Bunching and Non-Bunching at Kink Points of the Swedish Tax Schedule

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Abstract

Recent microeconometric studies of taxpayers’ responsiveness to taxation have shown that intensive margin labor supply and earnings elasticities typically are modest and sometimes equal to zero. However, a common view is that long-run responses might still be large since micro-estimates are downward biased owing to optimization frictions. In this paper we estimate the taxable income elasticity at a very large kink point of the Swedish tax schedule using the bunching method. During the period of study the change in the log net-of-tax rate reached a maximum value of 45.6%. Interestingly, we obtain a precise elasticity estimate of zero for wage earners at this large kink. The size of the kink allows us to derive tighter bounds on the long-run elasticity than previous studies. If wage earners on average tolerate 1% of their disposable income in optimization costs, the upper bound on the long-run taxable income elasticity is 0.39. We also evaluate the performance of the bunching estimator by performing Monte Carlo simulations.

Keywords: bunching, taxable income, bounds, optimization frictions

JEL Classification: H21; H42

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1 Introduction

Recent microeconometric studies of taxpayers' responsiveness to taxation have shown that intensive margin
labor supply and earnings elasticities typically are modest and sometimes equal to zero (Chetty 2011, Chetty
et al 2011a, Saez 2010, Saez et al 2010). However, a common view is that long-run responses might still
be large since micro-estimates are downward biased owing to optimization frictions. In this paper we make
use of population-wide register data sets covering the time period 1998-2008 to estimate the taxable income
elasticity at a particularly large kink point in the upper middle part of the Swedish income distribution (the
first central government kink point). During this period, the percentage change in the net-of-tax rate at
the kink reached a maximum value of 45.6%. This is a substantially larger change in marginal incentives
in comparison to similar kinks studied elsewhere for the purpose of estimating behavioral elasticities.1 The
size of the kink allows us to derive tighter bounds on the long-run elasticity than previous studies.

The behavioral parameter of interest in this paper is the taxable labor income elasticity with respect to
the net-of-tax rate. As taxable labor income is obtained by subtracting deductions from gross labor income,
the estimated elasticity does not only include hours of work responses to taxation but also responses related
to deduction behavior.2

Owing to the size of the central government kink point, the standard frictionless model predicts a sub-
stantial amount of bunching even if the elasticity is very small. In Figure 1 we have simulated taxable income
distributions under the 2002 Swedish tax schedule for two very modest values of the compensated elasticity,
0.01 and 0.1.3 Both values of the elasticity are below the elasticity estimates obtained in most empirical
studies of the taxable income elasticity when tax reforms are used for identification.4

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1The jump in marginal tax rates at the first central government kink point was 17.4-23.3 percentage points 1998-2008. The
threshold for the top tax in Denmark analyzed by Chetty et al (2011a), 1994-2001, amounted to 12.5 – 15 percentage points
during the period of study, and the second federal tax kink in the U.S. 1988-2002 examined by Saez (2010) implied a marginal
tax rate increase of 13 percentage points.

2The standard static labor supply model can be generalized to a model for taxable income. In the labor supply model the
individual equates the marginal disutility of work and the marginal net-of-tax hourly wage rate in optimum. In the taxable
income model the individual instead supplies taxable income until the marginal disutility of supplying taxable income is equal
to the marginal net-of-tax rate (1-marginal tax rate). The taxable income elasticity captures more margins than hours of work
(e.g. effort per hour, tax avoidance and tax evasion).

3In the simulations the income supply function is \( z = z_0(1 - \tau)^e \), where \( z \) is taxable income supply, \( z_0 \) is ‘potential income’
, \( \tau \) is the marginal tax rate and \( e \) is the taxable income elasticity. This supply function has been derived from an iso-elastic
utility function (see the parametric example in Saez 2010). The log normal ‘potential income’ distribution is calibrated such
that the mean and variance of the realized simulated income distribution coincides with the mean and variance of the actual
2002 income distribution.

4See Saez et al (2010) for a US centered overview of the taxable income literature and section 4 of Pirttilä and Selin (2011)
for a survey of the Swedish literature.
Figure 1: Simulated income distributions assuming an elasticity of 0.01 (top panel) and 0.1 (bottom panel) shown together with the marginal tax schedule under the Swedish income tax in 2002. The large and salient federal income tax kink appears at an earnings level of 290,100 SEK and generates a sharp empirical prediction for the taxable income distribution.
The figure demonstrates that, even when the elasticity is as low as 0.01, there is a clear spike at the first central government kink of the Swedish tax schedule. When the elasticity is 0.1 the spike is huge. In the real world, one cannot expect individuals to bunch exactly at the segment-limit, although if the distribution of deviations between actual and desired income choices of individuals is smooth and symmetric we should observe a hump rather than a spike at the kink point.

Given the simulated income distributions of Figure 1 our empirical results are surprising. Among wage earners we find no economically significant bunching of taxpayers at the large central government kink point. This implies an observed compensated elasticity of zero. The self-employed, on the other hand, display sharp and statistically significant bunching at the first central government kink point. However, the implied elasticity estimates are small, around 0.02 for broader groups of self-employed individuals and around 0.05 for the ‘purely self-employed’ who only earn income from the firm they own. It is interesting to note that the ‘purely self-employed’ also display bunching at the second central government kink point which is noticeably smaller and located further up in the income distribution.

The absence of bunching at the large central government kink point suggests that the local behavioral responses to changes in the tax schedule in these income ranges are negligible in the short run. This finding is consistent with the well documented empirical regularity that labor supply tends to be inelastic along the intensive margin. Since we study the taxable labor income elasticity, which also includes deduction-related responses, this paper lends even stronger support to the earlier finding that taxpayers’ short run responses to changes in marginal tax rates are small.

Our finding that the observed compensated elasticity is zero does not however lead us to believe that taxes are without distortions. While it is costly for taxpayers to respond to tax changes in the short run due to adjustment costs, hours constraints and inattention, long-run responses to tax changes might still be substantial. In particular, long-run responses to taxes do not need to take place at the individual level and as such, cannot be captured by conventional micro-economic models. Other potentially important channels are collective agreements and the regulatory structure of the labor market.

Chetty (2011) recognizes the discrepancy between structural elasticities (the elasticities we would observe in a world without optimization frictions) and observed elasticities. By making assumptions about the extent
to which individuals tolerate utility losses by ignoring tax changes, Chetty is able to put bounds on the structural elasticity based on observed behavior. If we adopt Chetty’s procedure and baseline assumption that individuals on average tolerate one percentage point of their disposable income in utility losses, we find an upper bound on the long run elasticity of 0.39. Noticeably, this upper bound on the compensated elasticity lies below the upper bounds implied by the elasticity estimates obtained by both Blomquist and Selin (2010) and Gelber (2011), two other recent studies on Swedish data. As our kink is very large, and technically the upper bound shrinks at a quadratic rate in the change in the log net-of-tax share, a substantial amount of information is contained in our estimates.

Finally, we contribute methodologically to the bunching literature by analyzing the potential impact of income effects on bunching estimates. Previous papers have not systematically taken into account the possibility that income effects may bias the compensated elasticity estimates downwards. While it is possible to prove analytically that income effects do not influence the bunching estimator when the tax change is small, it is difficult to appeal to this result when analyzing a kink point as large as in the present paper. To address the issue, we perform Monte Carlo simulations, where the data is generated by a utility function implying a constant compensated elasticity but a nonzero income effect term. This exercise reveals that although obtained elasticity estimates are somewhat smaller, the impact of income effects is practically insignificant.

The paper is organized as follows. Section 1.1 briefly summarizes previous literature on bunching estimation. Section 2 and 3 present the underlying model and estimation framework. Section 4 describes the institutional setting and data, whereas section 5 presents the main empirical analysis. Section 6 performs bounds calculations. Section 7 describes our Monte-Carlo exercise with income effects. Finally, section 8 concludes.

