Understanding Vacancy Yields: Evidence from German Data^{*}

Carlos Carrillo-Tudela[†] Hermann Gartner[‡] Leo Kaas[§]

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Abstract

Firms' search intensity is important for labor market matching as well as for aggregate labor market dynamics. Yet there is only limited research on the recruitment behavior at the microeconomic level. Using the Job Vacancy Survey of Germany's Institute for Employment Research (IAB), linked with the administrative employment histories, we explore the relationships between hiring, job-filling rates and recruitment policy. We find that faster hiring goes along with higher recruiting intensity, lower hiring standards and higher wages. We develop a quantitative competitive search model with heterogeneous firms and match-specific productivity in which firms decide about recruiting intensity, hiring standards and wage offers. A calibrated version of the model allows to assess the relative importance of these recruitment dimensions for the variation of job-filling rates across firms, as well as for the variation of matching efficiency across local labor markets.

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[†]University of Essex, cocarr@essex.ac.uk

[‡]IAB Nuremberg, hermann.gartner@iab.de

[§]Goethe University Frankfurt, kaas@wiwi.uni-frankfurt.de

1 Introduction

Recent evidence documents substantial and systematic variation in job-filling rates across firms. This is hard to reconcile with a standard aggregate matching function which predicts that the job-filling rate is a function of the vacancy-unemployment ratio in the relevant labour market but is otherwise unrelated to the characteristics of the firm. Variation in jobfilling rates is particularly large with respect to the firm's growth rate. Indeed, the hires rate of a firm increases more than one-to-one with its growth rate, whenever the latter is positive, and this increase is predominately accounted for by an increase of the job-filling rate (see Davis et al. (2013). Such variation also seems to matter for aggregate outcomes: changes of firms' "recruiting intensity" may account for much of the shift of the Beveridge curve in the aftermath of the Great Recession (see e.g. Gavazza et al. (2017)). More generally, while the job-filling rate is counter-cyclical (it is easier to hire in bad times), its volatility is smaller than predicted by a simple matching function which dictates how the job-filling rate depends on the aggregate vacancy-unemployment ratio. This suggests that "recruiting intensity" is pro-cyclical, which dampens the increase of the job-filling rate in recessions.

Different mechanisms can possibly explain these observations. Faster-growing firms may invest more in recruitment or screening intensity and hence fill jobs more quickly, they may pay higher wages (or offer more attractive non-pecuniary job benefits) to attract more workers (e.g. Kaas and Kircher (2015)), or they may reduce their hiring standards (e.g. Sedlacek (2014)). Other explanations, unrelated to the choices of firms, can be measurement issues due to time aggregation (since the vacancy stock is observed infrequently, some hiring may occur without a reported vacancy) or composition effects (for instance, firms that grow faster may be those firms that create jobs with lower skill requirements that are easier to fill). Without detailed information about the recruitment process or about specific characteristics of the hired workers, it is difficult to assess which of these channels are responsible for the observed variation in job-filling rates across firms.

In the absence of appropriate data on the recruiting process of firms, it is difficult to discriminate between such different explanations. We also know little about the implications of variation in recruiting intensity for the hired workers: (i) Do firms which recruit more intensively hire more workers from other employers, do these workers have more experience, or do they earn more? (ii) Do workers hired by firms that recruit more intensively experience different careers in these firms? Our paper aims to address these questions. We explore data from the Job Vacancy Survey (JVS) of the Institute for Employment Research (IAB) in Germany which we can link to the administrative labor-market histories of the workers in the surveyed firms. The JVS contains information on the stock of vacancies at the day of interview, which is further broken down into three skill levels. From the administrative data, we can measure the flow of hires in the period after the interview and hence calculate hiring rates and vacancy yields (hires per vacancy) in a similar fashion as Davis et al. (2013) do using JOLTS data for the U.S.

Using these data, we can analyze to what extent such variation reflects wages, hiring standards, or recruitment effort. First, we examine whether the observed characteristics of the new hires (such as employment status, skills or age) vary systematically with the firms' hiring rate. This allows us to see if the variation in job-filling rates across firms is partly accounted for by composition effects. Second, the JVS contains detailed information about the recruitment behaviour for the last case of a hire which we can also link to the hiring rate of the establishment as well as to other establishment characteristics, such as industry, size, or the local labour market conditions, as well as characteristics of the specific job (e.g. skill or experience requirements). We relate various dimensions of recruitment behaviour, such as search channels, recruitment costs, screening intensity (for instance, job interviews per suitable applicant), as well as information about hiring standards and wage policies to the hiring rate of the establishment. The results of this analysis show to what extent firms employ specific recruitment policies in order to hire faster. Finally (*still to be done*), we can relate the wages of new hires, after controlling for observed worker characteristics and worker-fixed effects, to the wages of existing workers in the same establishment, in order to see whether a high job-filling rate goes hand in hand with higher pay.

Our empirical findings give us quantitative estimates on the importance of wages, hiring standards, and other dimensions of recruitment of screening intensity for the variation of jobfilling rates across firms. These contribute to overall matching efficiency in different ways, and they may have different implications for matching outcomes and for workers' labour market mobility. To understand the labour market implications of recruitment behaviour, we develop a quantitative model of labour market matching in which firms can use recruiting intensity, screening intensity, wages and hiring standards. We show that, indeed, more productive firms hire more workers and they achieve higher job-filling rates by making use of all three margins.

We calibrate this model to the German labor market in order to explore to what extent the different recruitment dimensions matter for the variation of job-filling rates across firms and how this variation translates into differences in matching efficiency across local labor markets. We segment the labor market in twelve regions (German federal states) and three skill levels, so that we analyze variation across 36 such labor markets. **Describe results** ... **Literature**

Recruitment and vacancies (Empirics) Barron and Bishop (1985), Van Ours and Rid-

der (1992), Burdett and Cunningham (1998), (Davis et al., 2012, 2013), Faberman and Menzio (2017), Modestino et al. (2015), Kettemann et al. (2017)

- Recruitment and macroeconomics Acharya and Wee (2016), Gavazza et al. (2017), Leduc and Liu (2017), Sedlacek (2014), Wolthoff (2017)
- **Directed search** Moen (1997), Garibaldi and Moen (2010), Hawkins (2013), Kaas and Kircher (2015)
- Worker search intensity Shimer (2004), Krueger and Mueller (2010), Mukoyama et al. (2018), Faberman and Kudlyak (2017), Faberman et al. (2017)

2 Empirical Findings

Data

We use the Job Vacancy Survey (JVS) of the Institute for Employment Research (IAB) which is a representative survey of establishments in Germany (for a detailed description, see Kettner et al. (2011)). The main purpose of the survey is to measure the number of vacancies at these establishments, over and above those that are officially reported at the Federal Employment Agency, and to obtain information about the hiring behavior of these establishments. While the survey is conducted annually since 1989, establishment IDs can be obtained and linked to administrative records only from the year 2010 onward. For this reason we work with the pooled sample for the years 2010–2014, for which we observe around 13,000-15,000 establishments per year.¹

The survey is conducted in the last quarter of each year and consists of two parts. The first part contains general information about the establishment, including employment, location, industry, and changes of employment (hires and separations) during the last year. Importantly, this part of the survey also contains the stock of vacancies (defined as "open positions to be filled immediately or to the next possible date"), broken down by three levels of education requirements (no formal education, vocational training, and university degree). The specific date of interview varies across establishments, Figure 1 shows a generic day in the last quarter on which these vacancy stocks are reported.

