Wage and Employment Discrimination by Gender in Labor Market Equilibrium

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Abstract

This paper develops an equilibrium search model to study the mechanisms underlying the lifecycle gender wage gap: human capital accumulation, preference for job amenities, and employers’ statistical discrimination in wage offers and hiring. In the model, men and women differ in turnover behaviors, parental leave lengths, and preference for amenities before and after having children. Capacity-constrained firms anticipate these gender differences when setting wages and making match decisions. Estimating the model on administrative employer-employee data combined with occupational level survey data on amenities from Finland, I find that a large proportion (44%) of the gender wage gap in early career is attributed to employers’ statistical discrimination based on fertility concerns, whereas gender differences in labor force attachment explain the majority of the gap (70%) in late career. Both hiring discrimination and preference for amenities draw women to low-productivity jobs in early career, and slow down their career progression in the long run. Counterfactual simulations show that shifting two parental leave months from women to men shrinks the wage gap by 13%. A gender quota at top jobs improves women’s representation in high-productivity positions, but firms undo this policy by exerting more wage discrimination. An equal pay policy counterfactual shows that requiring firms to pay men and women the same wage closes the wage gap by 15% on average, but has unintended consequences as employers adjust on the hiring margin.

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1 Introduction

The gender wage gap expands substantially over the lifecycle, especially for highly educated men and women.\(^1\) An extensive literature emphasizes the role of child-related career interruptions and gender differences in labor force attachment in driving the divergence in wages.\(^2\) However, less is known about the extent to which employers respond to the different labor market behaviors of men and women, and the mechanisms through which employers’ choices affect gender disparities in career trajectories. Since match formation and wages are influenced by both workers and firms in a labor market with search frictions, it is important to consider both labor supply and demand sides when designing policies aimed at reducing gender inequality. On the one hand we have policies to foster labor market opportunities for women, their stable employment after childbirth and access to top-level jobs; but on the other hand, the same policies could have unintended consequences when employers’ counteractions are taken into account.

My paper studies both the worker- and employer-side mechanisms underlying the gender wage gap: worker’s human capital accumulation, preference for job amenities, and employer’s statistical discrimination in wages and employment. First, women might spend more time out of the labor force after having children, and thus accumulate less human capital than men on the job. Second, women might sort into jobs that pay lower wages but offer more flexibility and other non-wage amenities that allow them to balance work and family. Third, employers might anticipate women to have more family-related separations and absence than men, and statistically discriminate against women. Since the seminal paper of Becker (1962), economists have been aware that labor market frictions make turnover costly to both workers and firms. Given that finding a replacement is time-consuming and costly in a frictional environment, employers might transfer the future costs of turnover into lower wages for women, or avoid hiring women altogether and/or sort them into less productive jobs.

In order to study both worker and firm behavior in the presence of frictions, I develop an equilibrium search model to quantify the above mechanisms and their interactions. The model allows male and female workers to have different turnover behaviors and preferences for amenities in each of the three stages in life – before having children, after having children, and before the next major life event.

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\(^1\) Barth, Kerr and Olivetti (2017) and Goldin, Kerr, Olivetti, Barth et al. (2017) document the lifecycle wage patterns of college and high school-educated men and women in the US.

\(^2\) See Blau and Kahn (2017) and Altonji and Blank (1999) for comprehensive reviews on the explanations of the gender wage gap.
after children, and in infertile ages. Workers are heterogeneous in human capital and preference for amenities, and employers are heterogeneous in productivity and provision of amenities. Workers’ human capital grows during employment but not during unemployment or parental leave. When the worker is on parental leave, the firm continues production but at a lower productivity, and the job is kept for when the worker returns. Therefore, hiring a woman can be associated with a lower match value for several reasons. First, she is more likely to separate into unemployment and the employer has to pay vacancy costs for some periods before hiring another worker. Second, she is more likely to take a longer parental leave, during which the job suffers from a loss of production and a lack of growth in her human capital. Third, a job with a low amenity value risks losing the woman to high-amenity jobs, whereas the employer faces less of such risks if matched with a man. All these considerations might serve as a basis for employers to statistically discriminate against women (some employers more than others).

A novel feature of the model is that workers and employers make decisions on both the wage margin and the employment margin. Firms have capacity constraints, where each firm has only one position to fill. Unlike existing search models analyzing gender wage differentials (Bowlus (1997), Flabbi (2010), Amano, Baron and Xiao (2020), Morchio and Moser (2019), Bagger, Lesner and Vejlin (2019)), the capacity constraint in my model puts men and women in direct competition with each other as they search for the same jobs. This competition between the genders would be absent in any job ladder model where firms have unlimited capacity and operate under constant returns to scale. With a scarcity of jobs to allocate, employers in my model have to carefully consider the trade-offs between hiring a woman versus a man. More productive jobs (such as managerial positions) might be especially concerned about hiring women since these jobs forgo more production per period when the worker leaves. Employers at the top may choose not to match with women if the opportunity cost of hiring a female worker outweighs vacancy costs – for example when complementarity is strong, when human capital grows fast, or when the firm’s bargaining share of the match surplus is high.

The human capital and sorting channels offer further insights. First, women might sort into low productivity jobs if high-amenity firms are less productive, or if highly productive firms offer fewer opportunities for women (or both), so the job choices we see in the data might not be determined by workers’ decisions alone. Second, if workers gain skills more quickly in a highly productive environment, then part of the gender produc-

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3Even though a capacity constraint on the firm side is not necessary to generate sorting in the multi-dimensional settings (Lindenlaub and Postel-Vinay (2017)), the question I analyze naturally calls for a capacity constraint, so that men and women compete in the same market for the same jobs.
tivity difference could be due to women being stuck in low-end jobs where human capital grows slowly. These insights highlight the limitations of using reduced-form approaches (such as Mincer-type regressions or AKM-style fixed-effects regressions) to decompose the gender wage gap, since human capital accumulation, statistical discrimination and their interactions are unobserved in the data.

Using administrative matched employer-employee data combined with occupational level data on amenities from Finland, I first document gender differences in labor market behaviors around childbirth. I find that women are more than twice as likely as men to transition from employment to unemployment after having children. Compared to men, women are also more likely to reduce hours, switch to part-time jobs, and move to jobs with high amenity values after childbirth. Women in Finland spend on average 18 months in parental leave for each child, whereas men spend only 2 months. Over the lifecycle, the unconditional wage gap between highly educated men and women increases from 12 log points at labor market entry to 20 log points after 10 years, and then decreases to 15 log points in late career.

I estimate the model by the method of simulated moments, and find that 44% of the gender wage gap in early career is attributed to employers’ statistical discrimination based on fertility concerns. A large part of the statistical discrimination is in wages – 37% of the gender wage gap in the first 3 years of the lifecycle is driven by the different wages offered to men and women of the same type working in the same job. Statistical discrimination in employment accounts for only 7% of the early wage gap because it affects a small group of people – it comes from highly productive firms not matching with low-human capital women before they have children. As workers move beyond child-rearing ages, statistical discrimination fades away and a vast majority (over 70%) of the wage gap in late career is due to an accumulated shortage in women’s human capital. I find that women value amenities as much as men do before having children, but value them twice as much after children. This affects sorting patterns even before childbirth and is responsible for about 9% of the overall wage gap after having children. The residual wage gap, which could be due to employers’ taste-based discrimination or initial productivity differences between men and women, accounts for approximately 18% of the total gap.

I consider three policy counterfactuals aimed at reducing gender inequality – an expansion of “daddy months” in parental leave, a gender quota at top jobs, and an equal pay policy. Shifting 2 months of parental leave from women to men is not enough to correct hiring discrimination in early career, but it does change employer perceptions and reduces statistical discrimination in wages based on fertility concerns. The “daddy
“months” expansion closes the wage gap by 13% throughout the lifecycle, as a result of more equal wages and more human capital accumulation of women after childbirth. It can be funded by a small increase in tax rate from 2.80% to 2.88%. On the other hand, a gender quota policy improves women’s representation at top jobs, but firms undo this policy by exerting more wage discrimination – the gender wage gap increases by 3% in the first 10 years. The gender quota eliminates hiring discrimination in early career and allows young women to gain valuable skills at highly productive jobs, but it does not address the negative career impacts of motherhood. Women continue to forgo human capital accumulation after becoming mothers, and employers continue to statistically discriminate against women by offering them lower wages. Lastly, the equal pay counterfactual shows that requiring firms to pay the same wage to men and women of the same type closes the gender wage gap by 15% on average. However, the equal pay policy has unintended consequences as employers adjust on the hiring margin. Women are more likely to be unemployed, and the proportion of women in top job decreases from 39 to 38% two decades after labor market entry.

To sum up, my results suggest that it would be difficult to achieve gender equality at the workplace without more equality in family responsibilities (e.g. a more equal burden of child-related leave between men and women), given the sizable effect of employer statistical discrimination in both wages and employment in equilibrium.

This paper makes three contributions. First, it develops and estimates an equilibrium search model with employer capacity constraints, where men and women compete for the same jobs and employers may not match with both genders. While the capacity constraint is a natural feature in this context, it makes the problem considerably more complex, since it requires the solution of fixed point problems in not only the match surplus values, but also in the allocations of matched and unmatched agents.

Second, this paper is the first to bring together all three mechanisms – human capital, job preferences, and statistical discrimination in wages and employment – in one unified framework, opening an avenue to study the rich interactions between the channels. For example, statistical discrimination could be based on expected human capital stagnation during parental leave and/or anticipated job switches driven by amenity preferences. In turn, both hiring discrimination and amenity preferences push women into low-productivity jobs, affecting their human capital growth. The research question at hand requires many model features that are typically not present in standard search models in the literature, for example multi-dimensional firm and worker types, life-cycle dynamics, and human capital accumulation. These features make the model very rich but
also post significant computational challenges.

Third, my paper combines administrative employer-employee data with survey data on job amenities, and documents workers’ sorting patterns across jobs of different observable amenity levels. Exploiting the employer-employee linked nature of the data, I use the mobility patterns of men and women across jobs, gender ratios within jobs, wages and wage growths at various transitions over the lifecycle to separately identify human capital, preference and production parameters.

1.1 Related literature

There is an extensive literature examining the explanations for the gender wage gap (see Altonji and Blank (1999) and Blau and Kahn (2017) for surveys). A growing recent literature highlights the importance of fertility-related career interruptions in explaining the gap. Angelov, Johansson and Lindahl (2016), Kleven, Landais and Søgaard (2019) and Andreessen and Nix (2019) document a large and persistent income penalty experienced by women after having children, along with lower participation, fewer hours worked, and a higher tendency to work in the public sector after childbirth. Erosa, Fuster and Restuccia (2016) and Adda, Dustmann and Stevens (2017) develop dynamic models of human capital accumulation, fertility and labor supply choices of women to estimate the impact of children on the gender wage gap. However, this body of work focuses only on the direct consequences of childbearing on female workers, but it does not consider the role played by employers who, by statistically discriminating, extend some of the consequences to women who will not have children or have not yet had children. In addition to modeling how childbirth directly affects women’s labor supply and human capital accumulation, my paper also considers how women’s behaviors around childbirth will in turn affect firms’ wage policies both before and after the fertility event.

Another important factor in the gender wage gap highlighted by the literature is the sorting of men and women into high- versus low-paying establishments and occupations. Blau and Kahn (2017) points out that even though occupational segregation by gender has declined in the US over time, it still remains as the largest single factor accounting for the wage gap. Adda, Dustmann and Stevens (2017) shows that women sort into occupations with lower “motherhood atrophies” even before childbirth, and Felfe (2012) shows that after childbirth women switch into jobs with less stress, fewer hours, options to work at night or with flexible schedules. Notably, Goldin (2014) points to the temporal flexibility of work as the last step towards gender equality in the labor market, and Wiswall and Za-
far (2017) shows that women’s higher willingness to pay for work flexibility can explain a quarter of the gender wage gap in early career. In light of this literature, my model incorporates non-wage amenities that help balance work and family, and investigates how these preferences affect sorting across jobs and wages in an equilibrium framework. A strand of literature uses a revealed preference approach to study the importance of job amenities (Sorkin (2018), Lamadon, Mogstad and Setzler (2019) and Taber and Vejlin (2020)). Instead, I will use a more direct approach by focusing on observed amenities related to flexibility that are important for women’s occupation choice.

It might then be crucial to distinguish differential sorting of men and women across high- and low-wage jobs versus gender differentials in wage rates and wage growths within an establishment. Blau (1977) and Groshen (1991) find a large role played by firm-specific premiums and segregated employment of men and women across firms in accounting for the gender wage gap. With greater availability of matched employer-employee data, Card, Cardoso and Kline (2016), Goldin, Kerr, Olivetti, Barth et al. (2017) and Barth, Kerr and Olivetti (2017) study the relative importance of the across- and within-firm channels and find both to be important. Furthermore, women and men also tend to be employed at different levels of hierarchy within the firm, often termed the “glass ceiling”. Barth, Kerr and Olivetti (2017) shows that for the college-educated group, much of the lifecycle gender gap could be attributed to men receiving higher earnings growth within the establishment. Bronson and Thoursie (2019) also finds that the internal promotions gap between men and women is sizable in Sweden and importantly, that women are much less likely to be promoted than men especially in early career.

In all these empirical studies, however, it is difficult to determine the mechanisms driving the within-firm wage differentials between similar men and women. Card, Cardoso and Kline (2016) interprets the within-gap as a result of a smaller bargaining share obtained by women. Bronson and Thoursie (2019) shows that their findings are consistent with predictions of a statistical discrimination model where there are costs associated with promoting someone who might reduce future labor supply due to childbearing. I contribute to this literature by formalizing the forces driving worker mobility across jobs as well as pay-setting policies within each job in a unified equilibrium search framework, so that one is better-equipped to quantify the relative contributions of each channel.

