Estimated Dynamic Industry Equilibrium Model with Firing Costs and Subcontracting

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January 31st, 2020

Abstract

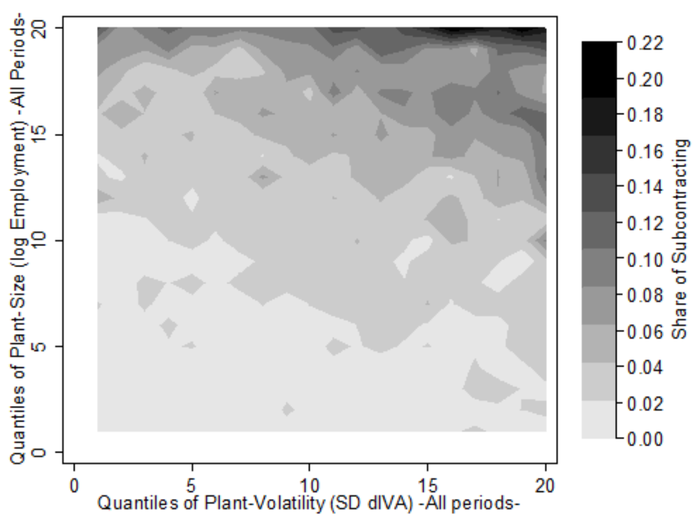
In Chile permanent workers are regulated by strict employment protection legislation. To regain flexibility this has triggered a widespread use of subcontracted workers. To assess the true costs of the regulation and of subcontracting, I estimate a dynamic industry equilibrium model in which firms optimally choose the division of labor between subcontracted workers that are totally flexible, and permanent workers that entail tenure-dependent firing costs. In the model, to overcome the potential costs of dismissing workers, firms hire permanent workers only up to the point where their expected firing costs equal the wage premium on subcontracted workers. Hence, firms' use of subcontracted workers is decreasing in their relative cost and increasing in the volatility of shocks. I use a simulated method of moments by fitting plant-level employment dynamics and the size distribution in the manufacturing sector in Chile. I find that severance payments are equivalent to seven monthly wages, and that workers get tenure after four years. Firms are willing to pay a "subcontracting wage premium" of 10% to substitute for hiring permanent workers. A naive researcher willing to estimate firing costs in this industry without considering the subcontracting margin of adjustment would conclude employment protection legislation is rather flexible. Allowing firms to use subcontracted workers in a heavily regulated environment increases output, employment and productivity; firms respond more aggressively to productivity shocks, which enhances the allocation of labor across firms and hence total factor productivity. Removing the regulation leads in steady state to an increase in average labor productivity around 1%. Restricting the use of subcontracting to improve working conditions would lead to a decrease in total output, employment and productivity.

KEYWORDS: employment protection legislation, subcontracted workers, labor market dynamics, composition of employment.

1 Introduction

In Chile permanent workers are regulated by strict employment protection legislation (EPL) that protects them against arbitrary actions and gives them higher job stability in the face of adverse economic conditions.[1] Even though these regulations can be welfare improving,[2] they raise firms' workforce adjustment costs, distorting the efficient allocation of labor within and across firms, while decreasing aggregate productivity.[3] To buffer temporary changes in employment needs employers have increasingly turned to nontraditional staffing arrangements, in particular, workers provided by contracting firms and temporary help work (a group I will refer to as 'subcontracting'). Subcontracting involves a specific type of contractual relationship: workers hired by a 'subcontract firm' are temporarily assigned to perform core production jobs at the customer's worksite ('user firm'), with machinery, inputs and raw materials provided by the user firm, creating a triangular relationship between the worker, the subcontract and the user firm. The user firm needs some level of sophistication (e.g. learn and adapt to the regulation of subcontracted workers) or installed capacity to be able to subcontract. Hence, manufacturing plants' use of subcontracted workers in Chile is increasing in plants' size and idiosyncratic volatility. Figure 1 plots Chilean manufacturing plants' use of subcontracted workers by plants' size and volatility in a contour diagram illustrating this fact. Consistent with the buffer motive, permanent workers' fluctuations are smoother and less frequent than adjustments in subcontracted workers.

Figure 1: Share of Subcontracted Workers by Plants' Size and Volatility



Note: this figure shows the relationship between the share of subcontracted workers and establishments' size and volatility. The establishments size is measured as the log of total employment, and volatility as the standard deviation of the annual growth rate of value added over a 5-year period.
Source: Author's calculations based on Micco and Munoz [2018].

The distortions created by the regulation have attracted attention in public policy circles. The common belief is that health and safety conditions of subcontracted workers are worse than for perma-

[1]Employment protection legislation usually regulates unfair dismissals, dismissals for economic reasons, mandatory severance payments, the use of fixed-term contracts, and minimum advance notice period in case of impending dismissal.

[2]Alvarez and Veracierto [2001] argue that in an economy with imperfect insurance markets and search frictions severance payments may be welfare improving. In a different vein, Feldstein [1976], and more recently, Blanchard and Tirole [2008] and Cahuc and Zylberberg [2008] claim that there is excess layoff of workers when employers have no incentives to internalize the social cost of their firing decisions.

[3]See, for instance, Alvarez and Veracierto [2001], Hopenhayn and Rogerson [1993], Samaniego [2006], Veracierto [2001], Poschke [2009].

nent workers,[4] and their excessive instability has an impact in productivity.[5] In this sense, many argue their use should be heavily regulated or at least limited to non-core production jobs. Despite these concerns, there is little work that formally models the interaction between these two policies when assessing the scope of EPL and evaluating its effects. In this paper, I propose and estimate a structural model of firms' dynamic in the tradition of Hopenhayn and Rogerson [1993] that endogenizes firms' division of labor between permanent and subcontracted workers. In the model, subcontracted workers are an equilibrium response of firms to introduce flexibility when permanent workers are subject to firing costs.

The model serves both a positive and a normative purpose. On the positive side, a structural model is needed to measure the true costs of the employment protection legislation and of subcontracting workers. It is not obvious how the regulation on permanent workers should be modeled given its complexity (discussed in Section 2). Firing costs increase with tenure on the job, but not every worker is entitled to severance payments when separated and in certain occasions depend on court's decision on case-by-case basis. The fact that the use of subcontracting depends on plants' size and volatility suggests that the costs of using subcontracting has both a variable and fixed component. On the normative side, what are the potential benefits of eliminating firing costs or restricting the use of subcontracted workers? The fact that the regulation increases firms' workforce adjustment costs, distorting the efficient allocation of labor within and across firms, suggests that productivity could be increased. As subcontracted workers restore some of the flexibility needed to better reallocate labor towards more productive firms, restricting their use might be worst. Because the model only considers a frictionless economy in which firing costs have no potential benefits but to distort the job reallocation process, I do not provide a full welfare analysis. I attempt to measure the steady-state employment, output and productivity cost of the regulations, and I abstract from transitional dynamics.

For this purpose, I estimate a structural model of firms' dynamic in the tradition of Hopenhayn and Rogerson [1993] with heterogeneous firms and endogenous entry and exit using a simulated method of moments. Firms can hire two types of workers: subcontracted workers that are totally flexible, and permanent workers that entail tenure dependent firing costs.[6] To overcome the potential costs associated with dismissing permanent workers, firms hire permanent workers only up to the point where their expected firing costs equal the wage premium on contracted workers. This way they buffer the regular workforce from economic fluctuations, smoothing the flow of inside workers at the expense of an increase in subcontracted employment volatility. Even when subcontractors' charges are higher than the firm's own production costs, high firing costs on permanent workers shifts labor demand towards more subcontracted workers.[7] This implies that firms' use of subcontracted workers is decreasing in their relative cost and increasing in demand volatility as the data suggests.[8]

For the estimation, I optimally choose the parameters of the model to reproduce a set of moments that combine time-series employment dynamics, and cross-sectional industry characteristics. By studying permanent and subcontracted workers dynamics, I am able to measure the costs for the firms of

[4]For instance, Weil [2014] reports that in the U.S. subcontracted worker in the mining sector have a 40% higher risk of a fatal work accident than regular workers.

[5]Temporary contracts generate a trade-off between flexibility and productivity gains as they reduce the workers' probability of receiving employer-paid training (Carpio et al., 2011; Jimeno and Toharia, 1996) and their effort when the probability of becoming a permanent workers is low [Dolado and Stucchi, 2008].

[6]In a new version of the paper I am currently working on, firms produce using permanent and subcontracted workers in a CES technology such that $y = z(an^{\gamma} + (1-a)s^{\gamma})^{\frac{\alpha}{\gamma}}$, where γ is the degree of substitutability of the two types of labor, $\alpha < 1$ returns to scale parameters, a share parameter, and z is firm's productivity. I relax the assumption that both workers are perfect substitutes in production. Also, I add the fixed cost of subcontracting. With this specification and new data on wages that is now available from ENIA, I am able to infer productivity differences between both types of workers.

[7]Most part of the literature has studied wage differences when outsourcing to business service providers takes place in areas not related to the firms' 'core competencies' (e.g. cleaning, security, logistics, human resources, or IT). In these cases, outsourcing potentially allows for reductions in wages for the contracted-out jobs [Goldschmidt and Schmieder, 2017].

[8]When temporary workers' charges are large enough and all permanent workers are subject to firing costs, the employment protection system studied in this paper reduces exactly to the separation tax regime in Hopenhayn and Rogerson (1993).

adjusting permanent workers, and the wage premium on subcontracted workers. I find severance payments in the manufacturing sector in Chile are equivalent to seven monthly wages, and that workers get tenure after 4 years in the job. Further, firms are willing to pay a rather large wage premium of 10% on subcontracted workers to substitute for hiring workers, and be able to buffer the regular workforce from economic fluctuations. A naive researcher willing to estimate firing costs in this industry without considering the subcontracting margin of adjustment would observe a rather flexible labor market (i.e. estimated firing costs between one and four months' wages, and that workers get tenure after 3 year on the job).

I embed the estimated model in a general equilibrium framework to quantify the aggregate costs of the regulation on permanent workers, and the potential benefits of removing it. Also, I measure the gains from subcontracted workers as a substitute for hiring workers when firms face strict job security regulations. Allowing firms to subcontract workers in a heavily regulated environment increases output, employment and productivity. To overcome the potential costs associated with dismissing permanent workers, firms subcontract to buffer the regular workforce from economic fluctuations. This way firms smooth out permanent employment fluctuations at the expense of an increase in subcontracted employment volatility. Provided subcontractors' charges are small relative to adjusting inside workers, subcontracting workers is an attractive alternative for the firms to cover peak demand or productivity shocks. When firms can subcontract they respond more aggressively to productivity shocks, which enhances the allocation of labor across firms and hence total factor productivity (TFP). In this context, the negative effects of firing costs on aggregate outcomes are less than previously estimated in the literature. If the government decided to eliminate firing costs instead of allowing subcontracting to introduce flexibility to the labor market, the increase in productivity and output of this policy would be even stronger. However, such a policy would eliminate subcontracted workers, being permanent workers the big winners of the change.