The second part of the survey contains detailed information about the recruitment behavior of the surveyed establishment. For that purpose, information about the last case of a successful hire within the last 12 months is collected. Not all surveyed establishments hired

¹Every year a new sample is drawn, so that we cannot follow establishments over time.



Figure 1: Data sources

in the last 12 months (or did not fill this part of the survey for other reasons). Thus we have information about the recruitment behavior for a subsample of around 9,000-10,000 establishments per year. Survey questions include the use of search channels, the number of applications and job interviews, search costs, search duration (as implied by the difference in days between the start of search and the decision for a particular candidate), information about the hired person (age, education, previous employment status, successful search channel, monthly starting wage), and a few general questions about the job (permanent/temporary, replacement hire etc) and about the hiring process. By construction, this search event is finished before the date of interview, sometimes with a considerable time lag. Figure 1 shows a generic day of a last hire for the surveyed establishment.

We can link the survey to the administrative records of individual employment spells which are collected by the Federal Employment Agency (*Integrated Employment Biographies*, IEB). Thus we can observe, for any particular day, all employed workers (with information about education, age, gender, nationality and daily earnings), and thus infer hires and separations during any arbitrary time interval.² To be as comparable as possible to the monthly JOLTS survey in the U.S., we measure hiring rates and employment growth rates over intervals of 30 days. For any surveyed establishment we do this for two such intervals (see Figure 1). First, we measure hires (and separations) during the 30 days after the day of interview. This allows us to calculate a measure of the *vacancy yield* (hires divided by the stock of vacancies at the beginning of the period), hence analogously to the monthly vacancy yield that Davis et al. (2013) calculate using JOLTS data. Second, to relate the hiring rate of an establishment to the actual recruitment behavior, we calculate hiring rates over a 30-day interval around the surveyed case of a last hire (see Figure 1).

 $^{^{2}}$ We exclude employer returns from hires and separations; that is, all workers whose employment spell is interrupted for a period less than three months, we do not count as hires or separations.

Variation of Vacancy Yields

We measure the employment growth rate over 30-days interval using average size at the beginning and at the end of the interval in the denominator (cf. Davis et al. (1998)). We partition these monthly employment growth rates into 15 bins around zero.³ We remove those (typically small) establishments which grow or shrink by more than 30 percent during the interval.

The left panel of Figure 2 shows the variation of hiring rates across employment growth bins, where the hiring rate is defined as hires in interval [t0, t1] divided by average employment. Formally, the hiring rate of an establishment is $\frac{H_{t0,t1}}{0.5(E_{t0}+E_{t1})}$ where t0 is the day of interview and t1 is 30 days after that. Each point on the blue curve shows a weighted average for establishments in a particular growth bin. This graph exhibits a very similar pattern as related graphs for the U.S. based on JOLTS data (e.g. Davis et al. (2013)): the hiring rate is rather flat for shrinking establishments which still hire for replacement purposes (hence hiring rates are positive and small on average for these establishments).



Figure 2: Hiring rate and vacancy rate by monthly establishment growth, with and without controls for industry and size.

The right panel of Figure 2 shows the variation of vacancy rates, defined as vacancies reported at the interview date V_{t0} divided by average employment at t0 and t1, again as a weighted average for each growth bin. In terms of magnitudes, these numbers are similar to those in Davis et al. (2013). In particular, vacancy rates increase from around two percent for stable

³These are: [-0.3, -0.2), [-0.2, -0.15), [-0.15, -0.1), [-0.1, -0.07), [-0.07, -0.04), [-0.04, 0.02), [-0.02, 0), zero, (0, 0.02], (0.02, 0.04], (0.04, 0.07], (0.07, 0.1], (0.1, 0.15], (0.15, 0.2], (0.2, 0.3].

establishments to over five percent for establishments that grow by more than 20 percent. In our data, however, there is more variation of vacancy rates in the negative growth range, whereas vacancy rates appear to be rather flat for shrinking establishments in JOLTS data. Next to the bin averages represented by the blue curves in each graph, we also show the regression coefficients on bin dummies where we also include controls for industry and establishment size. We do this in order to illustrate that these patterns are not merely induced by a changing industry or establishment size compositions across bins of establishment growth. In Figure 3 we show the variation of vacancy yields across employment growth bins where we define the vacancy yield for every growth bin as the ratio between the hiring rate and the vacancy rate in that bin which is equivalent to dividing total hires of all establishments in a particular growth bin by the total vacancies of these establishments.⁴ As in Davis et al. (2013) there is considerable variation of vacancy yields across growth bins. While vacancy yields are flat in the negative growth range, they increase steeply in the positive range (from values below one to over four). In other words, the variation of hiring rates (which is the product of the vacancy rate and the vacancy yield) is predominately accounted for by variation of vacancy yields rather than variation of vacancy rates: Firms that hire more do so by filling vacancies faster.

Do employers hire faster by recruiting more from those groups of workers whose labor market prospects are weaker? Figure 4 show to what extent the composition of hires changes when the establishment's employment growth rate varies from zero to 30 percent. The upper two graphs show that faster growing establishments hire more from non-employment (rather than hiring a worker who had an employment record at another employer prior to starting at the hiring establishment) and they hire relatively more females. There is no evidence, however, that firms hire more workers without German citizenship or above 50 years of age, two groups which are also considered to be disadvantaged in the labor market. In the next section, where we explore the recruitment behavior more deeply, we shed more light on whether employers with larger hiring rates indeed become less selective when deciding for a candidate to fill a vacant position.

Recruitment Behavior

We now turn to analyzing the hiring behavior of establishments based on the survey questions about the last case of a hire. In particular, we are interested in the relationship between the establishment's hiring rate and various recruitment policies. The previous section showed that higher hiring rates are mostly accounted for by a larger vacancy-fill rates (rather than

⁴Similar to Davis et al. (2013), we cannot calculate vacancy yields directly at the establishment level since a large fraction of establishments report zero vacancies.

Figure 3: Vacancy yield by monthly establishment growth, with and without controls for industry and size.

a larger vacancy rates) which is consistent with the U.S. evidence. The following analysis sheds light on the question how these establishments achieve faster hiring.

We measure the establishment's hiring rate based on a 30-day period around the case of a last hire. The underlying assumption is that the reported search behavior is sufficiently representative for the recruitment policy of this establishment in the period under consideration. We then regress various recruitment variables on six bin dummies for the establishment's hiring rate, ranging from zero to 30 percent to see whether faster hiring goes along with specific recruitment policies. In all regressions we control for year, establishment characteristics (industry and five size categories), job characteristics (1-digit occupation, three levels of skill requirements, dummies for experience and leadership requirements, and a dummy for a newly created job), and for local labor market tightness. In our benchmark results, we remove all observations with zero hires in the interval under consideration. This is for two reasons: First, zero hires should not occur in principle as we consider an interval around the *reported* day of last hire. Second, zero hire observations are predominately very small establishments, and controlling for establishment size categories may not be enough to rule out a spurious relationship. We show in Appendix A that our results do not change much when we include zero hires observations within the lowest hiring rate bin (i.e., hiring rates between zero and two percent). We also show in the Appendix that our results are similar

Figure 4: Hires composition by monthly establishment growth, with and without controls for industry and size.

when we remove the smallest establishments (those with less than 20 employees) from the sample. The presence of small establishments may be a concern because they can never have small and positive growth or hiring rates.⁵

Figure 5 shows the coefficient estimates of the establishment's hiring rate for three measures of recruitment intensity: the number of search channels (which we categorize in five groups: postings, network, public agency, internal, others), and the screening intensity as measured by the number of job interviews divided by the number of suitable applicants.⁶ Relative to the reference category, which are hiring rates between zero and two percent, faster hiring goes along with a broader use of search channels, with more international recruiting, and

⁵The lowest growth rate (hiring rate) of an establishment with 20 workers is 4.9%.

