From Secular Stagnation to Robocalypse? Implications of Demographic and Technological Changes*

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Abstract

Demographic changes and a new wave of innovation and automation are two main structural trends shaping the macroeconomy into the next decades. We present a general equilibrium model with a tractable demographic structure that allows the investigation of the main economic transmission mechanisms by which demography and technology affect the macroeconomy. Due to a trade-off between innovation and automation, population ageing lowers GDP per capita growth. Delaying of retirement age and assuming different scenarios for the roles of robots and labour may only partially compensate for the decrease of growth brought up by the projected demographic changes.

Key Words: Population Ageing, Automation, Innovation.

JEL Classification Codes: O31, O40, J11.

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1 Introduction

There are two structural forces that are about to frame the macroeconomic context in the next decades. One is the demographic change arising from the transitory effects of the baby boomers reaching retirement age, a permanent fall in fertility, and the continuous rise in longevity. Another is the technological change associated to the new wave of automation brought by developments in robotics and in artificial intelligence. The consequences of these structural changes for economic growth are subject of much debate. On the one hand, population ageing is found to be associated with lower interest rates, innovation activity and growth (Aksoy, Basso, Smith, and Grasl (2018), see also, Gordon (2012) and Derrien, Kecskés, and Nguyen (2017)). On the other hand, in a series of recent papers Acemoglu and Restrepo (2017b, 2018c, 2018a) argue that population ageing may give incentives to automation and, hence, to higher productivity growth, which may even increase long-run GDP per capita growth, although automation is also found to have negative employment effects.[1]

We analyse the macroeconomic consequences of demographic and technological changes employing a model in which population age structure, innovation, and automation interact with each other. Innovation involves the creation of new products and automation consists of the procedures that allow robots to replace labour in the production process. Although ageing boosts automation we find that demographic changes eventually lead to lower GDP per capita growth due to a trade-off between innovation and automation, which is often neglected in the analysis of the economic implications of robotics and artificial intelligence. Automation crowds out innovation, and as automation is a subsidiary activity of innovation, without innovation, automation cannot progress indefinitely.

[1]The empirical literature on the employment and wage effects of automation is increasing rapidly, providing a wide range of estimates of the "number of jobs that will be lost to automation". See, for instance Graetz and Michaels (2015), Acemoglu and Restrepo (2017a), Dauth, Findeisen, Sdekum, and Woessner (2017) and Frey and Osborne (2017).

In our set-up, one source of endogenous growth is innovation activity, which requires resources and labor to produce new ideas (varieties). New varieties are initially produced employing labour. Eventually, research and development activity geared towards generating automation procedures allow varieties to be produced employing robots, replacing labour. As robots are more productive than labour, automation promotes growth. With this general framework, we show how some demographic changes (fall in fertility, rise of longevity, delay of retirement age) impact innovation activity, automation, and, subsequently, economic growth. We identify three key channels. First, changes in labour supply affect the relative profitability of labour intensive and automated sectors, altering the incentive to automate and generate new products. Second, ageing affects savings and interest rate, and, therefore, the amount of resources available for investment in capital accumulation, innovation, and automation. Third demographic changes affect the efficiency of the research and development (R&D) sector insofar as it may depend on the age structure of population involved in innovation activities.

We start by looking at the long-run effect of population growth changes. We show analytically that if the economy is at a balanced growth path (with constant shares of the innovation and automation sectors) and GDP per capita growth is higher than the rate of growth of population, then a fall in labor supply leads the economy to a new balanced growth path with *lower* GDP per capita growth.

We then turn our attention to the medium-run dynamics of the demographic changes expected in the main advanced economies in the next decades. We utilize the UN demographic projections for the US and Europe and show how lower fertility and higher longevity leads to higher automation both in the US and in Europe, with a stronger effect in Europe. Automation supports growth, depresses wages and leads to lower labour income shares. However, the incentive to automate after a drop in the supply of labor leads to resources being diverted from innovation and, hence, the production of new ideas, which increases total factor productivity and makes it feasible the continuing automation of jobs into the future, is compromised. This effect, combined by the negative impact of lower supply of young workers on the productivity of the R&D sector eventually leads to lower rates of output growth, offsetting the gains from automation. Hence, our model delivers the result that, despite automation, population ageing leads to lower output growth, even without assuming that the discoveries of new ideas are harder to arise (as in Bloom, Jones, Reenen, and Webb (2017)).

In an attempt to offset the negative effects of demographic changes we modify the model altering both the role of workers in innovation and robots in production and innovation. Regarding the labour roles, we consider three cases: (*i*) we assume innovation no longer relies on labour input; (*ii*) we assume that upon observing a fall in wage after automation, workers migrate towards the R&D sector boosting labour supply and innovation activity; and (*iii*) we increase the retirement age as longevity increases. In all cases the negative effect of population ageing on growth is only partially offset. When retirement age increases to maintain the ratio of working life and retirement duration constant, demographic changes no longer generate an increase in automation.

Regarding the roles of robots we consider two alternative specifications: (*i*) automation leads to an increase in the relative productivity of robots; and (*ii*) we assume robots may replace labour also in the R&D sector. Once again we are only able to partially offset the negative effect on per capita output growth. In the first case, due to the presence of intermediate inputs in both labour and robot intensive sectors, higher relative productivity of robots increase total factor productivity in both sectors, reducing automation in the medium-run. In the second case automation is higher in the medium-run since the negative effect of resource reallocation on innovation is mitigated by the use of robots in R&D.

Even when we assume automation increases the productivity of robots we are not able to generate higher per capita growth. In order to ensure our economy converges back to a balanced growth path after a demographic change we assume the price of robots change to adjust the incentive for robot production and employment such that the weight of each sector in the economy does not asymptotically converge to zero. In that respect our framework embeds a "Baumol cost disease" of robots as discussed by Aghion, Jones, and Jones (2017). We altered the model offsetting this price adjustment mechanism in the medium-run (only enforcing it in the long-run). In this case, automation continues to increase and the share of the automated sector asymptotically approaches one. The labour share of income decreases substantially. Nonetheless, due to the trade-off between innovation and automation, innovation is neglected leading to a significant fall in per capita growth. Thus, a robocalypse scenario, resembling the immiseration equilibrium of Benzell, Kotlikoff, LaGarda, and Sachs (2015), arises.

In what follows, we describe the model (Section 2), discuss the characteristics of the balanced growth path, the assumptions needed for the economy to achieve it, and prove the main result regarding the reduction of economic growth after a fall in labor supply growth in the long-run (Section 3). Section 4 focuses on the medium-run effects of demographic changes using population projections for the US and Europe. We perform several quantitative exercises to illustrate the impacts of changes in fertility, mortality and delay of retirement age on the macroeconomy under different modelling assumptions. Comments on the lessons from these exercises about the economic consequences of demographic and technological changes and on the challenges for policy are given in the final section (Section 5).

2 The Model

We analyse an economy that consists of four main structures: a good production sector, a research and development (R&D) sector, a robots production sector, and households. The good production sector comprises of a final good producer, who aggregates intermediate goods produced by a continuum of intermediate good firms $i \in Z_t$ whose production processes employ a composite of goods from all firms (inputs), capital and either robots or labor (as in Acemoglu and Restrepo (2018c)). Thus, capital and labour are complementary factors of production, and so are capital and robots. However, robots substitute labour, but only in production processes for which automation knowledge has been generated. Thus, there is a worker displacement effect of robots.

The R&D sector comprises two activities: innovation and automation. Innovation creates new product varieties, being the key driver of endogenous growth (Romer (1990) and Comin and Gertler (2006)). We assume that newly invented varieties are added to the set Z_t of intermediate goods that can be produced using labour. Automation consists in the development of new procedures such that a variety i can then be produced employing robots. The set of varieties that can be produced using robots is denoted $A_t \subset Z_t$. Robots are assumed to be more productive than workers, and, hence, the introduction of robots increases productivity growth. Thus, robots are machines used in production while automation comprises the knowledge that allows robots to be employed in the production of a variety.

The household sector has a life-cycle structure, whereby individuals face two stages of life, mature (worker) and old (retirement). On this, we follow Gertler (1999) building a tractable framework that delivers closed-form solutions for consumption and allows us to investigate the implications of changes in the three main parameters that are driving current demographic trends: a fall in fertility, an increase of longevity, and the delay of the retirement age. Finally, there is a zero expected profit financial intermediary to facilitate the allocation of assets between the household and the production and innovation sectors, and to provide annuity services to the retired households.

2.1 Household Sector

There is a continuum of agents of mass N_t, divided amongst two age groups: workers (w) and retirees (r). $\omega^y_{t,t+1} N^w_t$ individuals are born every period as workers. Workers retire with a probability $1 - \omega^w$, and retirees die and leave the economy with a probability $1 - \omega^r_{t,t+1}$. As a result, the population dynamics are

$$N^w_{t+1} = \omega^y_{t,t+1} N^w_t + \omega^w N^w_t, \tag{1}$$

$$N^r_{t+1} = (1 - \omega^w) N^w_t + \omega^r_{t,t+1} N^r_t. \tag{2}$$

Workers and retirees decide their consumption to maximise welfare subject to a budget constraint. They face two idiosyncratic risks: i) loss of wage income at retirement and ii) time of death. There is a perfect annuity market allowing retirees to insure against time of death. They turn their wealth over to perfectly competitive financial intermediaries which invest the proceeds and pay back a return of $R_t/\omega^r_{t-1,t}$ for surviving retirees.

As in Gertler (1999), we assume that households are risk neutral. In this way, the uncertainty about the employment tenure does not affect optimal choices. Nevertheless, we keep a motive for consumption smoothing by assuming that individual preferences belong to the Epstein and Zin (1989) utility family, such that risk neutrality coexists with a positive elasticity of intertemporal substitution.

Thus, for $z = \{w, r\}$ we assume that the agent j selects consumption and asset holdings to maximise

$$V^{jz}_t = \left\{ (C^{jz})^{\rho_U} + \beta^z_{t,t+1} (E_t[V^j_{t+1} \mid z]^{\rho_U}) \right\}^{1/\rho_U} \tag{3}$$

subject to

$$C^{jz}_t + FA^{jz}_{t+1} = R^z_t FA^{jz}_t + W_t I^z + d^z_t \tag{4}$$

where $\beta^z_{t,t+1}$ is the discount factor, which is equal to β for workers and $\beta\omega^r_{t,t+1}$ for retirees, R_t^z is the return on assets, which is equal to R_t for workers and $R_t/\omega_{t-1,t}^r$ for retirees. W_t^j is the wage for worker j, and I^z is an indicator function that takes the value of one when $z = w$ and zero otherwise; thus we assume retirees do not work and workers' labour supply is fixed. FA_t^{jz} are the assets acquired from the financial intermediary and d_t^z is the dividend from the financial intermediary.

We assume a fixed share Sw_{RD} of new workers $\omega_{t,t+1}^y N_t^w$ enter in the $R\&D$ labour markets and the remaining $(1 - Sw_{RD})$ supply labour to intermediate good firms (we will consider below an extension where Sw_{RD} is endogenous). We also assume that at every period a fraction $drop_{RD}$ of existing R&D workers, who do not retire, are no longer able to work in this sector and thus start supplying labour to firms in the production sector. We do so to reflect the fact that the innovation productivity peaks during the first 10-15 years of a workers life (see Jones (2010)). As such, the set of workers at time t, N_t^w, are subdivided into N_t^{wRD} and N_t^{wL} such that

$$N_{t+1}^{wRD} = \omega_{t,t+1}^y N_t^w Sw_{RD} + (1 - drop_{RD})\omega^w N_t^{wRD}, \text{ and} \tag{5}$$

$$N_{t+1}^{wL} = \omega_{t,t+1}^y N_t^w (1 - Sw_{RD}) + \omega^w N_t^{wL} + (drop_{RD})\omega^w N_t^{wRD}. \tag{6}$$

The wage in the R&D labour market is W_t^{RD} and in the production labour market is W_t.

After aggregation, the key conditions that describe the individuals' behaviour are the consumption functions of workers and retirees, depicted below (the remaining equilibrium conditions of the household sector are described in the appendix)

$$C_{w,t} = \varsigma_t[R_t FA_{w,t} + H_{w,t} + D_{w,t}] \quad \text{and} \tag{7}$$

$$C_{r,t} = \varepsilon_t \varsigma_t[R_t FA_{r,t} + D_{r,t}], \tag{8}$$

where, $H_{w,t}$ is the present value of gains from human capital, $D_{z,t}$ is the present value of dividends for $z = \{w, r\}$. ς_t denotes the marginal propensity of consumption of workers and $\varepsilon_t \varsigma_t$ the one for retirees (where $\varepsilon_t > 1$). As marginal propensities to consume are different across ages, changes in the distribution of asset holdings across workers and retirees, as well as the population age structure, affect aggregate demand. Moreover, the marginal propensities to consume are functions of fertility (ω^y), longevity (ω^r) and time of retirement (ω^w). Thus, through changes in savings, these demographic variables end up affecting the equilibrium interest rate.

