

# A Joint Theory of Polarization and Deunionization\*

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## Abstract

Over the past 50 years, the U.S. and several European labor markets have undergone two most incisive developments: job market polarization and deunionization. In this paper, we argue that routine-biased technical change is not only the driving force behind polarization, as prevalently assumed, but that routine-biased technical change is the common driving force behind both deunionization and polarization. In a search and matching framework with endogenous occupational and endogenous union membership choices, we show that the shift in employment and income shares in favor of high-skill and low-skill occupations worsens the bargaining position of unions, which crucially depends on the occupational structure inside a firm. This directly affects the membership choice of workers, who base their decision on the potential union wage premium. The ensuing deunionization provides further incentives for middle-wage workers to switch occupations and thus amplifies both job market polarization and the increasing income inequality.

*Keywords:* Labor markets, Inequality, Search and matching, Unions, Occupational choice

*JEL classification:* E02; E24; J51; J62; J64; O33

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## 1. Introduction

Job market polarization and deunionization have radically changed the labor market over the last decades. Job market polarization refers to the falling employment shares in middle-skill occupations and increasing shares in low-skill and high-skill occupations.<sup>1</sup> The share of middle-skill employment in the U.S. has been continuously decreasing and is now almost 10 percentage points below the value in the 1980s. Deunionization describes the ongoing decline in union membership rates and is accompanied by an increase in inequality.<sup>2</sup> According to the Union Membership and Coverage Database constructed by Hirsch and Macpherson and described in Hirsch and Macpherson (2003), U.S. private sector union membership rates declined from 24.2% in 1973 to 6.5% in 2017. This decline is present throughout various industries and occupations.

In this paper, we argue that job market polarization and deunionization have a common driving force in the form of routine-biased technical change. Figure 1 plots the relative price for investment goods, the employment share of workers in routine occupations and the union membership rate for U.S. data between 1955 and 2005.<sup>3</sup> The relative price of investment goods has decreased since the 1970s. Both the share of routine workers and the union membership rate have declined since the 1970s, with the rate of change increasing in the 1980s.<sup>4</sup> From 1990 onwards, the decline of both series slowed down again.

The prevalent explanation for polarization is the routinization hypothesis, which relies on the assumption that machines or computers replace middle-wage workers in occupations performing routine tasks.<sup>5</sup> The non-routine

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<sup>1</sup>Empirical studies on this phenomenon include the seminal work by Autor et al. (2006), Goos and Manning (2007), Goos et al. (2009), Autor and Dorn (2013), and Kerr et al. (2016) among many others.

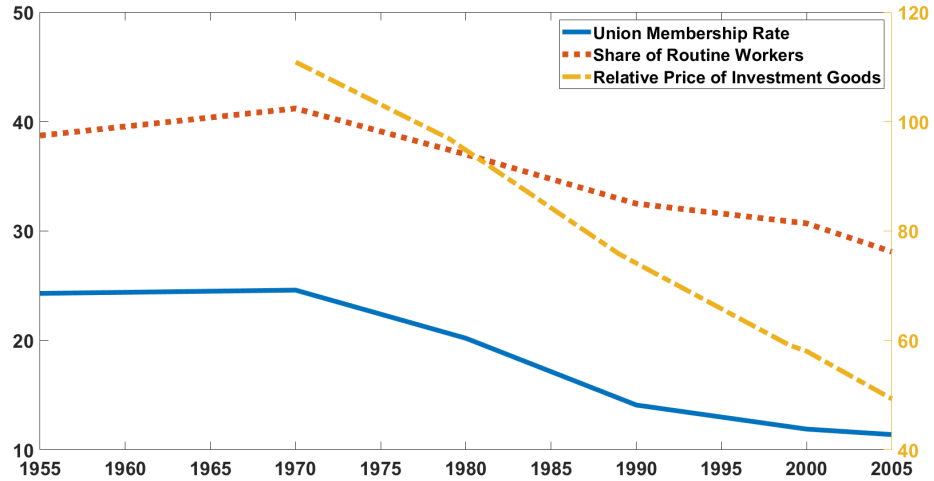
<sup>2</sup>Important contributions include Troy and Sheffin (1985), Waddington and Whitston (1997), Baldwin (2003), Checchi et al. (2010), and Frandsen (2012).

<sup>3</sup>The FRED series for the relative price of investment goods is measured as the investment deflator divided by the consumption deflator. The relative price for investment goods was chosen over the price for computer capital since data on the latter is more reliable and is available for a longer time period.

<sup>4</sup>Hubmer (2018) argues that there has been a substantial acceleration in the decrease of the relative price of equipment and software since 1982.

<sup>5</sup>See, for example, Autor et al. (2003), Autor et al. (2006), Autor and Dorn (2013), Michaels et al. (2014), and Feng and Graetz (2015). Other explanations are for example

Figure 1: Relative Price for Investment Goods, Share of Routine Workers, and U.S. Union Membership Rate



Note: The share of workers in routine occupations is constructed using the dataset and the occupational classification from Autor and Dorn (2013). Union membership rates are constructed using data from Mayer (2004), who merges data calculated by the CRS from the monthly Current Population Survey with data from the BLS and the Union Membership and Coverage Database. The relative price for investment goods is plotted as an index with 1977 = 100. Since computer capital played no major role before 1970, the series for the relative price of investment goods from the FRED is not displayed for the time period between 1955 and 1970.

nature of tasks performed by low-wage and high-wage workers means that their jobs are difficult to automate. Unlike for job polarization, no consensus has yet emerged regarding the source of deunionization.<sup>6</sup> Up until now, technical change as a cause for deunionization has received scant attention

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offshoring and changing institutions. Goos et al. (2009) develop and estimate a simple model to capture the effects of technology, globalization, institutions and product demand effects on the demand for different occupations in Western Europe. Their results suggest that the routinization hypothesis of Autor et al. (2003) is the single most important factor behind the observed shifts in employment structure.

<sup>6</sup>Explanations range from technical and organisational changes to globalisation, the decline of the manufacturing sector, the expansion of flexible forms of work, and population ageing.

in the literature.<sup>7</sup>

Deunionization and polarization have both proven to be especially harmful for middle-wage workers: job market polarization because the relative shifts in labor demand away from routine occupations have suppressed wage growth in that area, and deunionization because unionization rates are highest among middle-skill workers and those are also the workers that tend to be favored by union wage schedules. American middle class workers have been in focus for U.S. politicians not just since President Barack Obama declared himself "a warrior for the middle class" in his speech on the middle class on July 24, 2013. Even though the share of U.S. households classified as middle class by the American Institute for Economic Research has declined steadily since the 1980s, in 2013 still roughly 50% of households count as middle class. Thus, identifying and implementing suitable policies to support the middle class has become an ever more pressing issue for today's policymakers, especially considering the recent trends of political radicalization among this group.

To the best of our knowledge, we are the first to explain both job market polarization and deunionization by a common source and the first to combine routine-biased technical change with an endogenous union membership decision in an economic model. This allows us to study job market polarization and deunionization in a joint theoretical framework. While outlining our model setup, we give a detailed description of labor market facts on job market polarization, union structure, and deunionization that a theoretical model should take into account.

We introduce an endogenous occupational decision and an endogenous union membership decision into a search and matching model of the labor market. Workers are heterogeneous and differ with respect to their ability. When unemployed, previous routine workers can decide to remain routine workers or to switch to manual occupations. Similar to the structure proposed in Taschereau-Dumouchel (2017), employed workers decide via an election whether they want to form a union, and consequently a collective bargaining unit, or bargain individually about their wages with the firm.<sup>8</sup> If the majority of a bargaining unit votes in favor of the union, they receive

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<sup>7</sup>The few papers combining deunionization and technical change are Acemoglu et al. (2001), Açıkgöz and Kaymak (2014) and Dinlersoz and Greenwood (2016).

<sup>8</sup>A bargaining unit is commonly defined as a group of employees that shares a set of interests and may be reasonably represented by a collective bargaining agreement.

wage payments according to a union wage schedule through which the union distributes its share of the joint surplus.

The main mechanism behind our results is quite simple. Computer capital, which is able to replace routine tasks, becomes cheaper. This diminishes the demand for routine workers, whereas abstract and manual workers, who are complementary to routine tasks, are in great demand. The change in the labor demand structure implies that non-union wages for workers employed in manual and abstract tasks increase by more than non-union wages for routine workers. In line with the empirical literature, we assume that relatively unskilled middle-wage workers employed in routine occupations receive the highest wage premium. Manual workers, who benefit from the changing demand structure, are discouraged from voting in favor of a collective bargaining agreement. The wage gains for manual workers, that would be disproportionately given to routine workers by the union, lead to the least skilled workers being better off when bargaining individually with the firm.<sup>9</sup> Former routine workers, when faced with lower wages compared to manual workers, decide to switch occupations.

The model is calibrated to match U.S. data for the time period between 1977 and 2005. We simulate an economy with heterogeneous unions that differ with respect to their bargaining power. In such a setup, and in line with the empirical evidence, those unions with the lowest bargaining power and the lowest union wage premium will be the first to disappear. Hirsch and Schumacher (2004) estimate an increasing union wage premium in the early 1930s and in the late 1970s to early 1980s. Both periods were, according to Troy and Sheflin (1985), preceded by years with exceptionally large numbers of union termination. Thus, the model presented here is able to reconcile the falling union membership rates with a constant or even increasing union wage premium. Furthermore, since unionization rates among low-skilled and middle-skilled workers decrease, the average union member does not become less skilled over time. This accords with the empirical evidence in Farber et al. (2018), who show that union members became more rather than less skilled over the last decades. Models linking deunionization to skill-biased rather than routine-biased technical change, for example Acemoglu et al.

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<sup>9</sup>This is in line with Checchi et al. (2010) who argue that disillusion with respect to potential wage growth is the reason for declining membership rates among the least-skilled workers.

(2001), contrast strongly with this empirical observation.

Predicted changes in employment per capita, employment shares, and wages are all close to the data. Routine-biased technical change, through changes in the labor demand structure, leads to a drop of 18 percentage points in overall union density compared to a drop of 16 percentage points in the data. The simulation suggests that up to 25% of the changes in employment per capita for routine and manual workers are driven by deunionization, since the loss of their large union wage premia leads to switches of formerly unionized routine workers to manual occupations. In line with Frandsen (2012), Checchi et al. (2010), and DiNardo and Lee (2004), the overall effects of unions on inequality is small. Unions reduce inequality measured by the Palma ratio by 5% in our model, and roughly 7% of the simulated increase in the Palma ratio between 1977 and 2005 is caused by the termination of unions.<sup>10</sup> However, deunionization has substantial effects for the mid-wage workers favored by the union wage schedule. For the lower-skilled formerly unionized routine workers, the model predicts a wage increase of about 6% compared to an estimated counterfactual increase of over 14% in a scenario without deunionization. For that group of workers, over one third of the increase in the wage gap compared to the highest skilled workers is due to deunionization.

In our model, the way unions distribute their surplus across workers does not change over time. An adjustment of the wage schedule in favor of the lowest-skilled workers could lessen deunionization, polarization, and the increase in inequality. However, empirical evidence suggests that unions are troubled by rigid structures that partly prevent them from adjusting to recent developments on the labor market. Waddington (2005) argues that trade union practices are perceived as formal and old-fashioned and that the representative structures inside unions are often inappropriate for the participation of all members. This is in line with the evidence that the decline in membership rates can mostly be attributed to the failure of unions to recruit young members. While membership rates decline across all age groups, according to data from the Bureau of Labor Statistics, membership rates for workers aged between 16 and 24 declined at twice the rate of overall membership

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<sup>10</sup>The Palma ratio compares the richest 10% with the poorest 40%. This inequality measure is chosen over the Gini index because of the oversensitivity of the latter to changes in the middle of the income distribution.

between 2002 and 2012. Data on the evolution of the median age of union members points in the same direction. Dunn and Walker (2016) point out that over half of all U.S. union members are between 45 and 64 years of age. In addition, Bryson et al. (2016) argue that union representatives have long tenure and therefore do not represent the current membership composition. They show that unions in Britain have been slow to respond to current issues like gender equality. Thus, it seems that unions are mostly controlled and influenced by older members that might display a tendency to stick to established practices. The recent article "Technology may help to revive organised labour" in *The Economist* (2018) puts forth the argument that new technology could help unions to regain members. This argument is supported by evidence in Bryson et al. (2016) who argue that the decline in union membership rates across countries is strongly related to the degree of progressiveness of the unions. While the recent example of a union of Youtube employees that was formed by potential members joining a facebook group might be nothing more than a marketing gag, it seems that a more modern and progressive structure is needed in order for unions to attract more and especially younger members.

