Firm Dynamics, Labor Mobility, and Specific Human Capital

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First version: January 26, 2004
This version: June 5, 2007

Abstract

Firm-specific human capital loses its value if the firm exits the industry. This paper explores this simple but important link by developing a new firm-dynamics model that incorporates workers, their accumulation of specific human capital, and their mobility. In my model, a firm’s production efficiency is determined by the levels of its managerial capability and its workers’ firm-specific human capital. I demonstrate that the importance of managerial capability, through its connection to firm-specific human capital, systematically influences firm dynamics and employment practices. Equally important, the model offers a new perspective on the welfare consequences of apparently anticompetitive entry restrictions by investigating how such restrictions affect labor market characteristics.

JEL classification numbers: J41, J63, L10, L50, M20, M50.

Keywords: Entry restrictions, firm dynamics, labor mobility, managerial capability, specific human capital, turnover, wage structure, welfare.

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I would like to thank Kenzo Abe, Hiroyuki Chuma, Wayne-Roy Gayle, Arghya Ghosh, Junichiro Ishida, Shingo Ishiguro, Hideshi Itoh, Boyan Jovanovic, Takao Kato, Lawrence Katz, Edward Lazear, Kieron Meagher, Hiroyuki Okamuro, Hideo Owan, Pierre Richard, Bill Schworm, Kotaro Suzumura, Hirotaka Takeuchi, Katsuya Takii, Michael Waldman, Yijiang Wang, Jan Zabojnik and seminar participants at Chukyo University, Hitotsubashi University, Kansai Institute for Social and Economic Research, Nagoya City University, Osaka University, University of Sydney, UNSW, Fall 2004 Midwest Economic Theory Meetings, 2005 Econometrics Society World Congress, and 2007 Society of Labor Economists Annual Meetings for valuable discussions and comments, and Jonathan Lim for excellent research assistance. Part of this paper was written when I was a visiting scholar at the Institute of Economic Research of Hitotsubashi University, and I deeply appreciate their hospitality. Financial support from Australian Research Council and the Faculty Research Grants Program at the University of New South Wales is gratefully acknowledged.
1 Introduction

Firm dynamics have substantial impacts on labor mobility. Several studies of plant-level employment have found that a substantial amount of employment is lost due to plant contractions and failures even in expanding industries and regions, while a substantial amount of employment is created due to plant openings and expansions even in contracting industries and regions. At the level of individual workers, a substantial percentage of workers is displaced from the employer due to plant closings and contractions. See Section 2 for empirical findings concerning the connection between firm dynamics and labor mobility. At the same time, there are also important connections between labor mobility and firm-specific human capital investment, because specific human capital possessed by a worker loses its value if the worker leaves his/her current employer.

This paper develops a new model that captures the interconnections among firm dynamics, labor mobility, and specific human capital accumulation in a single theoretical framework. Models of firm and industry dynamics that allow for entry, exit, and firm heterogeneity and/or idiosyncratic shocks have been previously developed in the literature.\footnote{See Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995, among others.} For example, in his seminal contribution to this literature, Jovanovic (1982) developed a model in which the efficiency of firms in an industry is different across firms, and no firm knows its own true efficiency (which is captured as its true cost) \textit{ex ante}. True efficiency is gradually revealed through economic activity. In the equilibrium, efficient firms grow and survive, while inefficient firms decline and exit.

Despite significant contributions made by models previously developed in the firm-dynamics literature, the models do not explicitly incorporate one important aspect of reality, which is that most firms employ workers to produce outputs. On the other hand, although the connection between specific human capital and labor mobility has been previously explored,\footnote{See Parsons (1972), Mortensen (1978), Jovanovic (1979b), among others} models in this literature do not explicitly incorporate firm dynamics. The present paper is, to the best of my knowledge, the first to explore a model that captures interconnections among firm dynamics, labor mobility, and specific human capital accumulation in a single model.\footnote{In regards to theoretical analyses of job separations, several authors have previously developed models of labor-market search and matching (see Burdett, 1978; Jovanovic, 1979a), which provide explanations for the following established empirical finding: the probability of separation declines with both labor-market experience and firm-specific seniority. Complementary to these previous models that focus on voluntary separations, my model focuses on involuntary separations by incorporating firm dynamics into the analysis.}

An outline of my model is as follows: Consider an industry in a two-period setting, where the
industry faces a downward-sloping demand schedule in each period. Entry and exit of firms is free in each period. A firm must employ a worker to produce outputs, and can provide a level of firm-specific human capital with its employee. Each firm’s production efficiency is determined by its managerial capability and the levels of its worker’s firm-specific human capital, where managerial capability is interpreted representing the capability of a firm’s top management to develop an effective strategy and create a unique competitive position. I assume that no firm knows its own managerial capability \textit{ex ante}, and simplify the learning aspect of the model by assuming that each firm’s managerial capability becomes public knowledge at the end of the first period of its operation. If a firm continues to employ its first-period employee in the second period, the firm shares the return from its investment in specific human capital with the employee through wage bargaining. In the equilibrium, first-period entrants whose managerial capability turns out to be higher than a cut-off level survive, while other firms exit and some new firms enter before the second period starts.

The connection between firm dynamics and firm-specific human capital yields the following key result: the survival rate of firms decreases (or, equivalently, a firm’s exit rate increases) as the importance of managerial capability increases. Each first-period entrant has an advantage over second-period entrants, because it has a worker who has already accumulated a certain level of firm-specific human capital. As the importance of managerial capability increases, the advantage associated with firm-specific human capital becomes relatively less important, which results in a lower survival rate of the first-period entrants in the equilibrium.

The key result mentioned above naturally leads to the following labor market consequences: As the importance of managerial capability increases, the exit rate of firms as well as the separation rate of workers increase. Anticipating this, first-period entrants have lower incentives to train their workers, and hence the equilibrium level of firm-specific human capital becomes lower. This makes the tenure-wage profile less steep in the equilibrium. The model also yields similar comparative statics results concerning the importance of firm-specific human capital.

The comparative statics results outlined above yield empirical implications and predictions from a previously unexplored perspective, given that the importance of managerial capability can differ across industries. For example, in an industry undergoing revolutionary technological changes, a business’s success critically depends on the quality of its strategic decision making, because these industries face a high level of uncertainty about the needs of customers, the products and services that will prove to be the most desired, and the best configuration of activities and technologies to deliver them. Whereas in industries facing lower levels of uncertainty, strategic

\footnote{This assumption is consistent with the widely held view that the capability of a firm’s top management is mostly innate, and difficult to observe or assess \textit{ex ante}. See footnote 12 on page 6.}
decision making is less important. This argument suggests that the importance of managerial capability is higher in industries facing a higher level of uncertainty such as high-tech industries, while the importance tends to be lower in matured industries with lower levels of uncertainty. My model then predicts that firm’s exit rate and labor turnover rate are both high in high-tech industries such as the semiconductor industry, and evidences that support this prediction can be found in several case studies (see Section 3 for details).

Equally important, my framework offers a new perspective on the welfare consequences of entry restrictions. In Section 4, I consider an extension of the model in which the government can control firm dynamics to a certain degree by imposing entry restrictions, which in turn affects firms’ incentives to invest in specific human capital. Novelty of this analysis is that it captures the effects of entry restrictions on labor market characteristics. I demonstrate that entry restrictions can mitigate the underinvestment problem in specific human capital, which can result in a higher consumer surplus, as well as a higher total surplus. My approach is complementary to, but fundamentally different from, the approach taken by previous papers in the theoretical industrial organization literature (see Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987), which demonstrated the “excess-entry theorem” by focusing on the effects of entry restrictions on the strategic interactions among firms.

In reality, there are important interconnections among firms’ strategic decisions such as market entry and exit, the nature of their employment practices such as specific human capital investment, and resulting labor-market characteristics such as labor mobility. Nevertheless, in most previous theoretical analyses, firms’ strategic behaviors and their employment practices have been treated separately under industrial-organization theoretical models and labor-theoretical models, respectively. The present paper is one of several recent attempts to address such interconnections in a single theoretical framework.

As an important exception, there is a small literature that theoretically analyzes the effect of product-market competition on managerial incentives (see for example Hart, 1983; Hermalin, 1992; Schmidt, 1997; Raith, 2003). Schmidt (1997) pointed out that there are surprisingly few theoretical papers on this subject despite its importance.

For other papers along this line, see for example Chari and Hopenhayn (1991); Wang (2002); Mailath, Nocke, and Postlewaite (2004). Chari and Hopenhayn (1991) analyzed the connection between firms’ adoption of new technology and workers’ accumulation of “vintage-specific” skills (skills that are specific to a technology of a particular vintage), and explored its implications on the diffusion of new technology; Wang (2002) analyzed the connection between product-market conditions and job design, and explored its implications on explanations for heterogeneity of human resource management practices across countries, industries, and firms; and Mailath, Nocke, and Postlewaite (2004) analyzed the interaction between a firm’s choice of business strategy and its manager’s incentive for investing in “business-strategy-specific” human capital, and explored its implications on the organization of business activities.
The rest of the paper is organized as follows: Section 2 presents empirical findings on the connection between firm dynamics and labor mobility. Section 3 presents a model of firm dynamics that explicitly incorporates workers, their accumulation of specific human capital, and their mobility. It then characterizes the perfect foresight equilibrium of the model, presents comparative statics results, and discusses the real-world relevance of the results. Section 4 explores a new perspective on the welfare consequences of entry restrictions by analyzing how such restrictions affect labor market characteristics in my framework. Section 5 concludes the paper.

2 Related Empirical Findings

In this short section, I present previous empirical findings concerning the connection between firm dynamics and labor mobility. Firm dynamics have substantial impacts on labor mobility. Several studies of plant-level employment have found that gross employment flows, consisting of the number of positions added in new and growing plants and the number of existing positions lost in contracting and closing plants, are substantially larger than aggregate net employment growth. Dunne, Roberts, and Samuelson (1989) identified this pattern in US manufacturing employment over the 1963-1982 period using the Census of Manufacturers. They found that between 1977 and 1982, for example, total manufacturing employment declined by 3.8%. This net change was composed of an increase in employment of 17.6% and 11.7% due to plant openings and expansions respectively, and reductions of 15.4% and 17.7% due to plant contractions and closings respectively. Importantly, they found that over 70% of the turnover in employment opportunities occurs across plants within the same two-digit industry and geographic region. In other words, a substantial amount of employment is lost due to plant contractions and failures even in expanding industries and regions, while a substantial amount of employment is created due to plant openings and expansions even in contracting industries and regions.

