Abstract
In this paper we are interested in the organization of long-term care within a given population. Three care providers are identified: informal care supplied by the relatives (the child in our model), nursing home care financed by the government and private care financed by the individual’s income through a private insurance. Our interest lies in the effect of governmental intervention on the demand/supply of these three forms of LTC i.e. how state intervention affects the provision of LTC by the market and the family. Knowing that, we search for an efficient organization of LTC.

For that, we consider a heterogeneous society composed of different pairs of parent/child. Each parent has a probability of becoming dependent and he/she must receive appropriate cares if this happens. Children are active on the labor market. Additionally, they may devote part of their income to help their dependent parents. The population is heterogeneous. Parents differ according to their income level; children are altruistic or not. These characteristics cannot be observed by the government.

Information asymmetry is a serious constraint on what can actually be implemented by the government. We show that rich parents may not always subscribe to a LTC insurance, even if it is socially optimal that they do so. This non-purchase of insurance is due to the high opportunity cost of insurances, even if they are supplied on the market at an actuary fair price i.e. insurance is crowded-out by other forms of LTC.

As a consequence, the government will reduce the opportunity cost of insurances by decreasing its support to other forms of LTC provided directly by the state or by the family. Alternatively, the government can reduce the share of market financed LTC within the economy.

Keywords: Long-term care, crowding-out, dependency insurance, altruism.

JEL Classification: D64, H55, I18
1 Introduction

In OECD countries, the share of people over 65 and over 80 is increasing and demographic trends show that these proportions will continue to increase in the future. This demographic process is accompanied by an increase in the demand for long-term care (LTC) by elderly dependent at the end of their life.\(^1\) A major challenge for an ageing of the society is therefore to finance the provision of appropriate long-term care to dependent people.

Facing the problem of an ageing population and the associated increase in demand for LTC, countries have chosen different institutional solutions to tackle this problem (see Karlson et al., 2004 for a detailed comparison between Germany, Japan, Sweden, United States and UK and OECD, 2005). In Germany, the government introduced in 1995 a mandatory long-term care insurance program that covers most of the population. The system is financed by a new tax on wages equals to 1.7% of the salary. The LTC insurance is a PAYG system and it is managed as a part of the social security system. An elderly dependent can apply for LTC benefits; his/her dependency level determines the level of help he/she receives. Benefits are of three kinds: professional care at home, institutional care and cash. The right to these forms of help is independent of the income level. Introducing cash payment is meant to support the provision of informal care by relatives. By doing so, informal helpers can receive a compensation for their LTC provision. The German mandatory insurance is an exception and most of the countries do not have universal LTC coverage. In many countries, an elderly dependent person does not necessarily receive help from the state. In the UK for example, the local authorities provide care in residential homes and public intervention in LTC financing is targeted to low income people. Service provision and financial intervention by local authorities are subject to means testing and higher income individuals must self-finance their LTC needs with their own resources or must rely on informal help from their relatives.

Organizing LTC financing is a complex issue since many care providers and care financers are involved. Three different categories of long-term care can be distinguished: nursing home care, residential care provided informally by the relatives and paid residential care. Informal care is by large the most important source of LTC. For Sweden, Johansson (2000) estimated that two-thirds of the total volume of LTC is provided informally by relatives and friends. Bonsang (2007) documents on the basis of the SHARE-1 survey\(^2\) that 30.9% of adult children aged between 50

\(^1\)Long term cares refer to the provision of help/care to a dependent person for his/her activities of daily living (ADL). LTC excludes medical cares that are usually financed by other means.

\(^2\)The SHARE survey (wave 1) has been carried out in 2004 in 10 European Countries. It contains detailed information on a sample of individuals aged over 50.
and 69, with at least one living parent (not necessarily dependent) and not living with them, provide help in time to their parents, with an average of 25.1 hours per month. Help consists in personal care for 27.5% of the helping children. Direct financial help is much less common since it concerns only 2.6% of the children.

The share of private spending remains important in LTC financing. The OECD estimated that LTC spending accounts for 1.35% of GDP in Germany and for 1.37% in the UK. In both countries, the private spending represents 30% of the total expenses. Nursing home care is by large the most expensive form of LTC and it captures the largest fraction of LTC spending (more than 60% of the total public spending in most of the OECD countries). Because nursing home cares are costly for both the state and the individual and because elderly dependents prefer to stay at home (whenever it is possible), policies are settled to support informal care and paid residential care. These initiatives include in-kind benefit, budget for LTC care and financial support for informal helpers.

Private insurances could constitute an interesting alternative to public and private financing of LTC. But the market for dependency insurance is not very well developed. Several reasons may explain that. Pauly (1990) and Brown and Finkelstein (2004) show that there is an important crowding out of private insurances by the public financing of LTC, Medicaid in the US. Pestieau and Sato (2007) show that parents may prefer cares from their family to a private insurance, especially those who have a low income and those who anticipate an important help from their children. In light of that, a major problem for the organization of LTC by the state is that state intervention may seriously crowd-out LTC provided by the market and/or the family. This might be a serious concern for a financially constrained government facing an ageing population.

In this paper we are interested in the financing of LTC within a given population. Three sources of care financing are identified: family support provided by the relatives (the child in our model), private financed cares by the individuals either directly or through a private insurance and government financed cares. Government intervention is either direct: the government provides nursing home care places or indirect: the government supports LTC provision by the family and private insurances. Our interest lies in the effect of governmental intervention on the demand/supply of these three forms of LTC i.e. how state intervention affects the provision of LTC by the market and the family. Knowing that, we search for an efficient organization of LTC.

\footnote{Parents always prefer to buy a private insurance to insuring themselves through saving (at least for reasonable loading factor charged by the insurance company). Brown and Finkelstein (2004) document a load factor of 18\% for LTC insurance.}
For that, we consider in this paper a society composed of different pairs of parent/child. Each parent has a probability of becoming dependent and he/she must receive appropriate cares if this happens. Children are active on the labor market. Additionally, they may devote part of their income to help their dependent parents. The population is heterogeneous. Parents differ according to their income level. For simplicity, we consider two income levels i.e. we distinguish “rich” and “poor” parents. A child may or may not be concerned about the long-term care received by his/her dependent parent. That is, we distinguish altruistic and non-altruistic children. In our model, the government cannot observe the type of the parents (rich or poor) and of the children (altruistic or not). Information asymmetry is a serious constraint on what can actually be implemented by the government.

