Political Economy of Asymmetric Aging, Migration and Fiscal Policy

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This paper uses an overlapping generations model with international labor mobility and a politically responsive fiscal policy to examine aging in developed and developing regions. Migrant workers change the political structure composed of young and elderly voters in both labor-receiving and labor-sending countries. Numerical simulations show that the developed region benefits more from international labor mobility through the contribution of migrant workers as laborers, savers and voters. The developing region experiences significant growth in all specifications but benefit more under international capital mobility. Restricting political participation of migrant workers in the developed region produces inferior growth results.

**JEL classification:** E62, F21, F22, F43, H30, H52, H55, J10

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1. Introduction

Population aging has become a global phenomenon. Recent population projections show that the world median age will rise from 29.2 in 2013 to 36.1 in 2050, and to 41.2 in 2100. There has been an increasing focus on the international implications of the simultaneous aging of the populations of industrialized nations. However, global aging relates to developing regions as well as the developed regions of the world. According to recent projections from the United Nations Population Division\(^1\), the old-age dependency ratios\(^2\) in developed and developing regions will increase approximately by 4\% and 70\% between 2050 and 2100, respectively. By 2150, the difference between the old-age dependency ratios of the two regions will be quite small\(^3\). Thus, population aging is not confined to developed countries; sooner or later populations of all nations are expected to age.

An important consequence of population aging is increasing fiscal pressure through higher government spending on social security, health care and other welfare programs for the elderly.\(^4\) This may mean lower government spending for other programs that primarily benefit the young. The link between increasing political power of the elderly and government spending on such programs as education has been examined by Gradstein and Kaganovich (2003), Holtz-Eakin, Lovely and Tosun (2004), and Razin, Sadka and Swagel (2002). Since education is a major input to human capital accumulation, aging is expected to have significant growth consequences. Aging has also been analyzed in open economy by Cutler, et al. (1990), Börsch-Supan (1996), Kenc and Sayan (2001), Elmendorf and Sheiner (2000), Jelassi and Sayan (2004), Tosun (2001, 2003) and Van Groezen and Leers (2000). This literature shows the significance of the international spillover effects of aging. While there is a growing literature analyzing the link between population aging and international capital mobility, only few studies examined aging with labor migration (Leers, Meijdam and Verbon, 2004; Storesletten, 2000). In a dynamic model with international capital mobility only, wage rates do not necessarily equalize between countries in all periods of their transition to the long run steady state. This implies that, when allowed, labor will move to the country with the higher standard of living. This highlights the need for an analysis of aging in an open economy framework with international labor mobility. Galor (1992) argued that capital and labor have asymmetric characteristics. Labor mobility has a dual effect in the sense that it exhibits the characteristics of capital mobility as well. Young migrant workers contribute to the economy both as laborers, and as savers. What the aging literature has not addressed is that, when allowed to vote, migrant workers will change the political

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\(^2\) Old-age dependency ratio is the proportion of population aged 65 and older to population aged 15 to 64.
\(^3\) A medium fertility scenario shows that in 2150 the old-age dependency ratios will be 0.47 and 0.43 for the developed and developing regions, respectively.

\(^4\) See Heller (2003) and CSIS (2002) for recent discussions on the fiscal implications of global aging.
structure composed of young and elderly voters in both labor-receiving and labor-sending countries. In a majority voting mechanism for fiscal policy decisions, political shifts resulting from labor migration may have sizeable impacts on government programs, and in turn may have strong growth effects. This paper uses a two-region, two-period overlapping generations model with international labor mobility and a politically responsive fiscal policy to examine the growth effects of the aging trend in developed and developing countries. The paper contributes to the literature on the economic effects of population aging in two major ways. First, it brings out the political economy of aging through the aging – education - human capital link. The paper argues that this is particularly important in the context of international labor mobility with young migrant workers participating in the political system of developed countries. Second, the paper examines the significance of factor mobility as a policy choice by comparing the labor mobility model to an alternative open economy model with international capital mobility.

The paper is structured as follows. The next section gives a description of a two-region, two-period overlapping generations model with international labor mobility. This is followed by a transition analysis in section 3 that shows results from various numerical simulation exercises. The last section presents summary and concluding remarks.

II. The Two-Region Model

The model builds on a two-period overlapping generations model first developed by Diamond (1965)\(^5\). To examine open economy issues, the standard framework is extended to a two-region model with international labor mobility similar to Galor (1986, 1992) and Crettez et al. (1996, 1998)\(^6\). Another major extension is the modeling of fiscal policy decision-making through a political process.

Recent discussions on population aging have noted the potential generational conflict generated by the need to share society’s resources between non-working elderly and the younger working population. It is argued that increasing number of elderly voters could render changes in public expenditure patterns in favor of the elderly. This, in turn, could trigger serious generational conflict regarding government programs that enhances the productivity of the working young. One good example to this is government spending on public education. For simplicity, the productivity enhancing public program will be referred to as “education” throughout the text.\(^7\) The goal is to highlight the strong link between this type of government spending and human capital accumulation, which is considered to be one of the most important avenues for economic growth.

Empirical studies, particularly from the U.S., show the significance of the political economy consideration of the relationship between population aging and education spending. Among the earlier studies, Button (1992) suggests that generational conflict is quite apparent on education issues by examining the voting behavior of the

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\(^5\) However, the earliest overlapping generations models are described by Allais (1947) and Samuelson (1958). Children are not modeled in a two-period model.  
\(^6\) A two-country model with international capital mobility is shown by Buiter (1981).  
\(^7\) It should be noted that any other government program that is directed towards increasing the labor productivity of young could easily be used.
elderly in tax referenda in six Florida counties. Deller and Walzer (1993) found weaker evidence of such generational conflict based on a survey of residents in rural Illinois. Poterba (1997) provided empirical evidence using state-level data that older citizens prefer lower levels of public spending for education, an expenditure that primarily benefits the young. A recent study by Harris, Evans, and Schwab (2001) confirmed this finding using school-district-level data, however with a smaller estimated impact than Poterba’s estimates. While, the 2004 AARP Aging American Voter Survey indicates that a majority of older people support federal government’s responsibility in educating young people, the same survey shows that a large group of older people also became more conservative in issues such as government spending, bureaucracy and taxes.\(^8\)

Given the expected future aging trends in both developed and developing countries, generational conflict in education spending could become even more prevalent. Modeling generational conflict in education is also important when we consider immigration. Table 1 provides basic statistics on immigration and education spending. This shows that high-income (developed) countries that had positive net migration rates between 1995 and 2000 indeed had both higher average education spending per capita and higher percentage increase in education spending per capita in this period compared to both less developed regions and least developed countries. Hence, aging-education link may provide useful insight in a study on population aging and international labor mobility. To make the political process of fiscal policy determination for public education rich, interesting, yet tractable, a median-voter framework with voter heterogeneity is used. Voter heterogeneity is introduced by assuming a distribution of genetic ability levels for the working generation. The ability level of the individual will, in turn, determine the value she receives from public education. While the emphasis of the paper is political economy of education spending, a social security program is also introduced by having an exogenously fixed level of social security spending in the model. An income tax that is earmarked for social security adjusts through the periods to balance the social security budget. Thus, there are separate taxes for education and social security spending with voters deciding only on the education tax rate.\(^9\) For clarity, the model is presented for one region only. This is followed by a description of the two-region world equilibrium.

\textbf{A. Households}  
Individuals live for two periods and seek to maximize a utility function based on discretionary consumption in the first and second period of their lives,

\[ U = \ln C_{jt} + \left( \frac{1}{1+\delta} \right) \ln C_{jt+1}, \]  

\(^8\)See AARP (2004) for a summary of findings from this survey.  
\(^9\)The political process is modeled through a median voter framework because the conditions for the median voter theorem are satisfied. The choice of voters is over a single dimension since the preferred education tax rate is the only choice variable, and the voter preferences are single peaked. The property of single-peakedness has been demonstrated to ensure existence of a voting equilibrium (Black 1948).
here $j$ indexes individuals, $C_t$ is consumption when young, $C_{t+1}$ is consumption when old, and $\delta$ is the pure rate of time preference. The period-specific budget constraints in the first and the second periods are:

First period: $C_t(a_j) + S_t(a_j) = (1 - \tau_t - \mu_t) w_t l_t(a_j)$

Second Period: $C_{t+1}(a_j) = (1 + (1 - \tau_{t+1} - \mu_{t+1}) r_{t+1}) S_{t+1}(a_j)$, \hspace{1cm} (2)

where $S_t(a_j)$ is first period saving, $w_t$ is the wage rate individual $j$ faces, $l_t(a_j)$ is effective labor,\(^{10}\) where $a_j$ is the ability level of individual $j$, $r_{t+1}$ is the rate of return to capital, $\tau_t$ is the rate of income taxation that is applied to both capital and labor income. This tax is used entirely to finance education spending. An additional tax ($\mu_t$) is also applied to capital and labor income to finance social security spending by the government.