At the level of individual workers, a substantial percentage of workers is displaced from the employer due to plant closings and contractions. The Displaced Workers Surveys (DWSs) contain information regarding displaced workers in the United States, where displacement is defined as involuntary separation based on the operating decisions of the employer, such as a plant closing, an employer going out of business, or a layoff from which the worker was not recalled (Farber, 1997). Farber (1997) analyzed the seven DWSs from 1984 to 1996 to examine

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7 See Dunne, Roberts, and Samuelson (1989) and references therein.
8 The DWSs have been administered every two years since 1984 as a supplement to the monthly Current Population Survey. Each DWS from 1984-1992 asked workers whether they were displaced from a job at any
the incidence of job loss from 1981 to 1995, and found that adjusted three-year job loss rates
during the period ranged between 9% and 15%.\footnote{Farber (1997) computed three-year job loss rates as the number of workers who report having lost a job in the three calendar years before the survey date divided by employment at the survey date.} Also, a substantial percentage of job separation
is due to plant closings and contractions. According to an analysis of the Panel Study of Income
Dynamics (PSID) by Polsky (1999), on average 36% of job separators were job losers for the
periods 1976-1981 and 1986-1991 in the United States, and the percentage was higher for older
workers.\footnote{Polsky (1999) used a “reason for new position” question in the PSID to classify job separators into job losers and quitters. He classified a worker as a job loser if the worker gave “company folded, changed hands, employer moved out of town or went out of business” or “laid off or fired” as a reason for his or her separation. According to Polsky’s probit estimates, the probability of job loss conditional on a job separation for workers aged 45-54 rose 12% relative to workers aged 25-34.}

3 Firm Dynamics and Specific Human Capital

3.1 The Model

Consider an industry that produces a homogeneous good in a two-period setting. In each period
t (=1, 2) the industry faces a demand schedule given by \( Q_t = D(P_t) \), where \( P_t \geq 0 \) and \( Q_t \geq 0 \) denote the price and the aggregate output in period t respectively, \( D(P) \geq 0 \) and \( D'(P) < 0 \) for all \( P > 0 \), and \( \lim_{P \to 0} D(P) = +\infty \). Entry and exit of firms is free in each period, where each
firm is of measure zero so that it is too small to affect prices. The production requires labor
input; a firm can produce one unit of the good in a period if it employs one worker in that period,
and the firm can produce nothing otherwise. No firm can employ more than one worker. There
is a large number of ex-ante identical individuals, and in each period labor supply is perfectly
inelastic and fixed at one unit for each individual. Each individual can earn a reservation wage
of \( w > 0 \) per period in a competitive labor market outside this industry. Individuals display no
disutility of effort, and firms and individuals are both risk neutral. To keep the analysis simple,
they do not discount the future.

Each firm’s production efficiency is determined by its managerial capability and the level of
its worker’s firm-specific human capital, where managerial capability is interpreted representing
the ability of a firm’s top management to develop an effective strategy and create a unique
competitive position. Let \( a \) denote the managerial capability and \( a_t \) denote the realization of
time in the preceding five-year period, while the 1994 and 1996 DWSs asked workers whether they were displaced
from a job at any time in the preceding three-year period.

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9Farber (1997) computed three-year job loss rates as the number of workers who report having lost a job in the three calendar years before the survey date divided by employment at the survey date.

10Polsky (1999) used a “reason for new position” question in the PSID to classify job separators into job losers and quitters. He classified a worker as a job loser if the worker gave “company folded, changed hands, employer moved out of town or went out of business” or “laid off or fired” as a reason for his or her separation. According to Polsky’s probit estimates, the probability of job loss conditional on a job separation for workers aged 45-54 rose 12% relative to workers aged 25-34.
firm $i$'s managerial capability, which is a random draw from a uniform distribution between 0 and 1. Assume that $a_i$ is ex-ante unknown to all agents including firm $i$ itself and becomes common knowledge at the end of the first period of firm $i$'s operation.\footnote{One might argue that managerial capability could not be a significant driving force of firm dynamics by pointing out that, since managerial decisions are often made by a large number of executives, “average” managerial capability should not vary much between firms if each executive’s managerial capability is a random draw from a certain distribution. However, the managerial capability of a top management, which significantly differs across firms, is particularly important, because it is a top management who makes key strategic decisions. Furthermore, each executive’s managerial capability is not necessarily a random draw, because it is usually a top management who selects lower level executives and hence their selection is heavily influenced by the top management’s ability.}

This specification is consistent with the widely held view that the ability of a firm’s top management is mostly innate, and difficult to observe or assess \textit{ex ante}.\footnote{As Goleman (1998) puts it, “Every business person knows a story about a highly intelligent, highly skilled executive who was promoted into a leadership position only to fail at the job. And they also know a story about someone with solid - but not extraordinary - intellectual abilities and technical skills who was promoted into a similar position and then soared.” Mabey and Ramirez (2004) found, based on 1,400 telephone interviews in seven European countries (Norway, Denmark, Germany, France, Romania, Spain, and the UK), that the belief that managers and leaders are “born not made” continues to prevail in Europe: all countries except for Germany rate innate ability/personality as the most important factor in making an effective manager.}

In period 1, each firm $i$ can provide a level of firm-specific human capital denoted $h_i \in [0, H]$ with its period 1 employee by incurring a cost of $d(h_i) \geq 0$ per employee, where $d(.)$ is a convex function.\footnote{The qualitative nature of the results is unchanged under an alternative setup in which each worker acquires a level of firm-specific human capital by incurring costs.} To obtain closed form solutions in the analysis, let $d(h) = \frac{1}{2} h^2$. The level of firm-specific human capital is observable but not verifiable, and so wage contracts contingent upon it are not feasible. To keep the analysis simple, assume that firm-specific human capital affects the second-period production efficiency only. If a firm continues to operate in the second period, the return from its investment in specific human capital is shared with its employee through wage bargaining.\footnote{Gibbons and Waldman (1999) argue that this is a useful approach by pointing out that human-capital investment levels are typically not specified in contracts, and it is not clear that such investment levels are even contractible variables. Furthermore, they point out that post-training wages are not typically specified in a contract, and can often be renegotiated after training has taken place. This approach suggests underinvestment in specific human capital due to a hold-up problem.}

Since the return from firm-specific human capital is deterministic, managerial capability is the only source of uncertainty in this model. The qualitative nature of the results, however, is unchanged under an alternative assumption that the return from specific human capital is also uncertain. See the next subsection (third and fourth paragraphs)
after Proposition 2) for details.

Each firm $i$’s per-unit production cost (excluding the wage bill) is given by $c - x a_i$ in period 1 and $c - x a_i - \lambda$ in period 2, where $c > 0$ and $x > 0$ are given constants and $\lambda$ captures the relationship between firm-specific human capital and the production efficiency of a firm. In particular, assume that $\lambda = y h_i$ ($y > 0$) if firm $i$ employs worker $j$ in period 2 and employed the same worker in period 1, while $\lambda = 0$ if firm $i$ employs worker $j$ in period 2 but did not employ the worker in period 1. Here, $x$ and $y$ capture the importance of managerial capability and firm-specific human capital, respectively, for production efficiency. Assume that $c > x + y H$, which guarantees that the production cost is strictly positive.

The timing of moves in the game is as follows:

**Period 1:**

[Stage 1] Firms simultaneously make first period wage offers to the individuals. Each individual can apply to a firm for first-period employment. Each firm employs one individual from the applicants, or no individuals if there are no applicants. If an individual is not employed by the firm or if he/she has not applied for any firm, he/she can earn the reservation wage $w > 0$ for period 1 in a competitive labor market outside this industry.

[Stage 2] If firm $i$ employs worker $j$ in the first period, firm $i$ chooses a level of firm-specific human capital $h_i \in [0, H]$.

[Stage 3] Each firm $i$ that employed a worker at Stage 1 produces one unit of the good. At the same time, firm $i$’s managerial capability $a_i$ is realized and becomes common knowledge.

**Period 2:**

[Stage 4] Each firm that operated in period 1 can bargain against its first-period employee on his/her second-period wage, which is determined as the outcome of the generalized Nash bargaining process. Each employee’s bargaining strength is given by $b \in (0, 1)$. The outside option of the firm is to employ another individual at the wage of $w$ or to exit the industry, while the outside option of the worker is to earn the reservation wage $w$ in a competitive labor market outside the industry. At the same time, a firm that did not operate in period 1 can employ an individual at the reservation wage $w$ and enter the industry.

[Stage 5] Each firm $i$ that employed a worker at Stage 4 produces one unit of the good. At the same time $a_i$ is realized if firm $i$ did not operate in period 1.
3.2 An Analysis of the Model

Consider a perfect foresight equilibrium that is characterized by a price sequence \((P_1, P_2)\). I focus on equilibria in which a strictly positive measure of firms operates in each period.\(^{15}\) In the equilibrium all agents (firms, potential entrants, and individuals) make optimal decisions based on the anticipation of a particular price sequence \((P_1, P_2)\), and their behavior does in fact give rise to the same \((P_1, P_2)\). Given free entry and exit of firms, for every entrant in period \(t (=1, 2)\) the present discounted value of its expected overall profit is zero in the equilibrium. Also, given that there is a large number of ex-ante identical and risk-neutral individuals, and that every individual can earn a reservation wage \(w > 0\) per period in a competitive labor market outside the industry, the present discounted value of every individual’s expected overall wage is \(2w\) in period 1 in the equilibrium. The market clears in each period in the equilibrium.

I focus on perfect foresight equilibria in which a strictly positive number (measure) of firms enter and exit the industry at the beginning of period 2, given that in reality entries and exits of firms are common in most industries.\(^{16}\) Proposition 1 identifies necessary and sufficient conditions for such an equilibrium to exist, and characterizes the equilibrium. Proposition 2 and 3 then present comparative statics results on the exit rate of firms, the level of specific human capital, and the slope of the earnings-tenure profiles in the equilibrium. Throughout the analysis, I assume \(H > \max\{\frac{1}{2}(1-b)x_0, \frac{x_1}{2}\}\), which guarantees that the optimal level of firm-specific human capital is interior in the equilibrium. Note, proofs of the propositions are presented in the Appendix.

Suppose that there exists a perfect foresight equilibrium characterized by a price sequence \((P_1, P_2) = (P_1^*, P_2^*)\) where \(P_1^* > 0\) and \(P_2^* > 0\), in which a strictly positive measure of firms enter and exit the industry at the beginning of period 2. In what follows, we will first identify necessary conditions for such an equilibrium to exist.

In the equilibrium, each second-period entrant employs a worker at the reservation wage \(w\), and its expected production cost is \(c - \frac{1}{2}x\). Since each second-period entrant earns zero expected

\(^{15}\)There is a possibility of a trivial equilibrium in which a strictly positive measure of firms operates in period 1 but no firms operate in period 2. One way to rule out this equilibrium is to assume that \(D(P) > 0\) for all \(P \geq 0\).

