# Employment Protection, Product Market Regulation and Firm Selection<sup>\*</sup>

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#### Abstract

This paper analyzes the effect of labor and product market regulation in a dynamic stochastic equilibrium with search frictions. Modeling multiple-worker firms allows us to distinguish between the exit-and-entry (extensive) margin, and the hiring-andfiring (intensive) margin. We characterize analytically how both margins depend on regulation before we calibrate the model to the US economy. We find that firing costs matter most for the intensive margin. Fixed or set-up costs in the product market instead alter primarily the behavior of firms at the extensive margin. Moreover, we find important interactions between the policies through firm selection. Finally, the opposite effect of product and labor market regulation on job turnover rationalizes the empirically observed similarity of turnover rates across countries.

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### 1 Introduction

Product and labor market regulation differ substantially across OECD countries. Whereas Anglo-Saxon countries have flexible labor markets and deregulated product markets the opposite is the case for continental European countries.<sup>1</sup> However, continental European countries have attempted policy reforms to relax the stringency of their regulations in the past decades.<sup>2</sup>

Thus, it is important to understand the economics of both types of regulation in a unified framework in which product and labor market regulation each play a distinctive role but also interact endogenously by changing the costs and benefits of the respective other policy. In this paper we focus on important policies such as wasteful firing costs<sup>3</sup> in labor markets and administrative fixed and set-up costs in product markets. Our model with multiple-worker firms explicitly allows us to distinguish between the exit-and-entry (extensive) margin, and the hiring-and-firing (intensive) margin. We characterize analytically how both margins depend on the policies before we calibrate the model to the US economy. We find that firing costs primarily matter for adjustment at the intensive margin: incumbent firms that are exposed to exogenous changes in business conditions will hoard more or less labor depending on the adjustment costs. Fixed or set-up costs in the product market instead alter primarily the behavior of firms at the extensive entry margin and thus the total number of firms producing in equilibrium.

The model also allows us to highlight important interactions between the policies. Firing costs lower the asset value of the firm and thus encourage exit whereas product market regulation matters for labor hoarding through a selection effect. Higher fixed costs imply higher average firm productivity and a smaller number of firms with larger average size so

<sup>&</sup>lt;sup>1</sup>The correlation coefficient between summary indicators for EPL and PMR proposed in Nicoletti et al. (1999) is highly significant and equal to 0.72.

 $<sup>^{2}</sup>$ See Alesina et al. (2005), Figure 1, for the prevalence of product market deregulation from the 1970s to 1990s; and OECD (2004), chart 2.2, on the deregulation of EPL since the late 1980s including mostly marginal reforms that introduced more flexible contract types.

<sup>&</sup>lt;sup>3</sup>In reality, transfers between firms and workers are also an important component of employment protection legislation. For a recent discussion on the effects of severance payments see Garibaldi and Violante (2005).

that aggregate steady-state mobility costs decrease although job turnover per firm increases.

Since heterogenous firms decide whether to enter in the good state and can exit if a bad shock occurs, our model generates firm and job turnover in the steady state. In our numerical calibration we find that product and labor market regulation have quite different effects on firm and job turnover. Firing costs decrease job turnover but increase firm turnover because more firms exit in the bad state and default on firing costs. Fixed costs increase job turnover and especially job turnover per firm. Furthermore, fixed costs have a sizeable positive effect on firm turnover because more firms exit in the bad state. Not surprisingly, set-up costs have a small negative effect on firm and job turnover.

The opposite effect of firing and fixed costs on job turnover provides an alternative explanation to Bertola and Rogerson (1997) for why job turnover is similar across developed countries with different stringency of employment protection legislation (EPL). Whereas Bertola and Rogerson argue that rigid wages complement strict EPL in developed countries, we argue that the similar job-turnover rates can be explained by more product market regulation (PMR) in countries with stricter EPL.

The interactions between product and labor market regulations have received much interest in recent years. Blanchard and Giavazzi (2003) focus on the bargaining power of workers as labor market regulation. They argue that higher rents in regulated product market are complementary with more bargaining power in the labor market since workers try to appropriate some of the rents. Ebell and Haefke (2004) have extended the model to a dynamic context determining the type of bargain (individual or collective) as a function of product market regulation. In this paper, we take the type of bargain as exogenous and instead focus on employment protection legislation, a labor market policy which is very important in many OECD countries and at the same time quite heterogenous across them. Compared with the deterministic models mentioned above, we frame our analysis in a stochastic environment in order to analyze firing costs and turnover in a meaningful way.

We solve a dynamic stochastic equilibrium model with multiple-worker firms and frictions in the labor market. Imperfect labor markets with frictions imply realistic equilibrium unemployment and allow for a potentially positive welfare effect of market regulation. Although a dynamic model with multiple-worker firms and well-defined firm size is not easily solved, the distinction between administrative fixed and set-up costs per firm and firing costs per worker is most meaningful if multiple-worker firms have an intensive and extensive margin.<sup>4</sup>

Our paper builds on the model of Bertola and Caballero (1994), henceforth BC. We add an entry and exit decision to BC and maintain the assumption that workers are homogenous whereas firms are heterogenous. In our model firm heterogeneity also has a permanent component besides the standard stochastic component which fluctuates between two states, good and bad. Permanent productivity differences between firms allow us to determine two endogenous productivity thresholds: one above which firms decide to enter in the good state and another one below which firms exit in the bad state.<sup>5</sup>

As in BC, firm size is well defined because the production technology has decreasing returns to scale in labor and firms cannot hire immediately due to frictions in the labor market. Since wages are permanently renegotiated, this gives rise to intra-firm bargaining and overemployment. Firms exploit that an additional worker lowers the wage of all employed workers. This outcome of intra-firm bargaining has been derived in deterministic models such as the partial equilibrium analyses of Stole and Zwiebel (1996 a,b) and the general equilibrium analysis with multiple types of workers and capital of Cahuc et al. (2004) and their references.

The rest of the paper is structured as follows. In Section 2 we lay out the basic model and mention cross-sectional inefficiencies. We define and calibrate the equilibrium to the US economy in Section 3. Sections 4 provides a quantitative numerical analysis of the effect of product market regulation. Section 5 analyzes the impact of employment protection legislation. The interactions between the two types of regulation are detailed in section 6. We conclude in Section 7.

<sup>5</sup>This relates to the analysis of Hopenhayn and Rogerson (1993) who analyze the effect of firing costs in a neoclassical model with job and firm turnover in the steady state. In their calibration firing costs have a substantial negative effect on average productivity, employment and consumption.

<sup>&</sup>lt;sup>4</sup>For an analysis of hiring subsidies and firing costs in one-worker firms see Mortensen and Pissarides (2003) and Pissarides (2000), chapter 9. Since both policies affect the intensive hiring and firing margin in models with one-worker firms, they have a similar effect on the match surplus and hiring subsidies can be designed to offset the effects of firing costs.

### 2 Model

In this section we set up the model, provide analytical results on the firms' behavior and briefly discuss cross-sectional efficiency.

### 2.1 Set-up

The economy is populated by a continuum of workers. Workers are assumed to be homogenous and infinitely-lived. They are employed by a continuum of firms whose mass  $\mu$  is endogenously determined in equilibrium by the entry and exit conditions. Firms are indexed by the subscript *i* so that  $i \in [0, \mu]$ . Contrary to workers, firms are heterogenous and differ with respect to their permanent total factor productivity  $a_i$  and transitory differences in business conditions. Both firms and workers are risk neutral.

**Technology.** Each firm has access to a production technology that uses labor as the only input. The production technology has a fixed overhead component f and a variable component. The variable component has decreasing returns to scale. The firm's labor-demand schedule is characterized using a linearization of the marginal revenues

$$\rho_i^g = \eta_i^g - \sigma l_i \text{ with } \eta_i^g = \eta(a_i, \varepsilon_g)$$

and

$$\rho_i^b = \eta_i^b - \sigma l_i \text{ with } \eta_i^b = \eta(a_i, \varepsilon_b),$$

where the superscript denotes whether the firm is in the good or bad transitory state. We assume that  $\partial \eta / \partial a_i > 0$ ,  $\partial \eta / \partial \varepsilon_j > 0$  with  $\varepsilon_g > \varepsilon_b$ . For concreteness,

$$\eta_i^b = a_i \varepsilon_b$$
 and  $\eta_i^g = a_i \varepsilon_g$ .

