \) the game \( \Gamma (P, C_1, \alpha C_2) \) has equilibrium with \([I, S]\) played at the first stage.

This occurs for the following reason. As firm 2 is inefficient, firm 1, being a Stackelberg follower, achieves almost monopolistic profits. If it separates, its own and firm 2’s retailers compete more aggressively, which leads to an excessive output and significantly lowers firm 1’s profits. Thus, firm 1, operating even more efficiently, prefers to integrate given that firm 2 separates.

Combining Propositions 2 and 3, we arrive at a Corollary that states that two equilibria exist whenever at least one firm has a sufficiently steep marginal curve.

**Corollary 1** For any \( \alpha \geq \max\{\bar{\alpha}, \bar{\alpha} \} \) the game \( \Gamma (P, C_1, \alpha C_2) \) has two equilibria where one firm separates and another integrates.

Note that Corollary 1 does not imply existence of asymmetric equilibria in a symmetric game. To cover this case, we formulate the following proposition.

**Proposition 4** \( \exists \bar{\alpha} > 0 \) such that for any \( \alpha > \bar{\alpha} \) the symmetric game \( \Gamma (P, \alpha C, \alpha C) \) has two asymmetric equilibria with \([S, I]\) and \([I, S]\) played at the first stage.

Proposition 4 says that in the symmetric game, whenever the slope of marginal cost curve for each firm is steep enough, a loss from more aggressive retailers’ behavior dominates a gain from separation. Thus, each firm prefers to integrate given that its rival separates, and hence there exist two asymmetric equilibria in a symmetric game.

### 4 Linear demand and quadratic costs

This section provides analysis for the case of linear demand, \( P(Q) = 1 - Q \), and quadratic cost functions, \( C_i(q_i) = \gamma_i q_i^2 / 2 \), where \( \gamma_i \geq 0 \). This form of demand and cost functions allows us, first, to capture the effects of scale diseconomies on the equilibrium structure and, second, to obtain a closed-form solution.

Using (1) we get the solution of the \([I, I]\)-subgame:

\[
q_i^{*c} = \frac{1 + \gamma_j}{(3 + 2\gamma_i + \gamma_j\gamma_i + 2\gamma_j)} \quad \text{and} \quad \pi_i^{*c} = \frac{(2 + \gamma_i)(1 + \gamma_j)^2}{2(3 + 2\gamma_j + \gamma_j\gamma_i + 2\gamma_i)^2}, i, j = 1, 2.
\]
If firm 1 separates and firm 2 integrates, the retailers’ profit maximization problems (3) have the following solution:

\[
\begin{align*}
q^L_1 &= \frac{1+\gamma_2-\omega_1(2+\gamma_1)}{3+2\gamma_2} \\
q^F_2 &= \frac{1+\omega_1}{3+2\gamma_2}.
\end{align*}
\]  

(8)

Plugging (8) into firm 1’s profits and optimizing with respect to \(\omega_1\), we obtain:

\[
\omega^*_1 = \frac{(1+\gamma_2)(2\gamma_1 + \gamma_1\gamma_2 - 1)}{(2+\gamma_2)(2 + 2\gamma_2 + \gamma_1\gamma_2 + 2\gamma_1)}.
\]

Equilibrium quantities and profits are given, respectively, by:

\[
\begin{align*}
q^L_1 &= \frac{1+\gamma_2}{(2+2\gamma_2 + \gamma_1\gamma_2 + 2\gamma_1)} \\
q^F_2 &= \frac{1+\gamma_2}{(2+2\gamma_2 + \gamma_1\gamma_2 + 2\gamma_1)} \times \frac{1+\gamma_2}{(2+2\gamma_2 + \gamma_1\gamma_2 + 2\gamma_1)}.
\end{align*}
\]