Parents have the option to buy a private insurance that finance LTC in case of dependency. The parent decides the amount of insurance and, therefore, the care level in the case of dependency. We consider that the insurance market is competitive. Meaning that LTC insurances are offered for an actuary fair premium. If the insurance market is not competitive, the problems we enlighten in this paper are exacerbated.

Non-insured parents receive cares from their children if they agree to do so i.e. if the child is altruistic and if he/she prefers to help his/her parent to other forms of LTC. Parents of non-altruistic children must go to a public nursing home if the market does not insure them. We assume that these three forms of care are mutually exclusive. This means for example that an altruistic child does not help his/her parent if he/she is insured.

In this context, we search for the optimal policy mix, taking into account the reaction of private actors. Of course, the best policy depends on the instruments and the information available to the social planner. Consider that the LTC insurance is fair. If redistribution of the society’s resources can be done at no cost through lump-sum taxes and subsidies, the best policy consists in delegating the LTC financing to the market. All parents are insured (at a fair price) and the government cancels out ex-ante difference in wealth with appropriate income redistribution.

If the government cannot distribute all the resources, the market solution may no longer be the most efficient one. A possible limit in the government’s ability to redistribute resources is its inability to tax the wealth of the parents. As a matter of fact, parents wealth consists of assets that may not, for whatever reasons, be taxed. Moreover, the government may not have the ability to observe the wealth of the parents. This seriously limits the possibility of financing the insurance of the poor parents with a redistributive policy. And, financing a universal insurance with labor income taxes might be prohibitively costly mainly because rich parents will also be
subsidized.

Hence, the government will adopt another financing scheme for LTC. Rich parents will continue to rely on market mechanism but new solutions will be developed for the poor. Instead of redistributing income to finance private insurances, poor parents will be helped by their family or directly by the state; and, for that, the state finances nursing homes. Altruistic children help their parents and the government offers compensation to them. Parents of non-altruistic children receive LTC in a publicly financed nursing home. Hence, without perfect redistribution, the market, the family and the state will all contribute to LTC financing.

But once again, information constraint limits what can be actually implemented by the state. The main problem is that rich parents may prefer to receive cares from their family or from the state to subscribe to a private insurance, even if they would receive more cares in the latter case. The reason is that, even for a fair premium, the insurance cost might be considerable once opportunity costs are taken into account. If a parent is insured, he/she renounces to the other forms of LTC. Hence the LTC he/she might receive in the absence of insurance constitutes the opportunity cost of the insurance. We can then associate to this opportunity cost an implicit load factor for the insurance. This load factor might be considerable, discouraging the parents to subscribe to a private insurance. This rational non-purchase of LTC insurance, even for a fair premium, has been pointed first by Pauly (1990). We observe the same in our model. Because of the high implicit cost; rich parents may not subscribe to private LTC insurances.

The government cannot constraint the rich parents to be insured because wealth is unobservable. Hence, facing rich parents that do not have incentives to be insured, the government has two options. It can either reduce the share of the market in LTC financing and expend the family and the state financed support. The cost being that each dependent parent receives less for his/her LTC needs. Or, it can decrease the opportunity cost of insurances by reducing its support to LTC financing by the state and the family. This must be done in a way that preserve the incentives for the altruistic children to help their dependent parents. In both cases, poor parents suffer from the non-purchase of insurance by the rich ones.

Publicly financed nursing homes could crowd-out the informal care provided by the child and/or the insurance taken by the rich parents. If the nursing home is too generously financed, rich parents and/or parents of altruistic children will apply for, as they will then prefer this solution to the market or familial solution. Hence, because of information constraint, the state intervention in public home financing will be limited in order to avoid the crowding-out of private insurance by the rich and of informal help by the altruists. These information problems are exacerbated if altruism by the children is not perfect and if the insurance is not actuary fair.
In both cases, the optimal policy must be re-examined.

This paper is closely linked to Pestiau and Sato (2007) and Jousten et al. (2005). In their model, Pestiau and Sato (2007) consider a population of heterogeneous parent/child pairs. In particular, they focus on children with different labor productivities. This in turn affects the amount of help a child may provide to his/her dependent parent. And parents anticipating different levels of care by their families will have a different attitude towards other sources of LTC, provided by the state and the family. Optimal policies are derived in this context. In the current paper, we consider other sources of heterogeneity within the population: children differ with respect to their altruism; parents differ with respect to their initial wealth level. Jousten et al. (2005) develop a model where the only source of heterogeneity is the children's altruism. There is no LTC insurance in this model and it focuses on the impact of the altruism on the supply of institutional care by the government. If the government does not observe the degree of altruism, a too generous provision of publicly financed nursing home crowds-out informal help. Welfare consequences on each category of the population are then evaluated. This paper adds another source of heterogeneity on the parents side of the population.

2 Model

We consider an heterogeneous population of \( N \) parent/children pairs. Parents differ according to their wealth endowment; children differ according to their degree of altruism. The population is divided into four groups. Groups 1 and 2 contain the rich parents (wealth level \( I^H \)) and their respectively non-altruistic and altruistic child. Poor parents are in group 3 (altruistic child) and 4 (non-altruistic child). \( n_i, i = 1, 2, 3, 4 \) is the proportion of each group in the total population \( N \) that we normalize to 1.

The parents have an initial wealth level \( I \in \{I^H, I^L\} \), with \( I^L < I^H \). Independently of his/her wealth, each parent faces a probability of dependency \( \pi \). The utility \( (V) \) of a parent depends on his/her consumption level \( C_p \) and the help \( h \) he/she receives in case of dependency.

\[
V = v(C_p) + \pi H(h)
\]

Children are either altruistic or not. Both types of children have a utility level \( u(C^c) \) when they consume \( C^c \). In addition, altruistic children also care about the help \( h \) received by his/her parent in case of dependency (but not on his/her parent consumption if he/she remains in good health). The degree of altruism is measured by a parameter \( \beta \in (0, 1] \). For simplicity, we will consider that children are either perfectly altruist \( (\beta = 1) \) or non-altruist \( (\beta = 0) \). Perfect
Altruism means that there is no divergence of interests between the state and the child on the level of care that must be offered to his/her parent. The utility levels \((U)\) of altruistic and non-altruistic children are respectively:

\[
U = u(C_c^c) + H(h)
\]

\[
U = u(C_c^c)
\]

All the children have the same labor income \(w\).