⁶For all 0-1 outcome variables we use logistic regressions, and for discrete outcome variables we use negative binomial regression. Otherwise we use OLS.

with a larger screening intensity. Not all estimated coefficients are statistically significant, however, as shown by the 90 percent confidence bands.

Figure 5: Relationship between the hiring rate (relative to hiring rates below two percent) and measures of recruitment intensity.

Figure 6 shows the coefficient estimates of the establishment's hiring rate for four further indicators of recruitment policy. The first three of them relate to the "hiring standards" of the firm: Does the firm eventually hire a worker whose qualification or experience is below the one usually expected for this position? Panels (a) and (b) suggest this is more likely to be the case when the establishment hires faster. Panel (c) further shows that these fast-hiring establishments tend to be more willing to consider long-term unemployed applicants. The last panel (d) indicates that faster-hiring establishments also use higher wages to attract workers to their vacant job. The outcome variable here is the answer to a survey question whether it was required to pay more than originally intended for this position.

Figure 6: Relationship between the hiring rate (relative to hiring rates below two percent) and measures of hiring standards (a-c) and wage policy (d).

Our results show that faster-hiring establishments seem to make use of various recruitment tools: costly recruitment intensity (such as broadness of search and screening), reduced hiring standards, and wages. In the next section we build an equilibrium matching model of the labor market in which heterogeneous firms have these three recruitment tools at hand. We aim to calibrate this model in order to quantitatively asses to what extent these three channels matter for the cross-sectional variation in job-filling rates across firms, and how these channels contribute to overall matching efficiency. The model will also show to what extent recruitment policy responds to labor market conditions. Figure 7 shows that this is indeed the case empirically, at least for some of our measures: In tighter labor markets (as measured by the district-level vacancy-unemployment ratio), firms use more search channels, they are more likely to accept a candidate with inadequate qualification, and they are more willing to pay more than intended. They are less likely to consider a long-term unemployed worker, however, which may be due to the fact that the supply of long-term unemployed workers is much smaller in tighter local labor markets. Given that recruitment policy reacts to market conditions, our model will help us to quantify how much the recruitment channels amplify (or mitigate) exogenous variation in labor market conditions.

Figure 7: Impact of local labor market tightness on recruitment policy.

3 The Model

We develop a simple search-and-matching model where firms have different margins to fill vacancies. These are (i) wages, (ii) recruiting intensity (spending on advertising, screening, etc), (iii) hiring standards. The model should be able to quantify the contributions of these different margins to the between-firm variation in job-filling rates. It should also be helpful to understand the variation in job-filling rates across local labor markets.

Environment

Time is continuous and the interest rate r is constant and exogenous. There is a unit mass of risk-neutral firms which exit the economy at rate δ . To keep the stock of firms constant, there is an inflow of new firms of mass δ per unit time. A firm is a collection of multiple projects, each employing multiple workers. Labor productivity in a generic project is denoted p which is constant over time. A new firm operates projects at initial project productivity p drawn from distribution $\Pi(p)$ with support P. Over time, with flow probability χ , the firm draws productivity p for *new projects* from the same distribution Π . Productivities of all *old projects* in the firm do not change.

A measure L of risk-neutral, infinitely-lived workers search for jobs. Unemployment income equals b. Upon meeting, the worker-firm pair observe match-specific productivity $x \sim G(.)$ with support X. If they decide to form an employment relationship, the flow output of the match is $p \cdot x$ where p is the productivity of new projects in the firm. Workers cannot be shifted from one project to another within the same firm. Every employed worker leaves the firm into unemployment at exogenous rate s. Therefore the total separation rate is exogenous at $s + \delta$.

To operate $V \ge 0$ vacancies involves flow cost $c_V(V)$, where c_V satisfies $c'_V > 0$ and $c''_V > 0$. For any given vacancy, the firm chooses recruiting intensity $R \ge 0$ at cost $c_R(.)$ with $c'_R > 0$, $c''_R > 0$, $c_R(0) = 0$. Thus the total recruiting intensity of the firm ("effective vacancies") are RV which involve costs $c_V(V) + V \cdot c_R(R)$.

Search is competitive as in Moen (1997): Workers decide search for long-term contracts which are posted by firms. Unemployed workers and firms with vacant jobs meet in submarkets. In a given submarket a vacancy with recruiting intensity R is matched with a worker with flow probability $R \cdot m(\lambda)$, where λ are workers per unit of recruiting intensity in the submarket, and m(.) is an increasing and concave reduced-form matching function satisfying m(0) = 0. Consistency implies that a worker in this submarket is matched to a firm with flow probability $m(\lambda)/\lambda$.

Firms post a hiring threshold \bar{x} and (flat) wages⁷ conditional on hiring, denoted w(x). Workers observe these postings and choose in which submarket to search.

This model setup, with linear production technologies in multi-worker firms and convex vacancy costs is similar to Garibaldi and Moen (2010). It dispenses with job-to-job flows, however, while introducing firm productivity shocks, match-specific productivity and recruitment intensity to their framework.

⁷Wage schedules are indeterminate in this model with risk averse workers and firms. This concerns both the variation over time as well as over match productivity x. Limited commitment on either side of the market restricts the set of feasible wage schedules.

Value Functions

The profit value of a job in a firm with project productivity p filled with a worker with match-specific productivity x and wage w(x) satisfies

$$J(p, x, w(x)) = \frac{px - w(x)}{r + s + \delta}$$

A firm with project productivity p decides at any point in time about the vacancy stock V, recruiting intensity per vacancy R, and contract posting $(\bar{x}, w(.))$ for which it expects a meeting rate $m(\lambda)$ per unit of recruiting intensity. The firm's objective is to maximize the flow value

$$RVm(\lambda) \int_{\bar{x}} J(p, x, w(x)) \ dG(x) - C_V(V) - Vc_R(R)$$

Let W(w(x)) be the income value of an employed worker earning wage w(x), and let U be the income value of an unemployed worker. The worker surplus is

$$W(w(x)) - U = \frac{w(x) - rU}{r + s + \delta} ,$$

and the value of an unemployed worker searching in a submarket with posting $(\bar{x}, w(.))$ and meeting rate $m(\lambda)/\lambda$ satisfies $rU = b + \bar{\rho}(\bar{x}, w, \lambda)$ where the flow search value is

$$\bar{\rho}(\bar{x}, w, \lambda) \equiv \frac{m(\lambda)}{\lambda} \int_{\bar{x}} W(w(x)) - U \ dG(x) \ .$$

Equilibrium Definition

A stationary competitive search equilibrium specifies vacancies V_p , recruiting intensity per vacancy R_p , job postings $(\bar{x}_p, w_p(x)) \in Z \equiv X \times \mathbb{R}^X_+$ for all firms with project productivity $p \in P$, queue lengths in submarkets for different postings, defined by $\Lambda : Z \to \mathbb{R}_+$, s search value for unemployed workers ρ , unemployment rate u such that

1. All firms maximize expected profits: For all project productivities $p \in P$, vacancies V_p , recruiting intensity R_p and job postings (\bar{x}_p, w_p) solve

$$\max RVm(\lambda) \int_{\bar{x}} \frac{px - w(x)}{r + s + \delta} \ dG(x) - c_V(V) - Vc_R(R) \tag{1}$$

subject to $\lambda = \Lambda(\bar{x}, w)$.