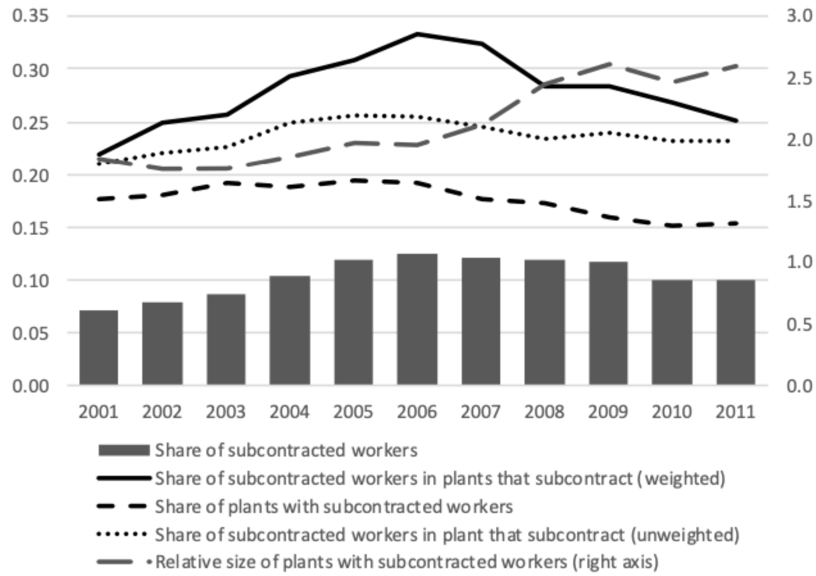
Chile provides a particularly interesting setting to investigate the interactions between firing costs and subcontracted workers. First, ENIA provides detailed establishment-level employment data independently on whether they are permanent or subcontracted workers. Even when the employer of record of the subcontracted worker is the 'subcontract firm', ENIA reports her as an 'employee' of the 'user firm' where she physically performs the core production job. Even though anecdotal evidence on the rapid increase of subcontracting, the economic literature is more limited because of the difficulties to measure in most data sets. ENIA contributes to overcome these data limitations. Second, in line with worldwide trends, subcontracting in Chile has exhibit a significant increase in the past decades.[9] As shown in Figure 2 the share of subcontracted workers in core production jobs almost doubled between 2001 and 2006 (7% to 13%).[10] Initially subcontracting emerged in routine- and low-skilled occupations, such as janitorial and security services, but rapidly became pervasive in key value-adding functions, such as logistics and accounting services, and high-skilled production-related occupations such as engineer and drafting services. In the intensive margin, the share of subcontracted workers in plants that subcontract reached 34% in 2006 up from 21% in 2001. This upward trend in the extensive and intensive margin broke in 2007 when a regulatory change (discussed in Section 2) increased the relative costs of subcontracting. The aim was precisely to curtail the rapid growth of subcontracting.

Related literature. This paper is related to a large body of literature that studies the impact of job security provisions on labor markets performance and aggregate productivity starting with the seminal work of Hopenhayn and Rogerson [1993]. While details differ somewhat, the key idea in this literature is that firing costs distort the allocation of labor across production units generating important losses in terms of aggregate TFP [Alvarez and Veracierto, 2001, Samaniego, 2006, Veracierto, 2001, Poschke,

[9] Katz and Krueger [2019] conduct their own survey for the U.S. economy to overcome data limitations based on the earlier Contingent Worker Survey and show a rise in the incidence of alternative work arrangements from 10.7% in 2005 to 15.8% in 2015. The fastest growing category of nonstandard work involves contracted workers that reached 3.1% in 2015 from 1.4% in 2005. Goldschmidt and Schmieder [2017] document a trend towards increasing reliance on domestic outsourcing for non-core production work in Germany in the past three decades.

[10] Core production jobs include consider engineering and drafting services, accounting services, blue-collar production, machine maintenance, storing and transport, and salesperson.

Figure 2: Share of Subcontracted Workers over Time



Note: The figure decomposes the aggregate share of subcontracted workers over time as the product of the weighted average of the share of subcontracted workers in plants that subcontract, the relative size of plants with subcontracted workers, and the share of plants with subcontracted workers.
Source: Micco and Munoz [2018].

2009]. Specifically, firing costs reduce labor reallocation from low-productivity firms (which should be shrinking by firing workers) towards high-productivity ones (which should be expanding by hiring workers) and prevents inefficient firms from exiting due to the large exit cost associated to firing the entire workforce. Hopenhayn and Rogerson [1993] show that empirically reasonable values for the tax on job destruction at the firm level -between 6 months' and 1 year's wages - could generate TFP losses on the order of about 2%. Relative to this literature, my contribution is to consider an alternative type of worker as a substitute for permanent workers when firms are subject to firing costs that decreases the costs of employment regulation.

There are several reasons why firms may choose to engage in subcontracting. They can serve as a flexible buffer for times of uncertainty or for demand fluctuations [Houseman, 2001, Houseman et al., 2003, Jahn and Bentzen, 2012], or can be used to circumvent employment protection regulation that makes labor adjustment costly [Boeri, 2011, Baumann et al., 2011, Autor, 2003]. Subcontracting can also allow employees to screen workers and as a springboard to a stable permanent job [Autor, 2001, Ichino et al., 2008, Jahn and Rosholm, 2013]. My paper belongs to the first strand of the literature and my contribution is to estimate a structural model of firms' dynamic in which subcontracted workers are an equilibrium response of firms to introduce flexibility when permanent workers are subject to firing costs. Using this structural model I estimate the true costs of the employment protection legislation and of subcontracting.

The studies closest in spirit to mine that focus on the interaction between labor protection policies and temporary workers are Alonso-Borrego et al. [2006], Veracierto [2007], Alvarez and Veracierto [2012] and Tejada [2017]. The main difference with this work is they assume search frictions and/or justify the use of temporary contracts exogenously (i.e. impose that all new jobs are temporary or model them as an exemption of firing costs and subsequently force firms to open permanent positions). They find that labor reforms that introduce temporary contracts allow firms to respond more aggressively to economic fluctuations, which enhances the allocation of labor and increases productivity. While they also produce an increase unemployment, the effects on welfare tend to be positive. To the best of my knowledge this is the first paper that estimates a structural model of firms' dynamic endogenizing firms' division of labor between permanent and subcontracted workers. The trade-off between both types of workers allows us to measure the true costs of the employment protection legislation and of subcontracted workers.

The remainder of this paper is organized as follows: Section 2 describes the data and presents evidence for the buffer motive. Section 3 describes the model economy, defines the equilibrium concept, and presents the calibration for the fixed parameters. Section 4 and the simulated method of moments for the estimation, and discusses the selection of moments. Section 5 shows the estimation results for different specifications of the benchmark model. Section 6 presents the results for the policy experiments in the general equilibrium framework, followed by the conclusion in section 7. The Appendix outlines the solution algorithm.

2 Motivating Evidence

I first describe briefly the institutional background, then present my data sources, and show some evidence of the buffer motive and the size effect for subcontracting.[11]

2.1 Institutional Background

The labor law regarding dismissals of permanent workers mandates a minimum period of advance notice (at least one month), regulates the causes considered as justified reasons, and the compensation payed to hired workers in case of dismissal for unjustified reasons.[12] The compensation amounts to one monthly wage per year of service with a maximum of eleven months. There is a penalty for firms that fail to prove just cause on court that ranges from 30 to 100% surcharge in the amount of the severance. Figure 3 shows Chile stands as one of the countries with the highest firing in OECD countries according to the economic cost of firing workers, computed as the equivalent in pay of the sum of the notice period, severance payment, and any other mandatory penalty directly related to the dismissal of the worker [Botero et al., 2004].

Instead, the use of subcontracted workers was completely liberalized during late-1970s: i) their use was extended to any occupation inside the firm; ii) every restriction preventing firms from using subcontracted workers for core-production jobs was eliminated; iii) any requirement to provide the same working conditions, salaries and social benefits to permanent and subcontracted workers was eliminated; iv) subcontracted workers were banned from joining a union in the user firm. In 2007 a labor law regulating the use of subcontracted workers was enacted with the aim of leveling the working conditions of permanent and subcontracted workers. This law called "Ley de Subcontratación" (Law No. 20,123) increased the *de facto* costs of subcontracting, making both parties -the user and the contract firm- jointly responsible for compliance with all employment liabilities (i.e. firing costs, unemployment insurance and social security contributions) of subcontracted workers at the user firms' worksite. However, the new regulation did not limit the possibility of subcontracting core-production jobs, and did not reinstate the requirement of providing the same working conditions to permanent and subcontracted workers.

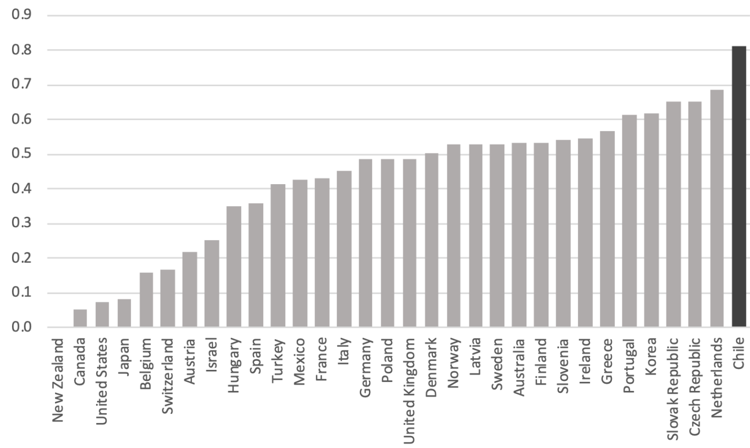
2.2 Data

The empirical analysis in the paper is performed using a panel of plant-level survey data from the Annual National Industrial Survey (hereinafter referred to by its Spanish acronym, "ENIA") collected by the National Institute of Statistics of Chile (INE). The survey encompasses all manufacturing

[11] For an assessment of the statistical significance of the buffer motive and the size effect see Micco and Munoz [2018].

[12] In 1990, firms' economic and financial needs were reclassified as just causes, but workers dismissed for these reasons were liable to severance pay.

Figure 3: Cost of Firing Workers in OECD Countries



Note: the cost of firing workers measures the cost of firing 20 percent of the firm's workers (10% are fired for redundancy and 10% without cause). The cost of firing a worker is calculated as the sum of the notice period, severance pay, and any mandatory penalties established by law or mandatory collective agreements for a worker with three years of tenure with the firm. If dismissal is illegal, the cost of firing is set equal to the annual wage. The new wage bill incorporates the normal wage of the remaining workers and the cost of firing workers. The cost of firing workers is computed as the ratio of the new wage bill to the old one.
Source: Botero et al. [2004].

establishments with at least 10 or more workers, and is updated annually incorporating all those plants that begin operating during the year plus the continuing plants, and excludes plants that stop operating or reduced their hiring below the survey's threshold. Each plant has a unique identification number which allows identification of entry and exit, and the computation of plant-level time-series.