For closed form solutions, we will consider a logarithmic specification for the functions \(v(.)\), \(u(.)\) and \(H(.)\).

The total welfare \(W\) is the sum of all utilities excluding the altruistic component of the children’s utility function to avoid double counting.

\[
W = \sum_{i=1}^{4} n_i (u(C_c^i) + v(C_p^i) + \pi H_i(h_i)))
\]

Rich and poor parents are endowed with an initial wealth level of \(I^H\) and \(I^L\); children have a labor income \(w\). So that, the total resources of the economy are \((n_1 + n_2)I^H + (n_3 + n_4)I^L + w\).

A benevolent government maximizes the total welfare \(W\). A major problem for the government comes from information asymmetries between the government and the population. In this paper, we consider that the government cannot observe the individual characteristics of the population. In particular, we consider that the government does not observe the wealth of the parents and the altruism of the children. This means that the policy cannot be contingent on the wealth of the parents (they can always pretend that they are poor) nor on the degree of altruism of the children (they can always pretend to be non-altruist). These information asymmetries seriously constraint the intervention in LTC financing by the government.

3 Provision of long-term care

In case of dependency, the parents can benefit from institutionalized (or public) and/or non institutionalized (or private) assistance. This assistance consists in either in-house care (food, nursing assistance,...) or in a nursing home. We distinguish three sources of LTC care financing: the market, the family and the state. Market financing of LTC consists of private insurance subscribed by the parents before dependency occurs. Dependent parents receive a payment

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4There is potentially a third source of information asymmetry between the government and the population if the dependency status is not perfectly observable. We left aside this and consider that the parents cannot cheat on their dependency status. See Kuhn and Nuscheler (2007) for an analysis of this case.
from their insurance company to finance their LTC needs. Family financing consists in financial transfers from children to their parents. Resources received from the family or the insurance company can be spent in LTC. The state intervenes directly and indirectly in the provision of LTC. It offers publicly financed cares for the persons in need. We will consider that this direct intervention consists of public nursing homes. In addition, it (may) subsidizes the provision of LTC by the market and/or the family. For example, the state may offers financial help to the helping children.\(^5\) In the remaining, we will consider that these three forms of care are mutually exclusive.

3.1 The market: private insurance

A private dependency insurance is available on the market. The insurance taker must decide on the premium paid and the corresponding payment in case of dependency. If the insurance is actuary fair, the premium is equal to the expected insurance payment, that is for a premium of \(\pi a\), the policy holder receives a payment of \(a\) in case he/she is dependent. The insurance is not actuary fair if for a repayment of \(a\), the premium exceeds \(\pi a\). We will assume that the market for LTC insurances is competitive. Hence LTC insurances are offered for a fair premium.

The insurance must be taken before dependency occurs. A parent, endowed with wealth level \(I\), that decides to buy an insurance chooses the amount of insurance \(a\) in order to maximize his/her expected utility:

\[
\max_a v(I - \pi a) + \pi H(a)
\]

With the ln formulation, the solution to this problem is:

\[
a^* = \frac{I}{(1 + \pi)}
\]

A parent buys insurance if its expected utility with an amount of insurance \(a^*\) exceeds his/her expected utility with another type of LTC, provided by either the state or the family. We will show that renouncing to other forms of LTC is the opportunity cost of the insurance. This opportunity cost implies that, even at a fair price, there is a positive load factor for the LTC insurance i.e. the premium exceeds \(\pi a\) once opportunity costs are taken into account.

3.2 The family: informal care provided by altruistic children

Parents of altruistic children may rely on their help if they need LTC. Those who anticipate family help will not subscribe to an insurance. If dependency occurs, the child will decide on

\(^5\)Various policies that support informal help are described in OECD (2005).
the amount of help he/she provides to his/her parent. A child endowed with resources $y$ will devote a part $s$ of his/her available income to help his/her parent. For a child with an altruism parameter $\beta = 1$, the optimal amount of help is found by solving:

$$\max_s u(y - s) + H(s)$$

Taking the ln formulation, $s^*$ is equal to:

$$s^* = \frac{y}{2}$$

3.3 The state: public nursing homes

The government finances public nursing homes. Parents that decide to go to the public nursing home do not receive help from insurance companies and their family. We consider that the production technology for nursing homes is imperfect: for an investment of $g$, the corresponding quality of LTC receives in a public home is $\gamma g$ with $\gamma \leq 1$.

4 First best

In the first best, the government decides on the consumption levels of the children ($C^c_i$), of the parents ($C^p_i$) and on the level of LTC ($h_i$). The government faces the following budget constraint:

$$\sum_{i=1}^{4}(C^c_i + C^p_i + h_i) = (n_1 + n_2)I^H + (n_3 + n_4)I^L + w.$$  

Welfare maximizing consumption and care levels are: for $i = 1, 2, 3, 4$:

$$u'(C^c_i) = v'(C^p_i) = H'(h_i)$$

In the first best situation, the government equates the marginal utility of consumption and of help for all individuals. The consumption and help levels are determined by the budget constraint.

4.1 Decentralization of the first best

Suppose that the government can make lump-sum transfers between all individuals. In this case, if the private insurance is fair, the first best can be decentralized with a generalized market financing of LTC. With fair insurance, we have $C^P = h$. More precisely, with an available income of $y$, we have $C^P = h = \frac{y}{1+\gamma}$. Hence, to decentralize the first best, the available income

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Pestieau and Sato (2007) consider that the children can devote part of their time or part of their income to provide cares to their parents.
of the parents must be π percent higher than the available income of the children. So that, the parents’ consumption after buying the insurance is equal to those of the children. Hence, the government can decentralize the first best with appropriate lump-sum transfers.

