Earnings Mobility, State Dependence and Unobserved Heterogeneity: the Case of the United-States in the 90's.

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Abstract

Modelling earnings quintiles dynamics with a dynamic multinomial logit with unobserved heterogeneity, the two dimensions of earnings mobility, state dependence and unobserved heterogeneity, are disentangled. Using in a first stage conditionnal maximum likelihood techniques, the presence of state dependence can not be rejected and it is showed that for every quintile but the first it creates more stability than mobility and it favours upward movements rather than downwards. In a second stage the law of the unobserved heterogeneity is supposed to be discrete and is estimated via the EM algorithm. The second stage shows essentially that each individual is attracted towards a specific quintile, which makes the quintile distribution very segmented. Moreover, men, white and the more educated are attracted towards the upper part of the distribution while women, non-white and the less educated towards the lower. Lastly, it is showed that the contribution of state dependence in earnings stability is not negligible when it is considered with respect to upward mobility (around 25%), and even more important when it is considered with respect to downward mobility.

Keywords: labor income mobility, state dependence, unobserved heterogeneity, dynamic multinomial logit.

JEL codes: J31, C33, C35.

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1 Introduction

The dramatic increase of earnings inequality in the United-States and the UK in the 80's has given rise to an important literature on earnings mobility. During twenty years, researchers have tried to determine if the increase of inequality had been, at least partly, offset by an increase of earnings mobility. (See for example for the United-States Buchinsky and Hunt (1999), Gottschalk and Moffit (1998) and Burkhauser et al. (1997)). The traditionnal summary of this literature is that "*Only a few studies have looked at changes in earnings mobility. Some have found declines, most have found no change, and none have found any increase*" (Gottschalk, 1997).

Individual earnings mobility has two dimensions. The first one is linked to individual heterogeneity: populations in each quintile at every period are selected on the basis of their fixed unobservable characterisitics, and have thus some propension to stay in the same quintile in the next period. For example, individuals who are in the lowest quintile today have unobservable characteristics that make them more likely to be in the lowest quintile tomorrow. The second dimension is structural and is called state dependence (Heckman, 1981a): the simple fact to be in a quintile one year has an influence on the probability to stay in this quintile next year. For example individuals who are in the lowest quintile could face a deterioration of their human capital that would make their rise more difficult. These two dimensions lead to different public policy instruments, and it is thus important to be able to identify them separately.

Recently, Capellari and Jenkins (2004), Stewart (2006) and Uhlendorff (2006) have studied the existence of state dependence in the transitions between unemployment, low pay and high pay. Their work shows the presence of state dependence and they find evidence of a "*low pay no pay cycle*". Moreover, as far as I am aware, the only paper to have studied state dependence not only in low pay but on the entire distribution of earnings quintiles is Weber (2002) with austrian data. This work studies essentially the state dependence dimension and shows its presence. The method used, the conditionnal dynamic multinomial logit developped by Honore and Kyriazidou (2000), doesn't allow the description of the unobserved heterogeneity, and the autor "*designs ar-*

tificial individuals with special unobserved and observed properties and investigate their simulated earnings profiles".

The first contribution of this paper is to study for the first time the state dependence in earnings mobility in the United-States. The dynamics of the entire distribution of quintiles (including non-employment) is modelled with a dynamic multinomial logit with unobserved heterogeneity. The presence of state dependence is tested in a first stage by Conditionnal Maximum Likelihood (Magnac, 2000) and its nature is characterized. The law of the unobserved heterogeneity is supposed to be discrete and is estimated in a second stage via the EM algorithm, solving the initial conditions problem à la Wooldridge (2005). The second stage estimates allow to identify types of individuals, and therefore to study earnings mobility for each group. Type specific transitions matrices are computed and stationnary type specific quintiles distributions are derived. Lastly, thanks to the whole model estimates, the relative contribution of state dependence in earnings stability is evaluated.

The main findings are the following. State dependence is significant in the earnings mobility process and its magnitude is upwards biased if unobserved heterogeneity is not taken into account. For every quintile but the first it creates more stability than mobility and it favours upward movements rather than downwards. These results are remarkably similar to those found in Weber (2002), which can be surprising since Austria and the United-States have two different labour markets. The second stage shows essentially that each individual is attracted towards a specific quintile, which makes the quintile distribution very segmented. Moreover, men, white and the more educated are attracted towards the upper part of the distribution while women, non-white and the less educated towards the lower. Lastly, it is showed that the contribution of state dependence in earnings stability is not negligible when it is considered with respect to upward mobility (around 25%), and even more important when it is considered with respect to downward mobility.

The paper is organised as follows. Data are described in section 2, state dependence is studied in section 3, individual heterogeneity in section 4 and section 5 concludes.