If both firms vertically separate, first-order conditions of the retailers’ profit maximization problems (4) give optimal outputs \(q_i, q_2\) as functions of \((\omega_1, \omega_2)\),

\[
q_i = \frac{1 - 2\omega^*_i + \omega^*_j}{3}, i, j = 1, 2,
\]

and the total output and the final price, respectively:

\[
Q = \frac{2 - \omega^*_1 - \omega^*_2}{3}, P = \frac{1 + \omega^*_1 + \omega^*_2}{3}.
\]

First-order conditions of firms’ maximization problem (6) determine firms’ reaction curves in the space \((\omega^*_1, \omega^*_2)\):

\[
\begin{align*}
\omega^*_i &= \frac{(-1 + 2\gamma_i)(1 + \omega^*_j)}{4(1 + \gamma_i)}, i, j = 1, 2.
\end{align*}
\]  

(9)

Note that \(\partial \omega^*_i / \partial \omega^*_j\) strictly increases in \(\gamma_i\) and does not depend on \(\gamma_j\). Thus, the degree of substitution between \(\omega^*_i\) and \(\omega^*_j\) decreases in \(\gamma_i\); if \(\gamma_i < 1/2\), \(\omega^*_i\) is a substitute for \(\omega^*_j\), and if \(\gamma_i > 1/2\), \(\omega^*_i\) is a complement to \(\omega^*_j\). Moreover, if \(\gamma_i > 1/2\) and \(\gamma_j < 1/2\), then \(\omega^*_i\) is a complement to \(\omega^*_j\), whereas \(\omega^*_j\) is a substitute for \(\omega^*_i\).

System (9) has the solution:

\[
\omega^*_{ij} = \frac{2\gamma_i - 2\gamma_j + 4\gamma_i\gamma_j - 1}{5 + 6\gamma_i + 6\gamma_j + 4\gamma_i\gamma_j}, i, j = 1, 2.
\]
Respective equilibrium quantities and profits are given by:

\[ q^S_i = \frac{2 + 4\gamma_j}{(5 + 6\gamma_i + 4\gamma_i\gamma_j + 6\gamma_j)}, \]
\[ \pi^S_i = \frac{2(1 + \gamma_i)(1 + 2\gamma_j)^2}{(5 + 6\gamma_i + 4\gamma_i\gamma_j + 6\gamma_j)^2}, \]
\[ i, j = 1, 2. \]

We can easily see that \( \pi_i^F > \pi_i^C > \pi_i^S \) and, as prescribed by Proposition 1, the outcome of the subgame \([I, I]\) is never played in SPNE. To determine the equilibrium outcome, we compare \( \pi_i^F \) and \( \pi_i^S \). Consider the set \((\gamma_1, \gamma_2)\) such that firm \(i\) is indifferent to separating and integrating, given that firm \(j\) separates:

\[ \pi_i^F = \frac{(1 + \gamma_i + 2\gamma_j + \gamma_i\gamma_j)^2}{2(2 + \gamma_i)(2 + 2\gamma_j + \gamma_i\gamma_j + 2\gamma_i)^2} = \frac{2(1 + \gamma_i)(1 + 2\gamma_j)^2}{(5 + 6\gamma_i + 4\gamma_i\gamma_j + 6\gamma_j)^2} = \pi_i^S. \]

(10)

It can be shown that \( \gamma_i = \theta_i(\gamma_j), i, j = 1, 2 \) determined by (10) are such that: (i) \( \gamma_i = \theta_i(\gamma_j), i, j = 1, 2 \) are strictly concave and have a unique maximum, (ii) \( \theta_i(0) > 0 \) and (iii) \( \exists \gamma_j < +\infty : \theta_i(\gamma_j) = 0 \). Figure 1 gives a graphical representation of \( \theta_i(\gamma_j) \) and \( \theta_j(\gamma_i) \).

\( \pi_2^F > \pi_2^S \) above the dashed line, \( \pi_1^F > \pi_1^S \) right of the dotted line.