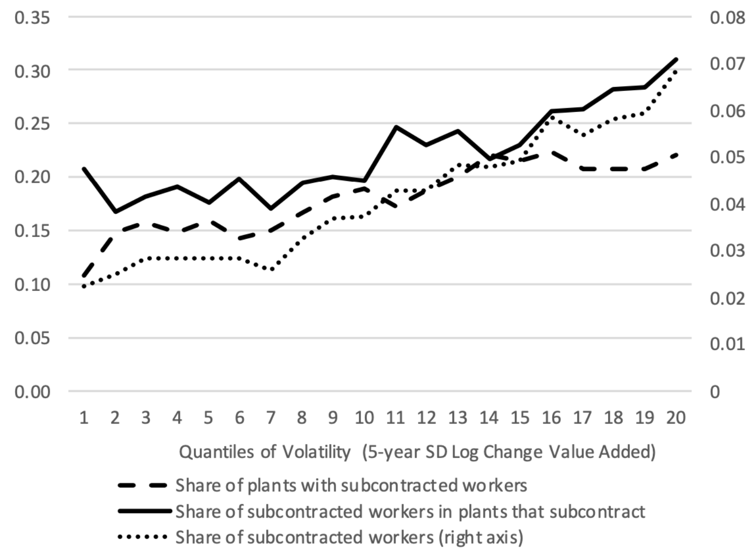
The dataset is available for the period 1996 to 2011, but panel-data information for subcontracted work is only available from 2001. Because of the regulatory change in 2007, I only use data until 2006. Plant-year observations are dropped if permanent employment is either zero or missing. I also excluded the tobacco industry and petroleum refineries from the analysis because they are organized as monopolies, operating with very few plants. This generates a sample of 10,906 plants and 69,938 observations with mean (median) employees of 72 (27). To ensure a reasonable sample size I run the estimation on the full panel, ignoring for now the specific industry to which the plants belong.

For each plant and year, the census collects detailed information on total number of employees, and asks whether there is contractual relationship between the establishment and the employee. Employers can hire workers under a permanent or open-ended contract, or subcontract to a third party the performance of a certain task or work with their own independent employees. ENIA defines subcontracted workers as "all personnel who do not have a direct contractual work-relationship with the firm and/or establishment but who perform administrative, service, sales, production or auxiliary activities for it". The survey reports plants' use of subcontracted workers in 6 different occupations: engineer and drafting services, blue-collar production, production assistant (i.e.machine maintenance, storage and transportation services), accounting services, blue-collar non-production (i.e. janitorial and secretarial services), and salesperson on commission.

2.3 Preliminary Data Analysis

Figure 4 plots Chilean manufacturing plants' use of subcontracted workers by manufacturing establishments' volatility pooling data for the entire period (2001-2011). Both in the intensive and extensive margin, the use of subcontracted workers (measured by their share of establishments' workforce) increases with plants' volatility. In the lowest quintile only 12% of the plants use subcontracted workers while in the largest quintile this figure increases to 22%. If I consider only the plants that subcontract, the share of subcontracted workers increases from 22% to 32% from the lowest to the largest quintile. This means firms exposed to higher volatility in idiosyncratic shocks, employ subcontracted workers in a larger proportion as they need to adjust labor more frequently and are more constraint by dismissal costs.[13]

Figure 4: Share of subcontracted workers by plants' volatility



Note: the share of subcontracted workers is the simple average of the share of subcontracted workers in each quantile. The share of plants with subcontracted workers is the fraction of plants with at least one subcontracted worker in each quantile. The share of subcontracted workers in plants that subcontract is the simple average of the share of subcontracted workers conditional on having at least one subcontracted worker.
Source: Author's calculations based on Micco and Munoz [2018].

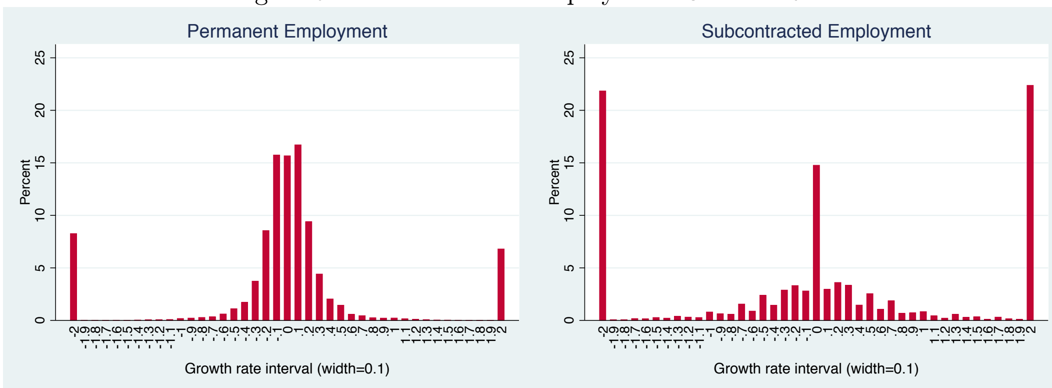
Consistent with subcontracted workers' role as a volatility buffer, permanent workers' fluctuations are smoother and less frequent than fluctuations in subcontracted workers. As observed in Figure 5, a large proportion of manufacturing plants that report mild or no changes in permanent employment per year coexists with many more plants adjusting subcontracted workers sharply. For instance, the share of plants changing permanent employment by less than 10% in a year is around 48%, whereas the share of plants adjusting subcontracting more than 50% is close to 62%.[14]

Figure 6 plots Chilean manufacturing plants' use of subcontracted workers by manufacturing plants' size. The share of plants with subcontracted workers monotonically increases with size-quantile suggesting the costs of subcontracting has a fixed-cost component: in the smallest quintile, 3% of the plants use subcontracted workers while in the largest quantile 45% of the plants subcontract. Once

[13]Micco and Pages [2006], Cingano et al. [2010] and Haltiwanger et al. [2014] show strict employment protection legislation constrains firms more strongly in sectors where idiosyncratic shocks have a larger variance.

[14]For subcontracted workers "exits" ($g_{it} = -2$) and "entries" ($g_{it} = +2$) do not necessarily correspond to plants that effectively entered or exit the market as is the case with permanent employment. These might be plants that start using subcontracted workers one period after not having employed any the period before (entry), and plants that fired all their subcontracted workers after having employed some the period before (exit). Further, these periods of sharp adjustment are usually followed by long periods of inaction.

Figure 5: Distribution of Employment Growth Rate



Note: the figure presents the fraction of plants expanding (contracting) at different growth rate intervals (as measured in the horizontal axis). Growth rate is computed according to the standard Davis and Haltiwanger [1992] definitions: $g_{it} = (x_{it} - x_{it-1})/(0.5 * (x_{it} + x_{it-1}))$, where x_{it} is the number of employees (subcontracted or permanent) in plant i at time t. The bars to the right of the origin correspond to job creation and to the left to job destruction. At the center, the proportion of plants for which employment remains unchanged, and exits (entries) correspond to the left (right) endpoint.

plants pay the fixed-cost to access to subcontracted workers the intensity with which they use subcontracted depends only on the wage costs. As seen in Figure 6, the relationship between size and use does not present a clear pattern when only plants that subcontracted are plotted.

3 Description of the Model

In this section I introduce the model for the estimation which is an industry equilibrium model in the tradition of Hopenhayn and Rogerson [1993] with heterogeneous firms and endogenous entry and exit, modified to include tenure dependent firing costs and two types of workers. For the estimation the model is set in partial equilibrium, and in Section 6, I embed it in a general equilibrium framework to perform the policy analysis.

I start by briefly motivating the elements in the theory:

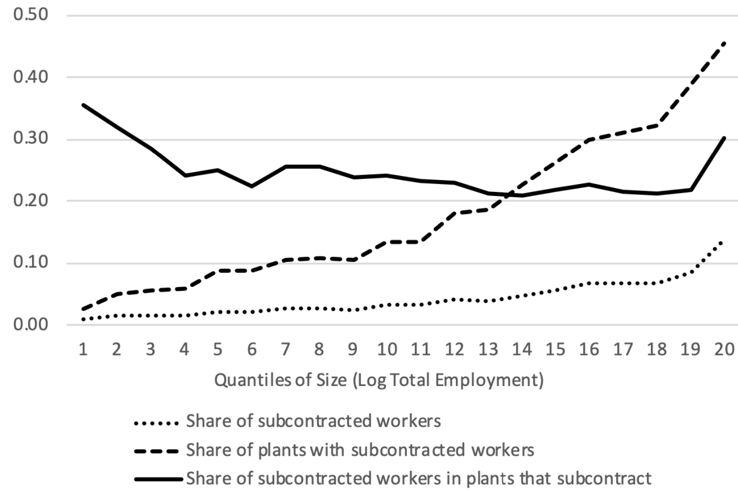
1. Firms produce output using two types of workers: subcontractors that are totally flexible, and permanent workers that entail firing costs that increase with seniority in the job. Both types are perfect substitutes in production, but permanent workers are relatively less expensive as subcontractors' charges are higher than the firm's own production costs. Firms decide the division of labor input between permanent and subcontracted labor as the optimal response to shocks;

2. I consider non-convex labor adjustment costs, in particular, piecewise linear adjustment costs. This specification can produce inaction and mimic the employment adjustment process: periods of sharp adjustment followed by long periods of inactivity;[15]

3. The regulation implies severance payments increase with seniority in the job in Chile. Instead of keeping track of the distribution of workers across tenure levels and increasing the dimension of the problem, I assume permanent workers randomly get tenure, and that only workers with tenure are entitled to severance payments;

[15] For more evidence of these patterns see Hamermesh [1989] and Caballero et al. [1997] for the U.S. For other countries, see for instance, Varejão and Portugal [2007].

Figure 6: Share of subcontracted workers by plants' size



Note: the share of subcontracted workers is the simple average of the share of subcontracted workers in each quantile. The share of plants with subcontracted workers is the fraction of plants with at least one subcontracted worker in each quantile. The share of subcontracted workers in plants that subcontract is the simple average of the share of subcontracted workers conditional on having at least one subcontracted worker.
Source: Author's calculations based on Micco and Munoz [2018].

4. There is a continuum of *ex ante* identical potential entrants, and selection occurs upon entry. Once firms enter the market they receive a random idiosyncratic productivity level, and they operate only if their first productivity draw is above the exit threshold. As the firm's productivity changes, it optimally chooses to grow, contract or exit the market. Since there are no aggregate shocks and the only source of uncertainty in the model is the firms' productivity, the distribution of firms over a size-productivity space is constant, and so all the aggregate variables.