1.1 Related Literature

This paper primarily draws on the seminal work by Saez (1999, 2002, 2010). Saez observed that predictions of the standard taxable income supply model can be tested by making use of large register data sets. In the past, non-linear budget set models have been estimated on small survey data sets, where it has been
impossible to disentangle measurement errors and optimization errors. Saez (2010) finds clear evidence of bunching at the first kink point of the U.S. earned income tax credit (EITC), the income level where the tax credit is maximized. The response is, however, concentrated among the self-employed and is interpreted as a consequence of reporting behavior, rather than real labor supply behavior.5

Saez (2010) also analyzes the U.S. federal income tax schedule, including the large second federal kink point. At this kink point the marginal tax rate jumps from 15 percent to 28 percent and is located at the upper middle part of the income distribution. Thus, it shares some similarities with the kink examined in the present paper. The kink point was kept constant at $54,350 (in 2008 dollars) for married taxpayers and $32,550 for single taxpayers during the period 1988-2002. Saez did not detect any bunching at this kink for any group (including the group of self-employed individuals).

By virtue of its transparency and its reliance on within-year variation (the between-year marginal tax variation at a given earnings level is often low whereas differences in marginal tax rates across two segments is sometimes high in progressive tax systems), the bunching method has recently gained popularity in the empirical public finance literature. A prominent example is the paper by Chetty et. al. (2011). Chetty et. al. set up a model with endogenous hours constraints and search costs. They test the predictions of the model on Danish individual level tax register data 1994-2001 using the bunching method. In the paper we briefly comment on how our results relate to the results obtained by Chetty et. al. on Danish data.

The bunching method has also been used in the context of randomized field experiments. Kleven et al (2010) analyze a tax enforcement field experiment, again in Denmark. They compare the excess mass at a large kink before and after a treatment (randomized audits and randomly assigned threat-of-audit letters). In a similar vein, Chetty and Saez (2009) compare bunching at the first EITC kink in the U.S. before and after informing randomly selected taxpayers about the tax system.

While the above mentioned studies estimate elasticities at convex kink points, Kleven and Waseem (2011) exploit discontinuous jumps in average tax rates (‘notches’) to estimate the taxable income elasticity on data from Pakistan. There is sharp bunching at the notches, especially among the self-employed. However, the implied compensated elasticities are small (close to zero for wage earners and at most around 0.1 for the self-employed).

5 Using a modified estimation approach, Weber (2011) detects significant bunching for both wage earners and self-employed at the second EITC kink, where the phase out region starts. This is partly explained by the fact that Weber defines the EITC kinks as a function of Adjusted Gross Income (AGI) rather than earned income.
self-employed). The method developed by Kleven and Waseem has also been used by Kleven et al (2011) who analyze the effects of income taxation on the international migration of top earners using the Danish preferential foreigner tax scheme.

2 Derivation of Bunching Formula

We now illustrate the theory underlying the bunching estimation technique, initiated by Saez (2010), in the simplest possible way. Consider a situation where each individual maximizes the quasi-linear utility function

\[ U(c, z) = c - v(z) \] subject to the budget constraint \( c = z - T(z) + m \), where \( c \) is consumption, \( z \) is taxable income, \( T(z) \) is the income tax function and \( m \) is non-labor income. Suppose a pre-reform situation where individuals’ taxable incomes are distributed according to a smooth density function \( h_0(z) \) and all individuals face a proportional tax schedule with a single marginal tax rate, \( T(z) = \tau_1 z \). A kink is introduced at an earnings level \( k \), so that for income \( z \geq k \) the tax rate \( \tau_2 > \tau_1 \) applies. This reform will transform the income distribution as individuals adjust their taxable income to the new tax system. Denote the density function for the post-reform earnings distribution by \( h(z) \). This hypothetical reform will have the following consequences:

1. The earnings distribution to the left of \( k \) is unaffected, i.e. \( h(z) = h_0(z) \) for \( z < k \).

2. Individuals who before the reform reported taxable incomes with \( z > k \) will reduce their earnings in response to the tax increase.

3. We will observe a spike in the income distribution. The specific mass of taxpayers \( B = \int_k^{k+\Delta z} h_0(z)dz \), will move to \( k \) where \([k, k + \Delta z]\) is the interval of taxpayers who choose to locate at the kink after the reform.

In the tradition of Feldstein (1995) we define the compensated taxable labor income elasticity, locally at \( k \), as

\[
\tilde{e}(k) = \frac{\text{percentage change in } z}{\text{percentage change in } (1 - \tau)} = \frac{\Delta z}{k} \frac{\Delta(1-\tau)}{(1-\tau_1)} \tag{1}
\]
Unless one is willing to impose further assumptions on the structure of preferences and abilities, \( \tilde{e}(k) \) can in general not be given a structural interpretation. However, it is possible to relate \( \tilde{e}(k) \) to the number of individuals who bunch at the kink. Note that

\[
B(\Delta z) = \int_{k}^{k+\Delta z} h_0(z) dz = \Delta z h_0(\xi)
\]

for some \( \xi \in [k, k+\Delta z] \).\(^6\) Hence inserting (2) into (1) and rearranging gives

\[
\tilde{e}(k) = \frac{B(\Delta z)}{k \times h_0(\xi) \times \frac{\Delta(1-\tau)}{(1-\tau_1)}}
\]

(3)

For small tax changes (\( \Delta \tau = d\tau \) and \( \Delta z = dz \)) we have \( \xi \to k \) and the number of individuals who bunch is \( B(dz) = h_0(k)dz \). Thus, we have that

\[
\lim_{\Delta \tau, \Delta z \to 0} \tilde{e}(k) = e(k) = \frac{dz}{d(1-\tau)} \frac{(1-\tau)}{z} = \frac{B(dz)}{k \times h_0(k) \times \log\left(\frac{1-\tau_1}{1-\tau_2}\right)}
\]

(4)

where \( e \) is the ‘structural’ compensated elasticity of taxable income. In (4) \( k \) and \( \log\left(\frac{1-\tau_1}{1-\tau_2}\right) \) are directly observable, while \( B \) and \( h_0(k) \) need to be estimated. Following Chetty et al (2011a) we refer to \( b = \frac{B}{h_0(k)} \) as the excess mass of taxpayers at \( k \). Hence, given that \( b \) can be estimated, the above method non-parametrically identifies \( e \) when the kink point is small. Note that the number of individuals who bunch at the kink is proportional to the compensated elasticity locally at \( k \).\(^7\) In Appendix A we show that this result holds also in the presence of income effects on labor supply.

\section{3 Estimation procedure}

The general idea of bunching estimation is to construct a measure of the excess mass of taxpayers at the kink by locally comparing the mass of individuals at the kink point with the mass of individuals at this same earnings level in the absence of a kink. The key methodological challenge is to remove the influence of the kink

\(^6\)This follows from the mean value theorem of integration calculus.

\(^7\)As shown by Saez (2010) this also holds true when elasticities differ between individuals at a given income level. Then the average elasticity at \( k \) is identified.
from the observed income distribution to obtain the 'counterfactual distribution'. Saez (2010) uses the actual (observed) income distribution to the left and to the right of the kink to infer the counterfactual distribution locally around the kink (where it is not observed). This procedure is simple but has two drawbacks.

First, individuals who bunch would have located to the right of the kink if there was no jump in marginal tax rates. Hence the observed income distribution close to the right of the kink is not necessarily a good proxy for the counterfactual mass of taxpayers at the kink. Moreover, as described above, the whole income distribution to the right of the kink is transformed and shifted to the left by the tax reform.

Second, Saez’ procedure assumes that the underlying distribution has a trapezoid shape. This is a good approximation in many cases but will bias estimates if the distribution is curved. To sidestep these issues, we use the refined estimation procedure proposed by Chetty et al (2011a) who estimate the counterfactual distribution using nonparametric methods. More specifically, $h_0(k)$ is estimated by fitting a polynomial to the income distribution, omitting an income band surrounding the kink and then adjusting the mass of the counterfactual distribution so that it integrates to one.