2 Data description

2.1 The Data

I use the Panel Study of Income Dynamics (PSID). The PSID is a representative panel of the US population which was started in 1968 and is still carried out today. The unit of obsevation is the household, the interviews were annual until 1997 and have been bisannual since then. There were 4800 families in 1968 and 7000 in 2001, totaling 65000 individuals obseved in 2001. In 1968 the PSID was composed of two separate samples, the first one (SRC sample) representative of the US population, and the second (SEO sample) specifically dealing with low income. Thus low incomes are over-represented in the whole sample and, therefore, I restrict the study to the SRC sample, without using weights as for example in Moffit and Gottschalk (2002). Because the goal is to obtain results for the most general population, I restrict the sample to the 25-60 years old (ie the 25-52 in 1990) in order to avoid student workers. In order to minimize measurement error, I use total annual labor income rather than wage rate because the number of hours worked is poorly reported in the PSID (see for example Bound and al. (1994) for a validation study of the PSID). Due to the bisannual periodicity of the PSID after 1996, I choose a step of two years on the period 1990-1998, and the final sample is composed of 3465 individuals who have no missing value. 53% of the individuals in the sample are women, 91% are white and 59% have spent at least 12 years in the educationnal system. The three quartiles of age in 1990 are 31, 37 and 42 years.

2.2 Descriptive statistics

To describe relative earnings mobility one of the most appropriate tool is the transition matrix between earnings quintiles. Following Buchinsky and Hunt (1999) I create the quintile zero to take into acccount zero wages (unemployed or out of the labor market). Table 1 shows that it is slightly easier to move from the quintiles 2 and 3 than from the first and the fourth (50%

vs 40%), and much more than from the quintiles 0 and 5 (35% and 20%). There is a positive correlation between the initial quintile and downward mobility and a negative correlation with upward mobility. Lastly the full transition matrix shows that the vast majority of the movements reach an adjacent quintile. These qualitative results are in line with the literature on relative earnings mobility (see for example Burkhauser and al. (1997), Moffit and Gottschalk (1998) or Buchinsky and Hunt (1999)).

Table 1: Two-Year Quintile Mobility Rates, 1990-1998

Origin	Destination Quintile							Direction			
Quintile	0	1	2	3	4	5	Sum	Down	Stable	Up	Sum
0	65.7	21.0	5.2	3.1	2.4	2.6	100.0	0.0	65.7	34.3	100.0
1	16.9	54.9	19.5	5.4	1.9	1.4	100.0	16.9	54.9	28.2	100.0
2	6.1	17.7	51.6	18.0	4.7	2.0	100.0	23.8	51.6	24.6	100.0
3	4.4	5.4	13.7	51.4	21.7	3.5	100.0	23.5	51.4	25.2	100.0
4	3.4	3.2	4.5	14.4	58.0	16.6	100.0	25.5	58.0	16.6	100.0
5	2.9	1.7	1.5	3.7	11.4	78.8	100.0	21.3	78.8	0.0	100.0

Notes: 13860 observations, PSID, annual earnings.

Thus a quintile and its lag are not independent, and being in one quintile one year increases the probability to be in the same quintile two years later. To see if there is some heterogeneity in this dynamics it is usefull to compute this matrix for some groups of the population. Table 2 presents the results by sex, education (College or not College), and race (white and non-white); to save space, the tables concerning age (25-29, 30-34, 35-39, 40-44, 45-49 and 50-59) are not reported. Here are the main features of these matrices. Men are over-represented in the upper part of the distribution, women in the lower part. In the whole distribution, women move up less, go down slightly more and are more

Table 2: Directions of the Two-Year Quintile Mobility by sex, race and education, 1990-1998

quintile	Down		Stable		Up		Total	
	Men	Women	Men	Women	Men	Women	Men	Women
0	0.00	0.00	62.86	66.72	37.14	33.28	6.89	17.35
1	19.09	16.37	33.17	59.59	47.73	24.04	6.46	26.50
2	20.82	25.24	44.56	54.98	34.62	19.78	11.63	21.11
3	22.48	24.43	47.17	55.39	30.35	20.19	17.43	15.98
4	23.45	29.04	57.70	58.43	18.85	12.53	24.14	11.90
5	19.41	28.84	80.59	71.16	0.00	0.00	33.45	7.14
Total	19.73	19.09	60.76	59.87	19.51	21.04	100.0	100.00
	White	Non White	White	Non White	White	Non White	White	Non White
0	0.00	0.00	65.79	65.15	34.21	34.85	12.10	16.18
1	16.42	20.07	54.75	56.27	28.83	23.66	16.58	22.79
2	23.76	24.15	51.28	54.24	24.96	21.61	16.42	19.28
3	23.11	27.14	51.93	45.71	24.96	27.14	16.61	17.16
4	25.08	29.70	58.23	54.95	16.69	15.35	17.74	16.50
5	20.52	40.40	79.48	59.60	0.00	0.00	20.55	8.09
Total	19.13	22.06	60.75	55.56	20.13	22.39	100.00	100.00
	College	No College	College	No College	College	No College	College	No College
0	0.00	0.00	63.92	67.56	36.08	32.44	10.70	14.98
1	16.62	17.05	52.41	57.11	30.97	25.84	13.49	22.33
2	25.18	22.55	48.54	54.32	26.28	23.13	13.43	21.32
3	22.69	24.41	47.29	56.22	30.02	19.37	15.39	18.47
4	24.06	28.11	58.19	57.53	17.75	14.35	19.61	14.79
5	18.49	34.63	81.51	65.37	0.00	0.00	27.38	8.11
Total	18.90	20.09	61.43	58.65	19.67	21.26	100.00	100.00