2.2 Production

Final good producers combine intermediate varieties to produce a final good. The production function is given by

$$y_t = \left[\int_0^{Z_t} y_{i,t}{}^{\frac{\psi-1}{\psi}} di \right]^{\frac{\psi}{\psi-1}} \tag{9}$$

As such, demand for each variety and the price of final goods are given by

$$y_{i,t} = \lambda \left(\frac{P_{i,t}}{P_t} \right)^{-\psi} y_t, \qquad P_t^{1-\psi_c} = \left[\int_0^{Z_t} P_{i,t}{}^{1-\psi} di \right] \tag{10}$$

where $P_{i,t}$ is the price of each variety.

Each firm $i \in [0, Z_t]$ produces a specialised good that is sold to final producers. Specialised good or varieties are different in respect to the production process that can be adopted. A subset of varieties $i \in A_t$ the technological frontier is such that they can be produced using final goods as inputs ($\Upsilon_{i,t}$), an amount of rented capital ($K_{i,t}$) and robots ($M_{i,t}$) or labor ($L_{i,t}$). We assume that robots are more productive than labour and thus if a good can be produced utilising robots, the firm selects to do so. For the remaining varieties $i \in Z_t \setminus A_t$, production can only be done using inputs ($\Upsilon_{i,t}$), an amount of rented capital ($K_{i,t}$) and labor ($L_{i,t}$). Intermediate production therefore is given by

$$\begin{cases} y_{i,t} = ((K_{i,t})^{\alpha}(\theta_t M_{i,t})^{1-\alpha})^{1-\gamma_I}\Upsilon_{i,t}^{\gamma_I} \text{ for } i \in A_t \\ y_{i,t} = ((K_{i,t})^{\alpha}(L_{i,t})^{1-\alpha})^{1-\gamma_I}\Upsilon_{i,t}^{\gamma_I} \text{ for } i \in Z_t \setminus A_t \end{cases} \tag{11}$$

θ_t denotes the relative productivity of robots versus labour. We initially set $\theta_t = \bar{\theta} > 1$ in the benchmark model. We also consider a case where θ_t increases as a function of the pace of automation in the economy (in this scenario the productivity of robots relative to labour increases as the economy automates more). Note that capital in our set-up is complementary to labour and thus technological progress that increases the productivity of capital may in fact increase wages. On the contrary, robots are assumed to replace labour, and, thus, as the share of varieties that can be produced using robots increase, wages are depressed (see Acemoglu and Restrepo (2018b) for a discussion).

2.3 Research and Development Sector

R&D is divided into two activities. The creation of varieties (*innovation*) and the development of procedures that make it possible for robots to be used in the production process for a variety (*automation*).

Let Z_t^p be the stock of varieties for innovator p, who at each period spends S_t^p and employs labour $(L_{I,p,t})$ to invent $\varphi_t(S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L}$ new varieties. Thus, Z_{t+1}^p is given by

$$Z_{t+1}^p = \varphi_t(S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L} + \phi Z_t^p, \tag{12}$$

where ϕ is the product survival rate. In Comin and Gertler (2006) the productivity of new inventions φ_t is assumed to be given by $\varphi_t^{CG} = \chi Z_t[\tilde{\Psi}^{\rho}(S_t)^{1-\rho}]^{-1}$, where χ is a scale parameter. Thus, it depends on the aggregate stock of prototypes (Z_t); so there is a positive spillover as in Romer (1990), and on a congestion externality via the factor $[\tilde{\Psi}_t^{\rho}(S_t)^{1-\rho}]^{-1}$, such that a balanced growth path exists, and the R&D elasticity of new technology creation in equilibrium is ρ. To give rise to a direct link between population and innovation we assume that R&D requires investment and labour to create new goods.[2] Hence, we take the productivity of invention as given by $\varphi_t \equiv \chi Z_t[\tilde{\Psi}^{\rho}(S_t)^{\kappa_{RD}-\rho}(N_t)^{\kappa_L}]^{-1}$, where we additionally include a measure of total population (N_t) in the congestion factor to ensure that a balance growth path exists. As discussed in Jones (1995) and more recently Bloom, Jones, Reenen, and Webb (2017), models of endogenous growth where an increase in the growth rate of the stock of workers employed in R&D (due to population growth) generates faster steady state output growth are inconsistent with the data. Finally, parameters κ_{RD} and κ_L denote the relative importance of labour and final good investment for total R&D.

We assume that innovators borrow S_t^p from the financial intermediary. Owning the rights of a variety allows investors to charge a fraction ϑ of the profits of the intermediate good firm who produces that variety and thus the value of an invented variety J_t is given by

$$J_t = \vartheta \Pi_{i,t} + (R_{t+1})^{-1}\phi E_t J_{t+1}, \text{ for } i \in Z_t \setminus A_t \tag{13}$$

where $\Pi_{i,t}$ for $i \in Z_t \setminus A_t$ is the profit of the intermediate good firm producing the newly created variety.

The innovator p will then invest $IS_{p,t} = (S_{p,t})^{\kappa_{RD}}(L_{I,p,t})^{\kappa_L}$ until the marginal cost equates the expected gain. Defining $\tau_{S,t}$ as the shadow price of $IS_{p,t}$, we have

[2]Several contributions look at the relevance of demographics and innovation (a non-exhaustive list is Kremer (1993), Feyrer (2008) and Aksoy, Basso, Smith, and Grasl (2018))

that

$$\phi E[J_{t+1}] = \frac{R_{t+1}\tau_{S,t}}{\varphi_t}, \quad (14)$$

$$S_{p,t} = IS_{p,t}\tau_{S,t}\kappa_{RD} \quad (15)$$

$$L_{I,p,t}W_{RD,t} = IS_{p,t}\tau_{S,t}\kappa_L \quad (16)$$

Total profits of innovators ($\Pi_{I,t}$) is given by the fraction of the profits acquired from the labour intensive intermediate good firms net of investment and labour costs.

The key mechanism driving the creation of new varieties relies on the changes in the profits in the labour intensive sector. If innovators expect $\Pi_{i,t}$ for $i \in Z_t \setminus A_t$ to increase, $E[J_{t+1}]$ goes up, increasing the incentives to invest ($S_{p,t}$) and to hire more labour ($L_{I,p,t}$). This leads to an increase in Z_t, ultimately increasing total output.

Automation investors (q) invest $\Xi_{q,t}$ and hires $L_{A,q,t}$ to transform a Z_t^q variety into a A_t^q variety, which then becomes part of the set of varieties that can be produced using robots.[3] This conversion process succeeds with probability $\lambda_t = \lambda\left(\frac{(Z_t^q - A_t^q)^{\kappa_{RD}+\kappa_L}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}}\Xi_{q,A,t}\right)$ and $\lambda'(\cdot) > 0$; thus more investment in automation yields more varieties whose production process employs robots. If unsuccessful, the good remains a labour intensive variety. Once automation is successful the investor gains the right to charge a fraction ϑ of the profits of the intermediate good firm whose production process is robot intensive.

$$V_t = \vartheta\Pi_{i,t} + (R_{t+1})^{-1}\phi E_t V_{t+1}, \text{ for } i \in A_t. \quad (17)$$

Thus, automation investors select $\Xi_{q,A,t} = (\Xi_{q,t})^{\kappa_{RD}}(L_{A,q,t})^{\kappa_L}$ to

$$\max_{\Xi_{q,A,t}} -\tau_{A,t}\Xi_{q,A,t} + (R_{t+1})^{-1}\phi E_t[\lambda_t(V_{t+1} + (1-\lambda_t)J_{t+1}]. \quad (18)$$

[3] We also consider an extension of the model in which robots can also be used as inputs in the automation of tasks, which resembles the artificial intelligence model in Aghion, Jones, and Jones (2017)

where, $\tau_{A,t}$ is the shadow price of $\Xi_{q,A,t}$.

Assuming the elasticity of λ_t to changes in its input is constant, thus $\epsilon_\lambda = \frac{\lambda'}{\lambda_t}\frac{(Z_t^q - A_t^q)^{\kappa_{RD}+\kappa_L}\Xi_{q,A,t}}{\tilde{\Psi}_t^{\kappa_{RD}} N_t^{\kappa_L}}$, then we obtain

$$\begin{aligned} \Xi_{q,t} &= \epsilon_\lambda \lambda_t R_t^{-1}\phi[V_{t+1} - J_{t+1}] && (19) \\ L_{A,q,t}W_{RD,t} &= \Xi_{q,t}\frac{\kappa_L}{\kappa_{RD}} && (20) \end{aligned}$$

The stock of labour intensive varieties at t, for which automation is feasible, is $(Z_t^q - A_t^q)$. Thus,

$$A_{t+1}^q = \lambda_t \phi (Z_t^q - A_t^q) + \phi A_t^q. \quad (21)$$

The expenditure in consumption goods of automation investors, financed by borrowing, is $\Xi_{q,t}(Z_t^q - A_t^q)$. Profits of automators $(\Pi_{A,t})$ is given by the fraction of the profits acquired from the intermediate good firms that employ automated procedures net of investment and labour costs.

The degree of automation depends on the relationship between the value of a labour intensive variety (J_t) and a robot intensive variety (V_t). These are functions of the profitability in each of these sectors. As the profits in the automated sector increase relative to profits in labour intensive firms, $E[V_{t+1} - J_{t+1}]$ increases, raising the incentive to invest in automation. As robots are more productive than labour, that leads to higher output. However, as automation increases, the incentive to create new varieties decreases. As such the stock $(Z_t - A_t)$ shrinks, reducing the potential for automation in the future.

2.4 Robots Producer Sector

Robots (machines that substitute labour) are needed to be employed in the production process of A_t varieties. We assume there are robot producers who invest Ω_t final goods to produce $M_t = \varrho\Omega_t^\eta$ robots. They sell these robots to intermediate good producers whose varieties are automated changing $P_t q_t$ for each robot. Robots producers select Ω_t to maximise profits, thus

$$\max_{\Omega,t} \Pi_{\Omega,t} = q_t P_t M_t - P_t \Omega_t \quad s.t. \quad M_t = \varrho \Omega_t^{\eta}. \tag{22}$$

2.5 Financial Intermediary

The financial intermediary sells assets to the households (FA_t^w, FA_t^r), owns capital (K_t) and rents it to firms (charging r_t^k) and lends funds (B_{t+1}) to innovators and automation investors to finance their expenditure (given by S_t and $\Xi_t(Z_t - A_t)$, respectively). Finally, we assume it owns the innovation plants, robots and good producers receiving their dividends at the end of the period. Thus, financial intermediary profits are

$$\begin{aligned} \Pi_t^F &= [r_t^k + 1]K_t + R_t B_t - R_t(FA_t^w + FA_t^r) - K_{t+1} - B_{t+1} \\ &\quad + FA_{t+1}^w + FA_{t+1}^r + (\Pi_{A,t} + \Pi_{RD,t} + (1-\vartheta)\int_{Z_t} \Pi_{i,t} di + \Pi_{\Omega,t}), \end{aligned} \tag{23}$$

where $B_{t+1} = S_t + \Xi_t(Z_t - A_t)$ and $FA_t = FA_t^w + FA_t^r$.

2.6 Market Clearing Conditions

The market clearing conditions that guarantee that a general equilibrium exists are the following:

Final Good: $y_t = C_{w,t} + C_{r,t} + \int_0^{Z_t} \Upsilon_{i,t} di + \Omega_t + I_t$.