It is assumed that there is a continuous distribution of abilities that is replicated in each new generation. The ability level of individual $j$ is indexed by $a_j$, which ranges from 0 to 1. The density function of abilities is denoted by $f(a)$ where by definition:

$$\int_0^1 f(a) da = 1.$$ \hspace{1cm} (3)

Human capital is accumulated from the interaction of ability level ($a_j$) of the individual and government spending per young ($g^r_t$) on education:

$$l_t(a_j) = \Phi \left[ a_j g^r_t + 1 \right]^\psi,$$ \hspace{1cm} (4)

where, $\Phi$ denotes an index on human capital efficiency and $\psi$ is a parameter indicating the return to human capital from the inputs ($a_j$ and $g^r_t$).\(^{11}\) The form of the human capital function is chosen so that even individuals with the lowest ability ($a_j = 0$) will contribute to the economy in terms of human capital (see Holtz-Eakin, Lovely, and Tosun 2004).

From the maximization of (1) subject to (2) and (4); we get the familiar first order condition:

$$C_t(a_j) = \frac{1 + \delta}{(1 + r_{t+1} (1 - \tau_{t+1} - \mu_{t+1}))} C_{t+1}(a_j).$$ \hspace{1cm} (5)

Using (5) and (2), we derive the optimal saving of an individual $j$:

$$S_t(a_j) = \frac{1}{2 + \delta} (1 - \tau_t - \mu_t) w_t l_t(a_j).$$ \hspace{1cm} (6)

\(^{10}\)Here, young supplies one unit of time to the economy. Note that, making the allocation of time between “schooling” and supplying labor endogenous does not change this analysis.

\(^{11}\) $\psi$ should be less than or equal to unity to prevent increasing returns from government spending.
Saving of an individual depends on net labor earnings but it is independent of the interest rate. This is due to the Cobb-Douglas form of the utility function. Given (5) and (6), it is straightforward to derive consumption functions in each period:

\[
C_{jt}(a_j) = \frac{1+\delta}{2+\delta} \left(1 - \tau_t - \mu_t\right) w_t l_t (a_j)
\]

\[
C_{jt+1}(a_j) = \frac{(1+r_{t+1}(1-\tau_{t+1} - \mu_{t+1}))((1-\tau_t - \mu_t) w_t l_t (a_j))}{2+\delta}.
\]

\[
(7)
\]

**B. Political Process of Fiscal Policy**

It is assumed that there is a predetermined level of social security spending. Thus the social security tax \(\mu_t\) is simply determined by the government budget constraint (\(\mu_t y_t = g^*_t\)) where \(g^*_t\) is the social security spending per young person.12

However, the fiscal policy for education is determined through a political process for which a median-voter framework with voter heterogeneity is used. Voter heterogeneity is introduced by assuming a uniform distribution of genetic ability levels for the working generation. The ability level of the individual will, in turn, determine the value she receives from the public good.

The consumption and saving decisions, as seen above, depend on human capital, which is in turn determined by government spending (see equation 4). By plugging these into (1), we get the indirect utility function, which each voter maximizes, in determining his or her preferred tax rate, subject to the government budget constraint for this type of government spending (\(\tau_t y_t = g^*_t\)).13 The preferred tax rate of individual \(j\) when young is:

\[
\tau_{jt}(a_j) = \frac{a_j y_t (1-\mu_t)-1}{(1+\psi) a_j y_t}.
\]

Equation (8) is the tax rate each individual prefers based on her ability level. This preferred tax rate is increasing in both ability level \(a_j\) and in income per young \(y_t\) but decreasing in the social security tax \(\mu_t\). Thus, the existence of a social security system depresses education spending through a lower preferred education tax rate. In addition, because the old do not derive any benefit from publicly provided education and there are

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12 Social security spending consists of equal payments to elderly population to finance a programmed elderly consumption such as health care. This consumption is assumed to be separate from the discretionary consumption decision and therefore it is not shown as part of households' utility maximization depicted in equations (1) through (7).

13 It is assumed in each period that government uses the entire revenue from this tax to finance the public good for all young equally, regardless of their ability level (Bearse, Glomm, and Ravikumar 2000).
no bequests in the model, they incur a cost without enjoying any benefits. Therefore, their preferred education tax rate will always be zero, regardless of their ability.

Total population in each period is \( N_{t-1} + N_t \) where \( N_t \) is composed of both newly born nationals and migrant workers. Given this, the median voter is defined by

\[
N_{t-1} + N_t \int_0^a f(a) \, da = \frac{N_{t-1} + N_t}{2},
\]

where \( a_m \) is the ability level of the median voter.

In the absence of migration, the median voter becomes a person with lower ability when population ages. In turn, the preferred tax rate of the median voter is lower. In other words, with population aging older people need fewer young voters to form a majority. Since these young voters are the ones at the lower end of the ability distribution, they prefer lower taxes than higher ability people because their return from public education is lower.

When labor migration is allowed, an aging country will experience an inflow of labor due to a higher wage rate than the rest of the world. This will change the age distribution of population in favor of the young generation. The identity of the median voter will be different from the case without migration (see Appendix I). Now, the ability of the median voter will be higher compared to median voter’s ability in the case without migration. However, whether the ability of median voter with migration can be greater than the pre-aging level is uncertain.\(^\text{14}\)

\section*{C. Producers}

Each country produces a single good using a Cobb-Douglas production technology.

\[
Y_t = \Lambda K_t^\alpha H_t^{1-\alpha},
\]

here \( \Lambda \) is the productivity index, \( K \) is capital stock and \( H \) is aggregate supply of human capital. The aggregate supply of human capital is:

\[
H_t = N_t \int_0^1 l(a) f(a) \, da.
\]

Human capital per worker, using (4) and (11), is

\[
h_t = \Phi \int_0^1 (ag_t + 1)^w f(a) \, da.
\]

Competitive factor markets require that real wage and interest rates are equal to the marginal products of labor and capital respectively. Therefore, factor demand equations are:

\[
w_t = (1-\alpha) \Lambda \left( \frac{k_t}{h_t} \right)^\alpha
\]

\(^{14}\) In reality, there are barriers to labor migration that may rule out such a case.
\[ r_t = \alpha \Lambda \left( \frac{k_t}{h_t} \right)^{\alpha-1}. \]  

(14)

Here, \( k_t = K_t / N_t \) and \( h_t = H_t / N_t \) are capital stock per worker and human capital per worker, respectively.

Using (6) and (12), saving per worker can be expressed as

\[ s_t = \left( \frac{1}{2 + \delta} \right) (1 - \tau - \mu) w_t \Phi \int_0^1 (ag_t + 1)^\nu f(a) da. \]  

(15)

**D. International Goods Market Equilibrium and Labor Flows**

In the absence of international capital mobility, capital market equilibrium requires that saving in each period equals to accumulated capital in the following period. Capital market equilibrium conditions for each region can be depicted as

\[ k_{t+1}^A = \frac{N_t^A S_t^A}{N_{t+1}^A}, \]  

(16)

\[ k_{t+1}^B = \frac{N_t^B S_t^B}{N_{t+1}^B}, \]  

(17)

where, superscripts \( A \) and \( B \) denote regions.

To close the dynamic model, international labor market equilibrium must be specified. For simplicity, I assume that there is perfect international labor mobility.\(^{15}\) International labor market equilibrium requires

\[ N_{t+1}^A + N_{t+1}^B = (1 + \eta_{t+1}^A) N_t^A + (1 + \eta_{t+1}^B) N_t^B. \]  

(18)

where, \( \eta_{t+1}^A \) and \( \eta_{t+1}^B \) are the population growth rates in region A and region B, respectively. Labor income is taxed where income is earned. Thus, source based income taxation is used for both regions.\(^{16}\) This implies that net-of-tax wage rates are equalized in equilibrium. Therefore, the international labor flow constraint is:

\[ w_{t+1}^A h_{t+1}^A (1 - \tau_{t+1}^A - \mu_{t+1}^A) = w_{t+1}^B h_{t+1}^B (1 - \tau_{t+1}^B - \mu_{t+1}^B). \]  

(19)

It is assumed that only the members of the young generation moves between regions. Both regions are assumed to have “uniform” ability distributions, which mean that migration does not have any effect on the ability distribution in these regions.\(^{17}\)

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\(^{15}\) A recent study by National Research Council shows that total stock of migrants increased quite dramatically in late 1980s and early 1990s (see National Research Council, 2000, pp. 157-159). In addition, in my model one period corresponds to 30 years, which makes perfect labor mobility a viable assumption.

\(^{16}\) Under a source system, labor income is taxed where income is earned. The model tax treaties of the OECD and the United Nations both give source countries the first rights to tax income accrued within their borders.

