Spousal Bargaining Power:
Decoupling Gender Norms and Earning Status

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Abstract

The collective model provides a spousal bargaining framework of household labor supply, but most empirical studies assume that husbands bargain with wives, leaving unclear the extent to which gender norms surrounding labor supply interact with empirical estimates. I estimate collective labor supply models for different-sex and same-sex married couples in order to quantify the role of gender norms in spousal bargaining as distinct from that of earning status within the couple. I find that wives in different-sex couples have a significantly larger Pareto weight on their utility relative to their husbands, but I find no significant evidence that gender norms affect spousal bargaining power within the couple. My findings suggest that observable differences between men’s and women’s intensive margin labor supplies in different-sex couples are not significantly influenced by traditional gender norms within the couple, but may be influenced by external factors, such as the gender wage gap, that create different likelihoods that a man or woman is the primary earner in their household. Ongoing work includes extending the present analysis to include labor force participation decisions.

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1 Introduction

The collective labor supply model introduced by Chiappori (1988, 1992) provides a spousal bargaining framework of household labor supply that requires the researcher to divide couples along one dimension so that one spouse can bargain with the other. Most empirical studies divide different-sex couples by sex (e.g., Chiappori, Fortin, and Lacroix 2002; Moreau and Donni 2002; Vermeulen 2006; Blundell et al. 2007; Donni and Moreau 2007; Cherchye, De Rock, and Vermeulen 2012; Gayle and Shephard 2019 among others). However, institutional factors in the labor market, such as the gender wage gap and traditional gender norms surrounding labor supply, suggest that husbands are more likely to be primary earners in their households, meaning that husbands’ estimated bargaining power may reflect both gender norms and earning status in the household. It is, therefore, unclear to what extent gender norms influence bargaining power within couples separately from each spouse’s earning status in the household. For example, Bartels and Shupe (2018) conclude that earning status in the household, rather than sex, is a more influential driver of responses to work incentives, and Baldwin, Allgrunn, and Ring (2011) suggest that the traditional male-female division in household labor supply has become less useful over time.

Disentangling the role of gender norms in spousal bargaining power and labor supply can not only illuminate the extent to which gender inequality drives intra-household inequality and economic outcomes, but can also help inform policies aimed at decreasing inequality and inform expectations about labor supply elasticities. For example, if observed differences in men’s and women’s labor supply are entirely attributable to gender norms, then policies aimed at addressing institutional inequalities, such as reducing marginal tax rates for secondary earners, may have little effect on women’s labor supply leading to small elasticity estimates. On the other hand, if observed labor supply differences are not affected by gender norms, then policies aimed at addressing institutional inequalities may be particularly effective.

In this paper, I estimate collective labor supply models for different-sex and same-sex married couples to quantify the role of gender norms in spousal bargaining power. Although this is my main goal and contribution in this paper, there are two other contributions to the literature. First, I
provide updated collective labor supply estimates for same-sex couples relative to the pathbreaking work by Oreffice (2011), who used data on same-sex cohabiting couples from the 2000 U.S. decennial census. Although valuable, the institutional context Oreffice (2011) studies pre-dates any legal access to same-sex marriage in the U.S., meaning the comparison of same-sex cohabiting partners’ collective labor supply parameters to different-sex married spouses’ parameters does not as cleanly identify the role of gender norms.\footnote{Massachusetts was the first state to legalize same-sex marriage, and did so in 2004.}

Second, I provide updated collective labor supply estimates from the model outlined by Donni (2003), which allows for non-linear budget constraints due to taxation. Moreau and Donni (2002) and Bloemen (2010) are the only others to estimate this model, to the best of my knowledge, and did so using French data from 1994 and Dutch data from 1990-2001, respectively. This model is useful in my context because there were substantial tax changes for same-sex married couples during my sample period, for which the model can account, and which I use to identify the unrestricted labor supply parameters.\footnote{Friedberg and Isaac (2019) and Isaac (2019) study these tax changes in more detail and estimate their effects on marriage and labor supply, respectively.}

I use the 2012–2017 American Community Surveys to construct a sample of different- and same-sex married, childless, dual-earner couples in which both spouses are between 25 and 60 years old. The 2012 American Community Survey is the first of the U.S. Census Bureau surveys to explicitly identify same-sex married couples in the data, whereas prior Census Bureau surveys suffered from substantial measurement error that made it difficult to reliably identify same-sex married couples (Black et al. 2007; Gates and Steinberger 2010). I divide different-sex couples by sex, as is common in this literature, and use a machine learning LASSO approach to divide same-sex couples by predicted earning status in the household. Identification of the sharing rule rests upon a distribution factor, defined as “variables that affect the household members’ bargaining position but not preferences or the joint budget set” (Chiappori, Fortin, and Lacroix 2002). I use the difference in years of education between the two spouses as a distribution factor in this paper.\footnote{Browning, Chiappori, and Weiss (2014) list the difference in years of education between spouses is listed as a distribution factor that has been used elsewhere (page 204). Chiappori, Iyigun, and Weiss (2009) also suggest that relative education may act as a distribution factor because education in their model increases the marital surplus share the spouse can extract.} Identification of the effect of gender norms on spousal bargaining power comes from the fact that bargaining between different-sex spouses necessarily includes differences in the spouses’
sexes, whereas these differences are not present during bargaining between same-sex spouses. My empirical strategy allows me to recover the structural Marshallian labor supply parameters as well as the relative Pareto weights on the utility functions of wives in different-sex couples and predicted lower earners in same-sex couples.

I estimate significant Marshallian hours elasticities for all spouses except predicted higher earners in same-sex couples, which suggest backward-bending labor supply curves for husbands in different-sex couples and predicted lower earners in same-sex couples. My estimates imply that wives in different-sex couples have a statistically significant 12.1% larger Pareto weight on their utility in the collective household maximization problem, indicating greater bargaining power for wives, relative to husbands, in different-sex couples. In contrast, I estimate no significant difference between the Pareto weights of same-sex spouses. In addition, my Pareto weight estimates for different-sex and same-sex couples are not statistically different from each other, meaning that I do not estimate any significant influence of gender norms on spousal bargaining power within the couple. It is important to note, however, that my present analysis is limited to dual-earner couples, and my results, therefore, cannot currently speak to the potential role of gender norms in labor force participation decisions. Ongoing work, however, includes extending the present analysis to include labor force participation decisions.

