Growth with a Digital Sector

Eva Nalbach and Werner Roeger

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Abstract

An important aspect of an emerging digital economy is the supply of electronic goods to consumers at zero marginal cost. Another important feature of current growth is its dependence on technical progress in the digital sector. The paper explores how in the context of a two sector growth model, distinguishing between physical and digital goods, digital technology shocks such as investment specific technical progress in IT hardware and innovations in platform design affect growth, the wage share, real wages and the digital skill premium. Our findings can be summarised as follows. Even though the specification of technology and the nature of technical progress differs fundamentally from the standard model, adverse wage effects are mitigated by free entry. In particular, ICT capital saving technical progress (Moore's law) and innovations in platform design (intangible capital saving technical progress), do not generally reduce the aggregate wage share and generally increase the real consumption wage. However, if we allow for a bias towards digital skills in the digital sector, digital technical progress goes along with a declining wage share of workers without digital skills as well as a reduction of their real consumption wage under certain conditions.

Keywords: Digital Economy, Two Sector Growth Model, Technological Change, Wage Share, Wage Dispersion.

JEL Code: O41, E10, J2

Affiliation: Eva Nalbach **(evanalbach@gmail.com)**;Werner Roeger EU Commission **(werner.roeger@ec.europa.eu).** The views expressed in this paper are those of the authors and should not be attributed to the European Commission. We have received useful comments from participants of the DIW macroeconomic seminar series.

Introduction

The digitisation of consumption is one of the most noticeable trends in advanced industrial economies. Consuming digital content on computers and smartphones has become an integral part of leisure and consumption activities. The economic importance of this trend also shows up in the market capitalisation of IT firms like Apple, Alphabet (Google) and Microsoft which focus on consumer goods and services. The OECD (2015) estimates that the value added share of the digital sector now amounts to about 8% of GDP. Future growth potential is also signalled by the R&D share of ICT in total business R&D, which amounts to about 33% (or 0.5% of GDP) in the OECD. Similarly, the amount of risk capital invested in internet specific start-ups is about 25% of all venture capital investments in the US. Market capitalisation, the focus of start-up financing and the concentration of research activities is signalling that future growth is likely to come from the digital sector of the economy. The recent OECD report states that the share of the digital sector has remained relatively stable in recent years, this should however not be misinterpreted as a sign of stagnation, it rather reflects the strong decline in relative prices of digital goods due to high rates of technical progress in the digital sector. Digitisation is of course not restricted to consumption but also affects the business sector. Nevertheless we focus in this paper on the digitisation of consumer services. It can be argued that the phenomena we want to stress in this paper, namely the scalability of digital services can more easily be achieved with consumer oriented services because the preferences of private households are more similar within a country or even across the world, while companies often ask for customised products.

In recent work Dhar and Sundararajan (2007) and Sundararajan (2016) identify three fundamental technological forces which distinguish digital technologies from technological developments preceding them. The first factor is what Dhar and Sundararajan call 'rendering of things as information and in particular, representing that information digitally'. What is meant here are all those recent technological developments which have made it possible to represent voice, music and video in a digital format. A second force is the sustained rate of technical progress in IT hardware, bandwidth and storage which has led to a miniaturisation of digital devices and greatly facilitated their portability and use in everyday life. This technological trend is most well known as "Moore's law" according to which the price-performance ratio of digital devices would half every two years. The third factor is the continuous increase in 'programmability in a modular' way. This trend allows to offer entirely different things on standardised software platforms, such as adding new capabilities to existing digital devices (i. e. add new apps to your smartphone).

These three factors are important technological developments by themselves but they are also interacting with each other. For example, the digitisation of music is possible since a long time and tangible equipment (such as a CD player) as a high capacity storage medium was used to play music.

But, the separation of the digital information contained in the song from the tangible devise became possible only when there was sufficient bandwidth to send and receive these millions of bits and when sufficiently powerful consumer owned general purpose devices (computers and smart phones) became available, equipped with the relevant software.

The separation of information from a physical object across a number of goods (music, videos, apps etc.) brings about fundamentally new ways of consumption and production. There is one crucial economic aspect which makes these digital goods differ from physical goods and services, namely their scalability. While for traditional goods a constant returns to scale production function is a good representation of the technology, digital goods have a fundamentally different cost structure. Digital goods are easily duplicated, once a prototype has been created. Their distribution can be organised via platforms. Setting up a platform requires an initial investment, which transforms a business idea into a software which allows to link providers of digital goods with customers. Operating the platform may itself be subject to fixed per period operating cost. However, once the platform is set up, sales can be adjusted to demand at zero marginal cost. This aspect of the digital economy has been stressed by various authors (e. g. Rifkin, Mason), but to the best of our knowledge, the possible implications of an expanding digital sector on growth, employment and income distribution has not been analysed formally yet.

In this paper we present a two sector economy which is producing physical goods as well as digital goods, which we will henceforth denote as physical and digital sector respectively. Our model economy can be described as follows:

The physical sector is characterised by a CRS technology and perfect competition and it produces rival goods. Technical progress is exogenous and labour augmenting. Consistent with the technological trends outlined above, we divide the digital sector into three subsectors. First, there are the IT hardware producers, which provide the physical infrastructure used by platforms. Production of IT equipment is subject to constant returns to scale and exogenous technical progress. Another important input is intangible capital in the form of software and R&D. Both activities allow to transform business ideas into a computer code which then allows the platform to respond to customer demand with zero marginal cost. Firms in the digital sector offer different varieties of digital goods. Production is characterised by fixed per period operating cost of a platform and zero marginal cost. Given consumer preferences for varieties of digital goods, each platform has the power to set prices such as to maximise profits. In order to obtain interior solution to the maximisation problem of the platform, we assume quadratic preferences which exhibit declining price elasticity as demand increases.

Technical progress in the digital sector can thus take three forms. First there can be a positive technology shock to the creation of new platforms, i. e. all activities related to the creation of software and R&D. These types of innovations lower the cost for creating new platforms which reduces entry costs. A second form of innovation are improvements in IT hardware (speed and storage capacity), which the platform rents. Innovations of this type reduce the period operating costs of the platform. It is assumed here that each platform needs a fixed amount of computer hardware, irrespective of the scale of operation plus overhead labour. Therefore a third form of technical progress in the digital sector are organisational innovations which reduce the number of overhead workers.

Technology in the digital sector requires to model entry explicitly. On the one hand, incumbent platforms have an advantage since a large fraction of customers are likely to become habituated shoppers enjoying the convenience value of well-known search and payment rules. These lock in effects are associated with increasing switching costs new platform providers have to incur when entering the market. However, as emphasised by Gilbert (2015), entry is not impossible as can be seen from the co-existence of various electronic goods providers, which offer slightly different service variants (e. g. by the degree in which e-book readers are locked into the use of its proprietary e-reader). Also, customers often have a choice between electronic and non-electronic variants. Finally, technical progress and new ideas can challenge incumbent positions (see Veughelers 2013). In this paper we consider two extreme positions. We will look at both an economy with free entry and an economy with entry costs. Concerning the modelling of entry into the digital sector we borrow heavily from the endogenous growth literature (see Romer (1990) or Grossman and Helpman (2003)). We also allow for two types of skills, namely digital and non-digital skills. Here we consider two extreme cases. In the first case we assume homogenous labour or perfect substitutability between workers with and without digital skill in all sectors of the economy. In a second case we assume that the individual sectors demand those skills in different, but fixed proportions.

It must be stressed that this paper does not cover all digital economic activities. In particular it leaves out two important activities, namely all activities related to sharing of physical goods, which can be coordinated via digital platforms. In a companion paper (Nalbach and Roeger (2017)) we discuss this case. Also free digital services such as obtaining information from search engines and interacting with others in social networks are not covered in this paper. Instead, we concentrate in this paper on the sale of electronic goods to consumers.

We use this model to study the following questions. If technical progress is predominantly taking place in the digital sector of the economy, how will this affect aggregate growth, (employment) and income distribution ((skill specific) wage share), given the specific nature of production and market structure in the digital sector?

In particular, technical progress in the digital sector is not labour augmenting in the traditional sense, i. e. it does not increase the marginal productivity of labour and the real wage. That the wage share could be negatively affected has also been discussed in the empirical literature. This question has especially gained attention in the US literature, where a significant decline in the wage share can be observed since early 2000.

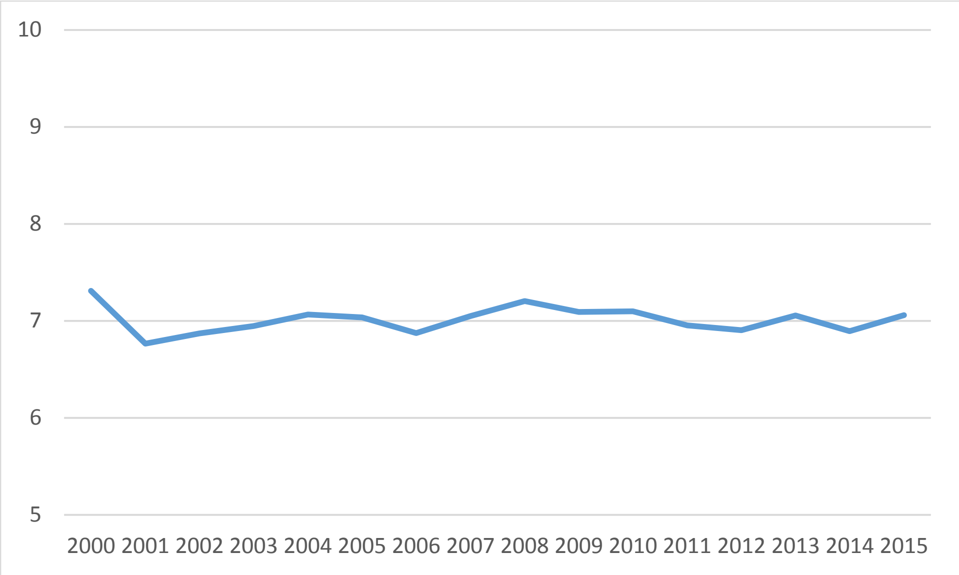
While this paper explores the aggregate growth and distributional effects of an emerging digital goods sector with distinct product and technology characteristics, there is a growing literature which analyses recent distributional trends with the emergence of so called superstar firms. Autor et al. (2016) find that rising firm mark ups in the US could be related to new production structures, especially associated with the proliferation of information intensive goods which are distributed via platforms. They call these firms superstar firms. These authors emphasize heterogeneity of firms (small and large firms) across all sectors. Korinek and Ng (2017) make an attempt to introduce superstar firms into an otherwise standard growth model, by introducing firms with high fixed and zero marginal cost and they analyse how sectoral output and productivity changes with the emergence of a superstar firm characterised by a certain level of fixed costs. These authors concentrate on changes in the structure of production, when a superstar firm enters. However, entry is exogenous.

The paper is organised as follows. Section 1 provides some stylised facts on the digital and non-digital sector of the US economy. Section 2 gives a detailed model description. Section 3 provides an analytical steady state solution for a special case, characterised by homogeneous labour and unit elasticity between physical and digital goods. This provides insights into the main mechanisms in which different types of technology shocks in the digital sector affect growth and wages. Section 4 assesses the quantitative effects of permanent digital technology shocks in a model calibrated to the US economy. In the annex we discuss the optimality of the market outcome, by comparing the social planning solution to the market solution and we also provide some sensitivity analysis.

