Type Search and Choice: True and Adopted Type Mismatch and the Generation of Frames

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[PRELIMINARY AND INCOMPLETE]

Abstract

A model of decision making is built where the type of the decision maker matters for how this process takes place. Individual’s type is assumed to be determined by Nature and ignored by individuals. Self-Type ignorance starts a process in which individuals commence an early search for a type to adopt. In this search process, individuals take into account the information in their current state, together with a net valuation function and a threshold, to determine when the search process must stop. The type-search process can produce a final type that may or may not coincide with individual’s true type. If type adoption happens to produce a type different to the true type, this adopted type is shown to function as a frame in an extended choice problem. In our choice framework, adopted types as frames can lead to sub-optimal choices with individual welfare implications. Possible applications of the model are suggested.

Keywords: Choice, Frames, Search, Types, Unknown Type.

1 Introduction

Decision processes can be overwhelming and costly in terms of the time invested and information search and acquisition, and given that there usually is certain level of information absence involved in decision making, even consciously selected options in a bounded setting may or may not, in the end, be the optimal for each individual. This is particularly true if knowledge about self-characteristics, areas of strength or capabilities are not well defined or are non-existent.

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We present a model in which individuals are born with an unknown type and start a type-search-process. Here, \( i \) is confronted with the problem of searching for her true type, the search process finishes with the adoption of the type the search process indicates at the time of stopping. Agents follow a search stopping rule defined on a threshold that interacts with a net valuation function to indicate the time of stopping, indicating a type to be adopted. This in the spirit of Simon (1955)’s concept of satisficing. Then, consideration sets are defined to correspond with each type, to evaluate under which circumstances adopted types function as frames and derive in suboptimal choices.

There are a number of approaches in the literature that address these and other related issues. Bounded rationality for example is one of the earliest attempts to do so, aiming to model choice behaviour considering boundaries to the unlimited capabilities of the rational man on information processing. A common reference for the origin of this line of research is Simon (1955), where the author exposes the problems and weaknesses of theories based on the rational individual (See Lipman (1995), Selten (1999), Rubinstein (1998) for a presentation of different models of bounded rationality).

Theoretical literature on type unawareness is rather scarce. Murayama (2010) for example develops a two sided search model where agents are not aware of their type and finds that this setting has some implications for welfare in equilibrium; Young (2008) builds a model where individuals do not have a clear idea of their identity when introducing self-image in individual’s utility function; and Boone and Shapiro (2006) build a model where the type of consumer changes over time as a function of previous consumption of goods, giving power to the producer on rent extraction. Also, Gul and Pesendorfer (2007) develop a model where agent’s preferences depend on other individuals’ characteristics and personalities; and Calvó-Armengol and Jackson (2009) model social environment influence on both parents and children and as a result the overlapping environment determines the correlation among the behaviours of this two.

Learning theory approaches situations similar to those of type ignorance or adjustment usually by departing from models that assume rational equilibrium or that are built in a game theoretical setting (Slembeck, 1998). In learning theory models, individuals adjust their behaviour incorporating information captured by social interaction in a way such that they optimize on payoffs via imitation of better strategies, or at least strategies that seem to be the best.

Another related line of research is the so called identity economics. This area of research has been recently developed by Akerlof and Kranton (2000), Bénabou and Tirole (2011), and Fryer and Jackson (2008), among others; and can be traced back to Sen (1985), Folbre (1994), Landa (1995), Kevane (1994). The main idea in this literature is that individual’s sense of self, and positive valuation towards fitting in a social environment and within the
particular groups, can affect economic outcomes. Regarding the applied literature, research has focused on type awareness instead of unawareness. Examples can be found in the works of Humlum et al. (2012) who use factor analysis methodology to extract how identity influences educational or career choices; Benjamin (2010) who implement experiments to capture the effects of race identity on patience in decision making; Shayo (2009) and Klor and Shayo (2010) develop a model of identity and then test it using experiments to determine the effect of identity on redistribution preferences, the former focusing on payoff maximization behaviour, and the later with class and national identities as focal point; Hoff and Pandey (2006) use two experiments in rural India to consider if social identity of individuals can explain cognitive performance and responses to economic incentives. Although this work focuses on identity and not on types, the latter can be related to former if type adopted is instead defined as identity (For other approaches see Durlauf (2001) and Glaeser et al. (1995): social interactions; Bisin et al. (2011): identity with social interactions; Benabou and Tirole (2011): identity driven by moral behaviour; Jamison and Wegener (2010): multiple selves).

1.1 Related literature

More recently, work on choice, frames, search and consideration sets has emerged with interesting results that expand the classical models of choice. Regarding the literature on frames and choice, Salant and Rubinstein (2008) develop a model of choice with frames where frames and the set of alternatives as a pair define the choice problem, and they axiomatically determine choice behaviour. Bernheim and Rangel (2007) suggest a framework with ancillary conditions that affect choices, focusing on welfare implications. Eliaz and Spiegler (2011) develop a model of consumer choice with consideration sets where entities with market power can affect choice via frames.

With respect to choice involving search processes, Nakajima and Masatlioglu (forthcoming) present a model of iterative search and decision making with reference points leading the search process. Horan (2010) offers a model where choice form lists. Dalton and Ghosal (2012) build a model where choices are driven by frames that are endogenously determined with a feedback process involved. The authors describe choice procedures under their framework and explore the effects on welfare under a number of assumptions that restrict the information on the part of the decision maker.

Other related literature includes Masatlioglu and Ok (2005) expand the classical choice theory to include the influence of status quo in choice behaviour departing from the revealed preference theory. Papi (2012) presents am axiomatic model of bounded rationality, making
use of the satisficing concept within the revealed preference framework. Additionally, Caplin and Dean (2011) build a model of choice with search in which search is costly and where the decision maker has a reservation utility that indicates when to stop searching.