The remainder of the paper is organized as follows. Previous research and the links between job market polarization and deunionization are discussed in the next section. The model is presented in Section 4 and Section 5 describes the quantitative analysis in detail. Policy implications are discussed in Section 6. To conclude, the results are summarized in Section 7.

## **2. Linking Polarization and Deunionization**

Since both polarization and deunionization are prevalent topics in the empirical literature, we are not the first to think about potential linkages between these two phenomena. Autor (2010) argues that while unions did contribute to the changing employment patterns, it is unlikely that deunionization is one of the main causes of job market polarization due to the fact that unions only have a very limited ability to affect employment levels. Goos et al. (2009) find that changes in general wage-setting institutions play only a minor role in explaining job polarization. However, as Firpo et al. (2009) propose, deunionization might have played a larger role for wage polarization: since unions tend to compress the distribution of earnings, falling unionization rates might lead to a widening of the wage gap. While it seems unlikely that deunionization caused job market polarization, the reverse appears to

be more plausible. With jobs and workers in the middle of the skill distribution disappearing, coalitions between workers of different skill groups are likely to become harder to maintain.<sup>11</sup>

We argue that job market polarization and deunionization have a common cause in routine-biased technical change. Overall union membership rates in the U.S. began to decrease in the late 1950s, which is usually explained by political resistance and the sharp increase in labor force participation of women who tend to be less unionized.<sup>12</sup> However, the number of private sector union members increased until the 1970s, with the increase in the 1960s being similar in size to the increase in the 1940s. Furthermore, the statistics on union creation and termination in Troy and Sheflin (1985) show that in no year since the late 1890s were more unions started than in 1970. The most terminations in recent decades are observed in 1980, while in the 1950s and 1960s almost no unions were terminated. This evidence on the declining union membership rates fits well with the starting point of job polarization. Job polarization, and to a lesser extent also wage polarization can be observed in the U.S. and several European countries at least since the 1980s.

Figure 2 plots the polarization indicator developed in Duclos et al. (2004) against the collective bargaining coverage for the U.S. and several European countries.<sup>13</sup> Despite the small sample size, the negative coefficient in the OLS regression of the polarization indicator on the collective bargaining coverage is statistically significant at the 5%-level. The coefficient of determination,  $R^2$ , is equal to 0.63.

Since the 1980s, the decline in U.S. union density has accelerated perceptibly. At about the same time, union membership rates began to decline

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<sup>11</sup>The former link has been put forward in multiple studies including but not limited to Alderson and Nielsen (2002), DiNardo et al. (1996), Freeman (1980), and Rueda and Pontusson (2000). The latter approach has received only very little attention in the literature.

<sup>12</sup>See, for example, Oh (1989) and Troy and Sheflin (1985).

<sup>13</sup>In contrast to the U.S., the differences between union membership rates and the percentage of workers covered by a collective bargaining agreement are large for most of the European countries. Thus, when looking at union influence, the share of workers covered by a collective bargaining unit seems to be more appropriate. The results also hold when exchanging the collective bargaining coverage for union density. The results are very similar when using changes in collective bargaining coverage instead of collective bargaining coverage.



Figure 2: Polarization and Collective Bargaining Coverage across Countries

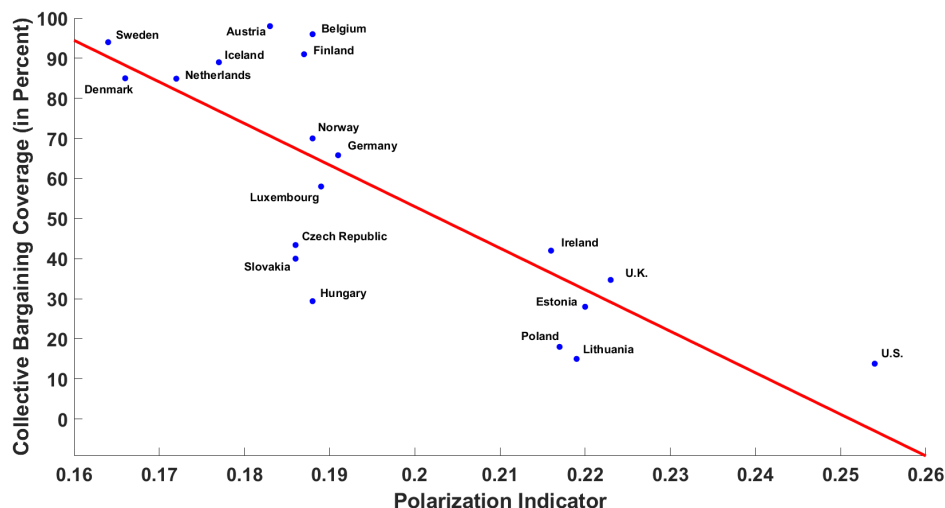


Figure 2 plots the polarization indicator developed in Duclos et al. (2004) against the collective bargaining coverage for the U.S. and several European countries. For all countries the polarization indicator is calculated for the year 2004. The collective bargaining coverage is the share of employed workers covered by a collective bargaining agreement in 2004 from the OECD data. The red line is the result of an OLS regression of the polarization indicator on the collective bargaining coverage. The coefficient of determination,  $R^2$ , is 0.63.

in many European countries. The decline in union membership rates is on average more pronounced in countries with larger degrees of job and wage polarization. This is visible when comparing the U.S. to Europe or Canada, but also within the group of European countries. The Nordic countries, which experienced upgrading rather than polarization, exhibit constant or even increasing union membership rates. Relatively stable union membership rates can be observed for Canada. In accordance to that observation, Green and Sand (2015) show that until 2005 the Canadian wage pattern exhibits increasing inequality with greater growth in high paid than middle paid occupations and greater growth in middle than low paid occupations.

The evidence presented in this section exposes that, contrary to the common believe, there is little discrepancy in timing between the two phenomena of job market polarization and deunionization. This motivates us to study the effect of routine-biased technical change on both phenomena in a joint theoretical framework. Our focus lies on the way in which technology influences

the occupational choice of workers and how this is reflected in the bargaining between firms, individual workers, and unions. For that reason, we employ a search and matching model à la Mortensen and Pissarides (1994) with heterogeneous workers, endogenous occupational choice, and endogenous union membership choice. The positive match surplus due to search frictions allows us to examine different bargaining regimes. Our work bridges the gap between the literature that deals with the changing employment structures and the literature on deunionization.

The empirical literature on job market polarization is quite extensive, starting with the seminal work by Autor et al. (2006) that first documents stronger wage and employment growth for low-wage and high-wage occupations in the U.S. Other influential papers include Goos and Manning (2007), Goos et al. (2009), and Autor and Dorn (2013). These papers establish the presence of job polarization for the UK, across European countries, and in the U.S., respectively. However, few studies analyze these developments in a theoretical framework. We follow the theoretical framework of Albertini et al. (2017), who develop a multi-sectoral search and matching model with endogenous occupational choice to examine the impact of task-biased technical change. Other related work includes Jaimovich and Siu (2012), Zago (2017), Nellas and Olivieri (2011), and von Brasch et al. (2018). Jaimovich and Siu (2012) use a simple search and matching model with occupational choice and routine-biased technical change to explain job polarization and jobless recoveries. Zago (2017) studies the effect of job polarization and the Great Recession on the allocation of skills across occupations in a model which combines elements of a Diamond-Mortensen-Pissarides search and matching framework with a model of cross-skill mismatch. Nellas and Olivieri (2011) analyze the joint effect of technology and institutions on labor market changes. von Brasch et al. (2018) analyze the effect of deunionization on job polarization in a small open economy model.

Technical change as a source for deunionization has received very little attention in the literature. Acemoglu et al. (2001) show that skill-biased technical change can trigger deunionization by increasing the outside option of skilled workers. In their model, deunionization is entirely driven by quitting high-skilled workers. This stands in sharp contrast to the empirical evidence in Açıkgöz and Kaymak (2014) and Farber et al. (2018). Açıkgöz and Kaymak (2014) show that union coverage declines over all skill types, while Farber et al. (2018) provide evidence for union members becoming more and not less skilled over time. Açıkgöz and Kaymak (2014) are the

first to study deunionization in a search and matching framework with an endogenous union membership decision. In their model it is a rise in the skill premium that encourages the most skilled workers to leave the union, while unions themselves decide to get rid of the least skilled workers. Dinlersoz and Greenwood (2016) focus on the connection between technology, unionization, and inequality. In a general equilibrium model of unionization with heterogeneous firms, skilled, and unskilled labor, they show that when the productivity of unskilled labor is high, the union decides to organize a lot of firms and demands generous wages for its members. However, both papers have troubles explaining the observation in Farber et al. (2018) that union members become more and not less skilled over time. In Dinlersoz and Greenwood (2016), union members are only drawn from the low-skilled workers. However, the inclusion of union members of other skill types would, as in Acemoglu et al. (2001) and in basically any model of skill-biased technical change, lead to union members becoming less skilled over time. In (Açıkgöz and Kaymak, 2014), unions decide not to represent the least-skilled workers as a consequence of skill-biased technical change. This contrasts with the idea of an industrial union that covers workers of different skills and with evidence in Checchi et al. (2010) who argue that disillusion about potential wage growth is the main driving force behind declining union membership rates among the least-skilled workers. In our model low-skilled workers endogenously decide to vote against union coverage based on economic incentives.

A large literature analyzes search and matching models à la Mortensen and Pissarides (1994) with collective wage bargaining, but only a few are dealing with the phenomenon of deunionization. Pissarides (1986) introduces a monopoly union into the Pissarides (1985) framework, and studies the impact on equilibrium outcomes in the labor market. Ortigueira (2013) provides an explanation for the initial establishment of collective wage bargaining and deunionization in a search and matching model. In his model, the setup of unions and its collapse can be accounted for in terms of the interplay of fiscal and technological links among different types of workers. In Taschereau-Dumouchel (2017) unions are created by a majority vote within a firm. The possibility of unionization distorts the behavior of non-union firms, who over-hire high-skill workers, who vote against the union. Bauer and Lingens (2010) study the welfare and employment effects of individual versus collective bargaining in a large firm search model with homogeneous workers. Krusell and Rudanko (2016) analyze a labor market with search and matching frictions, homogeneous workers, and decreasing returns to scale

in production where wage setting is characterized by collective bargaining. They find that wage solidarity leaves the unionized labor market vulnerable to potentially substantial distortions due to hold-up. Garibaldi and Violante (2005) and Boeri and Burda (2009) study the effects of employment protection policies, and Ebell and Haefke (2009) the effects of product market regulation. These papers generally introduce unions as an explanation for exogenous wage compression.

### **3. Unions in the U.S.**

This section provides a brief overview of how labor unions work in the U.S. These institutional features will be used when setting up the model in Section 4.

In the U.S., unions base their right to represent workers through collective bargaining on the voting decision of a so called bargaining unit. The National Labor Relations Act (NLRA) specifies the structure through which union organization and legal recognition takes place. This structure focuses on a system of elections to determine whether a majority of employees in the workplace wants to be represented by a union. The union then becomes the exclusive representative of all employees in the bargaining unit, whether they are union members or not. If a majority of the employees votes against union representation, the unit is not represented by the union no matter if workers individually choose to be union members or not.

The NLRA stipulates that only a union that demonstrates majority support in an appropriate bargaining unit can be certified as the collective bargaining representative. An appropriate bargaining unit is a group of employees in a workplace who meet the legal test of sufficient community of interest to be represented by the union, whereby managers and supervisors are excluded from any bargaining unit. According to the National Labor Relations Board (NLRB), professional employees who engage in predominantly intellectual and not in routine mental, manual or mechanical work are excluded from bargaining units with manual and routine workers, since they do not share a community of interests. Furthermore, the unionization rate for high-skilled abstract workers has stayed roughly constant over the last decades, while estimates of the union wage premium for these workers tend to be close to zero or even negative. As pointed out by Checchi et al. (2010), the reason for union membership among the highest-skilled are mostly non-monetary and related to their normative views on inequality.