The assumed production technology implies that each firm has decreasing returns in employing workers. Thus, firm size is a well-defined concept and allows us to analyze the effect of firing costs and product market regulation for firms with multiple workers. **Institutions.** Behavior in our economy is constrained by institutions in both the product and labor market. In the labor market, wasteful firing costs F constrain firms' layoff decisions. In the product market, firms face a regulatory burden. They have to pay a wasteful flow cost f in order to comply with regulation on licensing and other bureaucratic burden. We think of f as capturing the administrative procedures and economic regulations that impede firms in each period in which they produce. In reality, barriers to entrepreneurial activity also account for a significant part of product market regulation (see Nicoletti et al. 1999). In order to model this constraint, we assume that firms face a cost of entry equal to C.

The labor market. The labor market is characterized by search frictions as in the standard Diamond-Mortensen-Pissarides model. We consider a Cobb-Douglas matching technology with constant returns so that every vacancy is matched to an unemployed worker at Poisson rate

$$q(\theta_t) = \xi \theta_t^{\gamma}, \ -1 < \gamma < 0,$$

where  $\theta_t \equiv V_t/U_t$ ,  $V_t$  denotes the stock of vacancies at time t,  $U_t$  denotes the stock of unemployed workers and  $\xi$  is the scaling factor of the matching function.

The hiring process consumes time and resources. As in BC, we assume that open vacancies  $v_{it}$  imply a flow cost  $cv_{it}^2/2$  so that the marginal cost is  $cv_{it}$  and the number of posted vacancies is bounded.

### 2.2 Firm behavior

Our analysis focuses on the steady state so that time indices are dropped unless necessary. Prior to entering the market each firm knows its time-independent productivity parameter  $a_i$ . We assume that firms enter in the high-productivity state  $\varepsilon_g$  so that it is possible that firms enter and exit in steady state.

We first define the asset values of firms that always remain in the market and of firms that enter and exit. Then we determine below how this firm selection is endogenously determined. Firms that exit the market declare bankruptcy, fire all workers and default on the firing costs.<sup>6</sup> We define the asset value of a firm i in the bad and good state as  $A_i^{b,n}$  and  $A_i^{g,n}$ , where n is a discrete variable which takes value 0 when firm i declares bankruptcy in the bad state and 1 otherwise. We also apply superscripts 1 or 0 to the state and control variables in order to distinguish between firms that declare or will declare bankruptcy and those that do not exit the market in the bad state. The asset value of a firm in the good state  $A_i^{g,n}(\tau)$  depends on the time  $\tau$  spent in this state. This is because hiring takes time. Introducing  $\tau$  as a state variable enables us to keep track of the number of hired workers.

Asset values: no bankruptcy. Let us first characterize the asset values of a firm i that does not declare bankruptcy if hit by a bad shock. The asset value in the bad state is defined as

$$rA_i^{b,1} \equiv rA^1(l_i^1(0); a_i, \varepsilon_b) = \pi_i^{b,1} + \upsilon \left( A_i^{g,1}(\tau = 0) - A_i^{b,1} \right)$$

where v is the Poisson hazard of receiving a good shock and

$$\pi_i^{b,1} \equiv \int_0^{l_i^1(0)} \left(\rho_i^b(l) - w_i^b\right) dl - f \; .$$

Firms in the bad state pay wages  $w_i^b$  and hoard labor  $l_i^1(\tau = 0)$ . Because hiring frictions make it impossible to adjust labor immediately to its optimal level, employment in the bad state equals employment of a firm that has just received a good shock and has spent  $\tau = 0$ time units in the good state. Only firms in the good state post vacancies and incur total hiring costs  $cv_i^1(\tau)^2/2$ . Hence, the asset value of firm *i* in the good state reads

$$rA_{i}^{g,1}(\tau) \equiv rA^{1}(l_{i}^{1}(\tau), v_{i}^{1}(\tau); a_{i}, \varepsilon_{g}) = \pi_{i}^{g,1}(\tau) + \delta \left(A_{i}^{b,1} - A_{i}^{g,1}(\tau) - F(l_{i}^{1}(\tau) - l_{i}^{1}(0))\right) + \frac{d}{d\tau} A_{i}^{g,1}(\tau)$$

$$\tag{1}$$

where  $\delta$  is the Poisson hazard of receiving a bad shock and

$$\pi_i^{g,1}(\tau) \equiv \int_0^{l_i^1(\tau)} \left(\rho_i^g(l) - w_i^{g,1}(\tau)\right) dl - \frac{cv_i^1(\tau)^2}{2} - f \; .$$

The control variable of the firm is the number of posted vacancies so the envelope theorem implies that

$$\frac{d}{d\tau}A^{g,1}(l_i^1(\tau), v_i^1(\tau); a_i, \varepsilon_g) = \frac{\partial A^1(l_i^1(\tau), v_i^1(\tau); a_i, \varepsilon_g)}{\partial l_i^1(\tau)}\dot{l}_i^1(\tau)$$

<sup>&</sup>lt;sup>6</sup>This assumption is similar to Belviso (2005). However, in our model with heterogenous firms, it is optimal that not all firms avoid firing costs by declaring bankruptcy but only "small" firms choose to default on firing costs if they are hit by bad shock.

where a dot denotes a derivative with respect to time  $\tau$  spent in the good state. Inserting this expression for expected capital gains into the asset equation (1) allows us to rewrite it as a function of the optimal labor demand schedule

$$rA_{i}^{g,1}(\tau) = \pi_{i}^{g,1}(\tau) + \delta \left( A_{i}^{b,1} - A_{i}^{g,1}(\tau) - F(l_{i}^{1}(\tau) - l_{i}^{1}(0)) \right) + \frac{\partial A^{1}(l_{i}^{1}(\tau), v_{i}^{1}(\tau); a_{i}, \varepsilon_{g})}{\partial l_{i}^{1}(\tau)} q(\theta) v_{i}^{1}(\tau) ,$$

where we substitute  $\dot{l}_i^1(\tau) = q(\theta)v_i^1(\tau)$  using the assumptions on the matching technology.

Asset values: bankruptcy. We now characterize the asset values of a firm i that declares bankruptcy if hit by a bad shock. We assume that the ownership of the firm is lost after filing for bankruptcy so that the manager cannot use it as a way to avoid operational costs in the bad state until business conditions switch back to the good state. Therefore

$$A_i^{b,0} = 0$$

Bankruptcy is an attractive option because: (i) it allows to save on wages and fixed costs in the bad state; (ii) bankrupt firms default on firing costs. Thus, the asset equation in the good state is given by

$$rA_{i}^{g,0}(\tau) = \pi_{i}^{g,0}(\tau) + \delta\left(-A_{i}^{g,0}(\tau)\right) + \frac{\partial A^{0}(l_{i}^{0}(\tau), v_{i}^{0}(\tau); a_{i}, \varepsilon_{g})}{\partial l_{i}^{0}(\tau)}q(\theta)v_{i}^{0}(\tau)$$

where 0 has been substituted for the asset value in the bad state.

#### 2.2.1 Extensive margin and firm selection.

**Exit rule.** In steady-state some firms will decide to hoard labor while others will prefer to exit the market. This new alternative extends the choice set of the firm. The asset values derived in the previous section allow us to determine the permanent productivity threshold below which firms decide to exit the market. We solve the problem in a recursive way: the firm determine whether or not it will exit the market in the bad state, then it decides upon its optimal labor demand schedule.

In order to rule out inconsistent choice, we notice that the exit decision is based on the value of the firm in the bad state so that firms necessarily choose the alternative which yields the highest asset value when  $\varepsilon_i = \varepsilon_b$ . In other terms, the firm's value in the good state may

be higher if it could commit to hoarding labor in the bad state, but it will never implement this production plan if it does better in the bad state by declaring bankruptcy.