\(^{17}\) A more realistic case is allowing for migration of workers that have certain abilities (unskilled vs. skilled). However, this would conflict with the uniform ability distribution which assumes that ability levels in the distribution are chosen at random.
The model incorporates the interaction of household behavior, firm behavior, political process, and international labor flows. In the model explained above, a decrease in the population growth rate can affect labor flows and capital accumulation in two ways. First, it can affect “directly” by causing fewer workers in the economy, which leads to higher marginal product of labor, and thereby inducing labor inflows, *ceteris paribus*. Second, it can also affect “indirectly” through endogenous fiscal policy by changing the identity of the median voter. As a result of the aging trend, median voter becomes a lower ability person that votes for a lower tax rate.

3. Stability, Steady State, and Transitions

In this section, I further analyze the two-region model explained in section 2 in order to gain intuition about the effects of population aging. For this, I totally differentiate the equations given by (8), (10), and (12) through (15) for each region, and (16) through (19), which, by substitution, leads to the following reduced system:

\[
\begin{bmatrix}
    dk_{t+1}^A \\
    dk_{t+1}^B
\end{bmatrix} = \begin{bmatrix}
    M^I & M^I \\
    M^I & M^I
\end{bmatrix} \begin{bmatrix}
    dk_{t+1}^A \\
    dk_{t+1}^B
\end{bmatrix} + \begin{bmatrix}
    Z^I_4 Z^I_4 - Z^I_3 Z^I_3 \\
    Z^I_4 Z^I_4 - Z^I_3 Z^I_3
\end{bmatrix} \begin{bmatrix}
    da^A_t \\
    da^B_t
\end{bmatrix} + \begin{bmatrix}
    Z^I_2 Z^I_2 \\
    Z^I_2 Z^I_2
\end{bmatrix} \begin{bmatrix}
    d\eta^A_{t+1} \\
    d\eta^B_{t+1}
\end{bmatrix}. \tag{20}
\]

Equation (20) shows that the effect of a change in the natural population growth rate on capital stock per worker appear in two ways. First, there is a “direct” way, which is captured by the last term at the right hand side of equation (20). There is also an “indirect” way that is channeled through changing ability level of the median voter. This is captured by the second term at the right hand side of equation (20).

**Stability**

The vector difference equation (20) can be used to examine the stability of the dynamic system. For a constant identical population growth across periods in both regions, \(\begin{bmatrix}
    M^I & M^I \\
    M^I & M^I
\end{bmatrix}\) shows the relationship between \(k_t\) and \(k_{t+1}\), where

\[
M^I = \frac{\left(1 - \frac{\tau}{1 - \tau} \epsilon_{\gamma}\right)}{2P}, \quad \text{and} \quad P = 1 - \left[\left(1 - \alpha\right)\epsilon_{bg} \left(1 + \epsilon_{\gamma}\right)\right].
\]

Political equilibrium is locally stable when \(P > 0\) and the system is dynamically stable when \(2M < 1\) (see Appendix I for derivation of the stability conditions). These stability conditions are identical to the ones for the closed economy and international capital mobility models.

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18 See Appendix III.A for the steps involved in the derivation of this reduced system.

19 Due to Cobb-Douglas, \(\epsilon_{\gamma} < \frac{1 - \tau}{\tau}\) which means that \(0 < 2M < 1\) (see Appendix I for a proof).
Steady State

As \( t \to \infty \), \( da_t = \frac{d\eta_t}{2(1 + \eta^*)} \) and (20) can be written as

\[
\begin{bmatrix}
Z_4'' \\
Z_2'' + Z_1''
\end{bmatrix} + \begin{bmatrix}
Z_4' \\
Z_2' + Z_1'
\end{bmatrix} = \begin{bmatrix}
\frac{d\eta^A_t}{d\eta} \\
\frac{d\eta^B_t}{d\eta}
\end{bmatrix}
\]

(21)

As shown in Appendix III.B, at the steady state,\(^{20}\)

\[
\frac{dk^{A}_{t+1}}{dk^{B}_{t+1}} = \frac{Z_4'' + Z_4'}{1 - 2M'}
\]

(22)

where \( Z_2' = \frac{-k^*}{2(1 + \eta^*)} \), and \( Z_4' = \frac{(1 - \alpha)s^* \varepsilon_{h^a} \varepsilon_{m} \left(1 - \frac{\varepsilon_{h^a} \tau^*}{1 - \tau^*} - s^* \varepsilon_{m} \frac{\tau^*}{1 - \tau^*}P\right)}{2P \eta^* \left(1 + \eta^*\right)^3} \).

\( Z_2' \) is negative, while the sign of \( Z_4' \) is indeterminate which makes the sign of the derivatives in (22) indeterminate.\(^{21}\) \( Z_2' \) is the “direct saving effect”\(^{22}\) of a change in the population growth rate on capital per worker for a given ability of median voter. Holding the ability of the median voter fixed, a decrease in the population growth rate increases capital per worker by spreading the same saving over fewer workers. \( Z_4' \) is the “indirect saving effect”\(^{23}\) of a change in the ability of median voter on capital per worker. A decrease in the median voter’s ability, triggered by a decrease in the population growth rate, decreases wage income \((w; h)\) while decreasing the tax rate. Thus, the net effect on net labor income and saving depends on the relative magnitudes of the effects on wage income and the tax rate. The total effect of a decrease in the population growth rate on the steady state capital per worker depends on how the political economy related effect

\(^{20}\)Due to the perfect symmetry of the initial and the final steady states, this steady state formulation is identical to both closed economy and open economy with international capital mobility results. See Holtz-Eakin, Lovely and Tosun (2000) and Tosun (2000).

\(^{21}\)The denominator of (22) is positive since \(2M' < 1\) by dynamic stability.

\(^{22}\)By “direct” effect, I mean an effect independent of the change in the ability level of the median voter.

\(^{23}\)By “indirect” effect I mean an effect that is driven by a change in median voter’s ability. This effect carries the political economy of population aging.
(Z^4_i) compares to the direct saving effect (Z^4_j)^{24}. Notice that Z^4_i depends on the elasticity of the preferred tax rate with respect to income per worker (ε_y), elasticity of the preferred tax rate with respect to median voter’s ability (ε_m), and the elasticity of human capital with respect to government spending per worker (ε_{hg}).^{25} When ε_m = 0, Z^4_i is also equal to zero and steady state capital per worker unambiguously increases with a decrease in η. This is because zero elasticity eliminates completely the feedback effects from an endogenous fiscal policy. Z^4_i is increasing in ε_{hg} which means that a decrease in government spending decreases human capital per worker more as ε_{hg} gets bigger, leading to a greater decrease in labor income, ceteris paribus.

**Transitions**

In order to analyze transitions in response to aging, let’s consider the case where region A experiences a drop in its natural population growth rate in period 1. Since there are no changes in period 0 and population growth rate stays the same in region B, we can set da^A_0 = da^B_0 = dη^B_0 = 0 in (20). Also, as implied by (18) and (19), perfect labor mobility between regions ensures that populations of both regions should remain identical throughout the transition. This, in turn, implies da^A_1 = da^B_1. Thus, for a given capital stock per worker in both regions in period t (dk^A_0 = dk^B_0 = 0), this gives us

\[
\frac{dk^A_1}{dη^A_0} = \frac{dk^B_1}{dη^B_0} = Z^1_2 - \underbrace{Z^1_3}_{Z^1_2} da^A_1 + \underbrace{Z^1_1}_{Z^1_2} da^B_1 < 0. \tag{23}
\]

Notice that the indirect effect captured by \(Z^1_3/Z^1_1\) cancels out in (23). This is due to the fact that da^A_1 = da^B_1. However, there is still value to explore what \(Z^1_3/Z^1_1\) indicates. Here, the denominator \(Z^1_1 = \frac{α(1-α)}{khP} \left(1 - \frac{1 - ε_{hg}(1 + ε_y)}{ε_{hg}(1 + ε_y)}\right)\) can be thought of as a “direct net wage rate effect” (through capital changes) of a change in the population growth rate in one region. Under the stability conditions, \(Z^1_1 < 0\). For a given ability of the median voter, an increase in capital per worker in one region would raise

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24 In the case of an exogenously fixed fiscal policy, this uncertainty disappears and (22) reduces to

\[
\frac{dk^A_0}{dη} = \frac{dk^B_0}{dη} = \frac{2Z^1_2}{1-2M^f} < 0, \text{ where } M^f = \frac{α}{2(1-τ)}.
\]

25 ε_{hg} = \left[\frac{(1+g)^w (ψ_{g^*} - 1) + 1}{(1+g)^w + 1 - 1}\right] and ε_y = \frac{1}{a_m ψ_y - 1}.
income per worker, human capital per worker and the tax rate. However, the sign of the net effect on the net-of-tax wage rate is negative. 

\[ Z'_3 = -\frac{[\sigma(1 - \alpha)y_{e_m}e_{h_0} \sigma(1 - \tau) + (1 - \alpha)e_m)]}{ha_m P} \] 

wage rate effect” through a change in median voter’s ability. Under the stability of the political equilibrium (P>0), \( Z'_3 \) is negative. For a given level of capital per worker, a decrease in the median voter’s ability decreases both income per worker and human capital per worker. However, the marginal productivity of human capital will increase, creating a positive effect on the wage rate. Since, it also decreases the tax rate, the net wage rate will unambiguously increase. Thus, for a given level of capital per worker, a decrease in the median voter’s ability will encourage labor inflow to that region.