As an alternative to a generalized market financing, the first best can also be decentralized with a mixed family/market mechanism. If altruistic children are appropriately compensated, the first best can be implemented. A necessary condition for that is perfect altruism (β = 1). Finally note that if providing public nursing home places is frictionless (γ = 1), the first best can be also be decentralized by that mean.

To summarize, the first best can be decentralized if (1) the government can make any kind of lump-sum transfers between individuals and (2) either the private insurance is fair (θ = 1) or public home provision is efficient (γ = 1). If one of these conditions does not hold, the first best cannot be implemented.

4.2 Information constraint

In this paper, we assume that the government cannot observe the individual characteristics of the parents (their wealth level) and of the children (their altruism). These information constraints limit the possible actions of the government. Rich parents can always claim they are poor. The transfers needed to decentralize the first best are then unfeasible. Moreover, altruism being unobservable, altruistic children will help their parents only if they have an interest to do so.

5 Second best

5.1 Government intervention in LTC financing

Depending on the solution chosen for LTC financing, the amount of help received by a dependent parents is a∗, s∗ or γg. The government intervenes in the financing of LTC but its action is constrained by the unobservability of the individuals’ characteristic. In this paper, we consider two different interventions by the government: (1) a direct financing of public nursing home, (2) an intervention in the provision of informal LTC by the family. For that, the government pays a subsidy σ to the children that help their parents. Remember that even if altruism is not observable, the parent’s dependency and the provision of informal help can be observed.

To finance these policies, the government imposes a flat tax t on labor income. This means that only the children contribute to the financing of the governmental intervention. The total resources available for LTC financing are thus tw. The government must keep the budget
balanced and in the remaining, we denote by $\mu$ the Lagrange multiplier of the resource constraint.

To make thing simpler, we make the assumption that poor parents do not have access to private insurances. This assumption is not very demanding since, as we will see, rich parents may not subscribe to a LTC insurance.

5.2 Organizing LTC financing

There are many possible ways to finance and organize LTC in this economy. Parents of group 1 have two options to finance their LTC needs: They can go to the public nursing home or they can buy a LTC insurance. Parents of group 2 have a third possibility: they can benefit from child support. Parents of group 3 can either benefit from child support or go to the public nursing home. Finally, parents of group 4 have no other option than going to the public home. This means that there are 12 different ways of financing LTC.

Let us denote by $m^I$ the number of parents that buy a LTC insurance, by $m^A$, the number of parents that receive help from their altruistic child and by $m^P$ the number of parents that go to the public nursing home, with $m^I + m^A + m^P = 1$, $m^A \leq n_2 + n_3$ and $m^I \leq n_1 + n_2$. The government must pay a subsidy $\sigma$ to $m^A$ children and finance public nursing homes for $m^P$ parents. The budget constraint of the government writes as follow:

$$m^A \pi \sigma + m^P \pi g \leq wt$$

The objective of the government is to maximize the welfare $W$ defined as follow:

$$W = (n_1 + n_2)v(I^H) + (n_3 + n_4)v(I^L)$$

$$+ m^I[u(w(1-t)) + v(\frac{I^h}{1+\pi}) + \pi H(\frac{I^h}{1+\pi}) - v(I^H)]$$

$$+ m^A[(1-\pi)u(w(1-t)) + \pi u(\frac{w(1-t) + \sigma}{2}) + \pi H(\frac{w(1-t) + \sigma}{2})]$$

$$+ m^P[u(w(1-t)) + \pi H(\gamma g)]$$

The first term is the utility parents derived from consuming their wealth endowment. The second term is the utility of the parents that buy a LTC insurance and the utility of their child. Notice that parents that are insured do not consume $I^H$ but $\frac{I^h}{1+\pi}$ i.e. their wealth endowment minus the insurance premium. The third term is the utility of the helping children and their parents and the last term is the utility of the parents that go to the public home and their child.

The problem that the government faces is the following: it must decide on which form of LTC financing for each group of parents (among the 12 available) and it must decides on the tax level
$t$, on the subsidy level $\sigma$ and on the amount of financing for the public homes $g$. The government faces two types of constraint. First, the budget must be balanced. Second, the individuals must prefer the proposed solution to any other available solution. These second set of constraints emerges from the fact that the government cannot observe the individual characteristics (wealth and altruism) and therefore, the proposed LTC financing must be incentive compatible.

### 5.3 The unconstrained problem

Let us ignore for a while (until next subsection) the incentive constraints. We split the government problem into two sub-problems. First, we search for the 12 possible organizations of LTC, the optimal values of the tax rate, the subsidy to helping children and the public home financing. Second, we compare the welfare to determine the optimal LTC organization for the economy.

To determine the optimal values of $t, \sigma$ and $g$, we maximize the welfare $W$ subject to the government budget constraint. This constraint binds at the optimum. Hence, the Lagrangian of the problem can be expressed as:

$$L = W + \mu[m^A \pi \sigma + m^P \pi g - wt]$$

**LEMMA 5.1** For any $m^I, m^A, m^P$, the solution of the unconstrained problem is such that: $w(1 - t^*) = \sigma^* = g^*$

**PROOF:** The first-order conditions of the maximization problem read as follow:

$$\frac{\partial L}{\partial t} = (1 - \pi m^A) \frac{1}{w(1 - t)} + \pi m^A \frac{2}{w(1 - t) + \sigma} + \mu = 0 \quad (5.1)$$

$$\frac{\partial L}{\partial \sigma} = \frac{2}{w(1 - t) + \sigma} + \mu = 0 \quad (5.2)$$

$$\frac{\partial L}{\partial g} = \frac{1}{g} + \mu = 0 \quad (5.3)$$

Solving, we have: $w(1 - t^*) = -\frac{1}{\pi} = \sigma^* = g^*$.

**Lemma 5.1** has two implications. First, altruistic children are perfectly compensated for the help they give to their parents and so their consumption is not altered when they help their parents. Second, the government spends the same amount for a dependent parent in a nursing home than for a dependent parent who receives care from his/her family.\(^7\) Hence, lemma 5.1

\(^7\)Lemma 5.1 means that, as in the first best, the marginal utility of consumption is equalized for all children and for parents that receive a financial assistance from the state, at least for $\gamma = 1$. The difference with the first
implies that whenever $\gamma < 1$, the parents of an altruistic child are better-off if they receive familial support than if they go to a public nursing home. Together, these imply that parents of groups 2 and 3 have a higher utility when they receive help from their family than when they go to the public home. Then, the number of possible organization of LTC reduces to four. In table 1, we note the type of help received by each group of parents in the four possible solutions.