In zone A (low \( \gamma_1 \) and low \( \gamma_2 \)), both firms have relatively flat marginal cost curves. Each
firm prefers to separate given that its rival separates; hence, in the unique equilibrium, firms play \([S, S]\) at the first stage. The equilibrium profit of each firm is lower than in the Cournot equilibrium, yet higher than the Stackelberg follower’s profit: \(\pi_1^F < \pi_1^S < \pi_1^F\). Although inside zone A, firms may differ in efficiency, this difference ranks as sufficiently small. In zone C (low \(\gamma_1\) and moderate \(\gamma_2\)), firm 1 is more efficient than the firm 2, but the difference in efficiency is not too high. Then the strategy Separate is dominant for firm 1, while firm 2 chooses Integrate, \(\pi_1^F < \pi_1^S, \pi_2^F > \pi_2^S\). In zone D (low \(\gamma_1\) and moderate \(\gamma_2\)), the situation is the opposite of that in zone C: firm 2 is more efficient than firm 1, and the difference in efficiency is not too high. The strategy Separate is dominant for firm 1, while firm 2 chooses Integrate: \(\pi_1^F < \pi_1^S, \pi_2^F > \pi_2^S\). Finally, zone B is such that (either \(\gamma_1\) or \(\gamma_2\) or both are sufficiently high), either both firms are sufficiently inefficient, or the asymmetry in costs is very high. In this case, each firm integrates if its rival separates. Therefore, two asymmetric equilibria, with strategies \([I, S]\) and \([S, I]\), exist.

The following Proposition summarizes the above results:

**Proposition 5** At the first stage firms play

(i) \([S, S]\) if \((\gamma_1, \gamma_2) \in A \equiv \{(\gamma_1, \gamma_2) | \gamma_1 < \theta_1(\gamma_2), \gamma_2 < \theta_2(\gamma_1)\};

(ii) either \([S, I]\) or \([I, S]\) if \((\gamma_1, \gamma_2) \in B \equiv \{(\gamma_1, \gamma_2) | \gamma_1 < \theta_1(\gamma_2), \gamma_2 < \theta_2(\gamma_1)\};

(iii) \([S, I]\) if \((\gamma_1, \gamma_2) \in C \equiv \{(\gamma_1, \gamma_2) | \gamma_1 < \theta_1(\gamma_2), \gamma_2 < \theta_2(\gamma_1)\};

(iv) \([I, S]\) if \((\gamma_1, \gamma_2) \in D \equiv \{(\gamma_1, \gamma_2) | \gamma_1 < \theta_1(\gamma_2), \gamma_2 < \theta_2(\gamma_1)\}.

Intuition for this result is the following. If firm 1 separates and firm 2 integrates, then they get Stackelberg leader and follower profits, respectively. Let’s consider \(\tilde{\omega}_1\) and \(\tilde{\omega}_2\) which replicates this Stackelberg outcome in the \([S, S]\)-subgame, that is \(q_1^I(\tilde{\omega}_1, \tilde{\omega}_2) = q_1^{I*}\) and \(q_2^S(\tilde{\omega}_1, \tilde{\omega}_2) = q_2^{F*}\). Now, a deviation of firm 2 from strategy Integrate to strategy Separate is equivalent to switching from the outcome determined by \(\{\omega_1, \omega_2\}\) to the outcome determined by \(\{\omega_1^{I*}, \omega_2^{I*}\}\) in the \([S, S]\)-subgame. The deviation implies that \(\tilde{\omega}_2 > \omega_2^{I*}\), while either \(\tilde{\omega}_1 < \omega_1^{I*}\) or \(\tilde{\omega}_1 > \omega_1^{I*}\) depends on values of \(\gamma_1\) and \(\gamma_2\).