Notes: 13860 observations, PSID, annual earnings. The last two columns give the quintile distribution by group. The lines labelled "Total" give the distributions of movements by group. For example, the downward mobility rate among women is 19.09%. The other numbers are the probabilities of movement by group and origin quintile. For example, the downward mobility rate among women from the quintile 3 is 24.43%.

stable. These results are particularly massive in the first and the fifth quintile. Non-white are over-represented in the quintiles 0, 1 and 2, and under-represented in the top quintile. They move down more often, slightly in the quintiles 0 to 4 and more massively in the fifth quintile (40% vs 20%). In the quintiles one and two they move up slightly less and, interestingly, they go up slightly more in the third quintile. The less educated are over-represented in the quintiles 0 to 3 and under-represented in the fourth and fifth quintiles, with an impressive difference at the top (8% vs 27%). In the quintiles 0, 1, 2 and 3 they are more stable and they move up less, and they fall more out of the top quintile (35% vs 18%). Between 25 and 50 years old there is a slight positive correlation between age and quintile, but this trend is reversed after 50: the 50-59 are more often in the quintiles 0 and 1 and less in the 4 and 5 than the 45-49. Concerning age the only characteristic of the dynamics is that the 50-59 are different from the others: they are slightly more stable and move up less often.

Essentially these descriptive statistics give expected results: they show that there is some immobility in earnings quintile dynamics and that traditionnally disadvantaged workers are in a weaker position in this area too. But they allow to point out an interesting question. Consider for example the transition from the first quintile: there is globally a significant immobility rate, and the large majority of quintile 1 individuals are women, who are much more stable in this quintile than men. Where is the causality: are people in the first quintile stable because they are essentially women, or are women stable because they are in the first quintile? It could be for example argued that there is some discrimination against women that could explain at least partially why they don't go out from the first quintile. But it could be also argued that when people fall in the first quintile they can face a deterioration of their human capital that makes their rise more difficult. To answer this question these two effects must be separated, or differently said state dependence and unobserved heterogeneity must be disentangled (Heckman (1981)). That is one of the goal of this paper, and the econometric strategy is developped in the next section.

3 Earnings mobility and state dependence

3.1 The model

To disentangle state dependence from individual heterogeneity in earnings dynamics, I model quintiles dynamics with a dynamic multinomial logit with unobserved heterogeneity. More precisely, Note y_{it} ($y_{it} = 0...5$) the quintile of individual i ($i = 1...N$) at date t ($t = 1...T$). I suppose that

$$\forall k = 0...5\,,\ \forall t = 2...T\,,\ y_{it} = k \text{ if and only if } y^*_{ikt} = \max_{j=0...5}\{y^*_{ijt}\}$$

where

$$\forall k = 0...5\,,\ \forall t = 2...T\,,\ y^*_{ikt} = \sum_{j=0}^{5} \delta_{jk} \mathbb{1}_{\{y_{it-1}=j\}} + \alpha_{ik} + \epsilon_{ikt}$$

For usual identification purpose I take the non-employment as reference state and I suppose that $\forall j = 0...5$, $\delta_{j0} = 0$, $\forall i = 1...N$, $\alpha_{i0} = 0$ and $\forall k = 0...5$ $\delta_{0k} = 0$. I suppose that the $\epsilon's$ are type I extreme value distributed, and that they are independent accross alternatives, individuals and time, and independent of the α's. I note $\forall i = 1...N$, $\alpha_i = (\alpha_{i0}, ..., \alpha_{i5})'$. Consequently the probability of an individual i being in state k in period t ($t = 2...T$) conditionnal on being in state j in period $t-1$ is given by

$$P(y_{it} = k \,/ y_{it-1} = j\,,\, \alpha_i) = \frac{\exp(\delta_{jk} + \alpha_{ik})}{\sum_{l=0}^{5} \exp(\delta_{jl} + \alpha_{il})} \tag{1}$$

which implies that the the transition matrix is heterogenous between individuals. The law of y_{i1} is supposed to be an unspecified function of α_i.

The interpretation of the parameters is facilitated by writing

$$\frac{P(y_{it} = k \,/ y_{it-1} = j\,,\, \alpha_i)}{P(y_{it} = l \,/ y_{it-1} = j\,,\, \alpha_i)} = \exp((\delta_{jk} - \delta_{jl}) + (\alpha_{ik} - \alpha_{il})) \tag{2}$$

where the higher $(\alpha_{ik} - \alpha_{il})$ is, the higher the odds of being in state k with respect to state l, conditionnal on any lagged state j, are. Now note that if $\alpha_{ik} = \alpha_{il}$ then

$$\frac{P(y_{it} = k \,/ y_{it-1} = j\ ,\ \alpha_i)}{P(y_{it} = l \,/ y_{it-1} = j\ ,\ \alpha_i)} = \exp(\delta_{jk} - \delta_{jl}) \tag{3}$$

which means that $\delta_{jk} - \delta_{jl}$ is a direct measure of the log odds ratios for observations who have the same "individual propensions" to be in states k and l. It is thus a measure of state dependence and it provides a way to test its presence. If $\forall\, j, k = 1...5, \delta_{jk} = 0$ then (1) shows that the lines of the transition matrix for an individual are equal: there is no state dependence. If $\forall\, k, l = 1...5, k \neq l, \delta_{jk} = \delta_{jl}, j = 1...5$ then (3) shows that an individual such that $\alpha_{i1} = ... = \alpha_{i5}$ who is in the quintile j has the same probability to arrive in each quintile: there is no state dependence excluding the quintile zero.