Our estimation procedure, which draws on Chetty et al (2011a), proceeds as follows. First, a 'wide bunching window’ around the kink point is specified and taxable income is expressed in terms of the absolute distance to the kink point. This window specifies the sample to be used in estimating bunching and the counterfactual distribution. The data is collapsed into bins of width 1000 SEK and each bin $j$ is represented by an income level $Z_j$, defined as the mean absolute income distance between the observations falling within income bin $j$ and the kink point. In other words, $Z_j$ is the distance between bin $j$ and the kink point (measured in steps of 1000 SEK). Visual inspection of the histogram $\{Z_j\}$ guides the selection of a bandwidth $R$ and the associated ‘small bunching window’ $[-R, R]$. Ideally, this window should be chosen so as to capture exactly those individuals bunching. Choosing $R$ too small (large) will underestimate (overestimate) bunching and the associated behavioural response. The number of individuals in income bin $j$ is given by the nonparametric regression:

$$C_j = \psi(Z_j, R) + \eta_j$$

where $\psi$ is a polynomial in $Z_j$ including dummy variables for observations close to the kink (as measured by

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This is related to the question of identification. If the kink point would coincide with a peak in the counterfactual income distribution it is difficult or impossible to credibly estimate the counterfactual distribution and bunching is not identified.
$R$) and $\eta_j$ accounts for the error in the polynomial fit.

In our estimation we use the same iterative procedure as in Chetty et al (2011a) to estimate (5) and refer the interested reader to this paper for a more thorough description of this estimation procedure.\(^9\)\(^10\)

Denote by $\hat{C}_j$ the predicted values from regression (5). Bunching is estimated as the number of taxpayers at the kink (denoted by $\hat{B}$) relative to the average height of the counterfactual distribution in the band $[-R,R]$;

$$\hat{b} = \frac{\hat{B}}{\sum_{j=-R}^{R} \frac{\hat{C}_j}{R+1}}$$

Note that this measure is not unit-free and depends on the choice of binwidth $d$. When presenting our results we also report elasticities which are invariant to the unit of measurement and the binwidth $d$. When evaluating the elasticity, $k$ of equation (4) should be expressed in units of $d$. Standard errors are calculated using the bootstrap. We sample from the empirical distribution function associated with the observed income distribution and compute $\hat{b}$ repeatedly.

4 Institutional setting and data

4.1 Personal income taxation in Sweden

The basic structure of the Swedish statutory income tax system, which to a large extent is the result of the comprehensive 1991 reform, is simple. A proportional local tax rate applies to all earned income and taxable transfers. The mean local income tax rate in 2008 was 31.45%. The proportional local tax rate has been fairly constant during the period of study. For taxable labor income exceeding a certain threshold (SEK 340,900 in 2008, 1 USD = 7 SEK), the taxpayer also has to pay a central government income tax. This creates a large convex kink in the individual’s budget constraint, where the marginal tax rate increases by 20 percentage points in 2008.

\(^9\)We are grateful to John Friedman and Tore Olsen for making their code available to us.
\(^{10}\)As noted by these authors, a technical problem remains as this method will overestimate bunching because the counterfactual distribution does not integrate to 1. To see this, note that when estimating the counterfactual we use the observed income distribution of earnings excluding the income interval $[-R,R]$ around the kink. If there is bunching, the excluded bunching window contains excess mass consisting of individuals who, in the absence of the kink, would have located to the right of the kink. Hence it is necessary to increase the mass of the counterfactual distribution to the right of the kink until it integrates to one. Convex preferences guarantee that bunching individuals were located to the right of the kink point before the reform. It will be apparent from the Monte Carlo simulations of Section 7 that the correction procedure has minor impact on the results.
Since 1991 capital income is taxed separately from taxable labor income according to a proportional tax rate of 20 to 30 percent. Moreover, since 1971 the individual and not the family is the unit of personal taxation in Sweden. Thus, neither capital income nor spousal income is included in the tax base for the central government tax. Accordingly, uncertainty with respect to capital and/or spousal income does not translate into uncertainty with respect to the segment limits for taxable labor income.

During 1991-1998 the central government tax schedule contained only two brackets; the tax rate in the first bracket has always been zero. In 1995, the tax rate applying to taxable labor income exceeding the bracket cut-off was raised from 20% to 25%. In 1998 the jump in the marginal tax rate was 23.3 percentage points due to the existence of a tax deductible mandatory general pension contribution. The change in the log net-of-tax-share was 45.6% that year.\(^1\) In 1999, 20% (4%) of all taxpayers, or 37% (8%) of all full time employees earned taxable labor incomes above the first (second) central government kink point. Accordingly, the first central government is located centrally in the upper middle part of the income distribution. At the second central government kink point the income distribution is considerably thinner. Figure 2 shows the evolution of the central government tax schedule 1991-2008. Since 1995 there has been a steady, but non-dramatic, increase in the location of the first central government kink point.\(^1\)^2\(^1\)^3

How salient are these kink points to individual taxpayers? The central government kink points are salient in the sense that most taxpayers in these income ranges surely know about their existence. Still, it requires some degree of sophistication to trace out the exact locations of the bracket cut-offs as a function of taxable income. The reason is that the Swedish Tax Agency often reports segment limits using an income concept

\(^{11}\)In 1998 the average local tax rate was 31.65% and the general pension contribution was 6.95%. Since the general pension was deductible, the marginal tax rate for an individual located to the left of the central government kink point was \((1-0.0695)*0.3165+0.0695=0.364\). To the right, the marginal tax rate was \((1-0.0695)*(0.3165+0.25)+0.0695=0.597\).

\(^{12}\)The tax and benefit system also creates important (both convex and non-convex) kink points at lower parts of the income distribution. Some of those are generated by the basic deduction, which is phased in at lower income levels and phased out at higher income levels with consequences for the marginal tax rate facing individuals in these income ranges. Moreover, a system of housing allowances has for a long time been in place in Sweden. Housing allowances create large convex kinks (at the point where the phase-out of these allowances start) and large non-convex kinks (at the point where the entire allowance is taxed away). Other kink points are created by the study grants system and the social assistance system. The multitude of kinks at the bottom part of the income distribution – together with the substantial heterogeneity in budget sets across subpopulations – render bunching estimation problematic in these income ranges. In this paper we focus on the first and second central government kink points which are located in well-behaved parts of the income distribution.

\(^{13}\)As a general rule, the kink points of the central government schedule are “protected” against general real wage growth through indexation (Swedish Tax Agency (2010), p. 72 and the table at p. 92). Each year, the kink points are adjusted upwards by the inflation rate plus an additional 2 percentage points. However, in practice legislators have made small year-to-year deviations from this rule during the period of study. The kink points, expressed in nominal values of SEK, of the central government tax schedule of year \(t\) are legislated by parliament by the end of year \(t-1\). The kink points are assessed in terms of price base amounts (PBA). The PBA for year \(t\) is set based on the price level of June of year \(t-1\). Thus, information on the segment limits is publicly available to taxpayers before the start of the tax year.
Figure 2: The first central government kink point and the second central government kink point 1991-2008, segment limits are inflated by the consumer price index. (2008 prices). 1 USD = 7 SEK.

which does not correspond to the individual’s taxable labor income. Technically, to obtain the relevant bracket cut-off in terms of taxable income one needs to add back the so-called basic deduction and, before the tax year of 2006, the general pension contribution. Both the basic deduction and the deduction for the general pension contribution were mechanically provided by the tax authorities. The general pension contribution amounted to 6.95% of taxable labor income 1999-2000, but was gradually reduced 2001-2005 and, finally, completely abolished in 2006.

4.2 The components of taxable income

Table 1 shows a stylized characterization of the composition of the individual’s taxable labor income. Note that capital income, as well as deductions for interest expenses are absent. This is a consequence of the Swedish dual income tax system that taxes labor income separately from capital income. Notably, transfers, like unemployment insurance benefits, enter taxable income. As compared to a situation where these transfers were excluded from taxable labor income, the tax base becomes less volatile.