Our work builds on this literature to present a model of choice with frames and consideration sets when type unawareness is present, and type search is guided by a satisficing heuristic. The main contribution is the inclusion of self-type unawareness as an explanation of how frames are formed and how they interfere with choice behaviour. The model also aims to explain suboptimal equilibria.

In the following sections the main elements of the model of type search are presented, followed by the result of such search process. Then, the extended choice model is developed with adopted types as frames. Finally, some possible applications of the model are offered. A section with final comments including future extensions closes this work.

2 Model

2.1 Types and characteristics

Define $I \ni i$ as the set of all individuals. Let the set $\Theta := [\bar{\theta}, \bar{\theta}]$ be a compact metric space with typical element $\theta$, which we will call a type form now on, and $\bar{\theta}$ and $\bar{\theta}$ as the respective lower bound and the upper bound and a continuum of types between them. Assume there is a complete preorder $\succeq_{\Theta}$ on $\Theta$. Thus $\Theta$ contains finitely many types that can be ordered according to a criteria embedded in $\succeq_{\Theta}$. Define $\Omega := 2^\Theta \setminus \emptyset$ be the set all non-empty subsets of the set of types $\Theta$, and let $\Theta_h \in \Omega$ be one of those subsets, as $\Theta$ is finite $\Theta_h$ so is as well. Each $\Theta_h$ inherits the properties of $\Theta$, thus it is also a compact metric space and any preference $\succeq_{\Theta_h}$ respects $\succeq_{\Theta}$. Let $\Lambda \subseteq \mathbb{R}_+^L$ be the set of vector characteristics $\lambda \in \Lambda$, whose elements \{\lambda^1, \lambda^2, \ldots, \lambda^L\} indicate the magnitude of each characteristic\(^2\). Let $\preceq_{\Lambda}$ be a complete preorder for all elements in $\Lambda$. Define the pairs $(\Theta, \succeq_{\Theta})$ and $(\Lambda, \preceq_{\Lambda})$ as the corresponding complete preordered sets.

Each type $\theta'$ has a corresponding vector of characteristics $\lambda'$ that accompany that particular type and that indicate the characteristics that each type possesses and in which magnitude, with a type $\theta'$ being better allotted in terms of $\preceq_{\Lambda}$-ranked characteristics in comparison to any other type $\theta''$ if and only if $\theta' \succeq_{\Theta} \theta''$. As higher types are preferred to lower types as ranked by $\succeq_{\Theta}$, $\theta$ contains the lowest $\preceq_{\Lambda}$-ranked characteristics and $\bar{\theta}$ the highest $\preceq_{\Lambda}$-ranked

\(^2\)The value that each characteristic acquires indicates either lack of the characteristic, if the value is equal to zero, or the presence of the characteristic and its magnitude, when different form zero and higher values indicating higher magnitude.
characteristics. To formalise this observations the relationship between characteristics and types is specified as a correspondence in the following definition

**Definition 1** (Characteristics to types correspondence). Let $\rho : \Lambda \Rightarrow 2^\Theta \setminus \emptyset$ be an order-preserving mapping from the set of characteristics to the set of types, that is, $\rho$ defines which vector $\lambda' = \{\lambda'_1, \lambda'_2, \ldots, \lambda'_l\}$ of characteristics corresponds to the subset of types $\theta' \subseteq \Theta$.

The assumption of $\rho$ being order-preserving is imposed to assure that if $\rho(\lambda') = \theta'$ and $\rho(\lambda'') = \theta''$, then $\theta' \succcurlyeq_{\Theta} \theta''$ implies $\lambda' \succcurlyeq_{\Lambda} \lambda''$; that is $\rho$ will assign a higher $\succeq_{\Lambda}$-ranked vector of characteristics to higher $\succeq_{\Theta}$-ranked types. The intuition behind this is simple: for higher types, more characteristics and/or characteristics of higher magnitude are needed, as higher types are preferred to lower types, vectors of characteristics that lead to higher types must be preferred to those that lead to lower types. Definition 1 specifies the bridge between characteristics and types aiming to represent a mental process on the part of the agents, but such processes correspond to observations that could potentially be confirmed by data sets.

### 2.2 Agents’ types

Types lie on the continuum $[\theta, \theta]$ and are distributed across agents according to a density function $g(\theta)$ with c.d.f $G(\theta)$, each type with the correspondent vector of characteristics according to $\rho(\lambda_i) = \theta_i$. It is assumed that each of this types are unknown to the agents, and we will refer to them as the true types. Although the true type is unknown to each agent $i$, the agent receives a signal $\lambda'_o$ of the endowed characteristics, however, this signal is not complete and is not taken as the final set of characteristics that $i$ possesses.

Assume $\Theta_i \in \Omega$ is the set of all types $i$ could adopt given her characteristics. In order to complete the signal $\lambda'_o$, each $i$ searches for information on the types, and thus the characteristics, of other agents in the agent’s own environment $\mathbb{K}_i$ and other environments $\mathbb{K}_{-i}$. As information coming from each environment may be of different relevance to each agent depending on their own environment and on how close environments are to each other, we specify a weighting rule based on the agent’s perceived distance between the status quo of each environment as follows

**Definition 2** (Type-to-type distance). Let $\mathcal{M} : \Theta \times \Theta \rightarrow [0,1]$ be a metric on $\Theta$ that completes the metric space $(\Theta, \mathcal{M})$. Let $\mathcal{M}$ be the absolute distance. Define the type-to-type distance as the distance between two given types $\theta'$ and $\theta''$, and let it be specified by $\mathcal{M}(\theta', \theta'') = |\theta'' - \theta'|$.\(^3\)

\[^3\]We can think of type-to-type distance as a dissimilarity index, in which case $\mathcal{M}_i$ can be redefined as $\mathcal{M}_i = \frac{\eta_i(\mathcal{N}_j)}{\eta_i(\mathcal{N})}$. 