As the asset value  $A^{b,1}(a_i)$  is increasing in  $a_i$  whereas the bankruptcy option  $A^{b,0}$  is independent of  $a_i$ , there exists a threshold productivity  $a^*$  such that  $A^{b,1}(a_i) \leq A^{b,0} = 0$  as  $a_i \leq a^*$ . The firms with a permanent productivity below  $a^*$  are always better off in the bad state by declaring bankruptcy. Using the asset equations above to derive which firms will produce in equilibrium, we determine  $a^*$  with the equation

$$A^{b,1}(a^*) = \frac{\pi_i^{b,1} + \upsilon A^{g,1}(a^*, \tau = 0)}{r + \upsilon} = 0.$$
 (2)

It remains to pin down  $A_i^{g,1}(\tau = 0)$  for the marginal firm. Using  $A^{b,1}(a^*) = 0$  and inserting the analytic expression for the profit flow, the asset value of the marginal firm in the good state can be rewritten as follows

$$\begin{split} (\delta+r)A^{g,1}(a^*,\tau) &= \overbrace{\left(\eta^g(a^*) - \frac{\sigma}{2}l^1(a^*,\tau) - w^{g,1}(a^*,\tau)\right)l^1(a^*,\tau) - f - \frac{cv^1(a^*,\tau)^2}{2}}_{-\delta F(l^1(a^*,\tau) - l^1(a^*,0)) + \frac{\partial A_i^{g,1}(l_i^1(\tau),v_i^1(\tau);a^*,\varepsilon_g)}{\partial l_i^1(\tau)}q(\theta)v_i^1(a^*,\tau)} \end{split}$$

Optimal vacancy posting implies that

$$\frac{\partial A_i^g(l_i^1(\tau), v_i^1(\tau); a^*, \varepsilon_g)}{\partial l_i^1(\tau)} = \frac{cv^1(a^*, \tau)}{q(\theta)}$$

As in BC, the marginal value of employment is equal to the expected cost of posting another vacancy. Inserting this expression in the previous asset equation, we obtain

$$(\delta + r)A^{g,1}(a^*, 0) = \left(\eta^g(a^*) - \frac{\sigma}{2}l^1(a^*, \tau) - w^{g,1}(a^*, \tau)\right)l^1(a^*, \tau) - f + \frac{cv^1(a^*, 0)^2}{2} .$$
 (3)

This equation enables us to evaluate the entry condition (2) to solve for  $a^*$  using the optimal labor demand schedules derived below. Let us mention for future reference that firing costs F decrease the asset values and thus increase the productivity of firms that produce in equilibrium through the entry and exit decision. However, the effect is less direct than that of fixed costs f, since firing costs do not enter explicitly in equation (3). Firing costs only matter through their effect on the vacancy-posting policy  $v^1(a^*, 0)$  and hoarded labor  $l^1(a^*, 0)$ . This is due to the fact that the firm will have to pay firing costs solely in the distant future, when it will switch back from the good to the bad state.

Entry rule. We restrict our attention to the steady-state of the economy. In equilibrium, all the firms with a permanent productivity above  $a^*$  have already entered the market and remain in operation independently of their idiosyncratic business conditions. On the contrary, some firms might find it profitable to operate solely in the good state. Let  $a^{**}$  denote the lowest permanent productivity among firms which enter the market in the good state and exit in the bad state. The entry condition that determines  $a^{**}$  reads

$$rA^{g,0}(a^{**},0) = \pi_i^{g,0}(0) + \delta\left(-A^{g,0}(a^{**},0)\right) + \frac{\partial A^{g,0}(l_i^0(0), v_i^0(0); a^{**}, \varepsilon_g)}{\partial l_i^0(0)}q(\theta)v_i^0(a^{**},0) = rC$$

where C is the entry cost. As before, optimality implies that the derivative of the asset value with respect to labor is equal to  $cv^0(a^{**}, 0)/q(\theta)$ . Moreover, since  $l^0(a^{**}, 0) = 0$ , we have  $\pi_i^{g,0}(0) = -c\frac{v_i^0(a^{**}, 0)^2}{2} - f$ . Replacing these two expressions into the asset equation and simplifying yields

$$A^{g,0}(a^{**},0) = \left(\frac{1}{r+\delta}\right) \left(\frac{cv^0(a^{**},0)^2}{2} - f\right) = C$$

Therefore, the entry condition  $A^{g,0}(a^{**}, 0) = C$  is equivalent to finding a permanent productivity  $a^{**}$  such that

$$v^{0}(a^{**},0) = \sqrt{\frac{2\left((r+\delta)C+f\right)}{c}} .$$
(4)

Depending on the models parameters, and especially on C,  $a^{**}$  might be larger than  $a^*$ . Then there is no firm turnover. However, in most cases the equilibrium is characterized by the following cross-sectional distribution: the firms with a permanent productivity below  $a^{**}$ remain out of the market, all the firms with  $a_i \in [a^{**}, a^*]$  enter the market in the good state and declare bankruptcy in the bad state, while firms with a permanent productivity above  $a^*$  hoard labor in the bad state and never exit the market.

Note again that product market regulation (C, f) directly affects  $a^{**}$  whereas firing costs F only matter by changing the vacancy posting of firms, that is the function  $v^0(\cdot)$ .

#### 2.2.2 Intensive margin: hiring and firing.

We briefly mention how firms adjust at the intensive margin. This section is quite similar to BC but for the fact that firms differ with respect to their permanent productivity shifter  $a_i$  and that some firms do not hoard labor in the bad state. Search frictions in the labor market imply that hiring takes time so that firms cannot immediately adjust their stock of employed workers upwards. Instead, firing of workers is immediate. The stock of employed workers at firm i evolves according to

$$l_{it+dt}^n = l_{it}^n + q(\theta_t)v_{it}^n + \Delta l_{it}^n \tag{5}$$

where the firm i shed  $\Delta l_{it}$  workers if hit by a negative shock.

Dropping time indexes, the shadow value of employment reads

$$rS_i^n = \rho_i^n - \omega_i^n + \frac{d}{d\tau}S_i^n$$

where  $\omega_i^n(l^n(a_i, \tau), a_i)$  denotes the marginal cost of employment. Note that this marginal cost is not equal to the wage in our model since multiple-worker firms have monopsony power and take into account the effect of their marginal employment decision on the wages of all workers.

Labor demand schedule of "bankrupt" firms. The shadow value of an additional worker in the bad state depends on whether or not the firm declares bankruptcy. If  $a_i < a^*$ , so that a firm exits the market when hit by a bad shock, the shadow value is obviously equal to zero

$$S_{i}^{b,0} = 0$$

In the good state the firms decide to hire so that the shadow value equals the expected hiring cost

$$S_i^{g,0}(\tau) = \frac{cv_i^0(\tau)}{q(\theta)} .$$
(6)

Using the linearization of the revenue function, we find that

$$\eta_i^{g,0} - \sigma l_i^0(\tau) - \omega_i^{g,0}(\tau) + \delta \left( -\frac{cv_i^0(\tau)}{q(\theta)} \right) + \frac{d}{d\tau} \frac{cv_i^0(\tau)}{q(\theta)} = r \frac{cv_i^0(\tau)}{q(\theta)} .$$

$$\tag{7}$$

Given that bankrupt firms default on firing costs, their optimal labor demand schedule is independent of F. This means that the entry rule (4) is not directly affected by the stringency of EPL. There will be an equilibrium effect, however, as we will see in the numerical solution below. Labor demand schedule of "labor-hoarding" firms. If the model's parameters are such that firm i decides to both fire and hoard labor when hit by a bad shock, then the shadow value of an additional worker must be equal to the firing cost

$$S_i^{b,1} = -F$$

The shadow value in the good state is determined as before, so that

$$S_i^{g,1}(\tau) = \frac{cv_i^1(\tau)}{q(\theta)}$$

Since firing is instantaneous, each firm that fires has the same employment level conditional on permanent productivity  $a_i$ . The employment of firms in the good state depends on how long firms have been in the good state. Using the linearization of the revenue function and the firing condition, we find that firing in the bad state is determined by

$$\eta_i^{b,1} - \sigma l_i^1(0) - \omega_i^{b,1}(0) + \upsilon \left(\frac{cv_i^1(0)}{q(\theta)} - (-F)\right) = -rF$$
(8)

and hiring in the good state at time  $\tau$  is given by

$$\eta_i^{g,1} - \sigma l_i^1(\tau) - \omega_i^{g,1}(0) + \delta \left( -F - \frac{cv_i^1(\tau)}{q(\theta)} \right) + \frac{d}{d\tau} \frac{cv_i^1(\tau)}{q(\theta)} = r \frac{cv_i^1(\tau)}{q(\theta)}.$$
(9)