Considering \( Z'_3 \) and \( Z'_1 \) together, \( Z'_3/Z'_1 \) can be interpreted as the net wage rate effect of a change in median voter’s ability on capital per worker after controlling for the autonomous effect of capital stock changes on the wage rate (\( Z'_1 \)). This can be explained as follows: as \( Z'_1 \) indicates, for a given level of capital per worker, a decrease in the median voter’s ability would increase the net-of-tax wage rate, and holding other things constant \( Z'_1 \) shows that the net-of-tax wage rate can only be brought down by increasing the capital per worker in that period.

Therefore a decrease in the natural population growth rate in region A in period 1 will unambiguously raise capital per worker in both regions in that period. A decrease in the number of workers in a region means that period 0 savings will be utilized by a smaller number of workers in period 1, leading to an increase in \( k_1 \).

In addition, a decrease in the ability level of the median voter in period 1 will affect saving in period 1 and consequently capital accumulation in period 2. Building on the effects in period 1, this can be shown in the following expression:

\[
\begin{bmatrix}
dk^A_2 \\
dk^B_2
\end{bmatrix} = \begin{bmatrix} M^I & M^I \\ M^I & M^I \end{bmatrix} \begin{bmatrix} Z'_2 & Z'_2 \\ Z'_2 & Z'_2 \end{bmatrix} \begin{bmatrix} \sigma A^A_1 \\ \sigma A^A_1 \end{bmatrix} + \begin{bmatrix} Z'_3 & Z'_3 \\ Z'_3 & Z'_3 \end{bmatrix} \begin{bmatrix} \sigma A^B_1 \\ \sigma A^B_1 \end{bmatrix} + \begin{bmatrix} \sigma A^A_2 \\ \sigma A^A_2 \end{bmatrix} \begin{bmatrix} \sigma A^B_1 \\ \sigma A^B_1 \end{bmatrix} \begin{bmatrix} \sigma A^A_2 \\ \sigma A^A_2 \end{bmatrix}
\] (24)

Using the same assumptions that are used to derive the results in (23), and letting \( da^A = da^B = da_1 \), the changes in second period capital stock per worker can be written as

\[
dk^A_2 = dk^B_2 = 2M^I Z'_2 \sigma A^A_1 + 2Z'_3 da_1 + Z'_3 \sigma A^B_2.
\] (25)

The second term on the right hand side of (25) shows the effect of a drop in first period natural population growth rate on second period capital per worker through the
change in the identity of the median voter (indirect effect). While \( da_i < 0 \), the effect of a decrease in the ability of the median voter on \( k_2^A \) and \( k_2^B \) is still ambiguous since the sign of \( Z_4^i \) is indeterminate, as discussed above. The first term on the right hand side shows that the direct effect from the first period is discounted by \( 2M^i \) which is positive and less than 1 by dynamic stability. Similarly, the initial indirect effect \( (Z_4^i) \) will also be discounted for capital per worker changes in the following periods. The last term on the right hand side is the same as the direct effect shown in (23) but this time it comes from a change in the second period population growth. Hence, (25) hints that the initial effect of a population growth change on capital stock per worker dissipates over time.

(23) and (25) together show that the effect of a population growth rate decrease can be separated into two parts. The first, “direct saving effect”, which takes shape by spreading saving to fewer workers, increases capital per worker. The second, “indirect saving effect”, impacts through a decrease in the ability of the median voter. This effect tends to decrease both labor earnings and the tax rate, making the effect on saving and capital stock, which is a function of net labor earnings, ambiguous. Next, I will compare these results with results from alternative models.

**Model Comparisons**

In order to explore the significance of international labor mobility and endogenous fiscal policy effects, I compare the effects in (23) and (25) with analytic results from three alternative versions of this model. While a comparison with a closed economy model shows the influence of open economy in general, a comparison with an open economy model with capital mobility establishes the effects unique to my labor mobility model. In addition, examining an exogenous policy model indicates the importance of endogenous fiscal policy, and thus the political economy used in the current model. In the exogenous policy case, I assume that government spending for a productivity-enhancing public good \((g)\) is fixed. Solutions for these models are:

\[
\begin{align*}
\frac{dk_i^{A^r}}{d\eta_i^{A^r}} &= 2Z_i^c \quad \text{and} \quad \frac{dk_i^{A^c}}{d\eta_i^{A^c}} = 2\left(2MZ_i^c + Z_4^c\right). \\
\end{align*}
\]

(26)

Closed economy:

Open Economy with exogenous fiscal policy:

\[26\] This can actually be proven by using \( da_i^A = da_i^B \), where \( da_i = \frac{1}{1+\eta}dN_i - \frac{dN_{i-1}}{2N_i} \), the differentiated form of international labor market equilibrium condition (18), and the fact that population growth rate decreases in one region.

\[27\] Government spending, rather than the tax rate, is fixed because this enables me to do a comparison in the presence of inefficiencies stemming from choosing a sub optimal value of \( g \). Fixing the tax rate, however, allows for changes in government spending with changes in the tax base.
Open economy with capital mobility:

\[
\begin{align*}
\frac{dk^A_{i+1}}{\eta^A_{i+1}} &= \frac{dk^B_{i+1}}{\eta^B_{i+1}} = \frac{dk^B_{i+1}}{\eta^A_{i+1}} = \frac{dk^A_{i+1}}{\eta^B_{i+1}} = Z^f_2, \\
\frac{dk^A_{i+1}}{\eta^A_{i+1}} &= \frac{dk^A_{i+1}}{\eta^B_{i+1}} = \frac{dk^B_{i+1}}{\eta^A_{i+1}} = \frac{dk^B_{i+1}}{\eta^B_{i+1}} = 2M^f Z^f_2.
\end{align*}
\]

In (23) and (25) and (26)-(30), \( Z^f_2 = Z^f_3 = Z^f_4 \). Therefore, we can compare the direct saving effect without using superscripts. We see that the direct saving effect \( Z^f_2 \) is halved and it impacts equally on the two regions when these economies are open. It is also clear that an indirect net interest rate effect, \( \frac{Z^k_3}{Z^k_1} \), is unique to capital mobility model, making this a combined effect of capital mobility and endogenous fiscal policy. Since this effect changes capital per worker in A and B in opposite directions, it is a major factor in creating an asymmetry between the two regions throughout the demographic transition. A similar effect, which is explained before as the indirect net wage rate effect, is completely nullified in my labor mobility model.

Next, I compare the political economy effect (or the indirect saving effect) embodied in the term \( Z^f_4 \). In (25), (26), and (30), \( Z^f_4 = Z^f_4 = \frac{1}{2(1 + \eta^*)} Z^f_4 \). These equations show that this effect shows up only in the models with endogenous fiscal policy. Compared to closed economy model, the indirect saving effect is halved and shared equally by the two regions in the capital mobility model. However, it is not clear how the indirect saving effect in the labor mobility model compares to closed economy and capital mobility models. The reason for this is that we do not know by how much the ability of median voter decreases in response to a decrease in the natural population growth rate. Therefore, we can compare the direct saving effect without using superscripts. We see that the direct saving effect \( Z^f_2 \) is halved and it impacts equally on the two regions when these economies are open. It is also clear that an indirect net interest rate effect, \( \frac{Z^k_3}{Z^k_1} \), is unique to capital mobility model, making this a combined effect of capital mobility and endogenous fiscal policy. Since this effect changes capital per worker in A and B in opposite directions, it is a major factor in creating an asymmetry between the two regions throughout the demographic transition. A similar effect, which is explained before as the indirect net wage rate effect, is completely nullified in my labor mobility model.

In the next section, I examine the direction and magnitude of the effects mentioned above and how sensitive they are to model parameters.

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28 Also, \( M^f = M \).

29 In closed economy and capital mobility models \( da_i = \frac{d\eta_i}{2(1 + \eta^*)^2} \) but in the labor mobility model, change in ability depends on the change in actual population growth rate rather than the natural population growth rate.
4. Simulation Exercise

The simulation exercise aims to show transitions between two steady states in response to population aging in both regions. The two world regions consist of 28 developed and 141 developing countries. The list of these developed and developing countries are given in Appendix Table 1 and the population growth rates implied by the projections are shown in Figure 1. The simulations will be shown for two 30-year periods, 2000-2030 and 2030-2060 and for the entire period 2000-2060. The average population growth rates for the 1970-2000 period are used as a starting point. The population growth rate in developed countries decrease from an initial annual average rate of 0.8% to 0.67% for the 2000-2030 period and then to 0.38% for the 2030-2060 period. In developing countries, this rate goes down from an initial annual average rate of 2.47% to 1.39% for the 2000-2030 period and then to 1.27% for the 2030-2060 period.