My results suggest that although past studies have assumed that bargaining power is divided by sex in different-sex couples, this choice does not significantly bias their results. In addition, my results also suggest that external institutional factors in the labor market that make husbands more likely to be primary earners in their households, such as the gender wage gap, may be more influential in driving observable differences between male and female labor supply than gender norms.

2 The Collective Labor Supply Model

In this paper, I estimate the collective model of labor supply presented by Donni (2003), which extends of the models from Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002)
to allow for nonlinear budget constraints due to taxation. The collective labor supply model is empirically useful because, by first specifying a functional form for the spouses’ unrestricted labor supply functions, it is possible identify the Marshallian labor supply functions, the indirect utility functions, and the Pareto weight for each spouse’s utility, as demonstrated below. In this section I outline the collective model, along with its assumptions and restrictions, and reproduce the main propositions from Chiappori (1988, 1992), Chiappori, Fortin, and Lacroix (2002), and Donni (2003) below where necessary.

There are two individuals indexed by $i$ in the household ($i = 1, 2$), with vectors of preference factors $z$, labor supplies $L_i$, gross hourly wages of $w_i$, household non-labor income of $y$, and aggregate Hicksian consumption of $C_i$. Assume that the price of consumption is normalized to one and the total time available to each individual is normalized one, so that $1 - L_i$, with $0 \leq L_i \leq 0$, denotes individual $i$’s leisure.

Donni (2003) makes the following two assumptions:

**Assumption 1.** Each household member is characterized by specific utility functions of the form $u_i(1 - L_i, C_i)$. These functions are both strongly concave, infinitely differentiable, and strictly increase in all their arguments on $\mathbb{R}^3_{++}$, with $\lim_{C_i \rightarrow 0} u_i(1 - L_i, C_i) = \lim_{L_i \rightarrow 1} u_i(1 - L_i, C_i) = -\infty$.

**Assumption 2.** The outcome of the decision process is Pareto efficient

Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) also assume Pareto efficiency, and it forms the foundation of the collective approach.

Under assumptions 1 and 2 there exists a Pareto weight, $\mu$, such that household behavior is a solution to the problem:

$$\max_{L_1, L_2, C_1, C_2} u_1^1(1 - L_1, C_1, z) + \mu u_2^2(1 - L_2, C_2, z)$$

subject to $h(L_1, L_2; w_1, w_2, y) \geq C_1 + C_2$ \hspace{1cm} (P)

$$0 \leq L_1 \leq 1, \quad 0 \leq L_2 \leq 1, \quad C_1 \geq 0, \quad C_2 \geq 0,$$

where $h(\cdot)$ is infinitely differentiable, increasing in all its arguments, and concave in $L_1$ and $L_2$. 

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At this point, it should be noted that Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) provide canonical results of the collective model under linear budget constraints and assumptions 1 and 2. A key result from Chiappori (1988) is that the household problem $\bar{P}$ is equivalent to individual maximization problems in which the spouses split household non-labor income according to a sharing rule and then, conditional on the sharing rule, maximize their individual utilities subject to their relevant budget constraints. Chiappori (1992) builds upon this framework and provides testable restrictions on labor supply functions that allow for identification of the sharing rule. Chiappori, Fortin, and Lacroix (2002) introduce the concept of distribution factors, defined as “variables that affect the household members’ bargaining position but not preferences or the joint budget set,” which generate a new set of testable restrictions on labor supplies and allow for more straightforward identification of the sharing rule. It is here where Donni (2003) extends upon the framework from Chiappori, Fortin, and Lacroix (2002) to non-linear budget constraints.

Under a non-linear budget constraint, Donni (2003) defines the shadow wages and shadow income as:

$$\omega_1(w_1, w_2, y) = \frac{\partial h(L^1, L^2; w_1, w_2, y)}{\partial L^1}$$

$$\omega_2(w_1, w_2, y) = \frac{\partial h(L^1, L^2; w_1, w_2, y)}{\partial L^2}$$

$$\eta(w_1, w_2, y) = h(L^1, L^2; w_1, w_2, y) - \sum_i L^i \omega_i,$$

and puts forth the following lemma:

**Lemma 1.** Let $(\bar{L}^1, \bar{L}^2)$ be a pair of labor supplies consistent with collective rationality conditionally on the budget constraint in problem $\bar{P}$. Then, there exist a pair of functions $(\bar{C}^1, \bar{C}^2)$ and a pair of functions $(\bar{\rho}^1, \bar{\rho}^2)$, with $\sum_i \rho^i = \eta$, such that $(\bar{L}^i, \bar{C}^i)$ is a solution to:

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4. Using the notation of problem $\bar{P}$, the linear budget constraint would be $h(L^1, L^2; w_1, w_2, y) = L^1 w_1 + L^2 w_2 + y.$
\[
\max_{L,C} \quad u'(1 - L^i, C^i, x)
\]
subject to \(\gamma: L^i \omega^i + \rho^i = C^i\)
\((P^i)\)
\[0 \leq L^i \leq 1\]
for any \((w_1, w_2, y) \in \mathbb{R}^3_{++}\).