Many papers have theorized the link between child-related career interruptions and firms’ statistical discrimination. Barron, Black and Loewenstein (1993) and Thomas (2019) build a two-period model where employers’ uncertainty about workers’ labor force attachment in period 2 affects who gets trained in period 1 and who gets assigned to high-
paid jobs subsequently. In the context of the Family and Medical Leave Act of 1993, Thomas (2019) finds evidence that women hired after the leave expansion are less likely to get promoted. Albanesi and Olivetti (2009), Gayle and Golan (2012) and Tô (2018) formulate models where workers have private information about their labor market participation costs, and show that statistical discrimination can be quantitatively important in explaining the gender gap. I take a different approach regarding statistical discrimination. In the presence of labor market frictions, statistical discrimination in my model could arise endogenously not only from employer’s anticipation of workers’ quit behaviors, but also their future human capital accumulation, as well as transition probabilities to other jobs with different amenity provisions. I depart from this literature by specifically modeling stages in life with and without children, so that one can examine how statistical discrimination in one stage could propagate to other stages in life through equilibrium effects.

My model is built on a body of search-matching literature with wage bargaining (Cahuc, Postel-Vinay and Robin (2006)), sorting (Lise, Meghir and Robin (2016) and Lindenlaub and Postel-Vinay (2017)), and with human capital accumulation (Herkenhoff, Lise, Menzio and Phillips (2018), Lise and Postel-Vinay (2015)). My paper is closest to the literature that analyzes the gender pay gap through the lens of equilibrium search models. Bowlus (1997) finds that frictions and gender differences in quit behaviors play a key role in generating the cross-sectional gender wage gap in equilibrium. Using data from the US and Denmark respectively, recent studies by Amano, Baron and Xiao (2020) and Bagger, Lesner and Vejlin (2019) introduce human capital accumulation, fertility and parental leave periods to the equilibrium search framework, and examine how much of the lifecycle gender pay differential is due to experience accumulation versus different endogenous piece rates being offered to men and women with the same productivity. A concurrent working paper Morchio and Moser (2019) uses data from Brazil to analyze the extent to which the gender wage gap is driven by gender differences in mobility versus employer heterogeneity in amenities and gender preferences, while workers have time-invariant productivities. My model adds to this literature by allowing men and women to compete for the same jobs, and introducing trade-offs faced by employers in the hiring margin. My paper is the first to provide a unified equilibrium analysis of three different explanations of the gender wage gap – human capital growth, preference for job amenities, and statistical discrimination in wages and hiring – that are prevalent in the literature.

The rest of the paper is organized as follows. Section 2 describes the datasets and empirical patterns. Section 3 develops the theoretical model. Section 4 discusses the iden-
tification and estimation strategy. Section 5 shows the gender wage gap decomposition and results from counterfactual policy experiments. Section 6 concludes.

2 Empirical Motivations

In this section, I will briefly describe the datasets and show a number of empirical patterns related to gender differences in the labor market.

2.1 Data and sample

The Finnish Longitudinal Employer-Employee Data (FOLK) is assembled by Statistics Finland from numerous administrative registers, and covers the entire resident population aged 15 to 70 between years 1988 and 2016. FOLK provides detailed employment histories for each worker. Using the start and end dates of each employment relationship, I create a monthly employment status for each worker – employed, unemployed, or on parental leave. Since FOLK can be linked to the official population register, I can also observe the birth date of each child of the worker and use it to infer the worker’s parental leave status when he/she starts collecting benefits around that date. Parental leave duration is inferred from the annual parental leave allowance and home care allowance received according to a schedule detailed in Appendix B.

The hourly wage data comes from the Structure of Earnings Statistics (SES). The SES consists of large-scale surveys collected by the Employers’ Association in the last quarter of each year from 1995 to 2013. It covers all public sector workers and 55 to 75 percent of private sector workers depending on the year. Since I do not include small firms with 2 workers or less, data coverage is not a big issue.

The advantage of the SES is that it contains an array of personnel information, including 4-digit occupation codes, part-time status, and (paid) contracted hours that are typically not available from tax registers. The drawback of SES is that the observations are on the yearly level as opposed to daily in FOLK, and some firms might not be surveyed in certain years. In the estimation, I will use sample weights in the simulations to account

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4The following groups in the private sector are either entirely excluded or at least severely under-represented: 1. small (less than 5 persons) enterprises; 2. the vast majority of non-organized (mainly small) enterprises; 3. agriculture, forestry and fisheries; 4. international organizations; 5. company management and owners and their family members; 6. the employment relationships beginning or ending during the reference month.
for potential missing data from small firms.

Since educated people experience the largest increase in the gender wage gap over the lifecycle, in this paper I will focus on individuals who obtained master’s degrees in the years 1988 to 2005 so that we observe at least 10 years of labor market activities. I drop those whose age is in the bottom or top 5 percentiles of the age distribution at graduation – workers in my sample are aged between 24 and 31 when they graduated master’s. I drop the firms that have never had more than 2 workers during the sample period.

I only include periods after the individuals have completed their master’s education. Unemployment of 2 months or less is counted as the final tenure of the previous spell. Similarly, employment of 2 months or less is counted as non-employment.

If the worker has wages from more than one employer in that quarter, I keep only the wage from the “main” job – the full-time job if there is one, or the job with the most earnings if all jobs are part-time. I trim the top 0.5% of the wage distributions in each year, which tend to be very thin and cover wide ranges. After sample selection, I have an unbalanced panel of 116,781 workers, and 25,951 distinct firm-occupations over the course of 18 years.

I remove macroeconomic fluctuations in wages and transition rates by taking out year fixed effects in all moments calculations. I also remove cohort fixed effects in order to control for any real wage growth over time for different graduating cohorts. What is left at this point are lifecycle profiles of men and women which I take to have the same patterns across all graduating cohorts.

2.2 Empirical decomposition of the gender wage gap

Women have overtaken men in educational attainment in Finland. In my sample, the number of women with master’s degrees is about one third more than men in the cohorts that graduated in 1988 to 2005.

However, women’s labor market outcomes do not seem to catch up with their male classmates. To investigate what underlies the expansion of the gender wage gap over the lifecycle, I first decompose the gap empirically by successively adding more controls. Figure 1 shows the difference between men and women’s log hourly wages by years of potential experience, (i) unadjusted (only with year fixed effects), (ii) adjusted for a quadratic in actual experience, and (iii) adjusted for a full set of dummies in 4-digit occu-
pation and firm interactions.

**Figure 1.**
Real log hourly wage gap between men and women

![Graph showing the real log hourly wage gap between men and women.](image)

**Notes:** The lines represent the coefficients on the male dummy interacted with potential experience. Shaded areas represent 95% confidence intervals. The coefficients are obtained from running a regression of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience in addition to (i); (iii) a full set of interactions of firm and occupation dummies in addition to (i) and (ii).

The unadjusted real hourly wage of women with master’s degree is on average 12 log points less than that of men in the first year when they entered the labor market. This unconditional gap increases to 20 log points 10 years into their careers, and then starts to decline but never all the way down to zero.

Taking actual experience as the cumulative number of months a person has worked after college graduation, I then add a quadratic in actual experience. Intuitively, the human capital explanation accounts for very little of the wage differentials in early career before any experience accumulation takes place. Absorbing the effect of all occupation-firms eliminates the gap almost entirely to only 0.02 log point in the first year after graduation – almost the entire gender wage gap in early career is driven by men and women sorting into different firms and occupations.

Since college-graduated men and women typically work for some time before obtaining master’s degrees, I look at their formal labor market experience after bachelors’ graduation, excluding short-term employment of 3 months or less and excluding summer internships. By the time they graduate with master’s degree, men have 1.9 years of actual experience while women have 1.6 years. The difference is not statistically significant.
As we follow men and women over the course of their working lives, human capital becomes a more and more important factor in explaining the wage gap, as women work fewer months than men every year and accumulate less and less actual experience over time. The unexplained portion of the gap rapidly increases and stays at about 4 log points throughout the lifecycle. The main takeaway is that both human capital and sorting across jobs are important components of the lifecycle gender wage differential – together they explain about 70 to 80 percent of the overall gap. However, there is still a sizable proportion of the total gap that is unexplained even after controlling for detailed observables, suggestive of unequal pay for equally qualified workers. Of course, there could also be unobserved productivity differences, such as quality of education degrees, effort devoted to the job versus family and so on. Moreover, one must also be cautioned against interpreting the coefficients on the explanatory variables, since actual experience, occupations and firms may themselves be a result of discrimination.

How much of the wage divergence happens after childbirth? Women’s wages might be negatively affected early in the lifecycle if they have children soon after labor market entry. In Finland, master’s men and women graduate at the age of 27 on average, and both have the first child only 4 years after finishing school. To examine the impact of fertility on the gender wage gap, I conduct the same decomposition exercise as above, but now against years around childbirth as opposed to years of potential experience.

Figure 2 shows that the unadjusted gap is 14 log points before the birth of the first child, and increases to 21 log points a few years afterwards. Most of the wage gap (67 percent) before childbirth could be explained by women sorting into low-paid firms and occupations, while actual experience accounts for only 10 percent of the gap prior to birth. However, human capital becomes an important factor immediately after birth, as women start taking parental leave and fall behind men in experience accumulation. Notably, about 22 percent of the pre-birth wage gap remains unexplained within narrow firm-occupation cells, suggestive of the existence of discrimination even before women give birth.

2.3 Gender differences in labor market behaviors

The Finnish parental leave system is very generous (see Appendix B for a detailed description). Master’s graduated women take on average 1.5 years of paid leave for each child compared to only 2 months taken by men with master’s degree. Figure 3a shows the striking labor supply reduction of women after the birth of their first child. While
FIGURE 2.
Real hourly wage gap around childbirth

![Graph showing the real hourly wage gap around childbirth.](image)

**Notes:** The lines represent the coefficients on the male dummy interacted with the number of years since first birth. Shaded areas represent 95% confidence intervals. The coefficients are obtained from running a regression of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience in addition to (i); (iii) a full set of interactions of firm and occupation dummies in addition to (i) and (ii).

The proportion of educated women who are working is comparable to that of men prior to childbirth (at about 80 percent), virtually all women take some months off in the year of childbirth. The female employment rate increases from 3 to 28 percent the year after birth, but takes time to recover to its pre-birth levels since many women have a second or third child. Eventually, women’s labor supply does go back to 80 percent, but only some 14 years after the birth of the first child. Educated men, on the other hand, only experience a small dip in labor supply in the year of childbirth, and do not seem to be affected afterwards.

In addition to parental leave uptake, there are some other marked differences between men and women’s labor market behaviors after childbirth. **Figure 3b** shows the monthly transition rate of men and women from employment to unemployment outside of parental leave. Since I observe the exact amount of parental leave benefits collected around the time of childbirth, I can pinpoint the month at which benefits run out. If a worker is not associated with an employer and is not collecting parental leave benefits in a particular month, he/she is considered to be unemployed.⁶ According to this mea-

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⁶If someone is unemployed for only two months or less after she stops collecting parental leave benefits, I consider it as measurement error in leave duration calculations and do not count the months as
Figure 3.
Transition rates around childbirth

(A) Employment rate (PL=not working)

(B) Separation rates outside of PL

(C) Job-finding rates outside of PL

(D) Firm-to-firm transition rates

Notes: The lines represent the coefficients obtained from regressions of outcome variables on the number of years since first birth, separately for men and women. Shaded areas represent 95% confidence intervals.

Sure, female separation rate is already a little higher than male’s prior to birth, but the big difference appears right after childbirth, where women’s separation spikes and remain well above men’s for many years after childbirth. This could be driven by voluntary or involuntary quits, although they cannot be distinguished in the data.

If the worker goes back to work immediately after parental leave, but does not go back to the same firm, it is considered as a firm-to-firm transition. Figure 3d shows an increase in women’s monthly firm-to-firm transitions after childbirth, whereas men’s transition rate remains relatively smooth around birth. The higher female firm-to-firm transition rate before childbirth could be driven by job switches in anticipation of fertility, unemployment. A separation is only indicated for unemployment of 3 months or more.

7There is no information on occupations in the monthly employment spell data, so this variable does not include any job change within the firm.
although it is difficult to ascertain without survey data. In the year right before childbirth, women do not change firms because leave benefits are often tied to collective agreements with the employer who might require

How much of the job changes could be attributed to women’s demand for jobs that are more compatible with child-rearing? I find some evidence of women’s increased preference for lower hours and part-time opportunities after having the first child. Figure 4a shows that women reduce weekly contracted hours from an average of 37 before childbirth to 35.5 immediately afterwards. While there is a downward lifecycle trend in men’s contracted hours, there is no sudden drop around childbirth. The proportion of women doing part-time jobs also increases from 5 percent prior to birth to 15 percent the year after birth and remains at that level for 10 years, as shown in Figure 4b. In contrast, the proportion of master’s men doing part-time remains flat at 3 percent around childbirth.

**Figure 4.**
Demand for amenities around birth

![Chart](image)

**Notes:** The lines represent the coefficients obtained from regressions of outcome variables on the number of years since first birth, separately for men and women, with individual fixed effects. Shaded areas represent 95% confidence intervals.

Even though Finnish workers are allowed to ask for reduced hours after having children, in practice the ability to do so might depend on specific employers. Out of those women who have always worked full-time before childbirth but have switched to part-time for at least one year afterwards, about 58 percent of them have to either change firms or change occupations within a firm in order to switch to part-time status. This is consistent with what Altonji and Paxson (1992) found for the US.

I will use the availability of part-time work in a firm-occupation cell as part of the
measure for job-specific amenities in section 2.4.