A decrease in \(\omega_2\) raises firm 2’s output and lowers firm 1’s. If \(\tilde{\omega}_1 < \omega_1^{I*}\), an increase in \(\omega_1\) has the same qualitative effect on outputs: \(q_2\) increases and \(q_1\) decreases. Both changes raise firm 2’s profit, and thus firm 2 strictly prefers to deviate from Integration to Separation. This holds on the subset of zone A with \(\gamma_1\) and \(\gamma_2\) close enough to zero.

In contrast, if \(\tilde{\omega}_1 > \omega_1^{I*}\), a decrease in \(\omega_1\) raises firm 1’s output, lowers firm 2’s output, and negatively affects firm 2’s profit. In this case, a total effect of both changes depends on values of \(\gamma_1\) and \(\gamma_2\). If both \(\gamma_1\) and \(\gamma_2\) are sufficiently low (the rest of zone A and zone D), the positive effect of the change in \(\omega_2\) dominates the negative effect of the change in \(\omega_1\) and the deviation is profitable. Otherwise, firm 2 prefers to integrate, given that firm 1 separates (zones C and B).

Symmetric arguments apply for the case when firm 1 chooses between separation and integration given that firm 2 separates. Therefore, zone A is the set of \((\gamma_1, \gamma_2)\) such that each firm strictly prefers to separate, and thus, \([S, S]\) is played in a unique equilibrium. In zone C separation is a dominant strategy for firm 1 while firm 2’s profit is higher if it integrates. Thus,
we have a unique asymmetric equilibrium. Similarly, in zone $D$, there exists a unique asymmetric equilibrium where firm 2 separates and firm 1 integrates. Finally, each firm prefers to integrate, given that its rival separates in the zone $B$, and there exist two asymmetric equilibria.

**Corollary 2** In the symmetric game with $\gamma_1 = \gamma_2 = \gamma$, there exists a unique $\hat{\gamma}$ such that, if $\gamma < \hat{\gamma}$, then in the unique equilibrium $[S, S]$-played at the first stage, and if $\gamma > \hat{\gamma}$, then there exist two asymmetric equilibria with $[I, S]$ and $[S, I]$ played at the first stage.

In particular, for given demand and cost specifications, we have that $\hat{\gamma} \approx 0.47$ and in equilibrium $(\partial \omega_i/\partial \omega_j)|_{\omega_1=\omega_2=\omega_*} \approx -0.01 < 0$, $i, j = 1, 2$. Thus, asymmetric equilibria in the symmetric game arise even if the wholesale prices are not strategic complements, but given that the degree of substitution between them is sufficiently low.

### 4.1 Welfare analysis

Using explicit solutions for outputs in every subgame we have that for any $\gamma_1, \gamma_2$ the consumer surplus is maximized if both firms separate, that is

$$q^{C}_1 + q^{C}_2 < q^{L}_1 + q^{F}_2 < q^{S}_1 + q^{S}_2.$$  

Thus, if $(\gamma_1, \gamma_2) \in A$, then firms’ actions $\{\text{Separate}, \text{Separate}\}$ are optimal from the point of view of the consumers surplus.

In respect to profits, we have reverse relationship

$$\pi^{C}_1 + \pi^{C}_2 > \pi^{L}_1 + \pi^{F}_2 > \pi^{S}_1 + \pi^{S}_2,$$

which implies that total industry profit maximized in the Cournot setting. Calculating total welfare as the sum of consumers surplus and firm profits for each subgame, arrive at the result that total welfare in the $[S, S]$-subgame ($W^{SS}$) is always greater than the one in both the $[S, I]$-subgame ($W^{SI}$) and the $[I, I]$-subgame ($W^{II}$):

$$W^{SS}(\gamma_1, \gamma_2) > W^{SI}(\gamma_1, \gamma_2); W^{SS}(\gamma_1, \gamma_2) > W^{II}(\gamma_1, \gamma_2)$$

for any $\gamma_1, \gamma_2$.