3.1 Firms and Technology

There is an industry composed of a continuum of firms that produce an homogeneous good. Firms behave competitively taking prices in the output and labor markets as given. Each firm operates a decreasing returns to scale, labor-only production function, using both permanent and subcontracted workers:

$$y_t = f(n_t, s_t, z_t) = z_t(n_t + s_t)^{\alpha} \tag{1}$$

where n_t are the workers with a permanent contract, s_t the workers with subcontracts, $\alpha \in (0, 1)$, and z_t is the exogenous productivity that takes values in the finite set $Z \equiv \{\underline{z}, ..., \overline{z}\}$. The process for z_t follows a First Order Markov Process with transition matrix $\Pi(z, z')$ and is *i.i.d.* across firms. This implies there is no uncertainty at the aggregate level.[16]

The two types of workers are perfect substitutes in production, but they differ in their wages and firing costs:

1. Permanent workers are those with contracts of indefinite duration, and entail severance pay in case of dismissal. Permanent workers earn wage w. To avoid increasing the dimension of the

[16]These disturbances could also reflect shocks on the demand side, where firms produce differentiated goods and the distribution of consumer tastes across this differentiated goods is stochastic over time. See Hopenhayn and Rogerson [1993] for a more detailed description of this alternative structure.

problem and keeping track of the distribution of workers across tenure levels, I assume permanent workers have $(1-\lambda)$ probability of getting tenure, and only workers with tenure receive severance payments in case of dismissal. Workers with a permanent contract fired before tenure do not accrue severance pay.

Thus, workers with a permanent contract evolve:

$$n_t = l_{t-1} + o_t \tag{2}$$

where l_{t-1} is the number of permanent workers with tenure employed last period, and o_t is the number of workers hired or fired in t. The law of motion for permanent workers with tenure is:

$$l_t = \begin{cases} l_{t-1} + (1-\lambda)o_t, & \text{if } o_t > 0 \\ l_{t-1} + o_t, & \text{if } o_t \leq 0. \end{cases} \tag{3}$$

Since the optimal decision of current employment depends on the number of permanent workers last period, l_{t-1} is a state variable for the firm.

Firing costs on permanent workers with tenure take the following form:

$$g(l_t, l_{t-1}) = \max\{0, \tau\,(l_{t-1} - l_t)\} \tag{4}$$

where τ is the fixed payment for every permanent worker laid-off. In principle, labor adjustment costs can consider the search, recruiting and training cost of hiring workers, but since the interest falls on the effect of severance payments I choose to ignore hiring costs for now. This specification for labor adjustment costs imply the marginal cost of changing employment is constant; hence, when the gains to changing the number of workers is small firms optimally choose not to adjust– marginal costs of adjustment do not go to zero as the size of the adjustment goes to zero, and there is no reason for the firms to smooth adjustment. In this setting, firms' labor adjustments are characterized by episodes of sharp adjustment followed by periods of optimal inactivity.[17]

2. Contracted workers are those with temporary contracts subject to no costs for laying them off. In turn, they are relatively more expensive than permanent workers as their charges are higher than the firm's own production costs. Firms can employ contracted workers for occasional or seasonal purposes, or jobs for absent, as well as jobs for carrying out a specific task or service for a determined period of time related to the production process. The subcontract firm legally employs the worker (signs the contract and pays the wage w), which in turn works on the premises of the user firm who pays a *fee* per worker to the subcontract firm.[18] Hence, subcontracted workers earn $w_s = w(1+f)$, where f is the fee or wage premium on subcontracted labor. Provided the cost of contracting workers is small relative to the cost of adjusting in-house workers $[1-\tau(1-\lambda)]$, contracting out is an attractive alternative for the firms to cover peak demand or productivity shocks.[19]

The operative profits of an active plant are given by

$$py_t - wn_t - w_s s_t - pc_f - g(l_t, l_{t-1}) \tag{5}$$

The timing of the model for incumbents is as follows:

[17] In my treatment of firing costs, the analysis is related to the seminal paper by Hopenhayn and Rogerson [1993], and the more recent ones by Alvarez and Veracierto [2001], Veracierto [2001], Samaniego [2006] and Poschke [2009].

[18] Subcontracted workers may be restricted by law or mutual agreements between firms and unions, so that firms are obliged to hire a certain amount of employees on a permanent basis. For example, it could be assumed that the ratio between permanent and subcontracted workers can never fall below a minimum threshold $\bar{\psi}$. To remain faithful to the regulatory framework in Chile for the period under study, I assume no restrictions on subcontracted labor and no hiring cost.

[19] The premium over contracted workers could also be justified on the basis of a compensation contracted workers demand to work on the firm considering their higher expected probability of losing the job.

1. Enter period t with last period's shock z_{t-1} and permanent workers with tenure l_{t-1}
2. Decide whether to exit. If the firm exits, pays the adjustment costs $g(0, l_{t-1})$ for firing all workers from last period, and receives zero profits in all future periods avoiding to pay c_f.[20]
3. If the firm stays, it pays $p_t c_f$ and receives this period's shock, z_t
4. Firm chooses labor demand and the number of workers to hire under each type of contract.

The timing for a potential entrant:

1. Pay the one-time entry cost $p_t c_e$ and then draw a productivity level z_t from $\nu(z_0)$ (which is independent across firms)
2. Decide whether to stay in the industry. If the first productivity draw is above the exit threshold the firm stays and produces as in 4 above.

3.2 Static Subproblem of the Firm

For any plant with $z \in Z$ the optimal level of contracted workers solves the following static problem:

$$\begin{aligned} P(n,s,z) &= \max_s \{pz(n+s)^\alpha - n - w_s s - pc_f\} \\ st : s &\geq 0 \end{aligned} \tag{6}$$

Note that the wage rate for permanent employees has been normalized $w = 1$, hence does not appear explicitly in the expression.

The solution implies that the optimal subcontracted labor choice is:

$$s(n,z) = \begin{cases} \left(\frac{\alpha p z}{w_s}\right)^{\frac{1}{1-\alpha}} - n, & \text{if } \alpha p z n^{\alpha-1} > w_s \\ 0, & \text{if } \alpha p z n^{\alpha-1} < w_s \end{cases} \tag{7}$$

Subcontracted work will be hired up to the point where its marginal product equals wage. If the constraint is not binding, subcontracted labor is payed its marginal productivity $\alpha p z(n+s)^{\alpha-1} = w_s$ and firms optimally choose to operate with contracted labor, $s > 0$. If the constraint is binding, marginal productivity of subcontracted work is not enough to cover its cost $\alpha p z(n+s)^{\alpha-1} < w_s$ and firms operate without contracted workers, $s = 0$.

Then, evaluating the profit function $P(n,s,z)$ at the optimal contracted labor decision $s(n,z)$, the operating profit of the plant $R(n,z)$ is:

$$R(n,z) \equiv P(n, s(n,z), z) = \begin{cases} \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha p z}{w_s^\alpha}\right)^{\frac{1}{1-\alpha}} + n(w_s - 1) - pc_f, & \text{if } \alpha p z n^{\alpha-1} > w_s \\ pzn^\alpha - n - pc_f, & \text{if } \alpha p z n^{\alpha-1} < w_s. \end{cases} \tag{8}$$

3.3 Dynamic Optimization

Given that all uncertainty is idiosyncratic, I study a stationary equilibrium where $p_t = p$. In this equilibrium, firm undergo change over time, with some of them growing or contracting, even exiting the market and others starting up. Since there are no aggregate shocks, despite all these changes the distribution of firms over a size-productivity space is constant, and so all the aggregate variables.

[20] Fixed operating costs make the exit decision meaningful; plants exit to avoid paying the fixed cost instead of simply waiting for a better realization of z and bearing an output of zero.

3.3.1 Incumbent Firms

The dynamic programming problem of an incumbent plant that employed l_{t-1} permanent workers last period, decided to remain in the industry this current period, and received the new value for its shock z_t is described by the Bellman equation:

$$V(l_{t-1}, z_t; p) = \max_n \left\{ R(n_t, z_t; p) - g(l_t, l_{t-1}) + \beta \max[E_{z_{t+1}} V(l_t, z_{t+1}; p), -g(0, l_t)] \right\}, \tag{9}$$

subject to equation (2) and (3), and labor adjustment costs as defined in equation (4).

$E_{z_{t+1}}$ denotes the expectation of z_{t+1} conditional on the current value of productivity z_t, and β is the discount factor. The value $V(l_{t-1}, z_t; p)$ is the expected discounted stream of profits from operating a plant with productivity z_t and previous employment level l_{t-1}. Given that the firm does not receive any new information between the current decision point and the time of the exit decision at the beginning of next period, it chooses now whether to exit tomorrow. Conditional on this period's employment decision, the firm stays if the exit cost, $-g(0, l_t)$, is larger than the expected value of staying, $E_{z_{t+1}} V(l_t, z_{t+1}; p)$.

In this framework, there are two decisions of an incumbent firm:

1. optimal composition of total employment $n_t = L(l_{t-1}, z_t; p)$, and $s_t = S(n_t, z_t; p)$, and
2. optimal exit decision next period $x_{t+1} = X(l_t, z_t; p) \in \{0, 1\}$ with convention that $X = 1$ corresponds to exit and $X = 0$ to stay.

3.3.2 Entry Decision

The decision whether to open a plant is also dynamic. It is profitable to open a new plant if:

$$V^e(p) = \int V(0, z; p) d\nu(z) \leq pc_e, \tag{10}$$

where the value of of operating a new plant with productivity z_t and no previous employment, $l_{t-1} = 0$, is:

$$V(0, z_t; p) = \max_n \left\{ R(n_t, z_t; p) + \beta \max[E_{z_{t+1}} V(l_t, z_{t+1}; p), -g(0, l_t)] \right\}. \tag{11}$$

subject to equation (3), (2) and labor adjustment costs as in equation (4).

That is, new plants are open as long as the discounted expected profits from operating a new plant are enough to cover the entry costs. In equilibrium with positive entry, the entry of new plants induces changes in the output price and the firm value until there are no gains from entering this industry, and the constraint is satisfied with equality.

3.4 Stationary Distribution

In this model the state of an individual firm is fully described by (z, l), and the state of the industry in turn is described by the distribution over the state variables for all firms. Let the incumbent firms at the beginning of the period be summarized by the measure $\mu(z, l)$ (after they have made their exit/stay decision and new realizations of z have arrived), and the mass of firms that enter be equal to M.

The law of motion for the distribution of firms is given by

$$\mu'(z, l) = \int_{z'} \int_z [1 - X(l, z; p)] F(z'/z) d\mu(z, l) + \int_{z'} M' d\nu(z) \tag{12}$$

A stationary equilibrium is such that this distribution reproduces itself, i.e. $\mu' = \mu$.

The equilibrium distribution of productivity and permanent employment is determined by the productivity of entrants, the stochastic process of productivity, the extent of selection, and the number of entrants. Once the distribution of the state variables has been determined it is possible to compute all aggregate variables.

Total supply in the industry is:

$$Q^s(\mu, M; p) = \int_{z^*} f(L(l, z; p), S(n, z; p), z) d\mu(z, l) + M \int_{z^*} f(L(0, z; p), S(n, z; p), z) d\nu(z). \tag{13}$$

Aggregate demand for this industry follows a standard representation: $Q^d = D(P)$.

