Welfare Effects of Short-Time Compensation

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Abstract

In addition to unemployment insurance (UI), most OECD countries operate short-time compensation (STC) systems which partially compensate workers for a loss of income due to a reduction in hours. In several countries the utilization of STC experienced a large increase during the Great Recession of 2008-2009, which has led to a renewed interest in this type of labor market policy.

This paper studies the welfare effects of UI and STC in a model of implicit contracts. Full insurance through employers is prevented because firms have only limited funds, leaving a welfare improving role for UI and potentially STC. The model is used to answer three qualitative questions concerning the joint design of UI and STC. First, for a given level of UI, does introducing a small amount of STC necessarily improve welfare? Second, starting from the optimal UI system without STC, does introducing STC with the same replacement rate as UI necessarily improve welfare? Third, if a reform replaces the optimal UI system without STC with the optimal system combining UI and STC, does this necessarily reduce layoffs? We find that the answer is negative for all three questions.


1 Introduction

In addition to unemployment insurance (UI), most OECD countries operate short-time compensation (STC) systems which partially compensate workers for a loss of income due to a reduction in hours. In several countries the utilization of STC experienced a large increase during the Great Recession of 2008-2009, which has led to a renewed interest in this type of labor market policy.

This paper studies the welfare effects of UI and STC in a model of implicit contracts. Full insurance through employers is prevented because firms have only limited funds. The model is used to answer three qualitative questions concerning the joint design of UI and STC. First, for a given level of UI, does introducing a small amount of STC necessarily improve welfare? Second, starting from the optimal UI system without STC, does introducing STC with the same replacement rate as UI necessarily improve welfare? Third, if a reform replaces the optimal UI system without STC with the optimal system combining UI and STC, does this necessarily reduce layoffs? We find that the answer is negative for all three questions.

Previous theoretical work on the joint design of UI and STC by Burdett and Wright (1989, henceforth BW) has restricted attention to a setting in which firms are risk neutral and can fully insure their employees. Thus laissez faire is socially efficient. Their analysis focusses on the different distortions associated with UI and STC, respectively: UI induces excessive layoffs as in Feldstein (1976), while STC distorts the choice of hours worked. At best both policies are neutral, which occurs if they are fully experience-rated. Under full experience rating the question of how to combine UI and STC is irrelevant. Thus BW provide insights into the distortions both policies create, but their analysis does not offer guidance to policy makers who face the problem of jointly designing UI and STC in a world without full insurance through employers. Cahuc and Carcillo (2011) argue that full experience rating is unlikely to be optimal as many employers — especially small ones — have limited access to financial market. If experience rating is incomplete, however, then UI and STC are no longer neutral and how to combine them optimally becomes an important policy question.

We build on the analysis of BW, extending their model by allowing for the possibility that employers cannot fully insure their employees due to limited funds. Here we follow Blanchard and Tirole (2008), who study the implications of limited employer funds for the design of UI in a model without an hours margin. Specifically, we augment the model


of BW with a shallow-pocket constraint, which precludes the cash-flow of the employer from falling below a certain exogenous threshold. This constraint implies that laissez-faire is no longer socially efficient, allowing for the possibility that UI and STC could be welfare-enhancing. Importantly, this constraint has the immediate implication that any experience-rated components of UI and STC are neutral, hence without loss of generality we restrict attention to UI and STC without experience rating.

The shallow-pocket constraint is the sole friction in our model, and it binds whenever the employer experiences a sufficiently severe adverse shock to productivity or demand. If it were implementable the optimal policy in this environment would directly insure employers against such adverse shocks. In this setting, the optimal joint design of UI and STC can then be viewed as an attempt to approximate this first-best policy using policy instruments which only condition on labor input choices. This provides a useful perspective for understanding why we find that the answer to the questions posed above is negative.

Considering the first question, it would be comforting for policy makers to know that augmenting an existing UI system with a small amount of STC can do no harm. However, we find that this is not always true in our model, for the following reason: The first-best policy calls for a transfer of resources to distressed firms. In the model distressed firms tend to lay off a large number of employees. Thus UI which conditions transfers on layoffs goes in the right direction for approximating the first-best policy. In contrast to layoffs, the model implies that hours per worker are U-shaped in the level of distress: employers with positive shocks have long hours, but so do distressed employers which combine large layoffs with long hours for retained workers. Thus STC which conditions transfers on hours per worker is not as effective in directing resources to distressed employers.