The structure of bargaining in the U.S. is highly decentralized, with the estimated number of separate collective bargaining agreements in the U.S. ranging between 170000 and 190000 according to the Bureau of Labor Statistics. With regard to the prevalent union type in the U.S., Oh (1989) documents a steady decrease in the importance of craft unions and an increase in the importance of industrial unions. While the former is mostly limited to workers of a specific craft (and therefore of a specific skill group), the latter aims at including all workers employed in certain industries (and therefore covers workers of different skill groups). Moreover, most collective bargaining in the private sector takes place at the level of the individual firm.<sup>14</sup>

#### 4. Model

In this section, we present a discrete time search and matching model with an endogenous occupational and an endogenous union membership choice. Workers are heterogeneous, differ with respect to their uniformly distributed ability  $\eta$ , and are born as either manual, routine, or abstract workers. For each ability level there is a continuum of workers. As depicted in Figure 3, when unemployed, workers formerly employed in routine tasks can choose to switch occupations and join the unemployment pool of manual workers.<sup>15</sup> Following Albertini et al. (2017), workers have homogeneous skills at performing manual tasks. This is consistent with the view that blue-collar workers differ in their ability to perform tasks on the assembly line, while for manual workers like janitors differences in ability do not translate into differences in productivity on the job. Abstract workers are assumed to be homogenous with respect to their ability to perform abstract tasks. In line with Smith (2013), who shows that the increase in abstract employment is mainly driven by increased educational attainment and not by occupational switches, labor supply of abstract workers is assumed to increase exogenously. In our model, in the spirit of Taschereau-Dumouchel (2017), unions arise endogenously through elections within firms.<sup>16</sup> When a sim-

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<sup>14</sup>See, for example, Traxler (1994) and Nickell and Layard (1999).

<sup>15</sup>To ease notation, and in line with the empirical evidence in Smith (2013), we abstract from other switches. Thus, in our model there will be 'overqualified' routine workers in manual occupations but we rule out the case of 'underqualified' manual workers in routine occupations.

<sup>16</sup>As the production function features constant returns to scale, in contrast to Taschereau-Dumouchel (2017), firms have no incentive to overhire high-wage and low-

ple majority of the respective bargaining unit votes in favor of a union, a collective bargaining agreement is formed and wages are bargained collectively between the respective firm and the union. The collective bargaining agreement covers all workers in the bargaining unit, regardless of whether or not the worker votes in favor of the union.<sup>17</sup> In our model deunionization works through within-industry shifts in unionization rates rather than between-industry shifts in employment. This is consistent with the evidence presented in Baldwin (2003) who shows that the decline in unionization rates is not mainly caused by employment shifting from highly unionized industries to industries with low unionization rates, but rather by decreases in union membership rates within the respective industries.<sup>18</sup> A firm uses computer technology  $K$  and workers in abstract  $L^a(\eta)$ , routine  $L^r(\eta)$  and manual jobs  $L^m(\eta)$  as input factors.<sup>19</sup> Routine workers can be substituted by computer technology  $K$ , whereas abstract and manual workers are complementary to routine tasks. Routine-biased technical change is introduced through falling computer capital prices.

#### 4.1. Labor Market Frictions

Labor markets are characterized by search and matching frictions à la Mortensen and Pissarides (1994). Search is directed, as there are labor submarkets for each of the three occupations. Within each pool, vacancies and unemployed workers are matched randomly in any period and firms learn about the ability level of a worker upon matching. Given the number of

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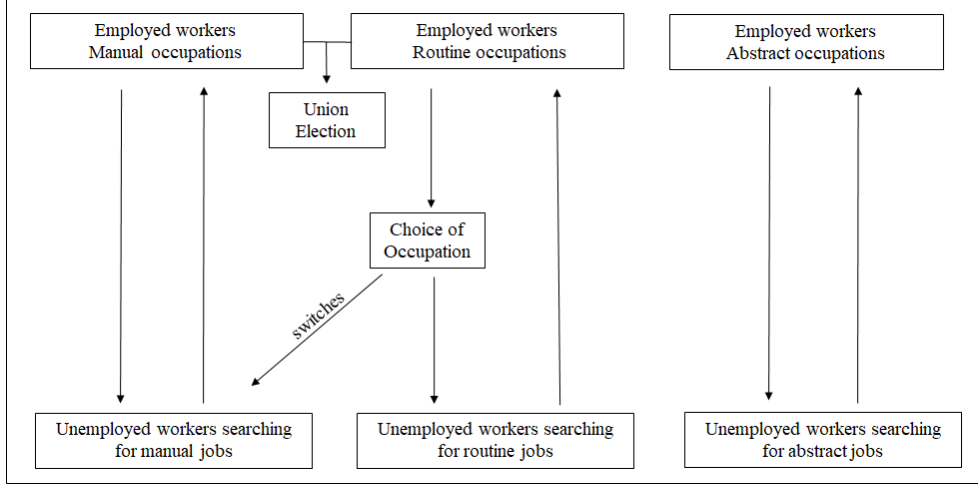
wage, and underhire middle-wage workers in our model.

<sup>17</sup>In the simulation the bargaining unit will, as depicted in Figure 3, consist of all manual and routine workers. However, the general model setup presented here allows for a wide range of different bargaining units. Subsection 4.6 takes a closer look at our specific choice of the bargaining unit.

<sup>18</sup>Data from the union database constructed by Hirsch and Macpherson (2003) provides evidence for the view that this is not only true for industry level data but does also hold at the occupational level.

<sup>19</sup>Evidence presented in Kerr et al. (2016) suggests that within-firm polarization is at least as important as between-firm polarization. Tüzemen and Willis (2013) show that job market polarization is mainly driven by changes in employment composition within industries, with changes across industries accounting only for a minor part. Contrary to conventional modeling strategies, the largest part of polarization is not explained by shifts away from industries such as manufacturing but by shifts from middle-skilled to low- and high-skilled jobs within industries

Figure 3: Labor Market Flows



vacancies  $v_i$  posted and the share of unemployed workers  $u_i$  for every occupation, the number of matches is determined by the following Cobb-Douglas matching technology

$$m_i = \Psi v_i^\psi u_i^{1-\psi} \text{ where } 0 < \psi < 1 \text{ and } i = a, r, m.$$

A vacancy is filled with probability  $q_i = \frac{m_i}{v_i}$  and the job finding probability is  $f_i = \frac{m_i}{u_i}$ . The labor market tightness is defined as the ratio  $\theta_i \equiv \frac{v_i}{u_i}$ . When the labor market is tight, many firms compete for few unemployed workers. The job finding probability is high but the job filling rate is low.

#### 4.2. Occupational Choice

Workers can either be employed in abstract, routine, or manual tasks. Existing jobs are destroyed at the exogenous rates  $s_i$ , with  $i = a, r, m$ . When fired, routine unemployed workers can choose to remain routine workers or to switch occupations and join the pool of unemployed workers looking for manual jobs. The value functions for union workers employed in the respective occupations are given by

$$\begin{aligned}
W_a^u(\eta) &= w_a^u(\eta) + \beta[(1 - s_a)(1_{u,+1}W_{a,+1}^u(\eta) + (1 - 1_{u,+1})W_{a,+1}^n(\eta)) \\
&\quad + s_a U_{a,+1}(\eta)], \\
W_r^u(\eta) &= w_r^u(\eta) + \beta [(1 - s_r)(1_{u,+1}W_{r,+1}^u(\eta) + (1 - 1_{u,+1})W_{r,+1}^n(\eta))] \\
&\quad + \beta s_r \max \{U_{m,+1}(\eta), U_{r,+1}(\eta)\}, \\
W_m^u(\eta) &= w_m^u(\eta) + \beta[(1 - s_m)(1_{u,+1}W_{m,+1}^u(\eta) + (1 - 1_{u,+1})W_{m,+1}^n(\eta)) \\
&\quad + s_m U_{m,+1}(\eta)],
\end{aligned}$$

where  $w_a^u(\eta)$ ,  $w_r^u(\eta)$  and  $w_m^u(\eta)$  denote the wage received by a union worker and  $w_a^n(\eta)$ ,  $w_r^n(\eta)$  and  $w_m^n(\eta)$  the wage received by a non-union worker with ability  $\eta$  in abstract, routine and manual tasks respectively.  $1_u$  is an indicator function with  $1_u = 1$  if and only if the worker is a union member. Thus, the term  $1_{u,+1}$  indicates whether a worker in the firm is covered by a collective bargaining regime in the next period. The term  $\max \{U_{m,+1}(\eta), U_{r,+1}(\eta)\}$  governs the occupational choice of routine workers when unemployed in the next period. The non-union workers' value functions are given by

$$\begin{aligned}
W_a^n(\eta) &= w_a^n(\eta) + \beta[(1 - s_a)(1_{u,+1}W_{a,+1}^u(\eta) + (1 - 1_{u,+1})W_{a,+1}^n(\eta)) \\
&\quad + s_a U_{a,+1}(\eta)], \\
W_r^n(\eta) &= w_r^n(\eta) + \beta [(1 - s_r)(1_{u,+1}W_{r,+1}^u(\eta) + (1 - 1_{u,+1})W_{r,+1}^n(\eta))] \\
&\quad + \beta s_r \max \{U_{r,+1}(\eta), U_{m,+1}(\eta)\}, \\
W_m^n(\eta) &= w_m^n(\eta) + \beta[(1 - s_m)(1_{u,+1}W_{a,+1}^u(\eta) + (1 - 1_{u,+1})W_{a,+1}^n(\eta)) \\
&\quad + s_m U_{m,+1}(\eta)].
\end{aligned}$$

When unemployed, workers lose their union membership.<sup>20</sup> Therefore, the value functions for unemployed workers are identical for former union and former non-union members and given by

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<sup>20</sup>This is in line with Lewis (1989) who finds that unions are not perceived to represent the interests of the unemployed.



$$\begin{aligned}
U_a(\eta) &= z_a(\eta) + \beta[(1 - f_a(\eta))U_{a,+1} + f_a(\eta)(1_{u,+1}W_{a,+1}^u(\eta) \\
&\quad + (1 - 1_{u,+1})W_{a,+1}^n(\eta))], \\
U_r(\eta) &= z_r(\eta) + \beta[(1 - f_r(\eta)) \max \{U_{m,+1}^n(\eta), U_{r,+1}^n(\eta)\} + f_r(\eta)(1_{u,+1}W_{r,+1}^u(\eta) \\
&\quad + (1 - 1_{u,+1})W_{r,+1}^n(\eta))], \\
U_m(\eta) &= z_m(\eta) + \beta[(1 - f_m(\eta))U_{m,+1} + f_m(\eta)(1_{u,+1}W_{m,+1}^u(\eta) \\
&\quad + (1 - 1_{u,+1})W_{m,+1}^n(\eta))],
\end{aligned}$$

where  $z_a(\eta)$ ,  $z_r(\eta)$  and  $z_m(\eta)$  denote the unemployment benefits received by abstract, routine and manual workers with ability  $\eta$ .

#### 4.3. Firms

Good-producing firms use three intermediates goods,  $Z_a$ ,  $Z_r$  and  $Z_m$ , as input factors to produce the final product  $Y$ .  $Z_a$  is produced with abstract jobs  $L^a$ ,  $Z_r$  with computer technology  $K$  and routine workers  $L^r(\eta)$  and  $Z_m$  with manual jobs  $L^m(\eta)$ . Routine workers and computer technology  $K$  are close substitutes, whereas abstract and manual workers are complementary to routine tasks. Due to constant returns to scale in production, the firms' maximization problem can be solved in two steps. The firms problem is given by<sup>21</sup>

$$\begin{aligned}
\Pi &= \max\{Y - p_{Z_a}Z_a - p_{Z_r}Z_r - p_{Z_m}Z_m\} \\
\text{s.t. } Y &\leq [(AZ_a^\alpha Z_r^{1-\alpha})^\rho + (A_m Z_m)^\rho]^{1/\rho}.
\end{aligned}$$

Firms maximize profits by choosing employment next period and the number of vacancies to be posted, subject to the firm-level employment constraint. Job creation comes at a flow cost of  $c_a$ ,  $c_r$ , and  $c_m$ . The behavior of firms in producing the intermediate good  $Z_a$  using workers in abstract non-routine cognitive jobs  $L_a$  is described by

$$\Pi^{Z_a} = \max \left\{ p_{Z_a}Z_a - 1_u w_a^u L_a - (1 - 1_u) w_a^n L_a - c_a v_a + \beta \Pi_{+1}^{Z_a} \right\}$$

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<sup>21</sup>This nested production function is chosen in order to generate larger complementarity in production between abstract and routine than between routine and manual tasks.

$$\begin{aligned} \text{s.t. } Z_a &\leq L_a \\ L_{a,+1} &= (1 - s_a)L_a + q_a v_a, \end{aligned}$$

$L_{a,+1}$  denotes the total abstract workforce next period.  $1_u$  is again the indicator function with  $1_u = 1$  indicating if the workforce in the firm is covered by a collective bargaining regime.