1. Some stylised facts

We define the digital sector as being composed of ISIC industries 26 (manufacture of computer, electronic and optical products), 58 (publishing activities), 60 (programming and broadcasting), 62 (computer programming, consultancy and related activities) and 63 (information service activities). The nominal share of the digital sector stayed roughly constant over the period 2000-2015 at close to 7% (see Figure 1).

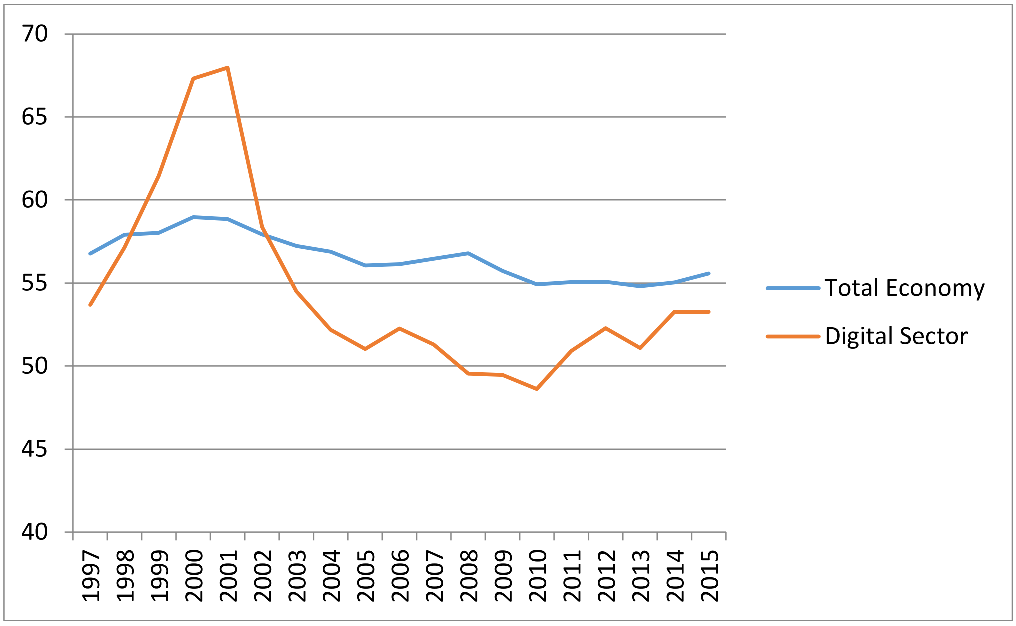
Figure 1: Digital Sector Share (Nominal GVA as % of GDP)



Source: BEA national accounts

Average labour productivity (real value added/employment) grew by 5.7% on average over the same period, compared to 1.3% for the total economy. Thus non-digital sector productivity only grew by ca. 0.9%. The wage share of the digital sector exceeded the total economy wage share in early 2000, but fell afterwards and reached a trough in 2010. It increased slightly in the following years but is still below the total economy wage share (see Figure 2).

Figure 2: Wage share – Total economy and digital sector (US)



Source: BEA national accounts.
Notes: Wage share defined as compensation of employees as per cent of GVA.

Information about the endowment of the workforce with digital skills is contained in the OECD's survey of adult skills (PIAAC). This survey suggests that about 50% of adults can carry out basic computer tasks (writing, emailing and searching the web). However only about 4% of workers are IT specialists (OECD 2016), i. e. workers who can code, develop applications, manage networks or analyse Big Data. Unfortunately data on the distribution of digital skills across sectors is scarce (see OECD 2016), however, some information is provided by Guellec et al. (2017). They show that employment with software related occupations are substantially higher in digital sectors.

Also in terms of the composition of the capital stock, the digital sector differs strongly from the total economy. While in th aggregate intangible assets are about a quarter of total assets and ICT and computer hardware constitute about 20% of machinery and equipment, the intangible asset share in the digital sector is 62% and machinery and equipment is practically completely made up of ICT and computer hardware[1]. Because of higher digital sector growth, intangible capital has grown more strongly compared to tangibles. Interestingly the average growth differential seems to a large extent due to a cleansing effect in the 2009 recession which shows a strong drop in tangibles, while intangibles remained flat (see Figure 3).

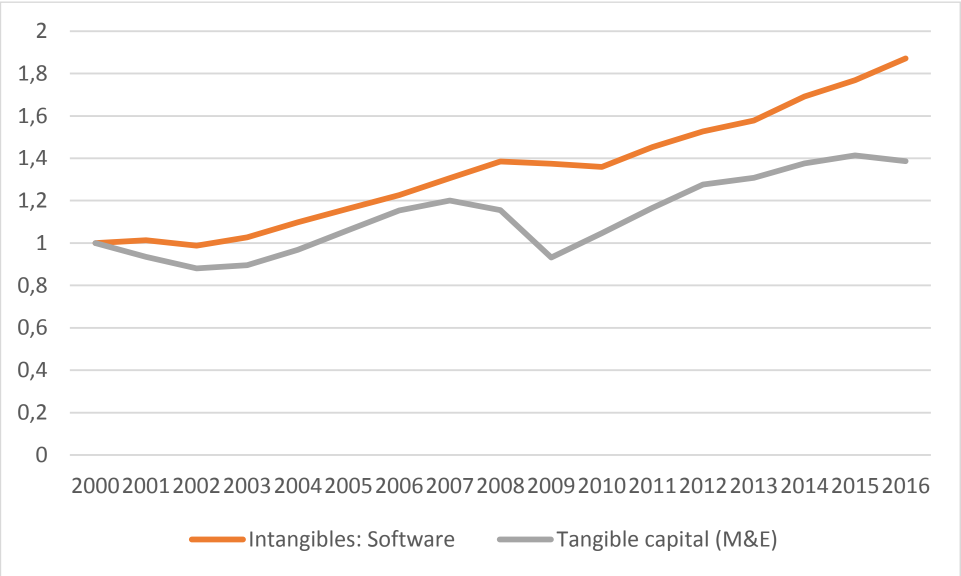
[1] See Haskel et al. (2018) for an analysis of intangible capital.

Table 1: Asset Structure – Total Economy and Digital Sector, as % of total Assets, US, 2016

	Total Economy	Digital Sector
Machinery & Equipment	33	27
ICT & Computer Hardware	7	27
Intangible assets	26	62
Software	11	27
R&D	13	25

Source: BEA national accounts

Figure 3: Tangibles (M&E) and Non Tangibles (Software), Index 2000=1



2. The model

We analyse a two-sector economy, with a sector for physical goods and a sector for digital goods/services. The physical goods sector is perfectly competitive. Physical goods are produced with a CRS production function. This sector sets prices according to marginal cost. The digital sector is comprised of a two stage production process. There is a knowledge production function for new platform designs which uses labour as input. The number of new designs is proportional to labour input. Platforms are sold to entrants in the digital sector. Platforms receive revenue from selling digital goods to households. Apart from the purchase price of the platform there is a per period fixed operating cost for the platform, but there are no other costs associated with the provision of digital goods, i.e. digital goods can be provided at zero marginal cost.

Household

There is a continuum of households h distributed over the unit interval $h\,\epsilon\,(0,1)$. The household maximises an intertemporal utility function over a consumption aggregate (C_t) and labour supply (L_t). We denote the supply of traditional skills with L_t^L and the supply of digital skills with L_t^H

$$U = \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{\omega^L}{1+\xi}\left(\frac{L_t^L}{L_t^{L*}}\right)^{1+\xi} - \frac{\omega^H}{1+\xi}\left(\frac{L_t^H}{L_t^{H*}}\right)^{1+\xi}\right) \quad with\, \beta = \frac{1}{1+\rho} \tag{1}$$

and, CES utility for physical (C_t^O) and digital goods (C_t^D)

$$C_t = \left[\gamma^{\frac{1}{\sigma}} C_t^{O\,\frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} C_t^{D\,\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

and the special case of unit elasticity

$$C_t = \frac{C_t^{O\,\gamma} C_t^{D\,1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \tag{3}$$

The utility function or digital goods $C^D(x_j)$ characterises preferences for varieties *j* of digital goods of the household sector. We assume that the utility of an individual household member *h* associated with the consumption of digital good *j* is given by

$$U(h) = (\phi_1 - \phi_2 h) \tag{4a}$$

if the household member is consuming the digital good and zero otherwise[2]. Let x_j be the share of household members which consume good j. Then the sum of utilities of all household members from consuming good j is given by a quadratic utility function

$$U(x_j) = \int_{h=0}^{x_j}(\phi_1 - \phi_2 h)dh = (\phi_1 x_{jt} - \frac{\phi_2}{2}x_{jt}^2) \tag{4b}$$

This utility function implies a price elasticity which is declining in x_{jt}. This is useful for solving the profit maximisation problem of the digital good provider. Household utility across digital variants is given by

$$C_t^D = \int_{j=0}^{A}(\phi_1 x_{jt} - \frac{\phi_2}{2}x_{jt}^2)dj \tag{4c}$$

The household receives labour income $W_t L_t$ from working in the three sectors as well as profit income π_{jt} from each of the A digital sectors and rental income ($r_{t-1}^K P_t^O K_{t-1}$) from physical capital. There is a one period bond B_t^N which can be traded between households (and which is in zero net supply). In order to simplify the optimisation we follow the endogenous growth literature (see Romer (1990)) and treat the index of different types of digital goods as continuous variable, this avoids taking explicit account of integer constraints

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\, U(C_t, L_t)&\\ -\sum_{t=0}^{\infty}\beta^t\,\lambda_t\Bigg(&B_t^N + \int_0^{A_t}P_{jt}^A dj - (1+i_{t-1})B_{t-1}^N - r_{t-1}^{K^O}P_t^O K_{t-1} + P_t^O(K_t - (1-\delta)K_{t-1})\\ &+ P_t^O C_t^O + \int_0^{A_t}P_{jt}^x x_{jt}dj + P_t^O\frac{\theta}{\upsilon}A_t^{\upsilon}\Delta A_t - W_t(L_t^O + A_t L_t^x + L_t^A) - \int_0^{A_t}\pi_{jt}dj - \int_0^{A_{t-1}}P_{jt}^A dj\\ &- r_{t-1}^{K^{IT}}P_t^{IT}K_{t-1}^{IT} + P_t^{IT}(K_t^{IT} - (1-\delta)K_{t-1}^{IT})\Bigg)\end{aligned}$$

There is a an entry cost $\frac{\theta}{\upsilon}A_t^{\upsilon}\Delta A_t$ which is a convex function ($\upsilon > 1$) of the number of existing platforms. This function admits two polar cases, namely free entry ($\theta = 0$) and zero entry ($\upsilon \to \infty$). We further assume that entry costs do not decline with technology improvements in the physical production sector, therefore we specify $\theta = \theta_0 u^O$. We choose the price of physical goods as numeraire ($P_t^O = 1$). The household makes consumption, investment and labour supply decisions. Concerning consumption the household optimises with respect to the physical good and A varieties of digital goods.