2.4 Non-wage amenities

In light of the results from subsection 2.2 and the literature highlighting women’s demand for reduced hours, part-time work and flexible work schedules after having children (Wiswall and Zafar (2017), Goldin (2014), Edwards (2014), Flabbi and Moro (2012), Felfe (2012) and Goldin and Katz (2011)), I investigate the importance of such non-wage amenities on women’s mobility decisions around birth.

I use several data sources to construct my amenity measure. The first dataset is the Finnish Quality of Work Life Survey (QWL). The QWL surveys are extensive studies that involve a representative sample of 4000 to 6000 wage or salary earners in Finland. It documents how people feel about their working conditions related to physical or social environment, job satisfaction, work orientation and so on. The survey began in 1977 has now been carried out seven times: 1977, 1984, 1990, 1997, 2003, 2008 and 2013.

I will use the 2013 wave since it has the most detailed occupation codes. The responses could be aggregated at most to 2-digit occupation level for questions related to flexibility (positive amenities) and over-working (negative amenities), listed below:

**Flexibility:**
- Have you agreed with the employer to work occasionally at home?
- Can you influence starting and finishing times for your work by at least 30 minutes?
- Can you use flexible working hours sufficiently for your own needs?
- Do have the possibility for brief absences from work in the middle of the working day to run personal errands?

**Overwork:**
- Do you sometimes work overtime without compensation?
- Have you been contacted about work outside of working hours during the last two months?
- Do you have to do more overtime work than you would like to?

2-digit occupations may not be detailed enough to give a fully comprehensive picture. However, due to the sample size of the surveys, we cannot go into more detailed occupations.

In order to get a sense of actual hours worked by female and male employees, it is not sufficient to look simply at contracted hours recorded in the SES, because it might be
typical for some occupations and industries to work long hours without specifying it in
the contract (e.g. law, finance, etc.). To this end, I turn to the Finnish Labor Force Survey
(LFS) for data on actual hours worked per week. The LFS samples approximately 12,000
persons aged between 15 and 74 every month about participation and working hours. I
obtain the average usual weekly hours of full-time employees in each 3-digit occupation,
and take the average across all available years from 2013 to 2018.

Finally, I obtain a measure of the extent of part-time opportunities in each occupation
within each firm. I calculate the proportion of part-time workers in each job cell in my
SES sample in each year, and take the average across all years the job has existed as a
long-term measure of the job characteristic.

I construct an amenity index for each job by extracting the first principal component of
standardized measures of all the above-mentioned variables – 7 QWL survey responses
about flexibility and overwork by 2-digit occupations, LFS usual hours worked by 3-digit
occupations, and the proportion of part-time workers on firm-occupation level.

**FIGURE 5.**
Amenity index of workers’ jobs around childbirth

![Amenity Index Chart]

**NOTES:** The lines represent the coefficients obtained from a regression of the amenity index on the num-
ber of years since first birth, separately for men and women, with individual fixed effects. Shaded areas
represent 95% confidence intervals.

Using this amenity index, several interesting patterns emerge. First, jobs with high
and very high amenity index values are relatively more abundant in the middle and lower
end of the wage distribution (see Figure 7). Second, in general men and women seem to
be moving from high- to low-amenity jobs over the lifecycle, possibly because they move to more demanding jobs that require more hours and overtime as they progress in their careers. Third, there is a break in trend in the amenity value of the jobs women take right after childbirth. Figure 5 shows that women are in jobs with slightly higher amenity index than men before childbirth, but the difference between the amenity values of the jobs taken by women versus men becomes more pronounced after childbirth. As men move to low-amenity jobs over time, women are more likely to be in high-amenity jobs especially after having children.

3 Model

Motivated by the empirical patterns of men and women’s labor market behaviors, the model will be outlined below. I first describe the characteristics of workers and firms, and their lifecycle stages. I then explain the matching process between workers and firms and the wage determination mechanisms. Lastly, the steady-state equilibrium of the labor market is characterized.

3.1 The environment

Workers Time is discrete and infinite. The labor market is populated by a continuum of female and male workers each of measure 0.5, as well as a continuum of jobs of measure 1. Workers are risk-neutral, and maximize the present value of their utilities, discounted at factor $\beta \in (0, 1)$. Workers are heterogeneous in the level of human capital $x$ and their value for amenity $\epsilon$. Human capital determines the worker’s contribution to output when employed and the worker’s home productivity $b(x)$ when unemployed.

Upon entering the labor market, workers of gender $g \in \{m, f\}$ draw their initial skills and value for amenities from an exogenous discrete distribution with probability mass function $\xi^g(x, \epsilon)$. Human capital evolution depends on the state of employment. The skills of a worker of type $x$ evolves according to a law of motion $p_e(x, y)$ in employment that depends on job productivity $y$. This captures the idea that workers might learn faster on the job when matched with more productive employers, either from knowledge spillovers by more productive coworkers (Nix (2019)) or from doing more complex tasks. In unemployment, the law of motion is $p_u(x)$. 
Employers  A job is an occupation within a firm. Each job maximizes the present value of its profit, also discounted at factor $\beta$. Jobs are heterogeneous in productivity $y$ and amenity provision $\alpha$ drawn from an exogenous distribution with joint density $\varphi(y,\alpha)$. If the job is vacant, it does not produce any output and has to pay a flow vacancy cost $c$. Importantly, each job can only match with one worker, and employers are not allowed to search for new hires when the job is filled. The distribution of jobs is fixed at $\varphi(y,\alpha)$ and there is no free entry of jobs. When an employer of type $(y,\alpha)$ matches with a worker of type $(x,\epsilon)$, they produce $f(x,y)$ units of output.

Life stages  Workers go through four age segments in life. All workers start their careers in a stage with no child (the NC stage). At an exogenous fertility rate $\chi$, the worker has a child and enters a stage with young child (the YC stage). Every time the worker has a child, he/she will enter a Parental Leave (PL) stage to stay home with the baby. Men and women might stay in the PL stage for different durations; at rates $\eta_m$ and $\eta_f$, men and women exit their PL and go back to their previous employers. Workers can have children repeatedly until they turn 40 (at rate $\gamma$), at which point they will be “Done” with children (D stage) so there is no more fertility shock in stage D. Workers retire at rate $\phi$ in stage D, and new workers enter the labor market at the same rate. Within each age segment $a \in \{NC, PL, YC, D\}$ of life, the search and matching process is analogous.

Workers of age $a$ and gender $g$ lose their jobs exogenously at rate $\delta^g_a$. Job destruction rates are allowed to differ by gender in stages NC and YC/PL, but are the same in stage D ($\delta^m_D = \delta^f_D = \delta$). In each period, workers in stage D exit the labor market at rate $\phi$, while simultaneously the same measures of new male and female workers enter the labor market.

3.2 Search and matching

At each point in time, workers can be matched to a firm or be unemployed. The aggregate number of meetings between vacancies and searching workers is determined by a standard aggregate matching function $m(\hat{U},V)$. This takes as inputs the total number of vacancies $V$ and the total amount of effective job seekers $\hat{U} = U + s(1-U)$, where $U$ is the total number of unemployed workers and $s$ is the search intensity in employment relative to unemployment. The matching function is assumed to be increasing in both arguments and exhibit constant returns to scale.
For easy exposition, let $\kappa = \frac{m(\hat{U}, \hat{V})}{\hat{U} \hat{V}}$ summarize the effect of market tightness, so that the arrival rate of jobs to unemployed workers is simply $\kappa \hat{V}$, and the arrival rate of workers to a vacancy is $\kappa \hat{U}$. $\kappa$ is constant in a stationary equilibrium, but it is not invariant to policy, and it is important to allow it to change when evaluating interventions or counterfactual regulations.

Let $u^g_a(x, \epsilon)$ denote the measure of unemployed workers of gender $g$, age $a$ and type $(x, \epsilon)$, and let $v(y, \alpha)$ denote the measure of vacancies of type $(y, \alpha)$. The joint distribution of matches between workers of type $(x, \epsilon)$ and jobs of type $(y, \alpha)$ is denoted as $h^g_a(x, \epsilon, y, \alpha)$. While unemployed, workers randomly sample offers from the vacancies distribution, and the instantaneous rate at which an unemployed worker meets a vacancy of type $(y, \alpha)$ is $\kappa \hat{V} \cdot v(y, \alpha)$. The instantaneous probability for any vacancy to make a contact with an unemployed worker of type $(x, \epsilon)$, age $a$ and gender $g$ is $\kappa u^g_a(x, \epsilon)$. Similarly, employed workers meet vacancies at rate $s\kappa v(y, \alpha)$, and vacancies meet employed workers at rate $s\kappa h^g_a(x, \epsilon, y, \alpha)$.

Upon a meeting between a worker and a job, a match will be formed if it generates positive surplus. In other words, match formation is assumed to be efficient.

Let $U^g_a(x, \epsilon)$ denote the lifetime value of an unemployed worker of type $(x, \epsilon)$, $\Pi_0(y, \alpha)$ denote the vacancy value of a job of type $(y, \alpha)$. Let $P^g_a(x, \epsilon, y, \alpha)$ denote the value of joint production of a match between worker $(x, \epsilon)$ and job $(y, \alpha)$. The surplus of a match is defined as $S^g_a(x, \epsilon, y, \alpha) = P^g_a(x, \epsilon, y, \alpha) - U^g_a(x, \epsilon) - \Pi_0(y, \alpha)$. A match is feasible and sustainable if the match surplus is positive, $S^g_a(x, \epsilon, y, \alpha) > 0$.

Workers have bargaining power denoted by $\sigma$ and obtain a share of the match rent, following the formulation in Cahuc, Postel-Vinay and Robin (2006). Let $W^g_a(w, x, \epsilon, y, \alpha)$ (and respectively $\Pi^g_a(w, x, \epsilon, y, \alpha)$) denote the value of a wage contract $w$ for a worker $(x, \epsilon)$ employed at a job $(y, \alpha)$ (respectively the firm’s profit). The surplus can then be written as:

$$S^g_a(x, \epsilon, y, \alpha) = W^g_a(w, x, \epsilon, y, \alpha) - U^g_a(x, \epsilon) + \Pi^g_a(w, x, \epsilon, y, \alpha) - \Pi_0(y, \alpha)$$

The way in which wage $w$ splits the surplus between the worker and the employer will be discussed in the following section.
3.3 Wage determination

To define wages and renegotiations, I follow the setup in Cahuc, Postel-Vinay and Robin (2006). Workers’ wages are determined by sequential auctions. Different wages are negotiated when a worker leaves unemployment, and when counteroffers are made for an employed worker upon poaching.

Wage bargaining with unemployed workers The starting wage $\phi_{0,a}^S(x,\epsilon,y,\alpha)$ obtained by a type-(x,\epsilon) unemployed worker when matched with a type-(y,\alpha) job is such that the worker receives the reservation utility $U(x,\epsilon)$ plus a share $\sigma$ of the surplus:

$$W_a^S(\phi_{0,a}^S(x,\epsilon,y,\alpha),x,\epsilon,y,\alpha) = U_a^S(x,\epsilon) + \sigma S_a^S(x,\epsilon,y,\alpha)$$

for jobs where surplus $S_a^S(x,\epsilon,y,\alpha)$ is positive.

Wage at job-to-job transitions When a worker of type (x,\epsilon) encounters an alternative job package (y',\alpha') that produces more surplus than her current job, she will transition from job (y,\alpha) to job (y',\alpha') with a wage $\phi_{1,a}(x,\epsilon,y,\alpha,y',\alpha')$ such that the value she receives at the new job (y',\alpha') is $W_{NC}^S(\phi_{1,a}^S, x,\epsilon,y',\alpha')$. In this scenario, the worker extracts the maximum value from the incumbent match $P_a^S(x,\epsilon,y,\alpha) - \Pi_0(y,\alpha)$ plus a $\sigma$ share of the surplus difference:

$$W_a^S(\phi_{1,a}^S(x,\epsilon,y,\alpha,y',\alpha'),x,\epsilon,y',\alpha') = P_a^S(x,\epsilon,y,\alpha) - \Pi_0(y,\alpha) + \sigma [S_a^S(x,\epsilon,y',\alpha') - S_a^S(x,\epsilon,y,\alpha)]$$

Wage renegotiation upon poaching If the poaching job (y',\alpha') generates a match surplus below that of the incumbent job, i.e. when $S_a^S(x,\epsilon,y',\alpha') < S_a^S(x,\epsilon,y,\alpha)$, the worker will stay in the incumbent firm. Incumbent employers will respond to outside offers and update wages only when there is a credible threat – when either the worker or the employer will credibly separate if they do not obtain an improved offer. In other words, wages will be re-negotiated when the poaching firm offers a value greater than what the worker currently receives, when $P_a^S(x,\epsilon,y',\alpha') - \Pi_0(y',\alpha') > W(w,x,\epsilon,y,\alpha)$. In this case, wages will be updated from $w$ to $\phi_{2,a}(x,\epsilon,y',\alpha',y,\alpha)$ such that the worker receives an updated value $W_a^S(\phi_{2,a}^S, x,\epsilon,y,\alpha)$ at the incumbent job (y,\alpha) that equals the maximum value.
the poaching employer is willing to offer:

\[
W^g_a(\phi^g_a(x, \epsilon, y', \alpha', y, \alpha), x, \epsilon, y, \alpha) = P^g_a(x, \epsilon, y', \alpha') - \Pi_0(y', \alpha') + \sigma \left[ S^g_a(x, \epsilon, y, \alpha) - S^g_a(x, \epsilon, y, \alpha') \right]
\] (3)

Note that when a worker’s human capital appreciates from \(x\) to \(x_+\) in the next period, her wage does not update until there is a credible outside option. Please refer to Appendix C for details of the workers’ values.

### 3.4 Value functions

In order to define an equilibrium, I will describe the value functions and the distributions of workers and jobs across employment states and life stages. These define the decision rules for each agent.