The behavior of firm in producing the intermediate good  $Z_r$  using workers in routine tasks  $L_r(\eta)$  and computer technology  $K$  is described by

$$\begin{aligned} \Pi^{Z_r} &= \max \left\{ p_{Z_r} Z_r - p_K K - 1_u \int_{\eta_m}^{\eta_a} w_r^u(\eta) L_r(\eta) - (1 - 1_u) \int_{\eta_m}^{\eta_a} w_r^n(\eta) L_r(\eta) \right. \\ &\quad \left. - c_r v_r + \beta \Pi_{+1}^{Z_r} \right\} \\ \text{s.t. } Z_r &\leq \left[ \left( (1 - \mu) \int_{\eta_m}^{\eta_a} \eta L_r(\eta) \right)^\sigma + (\mu K)^\sigma \right]^{\frac{1}{\sigma}} \\ L_{r,+1} &= (1 - s_r) L_r + q_r v_r \end{aligned}$$

where  $\eta_a$  denotes the exogenous ability threshold between workers in routine and abstract tasks and  $\eta_m$  the endogenous ability threshold between manual and routine workers. Workers born with an ability level greater than  $\eta_m$  but smaller than  $\eta_a$  work in routine occupations.

The behavior of firms in producing the intermediate good  $Z_m$  using workers in non-routine manual tasks  $L_m$  is described by

$$\begin{aligned} \Pi^{Z_m} &= \max \left\{ p_{Z_m} Z_m - 1_u w_m^u L_m - (1 - 1_u) w_m^n L_m - c_m v_m + \beta \Pi_{+1}^{Z_m} \right\} \\ \text{s.t. } Z_m &\leq L_m \\ L_{m,+1} &= (1 - s_m) L_m + q_m v_m. \end{aligned}$$

#### 4.4. First Order Conditions

Defining the value of a marginal worker in a abstract non-routine cognitive occupation for a firm as  $J_a$ , the first-order conditions for hiring and for vacancy posting are given by

$$\begin{aligned} c_a &= \mu_a q_a, \\ \mu_a &= \beta J_{a,+1}, \end{aligned}$$

where  $\mu_a$  is the Lagrange-multiplier on the employment constraint for workers in abstract occupations. The corresponding value of a marginal worker in abstract non-routine cognitive occupations for a firm is given by

$$J_a = p_{Z_a} - 1_u w_a^u - (1 - 1_u) w_a^n + (1 - s_a) \beta J_{a,+1}.$$

Defining the value of a marginal worker in a routine occupation for a firm as  $J_r$ , the first-order conditions for hiring workers in routine tasks and for vacancy posting are given by

$$\begin{aligned} c_r &= \mu_r q_r \\ \mu_r &= \beta J_{r,+1}, \end{aligned}$$

where  $\mu_r$  is the Lagrange-multiplier on the employment constraint for a worker in routine occupations. The corresponding value of a marginal worker in routine occupations for a firm is given by

$$\begin{aligned} J_r &= p_{Z_r} \bar{y}_r - 1_u \bar{w}_r^u - (1 - 1_u) \bar{w}_r^n + (1 - s_r) \beta J_{r,+1}, \\ \text{with } y_r(\eta) &= \frac{\partial Z_r}{\partial L_r(\eta)} = \eta(1 - \mu)^\sigma [(1 - \mu)^\sigma + (\mu k)^\sigma]^{\frac{1}{\sigma} - 1} \text{ and } k \equiv \frac{K}{\int_{\eta_m}^{\eta_a} \eta L_r(\eta)}, \end{aligned}$$

where  $\bar{y}_r$  is the expected marginal product of a routine worker,  $\bar{w}_r^u$  is the expected union wage, and  $\bar{w}_r^n$  the expected non-union wage.

Defining the value of a marginal worker with ability  $\eta$  in a non-routine manual occupation for a firm as  $J_m$ , the first-order conditions for hiring workers in manual tasks and for vacancy posting are given by

$$\begin{aligned} c_m &= \mu_m q_m, \\ \mu_m &= \beta J_{m,+1}, \end{aligned}$$

where  $\mu_m$  is the Lagrange-multiplier on the employment constraint for worker in manual occupations. The corresponding value of a marginal worker with ability  $\eta$  in manual occupations for a firm is given by

$$J_m = p_{Z_m} - 1_u w_m^u - (1 - 1_u) w_m^n + (1 - s_m) \beta J_{m,+1}.$$

#### 4.5. Job Creation Conditions

Since firms are subject to vacancy posting costs, the Job Creation conditions are given by

$$\begin{aligned}\frac{\tilde{c}}{\tilde{q}} &= \beta \tilde{J}_{+1} \\ \text{with } \tilde{c} &= c_a, c_r, c_m, \\ \tilde{q} &= q_a, q_r, q_m, \\ \tilde{J}_{+1} &= J_{a,+1}, J_{r,+1}, J_{m,+1}.\end{aligned}$$

Together with the values of marginal workers for firms, it follows that

$$\begin{aligned}\frac{c_a}{q_a} &= \beta \left[ p_{Z_a} - 1_{u,+1} w_a^u - (1 - 1_{u,+1}) w_a^n + (1 - s_a) \frac{c_a}{q_{a,+1}} \right], \\ \frac{c_r}{q_r} &= \beta \left[ p_{Z_r} \bar{y}_r - 1_{u,+1} \bar{w}_r^u - (1 - 1_{u,+1}) \bar{w}_r^n + (1 - s_r) \frac{c_r}{q_{r,+1}} \right], \\ \frac{c_m}{q_m} &= \beta \left[ p_{Z_m} - 1_{u,+1} w_m^u - (1 - 1_{u,+1}) w_m^n + (1 - s_m) \frac{c_m}{q_{m,+1}} \right].\end{aligned}$$

As we are mainly interested in the long-run effect of routine-biased technical change on the economy and on the wage bargaining regimes, we focus on the steady state of the economy. The steady state job creation conditions are given by

$$\begin{aligned}\frac{c_a}{q_a} &= \beta \left[ p_{Z_a} - 1_u w_a^u - (1 - 1_u) w_a^n + (1 - s_a) \frac{c_a}{q_a} \right], \\ \frac{c_r}{q_r} &= \beta \left[ p_{Z_r} \bar{y}_r - 1_u \bar{w}_r^u - (1 - 1_u) \bar{w}_r^n + (1 - s_r) \frac{c_r}{q_r} \right], \\ \frac{c_m}{q_m} &= \beta \left[ p_{Z_m} - 1_u w_m^u - (1 - 1_n) w_m^n + (1 - s_m) \frac{c_m}{q_m} \right].\end{aligned}$$

A firm hires workers of each type and each ability level  $\eta$  until the costs of labor are equal to the discounted expected marginal product. Here the costs consist of the vacancy posting costs plus the discounted expected wage minus the discounted cost of hiring next period.

#### 4.6. Wage Bargaining Regimes

We integrate features of the institutional environment for U.S. unions into the model. Workers can decide to form a union on the firm level which bargains with the firm and distributes the surplus according to a union wage schedule. Once a firm has hired its new workers, all manual and routine workers vote to decide whether to form a union or not. Abstract workers are excluded from the collective bargaining unit as they are generally not part of a bargaining unit consisting of manual and routine workers. The voting decision of an individual worker is endogenously determined and depends directly upon the potential union wage premium. Workers vote in favor of a union if the value of being a worker in a unionized firm is higher than the value of being a worker in a non-unionized firm, meaning when their respective union wage premium is positive. Therefore, a worker with ability  $\eta$ , taking the job finding rate as given<sup>22</sup>, votes in favor of a union if

$$W_i^u(\eta) > W_i^n(\eta), \text{ with } i = r, m.$$

There are two ability thresholds in the model regarding the union voting decision of workers denoted by  $\eta_{low}^u$  and  $\eta_{high}^u$  (with  $\eta_{high}^u > \eta_{low}^u$ ). All workers with ability levels between these two thresholds receive a positive union wage premium and decide to vote in favor of the union. Thus, whether a union is established or not depends crucially on the composition of the workforce in a firm. It follows that the model is characterized by the two different types of thresholds depicted in Figure 4: one concerning the occupational choice of workers and one concerning the union membership decision.

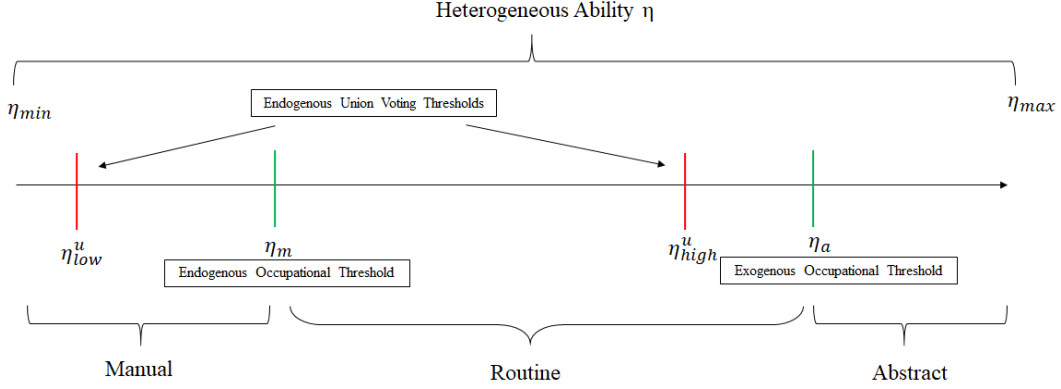
When a simple majority of the workers votes in favor of unionization, a union is created and wages are bargained collectively between the firm and all of its manual and routine workers.<sup>23</sup> The union then becomes the exclusive representative of all manual and routine workers and the collective bargaining agreement covers all manual and routine workers, regardless of whether or not the individual worker voted in favor of the union. If the majority of the bargaining unit votes against a collective bargaining agreement, manual and routine workers are not represented by the union and wages are negotiated

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<sup>22</sup>Since union terminations occur when the wage premium is close to zero, this assumption simplifies computation considerably without changing the results.

<sup>23</sup>The wages of abstract workers will still be bargained individually between these workers and the firm.

Figure 4: Occupational and Union Membership Choice



Note: The exact position of the voting thresholds crucially depends upon the union bargaining power and the union wage schedule.

individually. Union and non-union wages are both determined by generalized Nash bargaining over the match surplus. However, the surplus that is bargained over differs between the two bargaining regimes. Non-union workers bargain individually over their marginal product. The union bargains over the entire match surplus of all manual and routine workers, with surpluses accruing to the matched parties being split according to a rule that maximizes the weighted average of the respective surpluses.

### *Individual Bargaining*

If a majority of the manual and routine workers votes against a union, each worker bargains individually with the firm. Denoting the worker's weight in the bargaining process by  $\gamma^n \in [0, 1]$ , this implies the following sharing rule for individual bargaining

$$W_i^n(\eta) - U_i(\eta) = \frac{\gamma^n}{1 - \gamma^n} J_i^n(\eta),$$

with  $i = a, r, m$ ,

where  $W_i^n(\eta)$  is the asset value of employment for non-union members,  $U_i(\eta)$  is the value of being unemployed, and  $J_i^n(\eta)$  is the value of the marginal non-union worker of type  $i$  and ability  $\eta$  to the firm. This results in the wage

schedules for the three occupational types given below.<sup>24</sup>

Abstract Jobs:

$$w_a^n = \gamma^n p_{Z_a} + \gamma^n c_a \theta_a + (1 - \gamma^n) z_a$$

Routine Jobs:

$$w_r^n(\eta) = \gamma^n p_{Z_r} y_r(\eta) + \gamma^n c_r \theta_r + (1 - \gamma^n) z_r(\eta)$$

Manual Jobs:

$$w_m^n = \gamma^n p_{Z_m} + \gamma^n c_m \theta_m + (1 - \gamma^n) z_m$$

It follows that the wages resulting from individual bargaining are given by the sum of the marginal productivity of every  $\eta$  worker in every occupation, the search returns, and the outside option. This result is identical to the Nash-bargained wage in a standard Mortensen-Pissarides search and matching model.

As outlined above, we consider the influence of routine-biased technical change on the occupational choice and unionization decision of workers. Crucial for these decisions is the influence of routine-biased technical change on both bargaining regimes. When considering the wage schedules resulting from individual bargaining for manual, routine, and abstract workers, the positive relationship between labor market tightness and non-union wages is striking. This relationship implies that workers receive higher wages when they are relatively scarce. Routine-biased technical change reduces the demand for routine workers and increases the demand for manual and abstract workers. The total effect is a decline in both relative tightness and relative wages for routine workers and an increase in both relative tightness and relative wages for manual and abstract workers.