The elasticity of output with respect to capital input is set equal to one-third \( \alpha = 0.33 \). The annual rate of time preference is chosen to be 4 percent. The two parameters, the rate of time preference in the utility specification and the population growth rate, are adjusted to the length of the model period (30 years). In the simulations, the ability level, \( a \), is assumed to be distributed uniformly on the interval \([0,1]\).

As mentioned in section 2, social security spending \( g^s \) is exogenously set in the model. For simulations, social security expenditure for the developed and developing economies is compiled for the year 2000 using Government Finance Statistics by International Monetary Fund (2003). For simplicity, social security spending is assumed to grow in the future by the growth rate in population aged 65 and over. The United Nations population projections mentioned above enabled the derivation of social security spending for the periods 2000-2030 and 2030-2060.

A critical parameter in the model is the elasticity of human capital with respect to government spending on education and ability level \( \psi \). Since there is no known consensus on a possible value of this parameter, the literature on returns to education is briefly reviewed in the following section.

Search for a Human Capital Elasticity Parameter

Laitner (2000b) uses a human capital function that is similar to (4) and sets his human capital elasticity with respect to education equal to 0.1967. Based on an initial

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30 This elasticity estimate is consistent with the data from the United States. See Laitner (2000a) for an argument.
31 Caldwell, Favreault, Gantman, Gokhale, Johnson, and Kotlikoff (1999) argue that a premium of riskiness should be added to the widely used 2 percent rate. They use 3.5 percent as the discount rate which is the real safe return on indexed Treasury bonds. See Coronado, Fullerton and Glass (2000) for a recent argument on the variety of discount rates used in studies of social security. They assert that the selection of discount rates ranges between 2 to 5 percent.
value of the ability of the median voter, Laitner’s estimate corresponds approximately to \( \psi = 0.4 \) in my model. Studies on the estimates of returns to education show that the rates of return estimated for the United States generally fall in the range of 5-15%. Card (1995) gives a good survey of fairly recent rate of return estimates. While the OLS estimates are very low (highest estimate is 9%), the IV estimates range between 7-19%. Psacharopoulos (1985) gives estimates at international level as well. The author makes two key conclusions. First, the rate of return is considerably higher for low income and developing countries than for developed countries. Secondly, the rate of return is much higher for primary education than for secondary and high education. If we take developed and developing countries together in Psacharopoulos’ estimates, the upper bound will be close to 20%. Finally, Cawley, Heckman and Vytlacil (2001) look at wages and cognitive ability. They assert that without controlling for human capital measures such as education, measured ability explains 14-19.9% of wage variance. While, the above literature does not exactly point to a specific value for \( \psi \), \( \psi = 0.5 \) is chosen as a compromise given Laitner’s estimate and the large group of developing countries used in population projections.

**Full Labor Mobility Simulation Results**

In line with the model presented in section II, the full labor mobility simulation has perfect international labor mobility with migrant labor participating in the political system. Based on the population projections mentioned above, labor migrates from the developing region to the developed region. Figures 2 and 4 show this in reference to the change in the number of workers in both regions. Figure 2 shows that immigration of labor is a major component of the growth in the number of workers in developed countries. Labor immigration makes up 46% of the 41.1% growth between 2000 and 2030, and 61% of the 40% growth between 2030 and 2060. It accounts for more than half (56%) of the labor growth in developed countries between 2000 and 2060. Figure 3 shows how large the immigrant labor is relative to developed region population in 2000. Simulations here show that developed region receives increasing number of migrant labor through 2000-2030 and 2030-2060 periods. Overall, developed region receives 29% of its 2000 population between 2000 and 2060. This also amounts to 48% of its working-age population.

On the other hand, developing countries send labor to the developed region and thus experience lower domestic labor growth. However, Figure 4 shows that this loss in labor growth is still a relatively small portion of the overall labor growth. For example, out of the 51.3% labor growth that would have been seen in the developing region between 2000 and 2030, 11.5% was lost due to out-migration of labor leaving this region with a net 40% growth.

Figure 5 shows that this labor movement leads to opposite changes in capital stock per worker in developed and developing regions between 2000 and 2030. The influx of labor into the developed region does not translate into capital growth in that period since capital stock is determined by saving in the previous period (1970-2000), leading to a decrease in that period’s capital stock per worker. Developing region
experiences a significant rise in the capital stock per worker since the capital stock in the period 2000-2030 is utilized by fewer workers. The contribution of new labor to the growth in the developed region shows itself in the next period with a modest 4.2% growth in capital stock per worker. While the developing region still exhibits a strong growth in capital stock per worker between 2030 and 2060, this is significantly smaller than the growth recorded between 2000 and 2030. This is mainly due to considerable population growth decrease (from 2.47% to 1.39%) in developing countries coupled with labor migration between 2000 and 2030 leading to significant saving per worker and capital per worker increases. While this population loss gave an initial boost to the developing region in 2000-2030, it eventually showed itself in lower capital growth in 2030-2060 period. The outlook for the sixty year period from 2000 to 2060 shows a small overall decline in capital stock per worker in the developed region but a strong increase in the developing region.

The other important component of growth is the human capital accumulation. Figure 6 shows that the developed region fares much better in human capital worker between 2000 and 2030 with a 7.8% growth. This is mainly due to migrant labor offsetting the negative effect of a decrease in the median voter ability caused by aging. On the other hand, the developing region suffers from losing labor to the developed world by having a slight decrease in the human capital per worker between 2000 and 2030. As the labor growth falls by about ten percentage points in the developed region between 2030 and 2060, human capital per worker also drops. The decrease in the labor growth in the developing region in the same period is significantly smaller compared to the developed region. In addition, as shown in Figure 8, education spending recovers in the developing region between 2030 and 2060, leading to increases in human capital per worker. Nevertheless, both regions experience an overall growth in human capital per worker during the 2000-2060 period.

As shown in equations (4) and (12), human capital is a function of the government spending on education. Hence, the link between labor growth and human capital can be understood better with an examination of the changes in the education tax rate and education spending. These are shown in Figure 7 and Figure 8, respectively. As discussed in the political process of fiscal policy, aging causes median voter to become a lower ability person putting a downward pressure on the education tax rate. However, labor migration offsets this negative effect by increasing the number of young voters. The migrant workers do not only contribute to domestic production but they also participate in the political system by voting for fiscal policy decisions. Since they are young, they change the political scene in favor of the young generation. Thus, with labor migration the choice of the tax rate changes in favor of the young voters despite the aging trend. Figure 7 shows this for the developed region between 2000 and 2030. The education tax rate increases considerably in the developed region while it falls quite dramatically in the developing region. However, the education tax rate decreases in the developed region in the following period as labor growth diminishes. The developed region experiences an overall increase in the education tax rate between 2000 and 2060.
while the developing region records a significant decrease. Changes in education spending in Figure 8 closely match the tax rate changes in Figure 7 and the human capital changes in Figure 6. However, education spending in the developing region increases between 2030 and 2060 and between 2000 and 2060 despite the tax rate decreases in these periods. This is mainly due to income growth in this region (examined below) that outweighs the education tax rate decreases.

Results in Figure 5 and 6 for capital per worker and human capital per worker provide a good background for examining changes in income per worker. Figure 9 shows that both developed and developing regions exhibit income growth between 2000 and 2030. While strong growth in human capital per worker in the developed region barely dominates a decrease in the capital stock per worker, considerable increase in the capital per worker in the developing region generates a much stronger income growth in this region. The developed region suffers from a decrease in human capital in the following period that leads to a decrease in income per worker. This region exhibits an overall decrease in income per worker for the entire period. On the other hand, the developing region benefits from growth in both capital per worker and human capital per worker, which leads to an income growth in all periods considered.

In addition to income per worker, consumption as a measure of welfare is examined in Figures 10 through 12. Figure 10 presents the change in the consumption of young in developed and developing regions. As shown in equation 7, consumption of a person when young depends on the net labor earnings. As also seen in equation 19, perfect labor mobility dictates that the net labor earning of a young worker gets equalized between the two regions. Since labor flows from the developing to the developed region, net labor earnings must have gone up in the developing region and it must have gone down in the developed region. This would increase consumption of young in the developing region and decrease it in the developed region. This is seen clearly in Figure 10. Since labor migration to the developed region occurs in both 2000-2030 and 2030-2060 periods, the same consumption pattern is observed in both periods and in the entire period from 2000 to 2060.

Old-age consumption, different from the young-age consumption, also depends on the net return on saving. The increase in the capital per worker in the developing region shown in Figure 5 depresses the interest rate in this region, leading to a decrease in the net return on saving. Figure 11 shows that this leads to a decrease in the old-age consumption between 2000 and 2030. The developed region experiences the opposite and has an increase in the old-age consumption in that period. In the following period, the old-age consumption in the developing region (developed region) increases (decreases) due to an increase (decrease) in the net saving income. However, the old-age consumption decreases in both regions for the entire period.