#### 3.4.1 Value in unemployment

In the "No Child" stage of life, the utility of an unemployed worker of gender \(g\) and type \((x, \epsilon)\) is:

\[
U^g_{NC}(x, \epsilon) = b(x) + \beta \mathbb{E} \left[ \sum_{y, \alpha} \kappa v(y, \alpha) \left( U^g_{NC}(x_+, \epsilon) + \sigma \max \{ S^g_{NC}(x_+, \epsilon, y, \alpha), 0 \} \right) + \chi U^g_{PL}(x_+, \epsilon) + \gamma U^g_{D}(x_+, \epsilon) + (1 - \chi - \gamma - \kappa V) U^g_{NC}(x_+, \epsilon) \right].
\] (4)

The unemployed worker receives a flow value of \(b(x)\) in the current period. In the next period, the worker’s human capital level is \(x_+\), where the transition matrix from \(x\) to \(x_+\) is given by the law of motion \(p_u(x)\) in unemployment. The present discounted value takes the expected future payoff over the probability distribution \(p_u(x)\).

The worker randomly samples jobs of all \((y, \alpha)\) types, and the probability that he/she encounters a type-(\(y, \alpha\)) job is \(\kappa v(y, \alpha)\). With a human capital level of \(x_+\), the worker will take the job if the match generates positive surplus, i.e. when \(S^g_{NC}(x_+, \epsilon, y, \alpha) > 0\). Workers have bargaining power \(\sigma\), so will obtain unemployed value \(U^g_{NC}(x_+, \epsilon)\) plus \(\sigma\) share of the surplus upon match formation.

The worker can also experience lifecycle shocks in the next period. When an unemployed worker has a child at rate \(\chi\), he/she does not receive parental leave. The worker enters the “Young Child” stage still in unemployment, where the associated value is
The worker ages at rate $\gamma$, upon which he/she enters an infertile age with unemployment value $U_D^g(x_+, \epsilon)$. The unemployment values in YC and D stages are analogous to that in NC:

$$U_{PL}^g(x, \epsilon) = b(x) + \beta E \left[ x \left( \eta \ U_{YC}^g(x_+, \epsilon) + \gamma \ U_{D}^g(x_+, \epsilon) + (1 - \eta - \gamma) \ U_{PL}^g(x_+, \epsilon) \right) \right] \quad (5)$$

$$U_{YC}^g(x, \epsilon) = b(x) + \beta E \left[ \sum_{y, a} \nu(y, a) \sigma \max \{ S_{YC}^g(x_+, \epsilon, y, a), 0 \} \right] + \chi \ U_{PL}^g(x_+, \epsilon) + \gamma \ U_{D}^g(x_+, \epsilon) + (1 - \chi - \gamma) \ U_{YC}^g(x_+, \epsilon) \quad (6)$$

$$U_{D}^g(x, \epsilon) = b(x) + \beta E \left[ \sum_{y, a} \nu(y, a) \sigma \max \{ S_{D}^g(x_+, \epsilon, y, a), 0 \} + (1 - \phi) \ U_{D}^g(x_+, \epsilon) \right] \quad (7)$$

In stage D, individuals are infertile and will not have any additional child. Workers retire at rate $\phi$, upon which the joint value of the match is just the vacancy value.

### 3.4.2 Value of vacancy

A vacant job could potentially hire a male or female worker of any age $a \in \{ NC, YC, D \}$. The value of a vacancy of type $(y, a)$ is:

$$\Pi_0(y, a) = -c + \beta \left[ \sum_a \sum_g \sum_{x, \epsilon} \kappa u_0^g(x, \epsilon) \left( \Pi_0(y, a) + (1 - \sigma) \max \{ S_0^g(x, \epsilon, y, a), 0 \} \right) \right.$$

$$\left. + \sum_a \sum_g \sum_{x, \epsilon, y', a'} s \kappa h_0^g(x, \epsilon, y', a') \left( \Pi_0(y, a) + (1 - \sigma) \max \{ S_0^g(x, \epsilon, y, a) - S_0^g(x, \epsilon, y', a'), 0 \} \right) \right.$$  \quad (8)

$$\left. + \left( 1 - \kappa U - s \kappa (1 - U) \right) \Pi_0(y, a) \right]$$

where $c$ is a per-period cost of keeping a vacancy open, and $U$ denotes the aggregate unemployment.

Employers and unemployed workers meet at a rate determined by labor market tightness $\kappa$ and the measure of unemployed workers of each type $u_0^g(x)$. A meeting turns into a match if and only if the match surplus is positive. When a match is formed, the employer obtains its vacancy value $\Pi_0(y, a)$ plus $(1 - \sigma)$ share of the match surplus, while the unemployed worker receives $\sigma$ share as described in subsection 3.3.

Similarly, vacant jobs can also poach employed workers from other jobs. Employers Bertrand-compete for the employed worker, and the worker matches with the job that generates a higher surplus. If the vacant job successfully poaches the worker, it obtains $(1 - \sigma)$ share of the difference in surplus values.
3.4.3 Joint value of a match

In the “No Child” stage, the joint value of a match between worker \((x, \epsilon)\) and job \((y, \alpha)\) is:

\[
P^g_{NC}(x, \epsilon, y, \alpha) = (1 - \tau) f(x, y) + q^g(\epsilon, \alpha) + \beta \mathbb{E} \left[ \delta^g_{NC} \left( \Pi_0(y, \alpha) + U^g_{NC}(x, \epsilon) \right) \right]
\]

\[+ \sum_{y', \alpha'} s \kappa (y', \alpha') \left( \tilde{P}^g_{NC}(x, \epsilon, \alpha) + \sigma \max \left\{ S^g_{NC}(x, \epsilon, y', \alpha') - S^g_{NC}(x, \epsilon, y, \alpha), 0 \right\} \right)
\]

\[+ \chi \tilde{P}^g_{PL}(x, \epsilon, \alpha) + \gamma \tilde{P}^g_{D}(x, \epsilon, \alpha) + (1 - \delta^g_{NC} - \chi - \gamma - sk V) \tilde{P}^g_{NC}(x, \epsilon, y, \alpha) \]

In the current period, the match between worker of human capital \(x\) and job of productivity \(y\) produces \(f(x, y)\) units of flow output, and pays a proportional tax \(\tau\). The worker enjoys a flow utility of \(q^g(\epsilon, \alpha)\) if he/she values amenities at \(\epsilon\) and works at a job with amenity level \(\alpha\).

In the next period, the worker’s human capital level is \(x_+\), where the transition matrix from \(x\) to \(x_+\) is given by the law of motion \(p_e(x, y)\) in employment. Upon exogenous separation \(\delta^g_{NC}\), the match dissolves and the worker and the employer both receive their outside options. The worker searches on-the-job and encounters poaching employer \((y', \alpha')\) at rate \(s \kappa (y', \alpha')\). The poaching and incumbent employers Bertrand-compete for the worker, and the continuation value of the match is its current value \(\tilde{P}^g_{NC}(x, \epsilon, y, \alpha)\) plus \(\sigma\) fraction of the difference in the surpluses.

Lifecycle shocks could also happen in the next period. Upon having a child at rate \(\chi\), the worker enters parental leave and the match receives a continuation value of \(\tilde{P}^g_{PL}(x, \epsilon, y, \alpha)\). As the worker turns 40 at rate \(\gamma\), the continuation value is \(\tilde{P}^g_{D}(x, \epsilon, y, \alpha)\).

I assume efficiency in all matches. This implies that an existing match endogenously dissolves if the joint value of the match falls below the sum of the agents’ outside options in separation. There could be endogenous quits when human capital level \(x\) changes and at any age segment \(a\) in life:

\[
\tilde{P}^g_{a}(x, \epsilon, y, \alpha) = \max \{ P^g_{a}(x, \epsilon, y, \alpha), \Pi_0(y, \alpha) + U^g_{a}(x, \epsilon) \}, \quad a = \{ NC, PL, YC, D \}
\]
3.4.4 Modeling parental leave

When a worker has a child, several changes take place. The woman’s value for job amenities changes from $q^f(\epsilon, \alpha)$ to $q^f_{YC}(\epsilon, \alpha)$, whereas the men’s value stays the same. Separation rates also change from $\delta^g_{NC}$ to $\delta^g_{YC}$. The joint value in parental leave is:

$$P^g_{PL}(x, \epsilon, y, \alpha) = \frac{R f(x, y)}{\text{reduced flow output}} + \frac{q^g_{YC}(\epsilon, \alpha)}{\text{value for amenities}}$$

$$+ \beta E\left[ \delta^g_{YC}(\Pi_0(y, \alpha) + U^g_{PL}(x_+, \epsilon)) + \eta^g \tilde{P}^g_{PL}(x_+, \epsilon, y, \alpha) + \gamma \tilde{P}^g_D(x_+, \epsilon, y, \alpha) \right]$$

$$+ (1 - \delta_{YC} - \eta^g - \gamma) \tilde{P}^g_{PL}(x_+, \epsilon, y, \alpha)$$

Mimicking the institutional settings in Finland as closely as possible, the model assumes the following. First, the worker goes into parental leave immediately after having a child, and gets paid a wage that is fully funded by the government for the whole duration of leave. Second, the worker on leave enjoys job protection and the employer has to keep the job available for when he/she returns. Third, the job still produces a flow output when the worker is absent, but production is slashed to a ratio $R$ proportion of previous amount.

One could think of parameter $R$ as a reduced-form way of capturing various challenges that lead to a decrease in production whenever a worker goes on parental leave. Even though Finnish employers do not face direct costs of financing employees’ wages while on leave, they may still encounter difficulties and costs in finding a replacement worker and/or coordinating schedules of existing workers to keep production going, potentially at a lower productivity. Ginja, Karimi and Xiao (2020) quantifies these costs experienced by firms in Sweden.\textsuperscript{8}

During the parental leave period, the worker’s human capital evolves at the same rate as in unemployment with probability $p_u(x)$, and there is no on-the-job search during parental leave. Women go back to the previous employer at rate $\eta^f$, which is related to the length of wage-replaced parental leave. Men take parental leave as well, but they go back to work at a different rate $\eta^m$.

\textsuperscript{8}We find that firms hired temporary workers and increased incumbents’ hours when parental leave was extended by 3 months in Sweden. Even though firms did not have to pay wages to the person on leave, the total wage bill cost of the re-organization was on average equivalent to the salary of 1.5 full-time workers.
Once the worker goes back to work, production goes back to the previous level again, and he/she continues to accumulate human capital. The worker can have another child any time during fertile ages (including during parental leave). Upon having another child while employed, the worker will go into parental leave again.

The government runs a balanced budget. The tax rate \( \tau \) is set such that total government transfers to matches where workers are on parental leave are equal to the total tax revenues collected in stationary equilibrium:

\[
\sum_{g} \sum_{x, \epsilon, y, \alpha} \phi_{0,PL}^{g}(x, \epsilon, y, \alpha) h_{PL}^{g}(x, \epsilon, y, \alpha) = \sum_{g} \sum_{x, \epsilon, y, \alpha} \sum_{a=NC, YC, D} \tau f(x, y) h_{0}^{g}(x, \epsilon, y, \alpha)
\]

where \( \phi_{0,PL}^{g}(x, \epsilon, y, \alpha) \) denotes the flow wage in \( PL \) stage received by a worker of gender \( g \) and type \((x, \epsilon)\) at job \((y, \alpha)\).

The joint values of matches in “Young Child” and “Done with children” stages are analogous, and are listed below:

\[
P_{YC}^{g}(x, \epsilon, y, \alpha) = (1 - \tau) f(x, y) + q_{YC}^{g}(\epsilon, y, \alpha) + \beta \mathbb{E} \left[ \delta_{YC}^{g} \left( \Pi_{0}(y, \alpha) + U_{YC}^{g}(x_{+}, \epsilon) \right) \right] + \sum_{y', \alpha'} sk v(y', \alpha') \sigma \max \{ S_{YC}^{g}(x_{+}, \epsilon, y', \alpha') - S_{YC}^{g}(x_{+}, \epsilon, y, \alpha), 0 \}
\]

\[
+ \gamma \tilde{P}_{D}^{g}(x_{+}, \epsilon, y, \alpha) + \chi \tilde{P}_{PL}^{g}(x_{+}, \epsilon, y, \alpha) + (1 - \delta_{YC}^{g} - \gamma - \chi) \tilde{P}_{YC}^{g}(x_{+}, \epsilon, y, \alpha)
\]

\[
P_{D}^{g}(x, \epsilon, y, \alpha) = (1 - \tau) f(x, y) + q^{g}(\epsilon, \alpha) + \beta \mathbb{E} \left[ \delta \left( \Pi_{0}(y, \alpha) + U_{D}^{g}(x_{+}, \epsilon) \right) \right] + \phi \Pi_{0}(y, \alpha)
\]

\[
+ \sum_{y', \alpha'} sk v(y', \alpha') \sigma \max \{ S_{D}^{g}(x_{+}, \epsilon, y', \alpha') - S_{D}^{g}(x_{+}, \epsilon, y, \alpha), 0 \}
\]

\[
+ (1 - \phi - \delta) \tilde{P}_{D}^{g}(x_{+}, \epsilon, y, \alpha)
\]

The transition parameters and preference parameters in “Young Child” stage are the same as in “Parental Leave” stage, and one should think of these two stages as the period where workers have young children at home. The only difference is that individuals in “Parental Leave” stage are matched with some employers but are not working, whereas those in “Young Child” stage are actively participating in the labor force.

In stage D, individuals are infertile and will not have any additional child. Men and women have the same separation rate \( \delta \). Workers retire at rate \( \phi \), upon which the joint value of the match is just the vacancy value.