### *Collective Bargaining*

We consider a union which negotiates wages on behalf of both manual and routine workers within a firm. If the bargaining unit votes in favor of a

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<sup>24</sup>See Appendix A for a detailed derivation of the wage schedules.

union, the union bargains over the total surplus  $S^u$  of all union members. If no agreement on wages can be reached, all manual and routine workers go on a strike and the firm can only produce using abstract workers and computer capital.<sup>25</sup> With risk-neutral heterogeneous workers, our approach only pins down the total share of the surplus going to the workers, not how it is shared among them. In contrast to the approach of Taschereau-Dumouchel (2017), we need to impose an additional parametric structure on union wages due to the linear nature of the union bargaining problem.

For simplicity, and to keep the degrees of freedom in choosing the wage schedule small, we assume that the union sets a constant wage for each occupation. The share of the surplus used for manual workers grows at the same rate as the share of manual workers in the union.<sup>26</sup> This is in line with the idea that after certain union goals like equal pay for equal work are reached, the union mostly negotiates for across-the-board percentage wage increases. Empirical evidence is largely supportive of this view.<sup>27</sup> The total surplus is distributed according to the following wage schedule

$$\begin{aligned} w_r^u &= S^u * x^r / L_r \\ w_m^u &= S^u * x^m / L_m, \end{aligned}$$

where  $S^u$  is the total surplus of the union and  $x^r$  and  $x^m$  are the shares of the surplus that go to manual workers and routine workers, respectively.<sup>28</sup> Even though the chosen union wage schedule tends to favor the lowest middle-wage workers, the average union wage premium for manual workers is still

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<sup>25</sup>This contains the implicit assumptions that all workers are union members and that union members can be forced to strike by the union. Both of these assumptions are reasonable for the U.S. First, for U.S. data the difference between the union membership rate and the share of workers covered by a collective bargaining agreement is small. Second, under the NLRA workers who decide to work during a lawfully-called strike can be fined.

<sup>26</sup>The results are robust to other wage schedules as well.

<sup>27</sup>Recent examples of unions that negotiated across-the-board percentage wage increases for their members in particular firms include, among others, Communications Workers of America, United Auto Workers, and United Food and Commercial Workers. Furthermore, Checchi et al. (2010) show that at least since the 1960s, unions did not attempt to change existing earnings distributions for the fear of losing their highest-skilled workers. Additionally, Bryson et al. (2016) using the example of Britain show that unions are slow to address current problems. They argue that union representatives, who typically have long tenure, became less representative of the current membership over the last decade.

<sup>28</sup>Note that in line with empirical evidence, the union wage schedule, depending on  $x^r$  and  $x^m$ , leads to wage compression.



larger than the average union wage premium for routine workers. The data in Card et al. (2004) provides evidence for this type of union wage pattern.<sup>29</sup> Several studies support the view that the incentives for union membership are highest for middle-skilled workers. Checchi et al. (2010) show that the probability of union membership is largest for those workers earning roughly the median income. White (1982) finds that the wage structure inside the bargaining unit favors those earning the median income. Furthermore, in his literature review Schnabel (2002) shows that most of studies concerning union membership find a negative relationship between education and union membership. He argues that educated employees have greater individual bargaining power. Furthermore, he finds that workers in non-operative occupations are less likely to be union members than manual workers and that the probability of unionization first increases with earnings and drops off after a certain wage level. Estimates of the average union-nonunion wage differential across workers range from close to zero in Bryson (2002) and Frandsen (2012) to 25% in Hirsch and Schumacher (2004). Generally, more recent studies tend to find only very small wage premia on average.

Under collective bargaining, the outside option of a union member is not the value of being unemployed, but the value of being a union member during a strike.<sup>30</sup> Therefore, denoting the union's weight in the bargaining process by  $\gamma^u \in [0, 1]$ , the following surplus sharing rule holds in the case of collective

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<sup>29</sup>Some studies argue that there exists a negative linear relationship between worker skill levels and the union wage premium with the lowest-skilled workers profiting the most from union membership. These estimates might be biased by the union membership composition and by the presence of compensating wage differentials. Studies by White (1982), Schnabel (2002) and Checchi et al. (2010) all document that union members among low-skilled workers tend to be positively selected, while union members among middle-skilled workers tend to be negatively selected. Thus, the union wage premium for low-skilled workers is likely to be overestimated, as they would earn more than their non-union counterparts even in the absence of unions. For middle-skilled workers the reverse is likely to be true. In addition, Duncan and Stafford (1980) among others establish the presence of compensating wage differentials for union workers, generating an upward bias in the estimated union wage premium. It is plausible to assume that this effects low-skilled manual workers more than routine workers and thus adds to the overestimation of the wage premium for low-skilled workers.

<sup>30</sup>Since a match between a union-worker and a firm always generates a positive bilateral surplus the possibility of a strike is zero.

bargaining

$$\max \left( \sum_i \int_{\eta_{min}}^{\eta_a} L_i^u(\eta) [W_i^u(\eta) - W_i^{u,s}(\eta)] \right)^{\gamma^u} \\ \left( \sum_i \left\{ p_{Z_i} Z_i - p'_{Z_i} Z'_i - \int_{\eta_{min}}^{\eta_a} L_i(\eta) w_i^u(\eta) \right\} \right)^{1-\gamma^u}$$

with  $i = r, m$ ,

where  $W_i^u(\eta)$  is the asset value of employment for union members with productivity  $\eta$  and  $W_i^{u,s}(\eta)$  is the value of being a union member during a strike.<sup>31</sup>  $Z_i$  is again the production of each of the three intermediate goods produced with abstract, routine, and manual tasks.  $Z'_i$  is the production in each of the three sectors when manual and routine workers are on a strike and  $p'_{Z_i}$  is the corresponding price level.

It follows that the total surplus received by the union  $S^u$  is given by <sup>32</sup>

$$S^u = \gamma^u \sum_i (p_{Z_i} Z_i - p'_{Z_i} Z'_i) + (1 - \gamma^u) \sum_i \int_{\eta_{min}}^{\eta_a} L_i(\eta) w_i^{u,s}$$

with  $i = r, m$ .

where  $w^{u,s}$  denotes the wage received by a union worker during a strike, regardless of occupation and ability. The total union surplus is given by the sum of the production of all  $\eta$  workers in manual and routine occupation, the search returns, and the outside options. This is similar to the wage resulting from Nash bargaining in the standard Mortensen-Pissarides search and matching model.

As under individual bargaining, the union surplus is positively related to the number of vacancies posted. The increasing relative demand for manual workers in response to the drop in the price of computer capital increases the size of the share of the surplus the union can extract, while the decreased relative demand for routine workers works in the opposite direction. The surplus gain is distributed among all union workers, with middle-skill workers

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<sup>31</sup>The value of being a union member during a strike differs from the value of being a union member since wages are replaced by potential strike money.

<sup>32</sup>See Appendix B for a detailed derivation.

receiving the highest union wage premium.<sup>33</sup> Since the presence of routine workers lessens the increase of the union surplus, for low-skilled workers formerly indifferent between union and non-union wages the increase in non-union wages is larger than the increase in union wages. The opposite is true for middle-skilled workers. Thus, due to routine-biased technical change, the incentive to vote in favor of a collective bargaining agreement decreases for manual workers and increases for routine workers.

### *Comparing Collective and Individual Bargaining*

The wage schedules resulting from individual bargaining,  $w_i(\eta)$ , and the total surplus received by the union  $S^u$  exhibit similar structures. Both are a combination of a term related to the production and a term related to the outside option of the worker or workers. Under collective bargaining there is no term related to the costs of hiring because when collective bargaining breaks down, the worker goes on a strike.<sup>34</sup> The total surplus of the union is a function of the abilities of all manual and routine workers, while the non-union wage is a function of the individual ability of the respective worker. Most important for our analysis is the difference in the term concerning the production. Under individual bargaining, every worker with ability  $\eta$  behaves as if he is the last hired worker. In contrast, under collective bargaining the union bargains over the production of all workers. The union has a higher implicit bargaining power due to the fact that the average product of all workers employed in routine tasks with ability  $\eta$  exceeds the marginal product of a worker with ability  $\eta$  performing routine tasks.<sup>35</sup> Relative to individual bargaining, the union has a stronger threat point which leads to an increase in wages for workers within a specific range of abilities. This distinction between collective and individual bargaining is an important driver behind the union membership decision.

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<sup>33</sup>In the simulated model the union surplus always increases in response to falling capital prices.

<sup>34</sup>If we instead assume that all union workers become unemployed when bargaining breaks down, the difference in the solutions with respect to the outside option vanishes and both solutions contain a term related to the hiring costs.

<sup>35</sup>This is not true for manual workers, where the average and marginal product is identical.

#### 4.7. Household Preferences and Demand

Households consume the good produced by the firms with input factors  $Z_a$ ,  $Z_r$  and  $Z_m$ . There are no savings. For each worker the budget constraint is given by

$$C(\eta) = I$$

with  $I \in \{w_a^n, w_r^n(\eta), w_r^u, w_m^n, w_m^u, z_a, z_r(\eta), z_m\}$ .

#### 4.8. Government Expenditures and Transfers

Government expenditures are

$$G = z_a u_a + \sum_{\eta} z_r(\eta) u_r(\eta) + z_m u_m.$$

Firms can generate profits, which are given by

$$\Omega = \Pi^{Z_a} + \Pi^{Z_r} + \Pi^{Z_m}.$$

Therefore, the transfers received by households are

$$\Gamma = -G + \Omega.$$

Total Consumption in the economy is then given by the sum of individual consumption in addition to the transfers.<sup>36</sup>

#### 4.9. Equilibrium

With the model completely described, we define the equilibrium.

**Definition 1.** *An equilibrium is defined as a set of i) firm's policy functions; ii) household's policy functions; iii) a union wage schedule; iv) prices; and v) a law of motion for the aggregate states, such that: i) the firm's policies satisfy the firm's first order conditions and the job-creation conditions; ii) household's policy functions satisfy the household's first order conditions; iii) the wage is determined through individual or collective bargaining; iv) the aggregate states clear the markets; v) the law of motion for aggregate states is consistent with individual decisions and with the processes for capital prices.*

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<sup>36</sup>This allows us to abstract from the distribution of transfers to households. The results remain unchanged when lump-sum transfers are assumed instead.

## 5. Quantitative Analysis

In this section all the parameters discussed above are calibrated to match different aspects of U.S. data for the time period between 1977, the date from which on both polarization and deunionization can be observed in our dataset, to 2005. We use the calibrated model to assess the effects of a drop in investment capital prices on the occupational choice of workers and on union elections. For the simulation we choose a setting with heterogeneous unions that differ with respect to their bargaining power. We consider an economy that consists of a number  $N$  of independent islands that cannot interact with each other. All islands are identical except for the bargaining power of the potential union. The performance of the model is evaluated along several dimensions, especially with regard to the empirical evidence on deunionization in the U.S. The complete set of equations used to derive the steady states is given in Appendix C.

### 5.1. Calibration

The model is calibrated to quarterly frequencies. Table 1 lists the exact parameter values as well as the source that encourages the specific choice. We first calibrate the labor market variables. For the separation rates, we choose values of  $s_m = s_r = 0.1$ , and  $s_a = 0.05$ . These rates imply an average separation rate of 0.09 which is close to the actual separation rate constructed by the Bureau of Labor Statistics using the Job Openings and Labor Turnover Survey for the time period between December 2000 and June 2004 which is equal to 0.038 for the seasonally adjusted monthly time series.<sup>37</sup> The matching efficiencies are calibrated in order to match the targeted number of employed workers and the average quarterly job-finding rate between 1977 and 2005 in Shimer (2005). For manual and routine workers we target 0.54, a value slightly below the average quarterly job-finding rate of 0.6 between 1977 and 2005. Accordingly, the target for abstract workers is 0.77. The relative matching efficiencies of manual and routine workers are set in a way to generate identical steady-state job-finding rates. Vacancy posting costs are chosen to correspond on average to 30% of a workers steady state wage. This value is in line with the empirical evidence on worker recruitment, especially

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<sup>37</sup>The Job Openings and Labor Turnover Survey is only available since December 2000. A month is one third of a quarter. Thus, the average quarterly separation rate is roughly equal to 0.11.

when also accounting for training costs after hiring. Unemployment benefits and strike pay are both set to zero.<sup>38</sup> All production and skill specific parameters are calibrated jointly in order to match data on manual, routine, and abstract employment per capita in 1977, as well as abstract and total employment per capita in 2005.<sup>39</sup> This leaves manual and routine employment in 2005 as untargeted moments to gauge the performance of the model.