3.2 Estimation

To estimate such a model the main difficulty is to treat the initial conditions problem (Heckman, 1981b): it is an error to consider the period one quintile as exogenous, and the link between this quintile and unobserved heterogenity must be taken into account. There are two main families of solutions: the fixed effects and the random effects models.

The first models have been developped by Magnac (2000) and Honoré and Kyriazidou (2000). The idea is to condition the likelihood by sufficient statistics such that the conditionnal likelihood does not depend anymore on the individual effects. These methods are very attractive because they solve directly the initial conditions problem, they allow to make no asumption on the law of unobserved heterogeneity and they let free the correlation between the individual effects and potential time-varying covariates. The main problem of these methods is that they are data intensive because a consequence of conditionning the likelihood is to decrease the number of individuals who contribute to the likelihood. The main difference between both methods is that Honoré and Kyriazidou (2000) allows time-varying covariates (except time dummies), contrary to Magnac (2000). The cost of this improvement is the need of even more data.

To take advantage of these features, I estimate in a first stage the lag parameters with the method developped by Magnac (2000). Because of sample size I can't use Honore and Kyriazidou (2000), which explains that I have no time-varying covariates in the model. Setting Y_{ik} the number of times individual i has been in quintile k between periods 2 and $T-1$, Magnac shows in Appendix B that

$$P\left(y_{i2},...,y_{iT-1}\,/y_{i1},Y_{i0},...,Y_{i5},y_{iT}\right)=\frac{\exp\sum_{k>0}\sum_{j}\left(\sum_{t>1}\mathbb{1}_{\{y_{it-1}=j\}}\mathbb{1}_{\{y_{it}=k\}}\delta_{jk}\right)}{\sum_{B}\exp\sum_{k>0}\sum_{j}\left(\sum_{t>1}\mathbb{1}_{\{y_{it-1}=j\}}\mathbb{1}_{\{y_{it}=k\}}\delta_{jk}\right)} \tag{4}$$

where $B=\{b=(y_{i2},...,y_{iT-1})\,/\forall\,k>0,\ \sum_{t=2}^{T-1}\mathbb{1}_{\{y_{it-1}=j\}}=Y_{ik}\}$ is the set of all possible histories that are compatible with the number of visits to each state from 2 to $T-1$. This conditionnal likelihood doesn't depend anymore on α_i, which means intuitively that individuals who have the same number of visits to each state, but not at the same time, have the same level of unobserved heterogeneity. These observations can therefore be compared in terms of state dependence.

3.3 Results

The estimates of the first stage are reported in Tables 3 and 4. In Table 3 the estimates of the conditionnal fixed effects dynamic multinomial logit (Model 3) are contrasted with those of other models that don't take into account unobserved heterogeneity. Model 1 is a dynamic multinomial logit with Sex, Race, Education and Age at the first period as covariates. The model 2 is the model 1 plus the first period quintile as covariate. The three models give the same sign but not

Table 3: State Dependence Estimates

	Model 1[a]		Model 2[b]		Model 3[c]	
δ	coef	std	coef	std	coef	std
δ_{11}	2.26	0.09	2.01	0.09	1.26	0.15
δ_{12}	2.63	0.13	2.28	0.13	1.43	0.20
δ_{13}	1.88	0.17	1.65	0.18	1.02	0.26
δ_{14}	1.16	0.22	0.93	0.23	0.22	0.31
δ_{15}	0.80	0.24	0.83	0.25	0.81	0.40
δ_{21}	2.20	0.11	2.02	0.12	1.45	0.19
δ_{22}	4.64	0.15	4.04	0.16	2.72	0.24
δ_{23}	4.06	0.18	3.49	0.19	2.37	0.27
δ_{24}	2.97	0.20	2.51	0.22	1.65	0.31
δ_{25}	2.01	0.23	1.83	0.24	1.52	0.40
δ_{31}	1.43	0.15	1.39	0.17	1.22	0.24
δ_{32}	3.69	0.16	3.25	0.17	2.26	0.26
δ_{33}	5.45	0.19	4.57	0.20	3.17	0.31
δ_{34}	4.79	0.20	3.92	0.22	2.80	0.31
δ_{35}	2.82	0.21	2.39	0.24	1.94	0.38
δ_{41}	1.25	0.17	1.33	0.19	1.20	0.29
δ_{42}	2.87	0.18	2.65	0.21	2.05	0.30
δ_{43}	4.42	0.19	3.75	0.21	2.54	0.32
δ_{44}	5.95	0.21	4.81	0.23	3.00	0.34
δ_{45}	4.50	0.21	3.66	0.23	2.65	0.37
δ_{51}	0.94	0.19	1.16	0.23	1.04	0.33
δ_{52}	2.02	0.22	2.17	0.25	1.56	0.37
δ_{53}	3.24	0.21	3.10	0.25	2.45	0.35
δ_{54}	4.40	0.22	3.60	0.25	2.33	0.34
δ_{55}	6.04	0.23	4.91	0.24	3.25	0.40

[a] *Model 1:* Observed heterogeneity (Sex, Race, Education and Age in 1990) only without period one quintile, 13860 observations.