\(^{16}\)See Geroski (1995) for a survey of the empirical literature. For example, Dunne, Roberts, and Samuelson (1988) analyzed the pattern of firm entry and exit in 387 four-digit US industries in the period 1963-1982 by using a new data set that had been constructed from the individual plant-level data collected in the five Censuses of Manufacturers during the period. They found that, after deleting the smallest firms in each industry, the average entry rate varied from 30.7% to 42.7% across census years, while the average industry exit rate varied from 30.8% to 39.0%. Although there was substantial variation across industries, both entry and exit rates were at least 10% in most four-digit industries.
profit in the equilibrium, \( P_i^* = c + w - \frac{1}{2}x \) must hold. Consider firm \( i \) that employed worker \( j \) at Stage 1 and chose \( h_i \) at Stage 2 in the equilibrium. If firm \( i \) continues to employ worker \( j \) in period 2, its second-period production cost (excluding the wage bill) is \( c - xa_i - y h_i \) and the firm must pay at least \( w \) (the reservation wage) in order to employ worker \( j \) in period 2. Then firm \( i \) continues to operate in period 2 if and only if \( P_i^* - (c - xa_i - y h_i + w) \geq 0 \Leftrightarrow a_i \geq g(h_i, P_i^*) \), where \( g(h, P_2) \) (call it a cut-off managerial capability) is defined by

\[
g(h, P_2) \equiv \frac{c+w-P_2-yh}{x} \text{ if } h \leq \frac{c+w-P_2}{y}, \text{ and } 0 \text{ otherwise.}
\]

Suppose that firm \( i \)'s managerial capability \( a_i \) turns out to be greater than or equal to the cut-off level \( g(h_i, P_i^2) \) at Stage 3. Then, at Stage 4 firm \( i \) and worker \( j \) bargain over worker \( j \)'s second-period wage, where the worker’s bargaining strength is \( b \in (0, 1) \) and his/her threat point is the reservation wage \( w \). Firm \( i \)'s outside option is to exit the industry and earn zero profit or to employ another worker with wage \( w \) and continue operating in period 2. Firm \( i \)'s second-period profit under the latter option is \( P_i^* - (c - xa_i + w) \), and hence its threat point is \( \max\{P_i^* - (c - xa_i + w), 0\} \). We then find that worker \( j \)'s second-period wage (which is determined as the outcome of the generalized Nash bargaining process) is \( w_2(a_i, h_i, P_i^2) \), where \( w_2(a, h, P_2) \) is defined by

\[
w_2(a, h, P_2) \equiv w + bS(a, h, P_2).
\]

Here, \( S(a, h, P_2) \equiv P_2 - (c - xa - yh) - w - \max\{P_2 - (c - xa + w), 0\} \), which is the surplus gained by reaching an agreement in the bargaining.\(^{17}\) This in turn implies that firm \( i \)'s second-period production cost plus wage bill is \( C_2(a_i, h_i, P_i^2) \), where \( C_2(a, h, P_2) \) is defined by

\[
C_2(a, h, P_2) \equiv c - xa - yh + w_2(a, h, P_2).
\]

At Stage 2, firm \( i \) chooses the level of firm-specific human capital \( h_i \) without knowing its own managerial capability \( a_i \). Firm \( i \) makes this choice under the anticipation that it will continue to operate in period 2 and earn second-period profit of \( P_i^2 - C_2(a_i, h_i, P_i^2) \) if it realizes managerial capability \( a_i \geq g(h_i, P_i^2) \), and will exit the industry if \( a_i < g(h_i, P_i^2) \). Hence firm \( i \) chooses \( h_i \in [0, H] \) to maximize \( \pi(h_i, P_i^2) \), where \( \pi(h, P_2) \) is defined by

\[
\pi(h, P_2) \equiv \int_{\min\{g(h, P_2),1\}}^1 (P_2 - C_2(a, h, P_2))da - \frac{1}{2}h^2.
\]

Note, \( \pi(h_i, P_i^2) \) is firm \( i \)'s second-period expected profit minus its cost for providing the firm-specific human capital of level \( h_i \). Through the maximization exercise we find (see Claim 1 in

\(^{17}\)See for example Chang and Wang (1996) and Zábojník (1998) for similar formulations of worker-firm bargaining.
the Appendix for details) that \( x - (1 - b)y^2 > 0 \) must hold, and every firm \( i \) that employed a worker at Stage 1 chooses \( h_i = h^* \) at Stage 2 in the equilibrium, where

\[
h^* = \frac{1}{2} \frac{(1 - b)xy}{x - (1 - b)y^2}.
\]  

(4)

This implies that the firms’ exit rate is \( \alpha^* \) in the equilibrium, where

\[
\alpha^* \equiv \alpha(h^*, P_2^*) = \max \left\{ \frac{1}{2} \frac{x - 2(1 - b)y^2}{x - (1 - b)y^2}, 0 \right\}.
\]  

(5)

Since the exit rate is strictly positive in the equilibrium, the following condition must hold:

\[
x - 2(1 - b)y^2 > 0.
\]  

(6)

Note that under this condition we have that \( 0 < \alpha^* < \frac{1}{2} \).

Every firm that employs a worker at Stage 1 offers the same first-period wage (denoted by \( w_1 \)) in the equilibrium, given that firms and individuals are ex-ante identical. A worker employed by firm \( i \) at Stage 1 anticipates that his/her second-period wage will be \( w_2(a_i, h^*, P_2^*) \) if firm \( i \) realizes \( a_i \geq a^* \) at Stage 3, and \( w \) if \( a_i < a^* \). The first-period present discounted value of the worker’s expected overall wage is then \( w_1 + \int_0^{a^*} wda + \int_{a^*}^1 w_2(a, h^*, P_2^*)da \). Given that there is a large number of ex-ante identical and risk-neutral individuals, and that every individual can earn a reservation wage \( w > 0 \) per period outside the industry, firms in the equilibrium choose \( w_1 = w^*_1 \) such that \( w^*_1 + \int_0^{a^*} wda + \int_{a^*}^1 w_2(a, h^*, P_2^*)da = 2w \) holds. Hence we find

\[
w^*_1 = w - \int_{a^*}^1 (w_2(a, h^*, P_2^*) - w)da. \tag{7}
\]

In the equilibrium, every firm \( i \) that employs a worker at \( w^*_1 \) at Stage 1 provides the worker with level \( h^* \) of specific human capital by incurring \( \frac{1}{2} (h^*)^2 \) as a training cost. Since its expected production cost is \( c - \frac{1}{2}x \), the expected value of its first-period cost is \( c - \frac{1}{2}x + w^*_1 + \frac{1}{2}(h^*)^2 \equiv C_1^* \). Firm \( i \) operates in period 2 if it realizes the managerial capability of \( a_i \geq a^* \) with the second-period total cost of \( C_2(a_i, h^*, P_2^*) \). Hence the first-period discounted value of its overall expected cost is \( C_1^* + \int_{a^*}^1 C_2(a, h^*, P_2^*)da \), and the zero profit condition implies that the following condition must hold:

\[
P_1^* + \int_{a^*}^1 P_2^*da = C_1^* + \int_{a^*}^1 C_2(a, h^*, P_2^*)da. \tag{8}
\]

We then find (see Claim 2 in the Appendix for details) that

\[
P_1^* = c + w - \frac{1}{2}x - \frac{1}{8} \frac{x^2y^2}{[x - (1 - b)y^2]^2}.
\]  

(9)

\(^{18}\)The model allows a possibility for the first-period wage to take a negative value. This can be avoided by assuming that the reservation wage \( w \) is large enough. A sufficient condition for this is \( w > \frac{x^2y^2}{4(x - (1 - b)y^2)^2} \).
Given \( P^*_1 > 0 \), the following condition must hold:
\[
c + w > \frac{1}{2} x + \frac{1}{8} \frac{x^2[x - (1 - b)^2y^2]}{[x - (1 - b)y^2]^2}.
\] (10)

Given \((P_1, P_2) = (P^*_1, P^*_2)\), the demand for the good in period \( t \) (= 1, 2) is \( D(P^*_t) \). Since the market clears in each period and each operating firm produces one unit of the good, the measure of firms that enter and operate in period 1 is \( D(P^*_1) \) in the equilibrium. Since the exit rate is \( a^* \), \((1 - a^*)D(P^*_1)\) firms continue to operate in period 2. Since the second-period demand for the good is \( D(P^*_2) \), the measure of the second-period entrants is \( D(P^*_2) - (1 - a^*)D(P^*_1) \) in the equilibrium. Then the following condition must hold for a strictly positive measure of firms to enter at the beginning of period 2 in the equilibrium:
\[
D(P^*_2) - (1 - a^*)D(P^*_1) > 0.
\] (11)

Thus far we have found that conditions (6), (10) and (11) are necessary for the existence of a perfect foresight equilibrium in which a strictly positive measure of firms enter and exit the industry at the beginning of period 2. Concerning condition (11) we have that \( (1 - a^*)D(P^*_1) \) is strictly increasing in \( y \) while \( D(P^*_2) \) is independent of \( y \). We then find that conditions (6), (10) and (11) are all satisfied if the value of \( y \) is small enough (see Claim 3 in the Appendix for details). Proposition 1 below tells us that this condition is not only necessary but also sufficient, and the equilibrium is a unique equilibrium under this condition.

**Proposition 1:** For any given parameterization, there exists a unique value \( \bar{y} \geq 0 \) such that the following property holds: There exists a unique perfect foresight equilibrium in which a strictly positive measure of firms enter and exit the industry at the beginning of period 2, if and only if \( y < \bar{y} \). The equilibrium is characterized by the price sequence \((P_1, P_2) = (P^*_1, P^*_2) = (c + w - \frac{1}{2} x - \frac{1}{8} \frac{x^2[x - (1 - b)^2y^2]}{[x - (1 - b)y^2]^2}, c + w - \frac{1}{2} x)\). There exists a range of parameterizations in which \( \bar{y} > 0 \).