Contrary to the conditions for the extensive margin (2), (3) and (4) the conditions for the intensive margin (8) and (9) explicitly depend on firing costs F whereas fixed costs f or set-up costs C only have an implicit effect. Although higher fixed or set-up costs do not affect the optimal labor demand schedule of a given firm directly, they modify the distribution of operating firms through the selection effect at the extensive margin. As only more productive firms enter a more regulated market and the hiring and firing condition depend on  $a_i$ , fixed and set-up costs matter for aggregate labor hoarding. The selection effect also induces a general equilibrium effect by feeding back into the optimal policy of vacancy posting, and thus changes the aggregate unemployment rate and labor market tightness. We will illustrate this interaction further when we solve the model numerically in the next section.

The asset value of the worker, wages and the equilibrium are solved for quite similarly to BC so that we refer for these derivations to Appendix A. The model can be solved largely analytically but for the two conditions that determine hoarded labor,  $l_i^1(0)$ , and vacancies initially posted in the good state,  $v_i^n(0)$ . Compared with BC the permanent productivity shifter  $a_i$  implies that the two conditions also depend on the average number of vacancies posted.<sup>7</sup>

### 2.3 Cross-sectional efficiency

Before we solve the model numerically, we briefly want to point out the cross-sectional inefficiencies in our model by solving the social planner's problem. We characterize the socialplanner problem when the discount rate r = 0 and when firms cannot declare bankruptcy on F. This simplifies the notation and is not important for the main results. We mention below how the solution would differ if we allowed for firm bankruptcy.

Since the social-planner problem is similar to BC, we defer the explicit derivations to Appendix B and only mention the results which are important for our purposes. The social planner internalizes the congestion externality by making vacancy posting more costly for each firm in the good state, associated with the shadow price  $\mu$ . The social planner also decreases marginal revenues of firms to induce relatively more unemployment, associated with the shadow price  $\zeta$ .

As shown in Appendix B, the social planner seeks to maximize the total surplus and not the revenues net of wages. Hence, as in BC, the bargaining parameter does no longer determine the convergence rate of vacancy posting. It remains identical across firms but firms converge more quickly to their targeted employment rate than in the decentralized economy.

We briefly mention the solution for  $v_i^+(0)$  and  $l_i^+(0)$  where the superscript + denotes the social planner solution. The closed-form solutions afford further insights also for the decentralized equilibrium. We find that firms which just entered the good state post vacancies

<sup>&</sup>lt;sup>7</sup>This is because the wage of the employed worker in the bad state,  $w_i^b$ , depends on the outside option of unemployment which is a function of the average vacancies posted across firms in the good state. The posted vacancies in each firm summarize expected future employment changes in these firms which are important for expected future wages because of decreasing returns, intrafirm bargaining and the monopsony power of firms. See Appendix A for further details.

$$v_i^+(0) = \frac{q(\theta)}{c(\upsilon+\delta+\lambda^+)} \left[ \eta_i^g - \eta_i^b - (\upsilon+\delta)F - \delta \frac{\mu}{q(\theta)} \right] .$$

Firing costs F, turnover  $(\delta, v)$ , speed of convergence  $\lambda^+$ , vacancy costs c and a higher shadow price  $\mu$  decrease initial vacancy posting, whereas a higher marginal product in the good relative to the bad state and a quicker vacancy matching rate increase vacancy posting. Labor hoarding is given by

$$l_i^+(0) = \frac{1}{\sigma} \left[ \eta_i^b + \frac{\upsilon \lambda^+}{\upsilon + \delta + \lambda^+} F - b - \zeta + \frac{\upsilon}{\upsilon + \delta + \lambda^+} \left( \eta_i^g - \eta_i^b - \delta \frac{\mu}{q(\theta)} \right) \right] .$$

Conditional on  $\zeta$  and  $\mu$ , higher marginal revenue in the bad state and larger revenue gains (in case a good shock occurs) increase the amount of labor hoarded. Firing costs have an unambiguous positive effect on labor hoarding: they decrease firing and this effect outweighs the negative effect on vacancy posting and hiring in the good state.<sup>8</sup>

If we allow for bankruptcy, the solution for firms hoarding labor remains the same (see Appendix B). For firms exiting in the bad state, we find that the vacancy dynamics are the same but the initial amount of vacancies posted in the good state is

$$v_i^{++}(0) = \frac{q(\theta)}{c(\delta + \lambda^+)} \left(\eta_i^g - b - \frac{\delta}{q(\theta)}\mu\right)$$

where the superscript ++ denotes firms which exit in the bad state. Compared with the expression for labor-hoarding firms  $v_i^+(0)$ , vacancies posted by bankrupt firms do not directly depend on firing costs. Moreover,  $v_i^{++}(0)$  only depends on the marginal revenue in the good state (net of the worker's utility flow b) and is not discounted with the probability of being in the good state v since firms exit in the bad state. Thus, if  $\eta_i^b - (v+\delta)F > b$ ,  $v_i^{++}(0) > v_i^+(0)$  (see Appendix B).

The solution of the social-planner problem makes explicit in closed-form what we find numerically for the decentralized equilibrium (in which wage determination complicates the analysis). Firms that exit in the bad state post more vacancies in the good state (for a given  $a_i$ ) because these firms do not have to pay firing costs and do not hoard labor in the bad

<sup>&</sup>lt;sup>8</sup>As is well known, this result depends on the linear revenue schedule and does not necessarily hold for more general functional forms.

state. More generally, this points to the optimality of a firing tax schedule that differs across firms with different  $a_i$ . We explore these issues more in ongoing research.

For the purposes of this paper, it is important to note that the congestion externalities are rather unimportant for the parameter values of the calibrated model described below.<sup>9</sup>

### 3 Equilibrium

In this section we define the equilibrium, describe the numerical algorithm and calibrate our model to the US economy. Then we discuss the robustness of the calibration to some changes of important parameters.

**Equilibrium definition.** We define a search equilibrium for the economy as a set of aggregate quantities  $\{L, V\}$ , matching rate  $q(\theta)$ , permanent productivity thresholds  $\{a^{**}, a^*\}$  and infinite sequences for quantities  $\{l_i^n(\tau), v_i^n(\tau)\}_{\tau=0}^{\infty}$  and prices  $\{w_i^b, w_i^{g,n}(\tau)\}_{\tau=0}^{\infty}$  such that:

- Given the matching rate and prices,  $\{l_i^n(\tau), v_i^n(\tau)\}_{\tau=0}^{\infty}$  solve firm's *i* optimization problem.
- Wages  $\{w_i^b, w_i^{g,n}(\tau)\}_{\tau=0}^{\infty}$  are the solution of the Nash-bargaining problem.
- Permanent productivity thresholds  $\{a^{**}, a^*\}$  are determined by the optimal entry and exit decisions of firms.
- Aggregate quantities  $\{L, V\}$  result from the aggregation of firms' optimal labor demand schedules.
- The matching rate  $q(\theta)$  is given by the aggregate matching function.

<sup>&</sup>lt;sup>9</sup>The small vacancy cost c = 0.01 required to match empirically realistic unemployment rates and duration, implies that aggregate vacancy posting costs are an order of magnitude smaller than other aggregate welfare losses.