At an interior solution, the unrestricted labor supplies (\(\bar{L}^1\) and \(\bar{L}^2\)) can be re-written as Marshallian labor supplies:
\[
\bar{L}^1 = \lambda^1(\omega_1(w_1, w_2, y), \rho(w_1, w_2, y))
\]
\[
\bar{L}^2 = \lambda^2(\omega_2(w_1, w_2, y), \eta(w_1, w_2, y) - \rho(w_1, w_2, y)),
\]
where \(\rho = \rho^1\) and \(\eta - \rho = \rho^2\). Donni (2003) shows that we can further write \(\lambda^i\) as a function of only the shadow variables \((\omega_1, \omega_2, \eta)\) using the Implicit Function Theorem by making the following assumption:

Assumption 3. Labor supplies \(\bar{L}^1(w_1, w_2, y)\) and \(\bar{L}^2(w_1, w_2, y)\) and the budget constraint \(h(L^1, L^2; w_1, w_2, y)\)
are such that
\[
\begin{vmatrix}
\frac{\partial \omega_1}{\partial w_1} & \frac{\partial \omega_1}{\partial w_2} & \frac{\partial \eta}{\partial w_1} \\
\frac{\partial \omega_1}{\partial y} & \frac{\partial \omega_2}{\partial y} & \frac{\partial \eta}{\partial y}
\end{vmatrix}
\neq 0 \text{ for any } (w_1, w_2, y).
\]

If assumption 3 is satisfied, then we can write the labor supplies in (4) and (5) as:
\[
\hat{L}^1(\omega_1, \omega_2, \eta) = \lambda^1(\omega_1, \varphi(\omega_1, \omega_2, \eta))
\]
\[
\hat{L}^2(\omega_1, \omega_2, \eta) = \lambda^2(\omega_2, \eta - \varphi(\omega_1, \omega_2, \eta)),
\]
where \(\varphi(\omega_1(w_1, w_2, y), \omega_2(w_1, w_2, y), \eta(w_1, w_2, y)) = \rho(w_1, w_2, y)\).

Having defined the shadow wages, shadow income, and the unrestricted labor supplies, Marshallian labor supplies, and sharing rule (as functions of the shadow variables), the canonical results from Chiappori (1992) and Chiappori, Fortin, and Lacroix (2002) follow; namely, that the partial derivatives of the sharing rule are identifiable (with or without distribution factors) as functions of
the shadow variables. In this paper, I use a distribution factor, $s$, for identification. Before deriving the structural parameters, define:

$$A = \frac{\partial \hat{L}^1}{\partial \hat{L}^1}, \quad B = \frac{\partial \hat{L}^2}{\partial \hat{L}^2}, \quad C = \frac{\partial \hat{L}^1}{\partial s}, \quad D = \frac{\partial \hat{L}^2}{\partial s},$$

Chiappori, Fortin, and Lacroix (2002) present the proposition below, which I reproduce using the shadow variable notation from above:

**Proposition 1.** Take any point such that $\frac{\partial \hat{L}^1}{\partial \eta} \cdot \frac{\partial \hat{L}^2}{\partial \eta} \neq 0$. Then the following results hold: (i) If there exists exactly one distribution factor such that $C \neq D$, the following conditions are necessary for any pair $(\hat{L}^1, \hat{L}^2)$ to be solutions of $(P_i)$ for some sharing rule $\varphi$:

\begin{align}
\frac{\partial}{\partial s} \left( \frac{D}{D-C} \right) &= \frac{\partial}{\partial y} \left( \frac{CD}{D-C} \right) \\
\frac{\partial}{\partial \omega_1} \left( \frac{D}{D-C} \right) &= \frac{\partial}{\partial y} \left( \frac{BC}{D-C} \right) \\
\frac{\partial}{\partial \omega_2} \left( \frac{D}{D-C} \right) &= \frac{\partial}{\partial y} \left( \frac{AD}{D-C} \right) \\
\frac{\partial}{\partial \omega_1} \left( \frac{CD}{D-C} \right) &= \frac{\partial}{\partial s} \left( \frac{BC}{D-C} \right) \\
\frac{\partial}{\partial \omega_2} \left( \frac{CD}{D-C} \right) &= \frac{\partial}{\partial s} \left( \frac{AD}{D-C} \right) \\
\frac{\partial}{\partial \omega_2} \left( \frac{BC}{D-C} \right) &= \frac{\partial}{\partial \omega_1} \left( \frac{AD}{D-C} \right)
\end{align}

$$\frac{\partial \hat{L}^1}{\partial \omega_1} - \frac{\partial \hat{L}^1}{\partial \eta} \left( \hat{L}^1 + \frac{BC}{D-C} \right) \left( \frac{D-C}{D} \right) \geq 0 \quad (8g)$$

$$\frac{\partial \hat{L}^2}{\partial \omega_2} - \frac{\partial \hat{L}^2}{\partial \eta} \left( \hat{L}^2 + \frac{AD}{D-C} \right) \left( - \frac{D-C}{D} \right) \geq 0 \quad (8h)$$

(ii) Under the assumption that conditions (8a–8h) hold and for a given $z$, the sharing rule is defined up to an additive function $\kappa(z)$ depending only on the preference factors $z$. The partial derivatives of the sharing rule with respect to wages, non-labor income, and the distribution factor
are given by:

\[
\begin{align*}
\frac{\partial \phi}{\partial \eta} &= \frac{D}{D - C} \\
\frac{\partial \phi}{\partial s} &= \frac{CD}{D - C} \\
\frac{\partial \phi}{\partial \omega_1} &= \frac{BC}{D - C} \\
\frac{\partial \phi}{\partial \omega_2} &= \frac{AD}{D - C}
\end{align*}
\] (9)

Donni’s (2003) extension of the collective labor supply model to non-linear budget constraints, combined with proposition [1] above from Chiappori, Fortin, and Lacroix (2002), constitute the theoretical results needed for identification in this paper. In the next section, I assume a parametric specification for the unrestricted labor supply functions and derive the sharing rule implied by these functions.

### 3 Parametric Specification

For convenience, I follow Oreffice (2011) and assume the parametric specification of the unrestricted labor supplies below.

\[
L^1 = a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + a_5 z
\] (10)

\[
L^2 = b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + b_5 z
\] (11)

\(\omega_i\) is individual \(i\)’s net-of-tax hourly wage rate, \(\eta\) is the couple’s virtual income, \(s\) is the difference in years of education between the spouses (the distribution factor), and \(z\) includes controls for age, the state unemployment rate, and indicator variables for the individual’s level of education, race, year, state of residence, and whether the state recognizes same-sex marriage.\(^5\)

\(z\) additionally controls for the average gross hourly wage gap between spouses conditional on each spouse’s age, sex, and state of residence.\(^6\) Note, however, that equations (10) and (11) control

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5. The education level groups are “exactly high school education,” “some college education,” and “college education or more,” with the omitted category being “less than high school education.” The race groups are “black” and “hispanic,” with the omitted category being “non-black, non-hispanic.”