Thus, if $(\gamma_1, \gamma_2) \in A$, then the firm’s choice of separation at the first stage also maximizes total welfare.

### 5 Conclusion

This study investigates firms’ incentives to vertically separate (i.e., to sell their products through independent exclusive retailers) and to vertically integrate (i.e., to retail their own products) in a framework of Cournot duopoly with decreasing returns to scale. In contrast to existing
results, our analysis shows that the equilibrium market structure critically depends on a firms’ cost structure.

Under very mild assumptions of demand and cost functions, we demonstrate that vertical integration and separation coexist whenever diseconomy of scale is high enough for at least one firm. Moreover, two asymmetric equilibria may exist in a completely symmetric game.

The intuition for the results follows. If only one firm separates, then it makes a Stackelberg leader’s profit while its rival gets a Stackelberg follower’s profit. As the Stackelberg leader’s profit is always greater than a Cournot profit, each firm prefers to separate given that its rival integrates.

If both firms separate, then they compete at per unit prices. Retailers’ reaction curves in this case are steeper than those of integrated firms’ and, therefore, an increase in their own retailer’ output leads to a greater reduction in the output of the rival’s retailer as compared to the Cournot game. This leads to tough competition in per unit prices between separated firms and may lead to excessive production and low final profits.

If the separated firm, say firm 2, is inefficient enough (that is, it has a steep enough marginal cost curve), then firm 1 - the Stackelberg follower - makes an almost monopolistic profit. In this case, separation is not profitable for firm 1 as it would raise its retailer’s output beyond the monopolistic output and, thus, would lower firm 1’s profits. If, in contrast, firm 1, being more efficient, separates, then firm 2 prefers to integrate as a tough competition in per unit prices would lower its profit below the Stackelberg follower’s profit.

In the symmetric case, the higher the diseconomy of scale, the lower the gain each firm gets by manipulating its own retailer’s objectives; moreover, the higher the losses from tough competition on per-unit prices. Hence, if the diseconomy of scale is sufficiently high, each firm prefers to integrate given that its rival separates.

Furthermore, we provide a closed form solution for the case of quadratic costs and a linear demand. Under these assumptions, we show that in addition to previous results, one additional type of equilibrium arises. In particular, if diseconomy of scale is low for one firm and moderate for another, then there exists a unique equilibrium where the more efficient firm separates and the less efficient one integrates. We show also that both consumer surplus and total welfare are maximized if both firms separate, while the firms’ profits are higher under Cournot competition.

Our analysis provides a possible explanation for a widely observed asymmetry in firm sales strategies based on decreasing economies of scales and cost asymmetry. Notably, in the model, separation neither implies a change in the production function nor relates to additional costs. In this sense, we have shown the existence of asymmetric equilibria in a "pure" separation game.

The literature has applied the separation game to analyze a variety of issues, including mergers (Krakel and Sliewka, 2006; Ziss, 2001) and collusions (Lambertini and Trombetta, 2002; Spagnolo, 2005). All these applications employ the assumption of constant returns to scale. As our analysis shows, the equilibrium structure of the separation game itself is robust to this assumption only to a certain extent. Thus, we expect that substantially different results in these applications may
be obtained for cases when the diseconomies of scale are significant. We leave this for future
research.

A Appendix

Proof of Lemma 1. $q_2^F(q_1, \alpha)$ solves $P'q_2 + P - \alpha C_2'(q_2) = 0$ for all $\alpha > 0$ and all $q_1 \in [0, Q)$. Both $P'(\cdot)$ and $P(\cdot)$ are bounded, therefore $\alpha C_2'(q_2(q_1, \alpha))$ is bounded also for all $q_1 \in [0, Q)$. With necessity this implies that $q_2^F(q_1, \alpha) \to 0$ uniformly. Moreover,

$$
limit_{\alpha \to \infty} \alpha C_2'(q_2^F(q_1, \alpha)) = \lim_{\alpha \to \infty} P(q_1) > 0$$

for all $q_1 \in [0, Q)$.