Turning to the second question, countries with existing STC schemes often offer a similar replacement rate for both UI and STC. A natural reform to consider for countries contemplating the adoption of STC may then be to simply offer the UI replacement rate

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1 Many actual STC systems require employers to document that they are experiencing a temporary adverse shock, so there is an attempt to also condition directly on the level of distress. It would be interesting to extend the analysis in this direction, but here we follow BW in assuming that STC payments are a function only of labor input choices. Similarly, we follow BW in assuming that STC payments are a function only of the reduction of hours, not the associated loss of income. Allowing for the latter would be an important extension to the extent that actual STC systems also condition on the loss of income.

2 This may be a counterfactual implication of the model, however. We intend to investigate this question empirically.

3 See OECD Employment Outlook 2010.
for partial reductions in hours worked. The analysis of our first question already implies that this will not necessarily be welfare-enhancing for an arbitrary level of UI. But it could still be the case that it is always welfare-enhancing if the initial level of UI is optimal given the absence of STC. However, again we find that this is not the case. As discussed above, UI is more effective than STC in directing resources to distressed employers. By itself, this may suggest that there is no beneficial role for STC. However, as discussed earlier, UI distorts the level of employment. STC can mitigate this distortion. Thus we find that STC tends to enhance welfare when UI is sufficiently high, that is, when the distortion of employment is sufficiently severe. But nothing in this logic indicates that it is necessarily desirable to increase the generosity of STC up to the same replacement rate as UI. Indeed, we find that it is possible for such a policy to be worse than not having STC at all.

Finally, concerning the third question, one may expect that moving from an optimal UI system without STC to a system which optimally combines STC and UI should reduce layoffs. We find that this is not necessarily the case. Recall that the primary benefit of STC in the model is to mitigate employment distortions associated with generous UI. Thus allowing for STC as a second policy instrument tends to increase the optimal level of UI since it is now possible to mitigate the distortions it induces. We find that it is possible for the increase in optimal UI to be so large that layoffs are higher under the optimal combination of UI and STC than under optimal UI without STC.

The remainder of the paper is organized as follows. Section 2 lays out our augmented version of the Burdett-Wright model. Section 3 analyzes the contracting problem between the employer and its employees. Section 4 analyzes the qualitative questions concerning the joint design of UI and STC. Section 5 concludes.

2 Model

There is a single firm which has a given workforce of size \( N \) attached to it. The firm has the production function \( xf(Nnh) \) where \( x \) represents technological or other uncertainty, \( n \) is the probability that any given worker is employed, \( h \) denotes hours per employee and the function \( f \) is increasing and strictly concave. Employing a worker is associated with an additional fixed cost of \( F \), so net output is \( xf(Nnh) - NnF \). The distribution of \( x \) is given by the probabilities \( \theta(x) \). The firm is owned by a single risk-neutral individual.

Given workforce size \( N \), the firm chooses a labor contract \{\( n(x), h(x), w(x), b(x) \}\).
Here \( n(x) \) and \( h(x) \) denote employment and hours contingent on \( x \), respectively. The contract specifies contingent payments of \( w(x) \) and \( b(x) \) to employed and unemployed workers, respectively. These payments are inclusive of unemployment compensation and short-time compensation, to be discussed below.

Once the employer has offered a contract, workers choose whether to accept or reject. A worker who rejects the contract receives the exogenous payoff \( \bar{U} \). The preferences of a worker who chooses to stay with the firm are described by

\[
E[u(c) - v(h)]
\]

where \( u \) satisfies \( u' > 0 \) and \( u'' < 0 \), while \( v \) satisfies \( v' > 0, v'' > 0, \) and \( v(0) = 0 \). We assume throughout that gains from trade are sufficiently high such that all workers accept the contract offered by the employer in equilibrium.

The government has two policy instruments: unemployment compensation \( g \) paid to workers with zero hours (that is, the unemployed), and short-time compensation \( g_e \) paid for every hour that hours per employee fall below 1. There are no other government expenditures. The government imposes a lump-sum tax \( T \) on the employer to finance unemployment and short-term compensation.