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[2] This assumes that those members not buying good j can be excluded by the platform from using good j.

$$\frac{\partial \mathcal{L}}{\partial C_t^O} = \frac{\gamma}{C_t^O} - \lambda_t P_t^O = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial x_{jt}} = \frac{1-\gamma}{C_t^D}(\phi_1 - \phi_2 x_{jt}) - \lambda_t P_{jt}^x = 0 \tag{6}$$

The household can borrow or lend at the one period interest rate i_t.

$$\frac{\partial \mathcal{L}}{\partial B_t^N} = -\lambda_t + \beta\lambda_{t+1}(1+i_t) = 0 \tag{7}$$

The household invests in physical capital and platforms. There are two types of physical capital, namely physical capital used in the production of physical goods and IT hardware, with the two no-arbitrage conditions

$$\frac{\partial \mathcal{L}}{\partial K_t^j} = -\lambda_t P_t^j + \beta\lambda_{t+1} P_{t+1}^j \left(r_t^{K^j} + (1-\delta) \right) = 0,\ \ j = O, IT \tag{8}$$

Thus, the rental rate on capital ($r_t^{K^j}$) minus depreciation equals the interest rate.

$$r_t^{K^j} = (1+i_t)\frac{P_t^j}{P_{t+1}^j} - (1-\delta)) = i_t - \pi_{\ t+1}^{\ j} + \delta \tag{9}$$

The number of platforms is determined such that the PDV of profits generated by the platform is equal to the platform price, adjusted for marginal entry costs.

$$\frac{\partial \mathcal{L}}{\partial A_t} = -\lambda_t P_{jt}^A - \lambda_t \left(\theta A_t^{\nu-1}\Delta A_t + \frac{\theta}{\nu} A_t^{\nu} \right) + \lambda_t \pi_{jt} + \beta\lambda_{t+1}(P_{jt+1}^A - \theta A_{t+1}^{\nu-1}\Delta A_{t+1} + \frac{\theta}{\nu} A_{t+1}^{\nu}) = 0 \tag{10}$$

Optimisation w.r.t. labour yields the following labour supply equation

$$L_t^i = \left(L_t^{i*}\right)^{\frac{1+\xi}{\xi}} \left(\frac{1}{\omega^i} \frac{W_t^i}{P_t^C C_t} \right)^{\frac{1}{\xi}} \quad , i = L, H \tag{11}$$

Note, for $\xi \to \infty$ we get $L_t^i = L_t^{i,*}$ and the supply of labour becomes completely inelastic.

Using the two first order conditions one can represent the household demand for digital good j as

$$x_j = \frac{\phi_1}{\phi_2} - \frac{1}{\phi_2} \frac{P_{jt}^x}{P_t^O} \frac{C_t^D}{C_t^O} \frac{\gamma}{(1-\gamma)} \tag{12}$$

The price elasticity of demand is an increasing function of the price (or a declining function of demand) and varies between zero (for $P_{jt}^{x} = 0$) and infinity (for $x_{jt} = 0$). Let ε_i be the price elasticity of demand, then it is easy to see that $\lim_{x_i \to 0} \varepsilon_i \to \infty$ and the price elasticity declines monotonically as Pi declines towards zero .

Production and market structure for physical goods

We assume that the physical good is produced under perfect competition, with firms using a CRS production technology.

$$Y_t^O = u_t^O {L_t^O}^{\alpha} K_t^{1-\alpha} \tag{13}$$

Where u^O is an efficiency parameter and L^O is a composite labour input given by a Leontief aggregator

$$L_t^O = \min\left(\frac{1}{s^{OL}} L_t^{OL}, \frac{1}{s^{OH}} L_t^{OH}\right) \tag{14}$$

Where L_t^{OL} and L_t^{OH} is traditional and digital labour input in the physical sector. This implies that the two types of labour are used in fixed proportions

$$L^{OL} = s^{OL} L^O \tag{15}$$

$$L^{OH} = s^{OH} L^O \tag{16}$$

and $s^{OL} + s^{OH} = 1$.

We assume the skill structure to be non-digital intensive:

$$s^{OL} > s^{OH}.$$

With perfect competition, the maximisation problem of the firm yields

$$Max\ (P_t^O Y_t^O - W_t^O L_t^O - r_t^K P_t^O K_t) \tag{17}$$

The first order conditions for labour and capital are

$$\alpha \frac{Y_t^O}{L_t^O} = \frac{W_t^O}{P_t^O} \tag{18}$$

and

$$(1-\alpha)\frac{Y_t^O}{K_t^O} = r_t^K = (1+i_t)\frac{P_t^O}{P_{t+1}^O} - (1-\delta)) = i_t - \pi_{t+1}^O + \delta \quad (19)$$

This yields the following price equation for physical goods

$$P_t^O = \left(\frac{W_t^O}{u_t^O}\right)^{\alpha} (i_t + \delta)^{(1-\alpha)} \quad (20)$$

IT Hardware Production:

A subsector of the physical production sector produces IT hardware. Here we assume that perfectly competitive IT hardware producers use goods produced by the physical production sector as input (I_t^{ITO}) and transform them into IT equipment (I_t^{IT}). Production of IT equipment is subject to a technology shock u_t^{IT}.

$$I_t^{IT} = I_t^{ITO}(1 + u_t^{IT}) \quad (21a)$$

Because of perfect competition the price of computer equipment is given by,

$$P_t^{IT} = \frac{P_t^O}{1+u_t^{IT}} \quad (21b)$$

Production and market structure in the digital sector

Technology in the digital sector differs fundamentally from the technology in the physical sector. The goods sold via a platform can be adjusted to any level of demand at zero marginal cost. Firm j only requires a fixed number of workers L_j^x for operation and input provision and a fixed amount of computer hardware K_j^x which it rents from the household sector. Thus, the cost in period *t* for platform j is

$$Cost_{jt}^x = W_t L_j^x + r_t^{K^{IT}} P_t^{IT} K_j^x \quad (22)$$

We think of L_j^x as the time it takes to create platform applications. The fixed labour input per firm in the digital sector is given by the Leontief labour aggregate

$$L_{jt}^x = \min\left(\frac{1}{s^{xL}} L_{jt}^{xL}, \frac{1}{s^{xH}} L_{jt}^{xH}\right) \quad (23)$$

We assume the skill structure to be digital intensive:

$$s^{xL} < s^{xH}$$

As emphasised in the literature (see Rochet and Tirole (2003)) platforms are characterised by their two-sided nature. They interact with consumers and app developers. There can be a multitude of different business models, depending on the bargaining position of the platform vis-à-vis consumers and app providers, which in turn depends how participation on one side of the market affects participation on the other side of the market. Here we disregard such participation externalities. We assume that the platform provides digital goods in a monopolistically competitive fashion as implied by consumer preferences for digital goods. In addition we assume that app developers have zero market power and the platform pays the market wage[3]. We implicitly further assume that the apps created in period t are only sold in this period. In the next period, authors provide new apps to the platform. It is also often the case that digital goods and services that consumers possess electronic devices such as smartphones, laptops, consoles e-readers etc.). It is assumed here that the digital good provided by the platform is a composite of both an application and a digital device. Both assumptions require that we specify a period length which covers the time in which an electronic good is in fashion as well as the durability of the digital device. (ca. 5 years) . Profits are given by

$$\pi_{jt} = p_{jt}^{x} x_{jt} - W_t L_j^x - r_t^{K^{IT}} P_t^{IT} K_j^x \tag{24}$$

Since marginal cost are zero for the platform, profits are maximised by maximising revenue. Given the properties of the price elasticity of demand revenue for x_j is maximised for $\varepsilon(x_j) = 1$, This can easily be verified. The FOC w. r. t. P_{jt}^x is given by

$$\frac{dRev_{jt}}{dP_{jt}^x} = \frac{\phi_1}{\phi_2} - \frac{2}{\phi_2}\frac{P_{jt}^x}{P_t^O}\frac{C_t^D}{C_t^O}\frac{\gamma}{(1-\gamma)} = 0 \tag{25}$$

And the optimal monopoly price is given by

$$P_{jt}^{x^M} = P_t^O \frac{\phi_1}{2}\frac{C_t^O}{C_t^D}\frac{(1-\gamma)}{\gamma} \tag{26}$$

Substituting the optimal price into the demand equation yields the optimal monopoly output level

$$x_j^M = \frac{\phi_1}{\phi_2} - \frac{1}{2}\frac{\phi_1}{\phi_2} = \frac{1}{2}\frac{\phi_1}{\phi_2} \tag{27}$$

Under symmetry, revenue of platform j is given by

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[3] This is the simplest possible assumption we can make and is also consistent with the homogeneity restriction on labour assumed in this paper. Here it is implicitly assumed that the platform is transforming homogeneous labour input into a specific variety of a digital good. Alternatively one could assume that the platform is creating a digital good by combining varieties of distinct labour inputs, which are imperfectly substitutable. In this set up the app provider could charge a wage mark up to the platform.

$$Rev_{jt} = p_{jt}^{x\,M} x_j^M \quad = \frac{(1-\gamma)}{\gamma} \frac{P_t^O C_t^O}{A_t} \tag{28}$$

Notice the revenue of the platform is a negative function of the number of platforms. This results from the fact that consumer preferences require a declining price of digital goods as the supply of digital goods increases with the number of platforms. This limits entry into the digital sector. The number of platforms is determined by the arbitrage condition

$$P_{j,t}^A = \left(\frac{(1-\gamma)}{\gamma} \frac{P_t^O C_t^O}{A} - W_t L_j^x - r_t^{K^{IT}} P_t^{IT} K_j^x\right) + \frac{1}{1+i_t} P_{j,t+1}^A \tag{29}$$

The dynamics of A and $P_{j,t}^A$ can be characterised after describing the process of platform production.

Production and market structure in platform creation

We follow the endogenous growth literature (see Romer (1990) or Grossman and Helpman (2003)) and assume that platforms are created by research labs using a linear technology

$$\Delta A_t = u_t^A L_t^A \tag{30}$$

with efficiency parameter u^A and labour input L_t^A. We assume that research labs only employ workers with digital skills.

$$L_t^A = L_t^{AH} \tag{31}$$

Research labs are perfectly competitive. The zero profit condition determines the platform price

$$P_t^A = \frac{W_t^H}{u_t^A} \tag{32}$$

Equilibrium conditions

There is an equilibrium condition for labour of type L and type H

$$L_t^L = L_t^{OL} + A L_t^{xL} \tag{34}$$

$$L_t^H = L_t^{OH} + A L_t^{xH} + L_t^A \tag{35}$$

Goods market equilibrium for physical goods

$$Y_t^O = C_t^O + I_t^O + I_t^{ITO} + \frac{\theta}{\upsilon} A_t^{\upsilon} \Delta A_t \tag{36}$$

Where

$$I_t^{ITO}(1 + u_t^{IT}) = I_t^{IT} = K_j^x (A_t - (1-\delta) A_{t-1})$$

And equilibrium for A digital goods

$$x_{jt} = x_j^M \quad for\ j = 1, \dots, A \tag{37}$$

Given the non-standard technology in the digital sector and the deviation from perfect competition raises the question about optimality of the market solution. As shown in Annex C, the socially optimal supply of good j exceeds the revenue maximising supply of the monopolist in the market solution. However, the number of platforms under the market solution – with perfect competition in the market for physical goods and for platforms – does not differ from the social planning solution.

3. The steady state effects of technical progress in a simplified economy

Technical progress in this economy can take various forms. Besides a positive TFP shock in physical production $du_t^O > 0$, technical progress can occur via efficiency gains in platform creation $du_t^A > 0$; efficiency gains in hardware production (Moore's Law) $du_t^{IT} > 0$ and efficiency gains in the use of overhead labour for platform operation $dL_j^x < 0$.