Table 1: Calibrated Parameters

Symbol	Interpretation	Value	Source
$\beta$	Discount factor	0.9966	Interest rate of 4%
$c_m$	Manual recruiting costs	0.3	30% of wages
$c_r$	Routine recruiting costs	0.3	30% of wages
$c_a$	Abstract recruiting costs	0.5	30% of wages
$\Psi_a$	Abstract matching efficiency	0.5	Job-finding rate 0.77
$\Psi_r$	Routine matching efficiency	0.31	Job-finding rate 0.54
$\Psi_m$	Manual matching efficiency	0.35	Job-finding rate 0.54
$\psi$	Unemployment-elasticity of matching	0.5	Petrongolo and Pissarides (2001)
$s_a$	Abstract separation rate	0.05	Albertini et al. (2017)
$s_r$	Separation rate	0.1	BLS JOLTS
$s_m$	Separation rate	0.1	BLS JOLTS
$\gamma^n$	Worker's bargaining power	0.5	Midpoint of literature values
$\gamma^{u,l}$	Union bargaining power	0.83	Voting share Frandsen (2012)
$\gamma^{u,m}$	Union bargaining power	0.835 - 0.86	Wage Premium Bryson (2002)
$\gamma^{u,h}$	Union bargaining power	1	Voting share Frandsen (2012)
$x^m(0)$	Manual Share of Surplus 1977	0.165	Favors routine workers
$A$	Productivity routine and abstract input	3.4	Occupational shares in 1977
$A_m$	Productivity of manual input	0.3	Occupational shares in 1977
$\alpha$	Marginal return to abstract labor	0.3	Occupational shares in 1977
$\rho$	Production parameter	0.65	Occupational shares in 1977
$\sigma$	Production parameter	0.74	Albertini et al. (2017)
$\mu$	Production parameter	0.5	Albertini et al. (2017)
$\eta_a$	Upper bound on routine skill	1.44	Occupational shares in 1977
$\eta_{min}$	Lower bound on skill	0.48	Occupational shares in 1977
$\eta_{m,1977}$	Occupational threshold 1977	0.67	Occupational shares in 1977
$g_K$	Growth rate of investment prices	-0.025	Investment prices in 2005
$L_a(0)$	Abstract employment 1977	0.14	Abstract employment in 1977
$gL_a$	Growth rate of abstract employment	0.014	Abstract employment in 2005

The calibration of the union bargaining powers is mainly based on Frandsen (2012) who conducts a detailed analysis of union election data from the NLRB. In his study Frandsen (2012) reports that in about 11% of all union elections the share of pro-union votes is 90% or higher. Thus, for 11% of the islands the bargaining power of the potential unions is set to  $\gamma^{u,h} = 1$ , which

<sup>38</sup>The results are robust to alternative parameter choices.

<sup>39</sup>As Albertini et al. (2017) demonstrate, focusing exclusively on employment shares tends to blur polarization dynamics.

generates a pro-union vote share of 90% over the considered time period.<sup>40</sup>

For the remaining islands the union bargaining power and the share of islands with this respective union bargaining power are calibrated jointly to match overall, manual, and routine union density in 1977, an average union wage premium that falls in the interval of 3% to 6% as reported in Bryson (2002), and an average pro-union vote share of 56% as in Frandsen (2012). For 22% of the islands the union bargaining power is equally distributed between  $\gamma^{u,m} \in [0.835, 0.86]$ .<sup>41</sup> The share of islands with this bargaining power is chosen to match the overall union density in 1977 from the Union Membership and Coverage Database constructed by Hirsch and Macpherson and described in Hirsch and Macpherson (2003) as well as 1977 union density for manual and routine workers. The bargaining power is set to generate an overall union wage premium that falls in the interval of 3% to 6% reported in Bryson (2002). An overall wage premium that is positive but close to zero is also supported by DiNardo and Lee (2004), and Frandsen (2012). On all other islands the bargaining power of the potential unions is set to match the average pro-union vote share of 56% in Frandsen (2012). Thus, to generate a pro-union vote share of 49% on the remaining islands, the bargaining power is assumed to be  $\gamma^{u,l} = 0.83$ . The close elections are also in line with the evidence in Frandsen (2012), who reports that the vast majority of union elections are decided by only a few votes.

The share of the union surplus that goes to each occupational group is set in a way that favors routine workers, since those make up 80.5% of the bargaining unit but receive 83.5% of the surplus.<sup>42</sup> This share only adjusts to employment changes and not to changes in the economic environment. This suggests that, even though union membership rates were fast declining

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<sup>40</sup>The large differences between the union bargaining powers and the individual bargaining power of a worker are due to the fact that under collective bargaining workers are not lost to the firm when bargaining breaks down. If we instead assume that the firm loses its workforce when no agreement is reached, the union bargaining power is substantially lower than under individual bargaining.

<sup>41</sup>The results are robust to alternative intervals of the union bargaining power as long as the share of the surplus obtained by workers in the two occupations is adjusted accordingly. The results are identical when setting the bargaining power to the average value over the interval,  $\gamma^{u,m} = 0.8475$ . The interval is chosen because it allows us to judge the performance of the model in generating deunionization at different points in time.

<sup>42</sup>The results are robust with respect to the specific parameter choice, with changes in the parameter affecting the time at which deunionization occurs on an island.

since the 1980s, union officials did little to adjust union policies accordingly. A study by Checchi et al. (2010) indicates that this may indeed be the case. They show that at least since the 1960s, unions did not attempt to change existing earnings distributions for the fear of losing their highest-skilled workers. The wage schedule is also in line with the idea that after certain union goals like equal pay for equal work are reached, the union mostly negotiates for across-the-board percentage wage increases. Possible reasons for the lack of adjustments are listed in Waddington (2005): Trade union practices are perceived as formal and old-fashioned and the representative structures inside unions are often inappropriate for the participation of all members. This encourages the impression that trade union leadership has lost touch with current workplace realities. In addition, Bryson et al. (2016) argue that union representatives have long tenure and therefore do not represent the current membership composition. They show that unions in Britain have been slow to respond to current issues like gender equality. Thus, it seems that unions are mostly controlled and influenced by older members that might display a tendency to stick to established practices.<sup>43</sup>

## 5.2. Simulation Results

The timing of events is as depicted in Figure 5. First, given the initial calibration, the occupational thresholds are determined. Afterwards a union election takes place in all firms and all islands with a union bargaining power above  $\gamma^u = 0.835$  are unionized. Before routine-biased technical change, overall union density is equal to 25% and the union membership rates for manual and routine workers are both equal to one third. Capital prices fall and occupational thresholds in non-unionized firms change with former routine workers switching to manual occupations. A new union vote takes place and unions who fail to gain majority support are terminated. Afterwards, occupational shifts occur in the previously unionized firms.

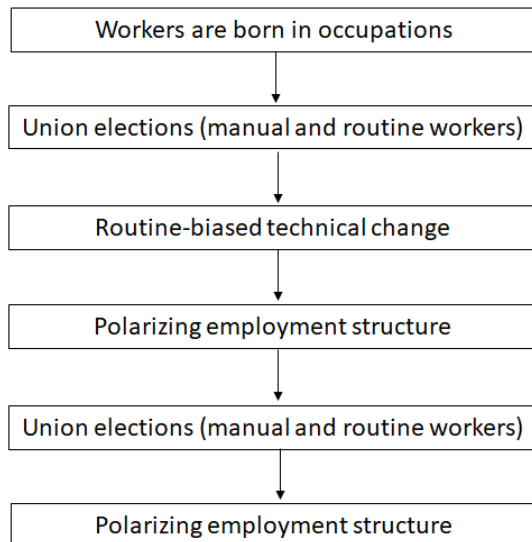
We first check the performance of the model with respect to matching the untargeted unemployment rates. The model predicts an overall unemployment rate of 7%, with slightly larger unemployment on unionized islands. The average unemployment rate estimated by the BLS over the time period

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<sup>43</sup>All other parameters are calibrated according to Table 1. Since the parameter choice is either very straightforward or common in the literature there is no value in discussing those at length.



Figure 5: Timing of Events



between 1977 and 2005 is 6.2%.<sup>44</sup> The ratio of the manual unemployment rate to the abstract employment rate stays roughly constant at 0.3 in the model between 1977 and 2005. This is in line with the empirical evidence on high vs. low educated unemployment rates in the U.S. presented in Broersma (2008).

Table 2: SIMULATED EMPLOYMENT IN 1977 AND 2005

Occupation	Employment Per Capita 1977	Employment Per Capita 2005
Manual	0.080 (14.5%)	0.1 (16.1%)
Routine	0.33 (60%)	0.31 (50%)
Abstract	0.14 (25.5%)	0.21 (33.9%)

Note: Employment per capita is the employment level divided by the aggregate population. Employment shares are given in the round brackets and calculated as employment levels divided by aggregate employment.

<sup>44</sup>We target the same employment levels on unionized and non-unionized islands. This is supported by Montgomery (1989) who finds that union coverage has little effect on aggregate employment, with the employment probability of the average worker decreasing from 0.884% to 0.881%.

As capital prices fall, employment in the non-unionized islands adjusts, with the lowest-skilled routine workers deciding to switch to manual occupations upon becoming unemployed. Since routine workers employed in unionized firms are better off not switching, the increase of workers employed in manual occupations is smaller compared to a model without unions. In the subsequent elections, the unions with the lowest bargaining power fail to gain majority support and are terminated.<sup>45</sup> This leads to employment adjustments in those firms identical to the ones happening in the non-unionized firms. The exact values for employment per capita and employment shares in the model are given in Table 2.<sup>46</sup> Figure 6 displays the percentage point changes in the employment share for each occupation.

The union wage schedule favors middle-skilled workers, as workers in routine occupations make up 80.5% of the bargaining unit but receive 83.5% of the surplus. Thus, the union wage schedule discourages employment reallocation and the employment shifts are more pronounced in the model with deunionization. For the counterfactual scenario without deunionization, the union wage schedule is slightly adjusted in favor of manual workers, such that all unions are able to maintain their majority support.<sup>47</sup> The untargeted changes in employment per capita and in employment shares for manual and routine workers are both close to the changes reported in the literature.<sup>48</sup> Even though the manual employment share appears roughly unchanged, there has been substantial employment reallocation with more than 5% of all routine workers in 1977 deciding to switch to manual occupations. 25% of the changes in manual and routine employment are triggered by the termination of unions. Thus, while the model is in line with the empirical

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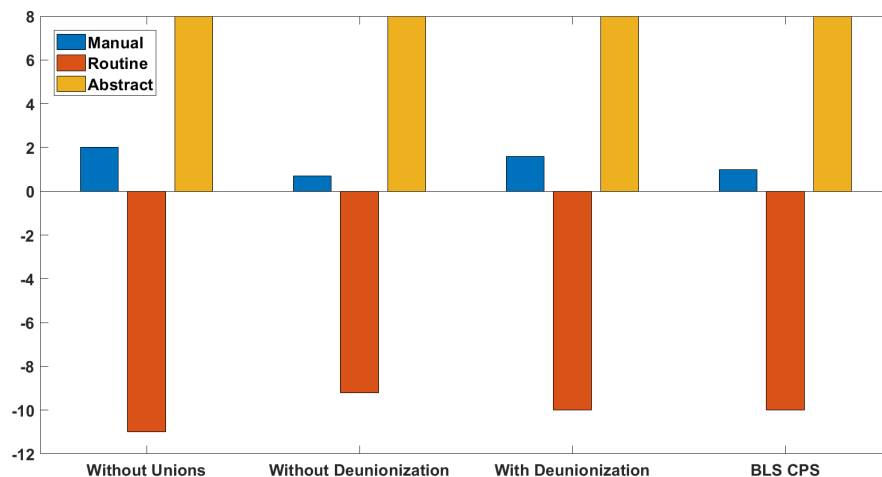
<sup>45</sup>This model prediction is supported by evidence in the 2004 NLRB Performance and Accountability Report. Going from 1994 to 2004, the number of filed representation petitions has dropped by 25% but the share of won elections has increased by over five percentage points.

<sup>46</sup>The model is calibrated to the employment shares in Albertini et al. (2017) which are at the upper end of the values reported in Jaimovich and Siu (2012). The results are virtually unchanged when calibrating the model to the employment shares in Autor and Dorn (2013), which are at the lower end of the values reported in Jaimovich and Siu (2012).

<sup>47</sup>Leaving everything else unchanged, the share of the union surplus that goes to manual workers is increased up to the point where a majority of the workers supports a union in 2005.

<sup>48</sup>See, for example, Jaimovich and Siu (2012) and Albertini et al. (2017).