In the equilibrium, \( D(P^*_1) \) firms operate in period 1. Each of the \( D(P^*_1) \) firms employs a worker at the wage of \( w^*_1 \) at Stage 1 and provides the worker with level \( h^* \) of specific human capital at Stage 2. At Stage 4, every firm \( i \) whose managerial capability \( a_i \) turns out to be greater than or equal to the cut-off level \( a^* \in (0, \frac{1}{2}) \) continues to operate in period 2 by employing its first-period employee at the second-period wage of \( w_2(a_i, h^*, P^*_2) \), while every firm \( i \) with \( a_i < a^* \) exits the industry. Hence the firms’ exit rate, which is equal to the labor turnover rate, is \( a^* \) in the equilibrium. Then, of the \( D(P^*_1) \) firms that operated in period 1, \((1 - a^*)D(P^*_1)\) firms continue to operate in period 2 and \( D(P^*_2) - (1 - a^*)D(P^*_1) > 0 \) new firms enter at the beginning of period 2.
I will now turn to comparative statics of the equilibrium exit rate $a^* = \frac{1}{2} \frac{\bar{y} x}{x - (1 - b) y}$, the equilibrium level of firm-specific human capital $h^* = \frac{1}{2} \frac{(1 - b) y}{x - (1 - b) y}$, and the steepness of the tenure-wage profile in the equilibrium. Concerning the tenure-wage profile, consider an individual who has been employed by firm $i$ at the first-period wage $w^*_1 = w - \int_{a^*}^{a_i} (w_2(a, h^*, P^*_2) - w) da$.

In period 2, the worker is employed by firm $i$ at the second-period wage $w_2(a_i, h^*, P^*_2)$ if firm $i$ realizes $a_i \geq a^*$, and earns the reservation wage $w$ elsewhere if $a_i < a^*$. Hence, his/her second-period expected wage conditional upon being employed by the first-period employer is $w'_2 = \frac{1}{1 - a^*} \int_{a^*}^{a_i} w_2(a, h^*, P^*_2) da$. I will interpret $w'_2 - w^*_1$ to be the steepness of the tenure-wage profile in the equilibrium. Note, (1) and (7) above together imply that the tenure-wage profile is upward-sloping in the equilibrium. This is because workers’ productivity (cost effectiveness) increases with tenure, and the return from the higher productivity is shared between the worker and the employer in period 2 which in turn implies that the second-period wage is higher than the first-period wage in the equilibrium.19

**Proposition 2:** As the importance of managerial capability (captured by $x$) increases, the firms’ exit rate increases, the level of firm-specific human capital investment decreases, and the tenure-wage profile becomes less steep in the equilibrium.

The key finding is that the firms’ exit rate is increasing in the importance of managerial capability. This result arises from the connection between firm dynamics and firm-specific human capital through the following logic: A first-period entrant continues to operate in period 2 if its second-period production efficiency is higher than the expected production efficiency of second-period entrants. Each first-period entrant has an advantage over second-period entrants because it has a worker who has already accumulated a certain level of firm-specific human capital. Hence, even if the managerial capability of a first-period entrant turns out to be less than average, it could still continue to operate in the second period. As the importance of managerial capability increases, the relative importance of firm-specific human capital declines. This reduces first-period entrants’ advantage associated with firm-specific human capital, and hence fewer first-period entrants with “lower than average” managerial capability survive in the second period.20 As a consequence, the firms’ exit rate increases.

19Complementary to the productivity-based reasoning, Lazear (1979, 1981) demonstrated that wages grow with experience, even if productivity does not. In his framework, senior workers receive high salaries not because they are so much more productive than junior workers, but because paying senior workers higher wages produces appropriate work incentives for junior workers. See also Salop and Salop (1976) for an explanation based on self-selection of workers.

20Regardless of the importance of managerial capability, every first-period entrant whose managerial capability turns out to be higher than average continues to operate in period 2.
To algebraically observe that the connection between firm dynamics and specific human capital is the driving force of the result, let us suppose $y$ (which captures the importance of specific human capital) is equal to zero. Then, the equilibrium exit rate becomes $a^* = \frac{1}{2} \frac{x - 2(1 - b)y^2}{x - (1 - b)y^2} = \frac{1}{2}$. That is, in the absence of firm-specific human capital, the firms’ exit rate is independent of the importance of managerial capability.

The result does not depend on the model specification that the firm’s managerial capability is the only source of uncertainty in the model. To see this, let us consider what happens if there is uncertainty regarding the returns from firm-specific human capital investment as well. In particular, suppose that if a first-period entrant firm $i$ continues to operate and employ its first-period worker in period 2, its second-period production cost (excluding the wage bill) is now given by $c - xa_i - y\theta_i h_i$. The uncertainty regarding firm-specific human capital is captured by $\theta_i$, which is a random draw from a known distribution function between $\theta$ and $\overline{\theta}$ ($\overline{\theta} > \theta \geq 0$). Assume that both $a_i$ and $\theta_i$ are ex-ante unknown and become common knowledge at the end of the first period (at Stage 3), and are mutually independent.

In this variant of the model, each first-period entrant firm $i$ continues to operate in period 2 if its second-period production cost $c - xa_i - y\theta_i h_i$ is lower than the expected production cost of second-period entrants $c - \frac{1}{2}x$. This condition is equivalent to $a_i \geq \frac{1}{2} - \frac{y}{x} \theta_i h_i$. On the other hand, the analogous condition is $a_i \geq \frac{1}{2} - \frac{y}{x} h_i$ in the original model, where the return from firm-specific human capital is deterministic (in particular, $\theta_i$ is equal to one for all $i$). Each of these two conditions tells us that each first-period entrant becomes less likely to survive as $x$ increases (note, the right-hand sides of these conditions, $\frac{1}{2} - \frac{y}{x} \theta_i h_i$ and $\frac{1}{2} - \frac{y}{x} h_i$, are both increasing in $x$). The key logic is that as the importance of managerial capability increases, the relative importance of firm-specific human capital (captured by $\frac{y}{x}$) declines, which in turn reduces first-period entrants’ advantage associated with firm-specific human capital. The logic does not depend on whether returns from human capital investment are uncertain or deterministic.

Other results of Proposition 2 naturally follow from the key result mentioned above. As the importance of managerial capability increases, the exit rate of firms as well as the separation rate of workers increase. Anticipating this, first-period entrants have lower incentives to train their workers in period 1, and this reduces the equilibrium level of firm-specific human capital investment. Since the return from firm-specific human capital is shared between a worker and his/her employer through the second-period wage bargaining, the lower level of specific human capital and the higher exit rate result in a lower second-period expected wage, which in turn implies that firms have to offer higher first-period wages to attract workers in period 1. The result is that the equilibrium tenure-wage profile becomes less steep as $x$ increases.

In what follows, I will discuss the real-world relevance of the comparative statics results
presented above, based on the idea that the importance of managerial capability can differ across industries. An argument that is consistent with this idea can be found in Porter (1996).\textsuperscript{21} Porter pointed out that the core role of a firm’s top management is to develop or re-establish a clear strategy, where the development of an effective strategy means the top management’s deliberate choice of a distinctive set of activities undertaken by the firm in order to deliver a unique mix of value to customers. He then argued that developing a strategy in a newly emerging industry or in a business undergoing revolutionary technological changes is particularly difficult, because in such industries the firm’s management faces a high level of uncertainty about customers’ needs, the products and services that will prove to be the most desired, and the best configuration of activities and technologies to deliver them.\textsuperscript{22}

This argument suggests that the importance of managerial capability tends to be higher in industries with higher levels of uncertainty, such as high-tech industries, while it tends to be lower in matured industries with lower levels of uncertainty.\textsuperscript{23} Proposition 2 then predicts that firm’s exit rate and labor turnover rate are both high in high-tech industries such as the semiconductor industry, and evidences that support this prediction can be found in several case studies. For example, in his study of labor markets in Silicon Valley, Benner (2002) pointed out as follows: “The rapid turnover and volatility in employment in Silicon Valley is integrally connected to the nature of competition in the region’s high-technology industries. In these industries, markets and technology change extremely rapidly and in unpredictable ways. Those firms that succeed are those that are able to innovate by developing both new products and improved production processes to shorten the time-to-market.” According to Benner, of the 100 largest Silicon Valley companies in 1985, only 19 still existed and were in the top 100 in 2000. While more than half of the top 100 companies in the 2000 listing of Silicon Valley’s largest

\textsuperscript{21}See also empirical studies by Ely (1991) and Hogan and Sigler (1998), who found that sensitivity of CEO compensation to firm performance significantly differs across industries. Assuming that the sensitivity captures the importance of managerial capability for firm performance, this finding suggests that the importance of managerial capability differs across industries.

\textsuperscript{22}A related discussion is found in Hayek (1945). Hayek pointed out that, “It is, perhaps, worth stressing that economics problems arise always and only in consequence of change. So long as things continue as before, or at least as they were expected to, there arise no new problems requiring a decision, no need to form a new plan”, where “planning” is defined as the complex of interrelated decisions about the allocation of available resources.

\textsuperscript{23}Consistent with this argument, in his study of the US semiconductor industry, Angel (1994, p. 4-5) pointed out that, “In an era of intensified global competition, it is the ability to anticipate and create new market opportunities, to develop new products ahead of competitors, and to reconfigure manufacturing processes rapidly in response to changing production requirements that offers the best prospect for long-term profitability of firms and industries.”
firms were not on the list only ten years previously.\textsuperscript{24}

I will now turn to the comparative statics results with respect to the importance of firm-specific human capital.

**Proposition 3:** As the importance of firm-specific human capital (captured by $y$) increases, the firm’s exit rate decreases, the level of firm-specific human capital investment increases, and the tenure-wage profile becomes steeper in the equilibrium.

Recall that each first-period entrant has an advantage over second-period entrants because it has a worker who has already accumulated a certain level of firm-specific human capital. This advantage becomes more substantial as the importance of firm-specific human capital increases, and hence this reduces the exit rate of first-period entrants. The lower exit rate, along with the higher return from firm-specific human capital (captured by $y$), implies that firms have higher incentives to train their employees in the first period. These two effects are mutually re-enforcing because the higher level of firm-specific human capital reduces firms’ exit rate by increasing the advantage possessed by first-period entrants. The result on the tenure-wage profile follows through the logic analogous to the one explained above for Proposition 2.

As a final point to the analysis section, note that in my model the exit of firms (that is, firm closure) is the sole source of job separation. This appears to capture only a fraction of involuntary separations, which in reality is also caused by firm contraction, lay-offs, and being fired. It can, however, be interpreted that my model captures job separation due to firm contraction as well. To see this, consider the following variant of the model: In the beginning of period 1, there exist a certain number of firms that conduct an existing business in an industry. The business is stable so that all firms will continue to operate in period 2 without contraction or expansion, no more firms enter, and no job separation occurs. Suppose that a new business opportunity arises in the industry at the beginning of period 1. The existing firms as well as new entrants can conduct the new business in period 1 where they are equally uncertain about their managerial abilities for the new business, and entry and exit of firms is free at the beginning of period 2.\textsuperscript{25}

Let us suppose that this new business has the same structure as the model analyzed above, where the demand schedule associated with this new business is given by $Q_t = D(P_t)$, $t = 1, 2$.

\textsuperscript{24}See also Saxenian (1996), who pointed out that, “By the 1970s, Silicon Valley was distinguished by the highest levels of job-hopping in the nation. Average annual employee turnover in local electronics firms exceeded 35% and was as high as 59% in the region’s small firms. It was almost unheard of for a technical professional in Silicon Valley to have a career in a single company.”