2. Workers search optimally: For all postings $(\bar{x}, w) \in Z$ and $\lambda = \Lambda(\bar{x}, w)$,

$$\bar{\rho}(\bar{x}, w, \lambda) \le \rho , \qquad (2)$$

with equality if $\lambda > 0$. Furthermore,

$$\int V_p R_p \lambda_p \ d\Pi(p) \le u\bar{L} \ , \tag{3}$$

with equality if $\rho > 0$.

3. Stationary unemployment rate:

$$(1-u)\overline{L}(s+\delta) = \int (1-G(\overline{x}_p))m(\lambda_p)R_pV_p \ d\Pi(p) \ . \tag{4}$$

Optimal search requires that workers earn the same expected income in all submarkets which they visit ($\lambda > 0$) which is entailed in (2). It also means that all unemployed workers decide to search in some submarket if they can obtain positive surplus, $\rho > 0$; otherwise unemployed workers are indifferent between search and inactivity, which is specified in the complementary-slackness condition (3). Condition (4) says that unemployment inflows (= separations) are equal to outflows (= hires). Note again that there is a unit mass of firms and that all firms' project productivities are drawn from $\Pi(.)$. Hence, the right-hand side of (4) aggregates hires over all firms.

Conditions (3) and (4) can be combined to

$$\int \frac{H_p}{s+\delta} + V_p R_p \lambda_p \ d\Pi(p) \le \bar{L},\tag{5}$$

where $H_p = R_p m(\lambda_p)(1-G(\bar{x}_p))V_p$ is the hires flow of firms with project productivity p. Thus, $H_p/(s + \delta)$ is aggregate employment in all projects of type p, and $V_p R_p \lambda_p$ are unemployed workers searching for jobs at firms with project productivity p. Hence equation (5) says that employed workers and unemployed searchers together cannot exceed the measure of workers \bar{L} , and they are equal to \bar{L} if all unemployed search.⁸

Because of $c_R(0) = 0 = m(0)$, firms will either choose $R = \lambda = 0$, or a positive value of $\lambda = \Lambda(\bar{x}, w)$ where

$$\frac{m(\lambda)}{\lambda} \int_{\bar{x}} \frac{w(x) - b - \rho}{r + s + \delta} \ dG(x) = \rho \ . \tag{6}$$

Note that the workers' search value ρ is an equilibrium object that depends, via equation

⁸It is straightforward to verify that the competitive equilibrium is constrained efficient.

(5), on labor demand (i.e. on the aggregate number of vacancies, recruiting intensity and selectivity) as well as on labor supply \bar{L} . A change of aggregate productivity, say a shift of p for all projects, takes an impact on all values of V_p , R_p , \bar{x}_p and λ_p , and therefore changes ρ as well.

The job-filling rate ("vacancy yield") for a firm with project productivity p is

$$y_p = H_p / V_p \equiv R_p \cdot m(\lambda_p) \cdot (1 - G(\bar{x}_p)) .$$
⁽⁷⁾

Variation in job-filling rates are accounted for by three factors: recruiting intensity R, wages as reflected through λ , and hiring standards as reflected through \bar{x} . In the cross-section, they all vary by a firm's current project productivity p.

Hiring rates and growth rates also vary by p, although this relationship is more subtle. A firm's employment stock N adjusts according to

$$N = H_p - sN \; .$$

Hence, the firm's growth rate $g = \dot{N}/N$ varies one-for-one with the firm's hiring rate $h = H_p/N$ according to g = h - s. Hiring rates in turn depend both on project productivity p and on the current employment stock N which varies across firms and depends on firm age as well as on the history of project productivities. Write $\Psi(N|p)$ for the cumulative distribution of firms by employment size, conditional on project productivity p. Hence $\Psi(N|p)d\Pi(p)$ is the mass of firms with current project productivity and employment less than or equal to N. The inflow into this group per unit time equals

$$\left\{\delta + \chi \int \Psi(N|p') d\Pi(p') + \Psi'(N|p) \max(0, sN - H_p) \right\} d\Pi(p) .$$

The three terms are the inflow of new firms (with initially zero employment), firms that change project productivity from p' to p, and a possible inflow of larger firms in those cases where separations sN exceed hires H_p . The outflow of the group $\Psi(N|p)d\Pi(p)$ is equal to

$$\Big\{\Psi'(N|p)\max(H_p-sN,0)+(\delta+\chi)\Psi(N|p)\Big\}d\Pi(p)\ .$$

Firms leave this group when hires H_p exceed separations sN at the margin, when firms exit (probability δ) or when they draw a new project productivity (probability χ). Equating inflows and outflows gives

$$\Psi'(N|p)(sN - H_p) = \delta(\Psi(N|p) - 1) + \chi \int \psi(N|p) - \psi(N|p') d\Pi(p') d\Pi(p')$$

This equation determines the (conditional) firm-size distribution in steady state.

Remark: I suggest to use a finite support for p (say $p_1 < \ldots < p_n$ with probabilities π_1, \ldots, π_n) and possibly to shut down entry and exit ($\delta = 0$). Then the support of N will be contained in the interval $(H_1/s, H_n/s)$ (because firm size converges to H_i/s if project productivity stays constant at p_i). The stationarity conditions are

$$\Psi'(N|p_i)(sN - H_{p_i}) = \chi \sum_j \pi_j \left[\psi(N|p_i) - \psi(N|p_j) \right] \,.$$

These are *n* differential equations that can be solved simultaneously subject to the boundary conditions $\Psi(H_1/s|p_i) = 0$ and $\Psi(H_n/s|p_i) = 1$ for all i = 1, ..., n (i.e. H_1/s and H_n/s are the lower and upper bounds for the support of all conditional distribution functions).

Drawing p and N from $\Pi(.)$ and $\Psi(.|p)$ then yields a cross-sectional distribution of hiring rates H_p/N and growth rates $H_p/N - s$ which can be related to vacancy rates V_p/N and vacancy yields $y_p = H_p/V_p$.