[b] *Model 2:* Observed heterogeneity only with period one quintile, 13860 observations.

[c] *Model 3:* Conditionnal logit with unobserved heterogeneity, 2019 observations.

the same magnitudes. For all the lag coefficients the model one give the highest estimates and the model three the lowest, which means that omitting unobserved heterogeneity tends to biais upwards all parameters. This bias is due to spurious state dependence (Heckman, 1981a): in the models without unobserved heterogeneity, the estimated parameters reflect both the effects of state dependence and unobserved heterogeneity which inflates the estimated retention rates in all quintiles. This result is classical in the state dependence literature. It is lastly interesting to note that the inclusion of the period one quintile in the Model 2 reduces the bias: this variable can be seen as a proxy of the unobserved heterogeneity that takes imperfectly its effect into account.

Table 4: State Dependence Parameters

Origin	Destination quintile				
quintile	1	2	3	4	5
1	1.26	1.43	1.02	0.22	0.81
2	1.45	2.72	2.37	1.65	1.52
3	1.22	2.26	3.17	2.80	1.94
4	1.20	2.05	2.54	3.00	2.65
5	1.04	1.56	2.45	2.33	3.25

Notes: Conditionnal logit with unobserved heterogeneity (*Model 3*), 2019 observations.

Focusing on Model 3 estimates the first result is that all the parameters are positive and significantly different from zero (but δ_{14}): non-employment is less likely to be the origin or destination quintile of all other quintiles. Now if I concentrate on the quintiles 1, 2, 3, 4 and 5, it is possible to test the presence of state dependence as shown in section 3.1 by testing the equality of parameters on each row (see Table 4). This Wald tests strongly reject for each row the null hypothesis of equality: for quintiles 1 to 5 the Wald statistics are equal respectively to

13.33, 45.65, 61.29, 34.19, 32.03 and are distributed under the null as $\chi^2(4)$. This important result shows the presence of state dependence in earnings mobility in the United-States in the 90's. Now what is its nature? Does it create movement or stability? For all the quintiles but the first, the highest coefficient on each row is the stability one: the effect of state dependence is to increase the probability to stay in the same quintile. But interestingly, this result is not true for the first quintile, in which state dependence is such that it is easier to move up one quintile than not to move. Lastly, it must be noted that on all the rows the upwards parameters are greater than the downwards ones: state dependence favours upward movements rather than downwards.

To sum up these results, this first stage shows that state dependence exists in earnings mobility in the US, that for every quintile but the first it creates more stability than mobility and that it favours upward movements rather than downwards. This picture is remarkably similar to that concerning Austria found in Weber (2002), which can be surprising since both labor markets are very different. The main difference between both countries is that in Austria the first quintile is similar to the others.

4 Earnings mobility and unobserved heterogeneity

4.1 Estimation

To estimate the law of unobserved heterogeneity, ie the law of α_i, assumptions must be done and thus random effects techniques must be applied. To minimize the impact of distributional assumptions I follow Heckman and Singer (1984) and I choose a discrete law, the number of support points (or number of types) being determined with an increasing iterative process stopped by a good fit of the model. To solve the initial conditions problem I adopt the solution proposed by Wooldridge (2005): I maximise the likelihood conditionnal on the initial conditions and I let the law of α_i depend on it. More precisely, I maximize

$$P\left(y_{iT},...,y_{i2}\,/y_{i1},\hat{\delta}\right)=\sum_{l=1}^{L}\left\{P(\alpha_i=\alpha_i^l/y_{i1})\prod_{t=2}^{T}P(y_{it}/y_{it-1},\alpha_i=\alpha_i^l,y_{i1},\hat{\delta})\right\} \tag{5}$$

where l denotes one of the L types and $\hat{\delta}$ represents first stage estimates. It must be noted that this likelihood is conditionnal on y_{i1} and that the term $P(\alpha_i=\alpha_i^l/y_{i1})$ shows that the law of α_i depends on y_{i1}. The parameters to be determined are $\forall\ l=1...L$ and $\forall\ k=0...5$, α_i^l and $P(\alpha_i=\alpha_i^l/y_{i1}=k)$ (that I note π_{lk}). This likelihood is maximized via a standard EM alogorithm, by iterating the two following steps.

- E-Step

For initial values $(\alpha_i^l)^{(r-1)}$ and $\pi_{lk}^{(r-1)}$, for each type $l=1...L$ and each individual i in the sample compute the posterior probability that i belongs to type l:

$$P(\alpha_i=(\alpha_i^l)^{(r-1)}/y_{iT},...,y_{i2},y_{i1})=\frac{\pi_{ly_{i1}}^{(r-1)}\prod_{t=2}^{T}P(y_{it}/y_{it-1},\alpha_i=(\alpha_i^l)^{(r-1)},y_{i1})}{\sum_{j=1}^{L}\left\{\pi_{jy_{i1}}^{(r-1)}\prod_{t=2}^{T}P(y_{it}/y_{it-1},\alpha_i=(\alpha_i^j)^{(r-1)},y_{i1})\right\}}$$

that I note w_{il}.