Figure 12, brings the results shown in Figure 10 and 11 together and show changes in the sum of consumptions, in a given period, of representative persons from

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32 The results for the social security tax are not shown here but the rates do not change significantly across the periods mainly due to a fairly stable social security spending per young. Social security spending is forecasted outside the model and social security tax rate is driven only by income changes.
young and old generations. The figure shows that the developing region experiences an increase in the sum of consumptions between 2030 and 2060 and for the entire period from 2000 to 2060. However, the developed region has an increase in the sum of consumptions only between 2000 and 2030. It has a slight overall decrease between 2000 and 2060. Finally, it is also possible to get a sum of the developed and developing region consumptions to see the change in the “world” consumption. While this is not explicitly shown in Figure 12, the growth in the sum of consumptions in the developing region dominates and the world consumption decreases only slightly between 2000 and 2030 but increases between 2030 and 2060 and between 2000 and 2060.

Model Comparisons and Labor Mobility Scenarios

In this section, full labor mobility model is compared to an alternative open economy model where capital is internationally mobile without international labor mobility. Further comparisons are also made using alternative assumptions about labor mobility and the political process of fiscal policy. The goal is to shed light on the choice of factor mobility in the presence of population aging. Table 2 presents simulation results for these comparisons. Results shown in Figure 2 and Figure 4 through 12 are reproduced in columns 1 and 2 of Table 2.

Columns 3 and 4 in Table 2 show that the capital mobility model produces significantly inferior outcomes for the developed region, in terms of the economic variables used in the analysis, compared to the full labor mobility model. It appears that labor flows to the developed region offset the adverse effect of population aging particularly on the human capital per worker. As explained earlier, migrant workers participate in the political process of fiscal policy by voting favorably for the education tax, leading to enhanced education spending and human capital in the developed region. The developing region exhibits strong capital and income growth throughout the periods under both full labor mobility and capital mobility (without labor mobility) models. In the capital mobility model without labor mobility, the developing region does not lose labor to the developed region and benefits from the inflow of capital from the developed region. In this model, capital inflows to the developing region enable this region to increase investment, production and income while at the same time retaining its young productive workers, voters and human capital. Thus, the capital mobility model (without labor mobility) produces more favorable results for the developing region compared to the full labor mobility model.

The next set of comparisons is between the full labor mobility model and variants of this model shown in columns (5) through (10) in Table 2. The first is the case where labor mobility is only allowed after 2030. This scenario considers the possibility that doors remain closed in the developed region until aging becomes an even bigger problem in the developed region.\footnote{For example, in the U.S., the effect of the baby boom generation retirement will not be seen for another decade.} The results for this scenario in column (5) show that the developed region may fare slightly better particularly after 2030 if labor flow is delayed
until 2030. This is mainly due to lower overall labor growth in the developed region. With this relatively low labor growth net return on labor does not decrease as much as in the full labor mobility model in column (1) leading to slightly better capital, human capital, income and consumption results.

The next scenario allows for free labor mobility but puts a constraint on the political participation of migrant labor in the developed region for the first period. This simulates an extended delay in the naturalization of foreign workers into the political system of a country. After a thirty-year delay, workers eventually become citizens with eligibility to vote. Column (7) shows that this scenario produces rather adverse outcomes for the developed region. These are mostly driven by sharp decreases in education tax rate, education spending and human capital per worker and show a strong contrast with results in column (1). With aging in the developed region, lack of young migrant votes for the education tax rate leads to a decrease in this tax rate such that net labor earning differential between the regions can only be closed by a greater flow of labor from the developing to the developed region. This is the reason behind the interesting result of substantially high labor growth in the developed region. The results for the developing region in column (8) are quite similar to the ones in column (2) with slightly inferior results in the labor mobility model with delayed migrant voting.

Finally, the full labor mobility model is compared to an alternative model where education tax and spending are not determined through a political process but instead determined exogenously by the government. It is assumed here that the government fixes the education spending per worker. Results for the developed region in column (9) show that the direction of the income changes for 2000-2030 and 2030-2060 is reversed under this alternative model. Since there is no human capital feedback under this scenario, decreases in capital per worker between 2000 and 2030 decreases income per worker. However, as column (1) indicated, it is the human capital increase that leads to an increase in income per worker in this period under the labor mobility model. In addition, the labor mobility model in column (1) gives better consumption results (particularly the sum of young and old consumption) than the ones in column (9). Results for the developing region in column (10) show that the direction of changes in economic variables does not differ from the labor mobility model results in column (2). There are only slight changes in growth magnitudes.

Discussion

The simulation results presented show that the full labor mobility model is generally superior to other alternative models considered. Due to the perfect international labor mobility assumption, labor inflows to developed countries implied by these results may be high by historical standards. To get a feel for the magnitude of these labor inflows, actual inflows of foreign population to developed countries are compared to labor inflows predicted by the full labor mobility model. Table 3 presents this comparison using the simulation results in Figure 3 for the predicted labor inflows and

34 While not shown in Table 2, the decrease in the education tax rate is also balanced by a sharp increase in the social security tax rate.
actual foreign population inflows as reported by the OECD in 2000. The second row shows that predicted average annual number of labor migrants is fairly close to actual inflows of foreign population. It is even below the actual figure for the 2000-2030 period. OECD distinguishes between foreign population inflows and foreign worker inflows. The third row in Table 3 shows the same ratio for the foreign worker inflows. In this case, the predicted average annual number of labor migrants is almost double the actual foreign worker inflows in 2000 for the 2000-2030 period, and it is about 2.7 times greater for the 2030-2060 period. While the difference in this second comparison is fairly large, the predicted flows may well be consistent with dramatic population growth decreases expected in the developed countries in the next sixty years.

Another issue that may be important in the interpretation of paper’s results is that the benefits of labor inflows may be undermined by substantial transfers (other than education) to these migrants within the welfare system of developed countries. While such transfers are not modeled in the paper, it is not certain how this would change the results presented in this paper. First, welfare transfers would be important if the magnitude of the labor inflows is indeed large. The comparison of predicted to actual foreign population inflows discussed above tells us that this magnitude may not necessarily be different from current actual inflows. Second, one can also imagine labor migrants in developed countries voting for lower transfers (mainly social security) or lower social security taxes when they participate in the political system. Hence, this would put a counter pressure on welfare transfers. This paper focused only on the political economy of aging and education spending in order to highlight the importance of this link and also to attain simplicity in using the median voter model within the growth model.35

Finally, simulation results may be sensitive to a key parameter (ψ) in the model, which indicates the return to human capital from the ability level and education spending. A closed economy study by Holtz-Eakin, Lovely and Tosun (2004) showed that low values of this parameter (mainly ψ <0.34) produced immiserizing steady state growth results. It was argued earlier in section B that the literature on returns to education pointed to relatively higher parameter estimates particularly for developing countries. Hence, such low ψ values wouldn’t be anticipated in a two-region world with developed and developing countries. In a simulation with ψ =0.4, results didn’t change significantly to alter the main conclusions of the paper.36

IV. Summary and Conclusion

This paper examines the economic effects of aging trends using the population projections for the developed and developing regions of the world. Unlike the majority of studies on aging, the paper addresses the political economy of aging and education spending through a median voter model. While there is a growing literature that

35 Poutvaara (2004) recently addressed the political economy of both education and social security in an overlapping generations model. However, that study does not examine population aging.
36 Results for this simulation are not reported in this paper but available from the author upon request.
examines the link between population aging and international capital mobility, population aging has not been widely addressed in an open economy framework with international labor mobility. The paper fills this gap by modeling perfect international labor mobility.

It is argued that labor mobility has a dual effect on the economy. Besides the contribution of young migrant workers to the economy as laborers, these workers also contribute to capital accumulation as savers. This paper highlights another effect through migrant workers’ involvement in the political process of fiscal policy. When allowed to vote, migrant workers change the political structure composed of young and elderly voters in both labor-receiving and labor-sending countries. In a majority voting mechanism for fiscal policy decisions, political shifts resulting from labor migration may have sizeable impacts on government programs such as education, and in turn may have strong growth and welfare effects.

For the analysis, a two-region, two-period overlapping generations model with international labor mobility and a politically responsive fiscal policy is used. The numerical simulations based on United Nations population projections for the developed and developing regions show significant labor movements from the developed to the developing region throughout the 2000-2060 period. While labor inflows seem to help the developed region recover from the aging trend, this region incurs an overall decrease in income per worker and consumption. Nevertheless, a comparison with an alternative capital mobility model without labor mobility indeed shows the importance of migrant labor and their political participation. The developed region suffers from substantial income and consumption decreases under this capital mobility model, which is averted, to a large extent, in the labor mobility model. The developing region, on the other hand, seems to benefit more in the capital mobility model. However, this region experiences significant growth under both models. These comparisons show that the choice between labor and capital mobility indeed matters for the analysis of the effects of population aging.