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3.5 Steady-state balance flow conditions

In equilibrium all agents follow their optimal strategy. Denote the measure of workers of gender $g$ in age segment $a \in \{NC, PL, YC, D\}$ as $m^g_a$. Then the total measure of women of all ages should add up to 0.5, as in the case for men.

$$m^g_{NC} + m^g_{YC} + m^g_{PL} + m^g_D = 0.5$$  \hspace{1cm} (13)

Also, the flows into and out of each age segment should balance.

$$\chi (m^g_{NC} + m^g_{YC}) = (\gamma + \eta^g) m^g_{PL}$$ \hspace{1cm} (14)

$$\eta^g m^g_{PL} = (\chi + \gamma) m^g_{YC}$$ \hspace{1cm} (15)

$$\gamma (m^g_{NC} + m^g_{YC} + m^g_{PL}) = \phi^g m^g_D$$ \hspace{1cm} (16)

The equilibrium distribution of vacancies and matches will satisfy the following balance equation:

$$v(y, \alpha) + \sum_a \sum_{g=m,f} \sum_{x,\epsilon} h^g_a(x,\epsilon,y,\alpha) = \varphi(y,\alpha), \quad a \in \{NC, YC, PL, D\}$$ \hspace{1cm} (17)

Equilibrium distribution of workers must be such that flows into and out of any worker stock must balance for each worker type, in employed or unemployed state, in each age segment of life, across all job types (if employed). Please refer to D for details.

3.6 Definition of equilibrium

A stationary equilibrium is a tuple of value functions $\{U^m, U^f, P^m, P^f, \Pi_0\}$ together with a distribution of male and female workers across employment states and across job types $\{u^m, u^f, h^m, h^f\}$ as well as a distribution of job vacancies $v$ such that:

(i) The value functions satisfy Bellman Equations (4) to (12).

(ii) The distributions $\{u^m, u^f, h^m, h^f, v\}$ are stationary given the transitions implied by the value functions, and satisfy balanced flow conditions (13) to (17) and flow equations in D.

(iii) Equilibrium wages are determined by surplus sharing rules defined in (1) to (3).
Note that the equilibrium values and allocations (points (i) and (ii) above) can be solved without making any reference to wages, just like in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). This is because utility is transferable between the worker and employer, so joint values and surpluses do not depend on wages. Moreover, match formation and worker mobility decisions are determined only by the sign of surpluses or difference in surpluses between two jobs, so the equilibrium distributions also do not depend on wages. The advantage of this transferable utility (TU) framework is that it makes the model very tractable, and the computation of the equilibrium fairly straightforward.

4 Estimation

In this section, I estimate the model using Simulated Method of Moments (SMM). To this aim, I obtain a vector of moments from $N$ individuals in the data, $\hat{m}^D = \frac{1}{N} \sum_{i=1}^{N} m_i$, for example mean wages out of unemployment in the first five years after graduation, etc. Model counterparts to these moments, $\hat{m}^S(\theta) = \frac{1}{M} \sum_{j=1}^{M} m_j^D$, are obtained from $M$ simulated lives from the model based on a parameter vector $\theta$. The estimation involves finding the vector $\theta$ that brings the simulated moments as close as possible to the data moments, i.e. minimizing the criterion function

$$L(\theta) = (\hat{m}^D - \hat{m}^S(\theta))^T \hat{W}^{-1} (\hat{m}^D - \hat{m}^S(\theta))$$

where $\hat{W}$ is a weighting matrix.

Key parameters of interest are outlined below.

4.1 Model specification

I set the length of a period in the model to be one month. Human capital of the worker takes discrete values $x \in H = \{x_1, x_2, ..., x_N\}$ and $0 < x_1 < x_2 < ... < x_N$. Human capital accumulation is assumed to take the form

$$p(x_i, y) = \text{Prob}(x_{i+1}|x_i, y) = d_1 + d_2 y.$$  

See for example McFadden (1989) and Pakes and Pollard (1989). Constructing the likelihood function for this model is intractable.
where \( d_1, d_2 \in (0, 1) \). That is, every period an employed worker moves up by one category of human capital with a probability that is linear in his/her job productivity \( y \). This captures the idea that workers might learn faster on the job when matched with more productive employers.

Central to the model is the sorting of men and women across jobs, which is intimately related to the production function. I specify the production of a match to be a CES function in the worker’s human capital and the employer’s productivity

\[
f(x, y) = K \left[ a x^\rho + (1 - a) y^\rho \right]^{\frac{1}{\rho}}.
\]

This allows for various degrees of complementarity governed by the estimated value of \( \rho \). Home production is assumed to take the form \( b(x) = bx \).

Men and women draw their values for amenities \( \epsilon^m \) and \( \epsilon^f \) from normal distributions \( N(\mu^m, sd^m) \) and \( N(\mu^f, sd^f) \) respectively. In the No Child stage, value for amenities takes the simple form \( q^g = \epsilon^g \alpha \). Women’s value increases by \( M \) in motherhood, so that \( q^f_{YC} = (\epsilon^f + M) \alpha \) in YC and PL stages, whereas men’s values stay the same at \( q^m_{YC} = q^m \).

Finally, I assume the matching function has an elasticity of 0.5 and takes the functional form (see Petrongolo and Pissarides (2001)):

\[
m(\hat{U}, V) = \vartheta \sqrt{\hat{U} V}
\]

where effective job seekers \( \hat{U} = U_{NC} + s_U(U_{YC} + U_D) + s_E(1 - U_{NC} - U_{YC} - U_D) \). I allow search in unemployment to be different in early and late stages in life. The search intensity for the unemployed in NC stage is normalized to one, and that of the unemployed in YC and D stages will be \( s_U \). The relative search intensity of the employed is \( s_E \) and does not vary over the lifecycle.

In the next section I offer a heuristic argument on how the parameters are identified.

### 4.2 Estimation method and identification

Given the above specification, I estimate two sets of parameters in an iterative procedure. The first set of parameters involves separation rates \( \delta^g \) and parameters from the matching function, denoted by \( \lambda = (\delta^g, \vartheta, s_U, s_E) \). The second group includes model “core” parameters characterizing human capital processes, production functions, bargaining and
preferences, denoted by $\theta = (d_1, d_2, K, a, \rho, \sigma, b, \mu_m, \mu_f, M)$.

Note that separation rates, job-finding rates and job-to-job transition probabilities in the model depend on equilibrium surplus values and the equilibrium distribution of vacancies, and consequently cannot be obtained independently outside of the model. However, parameters in $\lambda$ are directly related to workers’ transitions in and out of work and between jobs given the equilibrium. So $\lambda$ can be identified given $\theta$. Estimating the two groups of parameters iteratively significantly reduces estimation time. For details of the estimation procedure and computation of standard errors, please refer to Appendix E.

Human capital growth rates $d_1$ and $d_2$ do not have a direct data counterpart since the assignment of workers to jobs is not random. However, with the aid of the full equilibrium structure of the model, these parameters can be related to the following aspects of the data. When a worker goes through an unemployment spell in the model, she falls off the job ladder and loses any “search capital” accumulated through job-to-job transitions. However, human capital is general and she will carry her accumulated experience to the next job. Comparing the wages immediately following a transition from unemployment to employment (UE wages) at different points of the lifecycle can inform us of the average human capital growth rate $d_1$ in the economy (Dustmann and Meghir (2005)).

Moreover, human capital growth in each productivity category $y$ is related to within-job wage growth. Since productivity groups are observed in the data, we can obtain the amount of within-job wage gain for people who have stayed in the same job category from one year to another, and compare the gains at high- versus low-productivity employers. However, wage gains within a job is also related to any renegotiation triggered by poaching firms. Since the amount of contact with poaching firms is disciplined by $s_E$ and $\theta$ that are pinned down in the previous step, the remainder would be related to human capital growth.

Key to identification of production function parameters is the sorting of men and women across jobs. When production is very complementary ($\rho$ very small or negative), the marginal return of employing a high-type worker is considerably higher for high-productivity jobs. In the presence of a capacity constraint of a firm, this implies that the match surplus might not be monotonically increasing in job productivity (Eeckhout and Kircher (2011)). Indeed, the values of match surplus might be an inverted-U shape (as shown in Figure 6) or even decreasing with respect to job productivity for a low-type worker. This is because the more productive the firm is, the higher its outside option compared to matching with the low-HC worker. So we might not see matches
of highly productive jobs with low-skilled workers in equilibrium, with implications on wage levels and variance within each job type. Moreover, women might face even fewer job opportunities at the top. This is because women quit more and generate less surplus in general, and a high option value of the top jobs means that employers would shut off matches with women before they shut off matches with equally skilled men.

**Figure 6.**
Surplus values of medium-skilled men and women in “No Child” stage – an example

Consider the contrary case where production is perfectly substitutable ($\rho = 1$), then there are no productivity gains from sorting compared to random matching. Surpluses will be monotonically increasing in job productivity for a given worker type. Since match values are typically lower for women than men, it would imply that the low-productivity jobs are the first ones not to match with women, and we would see different sorting patterns of men and women vis-à-vis the case where production is complementary.

Relative productivity of labor (parameter $a$) is closely related to human capital parameters and wage growth over the lifecycle. When human capital appreciates, production grows more when $a$ is high. Wages can increase more both when human capital upgrades more frequently and/or when there is a bigger wage boost at each upgrade. Although both $d_1$, $d_2$ and $a$ are positively related to wage growth moments, they could have opposite implications for UE wage levels. The intuition is that when $a$ increases, all jobs are much better off matching with high-HC workers when production is complementary, and top jobs are actually worse off matching with low-type workers given the increased
option value of hiring high-types. In contrast, an increase in $d_1$ or $d_2$ invariably raises surpluses and UE wages of all matches. As a result, in early career stages when most workers do not have much human capital, we will see lower UE wages when $a$ increases but higher UE wages when $d_1, d_2$ increase. The extent of this effect is of course dependent on the strength of complementarity.

Moreover, the human capital parameters and $a$ also have different implications for the gender wage gap. Since wages do not update whenever human capital grows (and only update when poaching firms post credible threat), a higher $d_1$ or $d_2$ implies that more wages will be front-loaded when the worker first starts the job. As a consequence, women’s lack of human capital accumulation (due to separations and parental leave) is less harmful for women as they can always get high wages out of unemployment. Therefore, higher human capital growth implies generally smaller wage gap between men and women. In contrast, when $a$ increases and low-type workers become less valuable to firms, low-type women become even less valuable than low-type men because women do not stay around long enough to become higher types.

Preference parameters $\mu_m$ and $\mu_f$ characterize how much men and women value job amenities, and are related to workers’ mobility patterns across jobs of high- and low-amenity types as well as the amount of wage cut one is willing to accept to work in high-amenity workplaces (compensating differentials). One caveat is that workers in high-amenity jobs might be positively selected with respect to productivity (both in the data and in the model). High-HC workers might not be willing to accept low-productivity jobs in general, but if the low-type job provides enough amenities it might be enough to push the match surplus above zero. The extent to which female workers are drawn to high-amenity jobs helps to identify the magnitude of $\mu_f$ relative to $\mu_m$. The increase in value for amenities during motherhood $M$ is closely linked to the proportion of women who switch into high-amenity jobs after childbirth.

I fix or calibrate the following parameters without explicitly using the model. Firstly, the exogenous distribution of jobs $\Gamma(y, a)$ in the model is fixed to the data distribution. The distribution of jobs along the productivity dimension is obtained through k-means clustering. For each firm-occupation cell, I proxy its long-term productivity by the average log wage of all workers who have worked in the job in all available years from 1995 to 2013. I then group the jobs into seven productivity categories by clustering on these long-term average wages using k-means. The support of the distribution is normalized so that the bottom group takes a productivity value of 1. Summary statistics on job productivity categories are provided in Table A1. The distribution of jobs along the amenities
dimension is obtained by ranking their amenity index constructed in subsection 2.4 and grouping them into 3 categories: very high amenity (above 90th percentile), high amenity (between 75 to 90th percentile) and regular jobs (below 75th percentile). The final distribution of jobs across both productivity and amenity dimensions are shown in Figure 7.

**Figure 7.**
Distribution of jobs by productivity and amenities

Secondly, I calibrate the lifecycle Poisson parameters. Fertility rate $\chi$ is calibrated to match the total number of children workers have, ageing rate $\gamma$ is set to match the number of years between graduation and age 40, and retirement rate $\phi$ is set so that individuals retire at age 60. The rates at which parental leave ends for men and women, $\eta_m$ and $\eta_f$, are calibrated to match the average length of parental leave taken for each child by men and women respectively.

Other calibrated parameters include $R$, $c$ and the initial human capital distributions of men and women. Recall that the flow production goes down to a ratio $R$ of previous levels during parental leave. I calibrate $R$ to the cost of extended parental leave estimated in Ginja, Karimi and Xiao (2020), where they use exogenous variations from a Swedish parental leave reform to quantify the costs faced by firms. The vacancy cost $c$ is calibrated to that in Lise, Meghir and Robin (2016). The initial productivity distributions of male and female workers are calibrated to match the initial wage distributions at labor market entry. The monthly discount rate $\beta$ is set to 0.988.
4.3 Results

Figure A1 summarizes the fit of the model on wages, wage growths, distributions across job types, as well as transitions.

The model fits the lifecycle wage profiles of men and women very well, and is able to replicate key moments of the data. Men have higher wages than women throughout the lifecycle, enjoy higher within-job wage growths, are less represented in low productivity jobs and more represented in high-end jobs (type 1 is lowest productivity and type 7 is highest). The proportion of women in high-amenity jobs increases after childbirth, and the gender wage gap increases in the first years after birth before coming down 10 years afterwards. All these important qualitative features of the data are captured by the model.

The distribution of women and men across jobs of different productivities is related to both human capital accumulation and the amount of statistical discrimination in the economy. While the model generally fits women’s progression across jobs over time, it does not seem to push men into high-end jobs fast enough. This could be due to three reasons: 1. men and women might have different rates of human capital accumulation in the data, whereas I force them to accumulate at the same speed governed by $d_1$ and $d_2$ in the model; 2. there might be some element of directed search in the data whereas the model is random search; and 3. the model does not generate enough hiring discrimination at top jobs because of the transferable utility framework.