The numerical algorithm. The algorithm proceeds in three steps. In Step 1, we set starting values for the average number of vacancies, labor market tightness  $\theta$  and the productivity  $a^*$  of the marginal firm. In Step 2, we solve for  $v^1(a^*, 0)$ ,  $l^1(a^*, 0)$  and use the solution for  $v^0(a^{**}, 0)$  to determine  $a^{**}$ . We then update the average number of vacancies and  $\theta$ . As long as these two values have not converged up to numerical precision of  $10^{-6}$ , we repeat Step 2. Otherwise we continue with Step 3 and update  $a^*$  using the steady-state condition  $A^{b,1}(a^*) = 0$ . Unless  $a^*$  has converged up to numerical precision of  $10^{-6}$ , we update  $\theta$  and the average number of vacancies, and restart the algorithm at Step 2. Our numerical results indicate that the equilibrium labor market tightness  $\theta$  is locally unique.<sup>10</sup>

**Calibration.** For our computations we assume a uniform distribution so that  $a_i \sim U(0; \overline{a})$ . The constant density facilitates the interpretation of the numerical results. The upper bound of the uniform distribution can be tied down using the normalization to 1 of total labor in the frictionless economy.<sup>11</sup>

We set the annual interest rate r = 0.05 (see Cooley, 1995). The utility flow in unemployment b = 0.05, which is 43% of the average wage in the flexible economy as we will see below.<sup>12</sup> This value is within the range of commonly assumed values. We check that the value b implies that workers in the frictionless economy find it optimal to supply labor in

<sup>11</sup>We restrict our attention to the case where firms in the frictionless economy operate solely in the good state. All labor is shed if a bad shock occurs. Since the workforce in the good state is equal to  $l(a) = \frac{a\varepsilon_g - b}{\sigma}$ , the normalization of employment to full employment in the frictionless economy implies

$$1 = \frac{v}{v+\delta} \frac{1}{\overline{a}-\underline{a}} \int_{b}^{\overline{a}} l(a) da = \left(\frac{v}{v+\delta}\right) \left(\frac{1}{\overline{a}-\underline{a}}\right) \left(\frac{1}{2\sigma} \varepsilon_{g} \left(\overline{a}^{2}-b^{2}\right) - \frac{b}{\sigma} \left(\overline{a}-b\right)\right) \quad .$$

Setting  $\underline{a} = 0$  implies that  $\overline{a}$  is the positive root of a quadratic equation

$$\overline{a} = \frac{\frac{\upsilon + \delta}{\upsilon} + \frac{b}{\sigma} + \sqrt{\left(\frac{\upsilon + \delta}{\upsilon} + \frac{b}{\sigma}\right)^2 + \left(\frac{b^2 \varepsilon_g(\varepsilon_g - 2)}{\sigma^2}\right)}}{\varepsilon_g / \sigma}$$

 $^{12}b$  is 1/5 of the wage paid at the firm with the highest permanent productivity  $\overline{a}$  in the initial good state and 1/2 at that firm in the bad state.

<sup>&</sup>lt;sup>10</sup>We calculate the slope of the feedback locus  $\theta'(\theta)$  when we compute  $\theta$  (in each iteration for each given value of  $a^*$ ). In the program we check that the locus  $\theta'(\theta)$  intersects only once with the 45-degree line. We have always found a unique equilibrium for positive  $\theta$  given the parameter values we considered.

the good state so that  $b \leq \eta_i^g - \sigma l_i$ , for all  $a_i$ . Indeed, we find that for  $\sigma = 0.4$  and  $\varepsilon_g = 1$  this condition is always satisfied in equilibrium.<sup>13</sup> We set  $\varepsilon_b$  equal to  $b/\overline{a}$ . This value implies that in the bad state there is no wage for which firms employ a positive amount of labor in the frictionless economy.

The dynamic transitions between good and bad states are parametrized as  $\delta = 0.5$  and v = 1. This implies that a created job has a 60% chance to persist for one year or more whereas the chance for a destroyed job is 40%. The former is consistent with evidence reported in Davis et al. (1996) whereas v is higher than suggested by their evidence. A higher v makes it more attractive for firms to hoard labor also at low levels of unemployment. We need this for technical reasons as further explained below. We assume a matching efficiency  $\gamma = -0.5$  which is in line with parameters commonly used in the literature (see Petrongolo and Pissarides, 2000, for a survey on estimates of the matching function). We set the bargaining power of workers to  $\beta = 0.2$  which is slightly smaller than the estimates reported in Flinn (2005) in order to be able to match empirically plausible unemployment rates. Below we check the robustness of our results for a bargaining power of  $\beta = 0.3$ .<sup>14</sup> Finally, we assume that the flow cost of an additional vacancy is c = 0.01 and the scaling factor of the matching function  $\xi = 2.5$ . Both parameters are set to match a reasonable unemployment rate, labor market tightness and thus unemployment duration. The value of c equals 1/12 of the average wage, which yields an average recruiting costs close to one month's wage (see Hamermesh, 1993). As we will discuss further below, the small value of c is crucial for the model to produce realistic values of unemployment duration. The scaling factor instead allows for realistic unemployment rates.

In the calibration of our model there is a tension between targeting realistically low unemployment rates and unemployment duration together with all firms with  $a_i \ge a^*$  hoarding labor. The latter is important because it simplifies the solution of the model since the shadow value of labor in the bad state is then determined by (8). However, we need small

<sup>&</sup>lt;sup>13</sup>For  $l_i = a_i = 1$ , these parameters imply a marginal-revenue elasticity of 2/3.

<sup>&</sup>lt;sup>14</sup>Note that efficiency could not be restored in this model if we set  $\beta = -\gamma$ , as the Hosios condition might suggest. As pointed out in BC, cross-sectional efficiency is more difficult to achieve because of the additional intra-firm bargaining distortions and heterogenous vacancy posting of firms. This holds *a fortiori* in this model with permanent productivity differences  $a_i$ .

Parameters Values										
$\sigma$	r	δ	v	$\gamma$	ξ	$\beta$	c	F	f	C
0.4	0.05	0.5	1	0.5	2.5	0.2	0.01	0.04	0.1	0.1
			E	Equilit	orium	Vari	ables			
U	nempl	oyme	nt r	ate $U$	í (in ¢	%)		7.62	22	
		Vaca	ncie	es $V$			0.341			
Tightness parameter $\theta$						4.472				
		Out	tput	Y			0.388			
		We	lfare	$e \Omega$				0.32	29	
	Produ	ctivit	y a	$_i$ (ave	rage)		0.895			
Prod. margins $(a^{**}, a^*)$					(0.486, 0.506)					
Average firm size					1.489					
Av. labor hoarded					0.193					
Average wage					0.115					

Table 1: Equilibrium values in the flexible "US" economy.

search frictions in the labor market which imply realistic values for the level and duration of unemployment but also less labor hoarding for all firms. In order to generate some labor hoarding for all firms, we calibrate fixed and firing costs in the flexible economy as f = 0.1and F = 0.04. This is not unrealistic compared with an average wage of 0.12 since even in relatively flexible economies such as the US, firms face some administrative costs to maintain operations and lay off workers if these lay-offs are considered "unfair" (see OECD, 1999, ch. 2). Our calibration implies that the firm with  $a_i = a^*$  just hoards a tiny amount of labor  $l_i(0)$ in the bad state. We calibrate set-up costs C = 0.1 so that the hazard rate of bankruptcy is equal to 0.8% per year, which is realistic for publicly traded firms in the US economy (see www.bankruptcydata.com).<sup>15</sup>

<sup>&</sup>lt;sup>15</sup>The incidence of bankruptcy is computed as the fraction of firms in the good state that are hit by a bad shock  $\delta v/(\delta + v)$  multiplied with the fraction of operating firms which declare bankruptcy if a bad shock occurs  $(a^* - \min\{a^{**}, a^*\})/(\overline{a} - a^* + v/(\delta + v)(a^* - \min\{a^{**}, a^*\}))$ .