The firm has limited funds. Specifically, profits contingent on \( x \) cannot fall below an exogenous limit \( B \). Due to this constraint we can abstract from experience rating without loss of generality.

### 3 Contracting Problem

Given workforce size \( N \), the firm chooses a labor contract \( \{n(x), h(x), w(x), b(x)\} \) to maximize profits subject to the reservation utility \( \bar{U} \) for workers, the shallow-pocket constraint and an employment constraint:

\[
\max \sum \theta(x) \left( x f(Nn(x)h(x)) - Nn(x)[w(x) + F - (1 - h(x))g_e] 
\right. 
\left. - N(1 - n(x))[b(x) - g] - T \right)
\]

s.t. \( N \{ \sum \theta(x) \{ n(x) [u(w(x)) - v(h(x))] + (1 - n(x))u(b(x)) \} - \bar{U} \} \geq 0 \),

\[
x f(Nn(x)h(x)) - Nn(x)[w(x) + F - (1 - h(x))g_e] \geq 0,
\]

\[
-x f(Nn(x)h(x)) + Nn(x)[w(x) + F - (1 - h(x))g_e] - N(1 - n(x))[b(x) - g] - T - B \geq 0,
\]

\[
N(1 - n(x)) \geq 0.
\]
Since \( w(x) \) and \( b(x) \) are defined to be inclusive of unemployment and short-time compensation, the latter are deducted from \( w(x) \) and \( b(x) \) in the firm objective.

Let \( \lambda^F, \phi^F(x)\theta(x) \), and \( \nu^F(x)\theta(x) \) denote the multipliers on these constraints, respectively. The first-order condition for \( w(x) \) is

\[
\theta(x) \left[ -Nn(x)(1 + \phi^F(x)) + n(x)\lambda^F Nu'(w(x)) \right] = 0
\]

which simplifies to

\[
(1 + \phi^F(x)) = \lambda^F u'(w(x)).
\]

Similarly, the first-order condition for \( b(x) \) simplifies to

\[
(1 + \phi^F(x)) = \lambda^F u'(b(x)).
\]

Thus \( w(x) = b(x) \). Due to the shallow-pocket constraint the firm may not be able to equalize marginal utility of workers from consumption across levels of \( x \), but for a given level of \( x \) it equalizes marginal utility of employed and unemployed workers. The additive separability assumed here then implies equal levels of consumption. The problem then simplifies to

\[
\max \sum \theta(x) \left\{ xf(Nn(x)h(x)) - Nw(x) - Nn(x)[F + g - (1 - h(x))g_e] + Ng - T \right\}
\]

s.t. \( N \left( \sum \theta(x) \left\{ u(w(x)) - n(x)v(h(x)) \right\} - \bar{U} \right) \geq 0, \)

\[
xf(Nn(x)h(x)) - Nw(x) - Nn(x)[F + g - (1 - h(x))g_e] + Ng - T - B \geq 0,
\]

\[
N(1 - n(x)) \geq 0,
\]

The first-order conditions with respect to \( n(x), h(x), \) and \( w(x) \) are

\[
[xf'(Nn(x)h(x))h(x) - (F + g) + (1 - h(x))g_e] (1 + \phi^F(x)) - \lambda^F v(h(x)) - \nu^F(x) = 0,
\]

\[
(4)
\]

\[
[xf'(Nn(x)h(x)) - g_e] (1 + \phi^F(x)) - \lambda^F v'(h(x)) = 0,
\]

\[
(5)
\]

\[
-(1 + \phi^F(x)) + \lambda^F u'(w(x)) = 0
\]

\[
(6)
\]

On occasion it will be useful to write (5) as

\[
[xf'(Nn(x)h(x))h(x) - h(x)g_e] (1 + \phi^F(x)) - \lambda^F h(x)v'(h(x)) = 0.
\]

\[
(7)
\]
Taking the difference between (4) and (7) yields
\[-(F + g) + g_e] (1 + \phi^F(x)) - \lambda^F [v(h(x)) - h(x)v'(h(x))] - \nu^F(x) = 0. \tag{8}\]