6. I calculate each individual’s weighted average gross hourly wage, conditional on the individual’s age, sex, and state of residence, while omitting the individual’s own wage (i.e., a leave-one-out calculation). This calculation uses the person weights in the American Community Survey.
for the individual’s age and state of residence, and the set-up of the regressions mean that the
constant term will effectively control for the individual’s sex in different-sex couples. Therefore,
the identifying variation in the average wage gap is due only to age differences in different-sex
couples. This will also be true in same-sex couples because there is no within-couple variation in
sex. This average wage gap measure, therefore, does not truly reflect the gender wage gap within
different-sex couples, even though the calculation is sex-specific, but it does additionally control
for differences between spouses that may drive labor supply.

Equations \(10\) and \(11\) lead to the following functions for \(A, B, C,\) and \(D\) under the definitions in
Section \(2\):

\[
A = \frac{a_2}{a_3 \omega_2}, \quad B = \frac{b_1}{b_3 \omega_1}, \quad C = \frac{a_4}{a_3}, \quad D = \frac{b_4}{b_3}
\]

These definitions imply that conditions \(8a-8f\) are automatically satisfied because the derivatives
are zero, but it does imply other testable restrictions. Namely, the condition that \(\frac{\partial \hat{L}_1}{\partial \eta} \cdot \frac{\partial \hat{L}_2}{\partial \eta} \neq 0\)
requires that \(a_3b_3 \neq 0\), and the condition that \(C \neq D\) requires that \(\frac{a_4}{a_3} \neq \frac{b_4}{b_3}\).

Let \(\Delta = a_3b_4 - a_4b_3\). If the above restrictions are satisfied, then the partial derivatives of the
sharing rule, \(\varphi\), are given by:

\[
\frac{\partial \varphi}{\partial \eta} = \frac{a_3b_4}{\Delta}, \quad \frac{\partial \varphi}{\partial s} = \frac{a_4b_4}{\Delta}, \quad \frac{\partial \varphi}{\partial \omega_1} = \frac{a_4b_1}{\omega_1 \Delta}, \quad \frac{\partial \varphi}{\partial \omega_2} = \frac{a_2b_4}{\omega_2 \Delta} \quad (12)
\]

Solving this system of differential equations yields the sharing rule:

\[
\varphi = \frac{1}{\Delta}(a_4b_1 \log \omega_1 + a_2b_4 \log \omega_2 + a_3b_4 \eta + a_4b_4 s) + \kappa(\varphi) \quad (13)
\]

It is also possible to derive the Marshallian labor supplies that are consistent with the unre-
restricted labor supplies in equations \(10\) and \(10\) These functions should take the following form:

\[
\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \varphi_3 \quad (14)
\]

\[
\lambda^2 = \beta_1 \log \omega_2 + \beta_2(\eta - \varphi) + \beta_3 \varphi \quad (15)
\]

\footnote{Appendix A presents the derivation of these parameters.}
Given the form of $\varphi$ in equation 19, the parameters above can be recovered as: $\alpha_1 = \frac{a_1b_4-a_4b_1}{b_4}$, $\alpha_2 = \frac{\Delta}{b_4}$, $\beta_1 = \frac{a_2b_2-a_3b_1}{a_4}$, and $\beta_2 = -\frac{\Delta}{a_4}$.

Finally, we can also recover the indirect utility functions and the Pareto weight, $\mu$. Stern (1986) shows that the Marshallian labor supplies in equations 14 and 15 correspond to the following indirect utility functions:

$$V^1(\omega_1, \varphi, z) = \left(\frac{e^{\alpha_2 \omega_1}}{\alpha_2}\right) \left(\alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 z\right)$$  \hspace{1cm} (16)

$$V^2(\omega_2, \eta - \varphi, z) = \left(\frac{e^{\beta_2 \omega_2}}{\beta_2}\right) \left(\beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 z\right)$$  \hspace{1cm} (17)

As noted by Chiappori (1988), and appealing to the Envelope Theorem, $\frac{\partial V^1}{\partial \varphi} = \delta$, where $\delta$ is the Lagrange multiplier from the household problem. Similarly, because the utility of individual 2 is multiplied by the Pareto weight ($\mu$), $\frac{\partial V^2}{\partial (\eta - \varphi)} = \frac{\delta}{\mu}$. It is, therefore, possible to identify the Pareto weight as:

$$\mu = \frac{\frac{\partial V^1}{\partial \varphi}}{\frac{\partial V^2}{\partial (\eta - \varphi)}} = \frac{e^{\alpha_2 \omega_1}}{e^{\beta_2 \omega_2}}$$  \hspace{1cm} (18)

The above definitions and derivations can refer to same-sex spouses. In what follows, I define analogous variables for different-sex spouses as $\tilde{x}$, where $x$ is the relevant parameter. In order to quantify the role of gender norms, it is possible to compare the Pareto weight of wives in different-sex married couples ($\tilde{\mu}$) to the Pareto weight of lower earners in same-sex couples ($\mu$). This comparison assumes that, all else equal, the only remaining unobserved influence on spousal bargaining power are gender norms between different-sex spouses, which are present in $\tilde{\mu}$ for wives in different-sex couples, but not present in $\mu$ for predicted lower earners in same-sex couples.

4 Data

I use data from the 2012–2017 American Community Surveys to construct a sample of different- and same-sex married couples. The model outlined in Section 2 and first proposed by Donni

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8. Appendix B presents the derivations of these parameters.
(2003), assumes an interior solution for each spouse’s labor supply, and therefore requires a sample
restricted to dual-earner couples. In addition, I further restrict my sample to childless couples
and to couples in which both spouses are between 25 and 60 years old. The first restriction is
to abstract away from the differential role of children on intensive margin labor supply, and the
second restriction is to restrict our attention to labor supply of prime-age workers.9 My main
sample includes 613,258 couples (604,859 different-sex couples and 8,399 same-sex couples),
and, therefore, 1,226,516 individuals.10

I divide spouses in different-sex couples by sex, as is common in this literature, so that husbands
bargain with wives. I use a machine learning LASSO approach to predict earnings in levels for
spouses in same-sex couples, and divide spouses in same-sex couples by predicted earning status.11
This approach enables me to include a large number of covariates and interactions while allowing
the LASSO to select the subset of variables that best fit the data. Variables that I included, but
which the LASSO may have ultimately ignored, include five year age groups, four education level
groups, dummies for race, sex, occupation, college major, and state, as well as pairwise interactions
between all of these variables. I limit the prediction sample to individuals observed in 2012 so
that predicted earning status does not reflect potential intra-household labor supply responses to
concurrent policies that vary during the sample period.