Equilibrium Characterization

Substitute (6) into (1) to rewrite the firms' problem:

$$\max_{V,\lambda,\bar{x},R} V \cdot \left\{ Rm(\lambda) \int_{\bar{x}} \frac{px - b - \rho}{r + s + \delta} \, dG(x) - R\lambda\rho - c_R(R) \right\} - c_V(V) \,. \tag{8}$$

The first-order conditions are

$$p\bar{x} = b + \rho , \qquad (9)$$

$$c'_R(R) = m(\lambda) \int_{\bar{x}} \frac{px - b - \rho}{r + s + \delta} \, dG(x) - \lambda\rho \,, \tag{10}$$

$$\rho = m'(\lambda) \int_{\bar{x}} \frac{px - b - \rho}{r + s + \delta} \, dG(x) , \qquad (11)$$

$$c'_V(V) = Rm(\lambda) \int_{\bar{x}} \frac{px - b - \rho}{r + s + \delta} \, dG(x) - R\lambda\rho - c_R(R) \,. \tag{12}$$

Combine (10) and (11):

$$c'_{R}(R) = \rho \frac{m(\lambda) - \lambda m'(\lambda)}{m'(\lambda)} .$$
(13)

This equation implies that across firms λ and R are positively related. Combine (9) and (11):

$$\rho = m'(\lambda) \frac{b+\rho}{r+s+\delta} \int_{\bar{x}} \frac{x}{\bar{x}} - 1 \ dG(x) \ . \tag{14}$$

This equation implies that across firms λ and \bar{x} are negatively related. Substitute (10) into (15) to obtain

$$c'_V(V) = Rc'_R(R) - c_R(R) . (15)$$

This implies that R and V are positively related across firms. Finally, from (9) we know that p and \bar{x} are negatively related.

Therefore, we can conclude that firms with higher project productivity p (i) have more vacancies, (ii) have lower hiring standards, (iii) choose higher recruiting intensity, (iv) set wages so as to attract more workers to every vacancy (conditional on choosing recruiting intensity and selectivity).⁹

Variation in job-filling rates that are induced by variations in firm types p are due to all three factors in (7) which are positively correlated across firms. To see how much each factor contributes, consider the decomposition

$$\frac{dy}{y} = \frac{dR}{R} + \frac{m'(\lambda)\lambda}{m(\lambda)} \cdot \frac{d\lambda}{\lambda} - \frac{G'(\bar{x})\bar{x}}{1 - G(\bar{x})} \cdot \frac{d\bar{x}}{\bar{x}} .$$
(16)

Write $\varepsilon_{f,x}$ to denote the elasticity of function f with respect to variable x. Rewrite ?? as

$$\frac{d\lambda}{\lambda} = \frac{1 - \varepsilon_{\Phi,\bar{x}}}{\varepsilon_{m',\lambda}} \frac{d\bar{x}}{\bar{x}} , \qquad (17)$$

where we define

$$\Phi(\bar{x}) \equiv \int_{\bar{x}} x - \bar{x} \ dG(x) = \int_{\bar{x}} 1 - G(x) \ dx \ .$$

Further, differentiate (13):

$$\frac{dR}{R} = -\frac{\varepsilon_{m',\lambda}}{(1 - \varepsilon_{m,\lambda})\varepsilon_{c'_R,R}} \cdot \frac{d\lambda}{\lambda} .$$
(18)

From (9) we have

$$\frac{d\bar{x}}{\bar{x}} = -\frac{dp}{p} . \tag{19}$$

Now substitute (17), (18) and (19) into (16) to obtain

$$\frac{dy}{y} = \frac{dp}{p} (1 - \varepsilon_{\Phi,\bar{x}}) \left\{ \frac{1}{(1 - \varepsilon_{m,\lambda})\varepsilon_{c'_R,R}} + \frac{\varepsilon_{m,\lambda}}{-\varepsilon_{m',\lambda}} + \frac{G'(\bar{x})\bar{x}}{(1 - G(\bar{x}))(1 - \varepsilon_{\Phi,\bar{x}})} \right\}.$$
 (20)

All three terms in curly braces are positive and represent the respective contributions of

 $^{^{9}}$ This generalizes Kaas and Kircher (2015) who show that vacancies and wages are positively related across firms.

recruiting intensity, wages, and selectivity to variation in job-filling rates. In the next section, we use this decomposition to assess quantitatively how much the three channels contribute to cross-firm variation in job-filling rates.

4 Quantitative Analysis (preliminary)

How does the recruitment behaviour of firms matter for the cross-sectional variation in jobfinding rates and how does it contribute to matching efficiency? To answer these questions, we parameterize this model and calibrate its parameters to match selected statistics in the German labor market. We explore variation in recruitment behavior across different "local" labour markets, which are defined by region and worker skills (job skill requirements). This allows us to shed some light on how recruitment behavior contributes to aggregate matching efficiency. Some parameters are set uniformly for all markets, whereas others are allowed to vary across local labor markets.

Parameterization

We use the following functional forms: Recruitment and vacancy cost functions are $c_R R^{\gamma}$ and $c_V V^{\Phi}$ with elasticity parameters $\gamma > 1$, $\Phi > 1$. The matching function is Cobb-Douglas, $m_0 \lambda^{\mu}$ with $\mu \in (0, 1)$. Match-specific productivity is Pareto distributed, $G(x) = 1 - (x_0/x)^{\alpha}$ for $x \ge x_0$, with $\alpha > 1$, and firm productivity types are distributed with $\Pi(p) = (p/\bar{p})^{\eta}$ for $p \in$ $[0, \bar{p}]$, with $\eta > 0$. We make sure that all firms face a non-trivial selection decision. Formally, the bounds on the productivity distributions must be set such that hiring thresholds are interior for all firms which requires $\bar{p}x_0 < b + \rho$. For these functional forms, the firms' policies \tilde{x}_p , λ_p , V_p and R_p , as well as several cross-sectional model statistics (in particular, the mean and the squared coefficient of variation of various outcome variables) can be all obtained in closed form; see Appendix B for details.

The following parameters are set globally: The annual real interest rate is r = 0.015. Elasticity parameters μ , γ and Φ are also identical across local labor markets. We further assume that the matching function efficiency m_0 and the scale of the recruitment cost function c_R are global parameters. All other parameters vary across local labor markets.

We consider variation across German states where smaller states¹⁰ are merged to larger neighboring states. This gives rise to 12 regions. We further split observations by three skill levels (no formal education, vocational training, and college degree). Thus we consider 36

¹⁰These are the three city-states Berlin, Bremen, Hamburg, as well as Saarland.

labor markets. We believe that these labor markets are sufficiently segmented so that we can safely abstract from mobility across these markets.¹¹

For any of these labor markets, we use data for the period 2010–2014 on employment, unemployment, number of establishments (employing workers of a particular skill), jobfinding rate, mean wage, mean and variance of vacancies (skill-specific), mean and variance of recruitment costs (monetary and non-monetary). Let *i* denote a generic local labor market. From the unemployment rate *i* and the job-finding rate f_i , we obtain the total separation rate from the stock-flow identity $s_i + \delta_i = f_i u_i/(1 - u_i)$, and we attribute one-third of separations to exits, and two-thirds to separations for continuing establishments (cf. Fuchs and Weyh (2010) who find that one-third of destroyed jobs are at exiting establishments).

We measure the job-finding rate as the monthly UE flow divided by the unemployment stock. Consistent with this measure, we also define the vacancy yield as hires from unemployment (UE flow) divided by the vacancy stock. Figure 8 shows the relationship between labor market tightness (vacancies divided by unemployment), job-finding rates and vacancy yields across our 36 labor markets. OLS regressions of the logged variables suggest that the elasticity of the job-finding rate with respect to tightness is 0.38, whereas the elasticity of the vacancy yield with respect to tightness is -0.62. If an aggregate matching function alone would account for cross-market variation, the elasticity parameter (for $m(\lambda) = m_0 \lambda^{\mu}$, the relation between vacancy yields and inverse tighness) should be set to $\mu = 0.62$. Our calibrated value is at $\mu = 0.33$ which suggests that some variation of vacancy yields across labor markets must come from the other recruitment margins.