Define
\[V(h(x)) \equiv h(x)v'(h(x)) - v(h(x)).\]

To interpret this function, note that at a given level of hours \(v(h(x))\) is the utility cost of increasing labor input by \(h(x)\) units through an expansion of employment. If instead an expansion of hours is used to achieve the same increase in labor input, the corresponding utility cost is \(h(x)v'(h(x))\). Thus \(V(h(x))\) is the incremental cost of achieving the expansion through hours rather than employment. We have \(V(h(x)) > 0\) and
\[V'(h(x)) = h(x)v''(h(x)) > 0.\]

Using this notation condition (7) can be written as
\[\lambda^F V(h(x)) = \nu^F(x) + [(F + g) - g_e] (1 + \phi^F(x)). \tag{9}\]

Depending on which combinations of the employment constraint and the shallow-pocket constraint are slack or binding there are four cases. The next step will be to describe how to solve each case for a given value of the multiplier \(\lambda^F\).

**Case 1: \(\nu^F(x) = 0\), \(\phi^F(x) = 0\).** In this case we have the system of equations
\[xf'(Nn(x)h(x))h(x) - (F + g) + (1 - h(x)) ge] - \lambda^F v(h(x)) = 0,\]
\[xf'(Nn(x)h(x)) - ge] - \lambda^F v'(h(x)) = 0,
\[-1 + \lambda^F u'(w(x)) = 0.\]

From the last equation we obtain
\[w(x) = u'^{-1} \left( \frac{1}{\lambda^F} \right). \tag{10}\]

In this case the first two equations imply the following version of equation (9):
\[\lambda^F V(h(x)) = [(F + g) - g_e].\]

Thus
\[h(x) = V^{-1} \left( \frac{(F + g) - ge}{\lambda^F} \right).\]

Finally, the first equation can be used to obtain the following formula for employment:
\[n(x) = \frac{1}{Nh(x)} f'^{-1} \left( \frac{\lambda^F v(h(x)) + (F + g) - (1 - h(x)) ge}{xh(x)} \right).\]
Case 2: $\nu^F(x) > 0, \phi^F(x) = 0$. In this case the first-order condition for employment is replaced by constraint (3). Thus we get the conditions

$$1 - n(x) = 0,$$

$$[xf'(Nn(x)h(x))n(x) - n(x)g_e] - \lambda^F n(x)v'(h(x)) = 0,$$

$$-1 + \lambda^F u'(w(x)) = 0. \quad (13)$$

So once again we get the wage from (10) while $n(x) = 1$. For hours we have to solve

$$[xf'(Nh(x)) - g_e] - \lambda^F v'(h(x)) = 0. \quad (12)$$

Case 3: $\nu^F(x) = 0, \phi^F(x) > 0$. In this case the firm is shallow-pocket constrained and not productive enough to employ everyone. Thus we have to add the shallow-pocket constraint to the set of conditions. This yields the system

$$[xf'(Nn(x)h(x))h(x) - (F + g) + (1 - h(x))g_e] (1 + \phi^F(x)) - \lambda^F v(h(x)) = 0,$$

$$[xf'(Nn(x)h(x)) - g_e] (1 + \phi^F(x)) - \lambda^F v'(h(x)) = 0,$$

$$-(1 + \phi^F(x)) + \lambda^F u'(w(x)) = 0,$$

$$xf(Nn(x)h(x)) - Nw(x) - Nn(x)[F + g - (1 - h(x))g_e] + Ng - T - B = 0. \quad (11)$$

Here the solution is a bit more involved. Solving the second equation for $(1 + \phi^F(x))$ yields

$$(1 + \phi^F(x)) = \frac{\lambda^F v'(h(x))}{xf'(Nn(x)h(x)) - g_e}. \quad (12)$$

Substituting into the first equation yields

$$[xf'(Nn(x)h(x))h(x) - (F + g) + (1 - h(x))g_e] \frac{\lambda^F v'(h(x))}{xf'(Nn(x)h(x)) - g_e} = \lambda^F v(h(x)).$$