We can derive a number of steady state results of digital technical progress in a simplified case with CD preferences over physical goods and the bundle of digital goods. In this section we also assume inelastic labour supply ($\xi \to \infty$). We first discuss the case without different skill requirements across sectors and zero entry barriers. In later subsections we discuss both sector skill bias and entry cost.

3.1 No skill bias and free entry

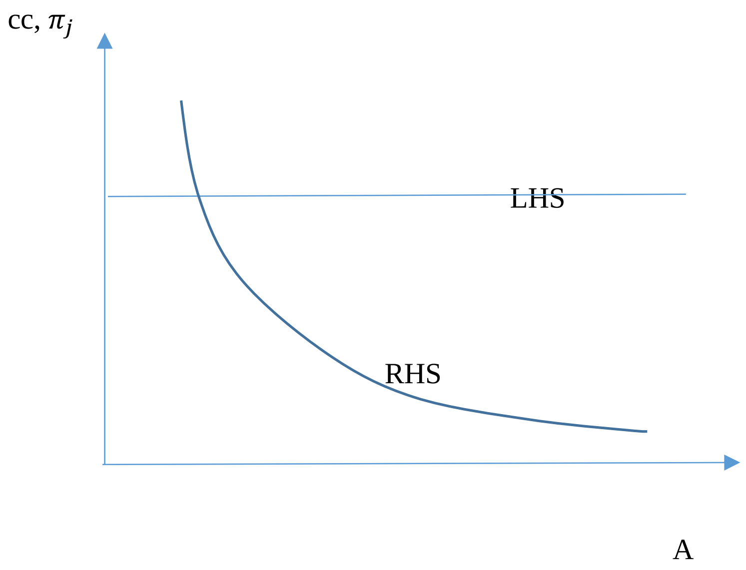
We first determine the number of platforms in the steady state, by using the platform arbitrage condition The platform price increases with the wage and declines with the technology term in platform design. Fixed wage costs are a positive function of the level of technology in the physical production sector. Revenue of the digital platform can be expressed via the resource constraint for physical goods (see Annex B). Thus we can write the steady state platform arbitrage condition

(38)

$$\rho P^A = cc = \rho \frac{\alpha u^O \chi_{Y^O}}{u^A} = \frac{(1-\gamma)}{\gamma} \frac{\chi_{C^O} u^O (L - AL^x) - \delta A K_j^x}{A} - \alpha u^O \chi_{Y^O} L^x - \frac{(\rho+\delta)}{1+u^{IT}} K_j^x = \pi_j$$

which shows that platform profits are a negative function of the number of platforms. For each constellation of technology and preferences, the number of platforms adjusts such that profits are equal to capital cost of the platform. In the steady state, the platform price is determined positively by the level of technology in the physical sector, since this determines the wage, and negatively by the technology in platform creation. These two technology terms, together with the rate of time preference determine profits of individual platforms in the steady state. The determination of the number of platforms can be shown graphically as the intersection between the horizontal line representing capital cost (LHS) of the platform and the downward sloping profit function (RHS).

Figure 3 Determining the number of digital variants – free entry



$du^O > 0$: Capital cost for the platform increases because higher wages paid in the physical production sector is increasing the platform price and therefore capital cost for intangible capital. Also wages for overhead labour increases. However, higher income generated in the physical production sector increases demand for digital goods. Cost and revenue increase offset each other and keep the number of platforms constant. This is represented by an identical upward shift of the capital cost line and the profit curve

$du^A > 0$: Capital cost for the platform decreases because of a declining price for the platform design. This increases entry until the PDV of profits are equal to capital cost for intangible capital and per period operating costs. This is represented as a decline of the capital cost line along a constant profit curve.

$dL_j^x < 0$: Platform overhead labour cost declines. Profits of incumbent platforms increases. Entry occurs until the PDV of revenues is equal to the cost of intangible capital. This is represented as an upward shift of the profit curve and a constant capital cost line. Thus entry will occur until profits are back at the baseline level

$du_t^{IT} > 0$: Period tangible capital cost for the platform declines and demand for consumption increases. Profit of incumbent platforms increases. Entry occurs until the PDV of revenues is equal to capital cost for intangible capital. This is represented as an upward shift of the profit curve and a constant capital cost line. Thus entry will occur until profits are back at the baseline level

Wage share

Assuming zero depreciation for tangible capital used by platforms allows for an analytical discussion of wage share results. Total nominal GDP in the steady state consists of physical consumption, physical investment (in the physical sector) and digital consumption. Wage income is the sum of labour income in the physical production sector and wage income of overhead labour in platforms. There is rental income from capital in the physical production sector and from IT capital used by platforms plus rental income from intangible platform capital.

Define the wage shares in the two sectors

$$WS^D = \frac{AWL^x}{P^D C^D} \tag{39}$$

$$WS^O = \frac{WL^O}{P^O Y^O} \tag{40}$$

The aggregate wage share can be written as

$$WS = \frac{AWL^x}{P^D C^D} s_D + \frac{WL^O}{P^O Y^O} s_O = WS^D s_D + WS^O s_O \tag{41}$$

Where s_D and s_O are the nominal sector shares. These shares are constant in the steady state under CD preferences

$$s_D = \frac{P^D C^D}{C^O + I^O + P^D C^D} = \frac{(1-\gamma)}{\gamma\left(1+\frac{\chi_I{}^O}{\chi_C{}^O}\right)+(1-\gamma)} \text{ and } s_O = \frac{\gamma\left(1+\frac{\chi_I{}^O}{\chi_C{}^O}\right)}{\gamma\left(1+\frac{\chi_I{}^O}{\chi_C{}^O}\right)+(1-\gamma)} \tag{42}$$

With a CD technology, the wage share in physical production remains unaffected. Therefore, the aggregate wage share is a positive function of the wage share in the digital sector. We can now show analytically how the wage share in the digital sector is affected by technology shocks.

The ratio of total profits to total wages in the digital sector is given by

$$\frac{A\pi_j}{A\alpha u^O \chi_Y{}^O L^x} = \frac{\pi_j}{\alpha u^O \chi_Y{}^O L^x} \tag{43}$$

And the wage share in the in the digital sector is

$$WS^D = \frac{A\alpha u^O \chi_Y{}^O L^x}{A\pi(u_o, u_a, L^x, u^{IT}) + A\alpha u^O \chi_Y{}^O L^x + \frac{(\rho+\delta)}{1+u^{IT}} A K_j^x} \text{ with } \frac{\partial \pi_j}{\partial u^O}\frac{u^O}{\pi_j} = 1, \frac{\partial \pi_j}{\partial u^A} < 0, \frac{\partial \pi_j}{\partial L^x} = 0, \frac{\partial \pi_j}{\partial u^{IT}} = 0 \tag{44}$$

From which it follows that the wage share declines with a reduction of L^x ,increases with an increase of u^A and also increases with capital saving technical progress. It also follows that a change in γ leaves the wage share in the digital sector unaffected.

From this we can conclude that a reduction of fixed costs lowers the aggregate wage share, while an increase in the efficiency of platform design increases the aggregate wage share. Finally, an increase in efficiency of physical production keeps the wage share in the digital sector unchanged, because digital profits and digital wages increase at the same rate.

Real consumption wage

Another important question is, how does technical progress and a preference shift affect the real (consumption) wage? Here it is useful to observe that the real wage in terms of the price of physical goods is always proportional to u^O. An increase of u^O increases W relative to P^O. This increases fixed costs in the digital sector, and the platform price which increases at the same rate as the wage. This in turn implies that the real consumption wage increases by less, since the increase in platform profit is associated with a price increase of digital goods. Thus even though the wage share is unchanged, the real consumption wage increases less because the price of digital goods increases relative to the price of physical goods. How the real consumption wage adjusts to technical progress in the digital sector can be inferred from the definition of period profit and making use of how firm profits adjust to technology and preference shocks in the steady state

$$P^x = \frac{\pi_j(u^O,u^A,L^x,u^{IT})+\alpha u^O \chi_{Y^O} L^x+\frac{(\rho+\delta)}{1+u^{IT}}AK_j^x}{x^M} \quad with\ \frac{\partial \pi_j}{\partial u^O}\frac{u^O}{\pi_j}=1, \frac{\partial \pi_j}{\partial u^A}<0, \frac{\partial \pi_j}{\partial L^x}=0, \frac{\partial \pi_j}{\partial u^{IT}}=0 \tag{45}$$

From this equation we can see that P^x declines (relative to P^O) in the case of a reduction in fixed costs and an increase in the efficiency of platform creation.

3.2 Entry barriers[4]

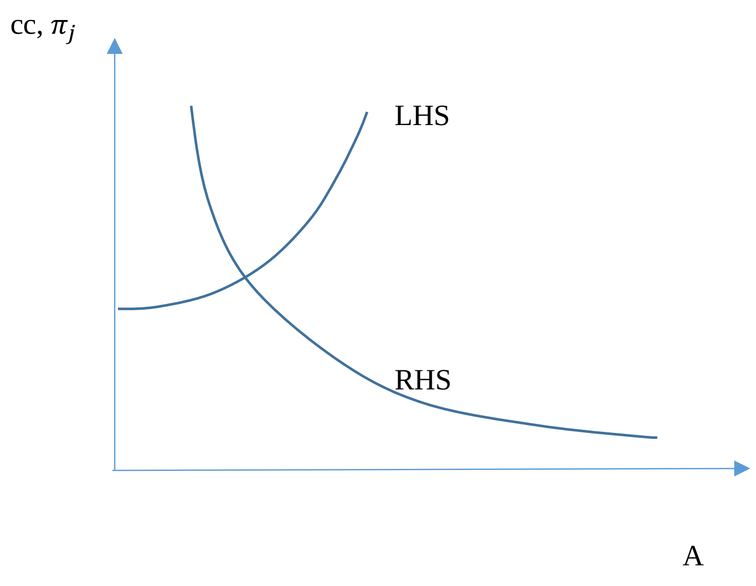
Now we assume that apart from buying the platform design, the household also faces convex entry costs $\frac{\theta}{\nu}A^\nu, \nu > 1$. These could be administrative costs, advertising costs (advertising costs could be large if there are network externalities). This makes capital cost for the platform a convex function of A

$$cc = \rho\left(\frac{u^O}{u^A} + \theta_0 u^O A^{\nu-1}\right) = \frac{1-\gamma}{\gamma}\frac{u^O(L-AL^x)}{A} - u^O L^x = \pi_j \tag{46}$$

Note:

$$\frac{\partial \pi_j}{\partial u^O}\frac{u^O}{\pi_j} = 1\ requires\ \theta = \theta_0 u^O$$

Figure 4 Determining the number of digital variants – with entry cost



[4] To simplify the analysis we only consider the case without IT hardware.

As can be seen from Figure 4, for sufficiently convex entry costs, A will not change strongly with shifts in technology and demand. A reduction of fixed costs digital goods increases profits of individual platforms permanently. However an increase in the efficiency of platform design still reduces steady state profits.

Also the effects on the wage share are likely to be stronger. In the case of a reduction of fixed costs the ratio of profits to wages increases both because of a decline in the denominator and an increase in the nominator. An increase in the efficiency of platform design reduces the numerator and keeps the nominator of the profit to wage ratio unaffected and thus has similar effects on the wage share. Notice, however, in all three cases we do not expect large quantitative differences, since larger profits per platform are accompanied by less entry.