Table 1 displays the equilibrium for the flexible economy with f = C = 0.1 and F = 0.04which we call the "US". The calibration matches the level and duration of unemployment in the flexible "US" economy quite well (see, for example, Abrahams and Shimer, 2001): the average unemployment duration is 2.3 month  $(1/\theta q(\theta))$  and an unemployment rate of 7.6% is realistic for the US in the last decades. Firms with permanent productivity  $a_i < a^{**} = 0.49$ do not produce in the market where for the firm with the highest permanent productivity  $a_i = \overline{a} = 1.3$ . Firms with  $a_i \in [0.49; 0.51]$  declare bankruptcy if hit by a bad shock.<sup>16</sup>

The output measure in Table 1 is defined net of steady-state mobility and vacancy costs. We then subtract fixed costs of all producing firms and take into account the utility flow of the unemployed for our measure of welfare (see Appendix A for the analytic expressions).

**Robustness.** Before we analyze changes in the policy parameters (f, C, F), we briefly investigate the robustness of the equilibrium to changes of some important parameter values.<sup>17</sup> In Table 2, column (1) we increase the utility flow during unemployment by 20% to b = 0.06. In column (2) we increase the bargaining power of workers to  $\beta = 0.3$ . In column (3) we decrease the monopsony power of firms by setting  $\sigma = 0.38$ . Finally, in column (4) we decrease the vacancy cost assuming a marginal flow cost c = 0.0095.

The main insights can be summarized as follows. Higher utility during unemployment b increases unemployment and unemployment duration (see column (1)). Operating firms are slightly more productive. They hoard less labor since the better outside option of workers renders labor hoarding more expensive. This increases productivity but also the steady-state mobility cost. The overall effect on output and welfare is negligible. A higher bargaining power of workers  $\beta = 0.3$  has a similar impact on unemployment and its duration but a stronger selection effect on  $a^{**}$  and  $a^*$  (see columns (2)). Less firms operate in equilibrium and average productivity increases.

If the marginal revenue of firms decreases less with higher labor demand ( $\sigma = 0.38$ ), more

<sup>&</sup>lt;sup>16</sup>Our model produces a left-skewed cross-sectional distributions for wages and a U-shaped distribution of employment over firms with different size which are roughly consistent with empirical data. Our model also generates a positive firm-size wage premium and smaller wage dispersion in larger firms as in Bertola and Garibaldi (2001). Results on the cross-sectional distribution are available on request.

<sup>&</sup>lt;sup>17</sup>We keep constant the other parameters which depend on  $\sigma$  or b (e.g.  $\overline{a}$ ).

Variables	b = 0.06	$\beta = 0.3$	$\sigma = 0.38$	c = 0.0095
	(1)	(2)	(3)	(4)
Unemployment $U$ (in %)	8.21	9.51	6.79	6.86
Vacancies $V$	0.32	0.28	0.39	0.39
Tightness $\theta$	3.93	2.94	5.74	5.72
Output $Y$	0.39	0.39	0.41	0.38
Welfare $\Omega$	0.33	0.33	0.35	0.33
average $a_i$	0.897	0.910	0.901	0.900
Prod. margins $(a^{**}, a^*)$	(0.49, 0.51)	(0.52, 0.54)	(0.50, 0.52)	(0.49, 0.51)
Av. firm size	1.48	1.51	1.52	1.52
Av. lab. hoarded	0.17	0.15	0.20	0.18
Av. wage	0.12	0.12	0.13	0.13

Table 2: Robustness of the equilibrium to changes in parameter values.

vacancies are posted so that the average firm size and amount of labor hoarded increase (see column (3)). The unemployment rate and duration fall substantially. Output and welfare are higher since a lower  $\sigma$  favors firms with higher permanent productivity  $a_i$  so that employment is more concentrated in these firms and only relatively more productive firms operate ( $a^{**}$  and  $a^*$  increase). Thus, the welfare loss due to fixed costs is smaller. Average wages increase both because of this selection effect and the higher revenues which augment the shared surpluses.

A slightly smaller flow cost of vacancy posting c = 0.0095 substantially lowers the unemployment rate and duration. It increases wages because of a better outside option (see column (4)). The lower cost of posting vacancies favors firms with a high permanent productivity  $a_i$  since they tend to post more vacancies. So in equilibrium only more productive firms remain in the market ( $a^{**}$  and  $a^*$  increase slightly). These firms have a larger average size and hoard less labor in the bad state since hiring has become less costly. Output and welfare fall slightly because firms bear higher steady-state mobility costs. Results which are not reported show that, quite intuitively, both a smaller c or  $\sigma$  imply that regulation has a

Equilibrium Variables	US			
F = 0.04	f = 0.1	f = 0.15	f = 0.1	f = 0.15
	C = 0.1	C = 0.1	C = 0.15	C = 0.15
	(1)	(2)	(3)	(4)
Unempl. rate $U$ (in %)	7.622	8.413	7.873	8.900
Vacancies $V$	0.341	0.305	0.328	0.285
Tightness parameter $\theta$	4.472	3.625	4.162	3.120
Output $Y$	0.388	0.384	0.386	0.381
Welfare $\Omega$	0.329	0.300	0.329	0.299
Productivity $a_i$ (average)	0.895	0.923	0.900	0.929
Prod. margins $(a^{**}, a^*)$	(0.486, 0.506)	(0.531, 0.579)	(0.514, 0.502)	(0.553, 0.573)
Average firm size	1.489	1.584	1.503	1.602
Av. labor hoarded	0.193	0.208	0.193	0.206
Average wage	0.115	0.106	0.112	0.101

Table 3: Equilibrium values for different fixed costs and set-up costs.

smaller effect on the unemployment rate.

In the following sections we analyze the effect of regulation on the equilibrium. We start with product market regulation in Section 4, continue with firing costs in Section 5 before we explain the interactions of both regulations in Section 6.

## 4 The impact of PMR

In this section we first investigate the effect of fixed costs and then compare it with the effect of set-up costs.

The selection effect of fixed costs. As can be seen from equations (A13), (A14) and (A15) in Appendix A, the fixed costs f do not directly enter in the optimal labor demand schedules. Since vacancy posting and labor hoarding decisions are based on workers' marginal

revenues, it is clear that fixed costs do not influence the behavior of a given firm if it produces.

However, fixed costs reduce firms' asset values. As the least profitable firm just breaks even, a tightening of administrative regulation drives it out of business. In terms of the model's parameters this means that  $a^{**}$  and  $a^*$  increase, as can be seen by comparing columns (1) and (2) or (3) and (4) in Table 3, where we increase fixed costs from 0.1 to 0.15 (for different levels of set-up cost C). Furthermore, the impact on  $a^*$  is stronger so that fixed costs increase the size of the interval  $[a^{**}; a^*]$  in which firms declare bankruptcy. Given that defaulting firms do not pay the fixed costs in the bad state, their asset values fall relatively less than the asset values of the firms that remain in operation. Quantitatively, for low set-up costs C = 0.1, higher fixed costs imply that the incidence of bankruptcy increases from 0.8% to 2%.

This selection effect on  $a^{**}$  and  $a^*$  decreases labor market tightness and thus reduces wages by lowering the outside option of workers. The operating firms take advantage of their stronger bargaining position through an increase in both hoarded labor and targeted employment in the good state  $l(a_i, \infty)$ . The new equilibrium is characterized by a smaller number of larger firms. Notice that labor hoarding remains nearly constant so that most of the adjustment is achieved through an increase of the firms' sizes in the good state. This implies that firms destroy on average more jobs when they are hit by a bad shock. This positive turnover effect on "labor hoarding" firms is reinforced by the fact that a larger share of firms declare bankruptcy and shed all their workers. Although the effect on firm size compensates the selection effect to a certain extent, the negative impact prevails so that labor market tightness decreases and unemployment increases substantially.