Write \bar{w}_i for the mean wage in the local labor market (relative to the average employmentweighted wage which we normalize to one). Applying the replacement rate $\beta = 0.46$ (cf.Krebs and Scheffel (2013)), we set unemployment income market-specific to $b_i = \beta \bar{w}_i$. While the matching-function elasticity μ is the same across markets, parameters α_i (Pareto scale parameter) and the variable ρ_i (endogenous search values) vary across markets. We calibrate (μ, α_i, ρ_i) jointly to match (see Appendix B for derivations):

1. Mean wage in market i,

$$\bar{w}_i = (b + \rho_i) \frac{\alpha_i + \mu - 1}{\alpha_i - 1}$$
 (21)

2. Unemployment (equivalently, the job-finding rate) in market i,

$$\frac{\rho_i(r+s_i+\delta)(\alpha_i-1)}{\mu(b_i+\rho_i)} = f_i .$$

$$(22)$$

¹¹In particular, most metropolitan areas are contained in only one of our 12 regions. Moreover, our skill groups are based on education acquired early in life so that workers usually do not move between them. The reported vacancies in the JVS are differentiated according to the same classification.

Figure 8: Market tightness, job-finding rates and vacancy yields (hires from unemployment divided by vacancies) in 36 (region×skill) labor markets in Germany (2010–2014).

3. Aggregate labor share equal to 2/3. When s_i is the employment share of market *i*, this condition says

$$\frac{\sum_{i} s_i (b_i + \rho_i) \frac{\alpha_i + \mu - 1}{\alpha_i - 1}}{\sum_{i} s_i (b_i + \rho_i) \frac{\alpha_i}{\alpha_i - 1}} = \frac{\sum_{i} s_i \bar{w}_i}{\sum_{i} s_i \bar{w}_i \frac{\alpha_i}{\alpha_i + \mu - 1}} = \frac{2}{3} .$$
(23)

These equations can be solved uniquely for the global parameter μ and for the market-specific variables (α_i, ρ_i) . Equilibrium search values $\rho_i > 0$ ensure that all workers in the labor market are either employed in a firm or join the unemployment pool. To explain how these parameters are calibrated, note that the matching-function elasticity μ determines which share of output goes to workers, hence it is pinned down by the labor income share target. Parameters α_i control the means of the Pareto distributions for x, and thus equilibrium wages in all local labor markets. Finally, the search values ρ_i (and therefore labor demand parameters) need to generate the targeted values of the job-finding rates for all labor markets. We assume a uniform productivity distribution ($\eta = 1$). Elasticity parameters of cost functions (γ and Φ) are key for generating variation in recruitment costs and vacancy postings across firms. They are set globally to match the dispersion of vacancies and recruitment costs across establishments. In particular, for each of the 36 local labor markets, we obtain coefficients of variation for reported vacancies and reported recruitment costs, and then calculate the employment-weighted averages of these measures. In Appendix B, we show how the corresponding model statistics depend on γ and Φ (as well as the calibrated values for $\alpha_i, \mu \text{ and } \eta$). Using these equations, $\gamma = 9.89$ and $\Phi = 1.024$ are pinned down uniquely from the calibration targets.

Two more parameters are allowed to vary across labor markets: \bar{p} , the upper bound of the

firms' productivity distribution, and c_V , the scale parameter of the vacancy cost function. We calibrate these two parameters to make sure that aggregate vacancies (per establishment) and the vacancy yield (hires per vacancy) correspond to the data targets. The closed-form expressions imply a unique solution for the parameter values. Intuitively, the scale parameter in vacancy costs controls vacancy postings for all firms, and the productivity level controls the extent to which firms use recruiting intensity and selectivity to boost hiring per vacancy. The remaining parameters m_0 (matching function scale), x_0 (lower bound for match-specific productivity) and c_R (recruitment cost scale) are set globally. These parameters do not matter for all the decomposition results presented in the next subsection, so we can set them to arbitrary values.¹²

4.1 Results

For the variation of vacancy yields across firms (20) in labor market *i* we obtain

Because all parameters in curly brackets are set globally, we obtain the same within-market decomposition of vacancy yields, as shown by the respective percentage shares. The majority of the variation is accounted for by selectivity. This is a result of the rather high calibrated value of γ which in turn follows from a rather modest variation of recruitment costs in the data.

We are further interested in the variation of matching efficiency across local labor markets. This can be studied by decomposing the job-finding rate as follows:

¹²The intuition is that all model aggregates depend on parameters m_0 , x_0 and \bar{p} only through the joint product $m_0 x_0^{\alpha} \bar{p}^{\alpha}$. Hence, only one of these three parameters can be identified from aggregate statistics, while the other two parameters are irrelevant for cross-market decomposition analysis. See Appendix B for details.

where

$$\begin{split} \bar{V} &\equiv \int V_p d \ \Pi(p) \ ,\\ \bar{R} &\equiv \int R_p \frac{V_p}{\bar{V}} \ d\Pi(p) \ ,\\ \bar{m} &\equiv \int m(\lambda_p) \frac{R_p V_p}{\bar{R} \bar{V}} \ d\Pi(p) \ ,\\ \bar{\lambda} &\equiv \frac{U}{\bar{R} \bar{V}} \ . \end{split}$$

 \bar{V} are aggregate vacancies, \bar{R} is a vacancy-weighted aggregate measure of recruiting intensity, and \bar{m} measures aggregate worker-firm meetings, weighted by firm-level recruiting effort RV. $\bar{\lambda}$ is the inverse of aggregate tightness (unemployed workers per aggregate recruiting intensity).

The four terms in the decomposition of the job-finding rate have the following interpretation: The first term is the one that arises in any standard search-and-matching model without search intensity in which the job-finding rate is a function of the vacancy-unemployment ratio. The second term captures the impact of aggregate recruiting intensity. The third term reflects the impact of heterogeneous wage policies across firms. It would be zero if the matching function was linear, but it is non-zero for a concave matching function (cf. Kaas and Kircher (2015)): Dispersion of wages (and the resulting meeting probabilities) reduces the average matching rate. The fourth term measures the impact of selectivity.

The four terms in this decomposition vary across markets due to variation of market-specific parameters and the resulting differences in firms' recruitment policies. The variance of the (logged) job-finding rate is 0.49. Table 1 shows the covariance variance matrix of the four (logged) terms in (24). Most of the variation of the total job-finding rate comes from labor market tightness and from selectivity. Recruiting intensity, as well as the dispersion of wages (meeting rates) across firms in a labor market play only a minor role. Covariances between all these terms are also quite small.

Total variance 0.49	Tightness	Recruiting intensity	Dispersion	Selectivity
Tightness	0.53	-0.05	-0.12	-0.06
Recruiting intensity	-0.05	0.01	0.01	0.01
Dispersion	-0.12	0.01	0.08	-0.09
Selectivity	-0.06	0.01	-0.09	0.47

Table 1: Covariance matrix of the four terms in (24) across local labor markets

This result is also reflected in Table 2 which shows the contribution of these four terms to

the total cross-market variance of the job-finding rate. Across all 36 labor markets, market tightness and hiring selectivity are the two dominant forces in accounting for the variation of job-finding rates. In contrast, dispersion of wages (and to some extent also recruiting intensity) contribute negatively to this variation: Labor markets with a higher job-finding rate tend to have more dispersed wages (and hence a lower average meeting rate), and firms choose a somewhat lower recruiting intensity in these markets.