<table>
<thead>
<tr>
<th>Group #</th>
<th>Solution 1</th>
<th>Solution 2</th>
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<tbody>
<tr>
<td></td>
<td>Market</td>
<td>Family</td>
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<td>1</td>
<td>x</td>
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</tr>
<tr>
<td>2</td>
<td>x</td>
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<tr>
<td>3</td>
<td>x</td>
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<tr>
<td>4</td>
<td>x</td>
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Table 1: The 4 possible organization of LTC

Now, we move to the first part of the problem to see which of these four solution gives the highest welfare. To obtain the optimal tax and subsidy levels and the optimal spending in nursing home, we must solve the last first order condition (the derivative of $L$ with respect to $\mu$) which, after integrating the results of lemma 5.1, can be expressed as

$$
\pi m^A \frac{1}{\mu} + \pi m^P \frac{1}{\mu} - (w - \frac{1}{\mu}) = 0 \Rightarrow -\frac{1}{\mu} = \frac{w}{1 + \pi (m^A + m^P)}.
$$

(5.4)

In solution 1, we have $m^I = n_1 + n_2$, $m^A = n_3$ and $m^P = n_4$. Hence, the optimal values for $t, \sigma$ and $g$ are:

$$
t_1^* = \frac{\pi (1 - n_1 - n_2)}{1 + \pi (1 - n_1 - n_2)}, \quad \sigma_1^* = g_1^* = \frac{w}{1 + \pi (1 - n_1 - n_2)}.\n$$

In solution 2, $m^I = n_1$, $m^A = n_2 + n_3$, $m^P = n_4$, then

$$
t_2^* = \frac{\pi (1 - n_1)i}{1 + \pi (1 - n_1)}, \quad \sigma_2^* = g_2^* = \frac{w}{1 + \pi (1 - n_1)}.\n$$

best is that the marginal utility of parents’ consumption is not equal to those of the children. With an imperfect technology $\gamma < 1$, the marginal utility of parents in a public home is equal to $\gamma$ times the marginal utility of a child.
In solution 3, \( m^I = n_2, m^A = n_3, m^P = n_1 + n_4 \), then
\[
t^*_3 = \frac{\pi(1 - n_2)}{1 + \pi(1 - n_2)}; \quad \sigma^*_3 = g^*_3 = \frac{w}{1 + \pi(1 - n_2)}.
\]

In solution 4, \( m^I = 0, m^A = n_2 + n_3, m^P = n_1 + n_4 \), then
\[
t^*_4 = \frac{\pi}{1 + \pi}; \quad \sigma^*_4 = g^*_4 = \frac{w}{1 + \pi}.
\]

It is immediate to see that the largest the number of elderly dependent that receive a financial assistance from the state, either directly through admission in a public home or indirectly through the compensation paid to helping children, the lowest is the public contribution per individual and the highest the tax rate. In other words, the largest \( m^A + m^P \), the highest the tax rate and the lowest \( \sigma \) and \( g \). That is if \( n_1 \geq n_2 \), \( t^*_4 > t^*_3 > t^*_2 \geq t^*_1 \) and \( \sigma^*_4 = g^*_4 < \sigma^*_3 = g^*_3 < \sigma^*_2 = g^*_2 < \sigma^*_1 = g^*_1 \). If \( n_2 > n_1 \), the ordering between solution 2 and 3 is inverted.

For each solution \( i \), let us denote by \( q_i = 1 + \pi(1 - m^I) \). To keep the problem simpler, we will assume that \( n_1 = n_2 \). Denote by \( W_i \) the welfare level when solution \( i \) is applied. The comparison of the welfare levels gives the following:

**Lemma 5.2** Define \( Z^{12} = \frac{1}{n_2 \pi}(q_1 \ln q_1 - q_2 \ln q_2) + \frac{1 + \pi}{\pi} \ln(1 + \pi) \) and \( Z^{24} = \frac{1}{n_1 \pi}(q_2 \ln q_2 - (1 + \pi) \ln(1 + \pi)) + \frac{1 + \pi}{\pi} \ln(1 + \pi) \). We have:

1. \( Z^{12} > Z^{24} \).

2. \( W_1 \geq \max[W_2, W_3, W_4] \) if \( \ln \frac{I^H}{w} \geq Z^{12} \).

3. \( W_2 \geq \max[W_1, W_3, W_4] \) if \( \ln \frac{I^H}{w} \in [Z^{24} + \ln \gamma, Z^{12}] \).

4. \( W_4 \geq \max[W_1, W_2, W_3] \) if \( \ln \frac{I^H}{w} \leq Z^{24} + \ln \gamma \).

**Proof:** Under the hypothesis that \( n_1 = n_2 \), we have \( W_2 - W_3 = -n_1 \ln \gamma \geq 0 \) and solution 3 is always weakly dominated by solution 2.

Solution 1 dominates if \( W_1 \geq W_2 \) and \( W_1 \geq W_4 \). Rewriting these two conditions, we have:

\[
W_1 - W_2 \geq 0 \quad \Leftrightarrow \quad \ln \frac{I^H}{w} \geq Z^{12} \tag{5.5}
\]

\[
W_1 - W_4 \geq 0 \quad \Leftrightarrow \quad \ln \frac{I^H}{w} \geq Z^{14} + n_1 \ln \gamma, \tag{5.6}
\]

where \( Z^{14} = \frac{1}{(n_1 + n_2)\pi}(q_1 \ln q_1 - (1 - n_1 - n_2)(1 + \pi) \ln(1 + \pi)) \). We can show that \( Z^{12} > Z^{14} \). This inequality is true if

\[
n_1 q_1 \ln q_1 + n_2 (1 + \pi) \ln(1 + \pi) > (n_1 + n_2) q_2 \ln q_2 \tag{5.7}
\]
 Taking $n_1 = n_2$ and defining $f(x) = (1 + \pi(1 - x))\ln(1 + \pi(1 - x))$, (5.7) is equivalent to:

$$f(2n_1) + f(0) > 2f(n_1)$$

Since $f(x)$ is a convex function, this inequality holds true and $Z^{12} > Z^{14}$.