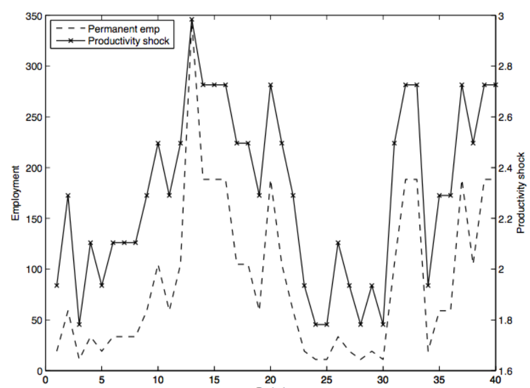
3.5 Definition of Equilibrium

A ***stationary industry equilibrium*** with positive entry and exit is a set of value functions and decision rules, a price p^*, a stationary distribution of firms μ^*, and a mass of entrants M^* such that:

1. Given prices, the value functions of the firms and the policy functions are consistent with firms optimization.
2. Markets clear: $p^* = D(Q^*)$ and $Q^* = Q^s(\mu^*, p^*, w^*)$.
3. There is an invariant distribution over firms: $\mu^* = T(\mu^*, M^*; p^*)$.
4. The free entry condition is satisfied: $V^e(p^*) = p^* c_e$.

Before going to the estimation of the model I discuss some properties of the labor policy function implied by the model. Starting with the model without firing costs ($\tau = 0$), subcontracted workers are meaningless in this setting as they are more expensive than permanent workers, but provide no advantage in terms of firing costs. Hence, firms choose permanent workers so that their marginal product equates the wage: $l_t = (\alpha p z_t / w)^{1/(1-\alpha)}$. To illustrate the firm optimal behavior, Figure 7 simulates the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock. It is clear that when current productivity is high, firms hire permanent workers, while if productivity is low they dismiss workers; current employment is determined entirely by the current value of the productivity shock.

Figure 7: Optimal Labor Decision: Model Without Firing Costs



Notes: the figure shows the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock when permanent workers do not entail firing costs. The parameters are given in Table 4 (Panel B, row 1, model with quick tenure) for $\tau = 0$.

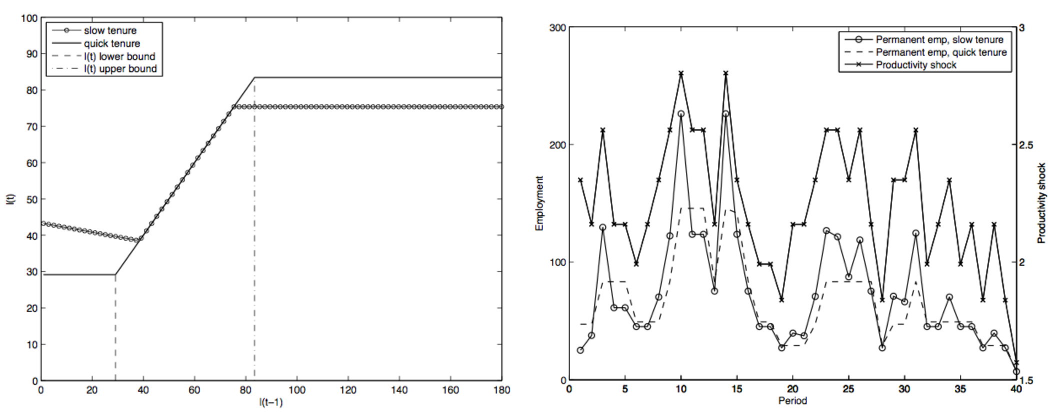
When the government introduces a positive firing cost, and no subcontracting is allowed yet, current employment also depends on last period's employment. In this setting, the optimal employment decision for permanent workers with tenure $l_t(z_t, l_{t-1})$ follows:

$$
\begin{aligned}
l_t(z_t, l_{t-1}) &= l_{t-1} \text{ if } l_{t-1} \in \left[\underline{l}(z_t), \bar{l}(z_t)\right] \\
l_t(z_t, l_{t-1}) &= \underline{l}(z_t) \text{ if } l_{t-1} < \underline{l}(z_t) \\
l_t(z_t, l_{t-1}) &= \bar{l}(z_t) \text{ if } l_{t-1} > \bar{l}(z_t),
\end{aligned}
\tag{14}
$$

where $\underline{l}(z_t)$ and $\bar{l}(z_t)$ are obtained from the first-order conditions of equation (9). Intuitively, $\underline{l}(z_t)$ is the largest amount of permanent workers a firm with productivity z_t wants to hire if it does not have to pay firing costs this period (i.e. is a firm that is expanding), and $\bar{l}(z_t)$ is the smallest amount of workers the firm hires if it has to pay firing costs this period (i.e. is a firm that is shrinking). For a firm with $l_{t-1} \in \left[\underline{l}(z_t), \bar{l}(z_t)\right]$, the gains from changing the number of workers is too small so they optimally choose not to adjust.

Figure 8 (left panel) illustrates this (s, S) type of rule for all workers with a permanent contract with quick tenure ($\lambda = 0$) and slow tenure ($\lambda > 0$).[21] All firms with employment last period below "l(t) lower bound" hire workers up to this lower bound, while all firms with employment levels above the "l(t) upper bound" reduce their employment levels down to this upper bound. Note also that the band is narrower when $\lambda > 0$; this is, when the firm hires permanent workers knowing that with probability $(1 - \lambda)$ they will actually get tenure.[22] The same figure, on the right, simulates the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock with quick tenure ($\lambda = 0$) and slow tenure ($\lambda > 0$). Consistent with the policy function, firms hire permanent workers only if the productivity shock is large enough, and I observe periods of sharp adjustment followed by long periods of inactivity. When $\lambda > 0$, the fact that not all workers get tenure gives the firm some flexibility to adjust employment to changes in productivity more often. Employment becomes more volatile in this case, and firms can use resources more efficiently.

Figure 8: Optimal Labor Decision: Model With Firing Costs and No Subcontracting



Notes: the figures illustrate the policy function for all workers with a permanent contract (on the left), and the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock when permanent workers entail firing costs (on the right). Two cases are plotted: quick tenure ($\lambda = 0$) and slow tenure ($\lambda > 0$). The parameters are given in Table 4 (Panel B, row 3, model with slow tenure).

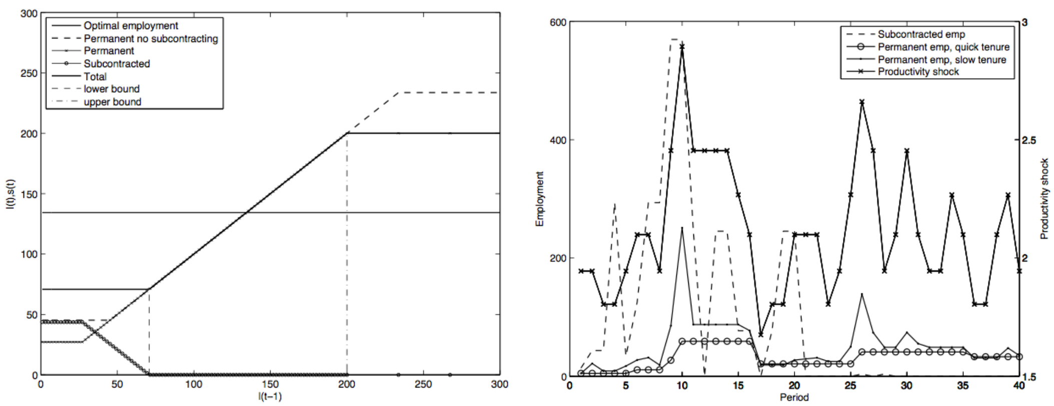
In the model economy with firing costs and subcontracted workers, firms use subcontracted workers to buffer the stock of permanent workers, and avoid their potential costs of dismissal during periods

[21] In the case when $\lambda = 0$, $n_t = l_t$ as there are no workers with permanent contracts that do not entail firing costs.

[22] The lower portion of the decision rule is downward slopping because smaller firms need to hire proportionally more permanent workers today to reach the "l(t) lower bound". Recall that when $\lambda > 0$, $n_t \neq l_t$.

of lower productivity. When the firm receives a positive shock, it responds by increasing the number of subcontracted workers. Only if the shock is large enough, the firms increase their hiring of permanent workers. In the case of a negative productivity shock, the firms start by firing as many contracted workers as possible, and when it runs out of contracted workers, starts firing permanent workers and bearing their dismissal costs (see Figure 9, right panel). Consistent with this dynamic, Figure 9 (left panel) illustrates the policy function for a firm subject to firing costs (quick tenure) and with the possibility to subcontract. When firms can subcontract, the "inaction band" narrows with respect to the case without subcontracting (compare the solid line labeled Total with the dashed line labeled Permanent no subcontracting) coming closer to reach the optimal level of employment without distortions. Hence, the extent to which resources are not allocated efficiently decreases. Also, the increase in employment up to the "lower bound", is attained by a combined increase of subcontracted and permanent workers. As explained before, firms begin subcontracting workers, and only if the productivity shock is large enough they increase their hiring of permanent workers.

Figure 9: Optimal Labor Decision: Model With Firing Costs and Subcontracting



Notes: the figures illustrate the policy function for all workers with a permanent contract and quick tenure (on the left), and the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock when permanent workers entail firing costs and plants can subcontract (on the right). The parameters are given in Table 3 (Panel B, row 3, model with slow tenure).

3.6 Solution Method

The model has no closed-form solution hence it is solved numerically. In the appendix I present a detailed characterization of the computation method used to solve the model.

The model period is one year. I assume firm's idiosyncratic shocks follow an AR(1) process of the form:

$$\log z_t = \mu + \rho \log z_{t-1} + \varepsilon_t \tag{15}$$

where μ is a constant, ρ the persistence of the shocks, and ε_t is a random variable with standard normal distribution. I approximate the distribution of the idiosyncratic shocks using the quadrature-based method developed in Rouwenhorst [1995], which has been shown to be more reliable in approximating highly persistent processes, and choose the number of grid points $g_z = 30$. The initial distribution $\nu(z_0)$ is chosen to be the stationary distribution of the z process which matches well the size distribution of the firms age 0-1 years in the data.

Industry demand is given by a decreasing function. For simplicity, take the following iso-elastic functional form:

$$p = Q^{-\frac{1}{\eta}}, \tag{16}$$

where p is output price, Q is the industry output, and $\eta > 0$ is the price elasticity of demand elasticity.

To discretize the state space for permanent employment I assign a log-linear grid with size $g_n = 300$. Because permanent employment n is an endogenous variable, I have to be careful that the choice of the number of points in the grid does not affect the results. Sensitivity analysis indicates the choice was adequate.

4 Estimation Method

In this section I propose a simple technique for the estimation of the model based on simulation, and the selection of moments that summarize key features of the data. Before I describe the data used for the estimations.

4.1 Simulated Method of Moments

Since the model has no analytical closed form solution I use an estimation technique based on simulation to estimate the parameters of the model. Specifically, the estimation of the parameters is achieved by simulated method of moments (SMM) ([McFadden, 1989, Pakes and Pollard, 1989, Duffie and Singleton, 1993]), which minimizes the distance between key moments from actual data and model-generated moments.