Donni’s (2003) model requires shadow wages and shadow income, \( \omega_1 \), \( \omega_2 \), and \( \eta \), respectively,
defined in equations 1–3. I follow Moreau and Donni (2002), and define, for a household with
taxable income in the \( k^{th} \) bracket, \( \omega_i = w_i(1-t_k) \) and \( \eta = y - T(B_k) + t_kB_k \), where \( t_k \) is the federal
marginal tax rate in bracket \( k \), \( B_k \) is the lower income limit of bracket \( k \), and \( T(B_k) \) is the federal
tax revenue corresponding to \( B_k \). I obtain the \( t_k \), \( B_k \), and \( T(B_k) \) parameters for each tax year from
the NBER TAXSIM program based on simulated married households with varying levels of earned
income.12 This process, therefore, takes into account numerous tax credits and deductions based

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9. Restricting to childless couples also reduces concerns that same-sex couples with children may have stronger preferences for children, which
may interact with their labor supply decisions and create divergent samples of different- and same-sex married couples.
10. If a same-sex couple reports themselves to be married even though they reside in a state that does not recognize same-sex marriages, then I
assume the couple married in a state that did recognize same-sex marriages.
11. The LASSO is a model selection method that uses a penalized regression to select the covariates that best predict earned income using
OLS (Tibshirani 2011). Friedberg and Isaac (2019) and Isaac (2019) also use a LASSO approach in similar contexts, and more detail about the
methodology can be found there.
12. These simulated households vary only in their total earned income; I do not consider other sources of income when obtaining these tax
on earned income when generating the tax brackets for each tax year, rather than using the statutory income tax brackets, which would result in much coarser measurement of the tax parameters. I also account for federal recognition of same-sex marriages when applying these definitions of \( \omega_i \) and \( \eta \) to same-sex couples.\(^{13}\)

Table 1 presents demographic and labor supply summary statistics for different- and same-sex couples in my sample. Same-sex couples are, on average, younger, more educated, have more dissimilar years of education, and more likely to live in states that recognized same-sex marriages. In addition, same-sex couples have higher earnings, work more hours, have more non-labor income, and exhibit a smaller hourly wage gap between spouses, on average, than different-sex couples.

### 5 Results

I estimate collective models for different-sex and same-sex married couples following the empirical specification in equations (10) and (11) for the unrestricted labor supply equations. Section 3 details the derivations of the Marshallian labor supplies, the derivatives of the sharing rule, and the Pareto weight on the second household member. In what follows, I will use “husband/wife” to refer to different-sex spouses and “predicted higher/lower earner” to refer to same-sex spouses.

#### 5.1 Unrestricted Labor Supply Parameters

Table 2 presents coefficient estimates for the unrestricted labor supply equations. Of primary importance are the coefficients on the distribution factor: the education difference between spouses. The coefficients on the distribution factor are both significant and negative among different-sex couples, rather than being opposite-signed for husbands and wives. They are, however, statistically different from each other, indicating that the distribution factor does differentially affect each spouse’s labor supply. The coefficients on the distribution factor are opposite signed among parameters. Figures of the tax brackets generated by this process are available upon request.

13. Same-sex married couples were still required to file federal taxes as two single individuals in tax years 2011 and 2012. Same-sex married couples were required to file joint federal taxes beginning in tax year 2013 following the United States v. Windsor Supreme Court ruling.
same-sex couples, although the coefficient for predicted higher earners is not statistically different from zero. These coefficients are statistically different from each other, again indicating that the distribution factor does differentially affect each spouse’s labor supply. The differential effects of the distribution factor on each spouse’s labor supply provides identification of the sharing rule, as outlined in section 2. The differences in how the distribution factor affects different-sex and same-sex spouses’ labor supplies in Table 2 is also consistent with Oreffice (2011), who finds that one of her distribution factors (the age difference between spouses) is opposite-signed for same-sex cohabiting couples relative to different-sex married couples.

The shadow wage and income variables vary for different-sex and same-sex spouses. The effect of own log net wage on the husband’s hours of work is negative and significant, but the effect of own log net wage on the predicted higher earner’s hours of work is positive and insignificant. These coefficients suggest possible backward bending labor supplies for husbands and predicted higher earners. The effect of own log net wage on the wife’s and predicted lower earner’s hours of work are both negative and significant, again suggesting possible backward bending labor supplies. The cross-net wage effect is negative and significant for husbands and predicted higher earners, and positive and significant for wives and predicted lower earners, suggesting that the wife’s and predicted lower earner’s labor supplies are at least partially substitutable for their spouses’, whereas the husband’s and predicted higher earner’s labor supplies are at least partially complementary to their spouses’.

The gross wage gap, perhaps surprisingly, is very small and insignificant among different-sex couples, but highly significant among same-sex couples. The coefficient on the gross wage gap is positive for husbands and negative for wives, but both are very close to zero and insignificant. Among same-sex couples, the coefficients indicate, as expected, that predicted higher earners work more hours and predicted lower earners work fewer hours if the gross wage gap is larger between same-sex spouses. Recall that, although the gross wage gap is the average wage gap conditional on the spouses’ ages, sexes, and state of residence, the identifying variation is due only to age differ-
ences between partners as a result of the other control variables and the regression set-up. These coefficients, therefore, suggest that age differences between spouses are not significant drivers of labor supply among dual-earning different-sex couples, but they are significant drivers of labor supply among dual-earning same-sex couples.

The remaining coefficients exhibit trends commonly supported in the literature: hours worked decreases with age, black and hispanic workers work fewer hours, on average, relative to others, and hours worked increases monotonically (except for predicted higher earners) with level of education. I do not find any significant influence of state same-sex marriage recognition on hours worked, and the coefficient on the unemployment rate is negative, as expected, but only significant for husbands.