\textsuperscript{25}The qualitative nature of the results would be unchanged under an alternative assumption that existing firms are more likely to have a higher managerial capability than new entrants.
In this variant of the model, the same results as the ones from the original model follow, except that job separation is now due not only to firm closure but also to firm contraction. That is, if an existing firm in the industry conducts new business in period 1 and exits from it at the beginning of period 2, then the job separation associated with it is due to firm contraction because the firm remains in the industry and continues to conduct the stable, existing business. On the other hand, if a new entrant in the industry conducts new business in period 1 and exits from it at the beginning of period 2, job separation is due to firm closure because the firm exits from the industry.

4 A Welfare Consequence of Entry Restrictions

This section explores a new perspective on the welfare consequences of entry restrictions by analyzing its effects on labor market characteristics in my framework. Is free entry desirable for social efficiency? This important question has been addressed in the theoretical industrial organization literature. Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) showed that in homogeneous final-product markets with Cournot oligopoly and fixed set-up costs, level of entry in the free-entry equilibrium is always socially excessive. It has often been argued that this theoretical result (often called “excess-entry theorem”) can provide a justification for enforcing entry regulations as a way of improving welfare.26

Strategic interaction among firms in the product market plays a central role in these earlier models. Complementary to, but fundamentally different from, this approach, my analysis focuses on the effects of entry restrictions on labor market characteristics.

Consider the following extension of my model. Recall that, in the original model where the second-period entry is free, a strictly positive measure of firms \( N_f \equiv D(P^*_2) - (1 - a^*)D(P^*_1) > 0 \) (the subscript \( f \) stands for free entry) enter the industry at the beginning of period 2 in the unique perfect foresight equilibrium when \( y < \bar{y} \). In this extension, everything is the same as in the original model except that a government can impose an entry restriction in the following way. At Stage 0 (before Stage 1) the government can announce \( N_r \in [0, N_f] \), which is the maximum measure of firms that are allowed to enter the industry at the beginning of period 2 (that is, at Stage 4). If the government imposes an entry restriction, the subsequent Stage 1 subgame

---

26Degree of entry regulations significantly differs across countries. Djankov, La Porta, Lopez-de-Silanes, and Shleifer (2002) presented new data on the regulation of entry of start-up firms in 85 countries around the world, where the data cover the number of procedures, official time, and official cost that a start-up must bear before it can operate legally. In their data, there are 13 countries in which the number of procedures is less than or equal to five, while there are 16 countries in which the number of procedures is greater than or equal to fifteen.
is called an entry-restriction subgame, and the Stage 1 subgame without entry restriction is called the free-entry subgame (which is the same as the original model). In an entry-restriction subgame, the government, at Stage 4, charges an entry tax per firm so that $N_r$ firms enter the industry and each second-period entrant earns zero expected profit in the equilibrium.

I will consider a perfect foresight equilibrium of each subgame, and show that entry restrictions can enhance welfare in this model.\(^{27}\) Note that in order to focus on the main logic behind welfare consequences of entry restrictions in this framework, the extension allows the government to impose an entry restriction for period 2 only. The qualitative nature of the results is however unchanged under an alternative setup in which the government can impose entry restrictions for both periods 1 and 2. See the second to the last paragraph of this section for details.

Lemma 1 characterizes the perfect foresight equilibria of entry-restriction subgames. Note, if the government chooses $N_r = N_f$, the entry restriction is not binding and so the corresponding perfect foresight equilibrium is the same as that of the free-entry subgame.

**Lemma 1:** Suppose that $y < \bar{y}$ holds, where $\bar{y}$ is as defined in Proposition 1. There exist functions $\hat{P}_1(N_r), \hat{P}_2(N_r)$ and $\hat{a}(N_r)$ with the following property:

(i) An entry-restriction subgame represented by any given $N_r \in [0, N_f]$ has a unique perfect foresight equilibrium characterized by the price sequence $(P_1, P_2) = (\hat{P}_1(N_r), \hat{P}_2(N_r))$ and the exit rate $\hat{a}(N_r)$, where $D(\hat{P}_2(N_r)) - (1 - \hat{a}(N_r))D(\hat{P}_1(N_r)) = N_r$ holds.

(ii) $\hat{P}_1(N) (\hat{P}_2(N))$ is continuous and strictly increasing (decreasing) in $N$ for all $N \in [0, N_f]$, where $\hat{P}_1(N_f) = P_1^*$ and $\hat{P}_2(N_f) = P_2^*$. Also, $\hat{a}(N)$ is continuous and strictly increasing in $N$ for all $N \in [0, N_f]$, where $0 < \hat{a}(N) < a^*$ for all $N \in [0, N_f]$, and $\hat{a}(N_f) = a^*$.

Lemma 1 tells us that, in the entry-restriction subgame represented by $N_r \in [0, N_f]$, the equilibrium second-period price $\hat{P}_2(N_r)$ is greater than $P_2^* = c + w - \frac{1}{2}x \equiv C_N$. In the free-entry equilibrium, the zero profit condition implies that the second-period price $P_2^*$ is equal to $C_N$, which is the expected cost (including the wage bill) of the second-period entrant. Under the entry restriction, the government charges a second-period entry tax of $\hat{P}_2(N_r) - C_N$ per unit so that every second-period entrant earns zero expected profits. In the equilibrium of any entry-restriction subgame, every entrant in period $t$ ($=1,2$) earns zero expected profits, and in period 1 the present discounted value of every individual’s expected overall wage is $2w$. Hence I define

\(^{27}\)An alternative way to incorporate entry restrictions in the model is that the government, at Stage 0, announces whether it will allow free entry or prohibit entry at the beginning of period 2. Under this alternative setup, it can be shown that there exists a range of parameterizations in which the prohibition of entry enhances welfare.
the equilibrium total surplus by

$$W(N_r) \equiv \int_{\hat{P}_1(N_r)}^{\infty} D(P)dP + \int_{\hat{P}_2(N_r)}^{\infty} D(P)dP + (\hat{P}_2(N_r) - C_N)N_r,$$

where \(\int_{\hat{P}_1(N_r)}^{\infty} D(P)dP\) is the consumer surplus in period \(t\), and \((\hat{P}_2(N_r) - C_N)N_r\) is the total entry tax the government receives. Since there is no entry tax in the free-entry subgame, the equilibrium total surplus of the free-entry subgame is defined by

$$\int_{\hat{P}_1(N_f)}^{\infty} D(P)dP + \int_{\hat{P}_2(N_f)}^{\infty} D(P)dP,$$

which is equal to \(W(N_f)\), given \(\hat{P}_2(N_f) = P^*_2 = C_N\). Also, let \(CS(N_r) \equiv \int_{\hat{P}_1(N_r)}^{\infty} D(P)dP + \int_{\hat{P}_2(N_r)}^{\infty} D(P)dP\), which is the equilibrium consumer surplus.

**Proposition 4:** Entry restrictions can increase the total surplus and the consumer surplus. More precisely, there exists \(N^*_r \in [0, N_f]\) such that \(W(N^*_r) > W(N_f)\) and \(CS(N^*_r) > CS(N_f)\).

The logic behind this result is as follows. Think back to the free-entry equilibrium. Recall that each first-period entrant firm \(i\) continues to operate in period 2 if its second-period production cost \(c - xa_i - yh_i\) is lower than the expected production cost of second-period entrants \(c - \frac{1}{2}x\), and exits the industry otherwise. This continuation/exit decision is socially optimal, which in turn means that, if each firm \(i\) could capture the entire return from its employee’s human capital in period 2, it would choose the socially optimal level of \(h_i\) in period 1. However, since each firm \(i\) must share the return from specific human capital with its employee through a second-period wage bargaining, it chooses a level of \(h_i\) that is below the socially optimal level. This is a version of the standard underinvestment problem in specific human capital when post-training wages are determined by bargaining.

An entry restriction represented by \(N_r \in [0, N_f]\) reduces the second-period supply of the good, which in turn implies that the equilibrium second-period price \(\hat{P}_2(N_r)\) becomes higher than the free-entry level \(P^*_2\) in order for the second-period market to clear. The higher second-period price makes the second-period operation more attractive, and hence reduces the equilibrium exit rate. This yields the following two welfare consequences. On the one hand, every firm anticipates a lower exit rate as a consequence of the entry restriction, which increases its incentive to provide specific human capital to its employee in the first period. This mitigates the underinvestment problem mentioned above. On the other hand, given that \(\hat{P}_2(N_r) > P^*_2 = C_N\) holds under the entry restriction, some first-period entrants whose second-period costs are higher than the expected cost of the second-period entrants \(C_N\) continue to operate in the second period. That is, continuation/exit decision is not socially optimal under the entry restriction.

Suppose that the government imposes a small entry restriction in the sense that \(N_r\) is strictly less than but close to \(N_f\), so that the equilibrium exit rate becomes slightly below the level in
the free-entry equilibrium. Then the increment of the level of specific human capital provided by firms is small, but this affects all firms that operate in the first period. On the other hand, since the equilibrium exit rate is slightly below the free-entry level, a small number of first-period entrants whose second-period costs are slightly higher than the second-period entrants’ expected cost survive in period 2. That is, the positive effect of the entry restrictions in mitigating the underinvestment problem in specific human capital is of first order, while its negative effect associated with the suboptimal continuation/exit decision is of second order. Hence, at the margin, the former dominates the latter, and hence entry restrictions can increase the total surplus. Also, given that the entry tax is zero when \( N_r = N_f \), entry restrictions can enhance the consumer surplus as well at the margin.

In summary, the government can control firm dynamics by imposing a certain degree of entry restrictions, which in turn affects labor mobility and hence firms’ incentives to invest in specific human capital. Entry restrictions can mitigate the underinvestment problem in specific human capital at the cost of the suboptimal continuation/exit decision, and this can improve welfare. This approach focuses on the welfare consequence of entry restrictions on labor market characteristics, while the complementary approach previously taken in the industrial organization literature (“excess-entry theorem”) has focused on the welfare consequence of the strategic interaction among firms in the product market. In my analysis, entry restrictions can enhance the consumer surplus as well as the total surplus, while in the excess-entry theorem entry restrictions enhance the total surplus at the cost of the lower consumer surplus.

I end this section by making two final points. First, as pointed out above, the qualitative nature of the results is unchanged under an alternative extension in which the government can impose entry restrictions for both periods 1 and 2. In particular, suppose that at Stage 0 the government announces \( N_{r1} \geq 0 \) and \( N_{r2} \geq 0 \), which denote the maximum measure of firms that are allowed to enter the industry at the beginning of periods 1 and 2, respectively. It can be shown that the total surplus is maximized when the government imposes entry restrictions for both periods 1 and 2. As in the extension considered above, the second-period entry restriction mitigates the underinvestment problem in specific human capital at the cost of lowering the firms’ exit rate below the socially optimal level, and the latter inefficiency can be mitigated by imposing an entry restriction in the first period. Details of the analysis are available upon request.