Solution 2 dominates if $\ln \frac{I_H}{w} \leq Z^{12}$ and $W_2 \geq W_4$. This condition can be expressed as:

$$W_2 - W_4 \geq 0 \iff \ln \frac{I_H}{w} \geq Z^{24} + n_1 \ln \gamma,$$

where $Z^{24} = \frac{1}{n_1\pi} (q_2 \ln q_2 - (1 + \pi) \ln(1 + \pi)) + \frac{1+\pi}{\pi} \ln(1 + \pi)$. Taking $n_1 = n_2$, $Z^{12} > Z^{24}$ if (5.7) holds.

Lemma 5.2 read as follow: if the wealth endowment of the rich parents is high enough compared to the labor income of the child, the welfare is maximized when the rich parents are left out of the state-financed LTC and rely on private insurance schemes to finance their LTC needs. Leaving aside the rich parents form state-financed LTC has two advantages: the tax rate is lower, which is beneficial to all the children, and the per-capita contribution of the state to dependent parents is higher, which obviously benefits to all the parents that received state-financed LTC.

When the wealth endowment of the rich parents declines relative to the labor income, the highest welfare is achieved in a generalized state-financed LTC system. But this switch from private insurance to state financed LTC is organized in two steps. Because the public provision of nursing homes involves resource losses, there are intermediate values of $\frac{I_H}{w}$ for which rich parents behaves differently depending if their child is altruistic or not. For these intermediate values, the rich parents of non-altruistic child will continue to finance their LTC needs with private insurance while the rich parents of altruistic child will be helped by their family in case of dependency. It is only when the ratio $\frac{I_H}{w}$ declines further that all the parents will depend on state-financed LTC and that the private insurance will no longer be bought (even at an actuary fair price).

The optimal solution is represented in figure 1: on the vertical axis, we represent the relative income of the rich parents compared to the labor income of the children, on the horizontal axis, we represent the technology for producing/financing public nursing homes. To draw this figure, we re-write conditions (5.5) and (5.8) by taking the exponential on both sides of the inequalities. These conditions can then be expressed as: $\frac{I_H}{w} \geq \gamma \tilde{Z}^{12}$ and $\frac{I_H}{w} \geq \gamma^{n_1} \tilde{Z}^{24}$, where $\tilde{Z}^{12} = e^{Z^{12}}$ and $\tilde{Z}^{24} = e^{Z^{24}}$.
5.4 Incentive constraints

We now introduce the incentive constraints in the above problem. The government cannot observe the wealth of the parents nor the altruism of the children. Rich parents can then pretend that they are poor and altruistic children can pretend that they are not. Hence, the LTC financing must be such that each group of parents/children prefers the preferred solution to any other possible way to finance LTC. We must then consider self-selection constraints.

Two sets of incentive constraints must be considered. First, if the organization of LTC prescribes that rich parents (or some of them) buy an insurance they must agree to do so. Rich parents may have incentives to mimic the behavior of the poor ones. By doing so, they save on private insurance and therefore enjoys a higher consumption and, in case of dependency, they do receive assistance from the state or from their family. Therefore, whenever the optimal organization of LTC calls for market mechanism for the rich, the planer must ensure that the rich parents indeed prefer the market solution to any other available one. This means that their utility with the LTC insurance must be higher than the other options they have for LTC financing: family support and public homes for parents of group 2 and public nursing home only for the parents of group 1. The corresponding incentive constraints write as follow:
\[ v\left(\frac{I^H}{1 + \pi}\right) + \pi H\left(\frac{I^H}{1 + \pi}\right) \geq v(I^H) + \pi H(\gamma g) \quad (IC1) \]
\[ v\left(\frac{I^H}{1 + \pi}\right) + \pi H\left(\frac{I^H}{1 + \pi}\right) \geq v(I^H) + \pi H\left(\frac{w(1-t) + \sigma}{2}\right) \quad (IC2) \]

Second, if the proposed LTC financing is such that altruistic children (or some of them) must help their dependent parents, they must agree to do so rather than mimicking the behavior of non-altruistic children. The corresponding incentive constraint writes as follow:

\[ U\left(\frac{w(1-t) + \sigma}{2}\right) + H\left(\frac{w(1-t) + \sigma}{2}\right) \geq U(w(1-t)) + H(\gamma g) \quad (IC3) \]

### 5.5 When is the unconstrained problem incentive compatible?

Take the unconstrained solution described in lemma 5.2. We first check if and when the unconstrained solution satisfies the corresponding incentive constraints.

Consider first solution 1. This solution is incentive compatible if for \( t = t^*_1, \sigma = \sigma^*_1 \) and \( g = g^*_1 \), the constraints (IC1), (IC2) and (IC3) are satisfied. By lemma 5.1, we know that (IC3) is satisfied for sure.

Define \( Z^{IC1} = \frac{1+\pi}{\pi} \ln(1 + \pi) - \ln q_1 \). (IC2) is satisfied if \( \ln \frac{I^H}{w} \geq Z^{IC1} \) and (IC1) is satisfied if \( \ln \frac{I^H}{w} \geq Z^{IC1} + \ln \gamma \). We can show that solution 1 is not always incentive compatible in the parameter space where it gives the highest welfare in the unconstrained problem. That is:

**LEMMA 5.3** \( Z^{IC1} > Z^{12} \)

**PROOF:** The inequality \( Z^{IC1} > Z^{12} \) can be simplified to \( 1 > -\frac{q_1}{n_2 \pi} \).

So for \( \ln \frac{I^H}{w} \in [Z^{12}, Z^{IC1}] \), the highest welfare would be achieved if rich parents are insured but, parents of group 2 prefers to receive help from their child. Moreover, if \( \ln \frac{I^H}{w} \in [Z^{12}, Z^{IC1} + \ln \gamma] \), parents of group 1 do not subscribe an insurance and apply for the public home. Clearly, the budget will not be balanced if this happens.