5.2 Sharing Rule and Marshallian Labor Supply Parameters

Table 3 presents coefficient estimates of the sharing rule derivatives. Among different-sex couples, a $1 increase in the husband’s net wage (an increase of $2,159 annually at the mean) translates into the transfer of $560 more income to himself, whereas a $1 increase in the wife’s net wage (an increase of $1,857 annual at the mean) translates into the transfer of $2,522 to herself. This suggests that wives receive a larger premium (in the form a larger transfer) when their net wage increases relative to their husbands. Among same-sex couples, a $1 increase in the predicted higher earner’s net wage (an increase of $2,135 annually) translates into a transfer of $205 to their spouse, although this estimate is not statistically significant. Additionally, a $1 increase in the predicted lower earner’s net wage (an increase of $1,935 annually) translates into the transfer of $547 to themself. These estimates suggest, again, that predicted lower earners receive a larger premium when their net wage increases relative to their spouses.

I also estimate that an additional $1 of virtual income translates into a transfer of $1.18 to husbands, but only $0.68 to predicted higher earners. Finally, one additional year of education for the husband relative to his wife translates to a $1,867 transfer to the wife, and one additional year

---

14. See Section 3 for further discussion of this variable.
of education for the predicted higher earner relative to their spouse translates to a $1,333 transfer to the spouse, although this estimate is insignificant.

Table 4 presents coefficient estimates of the structural parameters in the Marshallian labor supply equations. The coefficient on log own net wage ($\alpha_1$ and $\beta_1$ in equations [14] and [14]) are negative and significant for husbands and predicted lower earners, positive, but insignificant, for predicted higher earners, and positive and significant for wives. These coefficients imply negative and significant Marshallian elasticities for husbands and predicted lower earners, indicating backward bending labor supplies. In contrast, the Marshallian elasticity for wives is positive and significant, indicating the traditional upward sloping labor supply.

The share of unearned income also significantly affects all spouses except for predicted lower earners, although these coefficients are positive for both husbands and predicted higher earners. This indicates, unexpectedly, that a larger share of unearned income actually increases hours worked for husbands and predicted higher earners, whereas theory would predict the opposite. The coefficient on the share of unearned income is negative and significant, as expected, for wives, but is positive and insignificant for predicted lower earners.

5.3 Pareto Weights and the Role of Gender Norms

Given the structural parameter estimates above, it is also possible to estimate the Pareto weight on the wife’s utility ($\tilde{\mu}$) and on the predicted lower earner’s utility ($\mu$). The estimated difference between these Pareto weights, therefore, is my estimate of the effect of gender norms on bargaining power.

I estimate $\hat{\tilde{\mu}} = 1.122$ (s.e. = 0.029) for wives in different-sex couples, and $\hat{\mu} = 0.942$ (s.e. = 0.124) for predicted lower earners in same-sex couples. The Pareto weight for wives is statistically different than 1, indicating that wives have a significant 12.1% greater weight put on their utilities relative to their husbands. In contrast, the Pareto weight on predicted lower earners is not significantly different from 1, meaning that I cannot reject that the Pareto weights are equal for same-sex

---

15. Recall that $\mu = e^{\alpha_2/\beta_2}$, substituting in $\tilde{\alpha}_2$ and $\tilde{\beta}_2$ for different-sex couples.
spouses.

Finally, it is possible to obtain an estimate of the effect of gender norms on spousal bargaining power using the above estimates. This comparison assumes that, all else equal, the only remaining unobserved influence on spousal bargaining power are gender norms, which are present in $\hat{\mu}$ for wives, but not present in $\mu$ for predicted lower earners. Under this assumption, I estimate the difference between the Pareto weights to be $\hat{\mu} - \hat{\mu} = 0.179$ (s.e. = 0.127). In other words, I find no significant effect of gender norms on the bargaining power of wives in different-sex couples.

6 Discussion

In this paper, I estimate collective labor supply models for different-sex and same-sex married couples to quantify the role of gender norms in spousal bargaining power. In doing so, I also provide updated collective labor supply estimates for same-sex couples relative to the pathbreaking work by Oreffice (2011), who used data on same-sex cohabiting couples from the 2000 U.S. decennial census. I corroborate Oreffice’s (2011) conclusion that labor supply in same-sex couples is consistent with the collective labor supply model. Additionally, I provide updated collective labor supply estimates from the model outlined by Donni (2003), which allows for non-linear budget constraints due to taxation, and which is useful in my context because there were substantial tax changes for same-sex married couples during my sample period.

My estimates imply that wives in different-sex couples have a statistically significant 12.1% larger Pareto weight on their utility in the collective household maximization problem, indicating greater bargaining power for wives, relative to husbands, in different-sex couples. Note, however, that bargaining power in this context is specific to bargaining over labor supply in a dual-earner couple. A larger Pareto weight, therefore, suggests that wives have greater bargaining power to reduce labor supply relative to husbands. In contrast, I estimate no significant difference between the Pareto weights of same-sex spouses.

In addition, my Pareto weight estimates for different-sex and same-sex couples are not statistically different from each other, meaning that I do not estimate any significant influence of gender
norms on spousal bargaining power within the couple. This is not to say that gender norms play no role in the labor supply of married couples, but instead means that external institutional factors in the labor market that make husbands more likely to be primary earners in their households, such as the gender wage gap, may be more influential in driving observable differences between male and female labor supply than gender norms. Therefore, policies aimed at addressing institutional inequalities between men and women are more likely to reduce wives’ Pareto weights, leading to greater female labor supply or reduced male labor supply. My estimates also suggest that the assumption in past studies that bargaining power is divided by sex in different-sex couples may not significantly bias their results.

My current analysis is limited to dual-earner couples, and my results, therefore, cannot speak to the potential role of gender norms in labor force participation decisions. Ongoing work, however, includes extending the present analysis to include labor force participation decisions.