The complete set of parameter estimates is presented in Table A2. The estimate of $\rho$ shows that production is strongly complementary between worker and firm productivity. This implies that vacancy value increases substantially by job productivity, whereas output does not increase much when a low-type worker matches with a more productive firm. So match surplus is declining by job productivity for low-HC workers, leading to some matches in the extreme off-diagonals not to form.

The human capital accumulation rate is positively related to job productivity – worker skills upgrade much faster when they work at highly productive firms. The estimates imply that in the job category with the lowest productivity, human capital appreciates at the rate of 0.011, whereas at the high end the rate is 0.034. There will be a divergence in human capital levels of men and women over time, not only because men spend more time working and accumulating skills, but also because men are more represented at top jobs that offer great learning opportunities.

Men and women have similar valuations for amenities before having children, but
women’s value increases to almost twice as much after childbirth. However, this does not translate to women’s sudden switch into high-amenity jobs in the model as compared to the data. This is because opportunities to move to high-amenity jobs do not arise immediately after childbirth because of frictions in the model, so some women already sort into high-amenity jobs beforehand, and others gradually move into high-amenity jobs after having children.

These estimates imply an equilibrium allocation where the most productive jobs (category 7) do not match with low-HC women in the “No Child” stage, whereas these jobs do match with equally low-HC men. Such hiring discrimination against women in early career could have long-term consequences considering the different rates of human capital accumulation across high- and low-productivity jobs. In the “Young Child” stage, men of the highest HC type do not match with low productivity jobs, whereas high-HC women in YC stage are willing to take these jobs. This is because search is more effective in unemployment than in employment, and high-skilled men would rather wait for a great offer in unemployment than take a low-end job. In contrast, even though all high-end firms would also like to hire high-skilled women, these women would not turn down any offer from low-end jobs either, since they know they might not stay long on the job so it is worth it to accumulate some human capital whenever they get the chance to. In the “Done with children” stage where workers have moved beyond child-rearing ages, match formation decisions are the same for men and women.

5 Gender gap decomposition and policy counterfactuals

Given model estimates, I will offer a decomposition of lifecycle gender gaps and conduct two policy counterfactuals. I first decompose the gender gaps in wages and representation at top jobs into: a) a human capital component, b) a statistical discrimination component, and c) a preference component. Then I compare two common policies aimed at reducing gender inequality: 1) more parental leave months earmarked for fathers; 2) a gender quota at top jobs; and 3) equal pay for men and women of the same type in the same job.
5.1 Decomposition of the lifecycle gender wage gap

There is no straightforward way of decomposing the gender wage gap, since all three channels mentioned above interact with each other. In the following decomposition exercise, I will focus on the impact of child-related career interruptions on human capital accumulation and its interactions with statistical discrimination, while considering preference for amenities separately. One should however keep in mind that human capital growth could also be affected by sorting into high- and low-amenity jobs since high amenity jobs are more concentrated in low-productivity firms, and statistical discrimination could also be based on gender differences in taste for amenities.

I decompose the gender wage gap in three steps. First, I allow men and women to have the same child-related interruptions, while keeping equilibrium wages and employment decisions fixed. That is, men and women will have the same parental leave duration so that the total number of months spent with the child remains the same as before, and they will also face the same separation rates in NC and YC stages such that the total measure of employed workers is fixed to the estimated level. Since we do not consider equilibrium effects at this point, any wage change after equalizing parental leave duration will be due to human capital accumulation as women spend more time working (and men less). On the other hand, when separation rates decrease, women’s wages could increase because of two reasons. First, women now stay longer on the job and gain more human capital and second, they fall off the job “ladder” less and can extract more match surplus via poaching firms.

Figure 8 shows how much of the total wage gap is explained by each of these channels, and Table A3 shows the responding proportions. The top black solid line is the gender wage gap implied by model estimates. The orange dotted line and orange solid line show the decreased gaps as a result of equalizing parental leave and equalizing separation rates, respectively.

The human capital factor does not explain much of the gender wage gap in early career since educated men and women behave similarly before having children. However, the human capital effect of PL duration and separation rates compounds over time – it explains over half of the wage gap 10 years into the labor force, and is responsible for 3/4 of the gap in 20 years.

The second step is to measure the effects of child-related interruptions on (i) equilibrium employment and (ii) equilibrium wages. When parental leave durations and sep-
aration rates are equalized between men and women, employers will anticipate similar behaviors of male and female workers around childbirth and reduce statistical discrimination in both hiring and wage decisions.

**Figure 8. Gender wage gap decomposition**

Notes: The lines represent the gap between male and female wages over the lifecycle after sequentially closing additional channels. The top black solid line is the wage gap based on model estimates. The colored lines are the counterfactual wage gaps under: 1. Equal parental leave durations between men and women, while keeping the old equilibrium wages and employment. 2. Equal separation rates in addition to 1., without equilibrium effects. 3. Implement the new equilibrium job allocations implied by equal PL and equal $\delta$. 4. Implement new equilibrium wages implied by equal PL and equal $\delta$. 5. Equal preference for amenities between men and women, before and after childbirth.

In order to measure changes in equilibrium employment, I keep wage policies of each match fixed at the old equilibrium, but allow match formation and mobility to change to the new equilibrium. In this counterfactual simulation, average wages of men and women would change because the distribution of workers across jobs has changed. In the new equilibrium, jobs in the highest productivity category that did not hire low-type women now start matching with both men and women in NC stage. High-type men who did not accept low-end jobs in YC stage now start taking them. Even though match formation decisions only change for a handful of types of workers and firms, the effects
will propagate to the rest of the distribution. More women at top jobs implies some men would be “pushed” to lower jobs. Vice versa, more men being drawn to bottom jobs means women will contact these vacancies with lower probability now, and encounter vacancies elsewhere with a relatively higher probability. These changes in allocations, however, have only a small impact on the overall gender wage gap as shown in Figure 8. They explains about 6 percent of the gap on top of what was explained by fertility-related interruptions in the first step. This small effect might be driven by the fact that allocation changes only occur for a small group of people, who do not influence average wages considerably. Another reason might be that wages are kept to the previous equilibrium where there is still substantial wage discrimination especially at top firms, so women who gain human capital from the hiring decision change do not gain much in terms of wages.

Next, I implement new equilibrium wages under equal PL and separations on top of the new equilibrium employment changes. Employer’s statistical discrimination in wages explains much of the wage gap in early career – 37 percent in the first 3 years since labor market entry. As the employer anticipates men and women to spend the same amount of time in parental leave and separate at the same rate, the expected future costs associated with leave-taking and turnover also become equal whether the job is given to a man or a woman. As a result, in the new equilibrium employers revise wages downwards for men and upwards for women in early career stages (in both NC and YC life stages) when workers are prone to fertility events. Wage discrimination fades over time as more and more workers move beyond child-rearing ages. There is no reason for statistical discrimination in infertile ages, since in stage D men and women have equal separation rates and will not have additional children or enter parental leave.

In the third step, I compute a new equilibrium based on equal valuations of amenities between men and women and no change in preferences after childbirth, in addition to equal parental leave and separation rates. There are both wage and mobility changes in the new equilibrium, and altogether these changes explain an additional 9 percent of the gender wage gap in late career. Since men and women have very similar values for amenities in the “No Child” stage, preference for job amenities explains little of the gap in early career.

5.2 Under-representation of women at top jobs

Although wages are the most common statistic to investigate in issues revolving gender inequality, another relevant and related question is: why do so few women make it into
top-level positions compared to men? How much of the gender wage gap come from the top versus bottom of the productivity distribution?

**Figure 9.**
Counterfactual proportion female at top jobs

I answer the first question by investigating the share of women at the most productive jobs. These are jobs in categories 6 and 7 which are mostly management and professional positions in high productivity firms. In the estimated model, 35% to 39% of the workforce in top jobs are women, as shown by the bottom black solid line in Figure 9.

Similar to the decomposition in subsection 5.1, I proceed in 3 steps. First, introduce equal parental leave durations and equal separation rates between men and women without changing the equilibrium. Second, change to the new equilibrium implied by equal parental leave and equal separations. Third, change to the new equilibrium implied by equal preference for amenities between men and women, and before/after childbirth.

The gap between the black solid line and the green dotted line in Figure 9 shows that over half of the gender imbalance in late career (year 20 and onward) could be eliminated.

Notes: The lines represent the proportion of workers in the most productive two job categories who are women when sequentially closing additional channels. The bottom black solid line is the female share implied by model estimates. The colored lines are counterfactual female shares under: 1. Equal parental leave duration and equal separation rates between men and women, without equilibrium effects. 2. Implement the new equilibrium (less hiring discrimination) implied by equal PL and separations. 3. Equal preference for amenities between men and women, before and after childbirth, in addition to 1. and 2.
by the human capital channel alone. Since hiring discrimination at top jobs is only towards low-skilled, inexperienced women, it means that these jobs always hire a high- or medium-skilled woman whenever they encounter one. So it is unsurprising that most of the problem could be attributed to the human capital factor – there are simply not as many encounters between these top jobs and high-skilled women as compared to high-skilled men. Even though medium-skilled workers are also hired, complementarity forces tend to push them to other jobs.

The blue dotted line in Figure 9 shows the resulting female share at top jobs with the new equilibrium distribution – without hiring discrimination. Statistical discrimination in hiring based on fertility concerns starts years before childbirth and accounts for almost half of the gender disparity at top jobs in early career. Since women who do not get access to top jobs in early years also do not accumulate as much human capital as their male counterparts, the impact of hiring discrimination persists over time.

Preference for amenities does not seem to play much of a role in women’s under-representation at top jobs.

5.3 Counterfactual policy experiments

I will consider three policies that have the main goal of reducing the gender wage gap – a “daddy month” parental leave expansion, a gender quota at top jobs, and an equal pay policy. I will compute the new equilibrium and quantify the effect of each policy on the gender wage gap and on gender disparities in top positions over the lifecycle.

5.3.1 Daddy months

In Finland and many other Nordic countries, there is generous wage-replaced parental leave of durations from 6 months to over a year that could be shared between the parents, but it is almost always the mother who takes up all of the shared leave. Many of these countries have then introduced 1 to 3 months of “daddy months” to encourage fathers to spend more time with the baby, and policies have been under debate to expand it even more to replace shared leave (Dahl, Løken and Mogstad (2014)).

I consider a policy that expands daddy’s leave by 2 months per child and reduce mother’s parental leave by 2 months. To do this, I calibrate the parental leave termination shocks $\eta_m$ and $\eta_f$ so that men’s leave duration per child increases from 2 to 4 months,
while that of women’s decreases from 18 to 16 months.

The daddy month policy is quite effective in reducing the gender wage gap throughout the lifecycle. As shown in Figure 10a, the wage gap closes by 15% during the first 3 years of working, and over 10% afterwards. About half of the impact on wages comes from a reduction in statistical discrimination in pre-child years. Even though hiring discrimination still persists in years prior to childbirth, women’s wages are now closer to men’s when they are hired.

![Figure 10. Counterfactuals under daddy months policy](image)

Women gain more human capital during mid-career because they come back to work sooner after having children, while men accumulate less. This slightly balances the gender ratio in top jobs as the proportion of women increases from 39 to 41 percent by year 25 (see Figure 10b). Shifting 2 months of parental leave from women to men seems to push even more women into top positions than the gender quota policy, due to the long-term effect of human capital accumulation.

One caveat of this policy is that it might not result in a pareto improvement – the progress in women’s outcomes might come at the expense of men’s. In order to assess the overall social value of the policy, define social welfare (SW) as the sum of home production of the unemployed and production of the matched workers and firms net of the total cost of vacancies:

\[
SW = \sum_g \sum_a \sum_{x, \epsilon} b(x) u^g_a(x, \epsilon) + \sum_g \sum_a \sum_{x, \epsilon, y, \alpha} f(x, y) h^g_a(x, \epsilon, y, \alpha) - \sum_{y, \alpha} c \tau(y, \alpha)
\]

By the time men become fathers, they are already in slightly more advanced positions
than women and are producing more output, so the output loss of having men spend 2
months at home cannot be fully compensated by having women spend 2 more months
working. However, I find that the net loss in social welfare is very very small, only about
0.02%.

Since parental leave is wage-replaced and men typically earn more than women, mak-
ing men take a larger share in parental leave requires slightly more taxation to fund it. The
corporate tax rate on flow output increases modestly from 2.80% to 2.88%.

Overall, the daddy month policy is much more effective in reducing gender inequality
at the workplace compared to the gender quota policy. This is because gender quotas in
top jobs only address hiring discrimination at the surface without tackling its root cause –
career interruptions of women around childbirth. Daddy months, on the other hand, improve both the human capital shortage of women and reduce statistical discrimination
in wages based on fertility concerns.

### 5.3.2 Gender quota

To address the under-representation of women in top-earning jobs, many countries have
passed legislature to require a certain percentage of female board members in public com-
panies. Finland requires state-owned enterprises to reserve 40% of board seats to female
directors. However, the evidence on the effectiveness of these policies in reducing gender
gaps is mixed at best (Bertrand, Black, Jensen and Lleras-Muney (2018)).

There is no direct way of implementing a gender quota in the model since the propor-
tion of women in a particular job category depends not only on this job’s hiring decision,
but also on transition rates and mobility to all other jobs in equilibrium. Even if the em-
ployer hires both men and women of all types, the gender ratio may not be 0.5 because of
sorting.