The selection effect of barriers to entry. Table 3 also displays the equilibrium outcomes for higher set-up costs C = 0.15, again for different levels of fixed costs. Not surprisingly, higher barriers to entry decrease the number of operating firms and slightly increase average productivity and average firms' size.<sup>18</sup> The increase in unemployment and unem-

<sup>&</sup>lt;sup>18</sup>Average productivity is computed as productivity  $(a + \min \{a^{**}, a^*\})/2$  in the good state weighted with mass  $v/(\delta + v)$ , and productivity  $(a + a^*)/2$  in the bad state with mass  $\delta/(\delta + v)$ . Since  $\delta < v$ , the decrease of productivity in the bad state is weighted less than the increase of productivity in the bad state. Moreover, the fall of  $a^*$  is smaller in size than the increase in  $a^{**}$ .

ployment duration leads to a decline in wages. Set-up costs also decrease welfare and output. These negative effects are quantitatively smaller than for fixed costs because the set-up costs do not directly affect the asset value of the firm once it has entered the market.<sup>19</sup>

More interestingly, barriers to entry have opposite effects on the exit and entry margins: set-up costs raise  $a^{**}$  and lower  $a^*$ , so that less firms declare bankruptcy in the bad state. On the one hand,  $a^*$  decreases because the barriers to entry isolate operating firms from the competition of potential entrants. As set-up costs do not affect their revenues, the operating firms actually benefit from an increase in C. On the other hand, the effect on  $a^{**}$  is positive and significant as can be seen analytically from the entry rule (4). Column (3) shows that set-up costs can deter entry to such an extent that  $a^{**} > a^*$ . In other terms, for sufficiently high set-up costs, the equilibrium may exhibit no bankruptcy and no firm turnover.

The interactions between fixed costs and entry costs. By comparing the unemployment rates reported in Table 3, one can see that the two regulations interact negatively since their joint increase (see column (4)) leads to bigger job losses than the sum of their independent increases (see column (2) and (3)). Thus the negative impact of barriers to entry is intensified by the stringency of administrative regulation, and *vice versa*. Intuitively, the two selection effects reinforce each other by reducing both firms' incentives and capacities to enter the market.

### 5 The impact of EPL

The mechanism through which EPL affects the equilibrium is more intricate because firing costs also modify the labor demand schedules of firms directly. As explained in Bentolila and Bertola (1990), the *partial equilibrium effect* of firing costs yields less labor mobility and more labor hoarding. Firms respond to the change in labor market tightness by adjusting the number of posted vacancies. This *equilibrium effect* dampens the imbalance between the partial equilibrium effects on the hiring and firing margins. To the extent that the labor hoarding adjustments prevail, firing costs and employment are positively correlated.

<sup>&</sup>lt;sup>19</sup>The negative welfare effect remains if fixed and/or set-up costs are rebated at the aggregate level.

Equilibrium Variables	US	Partial	Equilibrium	Full effect
		(given $\theta, a^*, a^{**}$ )	(given $a^*, a^{**}$ )	
	F = 0.04	F = 0.09	F = 0.09	F = 0.09
fixed cost $f = 0.1$	(1)	(2)	(3)	(4)
Average labor hoarded	0.193	0.325	0.313	0.317
Average firm size	1.489	1.509	1.499	1.507
Unempl. rate $U$ (in %)	7.622	6.400	6.995	6.953
Labor market tightness $\theta$	4.472	4.875	4.462	4.552
Prod. margins $(a^{**}, a^*)$	(0.486, 0.506)	(0.486, 0.506)	(0.486, 0.506)	(0.482, 0.525)
fixed cost $f = 0.15$				
Average labor hoarded	0.208	0.369	0.328	0.331
Average firm size	1.584	1.628	1.593	1.602
Unempl. rate $U$ (in %)	8.413	5.832	7.859	7.803
Labor market tightness $\theta$	3.625	4.815	3.547	3.631
Prod. margins $(a^{**}, a^*)$	(0.531, 0.579)	(0.531, 0.579)	(0.531, 0.579)	(0.528, 0.598)

Table 4: Decomposition of the effect of firing costs.

Both partial equilibrium and equilibrium effects were already at work in BC. Since our model has an extensive adjustment margin, firing costs also have an additional *selection effect*. Table 4 decomposes the effect of firing costs into: (i) the partial equilibrium effect (for given  $a^{**}$ ,  $a^*$  and  $\theta$ ), (ii) the equilibrium effect through changes in  $\theta$  for given  $a^{**}$  and  $a^*$ , (iii) the selection effect on  $a^{**}$  and  $a^*$ . The upper part of the table analyzes the effect of increasing firing cost F from 0.04 to 0.09 for fixed cost f = 0.1. The lower part of the table repeats this exercise for higher fixed costs f = 0.15.

Consider first the case where fixed costs are low. Column (2) displays the partial equilibrium effect of higher firing cost. As expected, employment protection stimulates labor hoarding and the average firm's size increases. Hence, positive labor adjustments at the firing margin prevail over negative adjustments at the hiring margin. This is why the partial equilibrium effect on employment is positive. The fourth row displays the new labor market tightness. As the number of unemployed and posted vacancies decrease, the labor market becomes tighter. Obviously, the value of  $\theta$  reported in column (2) is not an equilibrium outcome since we assume that firms make their choice based on the value of  $\theta$  in column (1). The equilibrium adjustments resulting from the discrepancy between the two values of  $\theta$  are reported in column (3). As explained before, a higher labor market tightness induces firms to lower their labor demand, so that both labor hoarding and vacancy posting decrease. The equilibrium effect of firing costs on unemployment is positive and the equilibrium labor market tightness is substantially lower.

Of most interest to our analysis are the differences between columns (3) and (4) since they capture the selection effect that is new in our model. Although the selection effect of fixed and firing costs on  $a^*$  are qualitatively alike, their magnitude substantially differ.<sup>20</sup> Given that the decision to remain in the market is based on the asset value of the firm in the bad state, firing costs are heavily discounted since they will have to be paid in the remote future. Instead, fixed costs burden the profit of the firm at each instant so that they have a more noticeable influence on the extensive margin.

Conversely, the selection effect of fixed and firing costs on  $a^{**}$  are different in both quantitative and qualitative terms. Whereas fixed costs substantially increase  $a^{**}$ , the impact effect of firing costs is negative and quantitatively small. The reason is that defaulting firms are exempted from EPL. Hence, firing costs do not affect directly their asset values  $A_i^{g,0}(0)$ . Instead, "labor hoarding" firms  $(a_i > a^*)$  are hurt by firing restrictions and thus post less vacancies in the good state. *Ceteris paribus*, the firms which declare bankruptcy if a bad shock occurs  $(a_i \in [a^{**}; a^*])$  benefit from the increase in the rate of vacancy filling. This externality augments the incentives to enter the labor market in the good state so that  $a^{**}$ falls.<sup>21</sup>

Turning our attention to employment, we notice that the selection effect is positive. The sign of the relationship is due to the decrease in  $a^{**}$  and so crucially hinges on the assumption that firms can declare bankruptcy. On the contrary, when the model does not allow firms to declare bankruptcy, the selection effect unambiguously raises unemployment. Thus, the sign and size of the selection effect on unemployment depends importantly on whether small

<sup>&</sup>lt;sup>20</sup>Compare the values of  $a^*$  in columns (4) of Table 4 with its counterpart in column (2) of Table 3.

<sup>&</sup>lt;sup>21</sup>Note that the analytical solution for  $A^{g,0}(a^{**}, 0)$  does not depend on firing cost so that firing cost only matters through its effect on the vacancy posting policy  $v^0(a^{**}, 0)$ .

Equilibrium Variables	US			
	F = 0.04	F = 0.09	F = 0.04	F = .09
	f = 0.1	f = 0.1	f = 0.15	f = 0.15
	(1)	(2)	(3)	(4)
Job finding rate	5.287	5.334	4.760	4.764
Job flows	0.403	0.371	0.400	0.372
Job turnover rate	0.436	0.399	0.437	0.403
Job turnover rate / firm	0.645	0.601	0.692	0.646

Table 5: The effect of firing and fixed costs on turnover.

firms (with low permanent productivity  $a_i$ ) can "avoid" firing costs using the bankruptcy option. This motivates why, in countries with strict employment protection legislation like Italy or Germany, this legislation does not apply to small firms with employment below a certain threshold.<sup>22</sup>

Finally, it is worth emphasizing that the sign of the relationship between EPL and unemployment is ambiguous. Depending on the parameter values, it can be either positive or negative. Nevertheless, comparative statics around the proposed equilibrium show that the effects are locally robust, as can be seen from the lower part of Table 4 where f = 0.15.