Capital Flow Condition: $K_{t+1} = (1-\delta)K_t + I_t$

Capital Markets: $K_t = \int_0^{Z_t} K_{i,t} di$

Inputs: $\Upsilon_t = \int_0^{Z_t} \Upsilon_{i,t} di$

Robots Markets: $M_t = \int_0^{A_t} M_{i,t} di$

Labour Markets: $N_t^{wR} = \int_q L_{A,q,t} di + \int_p L_{i,p,t} di$, and $N_t^{wL} = \int_0^{Z_t \setminus A_t} L_{i,t} di$

2.7 Equilibrium and Balanced growth path

The symmetric equilibrium is a tuple of endogenous predetermined variables $\{FA^z_{t+1}$, K_{t+1}, A_{t+1}, Z_{t+1}, $B_{t+1}\}$ and a tuple of endogenous variables $\{C^z_t$, H^w_t, d^z_t, D^z_t, N^{wR}_t, N^{wL}_t, y_t, $y_{i,t}$, $y_{j,t}$, M_t, Ω_t, S_t, Ξ_t, $L_{A,t}$, S_t, $L_{I,t}$, V_t, J_t, λ_t, Π^i_t, Π^j_t, C_t, r^k_t, R_t, Π^{RD}_t, Π^A_t, W_t, $W_{RD,t}$, $P_{i,t}$, $P_{j,t}$, q_t, ε_t, $\varsigma_t\}$ for $z = \{w, r\}$, $i \in A_t$, $j \in Z_t \setminus A_t$ obtained such that:

*a.*Workers and retirees maximize utility subject to their budget constraint; *b.* Intermediate and final firms maximize profits; *c.* Innovators and automation investors maximise their gains; *d.* Robot producers maximize their profits; *e.* The financial intermediary selects assets to maximize profits, and its profits are shared amongst retirees and workers according to their share of assets; and *f.* Consumption goods, capital, labour, robots and asset markets clear.

We must set the efficiency of investment in the innovation sector ($\tilde{\Psi}_t$) such that as output grows investment in innovation and automation do not diverge. Comin and Gertler (2006), in a similar model, assumes $\tilde{\Psi}_t$ is equal to the value of the stock of capital, since in their setting the price of capital is determined at time t, $\tilde{\Psi}_t$ fluctuates accordingly to ensure stability. In our model there is only one final good sector and thus the price of capital and the value of the capital stock are constant at t, invalidating this choice of scaling factor. We therefore select the current value of automated goods as our scaling factor.[4] Thus,

$$\tilde{\Psi}_t \equiv V_t A_t. \tag{24}$$

Finally, in a balanced growth path the output shares of the labour intensive sector and of the robot sector remain both constant. The introduction of a robots producing sector, which implicitly determines the relative price of robots q_t, ensures

[4]We also verify the robustness of our results by setting $\tilde{\Psi}_t \equiv y_t$. Impulse responses are more persistent but the results are qualitatively similar.

the convergence towards the balanced-growth path. Therefore, as in Aghion, Jones, and Jones (2017), to avoid singularity, we are appealing to the "Baumol cost disease" of robots, so that as robots become abundant its relative price falls, so that the profitability of the production of robots, used to perform automated tasks, also falls.

3 Population growth under balanced-growth

The age structure of advanced economies is remarkably shifting towards older ages with both fertility and mortality decreasing. We analyse the impact of each of these forces in our model in the next section, but as one of the outcomes of these demographic changes is a lower share of labour supply in total population, we start by looking at the long-run effects of labour supply growth, and whether innovation and automation are able to offset them. Under balanced-growth, we can derive the following proposition.

Proposition 1. *After a reduction in labour supply growth, in the long run, as the economy converges to a new balanced growth path, per capita growth decreases when* $\eta < 1$.

Proof

At a balanced growth path Z_t must grow at the same rate as A_t. Moreover, $\int_{i \in A_t} y_{i.t} di$, $\int_{i \in Z_t \setminus A_t} y_{i.t} di$ and y_t must also grow at the same rate. Using the demand functions for each variety from the final good producer and aggregating across A_t and Z_t we obtain that the relative price in each sector $(P_{i,t}/P_t)$ for automated and $(P_{j,t}/P_t)$ for labour intensive, which are function of A_t and Z_t, also grow at the same rate. Thus,

$$g_{pM,t} = (g_t^A)^{\varphi_c - 1} = (g_t^{ZA})^{\varphi_c - 1} = g_{pL,t},$$

where $g_{pM,t} = \frac{P_t^M/P_t}{P_{t-1}^M/P_{t-1}}$,$g_{pL,t} = \frac{P_t^i/P_t}{P_{t-1}^i/P_{t-1}}$, $g_t^A = \frac{A_t}{A_{t-1}}$, $g_t^{ZA} = \frac{Z_t - A_t}{Z_{t-1} - A_{t-1}}$.

Using the marginal costs for each sector and the demand for labour and robots, the growth rate of the price of robots is equal to the ratio between the growth rates of output and of the labour force. Simply,

$$g_{pM,t} = g_{q,t}^{(1-\alpha)(1-\gamma_I)} = \left(\frac{g_t}{g_t^w}\right)^{(1-\alpha)(1-\gamma_I)} = g_{pL,t}, \text{ where } g_{q,t} = \frac{q_t}{q_{t-1}}, g_t^w = \frac{N_t^w}{N_{t-1}^w}, g_t = \frac{y_t}{y_{t-1}}$$

Using the production for robots, at the new balanced growth path, $(g_t)^{\eta-1} g_{q,t} = 1$. Define $gpc_t \equiv g_t/g_t^w$. If at initial balanced growth path, $g_t > g_t^N = g_t^w$ and thus $\eta < 1$ then $\frac{dgpc_t}{dg_t^n} < 0$. □

The key intuition behind this result is that under a balanced growth path, the output shares of the automated and labour intensive sectors converge to a constant. As each sector's output is produced by capital, inputs and machines or labour, the last two must eventually grow at the same pace. The price of robots q_t changes ensuring this result, and, thus, ultimately the growth rate of output in each sector is a function of labour supply growth. Through the lenses of our model, since the higher growth generated after a shift towards more automation cannot fully compensate for a fall in labour supply growth, there will be a negative impact of lower labour supply growth on output per capita growth.

4 Quantitative Analysis

We now look at the short and medium-run effects of a set of demographic changes (fall in fertility and increase in longevity). Before presenting the results we briefly describe the calibration.

4.1 Calibration

Throughout the calibration, we set one period of the model to correspond to one year. We define workers as the individuals between 20 and 65 years old and retirees are the individuals above 65 years old. Then, we define the parameters that control the law of motion of age group populations to match the average share of workers and retirees in total population in 1993 in the U.S. Matching these moments yields a birth rate of new young agents of $\omega^y = 0.0265$, a probability of the transition from mature to old of $1 - \omega^w = 0.022$, and a death probability for an old agent of $1-\omega^r = 0.07$. The share of workers in innovation (Sw_{RD}) is set such that N^{wRD}/N_t matches the share of R&D workers in US population, and $drop_{RD}$ is set to make the average age of R&D workers to be 40 (slightly lower than the average age of employed scientists reported in the Survey of Doctorate Recipients (SDR) of the National Science Foundation - 2013).

For the parameters that govern the innovation process, we follow Comin and Gertler (2006) closely. We set obsolescence (ϕ) and productivity in innovation (χ) so growth per working age person is 0.016, matching the data for the U.S. from 1970 onwards, and the share of innovation expenditures in total GDP is 0.012. The mark-up for intermediate goods is 15%. The elasticity of intermediate goods with respect to R&D (ρ) is 0.9. The rate of automation is set to $\lambda = 0.1$. The elasticity of this rate to increasing intensity (ϵ_λ) is set to 1. Finally we set κ_{RD} = 1, matching the framework in Comin and Gertler (2006). The link between demographics and innovation depends on the elasticity of invention to employed workers in R&D, parameter κ_L. We follow Aksoy, Basso, Smith, and Grasl (2018), who calibrate this parameter to $\kappa_L = 0.5$, reflecting the changes in productivity of individuals of different ages described in Jones (2010). Finally we set the standard macro parameters, in line with Comin and Gertler (2006)). The discount factor β= 0.96; the capital share $\alpha = 0.33$; the yearly depreciation rate $\delta = 0.08$ the share of intermediate goods $\gamma_I = 0.5$. Following Gertler (1999) we set the intertemporal elasticity of substitution $(1/(1-\rho_U)) = 0.25$. The robots production sector is introduced to ensure we obtain a balanced growth path and thus having output growth and population growth we obtain $\eta = 0.15$ such that $(g_t)^{\eta-1}(g_t/g_t^w) = 1$. A table with all parameters is shown in the appendix.

4.2 Demographic changes, automation, and growth

How does demographics affect technological changes and growth in our economy? The first mechanism is through changes in labour supply, which can be uncovered by analysing changes in fertility. To illustrate this mechanism we simulate a permanently decrease of fertility by 10%. Figure 1 shows the results (all responses are percentage changes from the initial balanced growth path). As fertility decreases, the new cohort of workers entering in the labour market also decreases, and equilibrium wages increase. As the profitability of labour intensive firms decreases and consequently the value of newly invented goods also decreases, investment in innovation will tend to fall, leading to less varieties and lower growth. Innovation is not only affected by the increase in profitability in the labour intensive sector. As the share of workers in innovation is more heavily influence by young workers, a drop in fertility implies that the pool of workers available for innovation decreases. As a result, a further push towards less creation of new varieties comes from the direct link between demographic and the R&D workforce. These effects compound to generate a fall in growth (1%) and interest rates (4%). As innovation investment decreases, the growth of Z_t falls below the growth of A_t and consequently, the share of output of the automated sector increases. Lower fertility increases the relative benefit from automation, partially offsetting the effect of the lower growth rate of varieties (%3 fall) on output.

Similarly, a temporary increase in fertility, resembling a baby boom, delivers a strong "demographic dividend" with growth and interest rates peaking while labour supply growth is at its maximum. Eventually, as the baby boomers come close to retirement, growth and interest rates fall while the incentive to automate increases.[5]

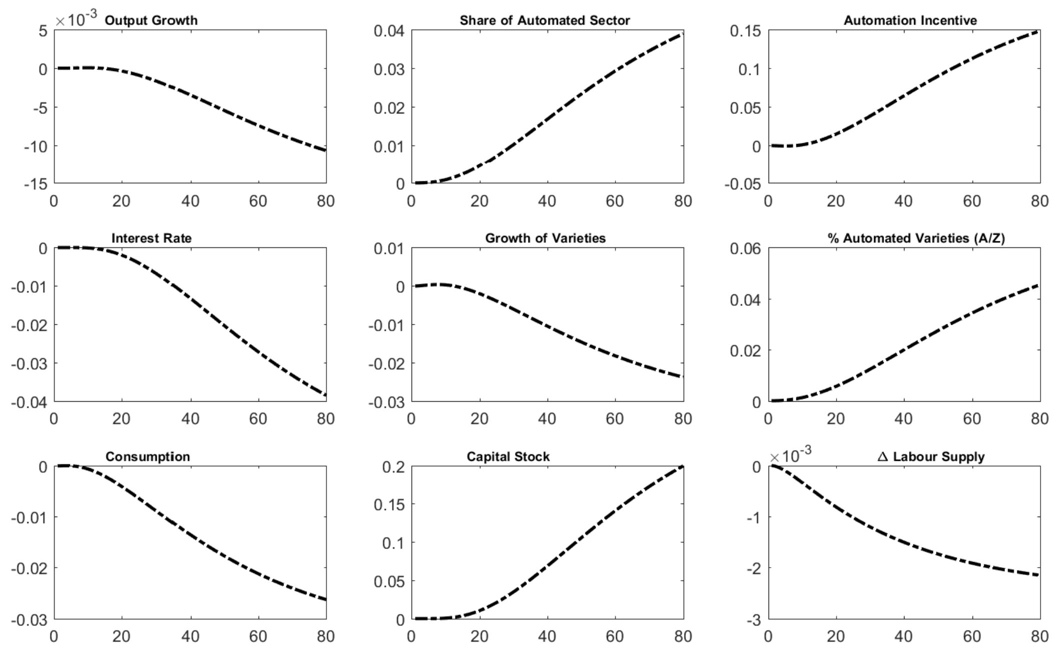

Figure 1: Permanent Fall in Fertility and Growth

The second mechanism relates to changes in the age composition of the population, which can be analysed when mortality is altered. In order to illustrate this mechanism we simulate our model economy while we permanently increase longevity by 5 years, while keeping population growth constant through the simulation. As longevity increases, working age individuals, expecting to live longer, decrease their marginal propensity to consume. That leads to a decrease in interest rates and an increase in savings. These are cheaply allocated to investment in innovation and automation (recall that innovators and automators borrow funds to invest) initially leading to an increase in growth. However, the ageing of the population affects the share of workers in innovation, compromising the productivity of innovation and automation activities. This effect is strong enough to eventually generate a reversal

[5]Results of this simulation are available from the authors upon request.

in growth rates. Automation initially increases as demographic changes imply a lower labour supply. Eventually, the effects of age composition on R&D activities reduced the relative stock of varieties ready to be automated ($Z_t - A_t$) leading to a decrease in pace of automation. Figure 2 shows the results (all responses are percentage changes from the initial balanced growth path).