The most important deductions from taxable income are those for commuting expenses and private

\[14 \text{Our taxable labor income concept corresponds to the administrative concept ‘taxerad förvärvsinkomst’ and not the administrative Swedish concept ‘beskattningsbar förvärvsinkomst’}.\]
Table 1: Components of taxable income

<table>
<thead>
<tr>
<th>Sources of income</th>
<th>Deductions</th>
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<tbody>
<tr>
<td><strong>Employment income</strong></td>
<td><strong>Deductions from employment income</strong></td>
</tr>
<tr>
<td>Wages and salaries*</td>
<td>Commuting expenses</td>
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<tr>
<td>Fringe benefits</td>
<td>Expenses for official journeys</td>
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<tr>
<td>Sickness insurance benefits</td>
<td>Work-related living costs</td>
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<tr>
<td>Unemployment insurance benefits</td>
<td>Other expenses</td>
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<tr>
<td>Parental leave benefits</td>
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<tr>
<td>Public pension benefits</td>
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<tr>
<td>Occupational pension benefits</td>
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<tr>
<td>Other sources of earned income</td>
<td></td>
</tr>
<tr>
<td><strong>Business income</strong></td>
<td><strong>Deductions from business income</strong></td>
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<tr>
<td>Profits from active and passive sole proprietorships and partnerships</td>
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<tr>
<td><strong>General deductions</strong></td>
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<tr>
<td>Private pension contributions</td>
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<tr>
<td>Alimony paid</td>
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<tr>
<td>Business deficits</td>
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</tbody>
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*Includes wages and salaries from the own firm for owners of closely held corporations.

Pension contributions. Almost one half of the taxpayers in the vicinity of the first central government kink point claimed deductions for deferrals to tax-favored savings accounts in our sample. 25 % of taxpayers around the first central government made deductions for commuting expenses.

Is it easy for taxpayers to fine tune their deductions in such a way that they bunch at the kink point? While income tax returns for the tax year $t$ typically are due in early May year $t+1$, the individual is only eligible for deductions for expenses that occurred in year $t$. If an individual wishes to make pension contributions up until she reaches the kink (where the marginal tax price for pension contribution increases) she needs to make these contributions before the end of year $t$. However, as recently shown by Engström et al (2011), some deductions (e.g. the deductions for ‘other expenses’) leave room for manipulation at the time when the individual files her income tax return.

4.3 The role of indirect taxation

A large share of total taxes on labor income in Sweden is levied on the employer side in the form of social security contributions (payroll taxes).\textsuperscript{15} The payroll tax rate, expressed as a percentage of the wage bill,\textsuperscript{13}

\textsuperscript{15}In 2008, SEK 393 billion were paid by the employers to the tax authorities as social security contributions (Table 8, The Swedish Tax Agency, 2010). In the same year, SEK 413 billion was collected in personal income taxation (net of tax reductions). Total tax revenues amounted to SEK 1,495 billion.
was fairly constant, around 32-33%, during the period of study. Consumption taxes are also quantitatively important. The effective value added tax (VAT) rate, expressed as a percentage of consumption, is currently around 21% (Pirttilä and Selin, 2011). Since our work is motivated by a static model of taxable income supply, where the entire tax incidence falls on individuals, a given tax on consumption or a payroll tax can equivalently be expressed as a personal income tax.

Neither payroll taxes nor consumption taxes do, however, affect the change in the log net-of-tax rate. To see this, note that the bunching formula (4) which is required to recover the elasticity locally at the kink point, includes the log of the net-of-tax ratio, \( \log \left( \frac{1 - \tau_1}{1 - \tau_2} \right) = \log \left( \frac{1 - \tau_1}{1 + \tau_{VAT}} \right) \frac{1 - \tau_2}{1 + \tau_{VAT}} \), where \( \tau_p \) is the proportional payroll tax rate and \( \tau_{VAT} \) is the value added tax rate as a percentage of consumption. Hence, given that the indirect marginal taxes do not change at the kink \( k \), the factors related to \( \tau_p \) and \( \tau_{VAT} \) cancel out in this expression.\(^\text{16}\) Still, as the payroll tax induces a difference between the tax base for personal income taxation and the individual’s true before-tax income, the payroll tax needs to be taken into account when evaluating the elasticity. Therefore, when calculating \( k \) in (4) we multiply the personal income tax base (‘taxerad förvärvsinkomst’) by \( (1 + \tau_p) \).

### 4.4 Data and sample selection

This study exploits two (partly overlapping) data sets. To study bunching behavior among wage earners we use administrative tax records covering the whole universe of Swedish taxpayers through the years 1998-2005.\(^\text{18}\) The data set entails variables corresponding to the boxes of the personal income tax return form. In addition, we have information on some demographic characteristics. Unless otherwise stated, in the empirical analysis of section 4 we remove ‘self-employed’ individuals from the sample of ‘wage earners’. For these purposes we define self-employment in the following way. We pool data for 1999-2005 and define those who either report positive active business income or are considered as being connected to a closely held

---

\(^{16}\)Using the notation of Section 2, while introducing VAT and payroll taxation, we can write a linearized budget constraint at segment \( i \) as \((1 + \tau_{VAT}) c = \left( \frac{1}{1 + \tau_p} \right) \left( \frac{1 - \tau_1}{1 + \tau_{VAT}} \right) + y \) or \( c = \left( \frac{1 - \tau_2}{1 + \tau_{VAT}} \right) \left( \frac{1}{1 + \tau_p} \right) z + \left( \frac{1}{1 + \tau_{VAT}} \right) y \), where \( z \) is the employer’s wage cost, i.e. the individual’s before tax return to work effort and \( y \) is virtual income.

\(^{17}\)Even though payroll taxes are proportional it can still be an issue that the social security contributions paid by the employer generate social benefits only up to a certain ceiling. In particular, individuals earn future social security benefits up to a taxable labor income level of 7.5 income base amounts (7.5 price base amounts up to 2000). During the period of study, 1998-2008, the mean distance between the pension kink and the first central government kink point, was SEK 17,250. The distance was the lowest in 2002 (SEK 990) and the largest in 1998 (SEK 40,470).

\(^{18}\)This is the same data set that was used by Selin (2011).
Table 2: The marginal tax change at the central government kink points 1998-2008

<table>
<thead>
<tr>
<th></th>
<th>First central government kink point</th>
<th>Second central government kink point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_1$</td>
<td>$\tau_2$</td>
</tr>
<tr>
<td>1998</td>
<td>0.364</td>
<td>0.597</td>
</tr>
<tr>
<td>1999</td>
<td>0.366</td>
<td>0.54</td>
</tr>
<tr>
<td>2000</td>
<td>0.34</td>
<td>0.53</td>
</tr>
<tr>
<td>2001</td>
<td>0.33</td>
<td>0.523</td>
</tr>
<tr>
<td>2002</td>
<td>0.317</td>
<td>0.514</td>
</tr>
<tr>
<td>2003</td>
<td>0.324</td>
<td>0.52</td>
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<tr>
<td>2004</td>
<td>0.327</td>
<td>0.524</td>
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<tr>
<td>2005</td>
<td>0.322</td>
<td>0.52</td>
</tr>
<tr>
<td>2006</td>
<td>0.316</td>
<td>0.516</td>
</tr>
<tr>
<td>2007</td>
<td>0.3155</td>
<td>0.5155</td>
</tr>
<tr>
<td>2008</td>
<td>0.3144</td>
<td>0.5144</td>
</tr>
</tbody>
</table>

corporation any of those years as self-employed.\textsuperscript{19} As in Chetty et al (2011a), we restrict the sample to individuals who are aged between 15 and 70.

We conduct a special analysis of the self-employed for the years 2000-2008 on a data set that is particularly suited for such an exercise. The FRIDA (‘Företagsregister och individdatabas’) contains individual level tax register data for the main groups of self-employed; sole proprietors, partnership owners and owners of closely held corporations. The two former organizational forms are taxed at the personal level, whereas closely held corporations are separable taxable entities. The FRIDA data contains the total Swedish population of self-employed individuals complemented with firm level data.

5 Empirical Analysis

Bunching estimation is a genuinely visual technique. Accordingly, we report each estimate of the excess mass and the implied elasticity in a figure, along with a graph of the corresponding income distribution locally around the kink point under study as in Chetty et al (2011a). For each year, we express the taxable income variable in the price level of 2008 (unless otherwise stated), and we redefine the taxable income variable such that it takes on the value of zero at the bracket cut-off. After that, we pool data from several years. The histogram is displayed as a series of dots. The solid line represents the polynomial fitted to the taxable

\textsuperscript{19}The relevant variables are \textit{nakte} ("inkomst av aktiv enskild näringsverksamhet"), \textit{nakthb} ("inkomst av aktiv näringsverksamhet för delägare i handelsbolag") and \textit{bfoab} ("kod för samgranskning med fämansföretag"). The \textit{bfoab} variable is not present in the 1998 data. For 1998 we therefore let the self-employment dummy take on the value of 1 if the individual reports positive business income in 1998 or if the individual is self-employed any of the years 1999-2005.
income distribution while excluding bins in the ‘small bunching window’. In our study we use an interval of [-SEK 5,000, SEK 5,000] around the kink point as our baseline.