5
In Definition 2 it is assumed, for simplicity, that $M$ is defined on the unit interval instead of the real numbers. The value of $M(\cdot)$ gives a measure of polarization across environments. A value of $M(\theta', \theta'')$ close to 1 indicates that the difference in status quo of types between environments $\theta'$ and $\theta''$ is as big as possible, indicating that one of the two is either at the top or the bottom, and the other at the opposite. Similarly, if the difference is close to zero, then we can infer that the two environments are close to each other according to this criteria. Thus, $M$ gives a non-negative measure of how apart types are form each other, including representative types of each environment (status quo), these measures will be particularized to each agent $i$ to focus on the perspectives of the agents. Notice as well that this measures are one-to-one comparisons and do not aggregate information, however aggregation can easily be done by summation over the status quo of all environments or particular types as is done later when needed.

It is argued later that the distances between types influence the determination of agents’ adopted types, together with $i$’s initial signal $\lambda_0^i$. As both elements carry relevant information both should bear some weight in agent’s type determinacy explanation.

2.3 Search environments

Assume now that agents in $I$ are distributed across a continuum of environments $R = [\mathbb{N}, \mathbb{R}] \ni \mathbb{N}_j$, each environment indexed by $j \in J$. Each agent is assigned, at a starting period, to a particular environment $\mathbb{N}_j$ according to a continuous differentiable cumulative distribution function $F : R \rightarrow [0, 1]$ with density $f$.

Define now $\eta_i \in I$ as the subset, with size $|\eta_i|$, of agents of type $\theta_i$ in $I$, and let $\eta_i(\mathbb{N}_j) \subseteq \eta_i$ be the agents of type $\theta_i$ located in environment $\mathbb{N}_j$, with $|\eta_i(\mathbb{N}_j)| = \#i$ with type $\theta_i \mid i \in \mathbb{N}_j$ being the size of such group. Fix $\eta_{i,j} = \frac{|\eta_i(\mathbb{N}_j)|}{|\mathbb{N}_j|}$ to be the fraction of types $\theta_i$ located in group $\mathbb{N}_j$. Each environment could have one or various types with higher frequency than the rest of the types present in such environment. Such over represented types constitute the status quo in that given environment, this observation is specified in the following definition.

**Definition 3** (Predominant type (Status quo)). A predominant type $\theta_{\mathbb{N}_j}$ in environment $\mathbb{N}_j$ is a type such that $\theta_{\mathbb{N}_j} = \{\theta_i \in \Theta \mid \text{card}(\eta_i(\mathbb{N}_j)) > \text{card}(\eta_{i'\neq i}(\mathbb{N}_j)) \quad \forall \eta_{i'\neq i}(\mathbb{N}_j) \in \mathbb{N}_j\}$. The type $\theta_{\mathbb{N}_j}$ represents the status quo of types in environment $\mathbb{N}_j$; where the status quo is the type of reference of those belonging to environment $\mathbb{N}_j$. If $\text{card}(\eta_i(\mathbb{N}_j)) = \text{card}(\eta_{i'\neq i}(\mathbb{N}_j))$ for some $\eta_{i'\neq i}(\mathbb{N}_j) \in \mathbb{N}_j$, then more than one status quo exists in such environment.
Predominant types work as *aggregators* of information on the composition of environments, indicating not only which type is the most representative in terms of number, but also a way to rank types in terms of representativity within and across environments. Using Definition 2, we can also define the status quo distance between the status quos in environment $\mathcal{N}_i$ and environment $\mathcal{N}_j$ as $M(\theta_{\mathcal{N}_i}, \theta_{\mathcal{N}_j}) = |\theta_{\mathcal{N}_i} - \theta_{\mathcal{N}_j}|$.

Now we turn to describe how agents process the information available in the environments. It is assumed here that agents have full awareness of the type’s space, and that they can form a complete ordering of such types that leads agents to be able to form a type set list, that is an ordered list $L$ of the elements of the types’ set, with the order of the elements corresponding to the order relation $\succsim_{\Theta}$, the following definition specifies ordered lists in the context of this work.

**Definition 4** (Type set list). Recall that $\Theta_K \subseteq \Omega := 2^\Theta \setminus \emptyset$. A list on the set of types $L_K = L(\Theta_K, \succsim_{\Theta}) = \{\theta_k, \theta_{k+1}, \ldots, \theta_K\}$ is a sequential order of the $\theta \in \Theta_K$, using $\succsim_{\Theta}$ as criterion of order, and meeting the condition that whenever $\theta'$ is placed after $\theta''$ in $L_K(\cdot)$, $M(\theta_K, \theta') > M(\theta_K, \theta'')$. Fix $L$ as the list of all elements in the set $\Theta$. A sub-list $L^H$ on a list $L_K$ is a list on the set of types $\theta \in \Theta^H$, with the following properties:

**Lemma 1.** For any $\Theta_H, \Theta_K \subset \Omega$ and $L_H, L_K \in L$; If $\Theta_H \subset \Theta_K$ then $L_H \subset L_K$.

**Proof.** (Lemma 1) Let $\Theta_H \subset \Theta_K$, then $\exists \{\theta'\}$ such that $\{\theta'\} \cup \Theta_H = \Theta_K$. From Definition 4 a list $L_D$ contains only the elements of set $\Theta_D$ in ascending order of preference. Thus the number of elements in $L_K$ must be larger than the number of elements of $L_H$. Then $\exists \{\theta'_i\} \in L_K$ such that $\{\theta'_i\} \cup L_H = L_K$, and then $L_H \subset L_K$.

**Remark 1.** If in Lemma 1 the equality part of $\subseteq$ is met, having instead that $\Theta_H \subseteq \Theta_K$, then $\Theta_K = \Theta_H$ and $L_K = L_H$.