	Variance JFR	Tightness	Recruiting intensity	Dispersion	Selectivity
Total	0.49	62%	-5%	-24%	67%
Low skill	0.42	14%	1%	-22%	106%
Medium skill	0.33	32%	0%	-1%	68%
High skill	0.34	13%	-3%	-22%	112%

Table 2: Relative contributions to the variation of job-finding rates across local labor markets (all markets and within each skill group).

We further explore to what extent variation across regions or across skill groups is driven by the different recruitment margins. Regarding variation across the 12 *regional* labor markets, the bottom three rows in Table 2 report the percentage contribution of the four channels to the variance of job-finding rates, separate for each of the three skill groups. Evidently, the cross-regional variance of the job-finding rate for each skill group is smaller than the total variance (first column of the table). Moreover, variation of tightness across regional markets plays a much less important role (except for the medium skill group). Instead, almost all variation in job-finding rates across regions comes from the selectivity margin.

Variation across skill groups is reported in Table 3. Medium- and high-skill labor markets have job-finding rates which are around 80 percent larger than those in low-skill labor markets. More than this gap is accounted for by differences in labor market tightness (2nd column). While labor market tightness does not matter much for cross-regional variation of job-finding rates (Table 2), they are the dominant factor in accounting for differences in job-finding rates between skill groups. Relative to medium-skill markets, high-skill labor markets are a bit less tight but they are also somewhat less selective while wages are more dispersed (4th and 5th columns).

	JFR	Tightness	Recruiting intensity	Dispersion	Selectivity
Medium skill	0.76	1.40	-0.10	-0.11	-0.42
High skil	0.82	1.22	-0.13	-0.53	0.26

Table 3: Average log differences to low-skill labor markets.

5 Conclusions

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Appendix

A. Robustness

Including zero hires observations

Figure 9 and Figure 10 show the relationships between the establishment hiring rate and various recruitment policies where we include zero hires observations to the reference category which are hiring rates between zero and two percent.

Figure 9: Relationship between the hiring rate and measures of recruitment intensity. The reference category are hires rates in the interval [0, 2%) (including zeros).

Removing establishments with less than 20 employees

Figure 11 and Figure 12 show the relationships between the establishment hiring rate and various recruitment policies where we exclude establishments with less than 20 employees from the sample.

Figure 10: Relationship between the hiring rate and measures of hiring standards (a-c) and wage policy (d). The reference category are hires rates in the interval [0, 2%) (including zeros).

B. Closed-form model solutions

For the parameterization of the calibration section, the decomposition (20) reads as

$$\frac{dh}{h} = \frac{dp}{p} \alpha \left\{ \frac{1}{(1-\mu)(\gamma-1)} + \frac{\mu}{1-\mu} + 1 \right\} \,.$$

Figure 11: Relationship between the hiring rate (relative to hiring rates below two percent) and measures of recruitment intensity. Establishments with less than 20 workers are excluded.

Further, there are closed-form constant-elasticity expressions for the firms' policy variables:

$$\bar{x}_p = (b+\rho)p^{-1} ,$$

$$\bar{\lambda}_p = \left[\frac{\mu m_0 (b+\rho)^{1-\alpha} x_0^{\alpha} p^{\alpha}}{(r+s+\delta)\rho(\alpha-1)}\right]^{1/(1-\mu)} ,$$

$$R_p = \left[\frac{\rho(1-\mu)}{c_R \gamma \mu}\right]^{1/(\gamma-1)} \cdot \lambda_p^{1/(\gamma-1)} ,$$

$$V_p = \left[\frac{c_R(\gamma-1)}{c_V \Phi}\right]^{1/(\Phi-1)} \cdot R_p^{\gamma/(\Phi-1)}$$

We can obtain closed-form expressions for a number of cross-sectional statistics. We make use of the following result.

Figure 12: Relationship between the hiring rate (relative to hiring rates below two percent) and measures of hiring standards (a-c) and wage policy (d). Establishments with less than 20 workers are excluded.

Lemma: Let $X_p = Ap^{\beta}$, for some parameters A, β , and let p be distributed with $cdf \Pi(p) = (p/\bar{p})^{\eta}$. Then the mean and the squared coefficient of variation of X_p (i.e., $SCV(X_p) \equiv var(X_p)/E(X_p)^2$), are

$$E(X_p) = \frac{A\eta}{\beta + \eta} \bar{p}^{\beta}$$
, $SCV(X_p) = \frac{\beta^2}{(2\beta + \eta)\eta}$.

Using this lemma and the above expressions, we obtain cross-sectional statistics for recruit-

ment costs per vacancy, $c_R R_p^\gamma$ and vacancies V_p :

$$\begin{split} E(c_{R}R_{p}^{\gamma}) &= c_{R} \cdot \left(\frac{\rho(1-\mu)}{\mu c_{R}\gamma}\right)^{\frac{\gamma}{\gamma-1}} \cdot \left(\frac{m_{0}\mu(b+\rho)^{1-\alpha}x_{0}^{\alpha}}{\rho(r+s+\delta)(\alpha-1)}\right)^{\frac{\gamma}{(\gamma-1)(1-\mu)}} \cdot \frac{\eta}{\eta + \frac{\alpha\gamma}{(\gamma-1)(1-\mu)}} \cdot \bar{p}^{\frac{\alpha\gamma}{(\gamma-1)(1-\mu)}} ,\\ SCV(c_{R}R_{p}^{\gamma}) &= \left(\frac{\alpha\gamma}{(\gamma-1)(1-\mu)}\right)^{2} \cdot \frac{1}{\eta(\eta + \frac{2\alpha\gamma}{(\gamma-1)(1-\mu)})} ,\\ E(V_{p}) &= \left(\frac{c_{R}(\gamma-1)}{c_{V}\Phi}\right)^{\frac{1}{\Phi-1}} \cdot \left(\frac{\rho(1-\mu)}{\mu c_{R}\gamma}\right)^{\frac{\gamma}{(\Phi-1)(\gamma-1)}} \cdot \left(\frac{m_{0}\mu(b+\rho)^{1-\alpha}x_{0}^{\alpha}}{\rho(r+s+\delta)(\alpha-1)}\right)^{\frac{\gamma}{(\Phi-1)(\gamma-1)(1-\mu)}} .\\ &= \frac{\eta}{\eta + \frac{\alpha\gamma}{(\Phi-1)(\gamma-1)(1-\mu)}} \cdot \bar{p}^{\frac{\alpha\gamma}{(\Phi-1)(\gamma-1)(1-\mu)}} ,\\ SCV(V_{p}) &= \left(\frac{\alpha\gamma}{(\Phi-1)(\gamma-1)(1-\mu)}\right)^{2} \cdot \frac{1}{\eta(\eta + \frac{2\alpha\gamma}{(\Phi-1)(\gamma-1)(1-\mu)})} . \end{split}$$