- M-step

Update the law of unobserved heterogeneity by averaging the posterior probabilities obtained in the E-step: $\forall\ l=1...L$, $k=0...5$,

$$\pi_{lk}^{(r)}=\frac{\sum_{i/y_{i1}=k}w_{il}}{\sum_{i=1}^{N}\mathbb{1}_{\{y_{i1}=k\}}}$$

Then update the support of the unobserved heterogeneity: $\forall\ l=1...L$,

$$\alpha_{il}^{(r)}=\operatorname*{argmax}_a\sum_{i=1}^{N}\sum_{t=2}^{T}w_{il}\ln P(y_{it}/y_{it-1},\alpha_i=a,y_{i1})$$

The latter expression is the likelihood of a multinomial logit weighted by the E-step posteriors and is thus easily maximized.

The standard errors are obtained by bootstrapping 50 times the first stage and the second stage of the procedure. Thanks to the estimates, it is possible to compute a transition matrix M_l for each type l. It is also possible to determine the stationnary distributions of quintiles for each type, ie the Q_l vectors such that $Q_l = M_l' Q_l$, ie the eigenvectors associated with the eigenvalue one of the transpose of the type-specific transition matrices.

4.2 Results

The minimum number of types needed to fit the data is six. The six types 0 to 5 represent respectively 12.9%, 16.0%, 12.5%, 19.0%, 21.0% and 18.6% of the population. Table 5 shows that the fit of this parsimonious model is excellent.

Table 5: Predicted Quintile Transition Matrix

Origin	Destination quintile						
quintile	0	1	2	3	4	5	*Sum*
0	65.8	19.6	5.3	3.2	3.0	3.1	100.0
1	16.5	56.3	17.8	5.7	2.0	1.7	100.0
2	6.0	18.1	51.6	18.0	4.5	1.9	100.0
3	4.1	5.9	13.6	51.2	21.5	3.6	100.0
4	3.5	3.4	4.5	14.0	57.1	17.5	100.0
5	3.2	1.6	1.3	3.8	11.5	78.5	100.0

Notes: This Table must be compared with Table 1

Table 6 presents the law of the types conditionnal on the initial conditions (ie the π_{lk}). A simple independence Chi-Square test (value 5728, 25 DF, p-value < 0.0001) indicates that the independence of the initial conditions and the unobserved heterogenity is strongly rejected. This result shows the importance to take into account the initial conditions problem. Moreover, each type is clearly associated with an initial quintile: among observations who are in 1990 in the quintile 0 (resp 1, 2, 3, 4, 5), 61% (resp 52%, 47%, 61%, 68%, 80%) are in the type 0 (resp 1, 2, 3, 4, 5).

Table 6: Type Distributions, by 1990 Quintile

1990	Type						
Quintile	0	1	2	3	4	5	Sum
0	0.61399	0.30543	0.03835	0.02618	0.00317	0.01289	1
	(0.02719)	(0.00750)	(0.05030)	(0.01036)	(0.02214)	(0.05135)	
1	0.16659	0.52329	0.18422	0.07868	0.04386	0.00337	1
	(0.04155)	(0.00291)	(0.03550)	(0.01148)	(0.05733)	(0.04962)	
2	0.09281	0.11036	0.47428	0.23001	0.07818	0.01437	1
	(0.08310)	(0.00586)	(0.02525)	(0.02404)	(0.06736)	(0.04267)	
3	0.05907	0.02947	0.05294	0.61137	0.22155	0.02560	1
	(0.04911)	(0.00657)	(0.02908)	(0.01170)	(0.02016)	(0.02065)	
4	0.04176	0.00001	0.02360	0.11730	0.68342	0.13392	1
	(0.03145)	(0.01901)	(0.00882)	(0.02776)	(0.01962)	(0.00641)	
5	0.02969	0.00144	0.00255	0.00544	0.15686	0.80402	1
	(0.00382)	(0.03176)	(0.01648)	(0.02853)	(0.00390)	(0.00493)	
Total	0.12900	0.15960	0.12525	0.18961	0.21010	0.18644	1

Notes: Standards errors between brackets.