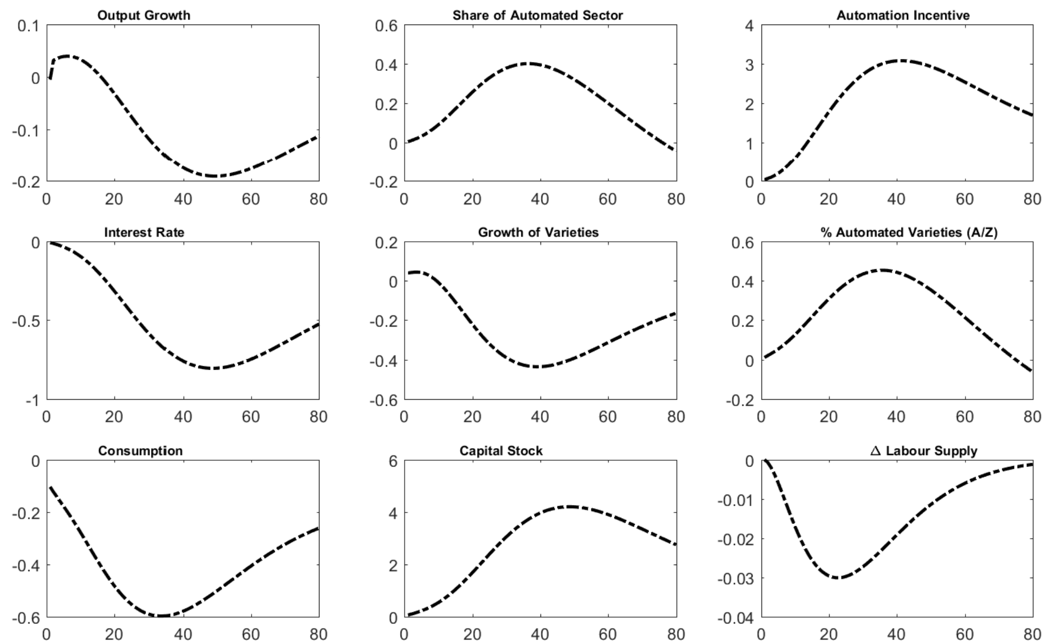

Figure 2: Mortality, Automation and Growth

4.3 Demographic Transition and Growth in Europe and in the US

Most advanced economies are experiencing demographic changes that are the consequences of the combination of baby boomers of the 1960s approaching retirement ages, a permanent fall in fertility, and a continuous rise in longevity. In most countries, the main implication of this combination is a sharp reduction in working age population growth. We use our theoretical model to analyse the consequences of demographic changes predicted for the U.S. and for Core Europe (defined as the sum of Germany, France, Italy and Spain) using the data from the UN World Population

Prospects, 2015 Revision (United Nations (2016). Since the life-cycle structure of the model comprises two distinct age groups, the workers (age 20-65) and the retirees (age above 65), we calculate population shares for each of these groups in the year 1993 and the projected shares in 2055 for each country/region. Based on these population shares we obtain the fertility and the survival probability (mortality) that are consistent with a stationary population distribution. We then simulate a transition path from population structure of 1993 to a population structure expected in 2055 that closely matches the projected population changes. We discard the first seven years of the simulated period to decrease the dependency of the simulation to the initial steady state and depict the results of the simulation from the year 2000 until 2040. Figure 3 shows the results (for growth rates we show the actual change while for the other variables we show the percentage changes from the initial balanced growth path).

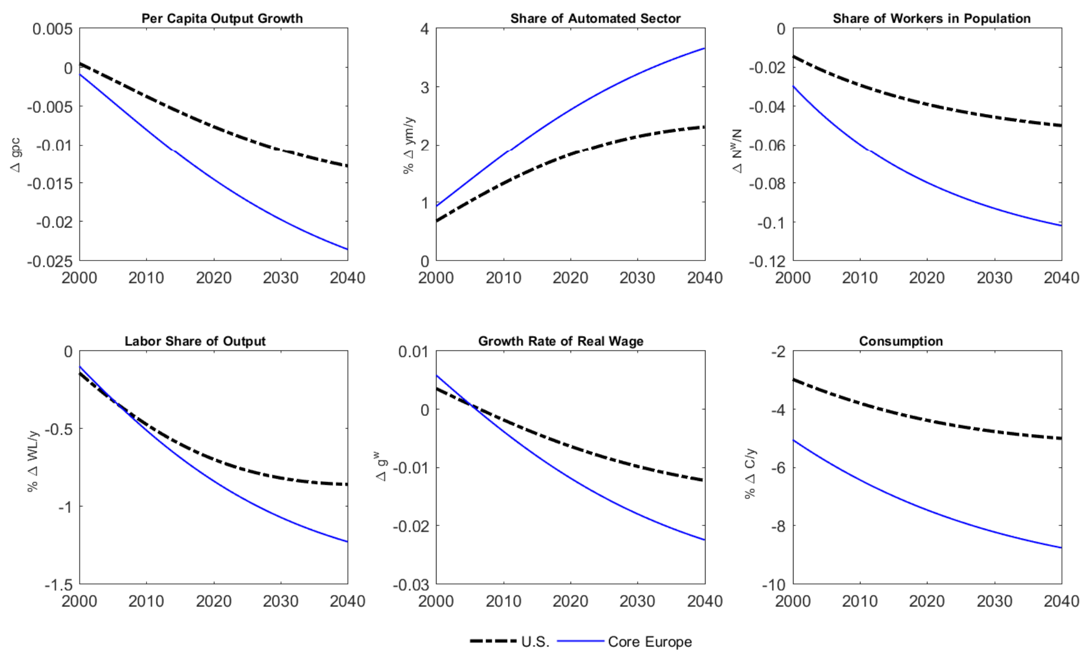

Figure 3: Demographic Transition: United States and Europe

As mortality decreases, savings increase and interest rates decrease, providing cheaper resources that are allocated to innovation and automation. That leads to an increase in the growth rate (per capita growth rate is initially positive). Nonetheless, as lower fertility becomes a main driver of the transition, the labour supply effect on R&D, reducing its productivity, is sufficiently strong to eventually reduce growth. Aksoy, Basso, Smith, and Grasl (2018) show that this link between demographics and innovation productivity is important to explain the negative effect of demographic transition on growth estimated using a Panel of OECD economies from 1970 to 2015. The fall in the share of workers in population leads to an initial increase in wage, given incentives to automation. The share of output in the robot intensive sector increases sharply in the first 30 years of this century.

Although in all countries mortality is decreasing in a similar vein, fertility is considerably lower in Europe. Thus the share of workers in the European economies decrease faster, boosting automation. This results is consistent with the data. During the period 2000-2015, automation, measured as the stock of robots by thousand of employees, increased from 1.55 to 2.7 in the four core European countries, with an increase from 2.28 to 4.24 in Germany, from 0.79 to 1.6 in Spain, from 0.81 to 1.17 in France, and from 1.7 to 2.5 in Italy, while in the US it increased from 0.64 to 1.55 (International Federation of Robotics (2017)). Finally, despite the initial increase in wage, as the economy becomes more automated, both wages and labour supply fall and as a result, the labour share of income decreases sharply in both regions. Automation therefore leads to lower wages (Acemoglu and Restrepo (2017a)).

As the growth of new varieties Z_t decreases, overall growth is reduced, hampering the pace of automation in the future and ultimately delivering lower per capita growth. The key trade-off behind our results is that although automation increases and generates growth, technological change is diverted from product creation to automation. As the initial effect of high savings and lower interest rates wears off, the reduction in invention of new varieties outweighs the benefits of automation leading to lower output growth. Using a cross-section data on patents and demographics, Acemoglu and Restrepo (2018a) confirm this opposing effect of demographics on automation and new product creation. They find that ageing leads to an increase in patents of classes related to robots, while decreasing patents of classes related to computer, software, nanotechnology and pharmaceutics.

Model Extensions

Given the importance of the link between demographics and innovation, we modify the model in two distinct ways to attempt to offset the overall effect of the demographic transition we observe. First, we eliminate the labour employment requirement in innovation, setting $\kappa_L = 0$. In this case innovation consists of a process that transforms an investment in final goods into a new or an automated variety. We denote this model as *No Labour in R&D*. Second, in our benchmark economy we assume a constant share of new workers are able to provide labour to the R&D sector. In this extension, denoted *Labour Choice*, we allow new workers (entrants) with differing inherited talent to R&D to select which sector they will supply labour for. Once this decision is done, workers drop from the R&D sector or retire at the same rate as in the benchmark model; workers cannot join the R&D sector during their working lives. In this extension Sw_{RD}, the share of new workers that join the R&D sector, is endogenous and a function of the wage differential between the R&D and the production sectors (W_t^{RD}/W_t). Details on this extension are shown in the Appendix. The simulation of the effect of the demographic transition contrasting these two models with our benchmark specification for the U.S. are shown in figure 4.

As expected, excluding the labour supply effect on the productivity of innovation offsets some of the negative effect on per capita growth of the demographic transition. The remaining effect is due to the lower population growth (as discussed in proposition 1). Consumption also decreases less in this scenario. Moreover, without the labour supply effect on innovation, reducing growth, automation incentives re-

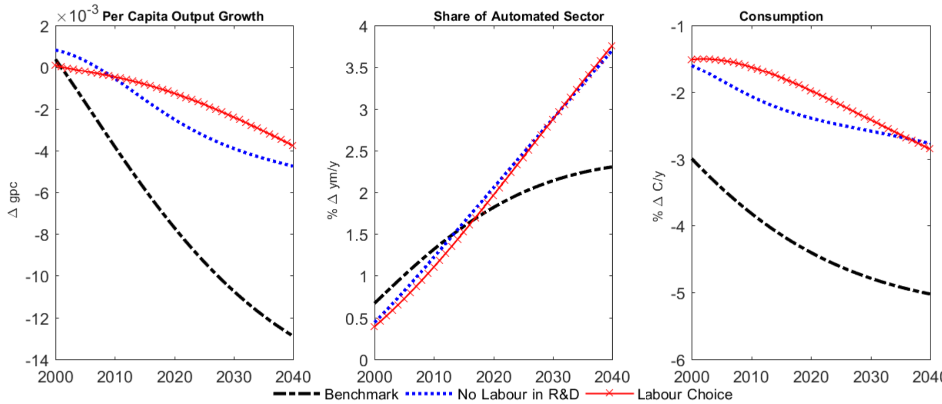

Figure 4: Demographic Transition: Labour in Innovation

main high and in the new balance growth the share of output from the automated sector is considerable higher, reaching its peak asymptotically.

We observe a similar outcome when we allow new workers to select their sector of activity when they enter the economy. As automation peaks up, the wage in equilibrium in the production sector falls. Wages in the R&D sector, given the lack of substitutes, do not fall and thus Sw_{RD} increases. Labour employed in automation and innovation increase, with the former increasing more. The trade-off between innovation and automation is still present, but is less pronounced as the economy diverts their labour resources towards R&D. A caveat is in order, as Bloom, Jones, Reenen, and Webb (2017) show, despite a sharp increase in labour employed in R&D, the productivity of ideas is decreasing; in their conclusion ideas seem to be harder to find. If that is indeed the case the endogenous labour mechanism might be less effective in dampening the effects of the demographic transition.

In the benchmark model we assume robots can only be used in production and their relative productivity in relation to labour is constant and thus $\theta_t = 1.2$. First we modify the model and consider that as varieties are added to the set of products for which robots can be used in production (A_t grows), the relative productivity advantage of robots over labour also grows and thus $\theta_t = A_t^{\mu}$, where we set $\mu = 0.1$. We denote this extension as *Robots Productivity*. Second, we modify the R&D

sector such that not only labour but robots can be used to innovate and generate automation procedures. We assume that investment in new varieties is given by $IS_t = (S_{p,t})^{\kappa_{RD}}((1-(A_t/Z_t))L_{I,t}^{\xi_{LM}} + (A_t/Z_t)M_{I,t}^{\xi_{LM}})^{\kappa_L/\xi_{LM}}$ and invest in automation is given by $\Xi_{A,t} = (\Xi_{q,t})^{\kappa_{RD}}((1-(A_t/Z_t))L_{A,t}^{\xi_{LM}} + (A_t/Z_t)M_{A,t}^{\xi_{LM}})^{\kappa_L/\xi_{LM}}$, where $M_{I,t}$ and $M_{A,t}$ are robots used in $R\&D$, produced by a similar robot production sector as in the benchmark model, and ξ_{LM} is the elasticity of substitution of robots and labour. Under this specification, as the economy becomes more automated, thus the ratio (A_t/Z_t) increases, robots replace a larger share of labour in innovation. This specification resembles the artificial intelligence model of Aghion, Jones, and Jones (2017), but as we assume a balance growth path, eventually the price of robots employed in R&D increases, and thus in the long-run robots and labour grow at the same rate avoiding a robocalypse scenario.