In practice, I impose that the top jobs (those in the highest productivity category) have
the same hiring policy towards a woman and a man of the same type, for the purpose of
prohibiting hiring discrimination at top-level positions. This policy essentially changed
hiring decisions of productive employers towards low-human capital women in the “No
Child” stage. Since these are matches that would not have been formed in the absence
of the quota policy, there is no standard wage protocol about how to split the (negative)
match surplus. In this exercise, I assume that the employer sets the wage to cover the
vacancy value of the job, and the worker gets the rest of the match value.
Unsurprisingly, banning hiring discrimination at top jobs corrects much of the gender imbalance in those jobs during the early years of workers’ professional lives. As shown in Figure 11b, proportion female at top job categories increases from 35.5 to 39.5 percent during the first 5 years of work. However, this effect is very short-lived. Since the gender quota policy does not address child-related career interruptions, women start falling behind men in human capital accumulation soon after childbirth, and are thus less likely to stay in highly productive jobs later in their careers due to positive assortative matching. The proportion female in top jobs almost falls back to baseline levels during child-rearing years. The overall effect of the gender quota on the share of women in top jobs is only slightly positive by the end of the lifecycle.

Even though the gender quota improves women’s representation at top jobs, firms will undo this policy by exerting more wage discrimination. Women hired under the gender quota policy receive much lower wages than men in the same job during the early years of the lifecycle. This is because matches are now required to form even though they generate negative surpluses, and the new female hires have to “compensate” the employers by accepting subpar wages. Since the new hires are a small proportion of the working population, the overall wage gap increases by very little. However, the human capital gain these women have obtained from being employed in productive jobs starts to pay off in later years, and the negative impact of the policy on women’s wages disappears from year 10 onward (see Figure 11a).
5.3.3 Equal pay policy

Many OECD countries have passed some form of Equal Pay Act that requires men and women in the same workplace be given equal pay for equal work. The Finnish Equality Act requires companies with 30 or more full-time employees to draft a gender equality plan, which should include an assessment of pay differences between men and women who perform work of equal value.\textsuperscript{10}

In the equal pay counterfactual, I require women of human capital level $x$ and amenity preference $\epsilon$ to have the same flow wage as men of the same type in the same job $(y, \alpha)$. I then calculate the equivalent lifetime value of the female worker $W_a^f(\phi_{0,a}^m, x, \epsilon, y, \alpha)$ implied by having men’s wages $\phi_{0,a}^m$ in each age segment $a$, everything else fixed. Recall that the match surplus takes the form:

\[
S_a^f(x, \epsilon, y, \alpha) = W_a^f(w, x, \epsilon, y, \alpha) - U_a^f(x, \epsilon) + \Pi_a^f(w, x, \epsilon, y, \alpha) - \Pi_0(y, \alpha) .
\]

Both the worker and the employer have to receive at least their outside options ($U_a^g(x, \epsilon)$ and $\Pi_0(y, \alpha)$ respectively) for the match to survive. When the worker’s value $W_a^f$ is required to increase, the employer’s portion $\Pi_a^g(w, x, \epsilon, y, \alpha) - \Pi_0(y, \alpha)$ might become negative, and the match would no longer form.

I simulate the workers’ careers with the equal wage policies, and impose that matches where the employer’s value $\Pi_a^g(w, x, \epsilon, y, \alpha)$ falls below its vacancy value $\Pi_0(y, \alpha)$ will not form. Note that even though wages within a $(x, \epsilon, y, \alpha)$ match are the same across gender, there could still be a gap between the average wages of men and women due to different compositions of worker types within each gender. As shown in Figure 12, the equal pay policy unsurprisingly reduces the gender wage gap throughout the lifecycle. However, some matches are destroyed in the stages after having children. As a result, women are more likely to fall off the “career ladder”, more likely to be unemployed, and they accumulate less human capital. The proportion of women in top jobs decreases (see Figure 12b).

The equal pay policy closes the gender wage gap by 28% in the first 3 years since labor market entry, and over 10% thereafter. However, it has unintended consequences as women are more likely to be unemployed, and the proportion of women in top jobs

\textsuperscript{10}Details of the Equality Act and related reforms can be found at: https://www.finlex.fi/en/laki/kaannokset/1986/en19860609
decreases from 39 to 38 percent by year 25.

**Figure 12.** Counterfactuals under equal pay policy

(A) Gender wage gap

(B) Female proportion at top jobs

6 Conclusion

This paper studies the mechanisms underlying the gender wage gap over the lifecycle – human capital accumulation, preference for amenities, and employer discrimination in wages and hiring. I propose an equilibrium search model with capacity constraints, production complementarities, fertility and parental leave, and taste for job amenities. The model is estimated using matched employer-employee data from Finland combined with occupation-level data on amenities from the Finnish Quality of Work Life Survey.

Men and women behave very differently in the labor market especially after having children. Employers take into account of these gender differences and statistically discriminate women even before they have children. The model estimates show that statistical discrimination based on fertility concerns explains a large portion of the gender wage gap in early career, while human capital accounts for the majority of the gap in late career.

The most effective policies in reducing gender gaps are those that alleviate women’s childcare responsibilities, for example childcare expansions that help to reduce women’s separation rates, and more parental leave for fathers. These policies would not only help women gain more human capital on the job, but also correct firms’ expectations and reduce statistical discrimination in both wages and employment. However, eliminating hiring discrimination at top jobs through a gender quota reduces women’s average wage in early career, and eliminating wage discrimination through an equal pay policy
reduces the proportion of women in top positions as employers adjust on the hiring mar-
gin. Taken together, the policy counterfactuals show that it would be difficult to achieve
gender equality at the workplace without more equality in family responsibilities, given
the sizable effect of employer statistical discrimination in equilibrium. Requiring equality
in one margin (either wages or employment) induces firms to counteract the policy on the
other margin, and does not address the main source of statistical discrimination – career
interruptions of women around childbirth.

A generalization of the framework would involve modeling separation rates as arising
from intra-household decisions that take into account of the husband and wife’s labor
market prospects. Employers’ decisions and women’s labor force attachment might be
interdependent. Employers’ priors that women are more prone to higher separations
might become a self-fulfilling prophecy if the resulting discrimination in wages and job
opportunities induce women to specialize in household production. This would be an
interesting area of future research.
References


# Appendix

## Appendix A  Tables and figures

<table>
<thead>
<tr>
<th>Job productivity types</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of workers</td>
<td>27,192</td>
<td>37,155</td>
<td>38,003</td>
<td>41,466</td>
<td>37,309</td>
<td>22,161</td>
<td>13,136</td>
</tr>
<tr>
<td>Number of workers per job</td>
<td>2.00</td>
<td>4.20</td>
<td>4.03</td>
<td>4.05</td>
<td>4.04</td>
<td>2.91</td>
<td>2.24</td>
</tr>
<tr>
<td>Mean log-wages</td>
<td>2.64</td>
<td>2.96</td>
<td>3.10</td>
<td>3.24</td>
<td>3.39</td>
<td>3.55</td>
<td>3.83</td>
</tr>
<tr>
<td>SD of log-wages</td>
<td>0.212</td>
<td>0.043</td>
<td>0.041</td>
<td>0.041</td>
<td>0.044</td>
<td>0.056</td>
<td>0.133</td>
</tr>
<tr>
<td>% Clerical jobs</td>
<td>33.51%</td>
<td>7.37%</td>
<td>4.49%</td>
<td>2.91%</td>
<td>1.47%</td>
<td>1.01%</td>
<td>0.70%</td>
</tr>
<tr>
<td>% Associates</td>
<td>23.03%</td>
<td>18.19%</td>
<td>28.42%</td>
<td>19.54%</td>
<td>13.02%</td>
<td>9.50%</td>
<td>3.46%</td>
</tr>
<tr>
<td>% Professionals</td>
<td>42.01%</td>
<td>72.26%</td>
<td>63.6%</td>
<td>70.03%</td>
<td>70.89%</td>
<td>59.97%</td>
<td>35.27%</td>
</tr>
<tr>
<td>% Managers</td>
<td>1.45%</td>
<td>2.17%</td>
<td>3.49%</td>
<td>7.52%</td>
<td>14.62%</td>
<td>29.52%</td>
<td>60.56%</td>
</tr>
</tbody>
</table>

Table A1. Summary statistics by job productivity types
### Table A2.
Parameter Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>SEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complementarity</td>
<td>$\rho$</td>
<td>-15.531</td>
</tr>
<tr>
<td>Relative productivity</td>
<td>$a$</td>
<td>0.856</td>
</tr>
<tr>
<td>TFP</td>
<td>$K$</td>
<td>29.230</td>
</tr>
<tr>
<td>Baseline HC rate</td>
<td>$d_1$</td>
<td>0.001</td>
</tr>
<tr>
<td>Proportional HC rate</td>
<td>$d_2$</td>
<td>0.010</td>
</tr>
<tr>
<td>Men’s value for amenities</td>
<td>$\mu_m$</td>
<td>0.783</td>
</tr>
<tr>
<td>Women’s value for amenities</td>
<td>$\mu_f$</td>
<td>0.867</td>
</tr>
<tr>
<td>Preference increase in motherhood</td>
<td>$M$</td>
<td>1.744</td>
</tr>
<tr>
<td>Worker’s bargaining</td>
<td>$\sigma$</td>
<td>0.522</td>
</tr>
<tr>
<td>Home productivity</td>
<td>$b$</td>
<td>5.164</td>
</tr>
<tr>
<td>Women’s separation rate in NC</td>
<td>$\delta_{NC}$</td>
<td>0.012</td>
</tr>
<tr>
<td>Women’s separation rate in YC</td>
<td>$\delta_{YC}$</td>
<td>0.016</td>
</tr>
<tr>
<td>Men’s separation rate</td>
<td>$\delta$</td>
<td>0.008</td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>$\vartheta$</td>
<td>0.107</td>
</tr>
<tr>
<td>Relative search intensity in unemployment</td>
<td>$s_U$</td>
<td>0.719</td>
</tr>
<tr>
<td>Relative search intensity in employment</td>
<td>$s_E$</td>
<td>0.531</td>
</tr>
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</table>
Table A3.
Proportion of gender wage gap explained by each channel

<table>
<thead>
<tr>
<th>Years in labor force</th>
<th>Equal PL duration</th>
<th>Equal separations</th>
<th>Hiring discrimination</th>
<th>Wage discrimination</th>
<th>Preference for amenities</th>
<th>Taste-based discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>11.80 %</td>
<td>6.79 %</td>
<td>6.75 %</td>
<td>36.85 %</td>
<td>-1.59 %</td>
<td>39.39%</td>
</tr>
<tr>
<td>6</td>
<td>24.68 %</td>
<td>19.16 %</td>
<td>6.94 %</td>
<td>21.72 %</td>
<td>3.93 %</td>
<td>23.57%</td>
</tr>
<tr>
<td>9</td>
<td>25.59 %</td>
<td>30.79 %</td>
<td>6.41 %</td>
<td>11.91 %</td>
<td>7.30 %</td>
<td>17.99%</td>
</tr>
<tr>
<td>12</td>
<td>27.20 %</td>
<td>37.87 %</td>
<td>5.87 %</td>
<td>5.85 %</td>
<td>8.65 %</td>
<td>14.55%</td>
</tr>
<tr>
<td>15</td>
<td>28.69 %</td>
<td>38.49 %</td>
<td>5.44 %</td>
<td>2.55 %</td>
<td>9.14 %</td>
<td>15.69%</td>
</tr>
<tr>
<td>18</td>
<td>29.00 %</td>
<td>41.98 %</td>
<td>4.58 %</td>
<td>0.61 %</td>
<td>9.06 %</td>
<td>14.76%</td>
</tr>
<tr>
<td>21</td>
<td>32.32 %</td>
<td>43.32 %</td>
<td>3.61 %</td>
<td>-0.49 %</td>
<td>8.65 %</td>
<td>12.60%</td>
</tr>
<tr>
<td>24</td>
<td>33.56 %</td>
<td>46.92 %</td>
<td>4.12 %</td>
<td>-1.04 %</td>
<td>8.55 %</td>
<td>7.89%</td>
</tr>
</tbody>
</table>
FIGURE A1.
Model fit

(A) Log hourly wage

(B) UE wages

(C) Decline in within-SD

(D) Proportion working

(E) EU transitions

(F) Gender wage gap around birth

(G) Initial distribution

(H) Compensating differential

(I) Gender gap in % high amenity

NOTES: The solid lines represent model-predicted moments, the dashed lines are data moments, and green denotes women while orange denotes men. Shaded areas correspond to 95% confidence intervals.
Appendix B  Parental leave system in Finland

The Finnish maternity allowance system was first introduced in 1964. Currently, parents are entitled to wage-replaced leave for a total of 12 months, in which 4 months are reserved for mothers, 2 months for fathers, and 6 months can be shared between the spouses. In addition, parents are entitled to Child Home Care Allowances until the child turns 3 years old. Both biological and adoptive parents are entitled to parental leave on the basis of permanent residence in Finland.

The amount of parental leave benefits is a piece-wise linear function of annual earnings in the previous employment, or social benefits collected in the case of unemployment. The rate of wage replacement depends on income tiers as shown in the following table:

<table>
<thead>
<tr>
<th>Annual earnings (€)</th>
<th>Calculation formula (annual amount in €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>up to 11,942</td>
<td>8,358</td>
</tr>
<tr>
<td>11,943 - 37,861</td>
<td>0.7 x annual earnings</td>
</tr>
<tr>
<td>37,862 - 58,252</td>
<td>26,503 + 0.40 x (annual earnings - 37,861)</td>
</tr>
<tr>
<td>over 58,252</td>
<td>34,659 + 0.25 x (annual earnings - 58,252)</td>
</tr>
</tbody>
</table>

After the parental leave is over, parents can continue to care for the child at home and receive the Home Care Allowances (HCA). The HCA may be paid to either parent, although it is predominantly the mother who takes up the allowance. The HCA benefit amount consists of two parts – there is a fixed amount of 338.34 euros per month for one child under 3, and a means-tested amount targeted at low-income families up to 180 euros per month. In addition, there is sibling extra and municipality-based supplements. For details of HCA, please refer to Kosonen (2014).