Rearranging yields $n(x)$ as a function of $h(x)$:

$$n(h(x); x) \equiv \frac{1}{Nh(x)} f^{-1} \left( \frac{[(F + g) - (1 - h(x))g_e] v'(h(x)) - g_e v(h(x))}{xV(h(x))} \right).$$

Rearranging the fourth equation gives a closed-form solution for the wage as a function of hours:

$$w(h(x); x) \equiv \frac{xf(Nn(h(x); x)h(x)) - Nn(h(x); x)[F + g - (1 - h(x))g_e] + Ng - T - B}{N}. \quad (10)$$

We can then use equation (9) combined with the third condition to obtain the equation

$$V(h(x)) - ((F + g) - g_e) u'(w(h(x); x)) = 0; \quad (8)$$
which can be solved for $h(x)$. Notice that over the range in which the case applies the wage is increasing in $x$. Thus hours are decreasing in $x$. In other words, hours will be higher if the firm is more distressed.

**Case 4:** $\nu^F(x) > 0$, $\phi^F(x) > 0$. In this case we again have $n(x) = 1$ replacing the first-order equation for $n(x)$. This leaves the conditions

$$[xf'(Nh(x)) - g_e](1 + \phi^F(x)) - \lambda^F v'(h(x)) = 0,$$

$$-(1 + \phi^F(x)) + \lambda^F u'(w(x)) = 0,$$

$$xf(Nh(x)) - Nw(x) - N[F + g - (1 - h(x))g_e] + Ng - T - B = 0.$$  

The last equation yields the wage as a function of hours

$$w(h(x); x) = \frac{xf(Nh(x)) - N[F + g - (1 - h(x))g_e] + Ng - T - B}{N}.$$  

This information can be used to put bounds on hours in what follows. Combining the first and second equation yields the following condition for hours:

$$[xf'(Nh(x)) - g_e] u'(w(h(x); x)) - v'(h(x)) = 0.$$  

Having solved for hours, employment, and the wage as functions $h(x, \lambda^F, T)$, $n(x, \lambda^F, T)$, and $w(x, \lambda^F, T)$, the multiplier $\lambda^F$ and the tax $T$ must then solve the system of equations:

$$\sum \theta(x) \left\{ u \left[ w \left( x, \lambda^F, T \right) \right] - n \left( x, \lambda^F, T \right) v \left[ h \left( x, \lambda^F, T \right) \right] \right\} = \bar{U}, \quad (14)$$

$$\sum \theta(x) \left\{ n \left( x, \lambda^F, T \right) [1 - h \left( x, \lambda^F, T \right)] g_e + [1 - n \left( x, \lambda^F, T \right)] g \right\} = T. \quad (15)$$

The first equation stems from constraint [1] of the contracting problem, and the second equation is the government budget constraint.

Evaluating expected profits at the solution to equations (14)–(15) yields profits as a function $\Pi(g, g_e)$ of the two policy instruments $g$ and $g_e$.

### 4 Three Questions on Optimal UI and STC

The objective of this section is to investigate qualitative properties of social welfare as a function of UI and STC, as well as properties of the socially optimal combination of the two policies.
Since the utility of workers is always equal to the exogenous payoff $\bar{U}$, social welfare is simply given by expected firm profits $\Pi(g, g_e)$. The Ramsey problem is therefore

$$\max_{g, g_e} \Pi(g, g_e).$$

Specifically, we will use a parametrization of the model to answer several qualitative questions concerning the optimal joint design of UI and STC. The parametrization is not intended as calibration for the purpose of quantitative analysis, rather we use it to disprove several qualitative statements about the social welfare function $\Pi(g, g_e)$ and the associated allocation.

### 4.1 Parametrization

The values of $x$ and the probabilities $\theta(x)$ are chosen to approximate a log-normal distribution with coefficient of variation 0.8, using 50 gridpoints. The mean of $\log(x)$ is normalized to zero.

The functional form of the production function is Cobb-Douglas $f(Nnh) = (Nnh)^\alpha$ and the parameter $\alpha$ is set to 0.85.

The functional form for $u$ is $u(c) = \log(c)$. The functional form for $v$ is $v(h) = -\log(1 - h)$. The fixed cost is set to $F = 0.04$, which equals about 10 percent of the laissez-faire expected gross wage. The reservation level $\bar{U}$ is set to the utility of consuming 0.275 and working 0.5 hours with certainty.