5.1 Wage earners

Figure 3a shows the taxable income distribution locally around the first central government kink point, 1999-2005, for the total population that includes both wage earners and self-employed. 1999-2005 the central government tax schedule was very stable and the reduction in log net-of-tax rate at the bracket cut-off was in the range 32.1% - 34.6% (see table 2). Figure 3a shows that there is a statistically significant excess density in an interval close to the first central government kink point. As explained above, the excess mass \( b \) should be interpreted as the number of individuals who bunch divided by the average frequency of taxpayers in the range [-SEK 5000, SEK 5000]. However, even though the elasticity is statistically significant it is not significant in an economic sense – the implied elasticity is 0.003. Figure 3b displays the same information when self-employed have been removed from the estimation sample. It is striking that there is no significant excess mass in the aggregate when wage earners is the sole scope of focus. Taken literally, in the frictionless model this estimate of the excess mass implies a precise estimate of the compensated taxable income elasticity of zero. In Section 5 we elaborate more on how one can interpret this zero estimate. It is also noteworthy that the income distribution takes on a trapezoid shape in the estimation window when all wage earners in different years are pooled.

We have also investigated bunching at the second central government tax kink, where the log net-of-tax rate decreases by around 11% 1999-2005. Not surprisingly, there is no evidence of bunching for high income wage earners at this substantially smaller second kink point.

It is a convention in the labor supply literature to examine different demographic groups separately. Therefore, we partition the sample of wage earners into single women, married women, single men and married men.\(^{20}\) Figure 4a-4d report similar graphs for these sub groups. Apparently, there is no significant bunching of taxpayers at the large and salient kink point for any of these demographic groups.\(^{21}\)

Real labor supply responses can be difficult to fine tune in a world with optimization frictions. Deductions,

\(^{20}\) A limitation of Swedish register data, probably without any practical importance in this context, is that cohabiting individuals without common kids are considered as singles.

\(^{21}\) In addition, we have performed bunching analysis of a number of other subgroups of wage earners with the same results.
Figure 3: All individuals vs. all wage earners.
Figure 4: Wage earners by group.
on the other hand, are under the taxpayers' direct control. To find out whether those who claim large
deductions bunch we separately examine wage earners with deductions over SEK 50,000, a group which
constitutes around one percent of the population. As can be seen from figure 5, there is no significant
bunching at the first central government kink among those who make large deductions.

\[
\begin{align*}
\text{Excess mass (b)} &= 0.067 \\
\text{Standard error} &= 0.127 \\
\text{Implied elasticity} &= 0.000
\end{align*}
\]

Figure 5: Wage earners with deductions > 50,000 SEK

In 1998 there was a 23.2 percentage points jump in the marginal tax rate at the first central government
kink point, implying a 45.6% reduction in the log net-of-tax rate at the kink. As we emphasize in section
6, the size of the kink is of great theoretical importance. Therefore, we analyze the 1998 data separately.
Figure 6 reveals that that the density of taxpayers did not display any major spike or hump around the
bracket cut-off in 1998 but there is an extremely small increase in the density at the kink. Due to the high
degree of precision in the polynomial fit, the excess density is statistically significant. However, the implied
elasticity estimate is 0.001 and not in any sense economically significant from zero. Thus, we consider the
zero result for wage earners to be very robust. \(^{22}\)

\(^{22}\)In addition, we have analyzed the time periods 1991-1994 (when the jump in marginal tax rates at the central government
kink was 20 percentage points) and 1995-1998 (when the jump was similar in size to the jump in 1998) using the smaller
register-based data-set "LINDA" which contains around 3% of the Swedish population. The zero result for wage-earners is very
robust.
5.2 Self-employed

The estimation sample for the self-employed contains the total Swedish population of owners of closely held corporations, sole proprietors and partnership owners. In the same spirit as above, we pool the years 2000-2008. Figure 7 reports histograms and elasticity estimates for the total group of self-employed individuals and for these three groups. In contrast to the wage earner sample, there is clear evidence of bunching at the first central government kink where the marginal tax rate jumps by 20 percentage points. But the implied compensated elasticity estimate is small. In the pooled sample the estimated elasticity at the first central government kink is 0.018.

There is some interesting heterogeneity across the subsamples in Figure 8. The corporate owners in panel (b) display a somewhat smaller elasticity than the other categories under the standard choice of bunching interval around the kink [-SEK 5,000, SEK 5,000]. On the other hand, the histogram plot clearly suggests that there is broader hump of corporate owners around the threshold for central government taxation. As mean incomes are higher for corporate owners than for the rest of the population it actually turns out that the central government kink coincides with the mode of the taxable income distribution for owners of closely held corporations 2000-2008. It lies beyond the scope of the present analysis to assess to what extent this phenomenon reflects 'broad bunching' (in the language of Chetty et al 2011) around the first central...
government kink point.

The taxable income distribution for the sole proprietors locally around the first central government kink, which is visualized in panel (c), is considerably more triangular shaped than its counterpart for corporate owners. There is a clear spike in the observed density in a narrow range around the bracket cut-off. The implied elasticity is 0.019, i.e. still very small. The partnership owners (panel d) exhibit a similar response as sole proprietors. A large spike is discernible at the first central government kink point, but the implied elasticity is only 0.018. For all groups of self-employed the elasticity estimate is statistically distinct from zero.

One might wonder how sensitive the estimation results are to the choice of the width of the ‘small bunching window’. The baseline, i.e. [-SEK 5000, SEK 5000], was chosen based on visual inspection of the histogram plots for the self-employed. Appendix C shows that deviating from the baseline choice only has minor consequences for the obtained results. When the ‘small bunching window’ increases, a larger number of bins are excluded from the polynomial fit. Hence, the standard error increases for larger windows. This effect is especially pronounced for owners of closely held corporations. We have also experimented with asymmetrical windows around the threshold with no major changes to the results (see appendix C). Finally, the results are also, by and large, robust to changes in the wide bunching window, where the baseline width is [-SEK 75,000 , SEK 75,000].

Our large sample sizes allow us to examine subgroups that can be expected to be particularly responsive to marginal tax rates. One such candidate is the subcategory of ‘purely self-employed’, i.e. those who do not simultaneously act as wage earners. We define ‘pure’ corporate owners as those whose earned income exclusively come from the firm they own. ‘Pure’ sole proprietors and partnership owners, who both are taxed at the personal level, are defined as exclusively reporting business income from the relevant organizational form and no wage income. Based on figure 8, panel (a), we infer that the excess mass at the first central government kink point almost increases by a factor three as compared to the case when examining the total aggregate of self-employed, 2000-2008.

Interestingly, in the sample containing the ‘pure’ self-employed there is also discernible bunching in the

---

23This interval was also used for wage earners. In practice, the choice of small bunching window for wage earners is of very little importance as there are no visual indications of bunching among wage earners.
Figure 7: Self-employed.
Excess mass (b) = 8.089
Standard error = 0.171
Implied elasticity = 0.052

Figure 8: Purely self-employed.
immediate vicinity of the second central government kink point, where the log net-of-tax share is reduced by almost 11%. Expressed in the price level of 2008, during the time period 2000-2008 the average value of the second central government bracket cut-off was SEK 487,000 (as compared to SEK 325,000 for the first central government kink). Statistically, owing to the small number of observations in these higher income ranges, the excess mass estimate is more uncertain. However, the estimate is indeed statistically significant from zero at a level of 1 percent (two-tailed confidence interval). Needless to say, the implied elasticity is still low, 0.012.

5.3 Comparison with other studies

It is worth noting that the results we obtain here are qualitatively different from those obtained by Chetty et al (2011a) on Denmark, a neighboring country that share many institutional features with Sweden (e.g. a high rate of unionization and a high tax-to-GDP-ratio). Chetty et al find clear bunching among Danish wage earners at the cut-off for the so-called top tax, even though the implied elasticities were small. Needless to say, with only two observations we lack means to assess the causal determinants of bunching at a cross country level. However, some aspects of Danish and Swedish tax law deserve to be discussed.