Figure 6: Simulated Percentage Point Changes in Employment Shares from 1977 to 2005



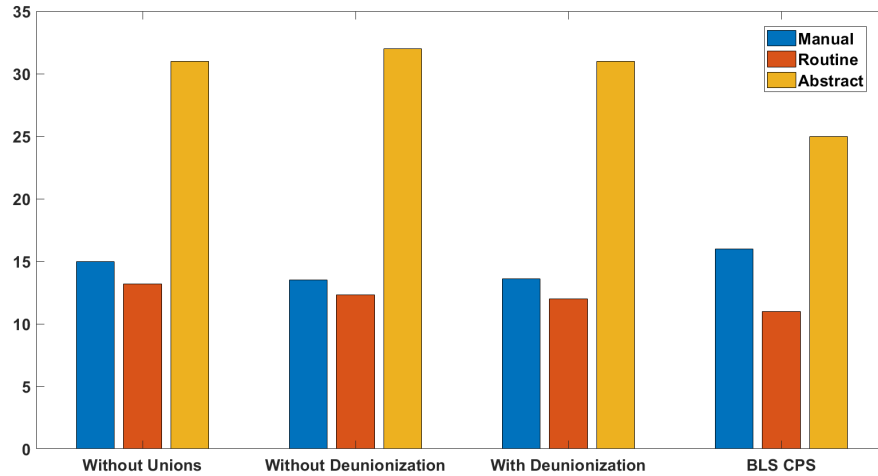
Note: BLS CPS denotes the changes in employment shares according to the Bureau of Labor Statistics Current Population Survey.

literature predicts routine-biased technical change to be the main explanation for job market polarization, deunionization seems to play an important role. Since the model is calibrated to match employment per capita in 1977, a model version without unions corresponds to the case of complete deunionization. As depicted in Figure 6, such a version tends to substantially overpredict employment changes.

The changes in employment are accompanied by changes in wages for workers in all three occupations. Depicted in Figure 7, the model predicts wages for manual workers to increase by 14%, wages for routine workers to increase by 12%, and wages for abstract workers to increase by 31%. These untargeted wage changes are close to the wage changes of 16%, 11%, and 25% reported in Autor and Dorn (2013) for the time period between 1980 and 2005.

In contrast to employment changes, deunionization has only modest effects on wage changes since the average union wage premium is small. Inequality, as measured by the Palma ratio, is roughly 5% higher without

Figure 7: Simulated Percentage Changes in Wages from 1977 to 2005



unions.<sup>49</sup> Going from 1977 to 2005, roughly 7% of the increase in the Palma ratio is caused by deunionization.<sup>50</sup> These rather small overall effects of deunionization on inequality accord with the empirical findings in DiNardo et al. (1996), Frandsen (2012), and Farber et al. (2018).

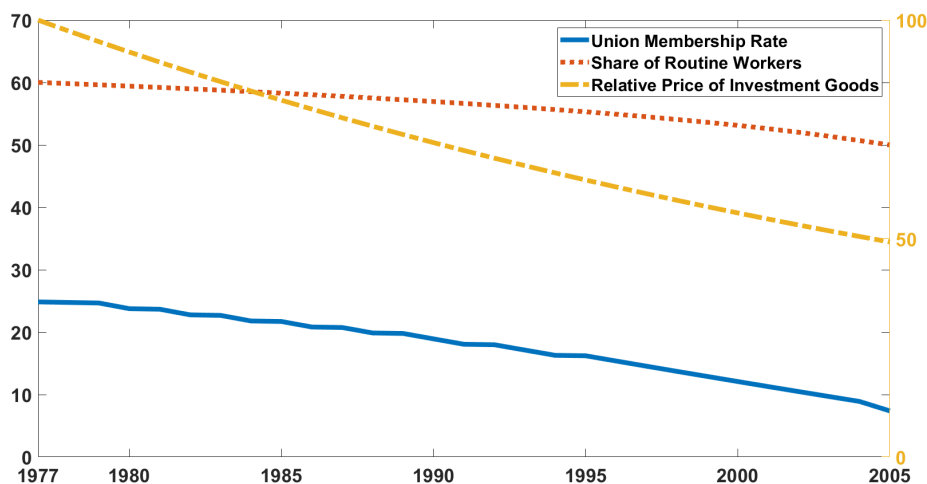
In contrast, the effects of deunionization are substantial for those groups that traditionally receive a high union wage premium. For the 50% lowest-skilled formerly unionized routine workers the simulated union wage premium amounted to roughly 20% before routine-biased technical change. This group makes up one third of all union members and 10% of the entire workforce in 1977, meaning that the results concern several million people. Since those workers lose their union wage premium going from 1977 to 2005, the average wage growth for that group is only 6%. In a counterfactual exercise where the

<sup>49</sup>The Palma ratio compares the richest 10% with the poorest 40%. This inequality measure is chosen over the Gini index because of the oversensitivity of the latter to changes in the middle of the income distribution.

<sup>50</sup>Since the Palma ratio completely ignores changes in the middle of the income distribution, the effects are slightly larger when looking at, for example, the 50:50 or 60:40 income ratios instead.

wage schedule is adjusted to maintain union support, wages for those workers grow by more than 14%. Furthermore, the relative wage of the highest-skilled workers compared to the group of formerly unionized routine workers grows by 28.3% over the considered time period, with over one third of the increase accounted for by deunionization.<sup>51</sup>

Figure 8: Relative Price for Investment Goods, Share of Routine Workers, and Union Membership Rate



Note: The number of islands is set to  $N = 90$  for the plot. The relative price of investment is plotted as an index with 1977 = 100.

Since the model predicts that the unions with the lowest bargaining power will be the ones that are terminated, union termination in the model is associated with increasing average union wage premia. Hirsch and Schumacher (2004) estimate an increasing union wage premium in the early 1930s and in the late 1970s to early 1980s. Evidence from Troy and Shefflin (1985) suggests, that both time periods were preceded by exceptionally large numbers of union termination. In contrast to models of skill-biased technical change, the model presented here is able to reconcile falling union membership density with constant or even increasing union wage premia.

<sup>51</sup>The increase in inequality is close to the data. U.S. real weekly wages for the top 10% have grown by 24% relative to the median real weekly wages over this time period.

Figure 8 is the model equivalent to Figure 1. The relative price of the investment good decreases by 51% between 1977 and 2005. The share of routine workers drops by 10 percentage points from 60% in 1977 to 50% in 2005. The union membership rate falls by 17.7 percentage points from 25% to 7.3%. In the data, the overall union membership rate decreases by 16.1 percentage points from 22% to 7.9%. For manual workers, the union membership rate falls by 24 percentage points from 33.3% to 8.9% in the model and by 22.5 percentage points from 35.5% to 13% in the data. For routine workers, the union membership rate falls by 21.5 percentage points from 33.3% to 11.8% in the model and by 22.4 percentage points from 35.5% to 13.1% in the data.<sup>52</sup> The model is unable to capture the observation that the decrease in the share of routine workers and in the union membership rates has flattened out since the late 1990s to early 2000s. This might have to do with the reversal in the demand for cognitive skills since the early 2000s reported in Beaudry et al. (2016). In the model, such a reversal would be able to generate a flatter decrease in both the share of routine workers and the overall union membership rates.

### 5.3. *Deunionization in a Single Firm*

In this section, in order to expose the underlying mechanisms, we focus on the occupational choices and union elections in a firm that generates union support in 1977 but fails to do so in 2005. The union bargaining power lies in the interval of  $\gamma^{u,m}$ . While this might appear to be a knife's edge scenario, Frandsen (2012) points out that the vast majority of union elections are decided by only a few votes. Using data on union elections between 1992 and 2001 in which at least ten votes were cast, Frandsen (2012) shows that the average voting share in favor of unions is 56.5%. Unions won 54.2% percent of the elections and the margin of victory was on average close to one vote.

Under the baseline 1977 calibration, two thirds of the manual and routine workers vote in favor of the union, as they are close enough to the

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<sup>52</sup>For the membership rates for routine and manual workers we use data on the union density of construction workers (manual) and manufacturing workers (routine). More detailed data on the union membership rates of workers in manual and routine occupations is only available since 1983. From 1983 onwards, union density for all manual workers and for construction workers as well as union density of all routine workers and manufacturing workers behave very similarly in terms of absolute values and changes over time.

median skill-level to profit from the union wage schedule.<sup>53</sup> The pro-union vote share corresponds to the average value for union elections in Frandsen (2012). Given the high prices for computer capital in 1977, wage inequality is relatively low with the highest skilled worker earning twice as much as the least skilled worker. Thus, for a majority of the bargaining unit the union wage schedule is beneficial. A union is formed and all manual and routine workers, regardless of their individual voting decision, are covered by the union contract. The collective bargaining agreement decreases inequality measured by the Palma ratio by 5%.

Going from 1977 to 2005, the price of investment capital drops by 51%. Routine workers can be substituted by computer technology  $K$ , whereas abstract and manual workers are complementary to routine tasks. The drop in the price for computer capital  $K$  leads to a relative wage drop for routine workers who can be replaced by computers. The high union wage premium for the lowest-skilled routine workers discourages those workers from switching to manual occupations. Since labor supply in the occupations does not adjust to the changing demand structure, the non-union wages for routine workers fall by more relative to the non-union wages of manual workers. While non-union wages of manual workers grow by 16%, union wages for those workers only grow by 14% and manual workers decide to vote against a collective bargaining agreement.<sup>54</sup> This is in line with Checchi et al. (2010), who argue that disillusion with respect to potential wage growth is the reason for declining membership rates among the lowest-skilled workers in the U.S. Since union wages for routine workers increase by more than non-union wages, more routine workers vote in favor of a union. However, the decrease in votes by manual workers more than offsets the increasing votes among routine workers and the pro-union vote share drops below 50%.<sup>55</sup> Routine workers lose their union wage premium and for some it is now beneficial to

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<sup>53</sup>Note, that under the assumed union wage schedule either all or none of the manual workers vote in favor of the union. The results are robust to other specifications of the wage schedule, as long as the decrease in the number of votes from manual workers is larger than the increase in the number of votes from routine workers.

<sup>54</sup>Deunionization occurs regardless of wage polarisation. When wages for routine workers grow relative to wages of manual workers, it will be routine workers who drive deunionization by voting against a collective bargaining agreement.

<sup>55</sup>This result is robust with respect to reasonable values for the surplus share of manual workers, with changes affecting the specific point in time at which the union fails to gain majority support.

switch occupations upon becoming unemployed. The ensuing employment reallocation leads to the same steady state as for the non-unionized firms.

## 6. Policy Implications

While routine-biased technical change hurts middle-wage workers, job market polarization in the sense of changing employment shares does not. In the model, the possibility to switch occupations allows labor supply to adjust to the changes in labor demand and thereby to partly offset the wage effects of routine-biased technical change. As shown by Kambourov and Manovskii (2009), Gathmann and Schnberg (2010) and Cortes and Gallipoli (2017), occupational switching costs could be large.<sup>56</sup> Therefore, as proposed for example in Autor et al. (2003), policies that simplify job switches or that aim at making them less costly for workers could serve to dampen the income inequality caused by routine-biased technical change.

The previous analysis has shown that while the overall effect of deunionization on income inequality is small, there are huge effects for those groups typically favored by union wage schedules. Taking into account evidence from Frandsen (2012), who reports that most union elections are very closely contested, even very small policy changes in favor of unions or union workers could lead to large effects on income inequality for lower-skilled mid-wage workers. We briefly consider the effects of three policies in our model that aim at preventing deunionization and increasing equality. The perhaps most straightforward policy simply abolishes union elections and maintains the established unions regardless of worker preferences. While this approach

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<sup>56</sup>Kambourov and Manovskii (2009), show that occupational experience is a major determinant of earnings and that it is considerably more important than either firm or industry tenure. They find that 5 years of occupational tenure are associated with an increase in wages of 12% to 20%. Gathmann and Schnberg (2010) show that task-specific human capital is an important source of individual wage growth, in particular for university graduates. For high-skilled workers at least 40% of overall wage growth over a ten year period can be attributed to task-specific human capital. For the low- and medium-skilled, task-specific human capital accounts for at least 35% and 25% of overall wage growth, respectively. Cortes and Gallipoli (2017) assess the role of task distance as a component of the cost of switching among any two occupations. They find that raising task distance by one standard deviation increases the cost of switching occupations by approximately 14%. In addition, if the switch involves moving across major task groups, mobility costs are raised much further. Additionally, despite the role of task content, they find that the largest share of occupational mobility costs is attributable to task-independent factors.



prevents deunionization and improves wages for low middle-skilled workers, it also prevents efficient deunionization in the sense that even unions generating a negative average wage premium would be maintained. The second policy lowers the necessary voting threshold for unions. For specific voting thresholds, this policy achieves the same results as the former policy with identical downsides. However, such an intervention is not well suited to stop the overall trend of declining union membership rates, as the threshold would have to be regularly adjusted to changes in the economy. Furthermore, low threshold values, apart from being difficult to justify, could in principle lead to the founding of further inefficient unions. The third policy aims at increasing the bargaining power of unions by increasing political support. Consider, for example, a scenario in which the bargaining power of all established unions is set to  $\gamma^{u,h}$ , the bargaining power of the high-power unions. In addition to at least delaying deunionization, this policy also raises wages for low-wage and middle-wage workers and reduces inequality. However, as with the policy lowering the voting threshold, deunionization would eventually occur if the trend of falling computer capital prices continues.