Moreover, for hires $H_p = m(\lambda_p)(1 - G(\bar{x}_p))R_pV_p$ we obtain

$$\begin{split} E(H_p) &= m_0 (\frac{x_0}{b+\rho})^{\alpha} \cdot \left(\frac{c_R(\gamma-1)}{c_V \Phi}\right)^{\frac{1}{\Phi-1}} \cdot \left(\frac{\rho(1-\mu)}{\mu c_R \gamma}\right)^{\frac{\Phi+\gamma-1}{(\Phi-1)(\gamma-1)}} \cdot \left(\frac{m_0 \mu (b+\rho)^{1-\alpha} x_0^{\alpha}}{\rho(r+s+\delta)(\alpha-1)}\right)^{\frac{\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}-1} \\ & \frac{\eta}{\eta + \frac{\alpha\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}} \cdot \bar{p}^{\frac{\alpha\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}} \ , \\ SCV(H_p) &= \left(\frac{\alpha\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}\right)^2 \cdot \frac{1}{\eta(\eta + \frac{2\alpha\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)})} \ . \end{split}$$

For vacancy yields, H_p/V_p we get

$$\begin{split} E(H_p/V_p) &= m_0 (\frac{x_0}{b+\rho})^{\alpha} \cdot \left(\frac{\rho(1-\mu)}{\mu c_R \gamma}\right)^{\frac{1}{\gamma-1}} \cdot \left(\frac{m_0 \mu(b+\rho)^{1-\alpha} x_0^{\alpha}}{\rho(r+s+\delta)(\alpha-1)}\right)^{\frac{1-\mu+\mu\gamma}{(\gamma-1)(1-\mu)}} \cdot \\ & \frac{\eta}{\eta+\frac{\alpha\gamma}{(\gamma-1)(1-\mu)}} \cdot \bar{p}^{\frac{\alpha\gamma}{(\gamma-1)(1-\mu)}} \ , \\ SCV(H_p/V_p) &= \left(\frac{\alpha\gamma}{(\gamma-1)(1-\mu)}\right)^2 \cdot \frac{1}{\eta(\eta+\frac{2\alpha\gamma}{(\gamma-1)(1-\mu)})} \ . \end{split}$$

Integrating over $V_p R_p \lambda_p$ over all firms, we further obtain an expression for aggregate unemployment

$$U = \left(\frac{c_R(\gamma-1)}{c_V\Phi}\right)^{\frac{1}{\Phi-1}} \cdot \left(\frac{\rho(1-\mu)}{\mu c_R\gamma}\right)^{\frac{\Phi+\gamma-1}{(\Phi-1)(\gamma-1)}} \cdot \left(\frac{m_0\mu(b+\rho)^{1-\alpha}x_0^{\alpha}}{\rho(r+s+\delta)(\alpha-1)}\right)^{\frac{\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}} \cdot \frac{\eta}{\eta + \frac{\alpha\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}} \cdot \bar{p}^{\frac{\alpha\gamma\Phi}{(\Phi-1)(\gamma-1)(1-\mu)}} \cdot$$

The above expressions make clear that parameters c_R , c_V (cost function scales), m_0 , x_0 and \bar{p} affect the means of several variables in the same way, so that they cannot be separately identified from aggregate statistics. To see why, consider the above expressions for mean

recruitment spending per vacancy and for the vacancy yield. They both depend on

$$c_R^{-\frac{1}{\gamma-1}} \left(m_0 x_0^{\alpha} \bar{p}^{\alpha} \right)^{\frac{\gamma}{(\gamma-1)(1-\mu)}}$$

Likewise, consider aggregate hires, aggregate unemployment, and aggregate spending on recruitment (i.e., the mean of $c_R R_p^{\gamma} V_p$). They all depend on

$$c_R^{-\frac{\Phi}{(\gamma-1)(\Phi-1)}} \left(m_0 x_0^{\alpha} \bar{p}^{\alpha} \right)^{\frac{\gamma \Phi}{(\gamma-1)(1-\mu)(\Phi-1)}} c_V^{-\frac{1}{\Phi-1}} .$$

Lastly, aggregate vacancies depend on

$$c_{R}^{-\frac{1}{(\gamma-1)(\Phi-1)}} \left(m_{0} x_{0}^{\alpha} \bar{p}^{\alpha} \right)^{\frac{\gamma}{(\gamma-1)(1-\mu)(\Phi-1)}} c_{V}^{-\frac{1}{\Phi-1}} .$$

The last three expressions show that the scale parameter for vacancy costs, c_V can be separately identified, for instance from aggregate vacancies. However, the remaining parameters c_R , m_0 , x_0 and \bar{p} all influence the above data targets (recruitment spending per vacancy, aggregate recruitment spending, mean vacancy yield, aggregate unemployment, aggregate hires) with the same log-linear proportions. Hence, only one of these parameters can be identified from the above data targets.

To obtain some intuition, a lower value of x_0 (less productive workers on the job) requires a higher productivity of firms \bar{p} to generate the same number of hires, unemployment, vacancy yield etc. A lower matching efficiency m_0 requires a higher value of $(x_0\bar{p})^{\alpha}$ to substitute a lower meeting rate with a higher selection probability so as to end up with the same number of hires, unemployment, vacancy yield, etc. The reason why c_R cannot be identified (say from recruitment spending) is more subtle: a higher value of c_R reduces recruitment effort R, and thus spending on recruitment, hires, unemployment etc. in the same proportion as a decrease of either m_0 or $(x_0\bar{p})^{\alpha}$ would do.

Because employment in firms of type p is $H_p/(s + \delta)$, aggregate employment is simply $E(H_p)/(s + \delta)$. The job-finding rate is given by aggregate hires per unemployed worker which simplifies to

$$\frac{E(H_p)}{U} = \frac{\rho(r+s+\delta)(\alpha-1)}{\mu(b+\rho)}$$

Regarding wages, the model neither pins down wage-tenure profiles nor the variation of wages across workers within the same firm. Assuming that individual wages are constant over time, they need to satisfy

$$\rho(r+s+\delta) = \frac{m(\lambda_p)}{\lambda_p} \int_{\bar{x}_p} w(x) - b - \rho \ dG(x) = m'(\lambda_p) \int_{\bar{x}_p} px - b - \rho \ dG(x) \ .$$

One wage schedule which is compatible with this condition and which also satisfies the limited commitment constraint that neither the firm nor the worker would dissolve the contract expost is

$$w_p(x) = (1 - \mu)(b + \rho) + \mu px$$
,

where μ is the constant matching function elasticity.¹³ Because expected match-specific productivity in firm p is $E(x|p) = \alpha \bar{x}_p/(\alpha - 1)$, the mean wage in a firm of productivity p is

$$E(w|p) = (b+\rho)\frac{\alpha+\mu-1}{\alpha-1} .$$

Output per worker (productivity) in this firm is

$$E(px|p) = (b+\rho)\frac{\alpha}{\alpha-1}$$
.

Because more productive firms employ less productive workers, average productivity and wages in all firms are identical. The labor share (wages divided by output) in any firm (and thus in the economy as a whole) is

$$\frac{E(w)}{E(px)} = \frac{\alpha + \mu - 1}{\alpha}$$

The squared coefficient of variation of wages within any firm is

$$SCV(w|p) = \frac{\alpha\mu^2}{(\alpha - 2)(\alpha + \mu - 1)^2}$$

¹³If the firm would provide perfect insurance to its applicants against realization of x, it would offer the same wage to all workers which is then $w(x) = w = (b + \rho)(\alpha + \mu - 1)/(\alpha - 1)$. Alternatively, the log-linear schedule $w(x) = px(\alpha + \mu - 1)/\alpha$ also satisfies the above condition. Both alternatives either violate limited-commitment constraints on either the worker (who prefers to quit when $w < b + \rho$) or the firm (who would like to layoff the worker ex-post if w > px).