We find that in both cases some of the negative effect on growth and consumption are partially offset (see figure 5). In the case robots are increasingly more productive in the intermediate good sector, we find that as automated varieties (A_t) increases total factor productivity increases (recall that both production process use final good as inputs). This productivity effect benefits the labour intensive sector, offsetting some of the negative impact of reduced labour supply on profits. Consequently, the incentive to automated is weaker and as a result the automated sector does not gain a significant share in relation to the labour intensive sector. In the case robots are used in R&D, the share of automation increases in both the production and the R&D sectors. The negative effect of resource reallocation on innovation is mitigated by the use of robots in R&D.

Delayed retirement age

Finally, we use the model to give some insights on how the delaying of the retirement age could compensate for the shortfall in labour supply associated to

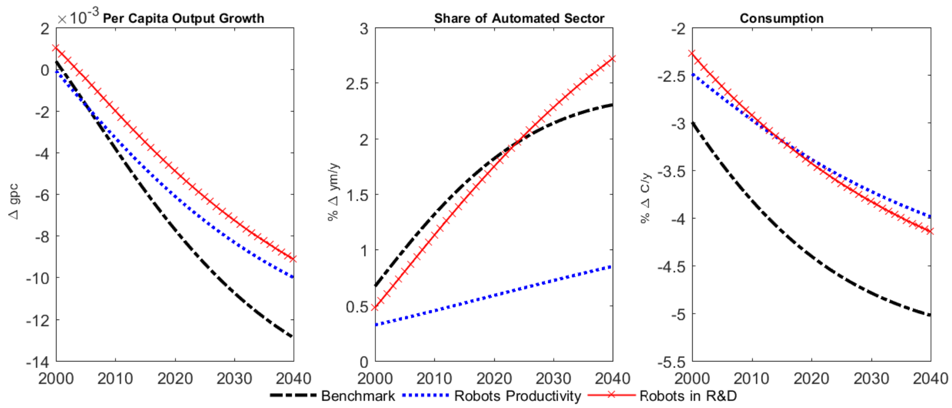

Figure 5: Demographic Transition: Robots vs Labour

the demographic transition in advanced economies. Under the UN demographic projections, life expectancy will increase by 14 years in the US by 2055. In this scenario we assume the retirement age eventually increases by 8 years, or two thirds of the increase in life expectancy. That way the ratio between the duration of working life and the duration of retirement is kept constant during the transition.

Figure 6 displays the results. Delaying the retirement age obviously delivers a lower fall in the share of workers to total population. As result, the incentives to automate are lower, and the fall in output growth is not so large as when retirement age remains constant (-0.4 pp instead of -1.2 pp). Thus, delaying the retirement age can partially offset the loss in output growth during the demographic transition in the US, but cannot completely compensate for it. Again, one important feature of the model driving this result is the link between population age structure and innovation. While delaying retirement age smooth out the fall in labour supply, it cannot avoid the negative impact on innovation activity. As less young workers are involved in the creation of new varieties, innovation is depressed during this demographic transition. As automation does not increase as much wage growth is sustained and as a result delayed retirement boosts the labour share of output.[6] This highlights another important mechanism in the interaction between demography and

[6] We do not explore whether the age structure within the working population has an effect on automation. If older workers are more/less replaceable relative to their younger counterparts our results may be altered.

technology arising from the way in which labour market institutions accommodate changes in both dimensions. Institutions leading to more inclusive employment and alignment of wages with productivity are more likely to deliver more incentive to innovation, less to automation, and, therefore, less wage stagnation and higher labor shares.[7]

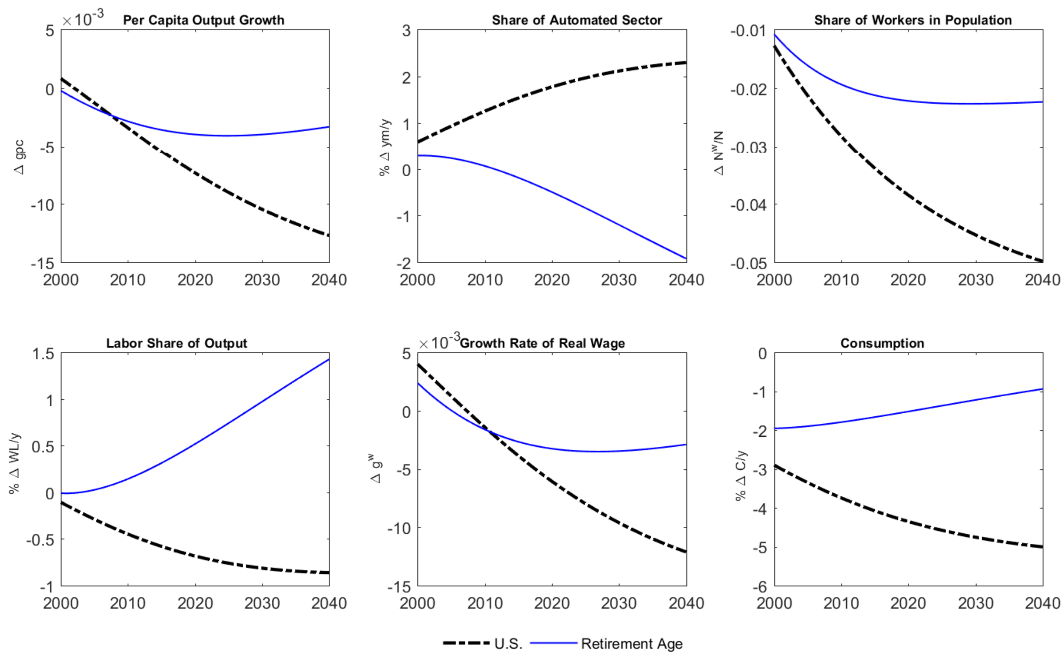

Figure 6: Longevity, Delay in Retirement Age, Automation and Growth

4.4 Divergence and Robocalypse

All simulations presented so far are obtained under the assumption that a mechanism ensuring a balanced growth path exists is in effect, so that the growth of varieties produced in each sector (labour intensive and robot intensive) is eventually the same. Hence, the share of each sector is not asymptotically negligible, implying the real shares of income of robots and labour, due to the adjusting effect of the price of robots q_t, are effectively growing at the same pace. This price adjustment mechanism restricts the potential economic impact of automation in the medium to

[7]See also Lordan and Neumark (2017) on minimum wages and automation

the long-run.

In order to consider the effects of demographics in a scenario where this adjustment in prices does not take place, we modify the robot producing sector. In the benchmark model the production of robots was assumed to be given by $M_t = \varrho\Omega_t^{\eta}$, where η was set to ensure that a balance growth path exists, and $\varrho = 1$. Now, we consider the case in which as the economy becomes more automated and (A_t/Z_t) increases, ϱ starts to trend up, pushing the price of robots down. Eventually, after more than 150 years, ϱ starts to converge to its new value ensuring the price mechanism becomes effective again (and the profitability of robots producers start to fall). Under this assumption we can focus on a medium-run demographic transition where the robot price effect is switched off. Figure 7 shows the results. In this case, as demographics triggers automation, robots are also produced more cheaply, increasing the incentive to reallocated resources from product creation towards automation. As this process continues robots total factor productivity continue to increase together with the ratio (A_t/Z_t). With product creation compromised, most of the output is produced by the automated sector but, without further product innovation, eventually output growth is negatively affected despite the higher productivity of robots. Under this scenario, if robots cannot invent new varieties, a demographic transition that generates automation and better processes to produce robots, fails to increase output growth, a result that resembles the immiseration equilibrium of Benzell, Kotlikoff, LaGarda, and Sachs (2015). This immiseration result remains even if we assume that automation also increases the productivity of robots in the production process of intermediate goods (*Robocalypse with Robots Productivity* - θ_t increases as A_t increases).

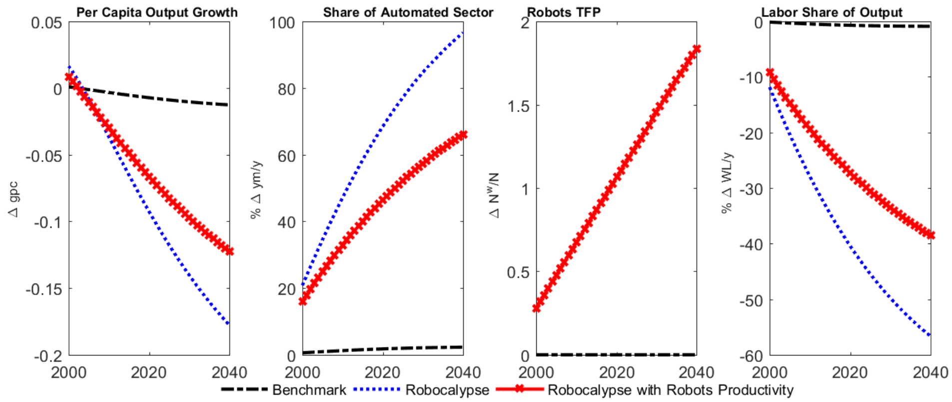

Figure 7: Demographics and Robocalypse

4.5 Conclusion

Demographic changes are bound to shape the macroeconomic landscape of the next decades. On the one hand, population ageing has strong implications for the natural interest rate and, hence, for the effectiveness of monetary policy in stabilising the economy under an effective lower bound constraint (Eggertsson, Mehrotra, and Robbins (2017)). Population ageing also affects the effectiveness of fiscal policy by altering the size of fiscal multipliers (Basso and Rachedi (2017). In the medium to the long-run demographic changes may restrain economic growth (Aksoy, Basso, Smith, and Grasl (2018)) and promote automation (Acemoglu and Restrepo (2018a)).

In this paper, we have analysed the main links between demographics and technology impacting the macroeconomy in the long-run. With it, we envisage productivity growth as a result of two R&D activities, innovation and automation. Investment in innovation yields new products (tasks/varieties) that are the primary source of economic growth. Automation increases productivity by substituting labour in production. However, it cannot sustain long-run growth, even though robots are more productive than labour, because automation is a subsidiary activity of innovation: without innovation, automation cannot progress indefinitely. In this world, demographics matters because of three main reasons: i) it affects savings and, therefore, the amount of resources available for investment in capital accumulation, innovation, and automation, ii) it affects the efficiency of the innovation sector insofar as it may depend on the age structure of population involved in innovation activities, and iii) changes in labour supply determine the relative profitability of investing in capital, innovation, and automation.

Our results show that under several alternative specifications of how innovation comes about and how it affects the macroeconomy, a decline in the share of the working population in total population leads to lower long-run productivity growth, even though it also increases automation and, thus, promotes the substitution of human labour for more productive machines. Assuming a balanced growth is achieved, where the share of the labour intensive and the robot intensive sectors do not diverge asymptotically, we show analytically that lower labour supply growth leads to lower output per capita growth. When using population forecasts for US and Europe, the model predicts a fall in output per capita growth and an increase in automation. The labour share of income is expected to fall reaching its minimum when the degree of automatization reaches its maximum.

Finally, we have also performed simulations that lead to the share of the automated sector growing asymptotically towards unity. In this case, growth falls substantially when the innovation sector does not deliver any more new tasks to be automated. In this "immiserisation equilibrium" labour becomes economically irrelevant and distribution cannot be performed through the markets of production factors. Admittedly, there are many uncertainties about how robotics and artificial intelligence will progress in the next decades. We have tried to consider alternative specifications of the R&D sector to cover them. Still, the main message of this exercise is that we may need to change conventional paradigms about economic growth and start thinking about non-conventional ways of distributing income.

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A Equilibrium Conditions

We start by looking at the factor markets with the final and intermediate firms decisions.

Good Production Sector

Intermediate good firms $j \in A_t$ select capital, robots and inputs to minimise total costs, $TC = P_t q_t M_t^j + (r_t^k + \delta)K_t^j + P_t \Upsilon_t^j$ given a level of production $Y_t^j = \left[(K_t^j)^{\alpha}(M_t^j)^{(1-\alpha)}\right]^{(1-\gamma_I)}\left[\Upsilon_t^j\right]^{\gamma_I}$.