Figure 1 illustrates the behavior of welfare, employment, hours per worker, and output as a function of the two policy instruments $g$ and $g_e$ for this parametrization. The horizontal axes in all plots display the level of STC ($g_e$). The plots in the left column show the outcome variables as a function of $g_e$ for several, relatively small values of $g$. To facilitate interpretation of magnitudes, throughout we express $g$ and $g_e$ as implicit replacement rates, that is, as fractions of the expected gross wage. The largest value of $g$ considered in the left column is the value which maximizes social welfare under the constraint $g_e = 0$. For future reference this value will be denoted $g^*_{NoSTC}$. Notice that over this range of $g$, any increase in $g$ results in an upward shift of the welfare schedule. Only values of $g_e \leq g$ are considered, hence plots with higher values of $g$ extend further to the right. The right column of plots shows the four outcome variables as a function

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4We refer to this as implicit replacement rates since in the model both $g$ and $g_e$ do not enter as a fraction of wages but are directly specified in terms of the consumption good.
of $g_c$ for relatively high values of $g$. The value $g^*_{NoSTC}$ is once again included, now as the lowest value of $g$. Over this range an increase in $g$ shifts down the welfare schedule.

The plots of employment, hours per worker, and output show the expected values of these outcomes, with the expectation taken across the different states $x$. As a point of reference: The first-best levels of employment and hours per worker are indicated through horizontal dashed lines. They have to be interpreted with caution, however: Even if a policy configuration implements the first-best levels of expected hours per worker and employment, this does not guarantee that hours and employment are allocated efficiently across the different levels of $x$. Indeed, in this parametrization no policy configuration comes close to implementing the first-best level of output.\footnote{Even the solution to the Ramsey problem implies an output loss of 21 percent vis-à-vis the first best. This is why the first-best level of output is not shown in the fourth-column panels of Figure 1.}

For low levels of $g$ expected employment is relatively high, and expected hours per worker are relatively low, when compared to the corresponding first-best levels. STC increases employment and reduces hours further, and this reduces both output and welfare.

High levels of $g$ induce the firm to offer a contract with low expected employment, which allows the firm and its employees to jointly take advantage of unemployment compensation. Lower expected employment is also associated with higher expected hours per worker. This imposes a negative externality on the government. While it is socially desirable to use unemployment compensation to direct resources to the firm in states of distress, high unemployment compensation also induces layoffs in states of the world in which the firm is not severely constrained by limited funds. In this situation STC can bring expected employment and hours per worker closer to their respective first-best levels. It is then possible for STC to raise expected output and welfare. The welfare-maximizing level of STC for given UI tends to be increasing in the level of UI.

4.2 Three Questions

We now use the parametrized model to discuss three questions concerning the optimal joint design of UI and STC.