For simplicity we consider a case with strong entry costs such that we can neglect changes of A. In this case we only have one form of technical progress, namely a reduction of fixed costs for platforms. Steady state profits of an individual platform can be represented as a function of preferences, fixed cost and the fixed number of platforms

$$\pi_j = \frac{P^D C^D}{A} - u^O L^x = \frac{1-\gamma}{\gamma}\frac{C^O}{A} - u^O L^x = \frac{1-\gamma}{\gamma}\frac{u^O L^O}{A} - u^O L^x = \frac{1-\gamma}{\gamma}\frac{u^O (L - AL^x)}{A} - u^O L^x \tag{47}$$

Unlike in the free entry case, a reduction of fixed costs and a preference shift towards digital goods is increasing profits of incumbent platforms. It is easy to see that total profits of the digital sector increases more strongly, compared to the free entry case. Total profits can be written as a negative function of A.

$$\pi^{TOT} = A\left(\frac{1-\gamma}{\gamma}\frac{u^O L}{A} - u^O L_j^x(\frac{1-\gamma}{\gamma} + 1)\right) = \left(\frac{1-\gamma}{\gamma} u^O L - A(u^A, L_j^x, \gamma) u^O L_j^x(\frac{1-\gamma}{\gamma} + 1)\right) \tag{48}$$

From this result it follows immediately that the wage share declines more strongly in the no entry case.

3.3 Sector skill bias and free entry

In order to simplify the discussion we assume in this section that there is only intangible capital but zero physical capital, i. e. physical goods are only produced with labour and the output elasticity of labour is equal to one ($\alpha = 1$). In this case, wages of the two skill groups and the number of platforms in the steady state are determined by the FOC in the physical sector

$$u^O = (w^L s^{OL} + w^H s^{OH}) \tag{49}$$

the free entry condition for platforms

$$\rho \frac{w^H}{u^A} = \frac{(1-\gamma)}{\gamma} \frac{u^O Min\left(\left(L^L - As^{xL}L^x\right), \left(L^H - As^{xH}L^x\right)\right)}{A} - (w^L s^{xL} + w^H s^{xH})L^x \tag{50}$$

and the two labour market clearing condition for workers with and without digital skills

$$L^H = As^{xH}L^x + s^{OH}L^O \tag{51}$$

and

$$L^L = As^{xL}L^x + s^{OL}L^O \tag{52}$$

These 4 equations determine $w^L, w^H, A,\ L^O$ with exogenous labour supply L^L and L^H. The two labour market equilibrium conditions determine A and L^O as functions of the skill intensity in the O and D sector.

$$A = \frac{s^{OL}L^H - s^{OH}L^L}{L^x\left(s^{xH}s^{OL} - s^{xL}s^{OH}\right)} \tag{53}$$

and

$$L^O = \frac{s^{xH}L^L - s^{xL}L^H}{\left(s^{xH}s^{OL} - s^{xL}s^{OH}\right)} \tag{54}$$

Positive solutions for A and L^O require certain restrictions, namely a digital skill bias in the digital sector

$$(s^{xH}s^{OL} - s^{xL}s^{OH}) > 0$$

A sufficiently small share of digital workers in the physical sector.

$$(s^{OL}L^H - s^{OH}L^L) > 0$$

And a sufficiently small share of non-digital workers in the platform sector.

$$(s^{xH}L^L - s^{xL}L^H) > 0$$

Using the free entry condition and the FOC for labour in the O sector we get

$$\frac{w^L}{w^H} = \frac{\left(\frac{\rho}{u^A L^x} + s^{xH}\right)\frac{s^{OL} L^H - s^{OH} L^L}{\left(s^{xH} s^{OL} - s^{xL} s^{OH}\right)} - \frac{1-\gamma}{\gamma}\frac{s^{xH} L^L - s^{xL} L^H}{\left(s^{xH} s^{OL} - s^{xL} s^{OH}\right)} s^{OH}}{\frac{1-\gamma}{\gamma}\frac{s^{xH} L^L - s^{xL} L^H}{\left(s^{xH} s^{OL} - s^{xL} s^{OH}\right)} s^{OL} - s^{xL}\frac{s^{OL} L^H - s^{OH} L^L}{\left(s^{xH} s^{OL} - s^{xL} s^{OH}\right)}} \tag{57}$$

Under restrictive labour market conditions (fixed supply of digital and non-digital skills), technical progress in platform design (u^A) does not affect the sectoral structure of production and leaves A and L^O unaltered, but in increases the skill premium. A reduction of overhead platform labour (L^x) increases A and leaves L^O unaffected and reduces the skill premium. I. e. the direct (skilled) employment reducing effect dominates the positive employment effect generated by increasing the number of platforms. These skill premia effects are, however mitigated by allowing for elastic skill specific labour supply.

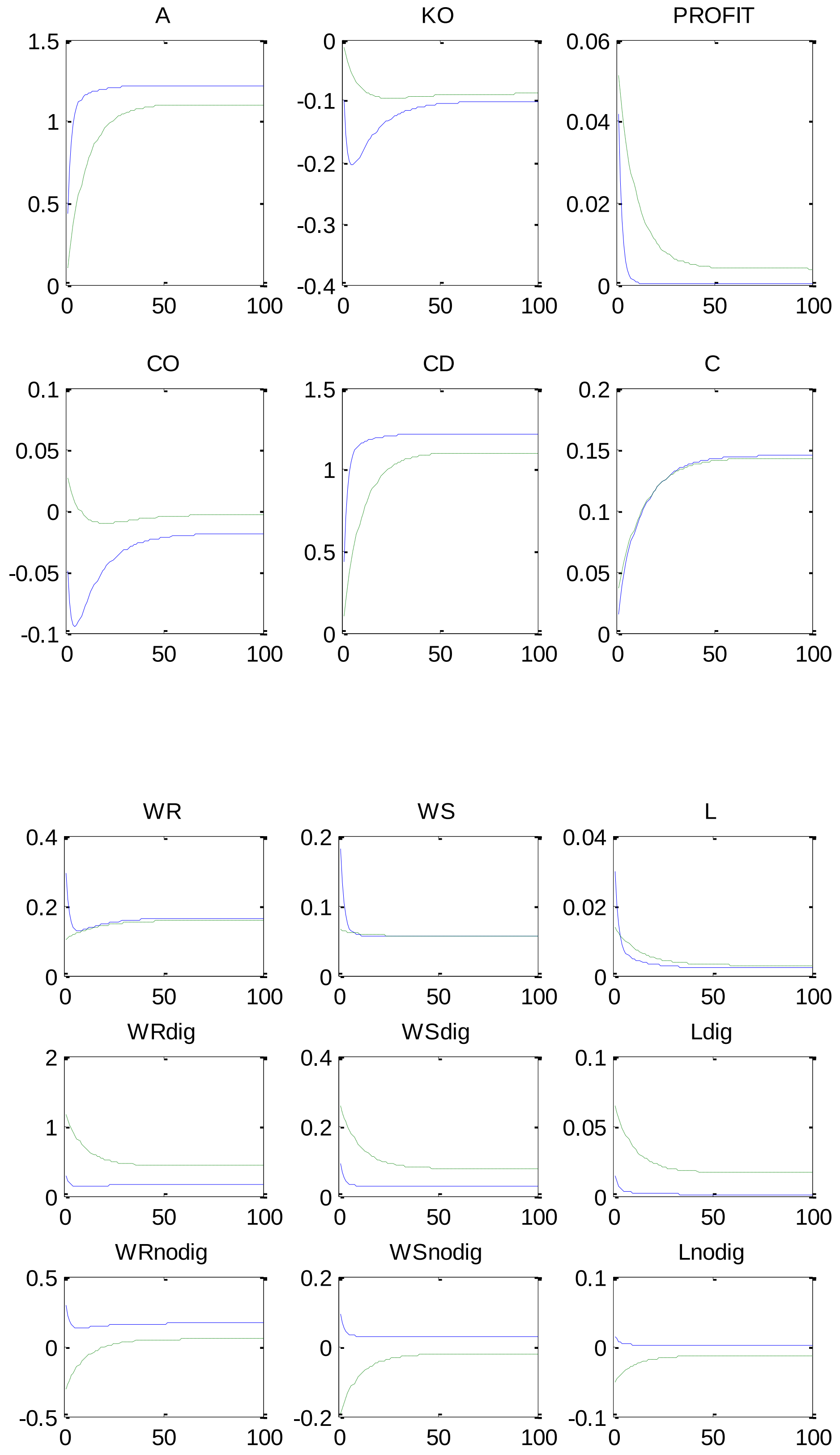
4. Impulse response analysis

In this section we are mainly interested in the quantitative impact of digital technology shocks on aggregate growth, the wage share, the real wage and the skill wage premium. In order to obtain realistic magnitudes we consider technology shocks which roughly double digital sector output in real terms. This resembles what can be observed in US data since 2000. In contrast to the simple analytical model, the model used in this section allows for elastic labour supply and is calibrated to match the shares of tangible and intangible capital in the digital sector as well as the share of IT hardware. The calibration details are contained in Annex A.

We conduct the quantitative analysis in three steps. As a benchmark we first consider the free entry case under the assumption of zero skill bias across sectors. This scenario comes closest to the analytical discussion in the previous section. We compare these results to a specification with sector specific skill bias. We are interested how this affects the aggregate results and the skill premia. In a third step we provide sensitivity analysis under the assumption that technical progress is associated with less entry, namely an increase in platforms not exceeding 10% compared to free entry.

The shocks we introduce are instantaneous permanent level shifts such that real output in the digital sector increases by 100% permanently.

Figure 5: Investment specific technical progress (IT hardware)



___: Symmetric skills; _ _ _: Sector skill bias

A: platforms (% dev.); KO: physical capital (% dev.); ,PROFIT: profit of single platform (abs. dev.): C: log of total consumption (abs dev); CO: log of physical consumption (abs dev); CD: log of digital consumption (labs. dev.); WR: log of real consumption wage (abs dev.); WS: wage share (abs. dev.); L: employment rate (abs. dev); WRdig: log of real consumption wage, digital skills (abs dev.); WSdig: wage share, digital skills (abs. dev.); Ldig: employment rate, digital skills (abs. dev); WRnodig: log of real consumption wage, non-digital skills (abs dev.); WSnodig: wage share, non-digital skills (abs. dev.); Lnodig: employment rate, non-digital skills (abs. dev);