Second, it seems important to note that these theoretical results do not automatically imply that the government in the real world can enhance welfare by restricting entry. In reality, there are a number of other factors to be taken into account. For example, as pointed out by Itoh,
Kiyono, Okuno-Fujiwara, and Suzumura (1991), the government may not be able to obtain sufficient information for effectively implementing entry restrictions. Also, even if it is possible, the government might have to incur substantially high costs to obtain such information. In real-world policy discussions, such negative aspects as well as potentially positive aspects of entry restrictions should be deliberately taken into account, and my contribution to this line of investigation is to offer a previously unexplored perspective on the welfare consequences of entry restrictions.

5 Conclusion

This paper has developed a new firm-dynamics model that incorporates workers, their accumulation of specific human capital, and their mobility. Models of firm and industry dynamics that allow for entry, exit and firm heterogeneity and/or idiosyncratic shock have been previously developed in the literature. Despite their significant contributions, they do not explicitly incorporate one important aspect of reality, which is that most firms employ workers to produce outputs. The present paper has attempted to fill this important gap in the literature by exploring a simple but important link that firm-specific human capital loses its value if the firm exits the industry.

I have demonstrated that the importance of a firm’s managerial capability, through its connection to firm-specific human capital, systematically influences firm dynamics, labor market variables, and employment practices. This has yielded empirical implications and predictions from a previously unexplored perspective, given that the importance of managerial capability can differ across industries. Equally important, my framework has offered a new perspective on the welfare consequences of entry restrictions. I have demonstrated this by analyzing an extension of the model in which the government can control firm dynamics by imposing entry restrictions. Novelty of this analysis is that it captures the effects of entry restrictions on labor market characteristics.

The importance of managerial capability can differ not only across industries but also across time and countries within the same industry. For example, Acemoglu, Aghion, and Zilibotti (2006) argued in their analysis of technology frontiers and firm selection that managerial skill is more important for undertaking innovative activities than for adopting and imitating existing technologies from the world technology frontier. They then pointed out that innovation becomes more important as the economy approaches the world technology frontier and there remains less room for adoption and imitation. This argument indicates that the importance of managerial capability tends to be higher in an industry that has reached or approached the technology
frontier than in an industry that is far behind it. Elaborating on this argument, in a future work I plan to explore an application of my framework to international differences in management practices.

6 Appendix

Proof of Proposition 1: Suppose that there exists a perfect foresight equilibrium characterized by a price sequence \((P_1, P_2) = (P^*_1, P^*_2)\) where \(P^*_1 > 0\) and \(P^*_2 > 0\), in which a strictly positive measure of firms enter and exit the industry at the beginning of period 2. In the text it has been shown that conditions (6), (10) and (11) are necessary for such an equilibrium to exist. Here, in Claims 1 and 2 below I present proofs of some mathematical/computational details that have not been presented in the text.

Claim 1: In the equilibrium every firm \(i\) that employed a worker at Stage 1 chooses \(h_i = \frac{1}{2} \frac{(1-b)xy}{y^2} = h^*\) at Stage 2, where \(x - 2(1-b)y^2 > 0\) must hold.

Proof: In the equilibrium there must exist a strictly positive measure of firms that operate in period 1 and exit the industry at the beginning of period 2 with a non-zero probability. Let firm \(k\) be such a firm. Following the same procedure and using the same notations as in the text, we find that

\[
\pi(h, P^*_2) = \int_{a(h, P^*_2)}^{1} (P^*_2 - C_2(a, h, P^*_2))da - \frac{1}{2} h^2 = \int_{a(h, P^*_2)}^{1} [P^*_2 - (c + w - xa - yh) - bS(a, h, P^*_2)]da - \frac{1}{2} h^2,
\]

where \(S(a, h, P^*_2) = [P^*_2 - (c - xa - yh) - w - \max\{P^*_2 - (c - xa + w), 0\}]\). Given \(P^*_2 = c + w - \frac{1}{2} x\), we have \(P^*_2 - (c + w - xa) \geq 0 \iff a \geq \frac{1}{2}\). This implies that

\[
\pi(h, P^*_2) = \int_{a(h, P^*_2)}^{1} [P^*_2 - (c + w - xa)]da - \int_{a(h, P^*_2)}^{1} b[P^*_2 - (c + w - xa)]da
+ (1 - a(h, P^*_2))(1 - b)yh - \frac{1}{2} h^2.
\]

Given \(a(h, P^*_2) = \frac{1}{2} - \frac{x}{2} h_k \in (0, \frac{1}{2})\) and \(P^*_2 - (c + w - xa(h, P^*_2)) = yh\), we find \(\frac{\partial}{\partial h} \pi(h, P^*_2) = (1 - a(h, P^*_2))(1 - b)y - \frac{(1-b)y^2}{x} h_k\). Suppose \(x - (1-b)y^2 \leq 0\) so that \(\frac{\partial}{\partial h} \pi(h, P^*_2) > 0\) for all \(h_k \in [0, \frac{x}{2y}]\). We then have that \(\pi(h, P^*_2) > \pi(h, P^*_2)\) for all \(h \in [0, \frac{x}{2y}]\).
Given $H > \frac{x}{2y}$ (by assumption), $h = h_k \in [0, \frac{x}{2y})$ cannot be a solution to the maximization problem $\max_{a \in (0, H]} \pi(h_i, P_k^*)$. Hence $x - (1 - b)y^2 > 0$ must hold. Note that $\frac{\partial}{\partial x} \pi(0, P_k^*) = \frac{1}{2}(1 - b)y > 0, \frac{\partial}{\partial x} \pi(h_k, P_k^*) = 0 \iff h_k = \frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2}$, and $H = \max \{ \frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2}, \frac{x}{2y} \}$. These together imply that $\frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2} < \frac{x}{2y}$ (which is equivalent to $x - 2(1-b)y^2 > 0$) must hold for $h = h_k \in [0, \frac{x}{2y})$ to solve the maximization problem. Under this condition, we have that $h_k = \frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2} = h^*$ is the unique solution to the maximization problem, and that $a(h_k, P_k^*) = \frac{1}{2} \frac{x - 2(1-b)y^2}{x-(1-b)y^2} > 0$.

Suppose that $x - 2(1-b)y^2 > 0$ holds. Then, following the same procedure as in the text, we find that every firm $i$ that employed a worker at Stage 1 chooses $h_i \in [0, H]$ at Stage 2 to maximize $\pi(h_i, P_k^*)$. Note that $a(h_i, P_k^*) > 0 \iff 0 \leq h_i < \frac{x}{2y}$. We then have that $\frac{\partial}{\partial x} \pi(h_i, P_k^*) = \frac{1}{2} (1 - b)y - \frac{x - (1-b)y^2}{x} h_i$ for all $h_i \in [0, \frac{x}{2y})$, while $\frac{\partial}{\partial h} \pi(h_i, P_k^*) = (1 - b) - h_i$ for all $h_i \in (\frac{x}{2y}, \infty)$. Then $x - 2(1-b)y^2 > 0$ and $H = \max \{ \frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2}, \frac{x}{2y} \}$ together imply that $h_i = \frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2} = h^*$ is the unique optimum for every firm $i$. Q.E.D.

Claim 2: Condition (8) presented in the text implies that $P_1^* = c + w - \frac{1}{2} x - \frac{1}{8} \frac{x^2(x-1-b)^2y^2}{x-(1-b)y^2}$. 

Proof: Condition (8) is equivalent to $P_1^* = (c + w - \frac{1}{2} x)$ $\iff c - \frac{1}{2} x + (2 - a^*) w - \int_a^{a^*} w(a, h^*, P_k^*) da + \int_a^{a^*} \left[ c - xa - yh^* + w_2(a, h^*, P_k^*) \right] da$. Using $P_k^* - c = w x_2^{*} + y h^{*}$, we find that $P_1^* = c + w - \frac{1}{2} x + \frac{1}{2} (h^*)^2 - \frac{1}{2} (1 - a^*)^2 x$. Then $h^* = \frac{1}{2} \frac{(1-b)xy}{x-(1-b)y^2}$ and $a^* = \frac{1}{2} \frac{x - 2(1-b)y^2}{x-(1-b)y^2}$ imply the result. Q.E.D.

Next we establish Claim 3.

Claim 3: For any given parameterization, there exists a unique value $\bar{y} \geq 0$ such that conditions (6), (10) and (11) presented in the text hold if and only if $y < \bar{y}$. There exists a range of parameterizations in which $\bar{y} > 0$.

Proof: Suppose $y < \frac{x}{\sqrt{2(1-b)^2}}$ so that condition (6) $(x - 2(1-b)y^2 > 0)$ holds. Using $1 - b > (1 - b)^2$ we find $\frac{d}{dy} \left\{ \frac{x - (1-b)y^2}{x-(1-b)y^2} \right\} > \frac{2(1-b)^2 y}{x - (1-b)y^2} > 0$, which implies that RHS of inequality (10) is strictly increasing in $y$. Next we show that $(1 - a^*) D(P_1^*)$ is strictly increasing in $y$. We have that $1 - a^* = \frac{x}{2(x-(1-b)y^2)}$, which is strictly increasing in $y$. Concerning $D(P_1^*)$, $\frac{d}{dy} \left\{ \frac{x - (1-b)y^2}{x-(1-b)y^2} \right\} > 0$ implies that $D(P_k^*)$ is strictly increasing in $y$, and hence $(1 - a^*) D(P_1^*)$ is strictly increasing in $y$. Given $D(P^*_2) = D(c + w - \frac{1}{2} x)$ is independent of $y$, this implies the first result, where $\bar{y} \in [0, \sqrt{\frac{x}{2(1-b)}}]$. Note that inequality (10) holds when $y = 0$, given $c > x + yH$. Note also that $(1 - a^*) D(P_1^*) \rightarrow \frac{1}{2} D(c + w - \frac{5}{2} x)$ as $y \rightarrow 0$. This implies that $\bar{y} > 0$ holds if and only if $D(c + w - \frac{1}{2} x) > \frac{1}{2} D(c + w - \frac{5}{2} x)$, which holds under a range of parameterizations. This implies the second result. Q.E.D.