Further comparisons are also made using alternative assumptions about labor mobility and the political process of fiscal policy. The full labor mobility model performed quite well particularly when it is compared to a scenario where labor migrates to the developed region but does not participate in the political system for the first thirty-year period. This scenario gives particularly inferior economic results for the developed region. Constraining political participation of young migrant workers does not seem to help when these young workers can vote for greater education spending that sets a growth process through human capital enhancement.
Table 1. Education Spending and Migration in Developed and Developing Regions

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Average of 23 High Income OECD Countries</td>
<td>12.26</td>
<td>1,490</td>
<td>10.85</td>
<td>2.37</td>
</tr>
<tr>
<td>Less Developed Regions (UN Definition)</td>
<td>9.09</td>
<td>122</td>
<td>5.27</td>
<td>-0.67</td>
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<tr>
<td>Least Developed Countries (UN Definition)</td>
<td>-0.40</td>
<td>7</td>
<td>1.60</td>
<td>-0.48</td>
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</tbody>
</table>

Source: International migration data is from United Nations Population Division, Department of Economic and Social Affairs, International Migration, 2002 (United Nations, 2002b). Education spending data is from World Development Indicators, World Bank, 2003 CD-ROM (World Bank, 2003). Migrant stock: For most countries, the mid-year estimate of the number of people who are born outside the country. For countries lacking data on place of birth, the estimated number of non-citizens. In both cases, migrant stock also includes refugees, some of whom may not be foreign-born. Net migration rate: The net number of migrants (annual number of immigrants less the annual number of emigrants, including both citizens and non-citizens), divided by the average population of the receiving country. It is expressed as the net number of migrants per 1,000 population.
Table 2. Comparisons with Alternative Factor Mobility Models\(^a\)

<table>
<thead>
<tr>
<th>Time Periods</th>
<th>Number of workers</th>
<th>Capital stock per worker</th>
<th>Human capital per worker</th>
<th>Income per worker</th>
<th>Education tax rate</th>
<th>Education spending per worker</th>
<th>Consumption of young</th>
<th>Consumption of old</th>
<th>Sum of young and old consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Developed Region</td>
<td>Developing Region</td>
<td>Developed Region</td>
<td>Developing Region</td>
<td>Developed Region</td>
<td>Developing Region</td>
<td>Developed Region</td>
<td>Developed Region</td>
<td>Developed Region</td>
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<tr>
<td>2000-2030</td>
<td>41.1</td>
<td>39.8</td>
<td>22.2</td>
<td>51.3</td>
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<tr>
<td>2030-2060</td>
<td>30.9</td>
<td>34.4</td>
<td>12.0</td>
<td>46.0</td>
<td></td>
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<tr>
<td>2000-2060</td>
<td>84.7</td>
<td>87.8</td>
<td>36.9</td>
<td>121.0</td>
<td></td>
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<tr>
<td>2000-2030</td>
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\(^a\)All numbers refer to percentage changes between the years indicated in the time period.
Source: Computed by author.
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<th>Capital stock per worker</th>
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<th>Income per worker</th>
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<th>Education spending per worker</th>
<th>Consumption of young</th>
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*All numbers refer to percentage changes between the years indicated in the time period.*

Source: Computed by author.
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<tr>
<td>% Ratio of Predicted Average Annual Number of Labor Migrants to Actual Foreign Population Inflows in 2000</td>
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<td>% Ratio of Predicted Average Annual Number of Labor Migrants to Actual Foreign Worker Inflows in 2000</td>
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Source: The figures in the first row are derived from simulations shown in Figure 3 for the developed region. Statistics on foreign population inflows and foreign worker inflows are from *Trends in International Migration*, OECD, 2002 Edition. Tables A.1.1 and A.2.1. are accessed online at [http://www.oecd.org/document/36/0,2340,en_2825_494553_2515108_1_1_1_1,00.html](http://www.oecd.org/document/36/0,2340,en_2825_494553_2515108_1_1_1_1,00.html)
Figure 1
Population Growth Rates in Developed and Developing Regions

Figure 2
Composition of the Change in the Number of Workers in the Developed Region
Figure 3
Ratio of Immigrant Labor to Developed Region Population

Figure 4
Composition of the Change in the Number of Workers in the Developing Region
Figure 5
Change in Capital Stock Per Worker in Developed and Developing Regions

Figure 6
Change in Human Capital Per Worker in Developed and Developing Regions
Figure 7
Change in Education Tax Rate in Developed and Developing Regions

Figure 8
Change in Education Spending Per Worker in Developed and Developing Regions
Figure 9
Change in Income Per Worker in Developed and Developing Regions

Figure 10
Change in Consumption of Young in Developed and Developing Regions
Figure 11
Change in Consumption of Old in Developed and Developing Regions

Figure 12
Change in Sum of Young and Old Consumption in Developed and Developing Regions
Appendix Table 1: List of Countries Used in Population Projections

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<th>Developing Countries</th>
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References


Appendix

I. Stability

Stability of the Political Equilibrium

The political equilibrium is locally stable if starting from a given level of government spending \( g \), the economy moves automatically to an equilibrium (Holtz-Eakin, Lovely, and Tosun 2000). For an analysis of stability, I assume a Marshallian-type adjustment rule:

\[
\frac{dg}{dt} = c \left[ \frac{\tau_p}{\tau} - 1 \right] = \varphi(g)
\]  

(A.1)

In this expression \( c \) is a positive constant. The variable \( \tau \) is the actual tax rate, which is a function of government spending per worker and income per worker. The variable \( \tau_p \) is median voter’s preferred tax rate for a given level of government spending and income. Equation (A.1) states that if median voter’s preferred tax rate exceeds actual tax rate, government spending will rise. For a political equilibrium, \( \varphi(\dot{g}) = 0 \) and \( \dot{g} \) is locally stable if and only if \( \varphi'(\dot{g}) < 0 \) (where \( \dot{g} \) is the equilibrium level of government spending). Differentiating (8), (10) and (14) in the text, we get:

\[
\frac{1}{y} (1 - \alpha) \varepsilon_{ng} \varepsilon_{\tau_y} dg_t,
\]  

(A.2)

where \( \varepsilon_{ng} = \left[ \left( 1 + g^* \right)^w \left( \psi g^* - 1 \right) + 1 \right] \bigg/ \left[ \left( 1 + g^* \right)^w + 1 \right] \) and \( \varepsilon_{\tau_y} = \frac{1}{m \psi y - 1} \). Differentiating the government budget constraint \( (\tau, y_t = y) \) and using the differentiated form of (10), we get:

\[
\frac{1 - (1 - \alpha) \varepsilon_{ng}}{y^*} dg_t
\]  

(A.3)

Finally, differentiating the right hand side of (A.1) and using (A.2) and (A.3), I get the stability condition for political equilibrium:

\[
1 - (1 - \alpha) \frac{\varepsilon_{ng} \left( 1 + \varepsilon_{\tau_y} \right)}{1 - (1 - \alpha) \varepsilon_{ng} \left( 1 + \varepsilon_{\tau_y} \right)} > 0
\]  

(A.4)

Stability of the Intertemporal Equilibrium

B.1 Intertemporal stability requires that the eigen value of the matrix \( \begin{bmatrix} M & M \\ M & M \end{bmatrix} \) in (20) is less than 1 in absolute value. Eigen value is found by solving

\[
\begin{vmatrix} M - \lambda & M \\ M & M - \lambda \end{vmatrix} = 0,
\]

where \( \lambda \) is the eigen value. The two possible values of \( \lambda \) are:
\[ \lambda = 0 \text{ or } \lambda = 2M^I = \frac{\alpha \left( 1 - \frac{\tau^*}{1 - \frac{\tau^* - \epsilon_{\tau}}{\tau} \cdot \epsilon_{\gamma} \cdot \frac{1 - \tau}{1 - \tau^*} \right)}{P} \]  

(A.5)

Using the nonzero root, intertemporal stability requires:

\[ |\lambda| < 1 \text{ or } |2M^I| = \left| \frac{\alpha \left( 1 - \frac{\tau^*}{1 - \frac{\tau^* - \epsilon_{\tau}}{\tau} \cdot \epsilon_{\gamma} \cdot \frac{1 - \tau}{1 - \tau^*} \right)}{P} \right| < 1 \]

However, since \( \epsilon_{\tau} < \frac{1 - \tau}{\tau} \) (as shown in the proof below) and \( P > 0 \) by political stability, \( 2M^I \) is positive and the intertemporal stability condition can be written as:

\[ \alpha \left( 1 - \frac{\tau^*}{1 - \frac{\tau^* - \epsilon_{\tau}}{\tau} \cdot \epsilon_{\gamma} \cdot \frac{1 - \tau}{1 - \tau^*} \right) < P \]  

(A.6)

B.2 \( 2M^I > 0 \)

Proof by contradiction: Let \( \epsilon_{\tau} \geq \frac{1 - \tau}{\tau} \),

Since \( \epsilon_{\tau} = \frac{1}{a_{m\psi y} - 1} \) and \( \frac{1}{a_{m\psi y} - 1} \geq \frac{1 - 1}{\tau} \),

then

\[ \frac{a_{m\psi y}}{a_{m\psi y} - 1} \geq \frac{(1 + \psi)a_{m\psi y}}{a_{m\psi y} - 1} \]

which cannot be true since \( a_{m\psi y} - 1 > 0 \) and \( \psi > 0 \), \( a_m > 0 \), \( y > 0 \). Then \( \epsilon_{\tau} < \frac{1 - \tau}{\tau} \), and under political stability \( (P > 0) \) this implies \( 2M^I > 0 \).