**Proof.** (Remark 1) Having $\Theta_H \subset \Theta_K$ implies $\Theta_H \neq \Theta_K$, and $\Theta_H = \Theta_K$ if and only if $\forall \theta \in \Theta_H$ also $\theta \in \Theta_K$. Thus, if $\theta' \in \Theta_H$, from Definition 4 such $\theta'$ is also in $L^H$, and form $\Theta_H = \Theta_K$ it must be that $\theta' \in \Theta_K$, and then $\theta' \in L_K$.

Form definition 4, it is possible to divide lists in sub-lists that contain only a fraction of the elements contained in the universal list. Notice that $\Theta_H \subset \Theta_K$ implies $\text{card}\{\Theta_H\} < \text{card}\{\Theta_K\}$, $L_H \subset L_K$ and thus $\#L_H < \#L_K$, and also that as sub-lists inherit all the properties of lists, all definitions and results that apply to one also apply to the other. This results will be used later when modelling the search process. For easiness in exposition we will refer to sub-lists only when the context requires this, but will work on lists for most of the definitions and results.
2.4 States and beliefs

The information that each agent \( i \) takes into account at each stage \( t \) is defined by the state \( \langle \lambda^i_t, \theta_{\text{si}}, M_i \rangle = \sigma^i_t \in \Sigma \), where \( \Sigma \) is the set of all possible states, with \( \theta_{\text{si}} \) and \( M_i \) being \( i \)'s environment status quo and a metric defined over \( \Theta \) according to \( i \) beliefs respectively. Notice that the metric is define for each \( i \) and thus we are assuming it can vary across agents; and also that the set of status quos and the metric remain constant with changes in \( t \). Let the initial state be \( \sigma^o_i \) be characterised by the triplet \( \langle \lambda^o_i, \theta_{\text{si}}, M_i \rangle \).

\( \sigma^i_t \) specifies the beliefs of agent \( i \) at \( t \). Thus, at each stage \( t \) the agent updates her beliefs given the current status, incorporating new information provided by \( \lambda^i_t \), that is the vector of characteristics at each stage. Notice that, for an agent \( i \) with beliefs \( \sigma^i_t \), \( \rho(\lambda^i_t) \) reports a subset of types \( \theta^i_t \), thus the probability form the point of view of the agent of having a true type \( \theta_k \subseteq \Theta \) given state \( \sigma^i_t \) is not zero as \( \theta_k \) is, by Definition 1, a subset of values and not a singleton\(^4\).

2.5 Type search process

Agent’s type search starts at each stage \( t \) with the information available to the agent at that stage. As already specified, this information defines a current state that is described by the characteristics at \( t \), the status quo in each environment, and the type-to-type and status quo distances. Agents use the information available at \( t = 0 \) to determine a point of departure from which they start their type-search process. Agents use updated informational structures to define a search-departure type at each stage \( t \). The type search process finishes with a final product \( \tilde{\theta}_i \) that is the adopted-type that \( i \) takes as if it were a true measure of her type. Formally, \( i \)'s expected type at stage \( t \), and given state \( \sigma^i_t \) is defined as

**Definition 5** (Search-departure type). Define agent \( i \)'s search departure type at stage \( t \) given beliefs \( \sigma^i_t \) as the type from which agent \( i \) initiates her type search process at that stage

\[ \theta^i_t = \rho(\lambda^i_t \mid \sigma^i_t) \]  

(1)

Define \( \theta^o_i \) as the \textit{initial search-departure type} for \( t = 0 \) in the specification above.

Form Definition 5, all the information \( i \) can initially possess is contained in \( \sigma^i_t \), this information gives \( i \) a biased perspective on the distribution of types across environments and summarises \( i \)'s beliefs.

\(^4\)Thus if the subset of types \( \theta_k \) includes types \( \{\theta_k, \ldots, \theta_k\} \), then the probability from the point of view of agent \( i \) of being of type \( \theta_k \) is given by \( \Pr(\theta_i = \theta_k \mid \sigma_i) = \frac{\int \theta f(\theta) d\theta}{\theta_k} \).
Besides having an initial type from which to start the type-search process, we assume other criteria is needed to define such process. These elements are related to the subjective valuation associated to the adoption of a particular type, and the costs associated with the adoption of that type. Regarding the valuation over types, we assume that agents have a positive and increasing at a decreasing rate valuation on types, and it is assumed that each $i$ has a subjective valuation of adopting type $\theta'_i \in \Theta$ equal to $\pi_i = \pi(\theta'_i) = \pi(\rho(\lambda'_i))$, with $\pi$ of class $C^2$ and agreeing with $\succsim_i \Theta$; and $\pi_{\theta} > 0$, $\pi_{\theta^2} \leq 0$.

On the other hand, type adoption is not a costless action, there are costs produced by the adoption of a type that are generated by the effort exerted to reach the possibility of adopting the type. These costs can derive from the actions involved in acquiring the characteristics needed by agents to be able to adopt a particular type. Agents’ costs depend on the characteristics, the status quo of their environment, and the distance defined over types. Let the costs of adopting type $\theta_k$ be represented by a class $C^2$ function $C_i = C(\lambda'_i, \theta_{R_i}, M_i)$ with $C_\lambda > 0, C_{\lambda^2} > 0, C_{\theta_k} > 0, C_{\theta^2_k} > 0, C_{M_i} > 0, C_{M^2} > 0$.$^5$

The net valuation agent $i$ has on adopting a type $\theta'$ is given by the difference between the valuation and the cost of adopting that particular type, that is $V(\theta') = \pi(\theta') - C(\lambda', \theta_{R_i}, M_i)$.