Claims 1-3, along with the analysis presented in the text, imply the following necessity

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part of the proposition: “Suppose that there exists a perfect foresight equilibrium in which a strictly positive measure of firms enter and exit the industry at the beginning of period 2. Then, $y < \bar{y}$ must hold.” We now check sufficiency. Suppose that $y < \bar{y}$ holds, and that $(P_1, P_2) = (c + w - \frac{1}{2}x - \frac{1}{8}(c - \frac{1}{2}y^2), c + w - \frac{1}{2}x) = (P_1^*, P_2^*)$. First note that $P_1^* > 0$ and $P_2^* > 0$ hold given $y < \bar{y}$. Consider firm $i$ that employed worker $j$ at Stage 1. Given $y < \bar{y}$ ($\Rightarrow x - 2(1-b)y^2 > 0$), firm $i$'s unique optimal choice at Stage 2 is $h_i = \frac{1}{2}(1-b)xy - (1-b)y^2 \equiv h^*$ as shown in the proof of Claim 1. Then, following the same procedure and using the same notations as in the text, firm $i$ continues to operate in period 2 if and only if $a_i \geq a^*$ (where $0 < a^* < \frac{1}{2}$), and worker $j$'s second-period expected wage is $\int_{a^*}^{\bar{a}} wda + \int_{a^*}^{1} w_2(a, h^*, P_2^*)da$. Since all individuals anticipate this at Stage 1, every firm must offer the first period wage $w_1 \geq w_i^*$ to hire a worker at Stage 1. Then the analysis presented in the text implies that, if $y < \bar{y}$, there exists a perfect foresight equilibrium with the following properties: (i) $(P_1, P_2) = (P_1^*, P_2^*)$ where $P_1^* > 0$ and $P_2^* > 0$, (ii) at Stage 1, measure $D(P_i^*)$ of firms employ workers at the first-period wage $w_1 = w_i^*$, (iii) at Stage 2, every firm $i$ that hired a worker chooses $h_i = h^*$, (iv) at Stage 4, every firm $i$ that operated in period 1 continues to employ its first-period employee at the second-period wage $w_2(a_i, h^*, P_2^*)$ if $a_i \geq a^*$, while every firm $i$ exits the industry if $a_i < a^*$, and (v) at Stage 4, measure $D(P_2^* - (1-a^*)D(P_1^*)$ of new firms enter and operate in period 2.

Finally we prove that the equilibrium described in the previous paragraph is the unique perfect foresight equilibrium if $y < \bar{y}$. In any perfect foresight equilibrium, $P_2 > c + w - \frac{1}{2}x \equiv P_2^*$ cannot hold because second-period entrants can make strictly positive expected profits if $P_2 > c + w - \frac{1}{2}x$. Then Claim 4 below proves the uniqueness and completes the proof of the proposition.

**Claim 4:** Suppose that $y < \bar{y}$ holds. Then there does not exist a perfect foresight equilibrium in which $P_2 < P_2^*$ holds.

**Proof:** Suppose such an equilibrium exists, and let $P_2 = \tilde{P}_2 = P_2^* - \psi$, where $\psi > 0$. Consider firm $i$ that employed worker $j$ at Stage 1 and chose $h_i$ at Stage 2 in the equilibrium. Following the same procedure and using the same notations as in the text, we have that $h = h_i$ solves the maximization problem $\max_{h \in [0, H]} \pi(h, \tilde{P}_2)$. Suppose $g(h_i, \tilde{P}_2) \geq 1$. Then given $\tilde{P}_2 < P_2^*$ there does not exist a strictly positive measure of firms that operate in period 2. This implies that $g(h_i, \tilde{P}_2) < 1 \Leftrightarrow h_i > \frac{2\psi - x}{2y}$ must hold. Then, noting $g(h_i, \tilde{P}_2) > 0 \Leftrightarrow h_i < \frac{2\psi + x}{2y}$, we find that $\frac{\partial}{\partial h} \pi(h_i, \tilde{P}_2) = (1 - g(h_i, \tilde{P}_2))(1-b)y - h_i = \frac{\tilde{P}_2 - (c + w) + x}{x} (1-b)y - \frac{x(1-b)y^2}{x} h_i$ if $h_i < \frac{2\psi + x}{2y}$, while $\frac{\partial}{\partial h} \pi(h_i, \tilde{P}_2) = (1-b)y - h_i$ if $h_i > \frac{2\psi + x}{2y}$. Then $x - 2(1-b)y^2 > 0$ (implied by $y < \bar{y}$) implies that $h_i = \frac{(1-b)y[P_2 - (c + w) + x]}{x - (1-b)y^2} = \frac{(1-b)y[\frac{1}{2}x - y]}{x - (1-b)y^2}$ is the unique optimum if $\psi < \frac{1}{2}x$, while $h_i = 0$ is the unique optimum if $\psi \geq \frac{1}{2}x$. Note, $(1 - g(h_i, \tilde{P}_2))(1-b)y - h_i = 0$ holds if $\psi < \frac{1}{2}x$. Suppose
ψ ≥ \frac{1}{2}x. Then every firm i that employed a worker at Stage 1 chooses h_i = 0 at Stage 2 in the equilibrium. This implies that firm i's minimum possible second-period cost (including the wage bill) is c + w − x, while ψ ≥ \frac{1}{2}x implies that P_2 = \hat{P}_2 ≤ c + w − x in the equilibrium. This implies that there does not exist a strictly positive measure of firms that operate in period 2, and hence ψ < \frac{1}{2}x must hold.

Thus far we have found that, if there exists a perfect foresight equilibrium in which P_2 = \hat{P}_2 < P_2^*, then ψ < \frac{1}{2}x must hold and every firm i that operates in period 1 chooses h_i = h(\hat{P}_2) \equiv \tilde{h} in the equilibrium, where h(P_2) \equiv \frac{(1-b)x[P_2-(c+w)+x]}{x-(1-b)y^2}. Then, following the same procedure and using the same notations as in the text, we find that the following property holds in the equilibrium:

(i) the exit rate is \tilde{a} \equiv a(\hat{P}_2) where a(P_2) \equiv a(h(P_2)), P_2 (note that we have h(\hat{P}_2) \in \left(\frac{2w-x}{2y} \frac{2w+x}{2y}\right), which implies 0 < \tilde{a} < 1), (ii) every firm that employed a worker at Stage 1 offers the first period wage \tilde{w}_1 \equiv w - \int_{\tilde{a}}^{1}(w_2(a, h, \hat{P}_2) - w)da, and (iii) every firm i that operates in period 1 has the expected first-period cost of \tilde{C}_1 \equiv C - \frac{1}{2}x + \tilde{w}_1 + \frac{1}{2}(\tilde{h})^2, and continues to operate in period 2 with the second-period cost of C_2(a_i, \tilde{h}, \hat{P}_2) if and only if a_i ≥ \tilde{a}. This implies that the equilibrium first-period price, denoted by \hat{P}_1, is uniquely determined by \hat{P}_1 + \int_{\tilde{a}}^{1}\hat{P}_2da = \tilde{C}_1 + \int_{\tilde{a}}^{1}C_2(a, \tilde{h}, \hat{P}_2)da.

Given 0 < \tilde{a} < 1 we have \hat{P}_2 - (c + w) + x\tilde{a} + y\tilde{h} = 0. Then, through the procedure analogous to the proof of Claim 2 above we find that \hat{P}_1 = c + w - \frac{1}{2}x + \frac{1}{2}(\tilde{h})^2 - \frac{1}{2}(1 - \tilde{a})^2x = c + w - \frac{1}{2}x - \frac{1}{2}(1 - \tilde{a})^2[x - (1 - b)^2y^2], where the second equality holds because (1 - \tilde{a})(1 - b)y = \tilde{h} = 0.

Define P_1(P_2) by P_1(P_2) \equiv c + w - \frac{1}{2}x - \frac{1}{2}(1 - a(P_2))^2[x - (1 - b)^2y^2]. Then the market clearing in period 2 requires D(\hat{P}_2) - (1 - a(\hat{P}_2))D(P_1(\hat{P}_2)) = 0, because \hat{P}_2 < P_2^* implies that no firms enter in period 2 in the equilibrium. We have that a(P_2^*) = \tilde{a} and P_1(P_2^*) = P_2^*, and hence D(\hat{P}_2) - (1 - a(\hat{P}_2))D(P_1(\hat{P}_2)) = N_f > 0. Since a(\hat{P}_2) is decreasing in P_2, we have that (1 - a(\hat{P}_2))D(P_1(\hat{P}_2)) is increasing in P_2, which in turn implies that there does not exist a value \hat{P}_2 < P_2^* such that D(\hat{P}_2) - (1 - a(\hat{P}_2))D(P_1(\hat{P}_2)) = 0. This is a contradiction. Q.E.D.

Proofs of Propositions 2 and 3:

(i) Note \tilde{a}^* \equiv \frac{1}{2}x - \frac{21(1-b)^2}{2(x-(1-b)y^2)}. We find \frac{\partial \tilde{a}^*}{\partial x} = \frac{(1-b)y^2}{2x-2(1-b)y^2} > 0 and \frac{\partial \tilde{a}^*}{\partial y} = -\frac{(1-b)xy}{(x-(1-b)y^2)^2} < 0.

(ii) Note h^* \equiv \frac{1}{2}x - \frac{(1-b)xy}{2x-(1-b)y^2}. We find \frac{\partial h^*}{\partial x} = \frac{1}{2}(1-b)^2y^4 \frac{1-x}{1-x(1-b)y^2} < 0 and \frac{\partial h^*}{\partial y} = \frac{1}{2}\frac{x-1-b^2y^4}{x-(1-b)y^2} > 0.

(iii) By equation (7) and definitions of w_2(a, h, \hat{P}_2) and S(a, h, \hat{P}_2), we find w_1^* = w - \int_{\tilde{a}}^{1} bS(a, h^*, \hat{P}_2^*)da = w - \frac{b}{2}y h_1^* where \phi_1 \equiv \frac{y h_1^*(x+yh_1^*)}{x}, and w_2^* \equiv \frac{\int_{\tilde{a}}^{1} S(a, h^*, \hat{P}_2^*)da}{1-a^*} = w + \frac{\int_{\tilde{a}}^{1} S(a, h^*, \hat{P}_2^*)da}{1-a^*} = w + b\phi_2 where \phi_2 \equiv y h^*. From (ii) above we have that \frac{\partial \phi_1}{\partial x} < 0 and \frac{\partial \phi_1}{\partial y} > 0.

Claim 5: \frac{\partial \phi_1}{\partial x} < 0, \frac{\partial \phi_2}{\partial x} < 0, \frac{\partial \phi_1}{\partial y} > 0, and \frac{\partial \phi_2}{\partial y} > 0 hold.