II. The Effect of Increasing Dependency Ratio on the Ability Level of the Median Voter

Recall that median voter is defined by \( N_{t-1} + N_t \int_0^a f(a) \, da = \frac{N_{t-1} + N_t}{2} \). Rewriting this: \( N_{t-1} + N_t F(a_m) - N_t F(0) = \frac{N_{t-1} + N_t}{2} \), which can be rearranged as:

\[ F(a_m) - F(0) = \frac{N_t - N_{t-1}}{2N_t} \cdot \] Differentiating both sides we get, \( F'(a_m) \, da_m = \frac{\dot{N}_t - \dot{N}_{t-1}}{2 \left( 1 + \eta^* \right)} \),

where \( \dot{N}_t = \frac{dN_t}{N_t}, \, \dot{N}_{t-1} = \frac{dN_{t-1}}{N_{t-1}}, \) and \( 1 + \eta^* = \frac{N_t}{N_{t-1}} \) evaluated at the initial steady state.
Finally this can be rearranged as $da_m = \frac{\hat{N}_t - \hat{N}_{t-1}}{F'(a_m)2(1+\eta)}$ which is negative when $\hat{N}_t < \hat{N}_{t-1}$.

III. Derivation of Equation (20) and Steady State

A. First, I totally differentiate (16) through (19) to get

\[
\begin{align*}
dk_A^{r+1} &= \frac{N_t^A}{N_{t+1}}ds^A_i + \frac{S^A_i}{N_{t+1}}dN^A_t - \frac{k^A_i}{N_{t+1}}dN^A_{t+1} \\
dk_B^{r+1} &= \frac{N_t^B}{N_{t+1}}ds^B_i + \frac{S^B_i}{N_{t+1}}dN^B_t - \frac{k^B_i}{N_{t+1}}dN^B_{t+1}
\end{align*}
\]

Next, I use (A.7) and (A.8) to substitute out $dN^A_t, dN^B_t, dN^A_{t+1}$, and $dN^B_{t+1}$ in (A.10). Evaluated at the initial steady state, this gives

\[
dk_A^{r+1} + dk_B^{r+1} = \frac{1}{1+\eta}ds^A_i + \frac{1}{1+\eta}ds^B_i - \frac{k^*}{1+\eta}d\eta^A_{t+1} - \frac{k^*}{1+\eta}d\eta^B_{t+1}
\]

Using differentiated forms of (8), (10), and (12) through (15) for each region, we can write $dw_{t+1}$ and $d\tau_{t+1}$ in terms of $dk_{t+1}$ and $d\tau_{t+1}$, and $ds_i$ in terms of $dk_i$ and $d\tau_i$ in equations (A.9) and (A.10)’. This gives us the reduced vector equation (20) in the text.

B. Let’s rewrite (20) as

\[
F_{t+1} = \Omega F_t + \hat{\Omega}_1 J^1_t + \hat{\Omega}_2 J^2_t
\]

where

\[
F_{t+1} = \begin{bmatrix} dk_A^{r+1} \\ dk_B^{r+1} \end{bmatrix}, \quad F_t = \begin{bmatrix} dk_A^r \\ dk_B^r \end{bmatrix}, \quad J^1_t = \begin{bmatrix} da^A_i \\ da^B_i \end{bmatrix}, \quad \text{and} \quad J^2_t = \begin{bmatrix} da^A_{t+1} \\ da^B_{t+1} \end{bmatrix}
\]

and

\[
\Omega = \begin{bmatrix} M^r & M^r \\ M^r & M^r \end{bmatrix}, \quad \hat{\Omega}_1 = \begin{bmatrix} Z^*_4 & Z^*_4 - Z^*_3 \\ Z^*_3 & Z^*_3 \end{bmatrix}, \quad \text{and} \quad \hat{\Omega}_2 = \begin{bmatrix} Z^*_2 & Z^*_2 \\ Z^*_2 & Z^*_2 \end{bmatrix}
\]

Expanding (A.11) for a fixed level of initial capital stock per worker, I can write (A.11) for $t = 0,1,2$ as:

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\[ F_1 = \Omega F_0 + \hat{\Theta}_2 J_0^1 + \hat{\Theta}_2 J_0^2 \]
\[ F_2 = \Omega^2 F_0 + \Omega \hat{\Theta}_1 J_0^1 + \Omega \hat{\Theta}_2 J_0^2 + \hat{\Theta}_1 J_1^1 + \hat{\Theta}_2 J_1^2 \]
\[ F_3 = \Omega^3 F_0 + \Omega^2 \hat{\Theta}_1 J_0^1 + \Omega^2 \hat{\Theta}_2 J_0^2 + \Omega \hat{\Theta}_1 J_1^1 + \Omega \hat{\Theta}_2 J_1^2 \]
\[ + \hat{\Theta}_1 J_2^1 + \hat{\Theta}_2 J_2^2 \]  
(A.12)

Then generalizing this for \( t > 2 \),
\[ F_{t+1} = \Omega^{t+1} F_0 + \Omega^t \hat{\Theta}_1 J_0^1 + \Omega^t \hat{\Theta}_2 J_0^2 + \hat{\Theta}_1 J_{t-1}^1 + \hat{\Theta}_2 J_{t-1}^2 + \hat{\Theta}_1 J_t^1 \]
\[ + \Omega \hat{\Theta}_2 J_t^2 + \Omega^{t-1} \hat{\Theta}_2 J_t^2 + ... + \Omega \hat{\Theta}_2 J_1^2 + \hat{\Theta}_2 J_1^2 \]  
(A.13)

where \[ F_0 = \begin{bmatrix} dk_0^A \\ dk_0^B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]

Notice that the coefficients of the terms in (A.13) resemble geometric series. In line with my assumptions that population growth rates are equalized and remain constant after period 2, I can assume that \( J_3^1 = J_4^1 = ... = J_{t-1}^1 = J_t^1 \) and \( J_3^2 = J_4^2 = ... = J_{t-1}^2 = J_t^2 \). Using this, I can write (A.13) as:
\[ F_{t+1} = \Omega^t \hat{\Theta}_1 J_0^1 + \Omega^{t-1} \hat{\Theta}_1 J_1^1 + \Omega^{t-2} \hat{\Theta}_1 J_2^1 + (\Omega^0 + \Omega^1 + \Omega^2 + ... + \Omega^{t-3}) \hat{\Theta}_1 J_t^1 \]
\[ + \Omega \hat{\Theta}_2 J_t^2 + \Omega^{t-1} \hat{\Theta}_2 J_t^2 + \Omega^{t-2} \hat{\Theta}_2 J_t^2 + (\Omega^0 + \Omega^1 + \Omega^2 + ... + \Omega^{t-3}) \hat{\Theta}_2 J_t^2 \]  
(A.14)

As \( t \to \infty \), the three terms preceding both parentheses in the above expression converge to zero and since all the elements of the matrix \( \Omega \) are less than 1 by dynamic local stability, the geometric series in both parentheses converges to \( (I - \Omega)^{-1} \). Thus, assuming that \( d\eta^A = d\eta^B = d\eta \) (identical changes in the steady state population growth rates) and \( da^A = da^B = da = \frac{d\eta}{2(1+\eta^3)} \), as \( t \to \infty \), (A.14) can be written as:
\[ F_{t+1} = (I - \Omega)^{-1} \hat{\Theta}_1 J_t^1 + (I - \Omega)^{-1} \hat{\Theta}_2 J_t^2 \]  
(A.15)

By using \( \Omega, \hat{\Theta}_1 \) and \( \hat{\Theta}_2 \), and converting (A.15) back to the scalar form, we get the change in capital per worker from the initial steady state to the post-demographic transition steady state:
\[ \frac{dk^A}{d\eta} = \frac{dk^B}{d\eta} = \frac{2(Z_2^l + Z_4^{l'})}{1-2M^l}, \]  
(A.16)

where \( Z_4^{l'} = \frac{Z_4^l}{2(1+\eta^3)} \).