

# Estimating Self-Productivity in Skill Formation During Childhood

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## Abstract

The aim of this paper is to estimate the different channels of the so-called self-productivity feature in the dynamic process of skill formation and thus to contribute to a better understanding of the technology of skill production in childhood. First, we present a model of skill formation including different channels of self-productivity, one of them being dynamic complementarity. Second, based on data of a broad battery of primary school students' skills and an exogenous variation of one of these skills, we estimate self-productivity and the implications for skill profiles. We discuss the implications for the intergenerational transmission of skills and SES and for policy recommendations.

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# 1 Introduction

A variety of skills (e.g., cognitive skills, socio-emotional skills) are fundamental for a broad range of economic and social success (e.g., Heckman et al. 2006, Lindqvist and Vestman 2011, Heckman 2007). The inequality in the skill distribution has been observed to strongly widen over time during childhood and adolescence and finally to cumulate in strong inequality in life success (e.g., Blomeyer et al. 2009). The reason for this increasing gap is highly complex and the process of skill formation is hardly understood yet. One of the key challenges in understanding the skill production function is the multiplicity of skills and the probably even higher multiplicity of possible investments shaping these skills. It most likely is impossible to measure all relevant investments and skills in order to consistently estimate the skill production function. Todd and Wolpin (2003) discuss the challenges in estimating the production function for skills.

One of the most prominent models of skill production has been presented by Cunha and Heckman (2007) proposing a dynamic process of skill formation featuring self-productivity—i.e., skills boost skills over time—and dynamic complementarity—i.e., the productivity (in terms of skill formation) of investments is higher for higher baseline skills. The existence of these features would explain the increasing skill gaps and corroborate the importance of early investments in skills. Actually, many studies found that skill gaps in children can be reduced by interventions taking place in early stages of childhood (see Cunha et al. 2006, for a discussion of this literature).

In this paper, we discuss the challenges of empirically estimating the features of self-productivity and dynamic complementarity in skill formation and we provide an attempt of estimating these features given the extensive data we have on a variety of skills for primary school students measured several times at similar distances and given an exogenous intervention boosting one specific skill.

What exactly means self-productivity in skill formation? According to Cunha and Heckman (2007) self-productivity in skill formation means that the stock of skills  $\theta$  in period  $t + 1$  is a function of the stock of skills in the previous period  $t$ , i.e.,  $\theta_{t+1} = f(\theta_t, I_t)$  (with  $I_t$  standing for other factors influencing the stock of skills  $\theta_{t+1}$ , the “investments” in skills) with the marginal productivity being positive, i.e.,  $\partial f(\theta_t, I_t) / \partial \theta_t > 0$ . The question arises what exactly are the channels of self-productivity, i.e., how could  $\theta_t$  affect  $\theta_{t+1}$ . One (the most obvious) channel would be the inertia of stocks implying that the stock of skills in some period is equal to the stock of skills in the previous period if there is no other influencing factor (no investment). Another channel arises if  $\theta_t$  positively affects the productivity of other influencing factors, i.e.,  $\partial^2 f(\theta_t, I_t) / \partial \theta_t \partial I_t > 0$ . This is what Cunha and Heckman (2007) call ‘dynamic complementarity’. Dynamic complementarity

can thus be interpreted as a channel of self-productivity in skill formation.