The full set of parameters necessary to compute the model is the vector:

$$\theta = \{\beta, \alpha, c_f, c_e, \rho, \mu, \sigma_\varepsilon, \tau, f, \lambda, \eta\} \tag{17}$$

where β is the discount rate, α the curvature of the production function, c_f is the fixed operating costs, c_e is the entry cost, ρ, μ, and σ_ε are the parameters that define the idiosyncratic shock, τ is the fixed cost the firm must pay for each permanent job destroyed, f is the wage premium on subcontracted workers, λ is the probability that a permanent workers gets tenure, and η is the price elasticity of demand. From the full set of parameters, 7 are estimated, and the remaining 3 are predefined.

To perform the SMM estimation a set of statistics of interest Ψ^A is selected from the actual data for the model to match. For an arbitrary value of θ, the solution to the model is used to generate S simulated data sets of size (N, T), where N is the number of firms and T is the number of periods.[23] The simulated moments $\Psi^S(\theta)$ are computed on each data set and then averaged out to compute the minimizing criterion function: $\Gamma(\theta) = [\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)]'\ W\ [\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)]$. I use the same random draw for the productivity shock throughout each simulation.

The parameter estimate $\hat{\theta}$ is obtained by searching over the parameter space to minimize the (weighted) distance between the moments implied by the model and those computed from the data:

$$\hat{\theta} = \arg\min_{\theta\in\Theta}[\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)]'\ W\ [\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)], \tag{18}$$

where W is a weighting matrix and Θ the estimated parameters space. $\hat{\theta}$ is consistent for any positive-definite weighting matrix (e.g. identity matrix) but the smallest asymptotic variance is obtained when the weighting matrix equals the inverse of the covariance matrix of the data moments, V. In this case, I use $W = \text{diag}(V^{-1})$ (diagonal elements equal to those of V and off-diagonal elements equal zero) because it has better small sample properties (see Altonji and Segal [1996]). V is calculated by bootstrap with replacement on the actual data.[24] To minimize the function I use Nelder-Mead simplex algorithm starting from $1,000$ different initial guesses to ensure the solution converges to the global minima.

[23] I set N=5,000 and T=200 which implies the number of firms in the simulation is approximately 10 times larger than in the data. I discard the first 50 periods of simulated data to start from the stationary distribution.

[24] To preserve the original time-series structure of the data to conduct inference I resample firm's complete time-series.

To generate the standard errors of the parameter point estimates, I compute the numerical derivatives of the simulated moments with respect to the parameters and using the standard SMM formula compute the asymptotic variance:[25]

$$SE(\hat{\theta}) = \left[(J'WJ)^{-1}\right]^{1/2}, \tag{19}$$

where $J = E(\partial\Psi^S(\theta)/\partial\theta)$ of dimension p (#moments) $\times$ q (#parameters). Given the underlying discontinuities of the value functions, I follow the methodology in Bloom [2009] to compute the numerical derivatives. I calculate the numerical derivative as $f'(x) = \frac{f(x+\varepsilon)-f(x)}{\varepsilon}$ for an ε of $\pm5\%$, $\pm2.5\%$, and $\pm1\%$ of the midpoint of the parameter space. Then, I simply compute the median value of these derivatives.

4.2 Predefined Parameters

The predefined parameters are shown in Table 1. Parameter β is set to be equal to 0.965, which is equivalent to annual real interest rate over the period of study of 3.62%. Because the curvature of the production function is difficult to identify in the data, I also set its value a priori. α not only captures the labor share in the total revenue, but also decreasing returns to scale and the elasticity of demand of firms' output. If capital is flexible, the elasticity of demand is infinite, and there is constant return to scale, then α should equal one. Relaxing any of these assumptions leads to an $\alpha < 1$ (See Gourio and Roys [2014]). I choose α equal to 0.85 so that for $\eta = 4$ the labor share is consistent with previous estimations for Chile.[26] The value of c_e is chosen so that the free-entry condition (10) holds under $p = 1$, and the wage rate of permanent workers is normalized to 1.

Table 1: Predefined Parameters in the Model

Parameter	Description	Value
β	Discount rate	0.965
α	Curvature production function	0.85
η	Price elasticity of industry demand	4

4.3 Selection of Moments

The choice of moments is guided by their "informativeness" regarding the underlying structural parameters to be estimated. In particular, the exact choice of moments is directed by a combination of cross-sectional industry characteristics and time-series employment dynamics. Heuristically, a moment is informative about a certain parameter if that moment varies when the parameter varies. Table 2 shows the elasticities of model moments with respect to the model parameters.

To pin down the fixed operating costs parameter I attempt to match the exit rate, the average firm size, and the firm size and employment distribution. An increase in fixed operating costs c_f increases the minimum level of productivity needed for incumbents firms to survive. This, in turn, intensifies market selection, and decreases entry barriers, resulting in a distribution of surviving firms with a larger proportion of high productivity establishments (see column (1) in Table 2). These same moments are also informative about the mean μ, persistence ρ and volatility σ_ε of the productivity process. An increase in μ or the volatility σ_ε, increase the exit rate, and decrease the average mean size of firms shifting the size distribution towards more small firms. Instead, the persistence parameter ρ increase the average size of firms and decreases the exit rate, shifting the size distribution towards more large plants (see columns (3), (4) and (5) in Table 2).

[25] See Gouriéroux and Monfort [1997].

[26] Estimations for the labor share parameter in Chile range from $0.53 - 0.6$. These estimates are somehow lower than those for the US economy because of a larger participation of natural resources in the GDP, and a low stock of human capital.

To study employment dynamics I use a modified definition of employment growth following Davis and Haltiwanger [1992]: $g_{it} = (x_{it} - x_{it-1})/(0.5 * (x_{it} + x_{it-1}))$, where x_{it} is the number of employees (subcontracted or permanent) in plant i at time t. This growth measure is symmetric about zero, and lies in the close interval $[-2, 2]$ with deaths (births) corresponding to the left (right) endpoints. The conventional growth rate measure (change in employment divided by lagged employment) does not allow for an integrated treatment of "exits" and "entries". However, a significant fraction of the adjustments in subcontracted employment corresponds to these cases so this information cannot be ignored; this is, plants that hire subcontracted workers this period after not having employed them the previous period ("entry"), and plants that cease to subcontract today after having hired subcontracted workers the previous period ("exit"), even when they still remain in operation. For consistency, growth in both types of employment is computed using this measure.

A key feature of the employment data is that permanent employment fluctuations are smoother and less frequent than fluctuations in subcontracted workers. It is transparent that the distribution of permanent employment growth rates is more peaked and with heavier tails, implying that there is a higher proportion of extreme events (even when sharp adjustments are still rare). Instead, the distribution of subcontracted employment growth rates indicates more smooth and persistent adjustment. Further, the permanent employment growth distribution has a considerable amount of mass around 0 (see Figure 5 in Section (2)). I select moments that describe these features of the distribution of both permanent and subcontracted growth rates; this is, volatility and kurtosis of the distribution, and inaction rate of permanent employment.

To pin down λ, τ and f I attempt to match the volatility and kurtosis of permanent and subcontracted employment growth, and the inaction rate of permanent employment growth. When τ increases, firms use more subcontracted workers as they rely more on these workers to buffer permanent employment from economic fluctuations. As a consequence, the volatility of permanent employment decreases, the inaction rate of employment growth increases, and the kurtosis increases (see column (7) in Table 2). In turn, when λ decreases (the probability of getting tenure for permanent workers increase), firms have to rely more on permanent workers increasing (decreasing) the volatility (kurtosis) of permanent workers growth rate (see column (2) in Table 2). Similarly, when the premium on subcontracted work f increases the volatility of subcontracted workers increases as firms use subcontract workers more infrequently (see column (6) in Table 2). The variance of permanent employment growth rate is informative about the mean, persistence and volatility of the productivity process.

Lastly, to complete the selection of moments I choose to match the proportion of subcontracted workers over the firm workforce as this is informative about the fixed lay-off cost τ (i.e. higher firing costs more subcontracting by the firms), the premium over subcontracted workers f (i.e. higher the premium less subcontracting), and λ (i.e. an decrease in the probability of getting tenure, decreases the adjustment costs of permanent employment, and the advantage of using subcontracted workers). Note also that the share of subcontracting is informative about the persistence (i.e. more persistent the risk decreases and firms use less subcontracted workers), and the volatility of the productivity process (i.e. an increase in the volatility increases the risk and firms rely more on subcontracted workers).

5 Empirical Results

In this section I present the estimates from the simulated method of moment. In Table 3, the column labeled Data reports the actual moments from ENIA, and next to it the associated standard errors. These show that permanent employment fluctuations are smoother and less frequent than fluctuations in subcontracted workers (the volatility of employment growth rate is more than two times for subcontracted work than for permanent work). Similarly, the higher kurtosis of the distribution of permanent employment growth rates indicates there is a higher proportion of extreme events, alongside long periods of no adjustments (the share of plants not changing permanent employment in a year is around 18%). Instead, the lower kurtosis of the distribution of subcontracted employment growth

Table 2: Sensitivity of Model Moments to Parameters

Moments	(1) c_f	(2) λ	(3) ρ	(4) μ	(5) σ_ε	(6) f	(7) τ
Average firm size	1.094	0.005	1.963	-0.201	-0.388	-0.207	-0.045
Exit rate	1.502	0.005	-7.903	0.736	1.550	0.003	-0.006
Fraction of plants in each bin:							
10-19 emp.	-1.225	-0.004	-2.240	0.073	0.253	-0.160	0.017
20-99 emp.	0.903	0.004	2.165	-0.055	-0.229	0.131	-0.006
100-499 emp.	1.219	-0.001	1.113	-0.029	-0.140	0.002	-0.004
500+ emp.	1.331	-0.001	0.933	-0.084	-0.166	-0.005	-0.001
Share of employment in each bin:							
10-19 emp.	-1.944	-0.033	-1.125	0.077	0.110	0.085	0.041
20-99 emp.	-0.127	0.029	1.276	-0.037	-0.218	0.216	-0.106
100-499 emp.	0.151	-0.008	-0.329	0.086	0.136	-0.064	0.032
500+ emp.	0.296	-0.008	-0.149	-0.001	0.051	-0.072	0.056
Volatility g_l	0.838	0.419	-4.293	0.378	0.842	0.056	-0.054
Volatility g_s	-0.456	0.117	-0.808	0.038	0.335	0.349	-0.330
Kurtosis g_l	-0.952	-0.241	5.196	-0.476	-1.095	-0.200	0.224
Kurtosis g_s	0.182	-0.057	1.617	-0.296	-0.553	-0.148	0.140
Share of subcontracting	0.406	-0.120	-4.916	0.304	0.796	-0.336	0.365
Inaction rate g_l	-0.302	0.000	1.648	-0.348	-0.464	-0.086	0.155

Note: this table presents elasticities of model moments with respect to the model parameters. To calculate the elasticities the numerical derivatives of the model moments with respect to the parameters are multiplied by the ratio of the baseline parameters to the baseline moments. The numerical derivative is the median value of the numerical derivatives $f'(x) = (f(x+\varepsilon) - f(x))/\varepsilon$ for an ε of $\pm5\%$, $\pm2.5\%$, and $\pm1\%$ of the midpoint of the parameter space.

rates indicates more smooth and persistent adjustments.[27]

The column labeled Slow Tenure in Table 3 presents the moments from the full model ('benchmark model') as presented in Section 3 evaluated at the estimated parameters. The model fits the data quite well with the exception of the kurtosis of both permanent and subcontracted employment distribution, and the inaction rate for g_l. The fact that the model cannot match these facts suggests the need to incorporate some restriction on the degree of substitutability between both types of labor, or some fixed cost to the use of subcontracted workers. Given that both types of labor are perfect substitutes in production, firms rely more on subcontracted workers, and adjustments of permanent employment are neither as frequent not as sharp as in the data. The fact that the volatility of subcontracted employment growth given by g_s fits well the data is also related to the fact that the model fits relatively high firing costs. In terms of fitting industry characteristics such as firm and employment distribution, the yearly exit rate and the average firm size the model performs well.