5

Whenever $V < 0$ the cost of adopting type $\theta'$ surpasses the payoff of adopting that type, and thus type $\theta'$ is not chosen. We don’t only require $V$ to be positive, but also that it reaches at least a minimum threshold to capture the idea of agents evaluating the worthiness of adopting a given type not only on the basis of private costs and payoff but also on the valuation that a type has in the environments. Define this threshold as $\Gamma_i = \Gamma(\lambda'_i, \theta_{R_i}, M_i)$ where $\Gamma$ is increasing in $\lambda$, that is, the higher the characteristics in the $\succsim^{\lambda}$-ranking the higher the threshold, and also increasing in $\theta_{R_i}$ as an indicator of what is acceptable in each environment, and what should the agent aim to. A measure of feasibility over types is captured here by $M$.

Now we define a stopping rule indicating when the agent is to continue searching for a type or stop and adopt the type reached at that stage of the type-search process. Clearly this stopping rule should, for a rational agent, require the net valuation to be positive. Additionally, we impose the net valuation to be higher than the threshold. The search rule is specified as a heuristic criteria $\Phi \left( L(\Theta, \succsim^{\Theta}), V, \Gamma \right)$ that indicates if search is to be stopped or continued, and is based on the satisfying criteria as described by Simon (1955). Define a search rule as follows

**Definition 6** (Type search rule). A type-search rule indicates to the agent whether to continue or to stop searching for a type to adopt based on the net payoff, the threshold, and the

$^5$Assume $\exists t : \Theta \rightarrow \mathbb{R}$. Thus, $C : \Lambda \times \Theta \times M \rightarrow \mathbb{R}$ or $C : \mathbb{R}^3 \rightarrow \mathbb{R}$. 

9
type alternatives at stake and the preferences over them represented by the list $L$

$$\Phi(L(\Theta, \succsim \Theta), \mathcal{V}, \Gamma) =$$

\[
\begin{align*}
&\text{i) Continue type search at stage } t \text{ if, for } \lambda^t \in \sigma^t_i, \theta^t = \rho(\lambda^t) \text{ is such that } \Gamma(\sigma^t_i) > \mathcal{V}(\sigma^t_i) \\
&\text{ii) Stop type search if } \theta^t \text{ is such that } \mathcal{V}(\sigma^t_i) \geq \Gamma_i(\sigma^t_i); \\
&\text{adopt } \tilde{\theta}^t = \theta^t \in L(\Theta, \succsim \Theta).
\end{align*}
\]

Although search processes can be done in different ways, here it is assumed that agents search sequentially, either progressively or regressively, from the list of types just described in Definition 4 and that they have perfect recall, that is agents know exactly where in the list they are positioned, where they have been, and retain all information derived from their past search in the list. This is clarified further below starting with the following assumption

**Assumption 1** (Sequential type search process). agent $i$’s type search process on a list $L(\Theta, \succsim \Theta)$ is sequential departing from a given type $\theta^o_i$, continuing progressively by testing types of higher order $\theta^{i+1}_t$, or regressively by testing types of lower order $\theta^{i-1}_t$. This search process starting from an initial type search $\theta^o_i$.

So far we have defined where the type search process starts, under which rules it operates and over which object takes place. Additionally, a search direction that indicates if the agent is searching progressively or regressively under the rules already specified has to be defined

**Proposition 1** (Search direction). Given a state $\sigma^o_i$, the type-search direction $\delta(\sigma^o_i)$ is from above and towards $\theta$ ($\nearrow$ $\theta$) if and only if $\Gamma^t > \mathcal{V}^t$ and condition $\frac{\partial \mathcal{V}(\theta^t)}{\partial \theta} < 0$ holds for $\rho(\lambda^t_i) = \theta^t$. Conversely type-search direction and from below and towards $\theta$ ($\searrow$ $\theta$) if and only if $\Gamma^t > \mathcal{V}^t$ and condition $\frac{\partial \mathcal{V}(\theta^t)}{\partial \theta} > 0$ is met, with $\theta^t < \theta^{t+1}$.

**Proof.** (Proposition 1) For any state $\sigma^o_i$, $\frac{\partial \mathcal{V}(\theta^t)}{\partial \theta}$ and $\theta$ if $\Gamma \leq \mathcal{V}$ then the search process stops as the agent has either exactly reached or surpassed the threshold value, thus to have a search direction we need $\Gamma > \mathcal{V}$. This proves the only if part. For the if part, notice that when $\Gamma > \mathcal{V}$ the agent has not reach a satisfactory type and the agent’s search process continues with either $\theta^t < \theta^{t+1}$ or $\theta^t > \theta^{t+1}$. Assume $\theta^t < \theta^{t+1}$, then two outcomes are possible, either $\mathcal{V}(\theta^t) > \mathcal{V}(\theta^{t+1})$ or $\mathcal{V}(\theta^t) < \mathcal{V}(\theta^{t+1})$, if the first inequality is true, then the gap between $\pi$ and $C$ is closing form $t$ to $t + 1$ and will keep closing with increases in $\theta$ as $\pi$ is strictly decreasing and $C$ is strictly increasing in $\theta$, this leads the agent to switch the direction of search either immediately or after some iterations in the same direction, with switching direction implying
that search direction is $\searrow \theta$. If the second inequality holds, then the agent has incentives to keep searching as the increase in $\nu$ with the increase in $\theta$ indicate that the gap between $\pi$ and $C_i$ is becoming wider. The direction of search in this case will be $\nearrow \theta$. Similar arguments can be given for the case in which $\theta^t > \theta^{t+1}$.

\section*{2.5.1 Search learning}

The initial departure type given beliefs $\sigma^o_i$ is given by $\theta^o_i = \rho(\lambda^o_i \mid \sigma^o_i)$, with $\sigma^o_i = \langle \lambda^o_i, \theta, \Theta, \mathcal{M}_i \rangle$. A key determinant of this initial stage is the characteristics signal that the agent receives, $\lambda^o_i$, which partially defines the search departure type $\theta^o_i$, but which is the only variable that can be modified by $i$ in the following stages if the proper incentives exist.\footnote{That is the payoff of increasing $\lambda$ should be at least equal to the costs.} Let such a change in $\lambda$ occurring at a stage $t$ be denoted by $\Delta \lambda^t_i$, thus we can define $\lambda^{t+1}_i = \lambda^t_i + \Delta \lambda^t_i$. It can be shown that a condition for such a change $\Delta \lambda^t_i$ to take place is $C(\Delta \lambda^t_i) \leq \pi(\rho(\lambda^{t+1}_i)) - \pi(\rho(\lambda^t_i))$.