Regarding the specific feature of dynamic complementarity, we ask the question of how plausible the assumption is that the productivity of *any* human capital investment monotonously increases in the baseline skill level. Alternatively one could think of the productivity of investments being inverse u-shaped in the baseline skill level, i.e., any investment activity having a peak in productivity for a specific baseline skill level and productivity of this investment activity being decreasing the farther one departs from this skill level in either direction. As an example, one could imagine that a very smart and imaginative kid is likely to benefit from being read a complex fancyful fairy tale, whereas a child with rather modest cognitive abilities or lower imagination skills or less knowledge about the backgrounds of the story does not understand much of it and is likely to even stop listening after a while. This implies that the investment of reading a complex fancyful fairy tale is more productive for the child with a high stock of skills than for the child with a low stock of skills. In contrast, the child with low skills might benefit from being read a simple fairy tale that on the other hand the smart child is quickly bored of. This implies that the investment of reading a simple fairy tale is more productive for the child with low skills than for the child with high skills. This example illustrates why the feature of dynamic complementarity is unlikely to be generally valid for the skill production function and for any type of investment. The observation that the most productive investment is not the same for every child (that is, differs depending on its current skill level) also explains why the practice of adaptive learning is currently being researched on (see, e.g., Alevin et al. 2016, Graf et al. 2014, Arroyo et al. 2014, Sampayo-Vargas et al. 2013).

Using a panel dataset of a detailed assessment of multiple skills measured repeatedly over equally distant periods during primary school age, we are able to trace the growth path of a variety of skills relevant at this and later stages in the life cycle. We illustrate the relationship between initial skill levels and the skill profile and compare the results for different types of skills. Comparing the actual skill profiles as a function of the initial skill levels to the different theoretical predictions based on the different channels of self-productivity, we are able to draw conclusions about their relevance in the actual skill formation process. Due to missing data about some relevant investment activities and the resulting endogeneity in the initial skill level, we employ an instrumental variable strategy using a randomized-controlled intervention as instrument for the initial stock of one particular skill. The intervention is special in the sense that it directly affects only this one specific skill, which is working memory capacity. Our strategy allows us to draw causal inference about the self-productivity (and dynamic complementarity as the key channel of self-productivity) of this skill.

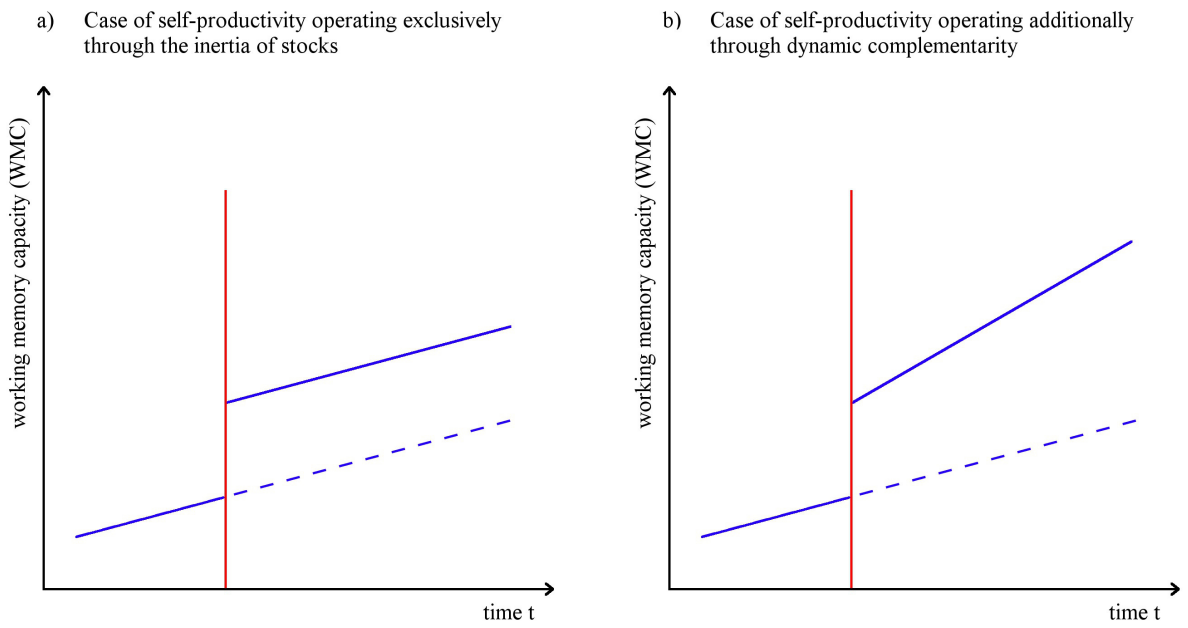
The skill under investigation for the initial period is working memory capacity—the ability to mentally store and process information (Baddeley 1999). Working memory capacity is a skill which is basic for a number of cognitive functions and in particular for the capacity of learning applied skills like mathematics or reading (Gathercole and Pickering 2000, Gathercole et al. 2001, Alloway and Alloway 2010, Raghubar et al. 2010, Dumontheil and Klingberg 2012, Shinaver III et al. 2014, Holmes et al. 2009, Bergman Nutley and Söderqvist 2017). There is a great number of experimental studies investigating the question whether working memory can be improved by targeted training. Many meta-studies in this field conclude that this is the case; the answer to the question of the transfer effects of such trainings are more controversial (see Au et al. 2015, Karbach and Verhaeghen 2014, Melby-Lervåg and Hulme 2013, Melby-Lervåg et al. 2016, Morrison and Chein 2011, Schwaighofer et al. 2015, Shinaver III et al. 2014, Shipstead et al. 2012, Weicker et al. 2016).