Let ν_t^j be the real marginal cost for firm j. Then

$$\nu_t^j = \frac{(r_t^k + \delta)^{\alpha(1-\gamma_I)} q_t^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)}\gamma_I^{\gamma_I}((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \tag{A.1}$$

$$K_t^j = \nu_t^j \frac{\alpha(1-\gamma_I)}{(r_t^k + \delta)} Y_t^j \tag{A.2}$$

$$\Upsilon_t^j = \nu_t^j \gamma_I Y_t^j \tag{A.3}$$

$$M_t^j = \nu_t^j \frac{(1-\alpha)(1-\gamma_I)}{q_t} Y_t^j \tag{A.4}$$

Given the demand of intermediate good j from final good producers

$$\frac{P_t^j}{P_t} = \frac{\varphi_c - 1}{\varphi_c}\nu_t^j \tag{A.5}$$

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\varphi}_{c} y_t \tag{A.6}$$

$$\Pi_t^j = \left[\frac{P_t^j}{P_t} - \nu_t^j\right] Y_t^j = \frac{1}{\varphi_c - 1}\nu_t^j Y_t^j \tag{A.7}$$

Intermediate good firms $i \in Z_t \setminus A_t$ select capital, labour and inputs to minimise total costs, $TC = W_t L_t^i + (r_t^k + \delta)K_t^i + P_t \Upsilon_t^i$ given a level of production $Y_t^i = \left[(K_t^i)^{\alpha}(L_t^i)^{(1-\alpha)}\right]^{(1-\gamma_I)}\left[\Upsilon_t^i\right]^{\gamma_I}$.

Let ν_t^i be the real marginal cost for firm j. Then

$$\nu_t^i = \frac{(r_t^k + \delta)^{\alpha(1-\gamma_I)}(W_t/P_t)^{(1-\alpha)(1-\gamma_I)}}{(\alpha(1-\gamma_I))^{\alpha(1-\gamma_I)}\gamma_I^{\gamma_I}((1-\alpha)(1-\gamma_I))^{(1-\alpha)(1-\gamma_I)}} \tag{A.8}$$

$$K_t^i = \nu_t^i \frac{\alpha(1-\gamma_I)}{(r_t^k + \delta)} Y_t^i \tag{A.9}$$

$$\Upsilon_t^i = \nu_t^i \gamma_I Y_t^i \tag{A.10}$$

$$L_t^i = \nu_t^i \frac{(1-\alpha)(1-\gamma_I)}{(W_t/P_t)} Y_t^i \tag{A.11}$$

Given the demand of intermediate good j from final good producers

$$\frac{P_t^i}{P_t} = \frac{\varphi_c}{\varphi_c - 1}\nu_t^i \tag{A.12}$$

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{\varphi}_{c} y_t \tag{A.13}$$

$$\Pi_t^i = \left[\frac{P_t^i}{P_t} - \nu_t^i\right] Y_t^i = \frac{1}{\varphi_c - 1}\nu_t^i Y_t^i \tag{A.14}$$

Robots Production Sector

Optimization of robots producers imply

$$\Pi_{\Omega,t} = q_t P_t M_t - P_t \Omega_t \tag{A.15}$$

$$M_t = \varrho \Omega_t^{\eta} \tag{A.16}$$

$$q_t = \frac{\Omega_t}{M_t \eta} \tag{A.17}$$

Innovation Process

One can easily determine the flow of the stock of total varieties (Z_t) and varieties for which robots can be employed in the production process (A_t), which are given by

$$\frac{Z_{t+1}}{Z_t} = \chi\left(\frac{S_t}{\tilde{\Psi}_t}\right)^{\rho}(L_{I,t}/N_t)^{\kappa_L} + \phi, \text{ and} \tag{A.18}$$

$$\frac{A_{t+1}}{A_t} = \lambda\left(\frac{(Z_t - A_t)^{\kappa_{RD}+\kappa_L}(\Xi_t)^{\kappa_{RD}}(L_{A,t})^{\kappa_L}}{\tilde{\Psi}_t^{\kappa_{RD}}N_t^{\kappa_L}}\right)\phi[Z_t/A_t - 1] + \phi \tag{A.19}$$

Investment in R&D (S_t) and labour demand in product creation is determined by (14), (16) which using (12) becomes

$$S_t = R_{t+1}^{-1}\phi E_t J_{t+1}(Z_{t+1} - \phi Z_t). \tag{A.20}$$

$$L_{I,t}W_{RD,t} = \frac{S_t\kappa_L}{\kappa_{RD}} \tag{A.21}$$

Profits are given by the total gain in seeling the right to goods invented as a result of the previous period investment S_{t-1} to adopters minus the cost of borrowing for that investment. Thus,

$$\Pi_{RD,t} = \vartheta\int_{i\in Z_t\setminus A_t}\Pi_t^i di - S_{t-1}R_t - L_{I,t}W_{RD,t}$$

Investment in automation (Ξ_t) is determined by solving (18). We thus obtain the following condition

$$\frac{Z_t - A_t}{\psi_t}\lambda' R_t^{-1}\phi[V_{t+1} - J_{t+1}] = 1 \tag{A.22}$$

where $\frac{Z_t - A_t}{\psi_t}\lambda' = \frac{\partial\lambda\left(\frac{A_t}{\tilde{\Psi}_t}\right)}{\partial\Xi_t\Xi_t}$. Assuming the elasticity of λ_t to changes in its input is constant, thus $\epsilon_\lambda = \frac{\lambda'}{\lambda_t}\frac{(Z_t^q - A_t^q)^{\kappa_{RD}+\kappa_L}\Xi_{q,A,t}}{\tilde{\Psi}_t^{\kappa_{RD}}N_t^{\kappa_L}}$, then we obtain

$$\Xi_t = \epsilon_\lambda\lambda_t R_t^{-1}\phi[V_{t+1} - J_{t+1}] \tag{A.23}$$

$$L_{A,t}W_{RD,t} = \Xi_t\frac{\kappa_L}{\kappa_{RD}} \tag{A.24}$$

Finally, the value of labour intensive varieties and automated varieties are given by

$$J_t = \vartheta\Pi_t^j + (R_{t+1})^{-1}\phi E_t[J_{t+1}], \text{ and} \tag{A.25}$$

$$V_t = \vartheta\Pi_t^i + (R_{t+1})^{-1}\phi E_t V_{t+1} \tag{A.26}$$

Profits for adopters are given by the gain from marketing specialised intermediated goods net the amount paid to inventors to gain access to new goods and the expenditures on loans to pay for adoption intensity.

$$\Pi_{A,t} = \vartheta \int_{j \in A_t} \Pi_t^j dj - \Xi_{t-1}(Z_{t-1} - A_{t-1})R_t - L_{A,t}(Z_t - A_t)W_{RD,t}$$

Household Sector

Retiree j decision problem is

$$\max V_t^{jr} = \left\{ (C_t^{jr})^{\rho_U} + \beta \omega_{t,t+1}^r ([V_{t+1}^{jr}]^{\rho_U}) \right\}^{1/\rho_U} \tag{A.27}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} + d_t^{jr}. \tag{A.28}$$

The first order condition and envelop theorem are

$$(C_t^{jr})^{\rho_U - 1} = \beta \omega_{t,t+1}^r \frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jr}} (V_{t+1}^{jr})^{\rho_U - 1}, \tag{A.29}$$

$$\frac{\partial V_t^{jr}}{\partial FA_t^{jr}} = (V_{t+1}^{jr})^{1-\rho_U} (C_t^{jr})^{\rho_U - 1} \frac{R_t}{\omega_{t-1,t}^r}. \tag{A.30}$$

Combining these conditions above gives the Euler equation

$$C_{t+1}^{jr} = (\beta R_{t+1})^{1/(1-\rho_U)} C_t^{jr} \tag{A.31}$$

Conjecture that retirees consume a fraction of all assets (including financial assets, profits from financial intermediaries), such that

$$C_t^{jr} = \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} FA_t^{rj} + D_t^{rj} \right]. \tag{A.32}$$

Combining these and the budget constraint gives

$$FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr}(1 - \varepsilon_t \varsigma_t) + d_t^{jr} - \varepsilon_t \varsigma_t (D_t^{rj}).$$

Using the condition above the Euler equation and the solution for consumption gives

$$(\beta R_{t+1})^{1/(1-\rho_U)} \varepsilon_t \varsigma_t [\frac{R_t}{\omega_{t-1,t}^r} FA_t^{rj} + D_t^{rj}] = \tag{A.33}$$
$$\varepsilon_{t+1} \varsigma_{t+1} \left[\frac{R_{t+1}}{\omega_{t,t+1}^r} \left(\frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr}(1 - \varepsilon_t \varsigma_t) + d_t^{jr} - \varepsilon_t \varsigma_t D_t^{rj} \right) + D_{t+1}^{jr} \right].$$

Collecting terms we have that

$$
\begin{aligned}
1-\varepsilon_t\varsigma_t &= \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\omega^r_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}}, && \text{(A.34)}\\
D^{jr}_t &= d^{jr}_t+\frac{\omega^r_{t,t+1}}{R_{t+1}}D^{jr}_{t+1}. && \text{(A.35)}
\end{aligned}
$$

One can also show that $V^{jr}_t=(\varepsilon_t\varsigma_t)^{-1/\rho_U}C^{jr}_t$.

Worker j decision problem is

$$
\max V^{jw}_t=\left\{(C^{jw}_t)^{\rho_U}+\beta[\omega^w V^{jw}_{t+1}+(1-\omega^w)V^{jr}_{t+1}]^{\rho_U}\right\}^{1/\rho_U} \tag{A.36}
$$

subject to

$$
C^{jw}_t+FA^{jw}_{t+1}=R_tFA^{jw}_t+W_t\xi_t+d^{jw}_t-\tau^{jw}_t. \tag{A.37}
$$

First order conditions and envelop theorem are

$$
\begin{aligned}
(C^{jw}_t)^{\rho_U-1} &= \beta[\omega^w V^{jw}_{t+1}+(1-\omega^w)V^{jr}_{t+1}]^{\rho_U-1}\left[\omega^w\frac{\partial V^{jw}_{t+1}}{\partial FA^{jw}_{t+1}}+(1-\omega^w)\frac{\partial V^{jr}_{t+1}}{\partial FA^{jw}_{t+1}}\right],\\
\frac{\partial V^{jw}_t}{\partial FA^{jw}_t} &= (V^{jw}_{t+1})^{1-\rho_U}(C^{jw}_t)^{\rho_U-1}R_t,\ \text{and} && \text{(A.38)}\\
\frac{\partial V^{jr}_t}{\partial FA^{jw}_t} &= \frac{\partial V^{jr}_t}{\partial FA^{jr}_t}\frac{\partial FA^{jr}_t}{\partial FA^{jw}_t}=\frac{\partial V^{jr}_t}{\partial FA^{jr}_t}\frac{1}{\omega^r_{t-1,t}}=(V^{jr}_{t+1})^{1-\rho_U}(C^{jr}_t)^{\rho_U-1}R_t. && \text{(A.39)}
\end{aligned}
$$

$\frac{\partial FA^{jr}_t}{\partial FA^{jw}_t}=\frac{1}{\omega^r_{t-1,t}}$ since as individuals are risk neutral with respect to labour income they select the same asset profile independent of their worker/retiree status, adjusting only for expected return due to probability of death.