According to Danish tax law, the movement in the top tax bracket from year $t$ to year $t+1$ is predetermined by the wage growth from year $t-2$ to $t-1$. This makes the movement in the kink more predictable from the perspective of the unions and employers’ organizations. In Sweden, on the other hand, discretionary year-to-year changes in the bracket-offs have been made during the period of study. Seen from a broader perspective, the most striking difference between Sweden and Denmark with respect to the taxation of labor incomes is that no payroll taxes are levied on the employer in Denmark (see p.260 in OECD (2011)). It is not clear, though, why this would lead to more bunching in the Danish case.

Bunching for self employed individuals is also more prevalent in the Danish case. Chetty et al (2011a) reports an elasticity of 0.24 for self employed, which is an order of magnitude larger than our corresponding estimate. Relatively large elasticity estimates on Denmark for the self employed have also been obtained by Kleven et al (2011) and Schjerning and le Maire (2011). As remarked by Slemrod and Kopczuk (2002), the taxable income elasticity is not only a function of utility parameters, but also a function of the tax system.
itself (e.g. the possibilities for tax avoidance). Hence, it is likely that cross country differences in bunching behavior among the self employed can be traced to specific features of the tax code (e.g. the rules governing retention funds).

6 Bounds on the taxable income elasticity

As mentioned in the introduction, the empirical prediction of the standard model regarding the shape of the income distribution around the large Swedish first central government kink point is extremely clear. In the absence of optimization frictions we should see a large spike in the taxable income distribution at the kink, even when the compensated taxable income elasticity is very small. The failure of the standard model in accurately predicting the behavior of agents around kink points does not necessarily imply that long run responses to taxation are absent. In fact, positive compensated elasticities can be consistent with zero bunching if one assumes that individuals make optimization errors. From the perspective of the econometrician, this introduces concerns about identification. Both the distribution of elasticities (preference parameters) and the distribution of optimization errors in the population are unobserved. In order to make progress, it is necessary to impose assumptions on the distribution of optimization errors, otherwise nothing can be said about the elasticity. In the non-linear budget set literature (e.g. Burtless and Hausman (1978) and Blomquist and Newey (2002)) point identification of the elasticity has been achieved by assuming that observed behavior is equal to desired behavior plus an additive error term which is orthogonal to the net-of-tax wage.

To our knowledge, the most comprehensive and systematic analysis of optimization errors in the labor supply context is to be found in the recent work by Chetty (2011). Influenced by the partial identification literature in econometrics, Chetty replaces the orthogonality condition with a bounded support condition. Chetty sets up a dynamic model of consumer demand, where individuals are allowed deviate from the frictionless optimum in an arbitrary fashion provided that the expected life time utility cost of doing so is less than $\delta$ percent of consumption expenditure. In accordance with the non-linear budget set literature, no particular structure is imposed on the mechanisms generating the error. In this way, the optimization error can be reconciled with a large number of frictions, including adjustment costs and inattention.
An outcome in Chetty’s framework is that because the utility cost for an individual of ignoring a large tax change is larger than the utility cost of neglecting a small one, elasticities estimated using large tax reforms are closer to structural elasticities than ‘observed elasticities’ obtained by using small tax reforms. In this manner Chetty (2011) is able to derive bounds on the size of the compensated labor supply elasticity as a function of the size of the tax change (captured by the log of the net-of-tax ratio) and the utility cost \( \delta \) individuals on average tolerate (expressed in percent of net-of-tax earnings). Of particular interest for our purposes is the case when the observed elasticity is zero. Then the upper bound of the compensated elasticity takes on the following simple form

\[
\varepsilon_U = \frac{8\delta}{(\Delta \log(1 - \tau))^2}
\]

Note that the upper bound increases linearly in \( \delta \), but shrinks at a quadratic rate in \( \Delta \log(1 - \tau) = |\log(1 - \tau_2) - \log(1 - \tau_1)| \). Thus, there is considerably more information inherited in large tax changes as compared to small ones. In our present application we have estimated an observed compensated elasticity of zero for wage earners. The two first rows of Table 4 report the upper bounds implied by our bunching estimation exercise for 1998 and 1999-2005, respectively, for three different values of \( \delta \). Chetty’s (2011) baseline assumption is \( \delta = 1 \). While adopting Chetty’s baseline assumption, the upper bound on the compensated elasticity is 0.7 for the 1999-2005 period, but only 0.39 for 1998. The latter is a surprisingly informative bound. Note also that the small standard error of the point estimate makes statistical imprecision irrelevant for the problem at hand. Following Chetty we calculate confidence intervals along the lines of Imbens and Manski (2004).

How do these bounds relate to the bounds implied by other studies? Recently, Blomquist and Selin (2010) estimated the compensated taxable income elasticity while exploiting Swedish panel data and unprecedentedly large marginal tax cuts for high income earners between 1981-1991. For top income earners, marginal tax rates were reduced by 34 percentage points. Blomquist and Selin obtained a compensated taxable income elasticity estimate of 0.24 for married men. The large variation generates tight bounds; the lower bound

\[\text{24 The bounds, expressed in Proposition 1 in Chetty (2011), are derived using a quadratic approximation to the utility function. Chetty also uses the assumption that the utility functions are iso-elastic and quasi-linear, but shows that Proposition 1 is still valid for more general utility functions if one imposes the local iso-elastic assumption on the structural Hicksian elasticity.}\]
being 0.12 and the upper bound being 0.5 for $\delta = 1$ when the statistical uncertainty is disregarded. Still, it is noteworthy that the upper bound of 0.39 obtained in our bunching analysis for the year 1998 is lower than the corresponding upper bound calculated based on Blomquist and Selin.\textsuperscript{25} This point can be pushed even further if one also takes the statistical uncertainty into account. Owing to the high degree of statistical precision in our estimates the upper end of the 95 % confidence interval virtually coincides with the upper end of the identified set of elasticities consistent with $\delta = 1$.\textsuperscript{26}

Chetty (2011, Table 1) reports bounds for $\delta = 1$ for the paper by Gelber (2011), a related study that exploits the Swedish 1991 reform to estimate compensated elasticities in a family model (including cross responses between spouses). A point estimate of 0.25 for males translates into a narrow interval of 0.12 to 0.54, i.e. a very similar band as that implied by the paper by Blomquist and Selin (2010).\textsuperscript{27}

When comparing our bunching analysis with Gelber (2011), Blomquist and Selin (2010) and Hansson (2007) the following cautious remark should be made. While the previous studies typically have estimated behavioral parameters for the whole population, our bunching analysis recovers a local elasticity estimate at a specific region of the upper middle part of the income distribution. Thus, in so far the male response is driven by income reporting responses of top income earners the two types of studies will reflect distinct behavioral parameters.

For married females, Blomquist and Selin found a compensated elasticity of 1.40 for married women. As shown in Table 4, this large intensive margin estimate translates into very high upper bounds, e.g. 2.26 in the baseline case.\textsuperscript{28} If the large female elasticities reflect choices along the half time / full time margin, those earlier estimates are not so relevant as a comparison for the local elasticity estimate obtained in our study. The reason is that the first central government kink point is located at an earnings level where, in principle, all individuals work full time.

\textsuperscript{25}The lower bound is always zero if the observed elasticity is zero.
\textsuperscript{26}For $\delta = 2$ (the rightmost column) the upper bound implied by the 1998 bunching analysis is 0.77, whereas the upper bound implied by the work by Blomquist and Selin is lower (at least when neglecting the statistical uncertainty). The reason is that the upper bound increases linearly in $\delta$ when the observed elasticity is 0, but grows at a decreasing rate in $\delta$ when the observed elasticity is positive.
\textsuperscript{27}Hansson (2007), who exploits the same data source and tax variation as Gelber (2011), obtains a taxable income elasticity of 0.29 for males (Table 2, second column). This estimate cannot, however, be interpreted as the compensated taxable income elasticity.
\textsuperscript{28}Even though the point estimate for females was significant at 10% the standard errors were considerably larger for females. On a larger data source, Gelber (2011) obtained a point estimate of 0.49 and Hansson estimated an elasticity of 0.76, both with smaller standard errors.
7 Monte-Carlo Simulations

In the last couple of years there has been a surge of papers exploiting the bunching methodology. Still, except from the early incarnations of Saez’s pioneering paper, i.e. Saez (1999, 2002) we are not aware of any systematic evaluation of the bunching estimator by the means of numerical simulations. The purpose of this section is to evaluate the bunching estimator by simulating data under various assumptions regarding the process generating the data. In each case we attempt to recover the taxable income elasticity by running the estimation method on the simulated data.