In the case that only the first channel of self-productivity (the ‘inertia of stocks’ as described above) is relevant, we expect to find for working memory capacity a skill profile with a one-time shift at the time of the exogenous intervention as illustrated in figure 1a.<sup>1</sup> In the case of the second channel (‘dynamic complementarity’ as described above) operating in addition, we expect to find a skill profile with a shift plus an increased slope at the time of the intervention as illustrated in figure 1b. For other skills not directly affected by the intervention, we expect to find no consequences for the skill profile in the case that only channel one but not channel two characterize the true production function of skills (illustrated in figure 2a). In the case, however, that channel two is relevant, the improved level of working memory capacity at period  $t$  improves the productivity of investments in other skills and we thus expect an increased slope (but no one-time shift) for other skills than working memory capacity at the time of the intervention as illustrated in figure 2b. Based on our empirical analysis, we find the skill profiles to resemble figure 1b and 2b rather than figure 1a and 2a. Thus, we provide evidence for the existence of dynamic complementarity—i.e., the second channel of self-productivity—in the process of skill formation.

Previous attempts to provide insight about the parameters of the production function of skills include Cunha and Heckman (2008) as well as Cunha et al. (2010). These authors provide a comprehensive model of skill production where skills and investments are dynamically interrelated across stages in childhood. They assume skills to reduce to two latent factors, one being measured by a mathematics and a reading recognition test and the other being measured by a

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<sup>1</sup>The reasoning is valid not only for the case of linear skill profiles shown in the figure. An alternative shape of the growth path would be concave, analog to the decreasing productivity of inputs theory in the production theory. However, unlike for the production of real goods, the stock of skills are not measured by a natural metric and it is therefore hard to pin down a shape for the growth path. We use tests increasing in difficulty adapted to the development of skills of primary school students. The test scores used in our analyses are standardized to mean zero and SD one. In that logic, the skill profiles would be flat (since the mean is always zero).



Note: The red vertical lines mark the time of the intervention targeted at improving working memory capacity (WMC).

Figure 1: Hypothesized skill profiles for working memory capacity



Note: The red vertical lines mark the time of the intervention targeted at improving working memory capacity (WMC).

Figure 2: Hypothesized skill profiles for other skills

behavioral problem index. The authors do not test the different channels of self-productivity, in particular not the channel of dynamic complementarity. In order to be able to do this, the key challenge is having an exogenous variation of one clear-cut skill without (direct) spill-overs to other skills. This is what we have and exploit in the present paper for the skill of working memory capacity.

The goal of this paper is to contribute to the understanding of the technology of skill formation which is crucial for understanding inequality in skills and outcomes across people. A better understanding of the skill formation process is also needed in order to optimally design policies aimed at improving human capital.

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