The type specific transition matrices are presented in Table 7. The main feature of this table is that each type attracts toward a specific quintile: for each type the mode of each conditionnal distribution is the same. In the type 0, 1, 2, 3, 4 and 5 individuals are respectively attracted toward the quintiles 0 1, 2, 3, 4 and 5. The type-specific stationnary quintile distributions in

Table 7: Type-Specific Transition Matrices

Origin	Destination Quintile, Type 0							Destination Quintile, Type 1							Destination Quintile, Type 2						
Quintile	0	1	2	3	4	5	Sum	0	1	2	3	4	5	Sum	0	1	2	3	4	5	Sum
0	81.2	12.5	3.0	1.4	0.8	1.1	100.0	31.0	56.4	9.1	2.7	0.8	0.0	100.0	22.1	27.4	39.4	8.8	1.6	0.7	100.0
1	56.0	30.4	8.7	2.6	0.7	1.7	100.0	11.2	71.9	13.9	2.7	0.4	0.0	100.0	7.1	30.9	53.0	7.8	0.6	0.5	100.0
2	39.7	26.2	22.4	7.2	2.1	2.4	100.0	7.0	54.4	31.3	6.5	0.9	0.0	100.0	2.6	13.9	70.9	11.2	1.0	0.4	100.0
3	39.3	20.6	14.0	15.8	6.6	3.6	100.0	8.0	49.5	22.7	16.4	3.4	0.0	100.0	3.0	12.7	51.5	28.6	3.5	0.7	100.0
4	41.5	21.4	11.9	8.9	8.4	7.9	100.0	9.1	55.4	20.8	9.9	4.8	0.0	100.0	3.8	16.0	53.4	19.5	5.6	1.8	100.0
5	44.3	19.4	7.8	8.7	4.6	15.3	100.0	11.3	58.4	15.9	11.4	3.0	0.0	100.0	5.2	18.3	44.2	24.2	3.8	4.3	100.0
Origin	Destination Quintile, Type 3							Destination Quintile, Type 4							Destination Quintile, Type 5						
Quintile	0	1	2	3	4	5	Sum	0	1	2	3	4	5	Sum	0	1	2	3	4	5	Sum
0	30.9	12.1	14.5	30.1	10.0	2.4	100.0	28.2	8.6	3.7	9.0	39.3	11.3	100.0	35.1	5.0	2.3	1.6	7.4	48.7	100.0
1	13.1	18.1	25.8	35.4	5.3	2.3	100.0	16.2	17.5	8.9	14.3	28.4	14.7	100.0	18.9	9.4	5.2	2.3	5.0	59.1	100.0
2	4.5	7.5	31.9	46.9	7.6	1.6	100.0	5.9	7.8	11.8	20.3	43.2	11.0	100.0	9.4	5.7	9.5	4.5	10.5	60.4	100.0
3	2.8	3.7	12.5	64.6	14.9	1.5	100.0	2.7	2.8	3.4	20.7	62.8	7.6	100.0	6.1	3.0	3.9	6.5	21.5	59.0	100.0
4	3.8	5.1	14.0	47.7	25.1	4.3	100.0	2.4	2.5	2.5	9.9	68.7	14.0	100.0	3.8	1.8	1.9	2.1	16.1	74.3	100.0
5	4.7	5.3	10.6	54.0	15.8	9.6	100.0	3.2	2.8	2.0	12.0	46.4	33.6	100.0	2.5	1.0	0.8	1.3	5.4	89.0	100.0

Notes: 13860 observations, PSID, annual earnings.

Table 8 confirm this result: at the equillibrium, in the type 0 (resp 1, 2, 3, 4 and 5), they are 72.7% (resp 66.0%, 63.6%, 55.4%, 60.9% and 85.1%) in the quintile 0 (resp 1, 2, 3, 4 and 5). For each type they are about 15% to be in an adjacent quintile and very few in the other quintiles. This means that at the stationnary equilibrium people will be locked between three quintiles, and essentially in one quintile, around two thirds of the time for the quintiles 1, 2, 3 and 4, three quarters of the time in the quintile 0 and even more in the highest one.

Table 8: Quintile Stationnary Distributions, by Type.

	Quintile						
Type	0	1	2	3	4	5	Sum
0	72.7	16.6	5.4	2.5	1.2	1.6	100.0
1	12.9	66.0	16.5	3.9	0.7	0.0	100.0
2	4.3	17.3	63.6	12.9	1.4	0.5	100.0
3	5.4	5.9	16.8	55.4	14.3	2.2	100.0
4	4.3	3.5	3.1	11.9	60.9	16.2	100.0
5	4.3	1.4	1.1	1.5	6.5	85.1	100.0

Notes: 13860 observations, PSID, annual earnings.

Are these unobserved types related with some time-invariant observable individual characteristics? To answer this question the EM alogorithm is very useful: thanks to the E-step posteriors it is possible to classify automatically each observation in a type and to cross it with observables. Table 9 shows that with respect to men, women are over-represented in the types associated with the bottom of the distribution and under-represented with those associated with the upper part. The same inequalities exist between White and Non-White people and between those who have spent at least one year in a College and those who have not. However the magnitude of these inequalities is lower than the gender related one.

Table 9: Type Distributions, by Individual Characteristics.

	Type						
	0	1	2	3	4	5	Sum
Women	17.3	26.6	17.1	18.6	13.7	6.8	100.0
Men	7.9	3.9	7.3	19.4	29.4	32.1	100.0
Non-White	15.0	24.5	15.0	18.0	21.2	6.2	100.0
White	12.7	15.1	12.3	19.1	21.0	19.8	100.0
No-College	15.6	21.8	17.8	21.8	16.4	6.7	100.0
College	11.0	11.9	8.9	17.0	24.2	27.0	100.0
Total	12.9	16.0	12.5	19.0	21.0	18.6	100.0

Notes: 13860 observations, PSID, annual earnings.