**Question 1: For a given level of UI, does introducing a small amount of STC necessarily improve welfare?**

It would be comforting for policy makers to be assured that augmenting an existing UI system with a small amount of STC can do no harm. However, it is already clear
from the upper left panel of Figure 1 that this is not true in our model. The reason for this can be explained with the help of Figure 2, which considers a specific illustrative low level of $g$, corresponding to an implicit replacement rate of 0.12. The left column of plots simply replicates some information from Figure 1 as a point of reference: the top panel shows welfare as a function of $g_e$ for this specific level of $g$, and the bottom panel shows employment, hours per worker, as well as total hours. The key panel is the top panel of the right column. It shows the net subsidies implied by a particular policy configuration for each level of $x$. Recall that the shallow-pocket constraint is the sole friction in the model, and it binds whenever the employer experiences a sufficiently severe adverse shock to $x$. If it were implementable the optimal policy in this environment would directly insure employers against such adverse shocks. As a point of reference, the dashed line in the upper right panel shows the net subsidy schedule which implements this optimal policy if the state-contingent lump-sum payments in states of distress are financed through a lump-sum tax imposed in all states. A useful way of understanding the costs and benefits of STC and UI in this context is to think of their joint design as an attempt to approximate this first-best policy using policy instruments which only condition on labor input choices. The remaining plots in the top right panel of Figure 2 show the net subsidy schedule implied by different values of $g_e$. The vertical axis for these plots is on the left, while the axis for the first best schedule is on the right. Notice that the scales differ by an order of magnitude: none of the policy configurations comes close to the large subsidies for states of distress prescribed by the first-best schedule. However, the schedule without STC ($g_e = 0$, solid line) at least has the correct shape, transferring resources only in the most distressed states. Introducing a small amount of STC ($g_e = 0.07$, dotted line) changes this subsidy schedule in a socially undesirable way. In states of distress the firm receives only a relatively small subsidy through STC. The reason for this is apparent from the bottom right panel of Figure 2, which displays hours per worker as a function of $x$ for the different policy configurations. As discussed in the analysis of Case 3 in Section 3, hours per worker are relatively high in states of distress, in which the firm relies heavily on layoffs while prescribing long hours for the remaining workers. Thus hours as a function of $x$ are U-shaped, since the firm also relies on long hours if $x$ is sufficiently high for it to be employment-constrained. The immediate consequence of this shape of the hours schedule is that the bulk of the subsidies from STC are paid in the event of an intermediate realization of $x$, rather than in states of distress.
as prescribed by the first best. These poorly targeted subsidies require an increase in the lump-sum tax, which is also borne by the firm in states of distress. Consequently the shallow-pocket constraint binds even more severely in states of distress, which induces the firm to raise production and employment in distressed states, resulting in an additional loss of UI subsidies. These adverse consequences of STC are magnified if the level of STC is increased further ($g_e = 0.12$, dash-dotted line).

**Question 2: Starting from the optimal UI system without STC, does introducing STC with a similar replacement rate as UI necessarily improve welfare?**

Countries with existing STC schemes often offer the same replacement rate for both UI and STC. A natural reform to consider for countries contemplating the adoption of STC may then be to simply offer the existing UI replacement rate for partial reductions in hours worked. The analysis of our first question already implies that this will not necessarily be welfare-enhancing for an arbitrary level of UI. But it could still be the case that it is always welfare-enhancing if the initial level of UI is optimal given the absence of STC. However, again we find that this is not the case. The reason for this can be explained with the help of Figure 3, which illustrates the effect of different reforms starting from the optimal UI system under the constraint $g_e = 0$, with associated UI replacement rate $g^*_{NoSTC} = 0.22$. In the upper left panel, moving from step 1 (no STC) to step 2 ($g_e = g^*_{NoSTC}$) along the horizontal axis traces out how welfare changes as the level of STC is increased up to the same replacement rate as offered by UI. Notice that a relatively small level of STC would be beneficial here. The reason for this has already been discussed in Section 4.1. STC can mitigate the negative externality associated with UI. If UI is sufficiently generous this positive impact of STC can outweigh the otherwise undesirable shape of the subsidy schedule associated with STC. But nothing in this logic indicates that it is necessarily desirable to extend STC up to the same replacement rate as offered by UI. And it does not in this parametrization of the model. The effect of doing so on the subsidy schedule can be seen in the upper right panel. The solid line (labeled NoSTC) shows the subsidy schedule for optimal UI with no STC. The dash-dotted line (labeled $g_e = g^*_{NoSTC}$) shows the subsidy schedule as STC is increased up to the same implicit replacement rate as offered by UI. Most subsidies are paid in states with intermediate values of $x$ rather than in states of distress. Once again this is due to the U-shape of the hours schedule, shown in the lower right panel.
Question 3: If a reform replaces the optimal UI system without STC with the optimal system combining UI and STC, does this necessarily reduce layoffs?