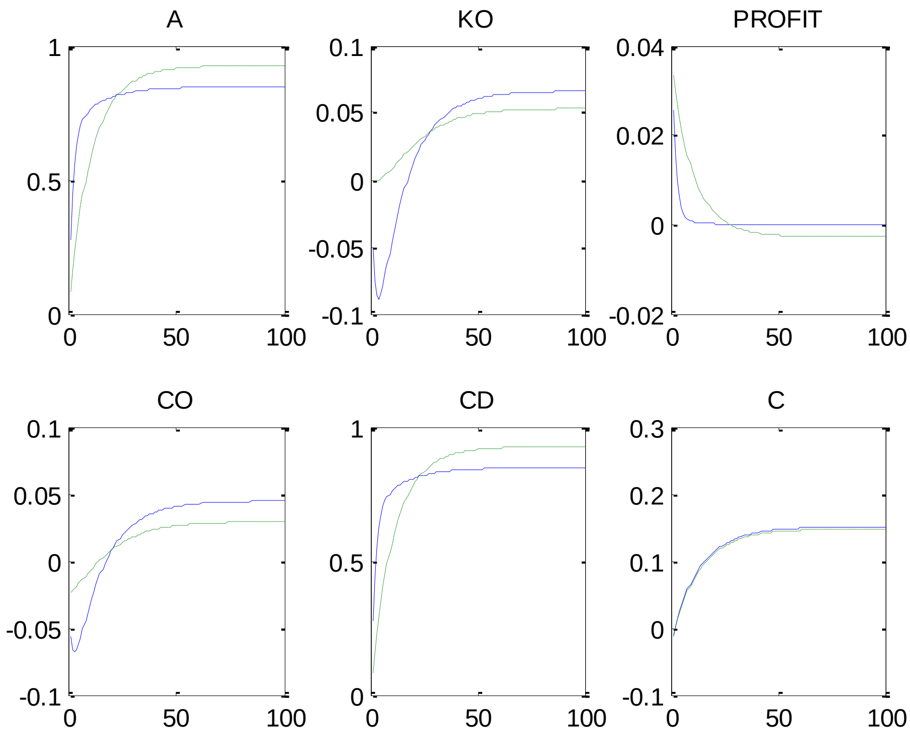
No sector skill bias:

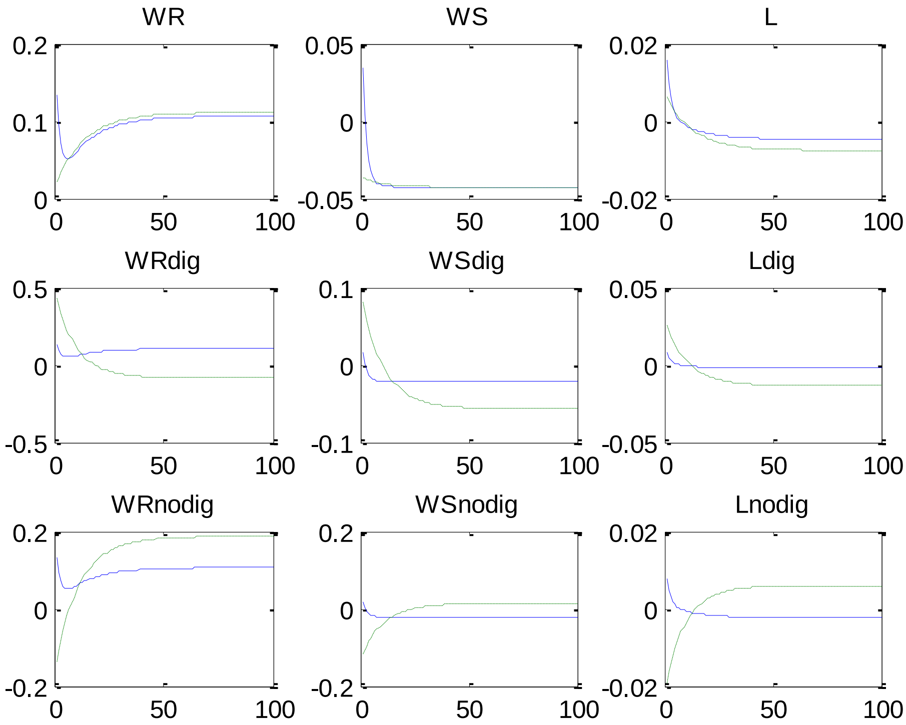
Technical progress in IT hardware reduces fixed capital costs and raises profits of incumbent platforms. At the same time wages remain constant relative to P^O because of unchanged technology in the physical goods sector and platform design sector. Thus the platform price (relative to P^O) does not change and entry occurs until profits have returned to the baseline level. However, entry increases the supply of digital goods, which reduces their relative price. This is how technical progress in IT hardware is transmitted into a higher real consumption wage of all workers. Also, the total economy wage share increases. This occurs because the wage share in the digital sector increases, since capital cost per platform declines while both profits and wages per platform stay constant. Because profits, labour and capital input increase proportionally with the number of platforms, the increase in the wage share per platform translates into an increase in the wage share of the platform sector. Finally, technical progress in IT hardware leads to an increase in intangible capital and a decline of tangible capital.

With sector skill bias:

The increase in the number of platforms leads to a relative increase in the demand for labour with digital skills (initially even more strongly because of platform creation). This increases the wage premium for digital skills. Since the FOC for labour requires that the ratio of wages to physical output prices remains constant, the demand for non-digital workers declines and output in the physical sector declines. The increase in the digital skill premium increases the platform price permanently. This implies that platform profits will stay above pre-shock levels. Associated with wage dispersion in favour of digital skills, relative employment/labour supply of workers with digital skills increases. The increase in the aggregate wage share is driven by the increased skill premium and higher employment. The wage share of workers without digital skills declines (largely because of a decline in employment). Though the real wage of workers without digital skills declines in terms of physical goods, the real consumption wage of non-digital workers does not decline with our calibration, since the price of digital goods drops relative to the price of physical goods. However, the evolution of the real consumption wage of workers without digital skills is sensitive to both the labour supply elasticity and the elasticity of substitution between physical and digital goods. In Appendix D we show that the real consumption wage of workers without digital skills increases with more elastic labour supply. Increasing the elasticity of substitution between between digital and physical goods increases the digital skill premium and reduces the real consumption wage of workers without digital skills.

Figure 6: Reduction in fixed labour costs





___: Symmetric skills; _ _ _: Sector skill bias

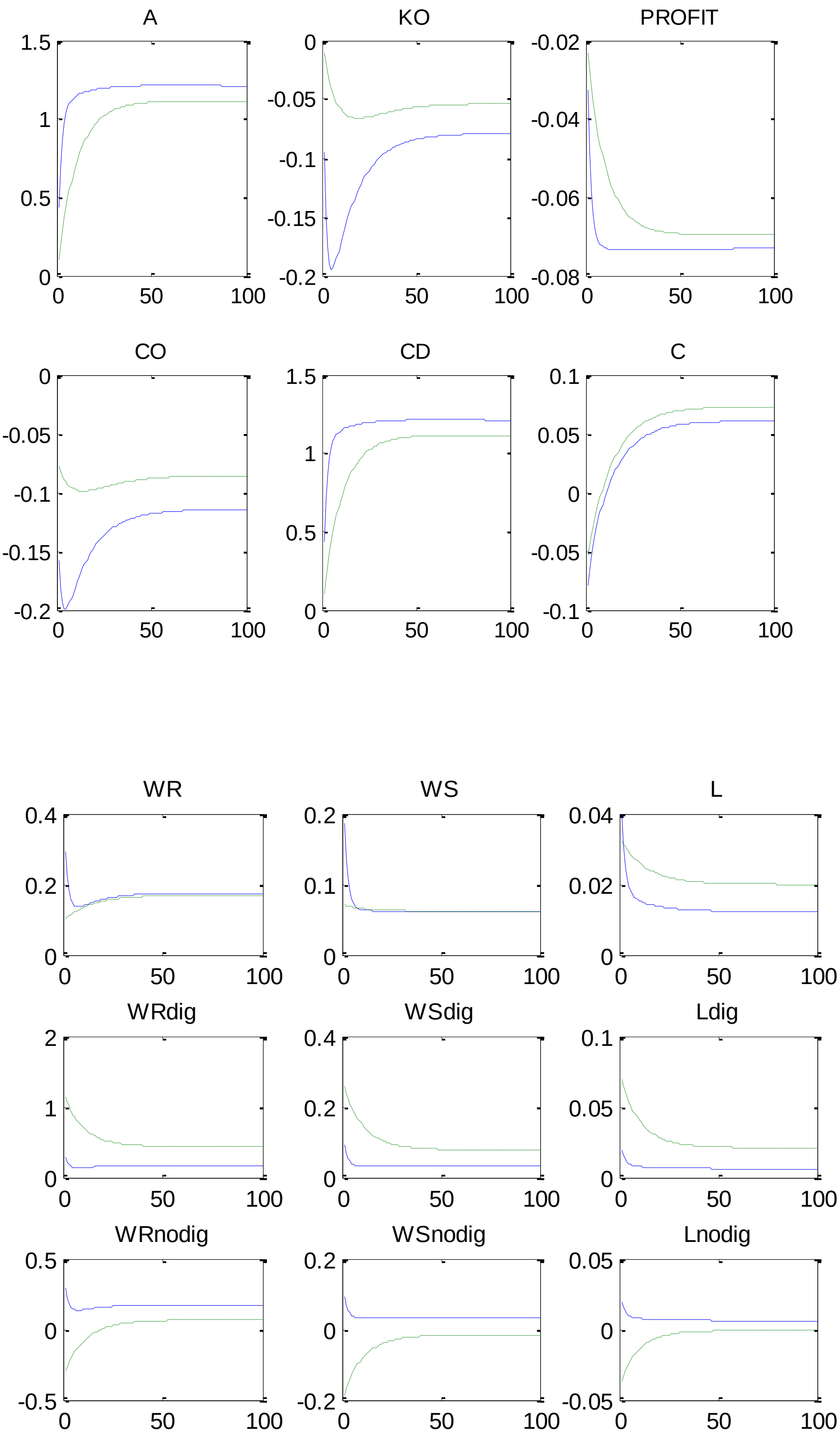
No sector skill bias:

If technical progress takes the form of reducing overhead platform labour, this also increases profits of incumbent platforms and generates entry until platform profits have returned back to pre-shock levels as in the case of reduced capital cost. What is different is the effect on the wage share, which declines in this case, because of labour saving in the platform sector. However, also in this case the reduction of fixed labour increases the consumption wage, due to an increase in the supply of digital goods which reduces their price. Finally, a reduction of overhead labour increases the ratio of intangible capital to physical capital. Notice, however, physical capital increases slightly as well, this is due to a reallocation of labour without digital skills to the physical production sector.

With sector skill bias:

Because of the digital skill bias in the platform sector the reduction in overhead labour reduces the demand for labour with digital skills more strongly compared to the case without skill bias in the long run. Initially there is a rising digital skill premium because of increased demand for digital labour engaged in the creation of new platform designs.

Figure 7: Increase in efficiency of platform creation



____: Symmetric skills; _ _ _: Sector skill bias

No sector skill bias:

An increase in the efficiency of platform design reduces the price of platforms and stimulates entry of new platforms until profits have fallen sufficiently. Because of no technical change in physical production both wage and capital cost for IT hardware remain unchanged. Therefore the share of wages in the platform sector increases. Increased entry increases the supply of digital goods and therefore prices. This in turn increases the consumption wage. The expansion of the digital sector relative to the physical sector increases intangible capital and reduces physical capital.