Now, we show that

$$
limit_{\alpha \to \infty} \alpha C_2'(q_2^F(q_1, \alpha)) \in (0, \infty)
$$

implies that

$$
limit_{\alpha \to \infty} \alpha C_2''(q_2^F(q_1, \alpha)) = \infty
$$

for all $q_1 \in [0, Q)$. If $C''(0) = 0$, then by the Taylor theorem $\exists \eta \in [0, q]: C_i'(q) = C_i''(\eta)q$. As $C_i''(q) > 0$ for any $q > 0$, we have that $C_i''(q)$ is increasing at $q = 0$ and, hence, $C_i''(\eta) < C_i''(q)$ for $q$ small enough. Thus,

$$
limit_{q \to 0} \frac{C_i'(q)}{C_i'(0)} \leq \lim_{q \to 0} \frac{C_i''(q)}{C_i''(0)} q \leq \lim_{q \to 0} q = 0.
$$

If $C''(0) > 0$, then $\lim_{q \to 0} C_i'(q)/C_i''(0) = 0$.

Therefore, for any $q_1 \in [0, Q)$ we have that

$$
limit_{\alpha \to \infty} \alpha C_2''(q_2^F(q_1, \alpha)) = \lim_{\alpha \to \infty} \left( \alpha C_2'(q_2) \frac{C_2''(q_2)}{C_2'(q_2)} \right)_{q_2 = q_2^F(q_1, \alpha)} =
$$

$$
P(q_1) \left( \lim_{\alpha \to \infty} \frac{C_2'(q_2)}{C_2'(q_2)} \right)^{-1} \bigg|_{q_2 = q_2^F(q_1, \alpha)} = \infty.
$$

Finally, this provides that $\frac{\partial q_2^F}{\partial q_1} = -\frac{P'q_2^F + P}{P'q_2^F + 2P - \alpha C_2'}$ uniformly converges to zero as $\alpha \to \infty$.

Proof of Propositions 2 and 3. We consider $[S, I]$-subgame. By Lemma 1, $\partial q_2^F / \partial q_1 \to 0$ and $q_2^F(q_1, \alpha) \to 0$ uniformly as $\alpha \to \infty$ and, therefore, (3) uniformly converges to the monopolist’s
first order condition:

\[ P'(Q)q_1 + P(Q) - C'_i(q_1) + P'(Q)q_1 \frac{\partial q_i^F}{\partial q_1} \to \frac{d}{\alpha \to \infty} P'(q_1)q_1 + P(q_1) - C'_i(q_1). \]  

(11)

Thus, \( \{q_i^L(\alpha), q_2^F(\alpha)\} \to \{q^*, M\} \) as \( \alpha \to \infty \).

Now, we consider \([S, S]-\)subgame. As there exists an internal solution of \( \partial \pi_2^S(\alpha) / \partial \omega_2 = 0 \) for all \( \alpha > 0 \), we have that \( \alpha C'(q_2^S) \) is bounded and thus \( q_2^S(\alpha) \to 0 \) as \( \alpha \to \infty \). Therefore, we have that \( -\frac{P''q_i^F + P'}{P''q_i^2 + 2P'} \to -\frac{1}{2} \) and, in the limit, firm 1’s first order condition takes a form:

\[ P'(Q)q_1^S + P(Q) - C'_i(q_1^S) + P'(Q)q_1^S \left( \frac{\partial q_i^S}{\partial q_1^S} / \frac{\partial q_1^S}{\partial q_1^S} \right) \to \frac{d}{\alpha \to \infty} P'(q_1^S)q_1^S / 2 + P(q_1^S) - C'_i(q_1^S) = 0. \]  

(12)

Comparing (11) and (12), we get that \( \lim_{\alpha \to \infty} q_1^S(\alpha) > q^* = \lim_{\alpha \to \infty} q_i^L(\alpha) \). Thus, if firm 2 separates, then firm 1’s output is higher than the Stackelberg leader’s output. As the firm 2’s profit, \( \pi_2(q_1, q_2) \), decreases in \( q_1 \) we have that \( \pi_2(q_1^L, q_2) > \max_{q_2} \pi_2(q_1^S, q_2) = \pi_2^S \) whenever \( q_1^S > q_1^L \). Thus, for \( \alpha \) big enough \( \pi_2^F \geq \pi_2^S \), which proves Proposition 2.