7.1 Income Effects

In the appendix we show analytically that income effects do not affect the bunching estimator when the tax change is small. Here we analyze how bunching estimates are affected by the influence of income effects in the decision to supply taxable income given an arbitrary tax change. As our benchmark we specify the utility function without income effects following Saez (2010)

\[ u(c, z) = c - \frac{1}{1 + z_0^{\frac{1}{\epsilon}}} z_0 \left( \frac{z}{z_0} \right)^{1+\frac{1}{\epsilon}}. \]  

(7)
Along any linear segment of the budget constraint with slope \((1 - \tau)\) an individual's taxable income is given by \(z = z_0(1 - \tau)e\) where \(z_0\) is an individual-specific parameter. The (compensated) taxable income elasticity is equal to \(e\). To introduce income effects we let preferences be represented by the utility function

\[
\begin{align*}
  u(c, z) = \log(c) - \log \left( 1 + \frac{1}{1 + \frac{1}{e} z_0} \left( \frac{z}{z_0} \right)^{1 + \frac{1}{e}} \right)
\end{align*}
\]  

This is the taxable income counterpart of the utility function used by Saez (2001) and is the simplest possible way to include income effects keeping the compensated elasticity constant and equal to \(e\). As is well known, the uncompensated elasticity \(\epsilon^u\) is related to the compensated elasticity \(e\) through the Slutsky relationship in elasticity form \(\epsilon^u = e + \eta\) where \(\eta = \frac{dz}{dy}(1 - \tau)\) is the income effect and \(y\) is virtual income.

The optimal choices of taxable income by individuals, whether given by maximization of (7) or (8), is determined by a structural equation \(z = z(z_0, e, \theta)\) where \(z_0\) is a vector of individual-specific parameters, \(\theta\) is a vector of tax parameters and \(e\) is the the scalar elasticity parameter. Let \(z^A\) denote the vector of optimal choices under the benchmark specification and denote by \(z^B\) the corresponding vector given specification (8). In the first case we know that \(z^A = z_0^A(1 - \tau)e\) and \(z_0^A\) has the convenient interpretation of taxable income in the absence of taxation \((\tau = 0)\). In the second case a closed form solution is not obtainable so we simply write \(z^B = g(z_0^B, e, \theta)\) for some vector-valued function \(g(\cdot)\) and compute these optimal decisions numerically.

Since the goal is to investigate the impact of income effects on bunching holding the economic environment constant, we proceed in the following way. We fix a baseline flat income tax system given by a constant marginal tax rate \(\tau = 0.3\) and pick a value of the elasticity \(e\). We sample \(z_0^A\) from a triangular distribution chosen in such a way so that the distribution of \(z^A = z(z_0^A, e, \tau)\) resembles the (actual) empirical taxable income distribution locally around an income level \(k\) where we wish to investigate bunching. The distribution of \(z^A\) under the flat income tax system is defined as the counter-factual distribution. Since we want to keep the counter-factual income distribution constant as we compare bunching with and without income
effects, we calibrate $z^B_0$ so that the distribution of $z^A$ and $z^B$ lie arbitrarily close under the flat income tax.\textsuperscript{32}

Next, given that $z^A_0$ and $z^B_0$ have been chosen appropriately, we introduce a kink into the tax system and calculate the optimal choices $z^A$ and $z^B$. Then, we estimate bunching around $k$ using the distribution of $z^A$ and $z^B$ to obtain the bunching estimates $\hat{b}^A$ and $\hat{b}^B$ respectively. This in turn allows us to recover estimates of $e$ through the formula

$$\hat{e} = \frac{\hat{b}}{k \log \left( \frac{1-\tau_1}{1-\tau_2} \right)}$$

The two top panels of figure 9 show the post-reform distribution of income under the kinked tax system; taxable income income exceeding 250000 SEK is taxed at the rate $\tau = 0.55$. For comparison, the bottom panels of figure 9 show the simulated taxable income distribution under the flat income tax where $\tau = 0.3$ applies on both segments. Note that we, unlike in the empirical exercise, plot the simulated income distribution against a logarithmic scale. Otherwise, the spike would simply have been too large to visualize in a conventional graph of reasonable size. The compensated elasticity is equal to 0.30 and the (local) uncompensated elasticity is approximately equal to 0.05.\textsuperscript{33}

In figure 10 we have applied the estimator described in section 3 on the simulated data without any correction of the integration constraint, and we use a width of the small bunching window of 1000 SEK. We find $\hat{b}^A = 33.18$ and $\hat{b}^B = 32.7$. As expected, the bunching estimate for the specification with income effects is slightly attenuated, but the effect is practically unimportant. The implied elasticity estimates are 0.3004 and 0.2960 respectively which both lie very close to the structural compensated elasticity of $e = 0.3$ used to simulate the data.\textsuperscript{34} We have carried out the exercise for other elasticity values with the same results. We conclude that the bunching estimator allows us to recover the elasticity parameter used to generate the data with good accuracy, even in the presence of income effects.

\textsuperscript{32}More precisely, we choose the vector $z^B_0$ so that $\|z^A - g(z^B_0, e, \tau)\|_\infty$ is small.

\textsuperscript{33}The income effect term is $\eta = -0.2578$ and is obtained through simulation using standard methods. It is defined as the average income effect across all agents within an interval of 100 000 SEK centered around the kink under the flat income tax system as they face a 10% tax rate reduction.

\textsuperscript{34}We have performed the estimation with and without the 'correction' of the integration constraint with minor differences in the resulting estimates. Hence the correction procedure was not necessary to obtain the correct elasticity and seems to be of minor importance, at least given the transformation of the income distribution implied by the utility function used in our simulations.
Figure 9: Simulated income distributions under flat income income tax (bottom panel) and reformed tax system (top panel), logarithmic scale.

7.2 Other Exercises

The framework of Chetty (2011) allowed us to put bounds on the structural elasticity consistent with a wide variety of structural primitives. Another approach mentioned in section 6 is to let individuals deviate from their optimal income choices through a process which is independent of the underlying tax variation (under such a process the observed elasticity should not depend on the size of the tax change for example). One such approach is to let individuals’ optimal income choices be subject to additive optimization errors. Such a procedure has been emphasized by the nonlinear budget set literature (such as Burtless and Hausman 1978, Blomquist 1983 and Heim 2009). We have performed simulations of a model where individuals’ taxable income is given by $\tilde{z} = z^* + \eta$ where $\eta \sim N(0,\sigma^2)$ is the difference between individuals’ actual (observed) income choices $\tilde{z}$ and desired income choices $z^*$. Note that under such a specification we should observe a hump in the income distribution at the kink rather than a clear spike. We have adopted an estimate of $\sigma$ of 1000 SEK calculated from Blomquist (1983) and based on the distribution of $\tilde{z}$ we were able to recover the
Figure 10: Estimation results for bunching with and without income effects.


8 Concluding Remarks

Economic theory predicts that, if preferences are convex and smoothly distributed in the population, we should observe an excess mass (bunching) of taxpayers at convex kinks of the budget constraint. In this paper we have estimated bunching of taxpayers at a particularly large kink point in the upper middle part of the Swedish income distribution. During the period of study, the percentage change in the net-of-tax rate at the kink reached a maximum value of 45.6%. By means of numerical simulation, we have illustrated that there is a huge discrepancy between the amount of bunching implied by the standard model and the shape of the actual income distribution locally around the first central government kink point. We found no economically significant bunching of wage earners at the large first central government kink point, implying a local estimate of the compensated taxable income elasticity of zero in these income ranges. The self-employed do bunch on the other hand, but the implied elasticities are small.

The main contribution of this study has been to examine a very large kink point in the upper middle part of the income distribution where a large number of taxpayers are located. This allowed us to derive upper bounds on the compensated elasticity along the lines of Chetty (2011) which are surprisingly tight. If wage earners on average tolerate 1% of their disposable income in optimization costs, the upper bound is 0.39 for the year 1998.