With sector skill bias:

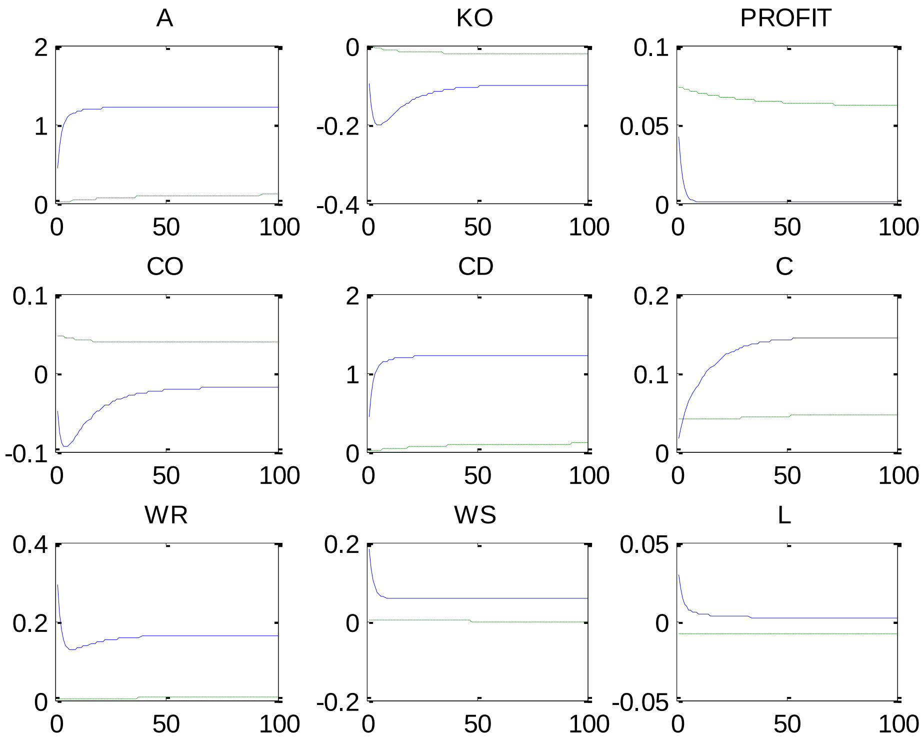
Like in the case of technical progress in IT hardware, technical progress in platform design and the resulting expansion of demand for workers with digital skills increases the digital skill premium and forces a reduction of wages of workers without digital skills, in order to satisfy the labour demand in the physical production sector (which requires a constant real wage (in terms of P^O) of the labour aggregate. The increase in the digital skill premium leads to a small decline of profits and therefore less entry into the platform sector. This reduces the extent of reallocation towards digital goods relative to the no skill bias case. Though the aggregate wage share increases in this case, the share of wages of workers without digital skills declines.

The importance of free entry for the results

Now we assume that apart from buying the platform design, the household also faces convex entry costs $\frac{\theta}{\nu}A^{\nu}, \nu > 1$. Here we only consider the case without sector specific skill bias.

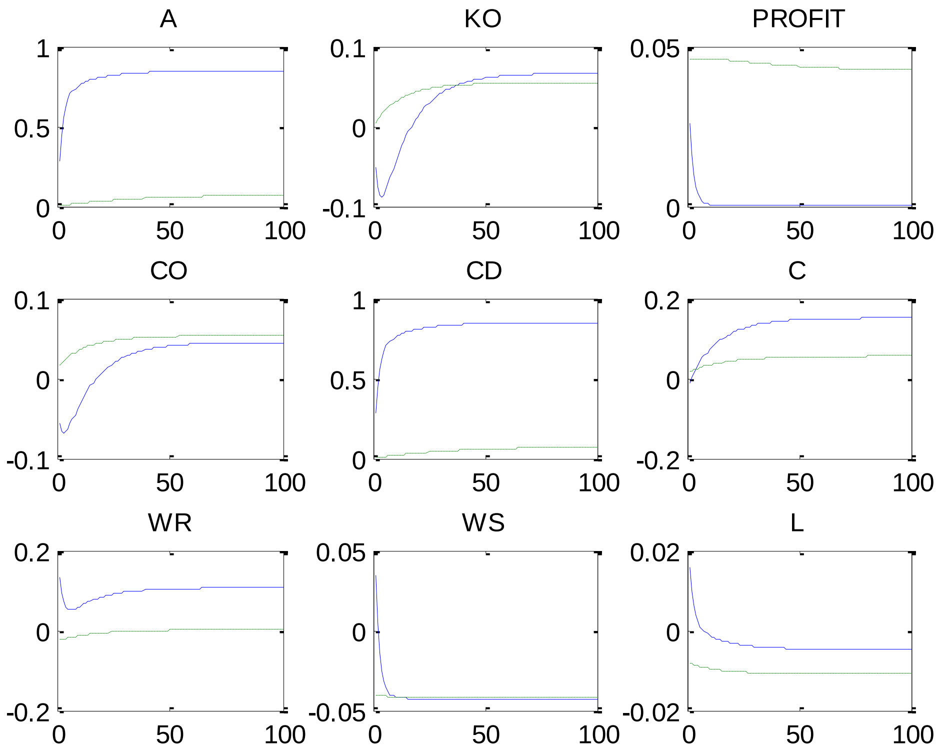
In the case of capital saving technical progress, entry barriers increase profits of incumbent platforms. Restricted entry limits the expansion of the digital sector. In fact labour saving technical progress reallocates labour to the physical production sector. In contrast to the solution with free entry, the wage share does not increase with entry cost.

Figure 8: Capital saving technical progress with free and restricted entry



___: Free entry; _ _ _: Restricted entry

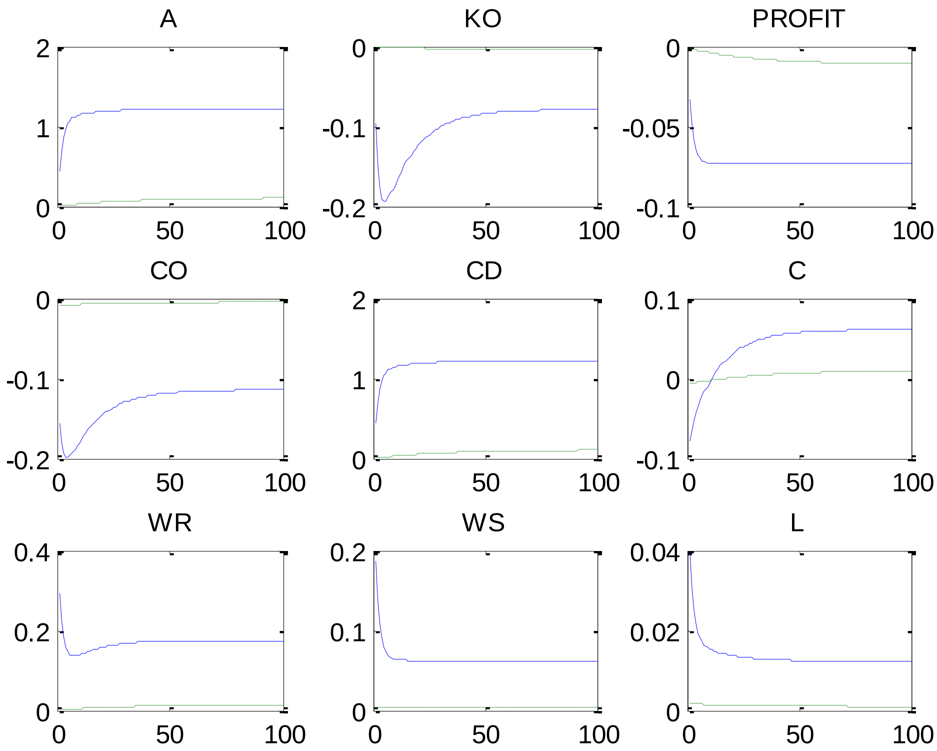
Figure 9: Reducing overhead platform labour with free and restricted entry



___: Free entry; _ _ _: Restricted entry

If digital firms save labour and there is no entry both profits per firm and total profits increase and the wage share is reduced. With limited entry digital output hardly increases, while physical output increases because of reallocation of employment towards the physical sector. Goods market equilibrium requires a decline of the price of physical goods. Because nominal wages are fixed in physical goods prices, the new equilibrium is characterised by a decline in the real consumption wage.

Figure 10: Technical progress in platform design with free and restricted entry



___: Free entry; _ _ _: Restricted entry

Entry costs prevent a decline of profits in the digital sector. Nevertheless, the physical production sector declines in the steady state, since the limited increase of platforms leads to a reallocation of labour from the physical sector to the digital sector. Higher profits in the digital sector restrict the increase of the wage share, which occurs under free entry.

Conclusion

An important aspect of an emerging digital economy is the supply of electronic goods to consumers at zero marginal cost. A second important feature of current growth is its dependence on technical progress in the digital sector. The paper explores how in the context of a two sector growth model, distinguishing between physical and digital goods, digital technology shocks such as investment specific technical progress in IT hardware and innovations in platform design affect growth, the wage share, real wages and the digital skill premium.

Our findings can be summarised as follows. First we show that for quadratic preferences there exists an interior solution to the profit maximisation problem of the platform. However, the market solution is characterised by undersupply of digital goods compared to the social planning solution. In the second part of the paper we characterise the response of this economy to digital technology shocks. Here we find that even though the specification of technology and the nature of technical progress, which is not labour augmenting, differs fundamentally from the standard model, adverse wage effects are mitigated by free entry. In particular, ICT capital saving technical progress (Moore's law) and innovations in platform design (intangible capital saving technical progress), do not generally reduce the aggregate wage share and generally increase the real consumption wage. However, if we allow for entry barriers into the digital sector, more adverse wage effects emerge. The paper also explores the consequences of digital technical progress on wage dispersion. Under the realistic assumption of a bias towards digital skills in the digital sector, digital technical progress goes along with a declining wage share of workers without digital skills as well as a reduction of their real consumption wage under certain conditions.

In order to get a better idea about the magnitude of macroeconomic effects originating from technology shocks in the digital sector, we calibrate the model to the digital and no-digital sector of the US economy. This allows us to ask the question to what extent the model can replicate important stylised growth facts in the US since the year 2000. First, our results show that digital technology shocks, generally have small negative real output effects in the non-digital sector, which is largely a consequence of falling relative prices of digital goods. This is consistent with the observed near constancy of nominal sector shares. All three types of technical progress in the digital sector generate faster growth of intangible capital, also consistent with the data. It appears that the free entry assumption over-predicts the divergence between growth of intangibles and tangibles, while the extreme entry barrier scenario leads to an under-prediction. This suggests that a more realistic entry scenario would lie somewhere in the middle. The free entry case also seems to over-predict the evolution of the wage share, especially when driven by positive shocks to hardware and platform design. Only with extreme entry restrictions can the model generate a non-increasing wage share. For the model to generate a decline in the wage share, technical progress in the digital sector must be associated with a reduction of overhead labour. However, this is unlikely to be a dominant form of technical progress. This suggests that the fall in the aggregate wage share is unlikely to be a consequence of technical progress in the digital sector, but is most likely related to technology trends in the non-digital sector of the US economy, as for example emphasised by Autor et al. (2017) and Korinek et al. (2017). Interestingly, the increase of the wage share in the digital sector since 2010 is consistent with investment specific technical progress and advances in platform design in the digital sector.

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Annex A: Calibration

Following closely the OECD convention we define the digital sector to be comprised of ISIC industries 26 (manufacture of computer, electronic and optical products), 58 (publishing activities), 60 (programming and broadcasting), 62 (computer programming, consultancy and related activities) and 63 (information service activities). We use data from the US Bureau of Economic Analysis to calibrate the preference and technology parameters in the model. We select the preference parameters such that the model replicates a nominal sector share of close to 7%. Given the rough constancy of the digital share since 2000, we assume an elasticity of substitution equal to one between physical and digital goods for most of the analysis. The calibration of technology in the physical sector is standard. We assume a Cobb Douglas production function with output elasticity of labour and capital equal to 0.6 and 0.4 respectively. We do not distinguish between tangible and intangible capital and we do not distinguish between ICT and non ICT tangible capital (software and R&D). For the digital sector the model is supposed to replicate a share of intangible capital of about 60%, while we allow that machinery and equipment consist nearly entirely of IT equipment. Also consistent with the observation of a somewhat smaller wage share in the digital sector, we determine sector specific employment rates. We assume that the digital sector employs a higher share of workers with specialised digital skills. In particular we assume a share of 25% for the physical production sector and 65% for the digital sector (platforms) and 100% for platform design. This corresponds to the share of intangible capital in both sectors. We further assume that both sectors (plus digital sub sectors) use workers with and without digital skills in fixed proportions. Assuming digital skill bias in the digital sector is consistent with the observation of higher wages peer employee (ca. 50%). Under this assumption, ca. 27% of all workers have digital skills. This is in the middle between PIAAC estimate according to which 50% of all employees have basic skills and a recent OECD estimate according to which 4% of all employees are IT specialists.