Lastly it is interesting to determine the relative contributions of state dependence and individual heterogeneity in earnings stability. Table 7 shows that in each type only one quintile can be considered as stable, the quintile that attracts the individuals. For example in type 2, the stability percentages are, from quintile 0 to 5, 22.1%, 30.9%, 70.9%, 28.6%, 5.6% and 4.3%: it is clear that this type is the quintile 2 stability group. Thus the idea is to study the stability of each quintile in its associated type, ie the stability of quintile k in type k, for $k = 0...5$. To do this decomposition, the logarithm of equation (2)

$$\ln\left\{\frac{P(y_{it} = k\,/y_{it-1} = k\,,\,\alpha_i = \alpha_i^k)}{P(y_{it} = j\,/y_{it-1} = k\,,\,\alpha_i = \alpha_i^k)}\right\} = (\delta_{kk} - \delta_{kj}) + (\alpha_{ik}^k - \alpha_{ij}^k) \tag{6}$$

with $j \neq k$ is used. The latter expression is the logarithm of the probability to stay in the quintile k with respect to the probability to do the transition k towards another quintile j, conditionnal on being in the type k, ie the quintile k stability type. It is the sum of $(\delta_{kk} - \delta_{kj})$, a state dependence term, and $(\alpha_{ik}^k - \alpha_{ij}^k)$, an individual heterogeneity term. The part S_{kj} of state dependence in earnings stability in quinitle k with respect to quintile j can thus be measured by

$$S_{kj} = \frac{(\delta_{kk} - \delta_{kj})}{(\delta_{kk} - \delta_{kj}) + (\alpha_{ik}^k - \alpha_{ij}^k)}$$

These contributions are reported in Table 10, which consists of three zones. The first zone is the first line: with respect to the rest of the table, these numbers are extremely low, which means that in the first quintile, earnings stability is essentially due to individual characteristics. The negative sign of S_{12} is due to the fact that, as it has been seen in section 3.3, $\delta_{11} < \delta_{12}$. The second zone

Table 10: Relative Contribution of State Dependence in Earnings Stability

$S_{kj}(\%)$	j				
k	1	2	3	4	5
1	-	-10.7	7.3	19.5	4.2
2	77.8	-	18.9	25.0	23.0
3	68.2	55.3	-	24.9	32.9
4	54.2	28.7	23.9	-	21.8
5	49.2	35.6	18.8	32.9	-

Notes: 13860 observations, PSID, annual earnings.

is the rest of the upper diagonal: percentages are rather homogenous around 25%. For the origin quintiles 2, 3 and 4 state dependence explains around one quarter of the ratio stability/upward mobility: this part is not negligible and could be the target of a public policy that would try to decrease the ratio stability/upward mobility by decreasing its state dependence part. The third zone is the lower diagonal, and the main feature is that the contributions are greater than in the upper part of the table, especially in the first column, ie for the ratios stability/quintile 1. It means that state dependence explains more the ratio stability/downward mobility than the ratio stability/upward mobility. These results show that for every quintile but the first state dependence play in earnings stability an important role, in particular by preventing from downward mobility. However, the stability in the lowest quintile is essentially due to individual heterogeneity.

In short, the results given by this section are the following. To describe the individual heterogeneity associated with earnings mobility, six unobserved types are necessary. Each type-specific transition matrix attracts towards a quintile, and at the stationnary equilibrium, individuals spend the vast majority of the time in that quintile. The types are correlated with observable time-invariant individual characterisitics: women, non-white and the less educated are over-represented in those associated with the bottom of the distribution; men, white and the more educated in those associated with the top. Lastly, it is showed that the contribution of state dependence in earnings stability is not negligible when it is considered with respect to upward mobility (around 25%), and even more important when it is considered with respect to downward mobility.

5 Conclusion

Modelling earnings quintiles dynamics with a dynamic multinomial logit with unobserved heterogeneity, the two dimensions of earnings mobility, state dependence and unobserved heterogeneity, are disentangled. Using in a first stage conditionnal maximum likelihood techniques (Magnac, 2000), the presence of state dependence can not be rejected and it is showed that for every quintile but the first it creates more stability than mobility and it favours upward movements rather than downwards. In a second stage the law of the unobserved heterogeneity is supposed to be discrete and is estimated via the EM algorithm, solving the initial conditions problem à la Wooldridge (2005). The second stage shows essentially that each individual is attracted towards a specific quintile, which makes the quintile distribution very segmented. Moreover, men, white and the more educated are attracted towards the upper part of the distribution while women, non-white and the less educated towards the lower. Lastly, it is showed that the contribution of state dependence in earnings stability is not negligible when it is considered with respect to upward mobility (around 25%), and even more important when it is considered with respect to downward mobility.

This picture suggests that there is still much between group inequality and that these inequalities will persist if the parameters process are left unchanged. Public policies could act on both dimensions of earnings mobility to reduce inequalities. On the one hand human capital policies could be implemented to improve unobserved heterogeneity of the individuals attracted towards the lower part of the distribution; on the other hand, it could be desirable to act on structural state dependence in order to make it more mobile, which requires to give an economic meaning to state dependence in earnings mobility.

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