Define an initial search neighbourhood as $\Theta^o_i = \{\theta \in \Theta : \theta = \rho(\lambda^o_i)\}$. Indeed this is the subset of types that correspond to the initial characteristics as perceived by the signal $\lambda^o_i$ and with $\Delta \lambda^t_i = 0$. An initial search neighbourhood produces the subset $\Theta^o_i \subseteq \Theta$ that corresponds to the signal $\lambda^o_i$. Then, given a search direction as specified in Proposition 1, the agent searches $\Theta^o_i$ until she finds $\theta' \in \Theta^o_i$ such that $\nu(\theta') \geq \Gamma(\theta')$ as the search rule indicates in Definition 6. If such a type is found, then $i$ adopts $\theta = \theta'$. Otherwise she continues searching in the next search neighbourhood defined as

**Definition 7.** Define a continuation search neighbourhood for $i$ at stage $t + 1$ as $\Theta^{t+1}_i = \{\rho(\lambda^{t+1}_i)\} \setminus \bigcup_{k \leq t} \Theta^k_i = \{\theta_{\Theta^{t+1}_i}, \theta_{\Theta^{t+1}_i}^\parallel\}$.

With
\begin{align*}
\lambda^{t+1}_i &= \lambda^t_i + \Delta \lambda^t_i, \quad \Delta \lambda^t_i = \text{change in } \lambda^t_i. \quad (3)
\end{align*}
\begin{align*}
C(\Delta \lambda^t_i) &\leq \pi(\rho(\lambda^{t+1}_i)) - \pi(\rho(\lambda^t_i)). \quad (4)
\end{align*}

In each search neighbourhood, $i$ searches for a type to adopt by testing types within the neighbourhood by use of a randomising mechanism $\alpha$ defined by the search direction $\delta(\sigma^o_i)$ at stage $t$.

**Definition 8.** A test type at stage $t$ is a type $\theta^t_i := \alpha(\delta(\sigma^o_i))$. Where
\begin{align*}
\alpha(\delta(\sigma^o_i)) &\sim \begin{cases} U[\theta_{\Theta^t_i}, \theta_{\Theta^t_i}^\parallel] & \text{increasing in the supert} \\
U[\theta_{\Theta^t_i}, \theta_{\Theta^t_i}^\parallel] & \text{decreasing in the supert} \end{cases} \\
\end{align*}
In this test type rule, agents search within stage \( t \) search neighbourhood picking types to test with uniform probability. This allows for identical agents confronting equal states \( \sigma \) at a given stage \( t \) to end up with different outcomes in terms of type adoption.

### 2.6 Type adoption

In this section we define the type search outcome. The result is an adopted type as product of the initial characteristics’ signal, the set of status quos, the metric over types, and the evolution of the agent’s characteristics. This is summarised in a history of states \( \sigma_i = \{\sigma_i^o, \sigma_i^1, \ldots\} \) that constitute the beliefs that drive agent \( i \)’s type choice.

**Lemma 2** (Non-existence of adopted type for pathological thresholds). If \( \Gamma > \forall \theta \in \Theta \), and any state \( \sigma_i^o \), and direction \( \delta(\sigma_i^o) \). Then for all stages and any \( i \) there is no \( \theta' \in \Theta \) such that \( \theta' = \bar{\theta}_i \).

**Proof.** (Non-existence of adopted type for pathological thresholds) First notice that if \( \Gamma > \forall \theta \in \Theta \) and \( \delta = \mathcal{N} \bar{\theta} \) then from Definition 8 the least value for the rule is \( \alpha(\delta(\sigma^o)) = \bar{\theta} \).

Since \( \Gamma(\bar{\theta}) > \forall(\bar{\theta}) \), the rule \( \alpha \) indicates to proceed searching in a subset \( \Theta^k = \{\theta \in \Theta : \theta > \bar{\theta}\} = \emptyset \), and thus \( \bar{\theta} = \{\emptyset\} \). Similarly, if \( \Gamma > \forall \theta \in \Theta \) and \( \delta = \mathcal{N} \bar{\theta} \) then \( \alpha(\delta(\sigma^o)) = \emptyset \) is the least value reached in \( \Theta \). Since \( \Theta^k = \{\theta \in \Theta : \theta < \bar{\theta}\} = \emptyset \) then \( \bar{\theta} = \{\emptyset\} \).

Pathological thresholds can be avoided by implementing a default rule indicating the agent to select either \( \bar{\theta} \) or \( \theta \) for \( \delta = \mathcal{N} \bar{\theta} \) and \( \delta = \mathcal{N} \theta \) respectively once an empty search neighbourhood is reached. In this case if \( \Gamma > \forall \theta \in \Theta \) and \( \delta = \mathcal{N} \bar{\theta} \) then \( \bar{\theta} = \bar{\theta} \). Similarly, if \( \Gamma > \forall \theta \in \Theta \) and \( \delta = \mathcal{N} \theta \) then \( \bar{\theta} = \theta \).

**Proposition 2** (Existence of adopted type). Given any state \( \sigma_i^o \), and direction \( \delta(\sigma_i^o) \), \( \exists \theta \in \Theta \) such that \( \theta = \bar{\theta}_i \) at some stage \( t \forall i \), for non pathological thresholds.