In Table 3 I also display the results for the model restricted to $\lambda = 0$; this is, to the case permanent workers gets tenure immediately after they are hired. As shown by the increase in the criterion function (from 1,342.5 to 5,265.9), in comparison to the full model the fit is worst. The reduction in fit is due both to the worst fit of firms and employment dynamics, suggesting that ignoring the tenure-dependency of firing costs is problematic. Given the cost of subcontracted workers, and the

[27] Even when it seems that the distribution of subcontracted employment growth rates would have the most kurtosis (it appears to have all of its mass in its tails as seen in Figure 5 in 2), being its variance a lot larger in fact it only has few mass in its tails. Instead, even when the distribution of permanent employment growth rates seems to have fewer mass in its tails, its kurtosis is larger because those events are much farther away from the mean.

proportion in which the plants use subcontracted workers, for the model to fit the low inaction rate for permanent workers it requires a rather low τ. In the benchmark model, much of the flexibility in employment adjustment is coming from the fact that only a fraction of workers get tenure, and not only from subcontracting. The low level of firing costs, in turn, produces an excessive volatility of g_l, and an even lower kurtosis of the distribution of permanent employment growth rates.

Table 3: Simulated Moments Estimations for the Full Model

Panel A: Moments

Moments	Data	S.E.	Simulated Moments	
			Slow Tenure	Quick Tenure
			($\lambda > 0$)	($\lambda = 0$)
Average firm size	71.95	1.8782	71.53	64.79
Exit rate	0.091	0.0012	0.098	0.135
Fraction of plants in each bin:				
10-19 employees	0.386	0.0049	0.398	0.457
20-99 employees	0.447	0.0049	0.436	0.407
100-499 employees	0.145	0.0038	0.148	0.121
More than 500 employees	0.022	0.0016	0.018	0.016
Share of employment in each bin:				
10-19 employees	0.064	0.0021	0.062	0.084
20-99 employees	0.260	0.0081	0.264	0.296
100-499 employees	0.417	0.0118	0.398	0.371
More than 500 employees	0.260	0.0173	0.275	0.249
Volatility g_l	0.688	0.0160	0.781	0.818
Volatility g_s	2.161	0.0618	2.118	2.519
Kurtosis g_l	5.144	0.0606	3.141	2.689
Kurtosis g_s	1.973	0.0273	1.645	1.704
Inaction rate g_l	0.181	0.0026	0.231	0.175
Share of subcontracting	0.247	0.0053	0.253	0.278
Criterion, $\Gamma(\theta)$			1,342.52	5,265.9

Panel B: Parameter Estimates

	c_f	λ	ρ	μ	σ_ε	f	τ
Quick tenure	4.807	-	0.903	0.023	0.139	0.095	0.160
($\lambda = 0$)	(0.0353)	-	(0.0197)	(0.0047)	(0.0198)	(0.0027)	(0.0421)
Slow tenure	6.384	0.758	0.913	0.029	0.129	0.101	0.593
	(0.0403)	(0.0284)	(0.0113)	(0.0025)	(0.0121)	(0.0025)	(0.0284)

Notes: Panel A reports the targeted moments and their corresponding standard errors, and the simulated moments evaluated at the estimated parameters. The bottom table reports the parameters' point estimates and their standard errors in parenthesis.

Panel B of Table 3 contains the point estimates of the parameters for both models with the associated standard errors. In the benchmark model with slow tenure, estimated firing costs are equivalent to seven months' wages, and on average workers get tenure after 4 years in the job. In terms of the wage premium on subcontracted workers, the model estimates are consistent with the data for manufacturing plants in ENIA. On average, subcontracted workers earned 8% more than permanent workers in the period 2001-2007.[28] Finally, shocks to productivity are estimated to be 14% per year, the mean growth rate of productivity 2.3% and the persistence of idiosyncratic shocks 0.903. As mentioned, for the model with quick tenure to fit well the relative flexibility of permanent employment as observed in the data moments, it requires firing costs that are substantially lower (only two months' wages). Consistent with the estimations for the benchmark model, the wage premium on subcontracted workers remains around 10%, and the rest of the parameters summarizing firm dynamics are also relatively stable.

For interpretation, Table 4 presents estimations for two additional restricted models. First, a model without subcontracting, and a positive probability of not getting tenure in the column labeled Slow tenure. I see the fit of the model is slightly worse in comparison to the benchmark case in spite of the reduction in the number of moments to fit. In terms of employment dynamics, the model also has problem fitting the volatility and the kurtosis of g_l when the inaction rate is too low as observed in the data. In the column labeled Quick tenure I present the estimates of a model that also restricts subcontracting, but assumes all workers get tenure. Note that this specification of the model is the same model as in Hopenhayn and Rogerson [1993]. For this model to fit such a low inaction rate for permanent employment growth rate is even more problematic. Panel B in Table 4 displays the point estimates of the parameters for both models with the associated standard errors. In the model with quick tenure, estimated firing costs are equivalent to one month' wages, while in the model with slow tenure they increase to four months' wages, as workers get tenure on average after 3 years on the job.

In conclusion, a naive economist that estimates firing costs from these data moments ignoring firms subcontract to substitute permanent workers would arrive to the conclusion that firing costs are rather low in this industry. However, the results from the benchmark model show they are rather high, and the flexibility observed in the data comes from subcontracted workers being used as an adjustment margin for firms to accommodate economic shocks.

6 Policy Experiments

In this section, I extend the partial-equilibrium model to a general equilibrium framework, and using the parameters' estimates I carry out several policy analysis. I use the estimations for the four models to analyze the implementation of two alternative labor market reforms: first, the elimination of subcontracted workers and, second, the reduction of firing costs to zero when suitable. This experiment is relevant in light of the debate that pits workers' demands to limit the use of subcontracting as a way to improve their working conditions, with those of the business community that advocate a reduction in firing costs. Finally, it is important to clarify that the model is not appropriate for welfare analysis as it only considers a frictionless economy in which firing costs have no potential benefits, but to distort the job reallocation process. The equilibrium allocation without government intervention is Pareto optimal, hence there is no space for improvement coming from firing costs.[29]

6.1 General Equilibrium Model

The economy is populated by a continuum of identical two member households: workers that supply labor under a permanent contract and subcontracted workers. Each household has preferences defined

[28] The wage of subcontracted (permanent) workers is computed as the total wage paid by the establishment to all subcontracted (permanent) workers divided by the number of subcontracted (permanent) workers employed by the establishment in that same period. The results are robust to the inclusion of bonuses on permanent workers' wages. The widespread perception that subcontracted jobs pay substantially less than permanent ones is largely contaminated by the decline in relative wages of low-skilled workers, and low-skilled jobs are still subcontracted in a larger proportion than permanent ones.

[29] See, for example, Alvarez and Veracierto [2001] and **?** who analyze the impact of firing costs in an economy with imperfect insurance markets and search frictions.

Table 4: Simulated Moments Estimations for the Model Without Subcontracting

Panel A: Moments

Moments	Data	S.E.	Simulated Moments Slow tenure ($\lambda > 0$)	Quick tenure ($\lambda = 0$)
Average firm size	66.76	1.7310	67.97	78.71
Exit rate	0.091	0.0012	0.100	0.113
Fraction of plants in each bin:				
10-19 employees	0.402	0.0049	0.418	0.321
20-99 employees	0.440	0.0049	0.434	0.482
100-499 employees	0.139	0.0038	0.130	0.173
More than 500 employees	0.019	0.0015	0.018	0.024
Share of employment in each bin:				
10-19 employees	0.071	0.0023	0.076	0.057
20-99 employees	0.272	0.0084	0.283	0.275
100-499 employees	0.423	0.0121	0.368	0.398
More than 500 employees	0.234	0.0177	0.274	0.270
Volatility g_l	0.688	0.0160	0.833	0.806
Kurtosis g_l	5.144	0.0606	3.035	2.834
Inaction rate g_l	0.181	0.0026	0.153	0.244
Criterion, $\Gamma(\theta)$			1,524.4	2,937.9

Panel B: Parameter Estimates

	c_f	λ	ρ	μ	σ_ε	f	τ
Quick tenure	7.756	-	0.871	0.048	0.144	-	0.133
($\lambda = 0$)	(0.0263)	-	(0.0092)	(0.0032)	(0.0068)	-	(0.0048)
Slow tenure	5.654	0.684	0.915	0.016	0.133	-	0.285
	(0.0546)	(0.0234)	(0.0283)	(0.0017)	(0.0247)	-	(0.0268)

Notes: Panel A reports the targeted moments and their corresponding standard errors, and the simulated moments evaluated at the estimated parameters. The bottom table reports the parameters' point estimates and their standard errors in parenthesis.

over consumption and labor supply given by:

$$\sum_{t=1}^{\infty} \beta_t [\log(c_t) - B\frac{n_t^{1+\phi}}{1+\phi}], \tag{20}$$

where $c_t > 0$ is total consumption, and n_t is labor effort. Parameters B and ϕ represent preferences for leisure, and the inverse of the Frisch elasticity of labor supply, respectively. Households take all of the income from all of the workers, and allocate it to the individuals within the household. Also, they allocate total hours worked independent of which workers performs the effort.

The output price is normalized to one, and the households supplies labor to the market at the wage $w = -u_n/u_c = Bcn^{\phi}$. As before, both members of the household are perfect substitutes in production, but permanent workers are relatively less expensive as subcontractors' charges are higher than the firm's own production costs. Subcontract firms incur in a real cost for "creating" subcontracted workers, and the premium they charge to the main firm per worker is just enough to cover the real cost c so that their profits are zero: $\pi(s_t) = (w_t^s - c)s_t = 0$. The cost for firms to subcontract a worker is $w_t^s = w_t(1 + f)$. I consider a stationary equilibrium, so all prices and aggregates in the economy are constant, and household maximization implies the interest rate satisfies $1/(1 + r) = \beta$.