We have also contributed methodologically by investigating the role of income effects in bunching estimation. While the bunching estimator recovers the compensated taxable income elasticity for infinitesimal tax changes, Monte Carlo simulations are necessary to show that this also holds true for large tax changes. This is especially relevant to our study since we are exploiting a large kink. Our results indicate that the bunching estimator is largely unaffected by the presence of income effects, even when the income effect term is large relative to the compensated elasticity. This result cannot, of course, be immediately generalized to

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35One source of uncertainty in taxable income is the uncertainty of wage earnings due to optimization errors in hours of work. Blomquist (1983) estimates a yearly standard deviation in annual hours of work due to optimization error/measurement error of 8.07. Hence for an individual with an hourly wage rate of 120 SEK, an estimate of $\sigma$ is $120 \cdot 8.07 = 968.40 \approx 1000$ SEK. In the estimation, the bandwidth must be chosen appropriately through visual inspection. In this case the interval $[-5000, 5000]$ turned out to be satisfactory. We performed this exercise both under the assumption that $\eta$ is unanticipated and under the assumption that $\eta$ is anticipated (so that individuals maximize expected utility) with negligible differences in the resulting estimates.
broader classes of utility functions. However, it clearly suggests that income effects create a second-order problem in the context of bunching estimation.

Historically, many studies of labor supply and taxable income responses have been undertaken on Swedish data. Most of these studies obtain behavioral elasticities that predict a certain amount of bunching at kink points. A final, perhaps trivial, contribution of our paper is that we are, to our knowledge, the first researchers to carefully examine the Swedish income distribution locally around kink points.

References


A Income effects with a small tax change

Consider without loss of generality a two segment piece-wise linear tax schedule. On each linearized segment of the budget constraint the optimal choice of $z$ is a function $z = z(1 - \tau, y_i)$ of the marginal net-of-tax rate $\tau_i$ and the virtual income $y_i$ facing the individual, $(i = 1, 2)$. Before the reform $\tau_1 = \tau_2 = \tau$ and $y_1 = y_2 = m$. After the reform $\tau_2 > \tau_1$ and $y_2 > y_1$. Suppose that the tax change is small so that $\tau_2 - \tau_1 = d\tau$. Consider an agent locating to the right of $k$ before the reform. The total derivative of the optimal supply function $z(1 - \tau, y)$ is
\[ dz = \frac{\partial z}{\partial (1 - \tau)} d(1 - \tau) + \frac{\partial z}{\partial y} dy \quad (9) \]

Using the Slutsky relationship \( \frac{\partial z}{\partial (1 - \tau)} = \frac{\partial z^c}{\partial (1 - \tau)} + \frac{\partial z}{\partial y} \) where \( z^c \) is the compensated supply function we can rewrite (9) as

\[ dz = \frac{\partial z^c}{\partial (1 - \tau)} d(1 - \tau) + \frac{\partial z}{\partial y} (dy + zd(1 - \tau)) \quad (10) \]

The textbook definition of virtual income for segment \( i \) is

\[ y_i = y_{i-1} + [(1 - \tau_{i-1}) - (1 - \tau_i)]k_i, \]

where \( k \) is the lower end point of the \( i \):th segment for all \( i > 1 \) (c.f. Blundell and MacCurdy 1999). On the first segment, \( y_1 = m \). Accordingly, virtual income on the second segment is

\[ y_2 = (\tau_2 - \tau_1)k + m = d\tau \cdot k + m. \]

Hence \( \frac{dy_2}{d\tau} = k \) which we rewrite as \( dy_2 = -d(1 - \tau)k \). Thus the relevant change in virtual income for an individual who before the reform reported \( z > k \) is \( dy = -d(1 - \tau)k \). Insertion into (10) yields

\[ dz = \frac{\partial z^c}{\partial (1 - \tau)} d(1 - \tau) + \frac{\partial z}{\partial y} d(1 - \tau) \left[ z - k \right] \quad (11) \]

The first term in the above equation is simply the compensated taxable income response whereas the second term arises because of the income effect. Intuitively, this term is larger, the larger is the share of income exceeding \( k \) (as reflected by \( [z - k] \)). However, for a small tax change, the interval of taxpayers who bunch at the kink point is \( [k, k + dz] \) i.e. the marginal individual who bunches at the kink point is located close to \( k \) before the reform. Hence, the second term in (11) contains the product of two infinitesimal terms and is therefore of second order.
B Proof that the compensated elasticity is equal to $e$.

Maximizing (8) with respect to $z$ leads to the first order condition

$$\frac{(1-\tau)}{z(1-\tau) + y} = \frac{1}{\left(1 + \frac{1}{1+\frac{1}{\tau}z_0} \left(\frac{z}{z_0}\right)^{1+\frac{1}{\tau}}\right)} \left(\frac{z}{z_0}\right)^{\frac{1}{2}}$$

Taking logs yields

$$\log(1-\tau) - \log(z(1-\tau) + y) = \frac{1}{e} (\log z - \log z_0) - \log \left(1 + \frac{1}{1+\frac{1}{\tau}z_0} \left(\frac{z}{z_0}\right)^{1+\frac{1}{\tau}}\right)$$

Differentiation of the above expression w.r.t. $(1-\tau)$ yields

$$-\frac{1}{(1-\tau)} + \frac{1}{e} \frac{dz}{d(1-\tau)} \frac{1}{z(1-\tau) + y} \left[(1-\tau) \frac{dz}{d(1-\tau)} + z\right] = \frac{1}{\left(1 + \frac{1}{1+\frac{1}{\tau}z_0} \left(\frac{z}{z_0}\right)^{1+\frac{1}{\tau}}\right)} \left(\frac{z}{z_0}\right)^{\frac{1}{2}} \frac{dz}{d(1-\tau)} \quad (12)$$

On the other hand, direct differentiation of the utility function (8) with respect to $(1-\tau)$ and setting it to zero yields

$$\frac{1}{(1-\tau)z + y} \left[z + (1-\tau) \frac{dz}{d(1-\tau)}\right] = \frac{1}{\left(1 + \frac{1}{1+\frac{1}{\tau}z_0} \left(\frac{z}{z_0}\right)^{1+\frac{1}{\tau}}\right)} \left(\frac{z}{z_0}\right)^{\frac{1}{2}} \frac{dz}{d(1-\tau)} \quad (14)$$

The above equation characterizes movements along a given indifference curve $U$. Combining (12) and (14) we obtain the marginal effect of a tax change, keeping utility constant, hence

$$\frac{1}{(1-\tau)z + y} \left[z + (1-\tau) \left.\frac{dz}{d(1-\tau)}\right|_U\right] = -\frac{1}{(1-\tau)} + \frac{1}{e} \frac{dz}{d(1-\tau)} \left|_U^{1-\tau}\right. \frac{1}{z(1-\tau) + y} \left[ z + (1-\tau) \left.\frac{dz}{d(1-\tau)}\right|_U\right]$$

Cancelling the common terms in the above equation and using the definition of the elasticity we obtain

$$\frac{dz}{d(1-\tau)} \left|_U^{1-\tau}\right. \frac{1}{z} = e$$
which was to be proven.

C Summary Statistics and Sensitivity Analysis


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<th>$z \in [5, 75]$</th>
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<td>0.652646</td>
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<td>0.660</td>
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Table 6: Sensitivity analysis with respect to small and large bunching window.

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<tr>
<th>Wide window (SEK)</th>
<th>Small window (SEK)</th>
<th>Partnership owners</th>
<th>Sole proprietors</th>
<th>Corporate owners</th>
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<td>[-75,000 , 75,000]</td>
<td>[-1,000 , 1,000]</td>
<td>1.915</td>
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<td></td>
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<td>(0.063)</td>
<td>(0.060)</td>
<td>(0.106)</td>
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<td>2.512</td>
<td>2.686</td>
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<td></td>
<td>(0.076)</td>
<td>(0.063)</td>
<td>(0.167)</td>
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<td>(0.559)</td>
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<td>(0.085)</td>
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<td>(0.298)</td>
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*Bunching (excess mass) is reported with standard errors in parenthesis.