The remaining model parameters are standard. We assume a rate of time preference of 1% per year and a labour supply elasticity of 0.4 for both workers with and without digital skills. We set the employment rate to 60%.

Annex B: Production of physical goods in steady state

Technology (physical goods)

$$Y_t^O = u_t^O {L_t^O}^{\alpha} K_t^{1-\alpha}$$

Where u_t^O is an efficiency parameter and L_t^O is labour input in the physical production sector. With perfect competition, the maximisation problem of the firm yields

$$Max\ (P_t^O Y_t^O - W_t^O L_t^O - r_t^K P_t^O K_t)$$

the first order conditions for labour and capital

$$\alpha \frac{Y_t^O}{L_t^O} = \frac{W_t^O}{P_t^O}$$

and

$$(1-\alpha)\frac{Y_t^O}{K_t^O} = r_t^K = (1+i_t)\frac{P_t^O}{P_{t+1}^O} - (1-\delta)) = \rho + \delta$$

using the fact that $P_t^O = 1$ and $i_t = \rho$ in the steady state.

Using the first order condition for capital, output can be represented as

$$Y^O = L^O u^O (\rho+\delta)^{\frac{\alpha-1}{\alpha}} = L^O u^O \chi_{Y^O}$$

And we obain the following expressions for physical capital and physical investment

$$K^O = \frac{(1-\alpha)}{\rho+\delta} Y^O = (1-\alpha) L^O u^O (\rho+\delta)^{\frac{-1}{\alpha}} = L^O u^O \chi_{K^O}$$

$$I^O = \delta \frac{(1-\alpha)}{\rho+\delta} Y^O = L^O u^O \chi_{I^O}$$

Using the goods market equilibrium condition we can express physical consumption as follows

$$C^O = Y^O - I^O - I^x = (1 - \delta \frac{(1-\alpha)}{\rho+\delta}) Y^O - I^x = L^O u^O \left(1 - \delta \frac{(1-\alpha)}{\rho+\delta}\right)(\rho+\delta)^{\frac{\alpha-1}{\alpha}} = L^O u^O \chi_{C^O} - I^x$$

And in the steady state, I^x is given by

$$I^x = \delta A K_j^x$$

From his it also follows that the ratio $\frac{I^O}{C^O} = \frac{\chi_{I^O}}{\chi_{C^O}}$ is a constant which does not depend on technology shocks

Annex C: Welfare Analysis: Comparing the market solution with the social planer solution

Since the production technology in the digital sector differs fundamentally from standard technological assumptions and perfect competition does not hold it is interesting to investigate which types of inefficiencies are associated with the market solution. Therefore it is useful to also present the Social Planner Solution. Here we look at the case without entry barriers, and no physical capital.

The social planner problem can be formulated in the following way

$$Max\,\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \log\left(\frac{C_t^{O^{\gamma}}\left(\int_{j=0}^{A}(\phi_1 x_{jt} - \frac{\phi_2}{2}x_{jt}^2)dj\right)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right) - \sum_{t=0}^{\infty} \beta^t\,\lambda_t(C_t^O - u_t^O L_t^O) - \sum_{t=0}^{\infty} \beta^t\,\theta_t(\Delta A_t - u_t^A L_t^A) - \sum_{t=0}^{\infty} \beta^t\,\phi_t\left(L_t - L_t^O - \int_0^A L_t^u dj - L_t^A\right) \quad \text{(C1)}$$

The social planner maximises total utility of consumption, taking into account the technology constraint for physical goods and the technology for producing new platforms and the resource constraint on labour. The social planer optimises w. r. t. C_t^O, A_t, L_t^O and L_t^A We consider the problem where the social planer takes the symmetry of the solution into account.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \log\left(\frac{C_t^{O^{\gamma}}\left(A(\phi_1 x_{jt} - \frac{\phi_2}{2}x_{jt}^2)\right)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right) - \sum_{t=0}^{\infty} \beta^t\,\lambda_t(C_t^O - u_t^O L_t^O) - \sum_{t=0}^{\infty} \beta^t\,\theta_t(\Delta A_t - u_t^A L_t^A) - \sum_{t=0}^{\infty} \beta^t\,\phi_t(L_t - L_t^O - A_t L_t^u - L_t^A) \quad \text{(C2)}$$

The resulting F.O.C. are

$$\frac{\partial \mathcal{L}}{\partial C_t^O} = \frac{\gamma}{C_t^O} = \lambda_t \quad \text{(C4)}$$

$$\frac{\partial \mathcal{L}}{\partial x_{it}} = \frac{1-\gamma}{C_t^D}(\phi_1 - \phi_2 x_{it}) = 0 \quad \text{(C5)}$$

$$\frac{\partial \mathcal{L}}{\partial A} = \frac{(1-\gamma)}{A} - \beta\theta_{t+1} + \theta_t + \phi_t L_t^x = 0 \quad \text{(C6)}$$

$$\frac{\partial \mathcal{L}}{\partial L^O} = \lambda_t u_t^O + \phi_t = 0 \quad \text{(C7)}$$

$$\frac{\partial \mathcal{L}}{\partial L^A} = \theta_t u_t^A + \phi_t = 0 \quad \text{(C8)}$$

Rearranging the FOCs yields the optimal number of platforms of the social planer

$$\left(\frac{u^O}{u^A}\rho + u^O L^x\right) = \frac{1-\gamma}{\gamma}\frac{u^O(L-AL^x)}{A} \quad \text{(C10)}$$

This condition is identical to the market solution under free entry. The equivalence to the market solution depends crucially on the assumption of perfect competition in the physical production sector and the platform design sector. Nevertheless the social planning solution differs from the market solution, because of the static optimality condition determining the level of output per platform. As can be seen from the FOCs, the planer sets the level of digital variant j to

$$x_j^S = \frac{\phi_1}{\phi_2} \quad \text{(C11)}$$

Notice, this level is equal to the level of x_j yielding maximum utility from variant j. It is twice the level set by the monopolist under the market solution (see eq. (27)). This level of x_j is not optimal for the platform monopolist since any increase of x_j bejond x_j^M is associated with a loss of revenue. It can be seen from the demand schedule for digital variants (eq. 6) that under the market solution this level of output of variant j could only be sold at a zero market price.

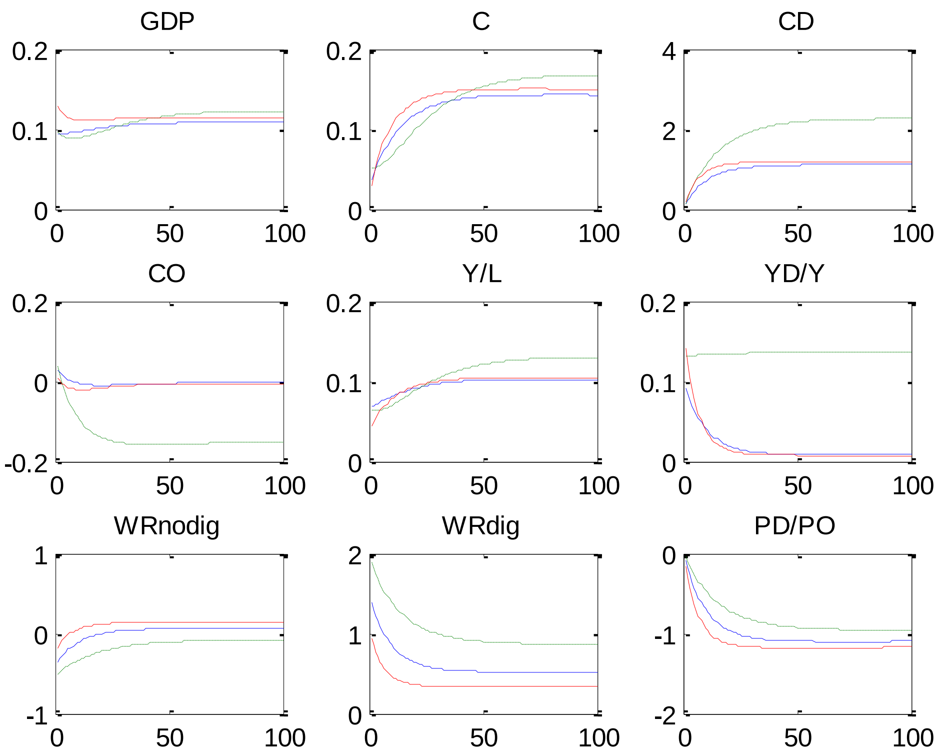
Annex D: Digital technical progress and aggregate growth

Despite a relatively rapid speed of IT innovations, aggregate growth has not increased in recent decades. In this section we explore demand and supply side conditions, which can potentially explain why aggregate growth has been small. The demand side restriction on aggregate growth is associated with the observation that the nominal share of the digital sector has remained roughly constant. This suggests that the elasticity of substitution between physical and digital goods is relatively small (around one). This prevents a strong expansion of the digital sector, since new firms can only enter by offering substantially lower prices. Restricted entry in turn puts a break on the reallocation of resources from low productivity growth physical sector to the high productivity growth digital sector.

An important supply side factor which could slow down the expansion of the digital sector is a low skill specific labour supply elasticity. With a higher labour supply elasticity the increased demand for workers with digital skills, associated with technology induced entry of new firms into the digital sector would be possible with a smaller increase in the digital skill premium and would allow more new platforms to operate.

In order to illustrate the quantitative importance of loosening these two constraints, we a compare both a high elasticity of substitution scenario (HEoS) and a high labour supply elasticity scenario (HLSE) scenario to the baseline scenario with free entry and sectoral skill bias. In the HEoS scenario we assume that the EoS between digital and physical goods is 2.5 (instead of 1) and in the HLSE scenario we assume a labour supply elasticity of 1 (instead of 0.25) .

Figure D1: Growth Effects IT Specific Technical Progress



__: Baseline; ---: HEoS; ____: HLSE

Figure D1 shows that the growth and productivity effects of a typical digital technology shock depends on both demand/preferences and supply conditions and varies positively with a higher EoS and a higher labour supply elasticity. In both cases, the stronger GDP growth effect is associated with an expansion of the digital sector (relative to baseline expansion) and a stronger decline of the physical production sector (relative to baseline reduction). It is interesting to observe that under the HEoS scenario, the expansion of GDP goes along with higher wage dispersion, implying even a decline in the real consumption wage of workers without digital skills, while under the HLSE scenario, the digital skill premium falls relative to the baseline scenario. In the HEoS case the fall of the real consumption wage is largely due to the strong crowding out of the traditional sector, i. e. the sector which uses workers without digital skills intensively. The wage compression under a higher labour supply elasticity is standard.