An individual firm that employed l_{t-1} permanent workers last period and draws a productivity shock z_t this period has expected adjustment costs given by:

$$r(l_{t-1}, z_t; w) = [1 - X(l_t, z_t; w)] \int g(n_{t+1}, l_t) dF(z_{t+1}, z_t) + X(l_t, z_t; w) g(0, l_t), \tag{21}$$

where $n_{t+1} = L(l_t, z_{t+1}; w)$. Integration yields aggregate adjustment costs given by $R(\mu, M; w)$. I assume proceeds from the regulation are rebated uniformly to all households as a lump-sum payment to households by the government. In fact, severance payments make up for the largest part of firing costs in Chile, and are paid entirely to the workers when they are fired. Aggregate adjustment costs do not appear in the resource constraint as they appear in both sides of the equation.

The demand curve in Section 3 is replaced by the resource constraint:

$$C = Y - Mc_e - F \tag{22}$$

where output is given by:

$$Y = \int_{z^*} [f(L(l, z; p), S(n, z; p), z) - c_f] d\mu(z, l) + M \int_{z^*} f(L(0, z; p), S(n, z; p), z) d\nu(z), \tag{23}$$

and the fees paid by the firms for the subcontracted workers are given by:

$$F = fw \left[\int_{z^*} S(n, z; w) d\mu(z, l) + M \int_{z^*} S(n, z; w) d\nu(z) \right]. \tag{24}$$

Finally, the clearing condition for the labor market is given by:

$$N^s(\mu, M; w) = \int_{z^*} [L(l, z; w) + S(n, z; w)] d\mu(z, l) + M \int_{z^*} [L(0, z; w) + S(n, z; w)] d\nu(z) \tag{25}$$

A ***stationary industry equilibrium*** with positive entry and exit is a set of value functions and decision rules, a wage w^*, a stationary distribution of firms μ^*, and a mass of entrants M^* such that:

1. Given prices, the value functions of the firms and the policy functions are consistent with firms optimization.
2. There is an invariant distribution over firms: $\mu^* = T(\mu^*, M^*; w^*)$.
3. The resource constraint (equation 22) and the labor market clearing conditions (equation 25) are satisfied.
4. The free entry (equation 10) is satisfied.

6.2 Results

In this section I present the results for the policy analysis. Few things to consider before presenting the results: first, I only compare steady-state values, and do not discuss the transitional dynamics. Second, I need to parametrize labor supply preferences: I set the elasticity of labor $\phi = 0.84$ (see Medina and Soto [2007] for estimations for Chile), and $B = 11.62$ so that total employment is 0.25.

6.2.1 Aggregate outcomes, prices and labor market

Table 5 reports the steady-effects of reducing firing costs in the four estimated models. The column label Full model/Slow tenure shows the effect of reducing firing costs in the benchmark model. Output goes up 3.54% when firing costs are eliminated, both due to an increase in productivity (+1.02%) coming from the better allocation of resources, and in total employment (+2.49%). The increase in permanent workers is even larger, as all the jobs previously assigned to subcontracted workers are

Table 5: Steady-State Effects of Eliminating Firing Costs

	Full model		No subcontracting	
	Quick tenure	Slow tenure	Quick tenure	Slow tenure
Output	2.87	3.54	4.20	2.24
Consumption	2.81	3.59	2.90	2.28
Average labor productivity	0.88	1.02	2.49	0.32
Total employment	1.97	2.49	1.67	1.92
Permanent	3.09	3.73	1.67	1.92
Wage permanent workers	4.51	5.75	4.34	3.92
Layoff costs/wage bill (before)	0.036	0.061	0.034	0.040
Subcontracting costs/wage bill (before)	0.087	0.092	-	-

Note: the table reports the steady-state percentage change if the firing costs are eliminated starting from each of the different estimated models.

reallocated to workers inside the firm. In the absence of firing costs, the wage of permanent workers goes up 5.75% as the distortions coming from firing costs disappear.

One of the main findings of the paper comes from the comparison of the effect of reducing firing costs between my benchmark model and the model without subcontracting/quick tenure. The column labeled No subcontracting/quick tenure in Table 5 presents the effect of reducing firing costs in a model that is equivalent to the framework in Hopenhayn and Rogerson [1993]. In this case, eliminating the regulation has also a positive impact on output, labor productivity and employment, though the effect is substantially larger in comparison to the effect of the same reform applied to my benchmark economy. In particular, the effect is larger on labor productivity, as subcontracting workers firms circumvent the regulation, and improve the allocation of labor in heavily regulated environments. The allocation of resources, therefore, in an economy where firms cannot subcontract is more inefficient, and the benefits of removing the regulation are larger. Even when firing costs (as a percentage of the wage bill of permanent workers) are substantially larger in the benchmark economy (i.e. 6.1 versus 3.4%) the firms in this economy use resources more efficiently, and better allocate labor due to the presence of subcontracted workers.

For completeness, the table also presents the results of removing the regulation in the benchmark economy, but when permanent workers get tenure quickly, and in the model without subcontracting when permanent workers slowly get tenure. The results are still consistent with the fact that firms manage risk better in the presence of subcontracted workers, as they buffer the regular workforce from economic fluctuations to avoid workers' firing costs by subcontracting workers.

Table 6 shows the results from the comparison between the benchmark economy (with quick and slow tenure), and the new stationary equilibrium associated with eliminating subcontracted workers.[30] The results show that output, employment and productivity go down when subcontracted workers are prohibited, as this change eliminates a margin that firms exploit to adjust to productivity shocks; firms fire subcontracted workers as a response to a negative shocks without paying firing costs. Instead, in the model without subcontracted workers, firms are forced to smooth their employment level over time to reduce firing costs. In the benchmark model, the lower output comes more from a decrease in the number of workers than from a decrease in average labor productivity. Instead, in the model with quick tenure the effects comes from a slow down in the reallocation of workers, and a decrease in productivity, and not so much from a decrease in employment. Firms in the economy with slow tenure use resources more efficiently, and already allocate employment better (i.e. subcontracted costs/wage bill are 0.092% in the economy with slow tenure versus 0.087% in the quick tenure economy). These

[30] In the model, to eliminate subcontracted workers I assume the fee charged by the subcontract firm becomes sufficiently high.

results come against the common view that subcontracted jobs are of lower quality, and that they decrease productivity. I see that the winners from this policy are permanent workers which increase in their hiring.

Table 6: Steady-State Effects of Eliminating Subcontracting

	Quick tenure	Slow tenure
Output	-0.146	-0.082
Average labor productivity	-0.095	-0.020
Mass of firms	-0.620	-0.227
Layoff costs/wage bill	1.738	0.674
Total employment	-0.051	-0.062
permanent	1.073	1.150
Wage permanent workers	-0.058	-0.018
Layoff costs/wage bill		
before	0.036	0.061
after	0.037	0.062
Subcontracting costs/wage bill	0.087	0.092

Note: the table reports the steady-state percentage change if subcontracted work was eliminated from both of models or, equivalently, if the wage premium on subcontracted workers was prohibitively high.

Row 5 reports the change in the wage of permanent workers in both models. When subcontracted workers are eliminated, there is a decrease in the wage of permanent workers coming from the increase in the number of permanent workers which lowers average labor productivity. As productivity decreases a lot more in the model with quick tenure, the effect on wages is also substantially larger. This lower wage compensates firms for the higher average adjustment cost of labor (i.e layoff costs/wage bill increase 1.74 and 0.67%).

7 Conclusion

In this paper, I analyze the effect of firing costs on aggregate outcomes when firms can circumvent the regulation subcontracting as a substitute for hiring full-time workers. In countries with strict job security regulations firms use flexible staffing arrangements to buffer the regular workforce from economic fluctuations and avoid workers' firing costs. I set up an industry equilibrium model in the tradition of Hopenhayn and Rogerson [1993] with heterogeneous firms and endogenous entry and exit, where firms can hire two types of workers: permanent workers that entail random firing costs, and subcontractors that are totally flexible, but carry a wage premium above the compensation permanent workers demand.

The results for the model estimations show that to match plant-level employment dynamics in the manufacturing sector in Chile subcontracted workers are needed. Put differently, a model that ignores this adjustment margin yields firing costs that are too low and very much at odds with empirical data. In the model with subcontracted workers firing costs are equivalent to seven monthly wages, and permanent workers get tenure after approximately 4 years in the job. Firms, in this framework, are willing to pay a rather large wage premium on subcontracted workers to be able to substitute for hiring workers (10%). Instead, in the model without subcontracted workers, firing costs are equivalent to only one monthly wage.

These findings are consistent with the results from the policy experiments which show that allowing firms to subcontract workers increases output, employment and productivity. Subcontracted workers allow firms to respond more aggressively to productivity shocks, which enhances the allocation of labor across firms and hence total factor productivity (TFP). Further, when firms can subcontract, the negative effects of firing costs in aggregate outcomes are less than previously estimated in the literature.

A Appendix

A.1 Solution algorithm

A.1.1 Partial equilibrium model

In this section, I present the solution algorithm for the partial-equilibrium model. Basically, the algorithm consists of two steps: 1) find the unique price p^* that is consistent with the free entry condition; 2) second, find the fixed point of .

Step 1 Iterate over p_i until the entry condition is satisfied at p^*:

[(a)]

1. For each p_i, compute $V_i(l, z; p_i)$ and $V_i(0, z; p_i)$
2. Let $EC(p_i) \equiv \int V(0, z; p)d\nu(z)/p_i - c_e$. If $EC(p_i) > 0$, then set $p_{i+1} < p_i$, otherwise set $p_{i+1} > p_i$.

Step 2 Iterate over (μ_i, M_i) until $Q^d = Q^s$ at (μ^*, M^*):

[(a)]

1. Letting $M_0 = 1$, solve for the stationary distribution μ_0^{ss} using the law of motion for the distribution of firms (equation 12)
2. Let $EQ(\mu_i, M_i) \equiv Q^d - Q^s(\mu_i^{ss}(M_i), M_i; p^*)$. If $EQ(\mu_i, M_i) > 0$, then set $M_{i+1} > M_i$, otherwise set $M_{i+1} < M_i$. When $EQ(\mu_{i+1}, M_{i+1}) \approx 0$ then $(\mu_{i+1}, M_{i+1}) = (\mu^*, M^*)$

A.1.2 General equilibrium model

To solve the general equilibrium model as explained in Section 6, the algorithm starts with Step 1 as before, but solving for the wage of permanent workers w_i instead of p_i. Then, I continue on to Step 2a:

Step 2a Iterate over (μ_i, M_i) until the resource constraint $C = Y - Mc_e - F$ and the labor market clearing condition $L^d = N^s$ are satisfied at (μ^*, M^*):

[(a)]

1. Letting $M_0 = 1$, solve for the stationary distribution μ_0^{ss} using the law of motion for the distribution of firms (equation 12)
2. Let $LMC(\mu_i, M_i) \equiv L^d(\mu_i^{ss}(M_i), M_i; w^*) - N^s[w^*, \Pi(\mu_i^{ss}(M_i), M_i; w^*)]$. If $LMC(\mu_i, M_i) > 0$, then set $M_{i+1} < M_i$, otherwise set $M_{i+1} > M_i$. When $LMC(\mu_{i+1}, M_{i+1}) \approx 0$ then $(\mu_{i+1}, M_{i+1}) = (\mu^*, M^*)$

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