Proof: Given \frac{\partial \phi_1}{\partial x} < 0, we find \frac{\partial \phi_1}{\partial x} = \frac{y}{x} (\frac{2 \partial \phi_1}{\partial x} + 2 x y h^* \phi_1^* - y(h^*)^2) < 0 and \frac{\partial \phi_2}{\partial x} = y \frac{\partial \phi_1}{\partial y} < 0. Also, \frac{\partial \phi_1}{\partial y} > 0 implies \frac{\partial \phi_1}{\partial y} > 0. Finally, given \frac{\partial \phi_1}{\partial y} > 0 we find \frac{\partial \phi_2}{\partial y} = h^* + y \phi_1 > 0. Q.E.D.
Noting that $w_2' - w_1' = \frac{b}{2}(\phi_1 + 2\phi_2)$, Claim 5 implies that $w_2' - w_1'$ is strictly decreasing in $x$ and strictly increasing in $y$. Q.E.D.

**Proof of Lemma 1:** Consider an entry-restriction subgame represented by $N_r \in [0, N_f)$.

Suppose that the game has a perfect foresight equilibrium characterized by the price sequence $(P_1, P_2) = (\hat{P}_1, \hat{P}_2)$, and let $\hat{\alpha} \in [0, 1]$ denote the exit rate in the equilibrium. Suppose $\hat{P}_2 < c + w - \frac{1}{2}x$. Then no firms enter the industry at the beginning of period 2 in the equilibrium for any given $N_r \in [0, N_f)$, and hence the entry restriction is not binding. Then Claim 4 (presented in the proof of Proposition 1) implies that $\hat{P}_2 < c + w - \frac{1}{2}x$ cannot hold, and hence $\hat{P}_2 \geq c + w - \frac{1}{2}x$ must hold in the equilibrium. Given this, define $\xi \geq 0$ by $\hat{P}_2 = P^{\ast}_2 + \xi$.

Suppose $\hat{\alpha} = 0$. We then have that $D(\hat{P}_2) - D(\hat{P}_1) = N_r \geq 0$, which implies $\hat{P}_1 \geq \hat{P}_2$. Suppose $\hat{P}_1 > c + w - \frac{1}{2}x$. Then a firm can make a strictly positive expected profit by employing a worker at the reservation wage $w$ and providing no firm-specific human capital in period 1. This is a contradiction, and hence $\hat{P}_1 = \hat{P}_2 = c + w - \frac{1}{2}x$ must hold. If $\hat{P}_1 = \hat{P}_2 = c + w - \frac{1}{2}x$ holds in a perfect foresight equilibrium of the entry-restriction subgame, then the original model should also have a perfect foresight equilibrium in which $P_1 = P_2 = c + w - \frac{1}{2}x$. However Proposition 1 implies that the original model does not have such an equilibrium. This implies that $\hat{\alpha} = 0$ cannot hold, and hence $\hat{\alpha} > 0$ must hold.

Then, following the analogous procedure and using the same notations as in the proof of Claim 4, we find that every firm $i$ that operates in period 1 chooses $h_i = h(\hat{P}_2) = \frac{(1-b)y(\hat{\phi}_1 + \hat{\phi}_2)}{x(1-b)^2} \equiv \hat{h}$ at Stage 2 in the equilibrium, where $h(\hat{P}_2) \equiv \frac{(1-b)y(\hat{P}_2 - (c+w) + x)}{x(1-b)^2}$. We also find that the equilibrium exit rate is $\hat{\alpha} \equiv \hat{a}(\hat{P}_2)$ where $\hat{a}(P_2) \equiv a(h(\hat{P}_2), P_2)$, and that the equilibrium first-period price is $\hat{P}_1 = P_1(\hat{P}_2)$ where $P_1(P_2) \equiv c + w - \frac{1}{2}x - \frac{1}{2}(1 - a(P_2))^2[\hat{\phi}_1 + \hat{\phi}_2]$. Then the market clearing in period 2 requires $\eta(\hat{P}_2) = N_r$, where $\eta(P_2) \equiv D(P_2) - (1 - a(P_2))D(P_1(P_2))$.

**Claim 6:** There exists a unique continuous function $\hat{P}_2(N)$ such that $\eta(\hat{P}_2(N)) = N$ for all $N \in [0, N_f]$. Also, $\hat{P}_2(N)$ is strictly decreasing in $N$ for all $N \in [0, N_r]$, where $\hat{P}_2(N_f) = c + w - \frac{1}{2}x$.

**Proof:** From the analysis of the original model we have that $\eta(c + w - \frac{1}{2}x) = N_f$. Note that $\hat{a}(P_2)$ is continuously differentiable and strictly decreasing in $P_2$ for all $P_2$ such that $\hat{a}(P_2) > 0$. This implies that $P_1(P_2)$ is continuously differentiable and strictly decreasing in $P_2$, which in turn implies that $\eta(P_2)$ is continuously differentiable and strictly decreasing in $P_2$ for all $P_2$ such that $\hat{a}(P_2) > 0$. Note also that there exists a unique value $P^{0}_2 (> c + w - \frac{1}{2}x)$ such that $\hat{a}(P^{0}_2) = 0$ for all $P_2 \geq P^{0}_2$. We have that $P_1(P^{0}_2) = c + w - \frac{1}{2}x - \frac{1}{2}[\hat{\phi}_1 + \hat{\phi}_2] < c + w - \frac{1}{2}x < P^{0}_2$. Hence $\eta(P^{0}_2) = D(P^{0}_2) - (1 - a(P^{0}_2))D(P_1(P^{0}_2)) < 0$. Then the Inverse Function Theorem implies the result. Q.E.D.
Note that, if \( N_r = N_f \), the entry restriction is not binging and hence \( \hat{P}_1 = P_1^*, \hat{P}_2 = P_2^* \) and \( \hat{a} = a^* \) hold where \( P_1^*, P_2^* \) and \( a^* \) are as defined in the text. Then Claim 6 implies the following result: Suppose that an entry-restriction subgame represented by \( N_r \) has a perfect foresight equilibrium characterized by a price sequence \((P_1, P_2) = (\hat{P}_1, \hat{P}_2)\) and an exit rate \( \hat{a} \). Then \( \hat{P}_1 = \hat{P}_1(N_r), \hat{P}_2 = \hat{P}_2(N_r) \) and \( \hat{a} = \hat{a}(N_r) \) hold, where \( \hat{P}_1(N) \equiv P_1(\hat{P}_2(N)) \) and \( \hat{a}(N) \equiv a(\hat{P}_2(N)) \). Note, Claim 6 and its proof imply that \( \hat{P}_1(N), \hat{P}_2(N) \) and \( \hat{a}(N) \) exhibit the properties described in Lemma 1.

Now, pick any \( N_r \in [0, N_f] \) and let \((P_1, P_2) = (\hat{P}_1(N_r), \hat{P}_2(N_r)) = (\hat{P}_1, \hat{P}_2)\) be given. Consider firm \( i \) that employed a worker at Stage 1. Then, following the same procedure and using the same notations as in the text, we find that at Stage 2 firm \( i \) chooses \( h_i \in [0, H] \) to maximize \( \pi(h_i, \hat{P}_2) \).

Note that \( \hat{P}_2 \geq c + \frac{w - \frac{1}{2}x}{2y} \) implies \( g(h_i, \hat{P}_2) < 1 \). Then, noting \( g(h_i, \hat{P}_2) > 0 \Leftrightarrow 0 \leq h_i < \frac{x - 2\xi}{2y} \), we find that \( \frac{g}{\partial h} \pi(h_i, \hat{P}_2) = (1 - g(h_i, \hat{P}_2))(1 - b)y - h_i = \frac{f_2 - (c + w)x}{x}(1 - b)y - \frac{x(1 - b)y^2}{x}h_i \) if \( h_i < \frac{x - 2\xi}{2y} \), while \( \frac{\partial g}{\partial h} \pi(h_i, \hat{P}_2) = (1 - b)y - h_i \) if \( h_i > \frac{x - 2\xi}{2y} \). From the proof of Claim 6 we have that \( \hat{a}(\hat{P}_2) > 0 \). Noting that \( \hat{a}(\hat{P}_2) = \frac{c + w - \frac{f_2}{x}}{2} - \frac{y}{2} \hat{h} \) where \( \hat{P}_2 = c + w - \frac{1}{2}x + \xi \), we find that \( \hat{a}(\hat{P}_2) > 0 \Rightarrow x - 2(1 - b)y^2 - 2\xi > 0 \). This in turn implies that \( h_i = h(\hat{P}_2) = \hat{h} \) is the unique optimum. Then, through the procedure analogous to the proof of Proposition 1 we find that any entry-restriction subgame represented by \( N_r \in [0, N_f] \) has a unique perfect foresight equilibrium characterized by the price sequence \((P_1, P_2) = (\hat{P}_1(N_r), \hat{P}_2(N_r))\) and the exit rate \( \hat{a}(N_r) \). Q.E.D.

**Proof of Proposition 4:** From the proof of Lemma 1 we have that \( \hat{P}_1(N_r) = P_1(\hat{P}_2(N_r)) \) and \( \hat{a}(N_r) = a(\hat{P}_2(N_r)) \), and find that \( \hat{P}_2(N_r) \) is continuously differentiable at \( N_r = N_f \) (which implies that \( \hat{P}_1(N_r) \) and \( \hat{a}(N_r) \) are also continuously differentiable at \( N_r = N_f \)). Given this we compute the derivative of \( W(N_r) \) evaluated at \( N_r = N_f \). Note that \( \hat{P}_2(N_f) - C_N = 0 \) and \( N_r = D(\hat{P}_2(N_r)) - (1 - \hat{a}(N_r))D(\hat{P}_1(N_r)) \). We then find that \( W'(N_f) = -D(\hat{P}_1(N_f))\hat{P}_1'(N_f) - (1 - \hat{a}(N_f))D(\hat{P}_1(N_f))\hat{P}_2'(N_f) \). Given \( \hat{P}_1(N_r) = c + w - \frac{1}{2}x - \frac{1}{2}(1 - a(\hat{P}_2(N_r)))^2[x - (1 - b)^2y^2] \), we find \( \hat{P}_1'(N_r) = (1 - a(\hat{P}_2(N_r)))[x - (1 - b)^2y^2]a'(\hat{P}_2(N_r))\hat{P}_2'(N_r) \). Also, by definition of \( a(P_2) \) we find \( a'(\hat{P}_2(N_r)) = -\frac{1}{x - (1 - b)y^2} \). We then find that \( W'(N_f) = D(\hat{P}_1(N_f))(1 - a(\hat{P}_2(N_f)))\hat{P}_2'(N_f)\frac{(1 - b)y^2}{x - (1 - b)y^2} \). Then \( \hat{P}_2'(N_f) < 0 \) and \( 0 < b < 1 \) together imply \( W'(N_f) < 0 \). Similarly we also find \( CS'(N_f) < 0 \). This implies the result. Q.E.D.
References