Combining these conditions above, and using the conjecture that $V^{jw}_t=(\varsigma_t)^{-1/\rho_U}C^{jw}_t$, gives the Euler equation

$$
\begin{aligned}
C^{jw}_t &= \left((\beta R_{t+1}\mathfrak{Z}_{t+1})^{1/(1-\rho_U)}\right)^{-1}[\omega^w C^{jw}_{t+1}+(1-\omega^w)\varepsilon_{t+1}^{\frac{-1}{\rho_U}}C^{jr}_{t+1}] && \text{(A.40)}\\
&\quad \text{where } \mathfrak{Z}_{t+1}=(\omega^w+(1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}).
\end{aligned}
$$

Conjecture that retirees consume a fraction of all assets (including financial assets, human capital and profits from financial intermediaries), such that

$$
C^{jw}_t=\varsigma_t[R_tFA^{jw}_t+H^{jw}_t+D^{jw}_t]. \tag{A.41}
$$

Following the same procedure as before we have that

$$
\begin{aligned}
&\varsigma_t[R_tFA^{jw}_t+H^{jw}_t+D^{jw}_t](\beta R_{t+1}\mathfrak{Z}_{t+1})^{1/(1-\rho_U)}= && \text{(A.42)}\\
&\quad\omega^w\varsigma_{t+1}\Big[R_{t+1}\Big(R_tFA^{jw}_t(1-\varsigma_t)+W_t\xi_t+d^{jw}_t-\varsigma_t(H^{jw}_t+D^{jw}_t)\Big)+H^{jw}_{t+1}+D^{jw}_{t+1}\Big]+\\
&\quad\quad\varepsilon_{t+1}^{\frac{-1}{\rho_U}}(1-\omega^w)\varepsilon_{t+1}\varsigma_{t+1}\Big[R_{t+1}\Big(R_tFA^{jw}_t(1-\varsigma_t)+W_t\xi_t+d^{jw}_t-\varsigma_t(H^{jw}_t+D^{jw}_t)\Big)+D^{jr}_{t+1}\Big].
\end{aligned}
$$

Collecting terms and simplifying we have that

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1} \mathfrak{Z}_{t+1})^{1/(1-\rho_U)}}{R_{t+1} \mathfrak{Z}_{t,t+1}} \tag{A.43}$$

$$H_t^{jw} = (W_t/P_t) + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} \text{ and} \tag{A.44}$$

$$D_t^{jw} = d_t^{jw} + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^{jr}. \tag{A.45}$$

Aggregation across households

Assume that for any variable X_t^{jz} we have that $X_t^z = \int_0^{N_t^z} X_t^{jz}$ for $z = \{w, r\}$, then

$$L_t = N_t^{wL}, \tag{A.46}$$

$$L_{I,t} + L_{A,T} = N_t^{wRD}, \tag{A.47}$$

$$H_t^w = (W_t/P_t)N_t^{wL} + (W_t^{RD}/P_t)N_t^{wRD} + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{A.48}$$

$$D_t^w = d_t^w + \frac{\omega^w}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{D_{t+1}^w N_t^w}{N_{t+1}^w} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \frac{D_{t+1}^r N_t^w}{N_{t+1}^r}, \tag{A.49}$$

$$C_t^w = \varsigma_t[R_t FA_t^w + H_t^w + D_t^w - T_t^w], \tag{A.50}$$

$$D_t^r = d_t^r + \frac{\omega_{t,t+1}^r}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r}, \tag{A.51}$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r]. \tag{A.52}$$

Note that $\omega_{t,t+1}^r$ is not shown in the last equation due to the perfect annuity market for retirees, allowing for the redistribution of assets of retirees who died at the end of the period.

Financial Intermediary

The profits of the financial intermediary are

$$\begin{aligned} \Pi_t^F = \; & [r_t^k + 1]K_t + R_t B_t - R_t(FA_t^w + FA_t^r) - K_{t+1} - B_{t+1} + FA_{t+1}^w + FA_{t+1}^r + \\ & + (\Pi_{A,t} + \Pi_{RD,t} + (1-\vartheta)\left(\int_{j\in A_t} \Pi_t^j dj + \int_{i\in Z_t \setminus A_t} \Pi_t^i di\right) + \Pi_{\Omega,t}), \end{aligned} \tag{A.53}$$

where $B_{t+1} = S_t + \Xi_t(Z_t - A_t)$ and $FA_t = FA_t^w + FA_t^r$.

The financial intermediaries selects capital and bonds such that it maximize profits and thus we obtain the standard arbitrage conditions whereby all assets must pay the same expected return, thus

$$E_t\left[r_{t+1}^k + 1\right] = R_t. \tag{A.54}$$

Also note that under a perfect foresight solution, by ensuring the financial intermediary behaves under perfect competition, this equality holds without expectations, $\Pi_t^F = 0$ and thus $d_t^r = d_t^w = 0$. If $\Pi_t^F \neq 0$, then we assume profits are divided based on the ratio of assets. As* such, $d_t^r = \Pi_t^F \frac{FA_t^r}{FA_t^r + FA_t^w}$ and $d_t^w = \Pi_t^F \frac{FA_t^w}{FA_t^r + FA_t^w}$.

The flow of capital is then given by

$$K_{t+1} = K_t(1-\delta) + I_t. \tag{A.55}$$

Where I_t is the investment in capital made by the financial intermediary.

Asset Markets

Asset Market clearing implies

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \tag{A.56}$$

Finally, the flow of assets are given by

$$FA_{t+1}^r = R_t FA_t^r + d_t^r - C_t^r + (1-\omega^w)(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{A.57}$$
$$FA_{t+1}^w = \omega^w (R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{A.58}$$

Clearing conditions

$$y_t = C_{w,t} + C_{r,t} + \Upsilon_t + \Omega_t + I_t \tag{A.59}$$
$$K_{t+1} = (1-\delta)K_t + I_t \tag{A.60}$$
$$K_t = \int_{j \in A_t} k_t^j dj + \int_{i \in Z_t \setminus A_t} k_t^i di \tag{A.61}$$
$$\Upsilon_t = \int_{j \in A_t} \Upsilon_t^j dj + \int_{i \in Z_t \setminus A_t} \Upsilon_t^i di \tag{A.62}$$
$$M_t = \int_{j \in A_t} M_t^j dj \tag{A.63}$$
$$N_t^{wR} = \int_q L_{A,q,t} di + \int_p L_{i,q,t} di N_t^{wL} = \int_{i \in Z_t \setminus A_t} L_t^i di \tag{A.64}$$
$$\tag{A.65}$$

B Detrended equilibrium conditions

This section shows the detrended equilibrium conditions. Note that $\bar{x}$ denotes the steady state of variable x_t.

$$w_t = ll_t + li_t + la_t \tag{A.66a}$$

$$h_t^w = w_t + \frac{\omega^w}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1}h_{t+1}^w}{g_{t+1}^w} \text{ where } h_t^w = \frac{H_t^w}{Y_t}, w_t = \frac{W_t L_t}{P_t Y_t}, g_{t+1} = \frac{Y_{t+1}}{Y_t}, g_{t+1}^w = \frac{N_{t+1}^w}{N_t^w} \tag{A.66b}$$

$$\tilde{D}_t^r = \tilde{d}_t^r + \frac{\omega_{t,t+1}^r}{R_{t+1}} g_{t+1} \frac{\tilde{D}_{t+1}^r \zeta_t^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^r = \frac{D_t^r}{Y_t}, \tilde{d}_t^r = \frac{d_t^r}{Y_t} \tag{A.66c}$$

$$\tilde{D}_t^w = \tilde{d}_t^w + \frac{\omega^w}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^w}{g_{t+1}^w} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^w = \frac{D_t^w}{Y_t}, \tilde{d}_t^w = \frac{d_t^w}{Y_t} \tag{A.66d}$$

$$c_t^w = \varsigma_t[R_t \frac{fa_t^w}{g_t} + h_t^w + \tilde{D}_t^w] \text{ where } fa_t^w = \frac{FA_t^w}{Y_{c,t-1}}, c_t^w = \frac{C_t^w}{Y_t} \tag{A.66e}$$

$$c_t^r = \varepsilon_t \varsigma_t[R_t \frac{fa_t^r}{g_t} + \tilde{D}_t^r] \text{ where } fa_t^r = \frac{FA_t^r}{Y_{c,t-1}}, c_t^r = \frac{C_t^w}{Y_t} \tag{A.66f}$$

$$1 - \varepsilon_t \varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)} \omega_{t,t+1}^r}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}} \tag{A.66g}$$

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1}\mathfrak{z}_{t+1})^{\frac{1}{(1-\rho_U)}}}{R_{t+1}\mathfrak{z}_{t,t+1}} \tag{A.66h}$$

$$\mathfrak{z}_{t+1} = (\omega^w + (1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}) \tag{A.66i}$$

$$g_{t+1}^w = \omega^w + (1-\omega^y)\zeta_t^y \tag{A.67a}$$

$$n_{t,t+1} = \frac{\zeta_{t+1}^y}{\zeta_t^y} (\omega^w + \zeta_t^y(1-\omega^y)) \tag{A.67b}$$

$$\zeta_{t+1}^r = \left((1-\omega^w) + \omega_{t,t+1}^r \zeta_t^r\right) (\omega^w + (1-\omega^y)\zeta_t^y)^{-1} \text{ and} \tag{A.67c}$$

$$g_{t+1}^n = (n_{t,t+1}\zeta_t^y) + (\omega^w + (1-\omega^y)\zeta_t^y) + ((1-\omega^w) + \omega_{t,t+1}^r \zeta_t^r)(1 + \zeta_t^r + \zeta_t^y)^{-1} \text{ where } g_{t+1}^n = \frac{N_{t+1}}{N_t} \tag{A.67d}$$

Note that all firms $j \in A_t$ take the same decisions, then $\int_{j \in A_t} k_t^j dj = A_t k_t^j$. A similar argument holds for firms $i \in Z_t \setminus A_t$.

$$k_{m,t} = \frac{\alpha(1-\gamma_I)}{(r_t^k+\delta)}\frac{\varphi_c - 1}{\varphi_c} y_{m,t} g_t \text{ where } k_{m,t} = \frac{A_t k_t^j}{Y_{t-1}}, y_{m,t} = \frac{(P_t^j/P_t) y_t^j A_t}{Y_t} \tag{A.68a}$$

$$\Upsilon_{m,t} = \gamma_I \frac{\varphi_c - 1}{\varphi_c} y_{m,t} \text{ where } \Upsilon_{m,t} = \frac{A_t \Upsilon_t^j}{Y_t} \tag{A.68b}$$

$$m_t = (1-\alpha)(1-\gamma_I)\frac{\varphi_c - 1}{\varphi_c} y_{m,t} \text{ where } m_t = \frac{A_t m_t^j q_t}{Y_t} = \frac{q_t M_t}{Y_t} \tag{A.68c}$$

$$g_{pm,t} = \left(\frac{(r_t^k+\delta)}{(r_{t-1}^k+\delta)}\right)^{\alpha(1-\gamma_I)} \left(\frac{\theta_{t-1}}{\theta_t}\right)^{(1-\alpha)(1-\gamma_I)} g_{q,t}^{(1-\alpha)(1-\gamma_I)} \text{ where } g_{pm,t} = \frac{(P_t^j/P_t)}{(P_{t-1}^j/P_{t-1})}, g_{q,t} = \frac{q_t}{q_{t-1}} \tag{A.68d}$$

$$\frac{y_{m,t}}{y_{m,t-1}} = g_t^A g_{pm,t}^{1-\varphi_c}, \text{ where } g_t^A = \frac{A_t}{A_{t-1}} \tag{A.68e}$$

$$\pi_{m,t} = \frac{1}{\varphi_c} y_{m,t} \text{ where } \pi_{m,t} = \frac{A_t \Pi_t^j}{Y_t} \tag{A.68f}$$

$$k_{L,t} = \frac{\alpha(1-\gamma_I)}{(r_t^k+\delta)}\frac{\varphi_c - 1}{\varphi_c} y_{L,t} g_t \text{ where } k_{L,t} = \frac{(Z_t - A_t) k_t^i}{Y_{t-1}}, y_{L,t} = \frac{(P_t^i/P_t) y_t^i (Z_t - A_t)}{Y_t} \tag{A.68g}$$

$$\Upsilon_{L,t} = \gamma_I \frac{\varphi_c - 1}{\varphi_c} y_{L,t} \text{ where } \Upsilon_{L,t} = \frac{(Z_t - A_t)\Upsilon_t^i}{Y_t} \tag{A.68h}$$

$$ll_t = (1-\alpha)(1-\gamma_I)\frac{\varphi_c - 1}{\varphi_c} y_{L,t} \text{ where } ll_t = \frac{(Z_t - A_t) L_t (W_t/P_t)}{Y_t} = \frac{(W_t/P_t) N_t^w}{Y_t} \tag{A.68i}$$

$$ll_t/ll_{t-1} = llpop_t/llpop_{t-1}(g_t^{wg} g_{t-1}^n)/g_t \text{ where } g_t^{wg} = \frac{W_t/P_t}{W_{t-1}/P_{t-1}} \tag{A.68j}$$

$$g_{pL,t} = \left(\frac{(r_t^k+\delta)}{(r_{t-1}^k+\delta)}\right)^{\alpha(1-\gamma_I)} \left(\frac{w_t}{w_{t-1}}\right)^{(1-\alpha)(1-\gamma_I)} \left(\frac{g_t}{g_t^w}\right)^{(1-\alpha)(1-\gamma_I)} \text{ where } g_{pL,t} = \frac{(P_t^i/P_t)}{(P_{t-1}^i/P_{t-1})} \tag{A.68k}$$

$$\frac{y_{L,t}}{y_{L,t-1}} = g_t^{ZA} g_{pL,t}^{1-\varphi_c}, \text{ where } g_t^{ZA} = \frac{(Z_t - A_t)}{(Z_{t-1} - A_{t-1})} \tag{A.68l}$$