**Proof.** (Existence of adopted type) Notice that condition \( \Gamma \leq \mathcal{V} \) must be reached in the closure of \( \Theta \). The proof limits to show that \( \exists \theta \in \Theta \) such that \( \bar{\theta} = \theta \) for \( \Gamma \leq \mathcal{V} \). In this case, for both \( \delta = \mathcal{N} \bar{\theta}, \delta = \mathcal{N} \theta \), \( \exists \Theta^t \subseteq \Theta \) for some \( t \) such that \( \Gamma \leq \mathcal{V} \). The proof is made here for \( \delta = \mathcal{N} \bar{\theta} \), a parallel proof can be made for \( \delta = \mathcal{N} \theta \) by following the same kind of argumentation. If such \( \Theta^t \) equals \( \Theta^o \), then \( \bar{\theta} \in \rho(\lambda^o) \), and \( \bar{\theta} \) is reached in the first stage. If this is not the case, then \( \Gamma \leq \mathcal{V} \) should be reached within \( \bar{\theta}^o, \bar{\theta} \). Assume now \( \bar{\theta} \in \Theta \setminus \{\bar{\theta}\} \), that is there is no such \( \bar{\theta} \) before \( \bar{\theta} \) is reached. Then it must be that \( \Gamma \leq \mathcal{V} \) is reached for \( \bar{\theta} = \bar{\theta} \), otherwise it would be the case of a pathological threshold, which has been already discarded.

If a \( \bar{\theta} \neq \theta^o, \bar{\theta} \) then \( \bar{\theta} \in \rho(\lambda^t) \) for some \( \lambda^t \neq \lambda^o \). Such \( \lambda^t \) must have been reached by means of additions of \( \Delta \lambda \) to all \( \lambda^h, h < t \), for which corresponding neighbourhoods
Lemma 2 and Proposition 2 show that, given a history of states $\sigma_i$ and a search direction $\delta(\sigma_i^o)$, an agent will adopt a type $\tilde{\theta}_i \in \Theta_i$ at some stage $t$, with the following possible outcomes for a given true type $\theta^*$

\begin{align}
\tilde{\theta}_i &> \theta^* \\
\tilde{\theta}_i &< \theta^* \\
\tilde{\theta}_i &= \theta^*
\end{align}

The next section covers the implications of this possibilities

### 2.7 Choices with types as frames

In this section we show how the type adoption process can lead to a mismatch between the agent’s adopted type and her true type, and that this subsequently produces suboptimal choices where a mismatched adopted type functions as a frame in a choice problem. To do so we define an extended choice problem that includes a set of frames that can alter the choice process without having any rational fundamental. We show how such frames arise in our type adoption model. For this we base our analysis on the work of Bernheim and Rangel (2007) and Salant and Rubinstein (2008) on ancillary conditions and frames respectively.

Let $X$ be the finite set of all available alternatives; $X := 2^X \setminus \{\emptyset\}$, a class in $X$ containing all non-empty subsets of $X$; and $A \subseteq X$ a consideration set, that is, a set that contains only the options to be considered by the decision maker. An extended choice set $\{X, f\}$ includes a choice set $X$ and a frame $f \in F$, with the set of all frames denoted by $F$. An extended choice set, “expands” the standard choice set with the inclusion of an additional criteria of relevance to the decision maker ($DM$), when selecting an option from a variety of alternatives. It is thus a useful tool for the analysis of decision making when the $DM$ restricts to a consideration set. The extended choice set requires a choice function that contemplates this “extension”. An extended choice correspondence $c(\{X, f\})$ selects a unique option $\{x\} \subseteq X$ from the choice problem $\{X, f\}$, notice that the choice $\{x\}$ can be a singleton or a subset of $X$.

We define now a consideration set as a pair formed by the set of all available alternatives and a frame. Such a set contains the alternatives that define the choice problem for the $DM$
under the presence of a frame, in this sense a consideration set is an extended choice set over which the choice correspondence above presented must be applied by the DM

**Definition 9.** Define a *consideration set* \( A := \{X, f\} \in \mathcal{X} \) as the set that contains only the choices from \( X \) that will be considered by the DM given the frame \( f \).

At this point, it is worth clarifying what is considered a frame in this context. Here, as in Salant and Rubinstein (2008), a frame is not additional information that can be of relevance for a rational decision to take place. In our type search framework \( i \)'s true type can (should) be of relevance when choosing from the set of viable alternatives, as the true type can reveal the preferences of the individual; thus, \( i \)'s true type is not considered as a frame. A different situation emerges if instead of the true type a distinct adopted type is used to define the set of choices to be considered. In this case the DM's adopted type can lead her to select choices she would not had consider from the set \( X \) of all available choices, had she adopted her true type.

Thus, when frames are absent, under full information, the choice problem is determined by the set of alternatives that are precisely available to the DM and that are in her budget set. When this is not the case, frames constitute distractors that can lead choice behaviour in a biased manner. We introduce this here as an assumption that explicitly requires each possible consideration set to be attached to a particular type

**Assumption 2. [Consideration set by type]** Given a type \( \theta' \in \Theta \), there exists a unique set \( A_{\theta'} \subseteq \mathcal{X} \) to be referred to as \( \theta' \) consideration set, this set contains only the alternatives that those \( i \)'s of type \( \theta' \) will take into account, given that they are a type of agent \( \theta' \).

According to Assumption 2 to every \( i \)'s true and adopted type there should correspond a consideration set. It remains to show when an adopted type can be considered a frame, and if such type-frames lead to biased choices under all possible scenarios

**Proposition 3. [Adopted type as frame]** An adopted type \( \tilde{\theta}_i \) can be considered a frame \( f \), if and only if it is not equal to \( i \)'s true type, \( \tilde{\theta}_i \neq \theta_i \) and \( x \in \arg \max \succ_i \notin A_{\tilde{\theta}_i} \cap A_{\theta_i} \).