The benefit amount of the parental leave allowance and the HCA claimed are separately reported in the FOLK data for each individual in each calendar year. This paper uses the pay schedule in Table A4 and the fixed HCA amount adjusted by inflation to infer the total number of months of parental leave taken for each worker.
Appendix C  Wage determination and workers’ values

To facilitate notation, define function \( A(\cdot, \cdot) \):

\[
A(v_p, v_I) = \begin{cases} 
  v_I + \sigma(v_p - v_I) & \text{if } v_p > v_I \\
  v_p + \sigma(v_I - v_p) & \text{otherwise}
\end{cases}
\]

where \( v_{g,a}^S(x, \epsilon, y', \alpha') = P_{g,a}^S(x, \epsilon, y', \alpha') - \Pi_0(y', \alpha') \) is the maximum value the poaching job offers, and \( v_{g,a}^I(x, \epsilon, y, \alpha) = P_{g,a}^S(x, \epsilon, y, \alpha) - \Pi_0(y, \alpha) \) is the maximum the incumbent job offers.

The equation below illustrates an example of the worker’s value when he/she gets a wage \( \phi_0 \) out of unemployment in the “No Child” stage:

\[
W_{NC}^g(\phi_{0,NC}(x, \epsilon, y, \alpha), x, \epsilon, y, \alpha) = U_{NC}^g(x, \epsilon) + \sigma S_{NC}^g(x, \epsilon, y, \alpha) \\
= \phi_{0,NC}(x, \epsilon, y, \alpha) + q^g(\epsilon, \alpha) + \beta \mathbb{E} \left[ \delta U_{NC}^g(x_+, \epsilon) + \gamma \tilde{W}_{PL}^g(w_+, x_+, \epsilon, y, \alpha) + \chi \tilde{W}_{PL}^g(w_+, x_+, \epsilon, y, \alpha) \right. \\
+ \sum_{y', \alpha'} s_k v(y', \alpha') \max \left\{ A(v_{NC,p}(x_+, \epsilon, y', \alpha'), v_{NC,I}(x_+, \epsilon, y, \alpha)), -\tilde{W}_{NC}^g(w_+, x_+, \epsilon, y, \alpha), 0 \right\} \\
\left. + (1 - \delta - \gamma - \chi) \tilde{W}_{NC}^g(w_+, x_+, \epsilon, y, \alpha) \right]
\]

where \( w_+ \) denotes the wage in the next period, and \( x_+ \) denotes the worker’s human capital type in the next period. When a worker’s human capital changes from \( x \) to \( x_+ \) in the next period, the wage does not update until there is a credible outside option. At any point in time, the match can dissolve endogenously if surplus falls below zero.

Appendix D  Steady-state balance equations

In a stationary equilibrium, flows into and out of any worker stock must balance. Every period is divided into 3 stages. Let \( u_a^-(x, \epsilon) \) and \( h_a^-(x, \epsilon, y, \alpha) \) denote the distributions of workers in unemployment and employment at the beginning of the current search period at age \( a \in \{NC, YC, PL, D\} \). In the human capital evolution stage (Stage I), the worker’s skill type changes from \( x \) to \( x_+ \) according to stochastic processes \( p_e(x_+ | x, y) \)
during employment (except in PL stage) and $p_u(x_+|x)$ during unemployment.

$$u_a^I(x, \epsilon) = u_a^I(x, \epsilon) + \sum_{x' \neq x} u_a^I(x', \epsilon) p_u(x|x') - \sum_{x' \neq x} u_a^I(x, \epsilon) p_u(x'|x)$$ (18)

$$h_a^I(x, \epsilon, y, \alpha) = h_a^I(x, \epsilon, y, \alpha) + \sum_{x' \neq x} h_a^I(x', \epsilon, y, \alpha) p_e(x|x', y) - \sum_{x' \neq x} h_a^I(x, \epsilon, y, \alpha) p_e(x'|x, y)$$

for stages $a \in \{NC, YC, D\}$. Workers in PL stage do not accumulate human capital, so

$$h_{PL}^I(x, \epsilon, y, \alpha) = h_{PL}^I(x, \epsilon, y, \alpha).$$

In the search stage (Stage II):

$$u_{NC}^{II}(x, \epsilon) = u_{NC}^{I}(x, \epsilon) \left(1 - \gamma - \chi - \kappa \sum_{y, \alpha} v(y, \alpha) \mathbb{1}[S_{NC}^{f}(x, \epsilon, y, \alpha) > 0]\right) + (0.5\phi D) \xi_0(x, \epsilon) + \delta_{NC} \sum_{y, \alpha} h_{NC}^{I}(x, \epsilon, y, \alpha)$$ (19)

$$h_{NC}^{II}(x, \epsilon, y, \alpha) = h_{NC}^{I}(x, \epsilon, y, \alpha) \left(1 - \gamma - \chi - \delta_{NC}\right) + \kappa u_{NC}^{I}(x, \epsilon) v(y, \alpha) \mathbb{1}[S_{NC}^{f}(x, \epsilon, y, \alpha) > 0] + \kappa \mathbb{1}[S_{NC}^{f}(x, \epsilon, y, \alpha) > S_{NC}^{f}(x, \epsilon, y', \alpha')]

- \kappa h_{NC}^{I}(x, \epsilon, y, \alpha) \sum_{y', \alpha'} v(y', \alpha') \mathbb{1}[S_{NC}^{f}(x, \epsilon, y', \alpha') > S_{NC}^{f}(x, \epsilon, y, \alpha)]$$

$$u_{PL}^{II}(x, \epsilon) = u_{PL}^{I}(x, \epsilon) \left(1 - \gamma - \eta\right) + \chi \left(u_{NC}^{I}(x, \epsilon) + u_{YC}^{I}(x, \epsilon)\right) + \delta_{YC} \sum_{y, \alpha} h_{PL}^{I}(x, \epsilon, y, \alpha)$$

$$h_{PL}^{II}(x, \epsilon, y, \alpha) = h_{PL}^{I}(x, \epsilon, y, \alpha) \left(1 - \gamma - \delta_{YC} - \eta\right) + \chi \left(h_{NC}^{I}(x, \epsilon, y, \alpha) + h_{YC}^{I}(x, \epsilon, y, \alpha)\right)$$
\[ u_{YC}^H(x, \epsilon) = u_{YC}^I(x, \epsilon) \left( 1 - \gamma - \chi + \kappa \sum_{y, a} v(y, a) 1[S_{YC}^f(x, \epsilon, y, a) > 0] \right) \\
+ \eta u_{PL}^I(x, \epsilon) + \delta_{YC} \sum_{y, a} h_{YC}^I(x, \epsilon, y, a) \]

\[ h_{YC}^H(x, \epsilon, y, a) = h_{YC}^I(x, \epsilon, y, a) (1 - \gamma - \delta_{YC} - \chi) + \eta h_{PL}^I(x, \epsilon, y, a) \]
\[ + \kappa u_{YC}^I(x, \epsilon) v(y, a) 1[S_{YC}^f(x, \epsilon, y, a) > 0] \]
\[ + s_k v(y, a) \sum_{y', a'} h_{YC}^I(x, \epsilon, y', a') 1[S_{YC}^f(x, \epsilon, y, a) > S_{YC}^f(x, \epsilon, y', a')] \]
\[ - s_k h_{YC}^I(x, \epsilon, y, a) \sum_{y', a'} v(y', a') 1[S_{YC}^f(x, \epsilon, y', a') > S_{YC}^f(x, \epsilon, y, a)] \]

\[ u_{D}^H(x, \epsilon) = u_{D}^I(x, \epsilon) \left( 1 - \phi - \chi + \kappa \sum_{y, a} v(y, a) 1[S_{D}^f(x, \epsilon, y, a) > 0] \right) \]
\[ + \gamma \left( u_{NC}^I(x, \epsilon) + u_{YC}^I(x, \epsilon) \right) + \delta \sum_{y, a} h_{D}^I(x, \epsilon, y, a) \]
\[ h_{D}^H(x, \epsilon, y, a) = h_{D}^I(x, \epsilon, y, a) (1 - \phi - \delta) + \gamma \left( h_{NC}^I(x, \epsilon, y, a) + h_{YC}^I(x, \epsilon, y, a) + h_{PL}^I(x, \epsilon, y, a) \right) \]
\[ + \kappa u_{D}^I(x, \epsilon) v(y, a) 1[S_{D}^f(x, \epsilon, y, a) > 0] \]
\[ + s_k v(y, a) \sum_{y', a'} h_{D}^I(x, \epsilon, y', a') 1[S_{D}^f(x, \epsilon, y, a) > S_{YC}^f(x, \epsilon, y', a')] \]
\[ - s_k h_{D}^I(x, \epsilon, y, a) \sum_{y', a'} v(y', a') 1[S_{D}^f(x, \epsilon, y', a') > S_{D}^f(x, \epsilon, y, a)] \]

In the endogenous quits stage:

\[ u_a^+(x, \epsilon) = u_a^I(x, \epsilon) + \sum_{y, a} h_a^H(x, \epsilon, y, a) 1[S_a^f(x, \epsilon, y, a) < 0] \quad (20) \]

\[ h_a^+(x, \epsilon, y, a) = h_a^H(x, \epsilon, y, a) (1 - 1[S_a^f(x, \epsilon, y, a) < 0]), \quad \forall \ a \in \{NC, PL, YC, D\} \]

After the dismissals (or endogenous quits) occur, \( u_a^+ \) and \( h_a^+ \) become the initial distributions for the next period. In stationary equilibrium, \( u_a^- = u_a^+ \) and \( h_a^- = h_a^+ \).
Appendix E  Estimation procedures and standard errors

I use the following iterative procedure to estimate two sets of parameters, the transition parameters $\lambda = (\delta_{NC}^{f}, \delta_{YC}^{f}, \delta, s_{U}, s_{E})$ and the core parameters $\theta = (d_{1}, d_{2}, K, a, \rho, \sigma, b, \mu_{m}, \mu_{f}, M)$.

**Step 1: Core moments given transition parameters**  Given a value for the transition parameters $\lambda$ obtained from the previous iteration (or an initial guess at the start), I estimate $\theta$ by minimizing the following quadratic distance

$$L_{1}(\theta|\lambda) = (\hat{m}_{1}^{D} - \hat{m}_{1}^{S}(\theta|\lambda))^{T} \hat{W}_{1}^{-1} (\hat{m}_{1}^{D} - \hat{m}_{1}^{S}(\theta|\lambda))$$

where $\hat{m}_{1}^{D}$ is a vector of data moments related to wage profiles of men and women, U-to-E wages and wage growths, proportion of men and women in high- and low-amenity jobs etc. that are described in section 4.2. The vector $\hat{m}_{1}^{S}$ are the corresponding model moments from simulations, taking $\lambda$ as given.

**Step 2: Transition moments given core parameters**  Given the estimate of $\theta$ obtained from the previous step, I update the estimate of $\lambda$ by matching appropriate moments related to transitions:

$$L_{2}(\lambda|\theta) = (\hat{m}_{2}^{D} - \hat{m}_{2}^{S}(\lambda|\theta))^{T} \hat{W}_{2}^{-1} (\hat{m}_{2}^{D} - \hat{m}_{2}^{S}(\lambda|\theta))$$

I iterate over these two steps using MCMC until the functions $L_{1}$ and $L_{2}$ are minimized and the estimates of $\lambda$ and $\theta$ converge. The estimation strategy is a good fit for my problem because MCMC is derivative-free, so it is able to handle the non-linearities in the criterion functions due to the discreteness in the model. MCMC can also deal with large parameter spaces and multiple local minima quite well.\(^{11}\)

I use the sandwich formula to estimate standard errors. Normally, the variance of the converged MCMC chain would provide a direct way to construct valid confidence intervals for the parameter estimates if the optimal weighting matrix is used. But I use a diagonally weighted approach. I will illustrate the computation for the core parameters $\theta$ below (the calculation is analogous for the transition parameters $\lambda$). The estimated

\(^{11}\)See the discussion in Chernozhukov and Hong (2003) for more details.
covariance matrix has the form

\[
\hat{\mathcal{V}}(\hat{\theta}) = \left(G'(\hat{\theta})\Omega G(\hat{\theta})\right)^{-1} G'(\hat{\theta}) \Omega \hat{\mathcal{E}} \left( (\hat{m}_1^S(\hat{\theta}) - \hat{m}_1^D)(\hat{m}_1^S(\hat{\theta}) - \hat{m}_1^D)' \right) \Omega G(\hat{\theta}) \left(G'(\hat{\theta})\Omega G(\hat{\theta})\right)^{-1}
\]

where \( \Omega \) is the weight matrix used in the estimation, \( G(\hat{\theta}) \) is the gradient matrix evaluated at the estimated parameters \( \hat{\theta} \).

Estimates for the gradient \( G \) are obtained through simulation. Suppose \( m_1 \) consists of \( K \) moments and \( \theta \) consists of \( J \) parameters. Then the numerical derivatives \( \hat{G}(\hat{\theta}) \) is a \( K \times J \) matrix where the \( j \)-th column is computed as:

\[
\hat{G}_j = \frac{m_1^S(\hat{\theta} + \hat{\theta}_j) - m_1^S(\hat{\theta} - \hat{\theta}_j)}{2 \hat{\theta}_j}
\]

where \( m_1^S \) is the vector of simulated moments evaluated at \( \hat{\theta} + \hat{\theta}_j \) and \( \hat{\theta} - \hat{\theta}_j \) respectively. The step size of deviation \( h \) is a vector of zeros except for one positive element at the \( j \)-th position equal to 1\%. \( \hat{\theta}_j \) is the \( j \)-th element of \( \hat{\theta} \).