$$\pi_{L,t} = \frac{1}{\varphi_c} y_{L,t} \text{ where } \pi_{L,t} = \frac{(Z_t - A_t)\Pi_t^i}{Y_t} \tag{A.68m}$$

$$m_t = \frac{\tilde{\Omega}_t}{\varrho\eta} \text{ where } \tilde{\Omega}_t = \frac{\Omega_t}{Y_t} \tag{A.68n}$$

$$\pi_{\Omega,t} = m_t - \tilde{\Omega}_t \text{ where } \pi_{\Omega,t} = \frac{\Pi_{\Omega,t}}{Y_t} \tag{A.68o}$$

$$\frac{m_t}{m_{t-1}} = \left(\frac{\tilde{\Omega}_t}{\tilde{\Omega}_{t-1}}\right)^{\eta} (g_t)^{\eta-1} g_{q,t} \tag{A.68p}$$

$$g_{t+1}^Z = \gamma_t^{\rho_{yw}} \chi \left(\frac{s_t}{\Psi_t}\right)^\rho (lipop_t)_L^\kappa + \phi \text{ where } g_t^Z = \frac{Z_t}{Z_{t-1}}, s_t = \frac{S_t}{Y_t}, \Psi_t = \frac{\tilde{\Psi}_t}{Y_t}, lipop_t = \frac{L_{I,t}}{N_t} \tag{A.69a}$$

$$g_{t+1}^A = \lambda_t \phi [1/a_{z,t} - 1] + \phi \text{ where } a_{z,t} = \frac{A_t}{Z_t} \tag{A.69b}$$

$$g_t^{ZA} = g_t^Z \frac{1 - a_{z,t}}{1 - a_{z,t-1}} \tag{A.69c}$$

$$az_t = az_{t-1} \frac{g_t^A}{g_t^Z} \tag{A.69d}$$

$$s_t = g_{t+1} R_{t+1}^{-1} \phi j_{t+1} \left(\frac{g_{t+1}^Z - \phi}{g_{t+1}^Z (1 - a_{z,t})}\right) \text{ where } j_t = \frac{J_t(Z_t - A_t)}{Y_t} \tag{A.69e}$$

$$li_t = s_t \frac{\kappa_L}{\kappa_{RD}} \text{ where } li_t = \frac{L_{I,t} W_{RD,t}}{Y_t} \tag{A.69f}$$

$$li_t / li_{t-1} = lipop_t / lipop_{t-1} (g_t^{wrd} g_{t-1}^n) / g_t \text{ where } g_t^{wrd} = \frac{W_{RD,t}/P_t}{W_{RD,t-1}/P_{t-1}} \tag{A.69g}$$

$$v_t = \vartheta \pi_{m,t} + (R_{t+1})^{-1} \phi \frac{g_{t+1}}{g_{t+1}^A} v_{t+1} \text{ where } v_t = \frac{V_t A_t}{Y_t} \tag{A.69h}$$

$$j_t = \vartheta \pi_{L,t} + (R_{t+1})^{-1} \phi \frac{g_{t+1}}{g_{t+1}^{ZA}} j_{t+1} \tag{A.69i}$$

$$\varpi_t = \epsilon_\lambda \lambda_t R_{t+1}^{-1} \phi g_{t+1} \left[\frac{v_{t+1}}{g_{t+1}^A} [1/a_{z,t} - 1] - \frac{j_{t+1}}{g_{t+1}^{ZA}}\right] \text{ where } \varpi_t = \frac{\Xi_t (Z_t - A_t)}{Y_t} \tag{A.69j}$$

$$la_t = \varpi_t \frac{\kappa_L}{\kappa_{RD}} \text{ where } la_t = \frac{L_{A,t} W_{RD,t} (Z_t - A_t)}{Y_t} \tag{A.69k}$$

$$la_t / la_{t-1} = lapop_t / lapop_{t-1} (g_t^{wrd} g_{t-1}^n) / g_t \tag{A.69l}$$

$$\lambda_t = \lambda \left(\frac{\varpi_t}{\Psi_t}\right) \approx \bar{\lambda} \left(1 + \epsilon_\lambda \left(\frac{\varpi_t - \bar{\varpi}}{\bar{\varpi}} - \frac{\Psi_t - \bar{\Psi}}{\bar{\Psi}} + \kappa_L \frac{lapop_t - la\bar{p}op}{la\bar{p}op}\right)\right) \tag{A.69m}$$

$$\pi_t^A = \vartheta \pi_{L,t} - R_t \varpi_{t-1} / g_t - li_t \tag{A.69n}$$

$$\pi_t^{RD} = \vartheta \pi_{L,t} - R_t s_{t-1} / g_t - la_t \tag{A.69o}$$

where ϵ_λ is the elasticity of $\lambda(\cdot)$

$$r^k_{t+1} + 1 = R_{t+1} \tag{A.70a}$$

$$\tilde{d}^r_t = \pi^F_t \frac{fa^r_t}{fa_t} \text{ where } \pi^F_t = \frac{\Pi^F_t}{Y_t} \tag{A.70b}$$

$$\tilde{d}^w_t = \pi^F_t \frac{fa^w_t}{fa_t} \tag{A.70c}$$

$$b_{t+1} = s_t + \varpi_t \text{ where } b_{t+1} = \frac{B_{t+1}}{Y_t} \tag{A.70d}$$

$$\pi^F_t = (Rk_t + 1)\frac{k_t}{g_t} + \frac{R_t}{g_t}b_t - \frac{R_t}{g_t}(fa_t) - k_{t+1} - b_{t+1} + (fa_{t+1}) + \pi^A_t + \pi^{RD}_t + (1-\vartheta)(\pi_{m,t} + \pi_{L,t}) \tag{A.70e}$$

$$llpop = \frac{\zeta^{wL}_t}{1 + \zeta^y_t + \zeta^r_t} \text{ where } \zeta^{wL}_t = \frac{N^{wL}_t}{N^w_t} \tag{A.71a}$$

$$lipop_t + lapop_t = \frac{\zeta^{wRD}_t}{1 + \zeta^y_t + \zeta^r_t} \text{ where } \zeta^{wRD}_t = \frac{N^{wRD}_t}{N^w_t} \tag{A.71b}$$

$$k_{t+1} = (1-\delta)\frac{k_t}{g_t} + i_t \text{ where } i_t = \frac{I_t}{Y_t} \tag{A.71c}$$

$$k_t = k_{m,t} + k_{L,t} \tag{A.71d}$$

$$\tilde{\Upsilon}_t = \Upsilon_{m,t} + \Upsilon_{L,t} \tag{A.71e}$$

$$1 = y_{m,t} + y_{L,t} \tag{A.71f}$$

$$1 = c_t + i_t + s_t + \varpi_t + \tilde{\Omega}_t + \tilde{\Upsilon}_t \text{ where } c_t = \frac{C_t}{Y_t} \tag{A.71g}$$

$$c_t = c^w_t + c^r_t \tag{A.71h}$$

$$fa^w_{t+1} + fa^r_{t+1} = k_{t+1} + b_{t+1} \tag{A.71i}$$

$$fa^r_{t+1} = \frac{R_t}{g_t}fa^r_t + \tilde{d}^r_t - c^r_t + (1-\omega^w)\left(\frac{R_t}{g_t}fa^w_t + w_t + \tilde{d}^w_t - c^w_t\right) \tag{A.71j}$$

$$fa_{t+1} = fa^w_{t+1} + fa^r_{t+1} \tag{A.71k}$$

$$\Psi_t = \upsilon_t \tag{A.71l}$$

$$fa^w_{t+1} = \omega^w\left(\frac{R_t}{g_t}fa^w_t + w_t + \tilde{d}^w_t - c^w_t - \tilde{\tau}_t\right)$$

C More on Calibration

This Section reports the values of the set of parameters of the model.

Table A.1: Calibration

Parameter	Value	Target/Source
Time Discount Factor	$\beta = 0.96$	Standard Value
Elasticity Intertemporal Substitution	$\eta = -3$	EIS = 0.25 (Gertler(1999))
Capital Depreciation Rate	$\delta = 0.08$	Standard Value
Capital Share in Production	$\alpha = 0.33$	Standard Value
Intermediate Share in Production	$\gamma_I = 0.5$	Comin and Gertler(2006)
Elasticity Substitution of Varieties	$\psi = 8$	Standard Value
Obsolescence	$\phi = 0.85$	Growth per Working age person
Productivity Innovation	$\chi = 5.67$	Share of innovation expenditure in GDP
Elasticity of Investment to Innovation	$\rho = 0.9$	Comin and Gertler (2006)
Elasticity of Final Goods to R&D Investment	$\kappa_{RD} = 1$	Comin and Gertler (2006)
Elasticity of Labour to R&D Investment	$\kappa_L = 0.5$	Aksoy et al. (2018)
Rate of Automation	$\lambda = 0.1$	Share of Automated Varieties
Robots Production Function	$\eta = 0.15$	Balanced Growth
Birth Rate	$\omega_n = 0.0265$	Share of Workers in Population
Probability Transition from Mature to Old	$1 - \omega_w = 0.022$	Avg. Number of Years as Worker: 45y
Death Probability of Old Agents	$1 - \omega_o = 0.07$	Share of Old in Population
Share of Workers in R&D	$Sw_{RD} = 0.07$	Share of R&D workers in Population
Probability Workers leaves R&D	$drop_{RD} = 0.07$	Average age of R&D workers

D Extension - Labour Choice Model

Under this extension, $Sw_{RD,t}$, the share of new workers that enter the economy and work in the R&D sector, is endogenous. In order to obtain that we assume that an individual, when entering her working life selects in which labour market (R&D or intermediate good production) to participate. At entry she is randomly assigned an efficiency level in R&D activity, denoted $\xi\tilde{\nu}_t^i$, where $\tilde{\nu}_t^i$ is drawn from a Pareto distribution with shape parameter $\epsilon > 1$ and support $[1, \infty)$. We denote the cumulative distribution by $F(\nu)$. The individual then compares the human capital gain under the $R\&D$ sector (H_t^{RD}) which is a function of the wage W^{RD} and the average efficiency of workers in the sector, denoted $\nu_{m,t}$, and the human capital gain in the production sector (H_t, which is a function of the wage W) and selects in which labour market to be active in.

There exists a cut-off point ν_t^* such that given H_t^{RD} and H_t the individual is indifferent between choosing each sector. Then, the share of individuals in $R\&D$ is given by

$$Sw_{RD,t} = \int_{\nu_t^*}^{\infty} dF(\nu) = \int_{\nu_t^*}^{\infty} \frac{\epsilon 1^\epsilon}{\nu^{\epsilon+1}} d\nu = \int_{\nu_t^*}^{\infty} \epsilon\nu^{-(\epsilon+1)} = (\nu_t^*)^\epsilon$$

The average efficiency of entrants in the R&D labour market is

$$\nu_{E,t} = \frac{\int_{\nu_t^*}^{\infty} \xi\nu dF(\nu)}{1 - F(\nu_t^*)} = \frac{\int_{\nu_t^*}^{\infty} \xi\epsilon\nu^{-(\epsilon)} d\nu}{1 - F(\nu_t^*)} = \xi\frac{\epsilon}{\epsilon - 1}\nu_t^*$$

The average efficiency of all workers in the R&D sector is then given by

$$\nu_{m,t} = \frac{Sw_{RD,t}\omega^{y}_{t,t+1}N^{w}_{t}}{N^{wRD}_{t+1}}\nu_{E,t} + (1 - drop_{RD})\omega^{w}N^{wRD}_{t}N^{wRD}_{t+1}\nu_{m,t-1}$$

Defining

$$\begin{aligned} H^{jw}_{t} &= (W_t/P_t) + \frac{\omega^{w}}{R_{t+1}\mathfrak{z}_{t,t+1}}H^{jw}_{t+1}, \text{ where } j \text{ works in production} \\ H^{iw_{RD}}_{t} &= (\nu_{m,t}W^{RD}_{t}/P_t) + \frac{\omega^{w}}{R_{t+1}\mathfrak{z}_{t,t+1}}H^{iw_{RD}}_{t+1}, \text{ where } i \text{ works in R\&D} \end{aligned}$$

And since $\nu_{m,t}$ is a function of ν^{*}_{t}, ν^{*}_{t} is such that $H^{jw}_{t} = H^{iw_{RD}}_{t}$. Finally, we calibrate ϵ and ξ to obtain the same effective wage in R&D and Sw_{RD} at steady state as in the benchmark model.