Finally, applying a similar argument to the \([I, S]-\)subgame, we get that \( \{q_1^F(\alpha), q_2^L(\alpha)\} \to \{q^*, M\} \) as \( \alpha \to \infty \). Therefore, \( \lim_{\alpha \to \infty} q_1^S(\alpha) > q^* = \lim_{\alpha \to \infty} q_i^F(\alpha) \) implies that \( \pi_1^F \geq \pi_1^S \) for \( \alpha \) big enough, which proves Proposition 3.  

**Proof of Proposition 4.** Suppose firm \( i \) separates and firm \( j \) integrates. As \( \alpha \to \infty \), both \( q_i^L(\alpha) \) and \( q_i^F(\alpha) \) converge to zero as well as

\[ \frac{\partial q_i^F}{\partial q_i} = -\frac{P''q_i^F + P'}{P''q_i^2 + 2P} - \alpha C'' \]

does. Thus, the term \( P'(Q)q_i^F(\partial q_i^F / \partial q_i) \) is of higher order of smallness than \( q_i \). Therefore, first order conditions of the Stackelberg game

\[
\begin{aligned}
P'(Q)q_i + P(Q) - \alpha C'(q_i) + P'(Q)q_i \frac{\partial q_i^F}{\partial q_i} &= 0 \\
P'(Q)q_j + P(Q) - \alpha C'(q_j) &= 0
\end{aligned}
\]

uniformly converge to first order conditions of the Cournot game:

\[
\begin{aligned}
P'(q_i^T + P - \alpha C &= 0 \\
i &= 1, 2
\end{aligned}
\]

(13)

This means that

\[
\lim_{\alpha \to \infty} q_i^F(\alpha) = \lim_{\alpha \to \infty} q_i^F(\alpha) = 1.
\]
Equilibrium values of \( \{q_1^S(\alpha), q_2^S(\alpha)\} \) converge to \( \{0,0\} \) as \( \alpha \to \infty \) and thus

\[
\frac{\partial q_i^S}{\partial \omega_i} / \frac{\partial q_i^S}{\partial \omega_i} = -\frac{P'' q_j^S + P'}{P'' q_j^S + 2P'} \to -1/2.
\]

Therefore, first order conditions of the \([S,S]\)-subgame

\[
\begin{align*}
& P' q_i^S + P - \alpha C_i^S + P' q_i^S \left( \frac{\partial q_i^S}{\partial \omega_i} \right) = 0, \\
& i, j = 1, 2, i \neq j
\end{align*}
\]

converge to the system

\[
\begin{align*}
& P' q_i^S + P - \alpha C_i^S - \frac{1}{2} P' q_i^S = 0, \\
& i = 1, 2.
\end{align*}
\]

(14)

as \( \alpha \to \infty \). Comparing (13) and (14) we obtain

\[
\lim_{\alpha \to \infty} \frac{q_i^C(\alpha)}{q_i^S(\alpha)} = 1 > \lim_{\alpha \to \infty} \frac{q_i^C(\alpha)}{q_i^S(\alpha)}.
\]

This implies \( q_i^L(\alpha) < q_i^S(\alpha) \) for \( \alpha \) big enough. Therefore \( \pi_j^L > \pi_j^S, \pi_j^L > \pi_j^S \), which proves Proposition 4. ■

References