**Proof.** Assume \( \tilde{\theta}_i \neq \theta_i \), then by Assumption 2 \( A_{\tilde{\theta}_i} \neq A_{\theta_i} \). Two possibilities arise, either \( A_{\tilde{\theta}_i} \cap A_{\theta_i} = \emptyset \) or \( A_{\tilde{\theta}_i} \cap A_{\theta_i} \neq \emptyset \). If \( A_{\tilde{\theta}_i} \cap A_{\theta_i} \neq \emptyset \) then \( \forall x' \in \arg \max \succ_i (A_{\tilde{\theta}_i}) \), \( x' \notin \arg \max \succ_i (A_{\theta_i}) \), thus it is not possible for \( i \) to make an optimal choice having adopted a type different from her true type.

If \( A_{\tilde{\theta}_i} \cap A_{\theta_i} = \emptyset \) then for \( x' \in \arg \max (A_{\tilde{\theta}_i}) \) either \( x' \in (A_{\tilde{\theta}_i} \cap A_{\theta_i}) \) or \( x' \notin (A_{\tilde{\theta}_i} \cap A_{\theta_i}) \). If \( x' \in (A_{\tilde{\theta}_i} \cap A_{\theta_i}) \) then for \( x'' \in \arg \max \succ_i (A_{\theta_i}) \) if \( x'' \in (A_{\tilde{\theta}_i} \cap A_{\theta_i}) \) then it must be the case that \( x' = x'' \), that is \( \tilde{\theta}_i \) is not a frame. If on the contrary \( x'' \notin (A_{\tilde{\theta}_i} \cap A_{\theta_i}) \) then
\( i \) chooses \( x' \notin \arg\max \propto_i (A_i) \) when adopting a type \( \hat{\theta}_i \neq \theta_i \). This includes both cases \( A_i \subset A_{\hat{\theta}_i} \) and \( A_i \not\subset A_{\hat{\theta}_i} \).

Proposition 3 shows that although adopted type and true type mismatch is a necessary condition for inefficient choices to arise, it is not a sufficient condition on its own. In our framework even if the adopted type does not coincide with the true type, it loses its biasing power if the optimal choice under the absence of frame is still reachable. This case emerges when both sets \( A_{\hat{\theta}_i} \) and \( A_i \) have a non-empty intersection, and optimal choices on both lead to the same element. A direct implication for individual welfare from Proposition 3 is presented next.

**Proposition 4.** Define a type extended choice set as \( \{X, \theta\} \). If \( \hat{\theta}_i \in F \), then \( \hat{\theta}_i \neq \theta_i \) and \( \{X, \theta\} = \{X, \hat{\theta}\} \); if \( \{X, \theta\} = \{X\} \) then \( \hat{\theta}_i = \theta_i \). Given an adopted type \( \hat{\theta}_i \), \( \{X\} \propto_i \{X, \hat{\theta}\} \).

**Proof.** First notice that if \( A_{\hat{\theta}_i} = A_i \) then \( i \) must be indifferent between the two sets, as they contain exactly the same elements. This case is equivalent to the absence of frame. Now for all cases where \( A_{\hat{\theta}_i} \neq A_i \) \( i \) is indifferent between any of the two sets if \( A_{\hat{\theta}_i} \cap A_i \neq \emptyset \) and for \( x' \in \arg\max(A_{\hat{\theta}_i}) \) we also have \( x' \in \arg\max(A_i) \). For the rest of the cases with non-empty intersection \( x' \notin \arg\max(A_i) \). From Assumption 2 this implies that \( A_i \succ_i A_{\hat{\theta}_i} \). Putting these two outcomes together leads to conclude that \( A_i \propto_i A_{\hat{\theta}_i} \).

The result form Proposition 4 reveals that agents have a weak preference for true-type’s consideration sets over those consideration sets that do not correspond to the true-type. The interpretation lies in the fact that when only relevant option are available, there is no possibility for options outside the corresponding true-type consideration set to be considered. Thus an optimal element should be selected. In opposition, when such elements are present, the case where adopted types are effectively frames, non-optimal options are chosen affecting individual welfare.

### 2.8 Pending questions

1. Pr of adopting a particular type if a fn of the freq. with which each type is present in \( i \)’s environment.

2. If the threshold is not reached, then the agent moves to another environment. The agent chooses form the available environment the one that minimises the cost of moving.

3. The probability of choosing a given type depends on which parameters?

4. Under which circumstances subtypes or supertypes are choosen? That is, when adopted type is not going to be equal to the true type.
2.9 Comparative statics

2.10 Applications

3 Final comments

We presented a model in which agents’ true type is unknown and a search process leads to type adoption. This process can result in type mismatch which can lead to suboptimal choices if the type adopted functions as a frame.

The proven results show that with the exception of $\tilde{\theta}_i = \theta_i$, some other possibilities can lead to poor decision making. In such cases, $i$’s choices are made from $\tilde{A}_i \subset X$ that does not include some $x \in A_i$ such that $x \in \arg\max_{x \in A_i} \succ_i$, unless $x \in \tilde{A}_i \cap \hat{A}_i \neq \emptyset$ and this maximal elements coincide in the intersection. Although such an $x$ could exist, under a large number of possibilities this should be not the case for most of the choice problems.

The model just described can be applied to settings where search neighbourhoods are interpreted as social environments, and provides some explanation on how and why inefficiency traps (poverty traps) can emerge. Second, the model can also be applied to the labour markets, by introducing a employer, types mismatch can lead to underemployment and underrepresentation of female workers in certain positions.

In a choice problem where educational attainment has to be chosen, $i$ true type could correspond to certain level of education, lets say secondary level, if $i$ adopts a subtype then the education of choice could be primary, in which case $i$’s capacity would be sub-utilized. On the contrary, if a over-type is adopted, then $i$’s capacity would be over her limits, underperforming and leading to resource wasting.

The effects of polarization of environments, inequality in the distribution of initial characteristics, and the implications of different metrics defined over the set of types are left for further research.

References


4 Appendix

\[ \Gamma, \nu \]

\[ \theta \]

2  4