One may expect that moving from an optimal UI system without STC to a system which optimally combines STC and UI should lead to a reduction in layoffs. After all, as discussed in Section 4.1, the effect of increasing STC is a reduction in layoffs. However, moving to the optimal combination of UI and STC in general will also entail a change in UI. If UI becomes sufficiently more generous, then layoffs could increase. The parametrization considered here demonstrates that this is indeed possible, which is illustrated in Figure 4. When discussing Question 2 we have seen that starting at $g^*_{\text{NoSTC}}$ (the optimal level of UI in the absence of STC), raising the level of STC to equal the UI replacement rate reduces social welfare. This does not imply, however, that this level of STC is excessive from a welfare perspective. The upper right panel illustrates the case with equal replacement rates as point of departure (solid line, labeled $g_e = g^*_{\text{NoSTC}}$). We then fix the corresponding level of STC and reoptimize the level of UI (step 3), yielding a replacement rate of $g = 0.35$. The move from step 2 ($g_e = g^*_{\text{NoSTC}}$) to step 3 in the upper left panel continues the journey begun in the corresponding panel of Figure 3: it traces out the welfare effect of gradually increasing $g$ to this reoptimized level for the fixed value of $g_e$. Notice that this achieves a higher value than the starting point of $g^*_{\text{NoSTC}}$ with no STC. To understand the source of this welfare improvement, the upper right panel show the subsidy schedule (dash-dotted line, labeled reopt. $g$) induced by this policy configuration. Notice that the reoptimization of $g$ undoes the adverse effects on the shape of the subsidy schedule associated with only increasing STC. The lower left panel traces out the effects of this sequence of reforms on labor inputs. The reoptimization of $g$ does not yet yield the solution to the Ramsey problem, but it turns out to be quite close. The move to the full Ramsey optimum involves further increases the replacement rates to $g = 0.47$ and $g_e = 0.38$, respectively. This is illustrated by the single move from step 3 to step 4 on the horizontal axes of the left-column panels. The implied subsidy schedule is given by the dotted line in the upper right panel. The additional changes in labor inputs from this step are relatively small, and there is a slight further improvement in the subsidy schedule. From step 1 to step 2 on the horizontal axis the increase in only $g_e$ results in higher employment and lower hours. The reoptimization of $g$ traced out in the move from step 2 to step 3 and then the move to the Ramsey equilibrium in step 4.

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6The horizontal axis of this panel is not in units of $g$ or $g_e$, rather it traces out the effect of a particular sequence of changes in policy.
reduce employment and increase hours. The combined result of moving from step 1 to step 4 is a drop in both components of labor input. In other words, the introduction of optimal STC accompanied by a reoptimization of UI can actually result in an increase in layoffs. Intuitively, better insurance reduces the shadow value of output in distressed states, reducing labor utilization in such states.

5 Conclusion

In this paper we have extended the implicit-contracts model of unemployment insurance and short-term compensation by Burdett and Wright (1989) to allow for the possibility that firms are constrained in their ability to insure workers due to limited funds.

We have used the model to address three qualitative questions concerning the optimal design of UI and STC. First, we found that it is possible for the introduction of a small amount of STC to reduce social welfare. Second, even if the initial level of UI is optimal given the absence of STC, introducing STC with the same replacement rate as offered by UI could reduce social welfare. Third, it is possible that a reform which replaces optimal UI without STC with the optimal combination of both STC and UI increases layoffs.

We conclude by discussing some directions for further research. First, a recurring concern with STC in policy discussions is that it may delay socially beneficial reallocation of labor in the economy. We would like to extend the model to be able to speak to such concerns by giving workers an additional productive use of time, namely search for alternative employment opportunities.

Second, the only friction in our model are limited funds by firms. There are no frictions interfering with risk sharing within the employment relationship. It would be interesting to consider frictions within the employment relationship such as limited commitment, and examine how they affect the problem of designing an optimal system of UI and STC. While in our present model marginal utility of employed and unemployed workers is equalized, such frictions within the employment relationship may also give rise to unequal treatment of laid-off workers. This may also imply that laid-off workers have stronger incentives to search for a new job than workers on short-time, so this feature may interact in interesting ways with the concern about reallocation discussed above.
Figure 1: Welfare, Hours, Employment, and Output as Functions of $g$ and $g_e$
Figure 2: Introducing small $g_e$ for given $g$ (Question 1)

welfare across $g_e$ at $g = 0.12$

net gov. subsidy across $x$

Employment across $g_e$ at $g = 0.12$

hours/worker across $x$
Figure 3: Introducing $g_e$ with replacement rate of optimal $g$ (Question 2)
Figure 4: From optimal $g$ with $g_e = 0$ to Ramsey Optimum (Question 3)
References