In the model, since unions could in principle always pay every worker just a bit above the individually bargained wage, deunionization can always be prevented by appropriately adjusting the wage schedule.<sup>57</sup> However, empirical evidence suggests that unions are troubled by rigid structures that partly prevent them from meeting today's challenges. Waddington (2005) argues that union practices are perceived as formal and old-fashioned and that the representative structures inside unions are often inappropriate for the participation of all members. This encourages the impression that trade union leadership has lost touch with current workplace realities. Additionally, the decline in membership rates can mostly be attributed to the failure of unions to recruit young members. While membership rates decline across all age groups, according to data from the Bureau of Labor Statistics membership rates for workers aged between 16 and 24 declined at twice the rate of overall membership between 2002 and 2012. Data on the evolution of the median age of union members points in the same direction. Dunn and Walker (2016) point out that over half of all U.S. union members are between

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<sup>57</sup>This of course abstracts from workers voting against the union simply because they are dissatisfied with the changes in the wage structure, as our analysis solely focuses on monetary incentives.

45 and 64 years of age. In addition, Bryson et al. (2016) argue that union representatives have long tenure and therefore do not represent the current membership composition. They show that unions in Britain have been slow to respond to current issues like gender equality. Thus, it seems that unions are mostly controlled and influenced by older members that might display a tendency to stick to established practices. The recent article "Technology may help to revive organised labour" in *The Economist* (2018) puts forth the argument that new technology could help unions to regain members. This argument is supported by evidence in Bryson et al. (2016) who argue that the decline in union membership rates across countries is strongly related to the degree of progressiveness of the unions. While the recent example of a union of Youtube employees that was formed by potential members joining a facebook group might be nothing more than a marketing gag, it seems that a more modern and progressive structure is needed in order for unions to attract more and especially younger members.

## 7. Conclusion

This paper explores how routine-biased technical change effects both the occupational and the union-membership choice of workers. To do so we develop a model that endogenizes both decisions in a search and matching framework.

We use the calibrated model to show that routine-biased technical change, represented by a sharp drop in computer capital prices, not only generates employment and wage polarization but also deunionization. The drop in computer capital prices reduces the demand for routine workers, while abstract and manual workers are in great demand. The changing demand structure influences the surplus the union can extract and thereby also the individual union wage premium of workers. Manual workers, who benefit from the changing demand structure, are discouraged from voting in favor of a collective bargaining agreement. The wage gains for manual workers, that would be disproportionately given to routine workers by the union, lead to the least skilled workers being better off when bargaining individually with the firm. Former routine workers, when faced with lower wages compared to manual workers, decide to switch occupations.

We demonstrate that this effect can lead to a change in the voting outcome, with the majority of the workforce of formerly unionized firms now voting against unionization and in favor of individual bargaining. In an economy

in which unions differ with respect to their bargaining power, routine-biased technical change leads to a large decrease in union membership, as those unions with the lowest bargaining power are terminated. This contributes substantially to employment polarization. While overall effects on income inequality are small, relatively low-skilled formerly unionized routine workers are severely affected.

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## Appendix

### Appendix A. Derivation of Wages

This section derives the non-union wages in the model. The first order conditions are given by

$$W_i^n(\eta) - U_i(\eta) = \frac{\gamma^n}{1 - \gamma^n} J_i^n(\eta),$$

with  $i = a, r, m$ .

#### *Abstract Workers*

After replacing the value function the Nash sharing rule for abstract workers is given by

$$\begin{aligned} w_a^n + \beta [(1 - s_a)W_a^n + s_a U_a] - z_a - \beta [(1 - f_a)U_a^n + f_a W_a^n] \\ = \frac{\gamma^n}{1 - \gamma^n} [p_{z_a} - w_a^n + (1 - s_a)\beta J_a]. \end{aligned}$$

After some rearrangement, we have

$$\begin{aligned} w_a^n = \gamma^n p_{z_a} + (1 - \gamma^n)z_a + \gamma^n(1 - s_a)\beta J_a \\ + (1 - \gamma^n)\beta [f_a(W_a^n - U_a^n) - (1 - s_a)(W_a^n - U_a^n)]. \end{aligned}$$

By using the job creation condition and  $\frac{c_a}{q_a} = \beta J_{a,+1}$  as well as the first order condition resulting from the Nash sharing rule:

$$(1 - \gamma^n)(W_a^n - U_a^n) = \gamma^n J_a^n = \gamma^n \frac{c_a}{\beta q_a}$$

we obtain the following wage equation

$$w_a^n = \gamma^n p_{z_a} + \gamma^n c_a \theta_a + (1 - \gamma^n)z_a.$$

#### *Routine Workers*

After replacing the value function, the Nash sharing rule for routine workers of ability level  $\eta$  is given by

$$\begin{aligned} w_r^n(\eta) + \beta [(1 - s_r)W_r^n(\eta) + s_r U_r(\eta)] - z_r(\eta) - \beta [(1 - f_r)U_r^n(\eta) + f_r W_r^n(\eta)] \\ = \frac{\gamma^n}{1 - \gamma^n} [p_{z_r} y_r(\eta) - w_r^n(\eta) + (1 - s_r)\beta J_r]. \end{aligned}$$

After some rearrangement, we have

$$w_r^n(\eta) = \gamma^n p_{Z_r} y_r(\eta) + (1 - \gamma^n) z_r(\eta) + \gamma^n (1 - s_r) \beta J_r \\ + (1 - \gamma^n) \beta [f_r (W_r^n(\eta) - U_r^n(\eta)) - (1 - s_r) (W_r^n(\eta) - U_r^n(\eta))].$$

By using the job creation condition and  $\frac{c_r}{q_r(\eta)} = \beta J_r(\eta)$  as well as the first order condition resulting from the Nash sharing rule:

$$(1 - \gamma^n) (W_r^n(\eta) - U_r^n(\eta)) = \gamma^n J_r^n(\eta) = \gamma^n \frac{c_r}{\beta q_r}$$

we obtain the following wage equation

$$w_r^n(\eta) = \gamma^n p_{Z_r} y_r(\eta) + \gamma^n c_r \theta_r + (1 - \gamma^n) z_r(\eta).$$

### *Manual Workers*

After replacing the value function the Nash sharing rule for manual workers is given by

$$w_m^n + \beta [(1 - s_m) W_m^n + s_m U_m] - z_m - \beta [(1 - f_m) U_m^n + f_m W_m^n] \\ = \frac{\gamma^n}{1 - \gamma^n} [p_{Z_m} - w_m^n + (1 - s_m) \beta J_m].$$

After some rearrangement, we have

$$w_m^n = \gamma^n p_{Z_m} + (1 - \gamma^n) z_m + \gamma^n (1 - s_m) \beta J_m \\ + (1 - \gamma^n) \beta [f_m (W_m^n - U_m^n) - (1 - s_m) (W_m^n - U_{m,+1}^n)].$$

By using the job creation condition and  $\frac{c_m}{q_m} = \beta J_m$  as well as the first order condition resulting from the Nash sharing rule:

$$(1 - \gamma^n) (W_m^n - U_m^n) = \gamma^n J_m^n = \gamma^n \frac{c_m}{\beta q_m}$$

we obtain the following wage equation

$$w_m^n = \gamma^n p_{Z_m} + \gamma^n c_m \theta_m + (1 - \gamma^n) z_m.$$

## Appendix B. Union Surplus

This section derives the union surplus. The first order condition in the collective bargaining problem is given by

$$\begin{aligned} & \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i^u(\eta) [W_i^u(\eta) - W_i^{u,s}(\eta)] \\ &= \frac{\gamma^n}{1 - \gamma^n} \sum_i \left\{ p_{Z_i} Z_i - p'_{Z_i} Z'_i - \int_{\underline{\eta}}^{\bar{\eta}} L_i(\eta) w_i^u(\eta) \right\}, \\ & \text{with } i = r, m. \end{aligned}$$

After replacing the value function and using the job creation condition  $\frac{c_i}{q_i} = \beta J_{i,+1}(\eta)$  the Nash sharing rule is given by

$$\begin{aligned} & \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i^u(\eta) [w_i^u(\eta) - w_i^{u,s}(\eta)] \\ &= \frac{\gamma^n}{1 - \gamma^n} \sum_i \left\{ p_{Z_i} Z_i - p'_{Z_i} Z'_i - \int_{\underline{\eta}}^{\bar{\eta}} L_i(\eta) w_i^u(\eta) \right\}. \end{aligned}$$

After some rearrangement, we have

$$\begin{aligned} & \gamma \left[ \sum_i \left\{ p_{Z_i} Z_i - p'_{Z_i} Z'_i \right\} + (1 - \gamma) \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i^u(\eta) w_i^{u,s}(\eta) \right] \\ &= \gamma \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i(\eta) w_i^u(\eta) + (1 - \gamma) \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i^u(\eta) w_i^{u,s}(\eta). \end{aligned}$$

Thus, the total union surplus is given by

$$\begin{aligned} S^u &= \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i^u(\eta) w_i^u(\eta) = \gamma^u \sum_i (p_{Z_i} Z_i - p'_{Z_i} Z'_i) + (1 - \gamma^u) \sum_i \int_{\underline{\eta}}^{\bar{\eta}} L_i(\eta) w_i^{u,s}(\eta) \\ & \text{with } i = r, m. \end{aligned}$$

## Appendix C. Steady State Equations

$$Y = [(AZ_a^\alpha Z_r^{1-\alpha})^\rho + (A_m Z_m)^\rho]^{1/\rho}$$

$$Z_a = L_a$$

$$Z_r = \left[ \left( (1 - \mu) \int_{\eta_m}^{\eta_a} \eta L_r(\eta) \right)^\sigma + (\mu K)^\sigma \right]^{\frac{1}{\sigma}}$$

$$Z_m = L_m$$

$$s_i L_r = q_i v_i$$

with  $i = a, r, m$

$$L_r = (\bar{\eta} - \eta_m) l_r$$

$$L_m = (\eta_m - \underline{\eta}) l_m$$

$$p_{Z_i} = \frac{\partial Y}{\partial Z_i}$$

with  $i = a, r, m$

$$p_K = \frac{\partial Z_r}{\partial K}$$

$$\frac{c_a}{q_a} = \beta \left[ p_{Z_a} - 1_u w_a^u - (1 - 1_u) w_a^n + (1 - s_a) \frac{c_a}{q_a} \right]$$

$$\frac{c_r}{q_r} = \beta \left[ p_{Z_r} \bar{y}_r - 1_u \bar{w}_r^u - (1 - 1_u) \bar{w}_r^n + (1 - s_r) \frac{c_r}{q_r} \right]$$

$$\frac{c_m}{q_m} = \beta \left[ p_{Z_m} - 1_u w_m^u - (1 - 1_u) w_m^n + (1 - s_m) \frac{c_m}{q_m} \right]$$

$$w_a^n = \gamma^n p_{Z_a} + \gamma^n c_a \theta_a$$

$$w_r^n(\eta) = \gamma^n p_{Z_r} y_r(\eta) + \gamma^n c_r \theta_r$$

$$w_m^n = \gamma^n p_{Z_m} + \gamma^n c_m \theta_m$$

$$y_r(\eta) = \frac{\partial Z_r}{\partial L_r(\eta)}$$

$$f_i = \Psi \theta_i^\psi$$

with  $i = a, r, m$

$$f_m = f_r$$

$$w_m^n = w_r^n(\eta_m)$$

$$Y = \sum_i w_i L_i + \sum_i c_i v_i + \Gamma$$

with  $i = a, r, m$

$$S^u = \gamma^u \sum_i (p_{Z_i} Z_i - p'_{Z_i} Z'_i)$$

with  $i = r, m$ .

$$w_r^u = S^u * x^r / L_r$$

$$w_m^u = S^u * x^m / L_m$$

$$w_m^u = w_m^n (\eta_{low}^u)$$

$$w_r^u = w_r^n (\eta_{high}^u)$$