References


<table>
<thead>
<tr>
<th></th>
<th>Different-sex couples</th>
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<th>Same-sex couples</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Husbands</td>
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<td>Predicted secondary earners</td>
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<td>0.053</td>
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<td>(0.234)</td>
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<td>(0.203)</td>
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<td>Hispanic</td>
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<td>47.051</td>
<td>44.815</td>
<td>44.354</td>
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<td>-0.331</td>
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<td>(2.503)</td>
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<td>0.682</td>
<td>0.882</td>
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<td>5.550</td>
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<td>(1.655)</td>
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<td>(1.483)</td>
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<td>Observed higher earner</td>
<td>0.677</td>
<td>0.323</td>
<td>0.721</td>
<td>0.279</td>
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<td>(0.449)</td>
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<td>Reported annual earnings</td>
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<td>85,589.463</td>
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<td>(67,846.027)</td>
<td>(44,687.204)</td>
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<td>(86,018.753)</td>
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<td>Annual hours worked</td>
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<td>(641.171)</td>
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<td>(625.016)</td>
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<td>25.247</td>
<td>40.650</td>
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<td>(100.587)</td>
<td>(79.763)</td>
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<td>(46.876)</td>
<td>(45.505)</td>
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<td>Gross hourly wage gap</td>
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<td>8.024</td>
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<td>(4.741)</td>
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<td>(4.319)</td>
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<td>After-tax hourly wage</td>
<td>25.299</td>
<td>19.463</td>
<td>30.297</td>
<td>23.102</td>
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<td>(67.893)</td>
<td>(52.845)</td>
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<td>(33.354)</td>
<td>(33.643)</td>
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<td>Reported non-labor</td>
<td>4,923.150</td>
<td>4,923.150</td>
<td>5,614.467</td>
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<tr>
<td>income</td>
<td>(21,893.175)</td>
<td>(21,893.175)</td>
<td>(24,698.936)</td>
<td>(24,698.936)</td>
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<tr>
<td>Virtual income</td>
<td>1.520</td>
<td>1.520</td>
<td>1.877</td>
<td>1.877</td>
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<td>($10,000s)</td>
<td>(2.448)</td>
<td>(2.448)</td>
<td>(2.834)</td>
<td>(2.834)</td>
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<td>Observations</td>
<td>604,859</td>
<td>604,859</td>
<td>8,399</td>
<td>8,399</td>
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</table>

Notes: The data come from the 2012–2017 American Community Surveys and include different-sex and same-sex married, childless couples in which both spouses are working and between 25—60 years old.
| Outcome: Annual hours of work | Different-sex couples | | Same-sex couples | | | |
|-----------------------------|-----------------------|--|------------------|--|------------------|
| | Husbands | Wives | Predicted higher earners | Predicted lower earners | |
| log(partner 1 net wage) | -46.715*** (2.486) | 62.429*** (2.520) | 13.218 (21.587) | 41.884** (19.279) |
| log(partner 2 net wage) | -56.315*** (1.801) | -96.852*** (1.888) | -39.923*** (12.809) | -111.628*** (15.605) |
| Virtual income | 13.580*** (0.586) | 8.087*** (0.594) | 21.543** (8.367) | 21.407*** (5.529) |
| Education difference | -2.142*** (0.481) | -8.226*** (0.481) | -4.214 (3.550) | 8.968** (3.931) |
| Gross wage gap | 0.389 (0.294) | -0.363 (0.267) | 6.028*** (1.956) | -5.375*** (2.004) |
| Age | -0.709*** (0.136) | -2.445*** (0.123) | -2.517*** (0.925) | -0.767 (0.936) |
| Black | -123.863*** (4.496) | 9.777** (4.347) | -100.494** (40.188) | -94.417** (46.032) |
| Hispanic | -80.698*** (3.973) | -45.058*** (4.123) | -68.061** (32.343) | -30.294 (26.002) |
| Exactly HS education | 114.483*** (5.712) | 111.984*** (6.979) | 134.029** (66.643) | 61.713 (65.950) |
| Some college education | 148.324*** (5.999) | 136.598*** (7.177) | 101.747 (67.199) | 113.197* (66.613) |
| College education or more | 222.095*** (6.327) | 233.411*** (7.518) | 113.619* (67.272) | 191.786*** (69.886) |
| State recognizes same-sex marriage | 4.620 (4.008) | -4.113 (4.007) | -48.830 (35.480) | 48.970 (39.879) |
| Unemployment rate | -9.297*** (2.204) | -0.862 (2.203) | -5.434 (21.947) | -9.739 (22.838) |
| Observations | 604,859 | 604,859 | 8,399 | 8,399 |

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. All specifications also include year and state fixed effects. The data come from the 2012–2017 American Community Surveys.
Table 3: Sharing Rule Derivatives

<table>
<thead>
<tr>
<th>Derivative with respect to:</th>
<th>Different-sex couples</th>
<th>Same-sex couples</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(partner 1 net wage)</td>
<td>560.170***</td>
<td>-205.562</td>
</tr>
<tr>
<td></td>
<td>(159.531)</td>
<td>(178.756)</td>
</tr>
<tr>
<td>log(partner 2 net wage)</td>
<td>-2,521.868***</td>
<td>-546.856**</td>
</tr>
<tr>
<td></td>
<td>(196.387)</td>
<td>(245.777)</td>
</tr>
<tr>
<td>Virtual income</td>
<td>1.184***</td>
<td>0.682***</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.234)</td>
</tr>
<tr>
<td>Education difference</td>
<td>-1,867.225***</td>
<td>-1,333.496</td>
</tr>
<tr>
<td></td>
<td>(511.334)</td>
<td>(892.449)</td>
</tr>
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</table>

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. Section details the derivations of the sharing rule derivatives above from the unrestricted labor supply equations. The derivatives with respect to \( \omega_1 \) and \( \omega_2 \) are calculated at the mean values of these variables.
Table 4: Marshallian Labor Supply Parameters

\[
\begin{align*}
\lambda^1 &= \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 z \\
\lambda^2 &= \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 z
\end{align*}
\]