Consider next solution 2. This solution is incentive compatible of for \( t = t^*_2, \sigma = \sigma^*_2 \) and \( g = g^*_2 \), the constraints (IC1) and (IC3) hold. Moreover, inequality (IC2) should be reversed. Define \( Z^{IC2} = \frac{1+\pi}{\pi} \ln(1 + \pi) - \ln q_2 \). We can show that solution 2, when it dominates the other possible solutions, is not always incentive compatible. That is:

**LEMMA 5.4** \( Z^{IC1} > Z^{IC2} > Z^{12} \)

**PROOF:** The inequalities \( Z^{IC2} > Z^{12} \) and \( Z^{IC1} > Z^{IC2} \) can be simplified to \( n_2 > 0 \).
Finally solution 4 is always incentive compatible. In figure 2, we represent the parameter space where the unconstrained solution does not satisfy the corresponding incentive constraints. In the vertically shaded area, constraint (IC1) is not satisfied. In the obliquely shaded area, (IC2) is not satisfied. To construct the figure, we call $\tilde{Z}^{IC1} = e^{Z^{IC1}}$.

\[ \frac{L^h}{w} \]

\[ \tilde{Z}^{IC1} \]

\[ \tilde{Z}^{IC2} \]

\[ \tilde{Z}^{12} \]

\[ \tilde{Z}^{24} \]

\[ \gamma \]

Figure 2: Incentive constraints in the unconstrained problem

Information asymmetry has for consequence that rich parents do not always subscribe to a private LTC insurance when it is optimal (for the society) that they do so. Pauly (1990) explained this rational non-purchase of private LTC insurance and his explanation fits our model very well. For a rich parent, buying an insurance means that consumption if he/she remains in good health decreases while consumption in case of dependency does not necessarily increase. LTC support is higher only if the insurance repayment is higher than any other form of care available. But even if the rich parents receive more cares when they are insured, they do not necessarily buy an insurance.

Because the three forms of care are mutually exclusive, a parent that subscribe to an insurance renounces to the other forms of care. Hence, even if the insurance is offered at a fair premium, the cost of the insurance could be quite high once opportunity costs are incorporated. For parents of group 1, the opportunity cost of a LTC insurance is the level of LTC they can receive in a public nursing home. For parents of group 2, it is the LTC received from their child.
Once opportunity costs are included, cost of insurance increases dramatically and discourage insurance subscription.

Even if the insurance company does not charge a load factor and offers the insurance at a fair price, there is an implicit load factor because parents renounce to other forms of help. For a repayment of $a$, the parents pay $\pi a$ and renounces to either $\gamma g^*$ (group 1) or $s^*$ (group 2). So the total cost of an insurance is $\pi a + \gamma g^*$ or $\pi a + s^*$. Differently, we can define a implicit load factor, $\tilde{\theta}_i$, for the parents of group $i = 1, 2$ equals to:

$$\tilde{\theta}_1 = 1 + \frac{\gamma g^*}{\pi a} \quad \text{(5.9)}$$

$$\tilde{\theta}_2 = 1 + \frac{s^*}{\pi a} \quad \text{(5.10)}$$

This modified load factor is the additional cost per unit of insurance paid by the parents. As it is clear from these formulations, the higher the help received by the parents either from the state or from their child, the higher this implicit load factor. And obviously, a high load factor discourage insurance taking by the parents.

High insurance costs implies that rich parents buy it only if they could expect a much higher quality of care if they are insured. This is the case if $I^H$ is high compared to $g^*$ and/or $s^*$. In our solution 1 we have $Z^{IC_1} > 1$. This means that if $\frac{I^H}{w} = 1$, the rich parents do not subscribe to a LTC insurance. It is only when the rich parents have a wealth level sufficiently higher than the children that they buy an insurance. This can be seen from expressions (5.9) and (5.10): When labor income increases, the implicit load factor of insurance increases. When the parent’s wealth increase, they buy more insurance (if they buy an insurance), and the load factor is inversely proportional to the insurance level.

The other source of information asymmetry does not create problem. Because altruistic children are perfectly compensated for the help they give to their parents, they have incentives to do so. In Jousten et al. (2005), because of distortionary taxation, altruistic children are worse-off than non altruistic ones. Hence altruistic children have incentives to behaves like non-altruistic ones.

Note that, even if the incentive constraint (IC3) is never binding in the above problem, it does not mean that this constraint is irrelevant in the design of a LTC financing scheme. We will see that this constraint must be taken into account in the problem. More in particular, when the government distorts the LTC financing to constraint the rich parents (or some of them) to subscribe to a private insurance, it must check that altruistic children continues to have the right incentives to help their parents.
As shown on figure 2, the unconstrained solution can not be implemented for \( \frac{h}{w} \in [Z^{12}, Z^{IC}] \cup [Z^{24}, Min[Z^{12}, Z^{IC2} + \ln \gamma]] \). In the parameter space where the unconstrained solution is not incentive compatible, the government has two options: it can either change \( t, \sigma \) and \( \gamma \) in order to make the proposed LTC financing system incentive compatible or it can switch to another LTC financing solution. We examine in turns these two alternatives.

5.6 The constrained problem

Suppose that the government wants to have all the rich parents insured (solution 1). For that, they must be prevented from relying on the help of their child (constraint (IC2)) and from applying to public homes (constraint (IC1)). The only way to do so is to reduce the state-financing of LTC, that is reducing the help to the altruistic children \( \sigma \) and the quality of public home \( g \). But by doing so, the state must take into account that reducing \( \sigma \) may have an impact on the behavior of the altruistic children of poor parents. If they receive a lower compensation for helping their parents they may be tempted to mimic non-altruistic children. So we must maximize the welfare \( W_1 \) subject to the budget constraint, the incentive constraints (IC1), (IC2) and (IC3) and check which constraint is binding.