<table>
<thead>
<tr>
<th>Coefficient on:</th>
<th>Different-sex couples</th>
<th>Same-sex couples</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Husbands</td>
<td>Wives</td>
<td>Predicted higher earners</td>
<td>Predicted lower earners</td>
</tr>
<tr>
<td>log(own net wage)</td>
<td>-62.975***</td>
<td>119.366**</td>
<td>32.899</td>
<td>-196.591**</td>
</tr>
<tr>
<td></td>
<td>(4.555)</td>
<td>(48.980)</td>
<td>(29.850)</td>
<td>(82.443)</td>
</tr>
<tr>
<td>Share of virtual income</td>
<td>0.001***</td>
<td>-0.004***</td>
<td>0.003**</td>
<td>0.007</td>
</tr>
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<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Conditional Marshallian hours elasticity</td>
<td>-0.029***</td>
<td>0.064**</td>
<td>0.015</td>
<td>-0.102**</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.026)</td>
<td>(0.014)</td>
<td>(0.043)</td>
</tr>
</tbody>
</table>

Notes: * *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. Section 3 details the derivations of the structural parameters above from the unrestricted labor supply equations. The Marshallian hours elasticity is conditional on the share of unearned income.
A  Derivation of Sharing Rule Derivatives

Under the assumption that conditions 8a–8h hold for a given \( z \), the derivatives of the sharing rule are given by:

\[
\begin{align*}
\frac{\partial \phi}{\partial \eta} &= \frac{D}{D - C} \\
\frac{\partial \phi}{\partial s} &= CD \\
\frac{\partial \phi}{\partial \omega_1} &= \frac{BC}{D - C} \\
\frac{\partial \phi}{\partial \omega_2} &= AD
\end{align*}
\]

Recall the definitions of \( A, B, C, \) and \( D \) are:

\[
A = \frac{\partial L_1}{\partial \omega_2}, \quad B = \frac{\partial L_2}{\partial \omega_1}, \quad C = \frac{\partial L_1}{\partial s}, \quad D = \frac{\partial L_2}{\partial s},
\]

Under the function form in Equations 10 and 11, these values are:

\[
\begin{align*}
A &= \frac{a_2}{a_3\omega_2}, \quad B = \frac{b_1}{b_3\omega_1}, \quad C = \frac{a_4}{a_3}, \quad D = \frac{b_4}{b_3}
\end{align*}
\]

Note that the denominator of the sharing rule derivates are the same \( (D - C) \), which can be written:

\[
D - C = \frac{b_4}{b_3} - \frac{a_4}{a_3}\frac{a_3b_4}{a_3b_3} - \frac{a_4b_3}{a_3b_3} = \frac{a_4b_3 - a_4b_3}{a_3b_3} = \Delta
\]

Where \( \Delta \equiv a_3b_4 - a_4b_3 \). Using this expression for \( D - C \), the sharing rule derivatives can be written as:

\[
\begin{align*}
\frac{\partial \phi}{\partial \eta} &= \frac{D}{D - C} = \frac{b_4}{b_3} \frac{a_3b_4}{\Delta} = \frac{a_3b_4}{\Delta} \\
\frac{\partial \phi}{\partial s} &= CD = \frac{a_3b_4}{\Delta} = \frac{a_4b_4}{\Delta} \\
\frac{\partial \phi}{\partial \omega_1} &= \frac{BC}{D - C} = \frac{a_4b_1}{\omega_1\Delta} \\
\frac{\partial \phi}{\partial \omega_2} &= AD = \frac{a_2b_4}{\omega_2\Delta}
\end{align*}
\]
The above expressions are those in Equation 12, and solving this system of differential equations leads to the sharing rule in Equation 19.

B Derivation of the Marshallian Labor Supply Parameters

Recall that the sharing rule is:

$$\varphi = \frac{1}{\Delta} (a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(z)$$  \hspace{1cm} (19)

The Marshallian labor supplies take the following form:

$$\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 z$$

$$\lambda^2 = \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 z$$

Beginning with $\lambda^1$, let $\alpha_2 = \frac{\Delta}{b_4}$. Expanding the expression for $\varphi$, we obtain:

$$\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 z$$

$$= \alpha_1 \log \omega_1 + \frac{a_4 b_1}{b_4} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \frac{\Delta}{b_4} \kappa(z) + \alpha_3 z$$

$$= \left( \alpha_1 + \frac{a_4 b_1}{b_4} \right) \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \frac{\Delta}{b_4} \kappa(z) + \alpha_3 z$$

$$= a_1 + \frac{\Delta}{b_4} \kappa(z)$$

In order for $\lambda^1$ to be consistent with $L^1$, it must be the case that $\alpha_1 + \frac{a_4 b_1}{b_4} = a_1$, implying that $\alpha_1 = a_1 - \frac{a_4 b_1}{b_4} = \frac{a_3 b_4 - a_3 b_1}{b_4}$. 

A-2
Similarly, moving to $\lambda^2$, let $\beta_2 = -\frac{\Delta}{a_4}$. Expanding the expression for $\varphi$, we obtain:

$$
\lambda^2 = \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z}
$$

$$
= \beta_1 \log \omega_2 - \frac{\Delta}{a_4} \eta + b_1 \log \omega_1 + \frac{a_2 b_4}{a_4} \log \omega_2 + \frac{a_3 b_4}{a_4} \eta + b_4 s + \frac{\Delta}{a_4} \kappa(z) + \beta_3 \mathbf{z}
$$

$$
= b_1 \log \omega_1 + \left( \beta_1 + \frac{a_2 b_4}{a_4} \right) \log \omega_2 + \left( \frac{a_3 b_4 - a_3 b_4 + a_4 b_3}{a_4} \right) \eta + b_4 s + \frac{\Delta}{a_4} \kappa(z) + \beta_3 \mathbf{z}
$$

$$
= b_1 \log \omega_1 + \left( \beta_1 + \frac{a_2 b_4}{a_4} \right) \log \omega_2 + b_3 \eta + b_4 s + \frac{\Delta}{a_4} \kappa(z) + \beta_3 \mathbf{z}
$$

In order for $\lambda^2$ to be consistent with $L^2$, it must be the case that $\beta_1 + \frac{a_2 b_4}{a_4} = b_2$, implying that

$$
\beta_1 = b_2 - \frac{a_2 b_4}{a_4} = \frac{a_4 b_2 - a_3 b_3}{a_4}.
$$