Let us denote by the \( \lambda_{ICi} \), the Lagrange multiplier of constraint (ICi), \( i = 1, 2, 3 \). The constrained solution 1 is the following:

\[
\tilde{t}_1 = \frac{\lambda_{IC3} + (1-n_3 \pi)(1-n_1-n_2-\lambda_{IC1}-\lambda_{IC2})}{(1-n_3 \pi)(1+\pi(1-n_1-n_2-\lambda_{IC1}-\lambda_{IC2}))} \tag{5.11}
\]

\[
\tilde{\sigma}_1 = \frac{w(\lambda_{IC3}(2-n_3 \pi) - \pi(\lambda_{IC2} - n_3)(1-n_3 \pi))}{n_3 \pi(1-n_3 \pi)(1+\pi(1-n_1-n_2-\lambda_{IC1}-\lambda_{IC2}))} \tag{5.12}
\]

\[
\tilde{g}_1 = \frac{w(\pi(n_4 - \lambda_{IC1}) - \lambda_{IC3})}{n_4 \pi(1+\pi(1-n_1-n_2-\lambda_{IC1}-\lambda_{IC2}))} \tag{5.13}
\]

In the appendix, we discuss when the Lagrange multipliers \( \lambda_{IC1} \) and \( \lambda_{IC3} \) are positive. \( \lambda_{IC2} > 0 \) for all values of \( \frac{h}{w} < Z^{IC1} \). Clearly, to make solution 1 incentive compatible, the government reduces the nursing home financing.

Suppose that the government wants to have only the rich parents of non-altruistic child to be insured (solution 2). When this solution is not incentive compatible, the government must lower the quality of its public homes. By doing so, it will prevent rich parents to apply to the public homes. Let us denote by \( \lambda_{IC2} \), the Lagrange multiplier of the constraint (IC2), the incentive compatible solution 2 is:

\[
\tilde{t}_2 = \frac{\pi(1-n_1-\lambda_{IC2})}{1+\pi(1-n_1-\lambda_{IC2})}, \quad \tilde{\sigma}_2 = \frac{w}{1+\pi(1-n_1-\lambda_{IC2})}, \quad \tilde{g}_2 = \frac{(n_4-\lambda_{IC2})w}{n_4(1+\pi(1-n_1-\lambda_{IC2}))}.
\]
In this problem, we have $\lambda_{IC2} > 0$ for $I_{hw} < Z^{IC2} + \ln \gamma$.

We can show that: $\frac{\partial \tilde{I}_2}{\partial \lambda_{IC2}} < 0$, $\frac{\partial \tilde{g}_2}{\partial \lambda_{IC2}} < 0$ and $\frac{\partial \tilde{\sigma}_2}{\partial \lambda_{IC2}} > 0$. Moreover $\frac{\partial \tilde{W}_2}{\partial \lambda_{IC2}} < 0$. This means that, to make solution 2 incentive compatible, the government reduces the public home quality.

This relaxes the resource constraint and the government uses this freedom to increase the subsidy paid to altruistic children and to lower the tax rate. As a consequence, the welfare is reduced.

5.7 Optimal policy

We conclude our analysis by establishing the optimal LTC financing.

PROPOSITION 5.1 There exists $Q_{12}$ and $Q_{24}(\gamma)$ such that,

1. For $\frac{I_{hw}}{w} \geq Q_{12}$, solution 1 is optimal.
2. For $\frac{I_{hw}}{w} \in [Q_{24}(\gamma), Q_{12}]$, solution 2 is optimal.
3. For $\frac{I_{hw}}{w} \leq Q_{24}(\gamma)$, solution 4 is optimal.
4. $Q_{24}(\gamma)$ is increasing in $\gamma$ and $Q_{24} \in [\tilde{Z}^{IC2} + \gamma, \tilde{Z}^{24} + \gamma]$.
5. $Q_{12} \in [\tilde{Z}^{12}, \tilde{Z}^{IC1}]$.

Proposition 5.1 reads as follow: Under asymmetric information, the government continues to use the same policy mix than in the unconstrained problem. This means that only solutions 1, 2 and 4 are considered as optimal solutions. But, to make these solutions incentive compatible, the government must distort its policy. And, in particular, it must reduces its nursing home subsidy, which hurts the welfare of those who do not have another solution than applying for public nursing home space. Because of that, the government compares the cost of making the solution incentive compatible to switch to another solution.

As it can be seen on figure 3, the parameter space where solution 4 is optimal increases. Second, the parameter space where solution 1 applies decreases. Together, it means that, because of information constraints, the market financing of LTC decreases.

6 Conclusions

In this paper, we considered the optimal organization of LTC financing in a society composed of pairs of parents-children with different individual characteristics. In particular, we considered rich and poor parents and altruist and non-altruist children. In a world where the parents cannot contribute to a generalized LTC financing system, as it is the case in a PAYG system where LTC
is financed by individual contributions and labor income taxes, LTC financing is split between the market, the state and the family.

The main problem comes from the high opportunity cost of private insurances and their significant crowding-out by family and government financed LTC. If rich parents do not want to buy a LTC insurance, the government has two options. Either, it diminishes the proportion of market financed within the economy at the cost that each parent receives less care or it changes its policy in order to provide the right incentives for the rich to be insured. This can be done only by reducing the state financing of LTC and makes the poor parents worse off. Hence rich parents who have multiple options for LTC financing exerts a negative externality on those who have less options.

In this paper, we made three important assumptions: (1) insurances are offered at a fair price, (2) children are perfectly altruistic and (3) poor parents do not have access to private insurance. We briefly review these three assumptions.

If insurances companies charge a positive load factor, this would have the following consequences on the optimal policy: on the one hand, the market mechanism will be used less often by the government, but, on the other hand, a higher insurance price would exacerbate the incentive problems. A high price will make insurance subscription by the rich even more complicated.
Without perfect altruism ($\beta < 1$), the government and the family diverge on the way resources must be distributed within the family. With the consequences that parents of altruistic children receive less than what the government expected. With imperfect altruism, the government will be reluctant to subsidy helping children and the number of possible LTC organization diminishes. This in turn would simplify the incentive problems we encountered.

Finally, we assumed that poor parents do not have access to LTC insurances. Clearly, this assumption is not very demanding since we observed that rich parents already found the insurance expensive. If poor parents would have access to insurances, the government could organize LTC financing with a generalized market financing. In this case, it would be necessary to provide income subsidies to the parents in order to guarantee them a decent level of cares if dependency occurs. Given our assumptions, these income subsidies should be financed with labor income taxes. Moreover, they would also be paid to the rich parents. Hence, such a system might turn to be prohibitively costly particularly if the proportion of rich parents in the population is consequent.
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