### 6 Interaction of labor and product market regulation

The interactions between firing and fixed costs. Before analyzing the interaction between fixed and firing costs, it is useful to remember that their effects in BC's framework are independent. The interaction between both regulations arises because of the adjustments at the extensive margin. According to the previous sections, the selection effects of product market regulation and employment protection are qualitatively similar: they both reduce the number of operating firms in the bad state, increase the firm turnover and average firm size.

<sup>&</sup>lt;sup>22</sup>The threshold is currently at 15 employees in Italy and 10 employees in Germany. Furthermore, in some countries entering firms are exempt from EPL for a limited time period.

Equilibrium Variables	US			
C = 0.1	F = 0.04	F = 0.09	F = 0.04	F = 0.09
	f = 0.1	f = 0.1	f = 0.15	f = 0.15
	(1)	(2)	(3)	(4)
Unempl. rate $U$ (in %)	7.622	6.953	8.413	7.803
Vacancies $V$	0.341	0.317	0.305	0.283
Tightness parameter $\theta$	4.472	4.552	3.625	3.631
Output $Y$	0.388	0.366	0.384	0.364
Welfare $\Omega$	0.329	0.307	0.300	0.279
Productivity $a_i$ (average)	0.895	0.897	0.923	0.925
Prod. margins $(a^{**}, a^*)$	(0.486, 0.506)	(0.482, 0.525)	(0.531, 0.579)	(0.528, 0.598)
Average firm size	1.514	1.563	1.654	1.709
Av. labor hoarded	0.193	0.317	0.208	0.331
Average wage	0.115	0.11	0.106	0.102

Table 6: Equilibrium values for different firing and fixed costs.

Quantitatively, firing costs are more important for labor hoarding (the intensive margin) whereas fixed costs have a larger effect on entry and exit (the extensive margin).

Let us first revisit how the effect of firing costs changes with higher fixed costs. We have already mentioned that higher fixed costs increase job turnover per firm because of the higher rate of vacancy filling. Conversely, it is well known that firing costs yield less labor mobility at both the aggregate and firm level. These insights are illustrated in Table 5. The table reports the rate of job finding  $q(\theta)\theta$ , the aggregate job flows<sup>23</sup> along with the rates of job turnover and job turnover per firm.<sup>24</sup> Table 5 shows that fixed costs decrease whereas firing costs increase the job finding rate. Firing costs do reduce job flows because of the lower unemployment rate. Fixed costs have almost no effect on aggregate job flows because

<sup>&</sup>lt;sup>23</sup>Notice that job flows and worker flows are indistinguishable in the current formulation of the model since we have excluded job-to-job transitions.

<sup>&</sup>lt;sup>24</sup>More precisely, the aggregate job flows are  $q(\theta)\theta U$ , the job-turnover rate is  $q(\theta)\theta U/L$  and the job-turnover rate per firm is  $q(\theta)\theta U\overline{a}/(\overline{a}-a^*)$  if  $a^{**} > a^*$  and  $q(\theta)\theta U\overline{a}/(\overline{a}-a^* + \left(\frac{v}{v+\delta}\right)(a^*-a^{**}))$  if  $a^{**} < a^*$ .

the smaller transition rate is compensated by the increase in the size of the unemployment pool. Since the flows out of the employment pool are nearly constant whereas the number of employees is smaller, it follows that the job turnover rate is an increasing function of fixed costs. The job turnover rate per firm is also increasing for the same reasons, but in this case the effect of fixed costs is strong enough so as to completely offset the "sclerosis" generated by EPL.

This implication of the model is *a priori* consistent with empirical evidence that turnover rates across countries are very loosely related to the stringency of the employment protection legislation. This empirical fact has led Bertola and Rogerson (1997) to argue that the greater compression of wages in Europe than in the US can compensate the differences in EPL and so explain the similarity of the turnover rates. The model proposed in this paper suggests that more product market regulation in Europe is an alternative explanation. In the light of Table 5, the lack of conclusive evidence might be partly explained by the countervailing effects of EPL and PMR.<sup>25</sup>

Let us now comment on the effect of fixed costs for different levels of firing costs. Table 5 shows that fixed costs increase the turnover rate and thus the steady-state mobility cost per firm. However, this does not necessarily induce additional welfare losses because the impact of both policies on firm selection is such that less firms need to pay the fixed costs or firing costs. This pure accounting effect reduces, and can even outweigh, the direct negative impact on welfare. Comparing the welfare in Table 6, column (1) with its counterparts in columns (2) and (3), it appears that the welfare losses due to independent increases in firing and fixed costs add up to 16.2% of the initial welfare. When regulations in both product and

<sup>&</sup>lt;sup>25</sup>Preliminary empirical results provide weak support for this prediction of the model. We regress job turnover statistics taken from the OECD Employment Outlook 1996 on cross-country indexes for both types of regulations (Nicoletti et al., 1999). Considered separately, the EPL and PMR indexes are not significant at all and have a negative coefficient. When both EPL and PMR indicators are included as regressors, the explanatory power of the regression increases. Moreover, the coefficients have the desired negative sign for EPL and positive sign for PMR. Nevertheless, both variables remain non-significant at conventional levels. The stylized nature of the indexes and, most importantly, the fact that they are nearly colinear probably explain the lack of conclusive evidence. Thus, although a preliminary look at the data does not contradict the model's prediction, further empirical research is needed in order to ascertain whether or not the positive relationship between job turnover and PMR can be documented in the data.

Equilibrium Variables						
f = 0.1	F = 0.04	F = 0.04	F = 0.09	F = 0.09		
	C = 0.1	C = 0.15	C = 0.1	C = 0.15		
	(1)	(2)	(3)	(4)		
Unempl. rate $U$ (in %)	7.622	7.873	6.953	7.345		
Vacancies $V$	0.341	0.328	0.317	0.295		
Tightness parameter $\theta$	4.472	4.162	4.552	4.021		
Output $Y$	0.388	0.386	0.366	0.363		
Welfare $\Omega$	0.329	0.329	0.307	0.306		
Productivity $a_i$ (average)	0.895	0.900	0.897	0.905		
Prod. margins $(a^{**}, a^*)$	(0.486, 0.506)	(0.514, 0.502)	(0.482, 0.525)	(0.508, 0.519)		
Average firm size	1.514	1.503	1.563	1.543		
Av. labor hoarded	0.193	0.193	0.317	0.317		
Average wage	0.115	0.112	0.11	0.105		

Table 7: Equilibrium values for different firing and set-up costs.

labor markets are combined, the welfare losses decrease to 15.8%. Hence, the coexistence of the two regulations slightly alleviates their individual costs.

In order to understand better the reason for this complementarity, we have decomposed the welfare changes reported in Table 6 into changes in aggregate vacancy costs, steadystate mobility costs and fixed costs. We find that both types of regulation reduce the cost of vacancy posting although these costs are an order of magnitude smaller than the cost of regulation for the chosen small parameter value of c. More importantly, for the case of the US in Table 6, the steady-state mobility costs decrease by 3.2% if fixed costs increase from 0.1 to 0.15. Less firms bear a higher steady-state mobility costs and the net effect reduces the welfare loss. Similarly, the selection effect implies that, for the case of the US in Table 6, higher firing costs increase the direct welfare losses resulting from fixed and set-up cost payments by 0.5%.<sup>26</sup>

<sup>&</sup>lt;sup>26</sup>For f = C = 0.1, the saved fixed costs of bankrupt firms exactly cancel the additional set-up costs of newly entering firms for given  $a^{**}$ . Since  $a^{**}$  falls slightly, more firms pay the set-up cost so that the welfare

The interactions between EPL and Barriers to Entry. From the point of view of the firm, defaulting can be seen as a way to avoid paying firing costs. Therefore, EPL makes bankruptcy a more attractive option. But for this option to be relevant in equilibrium, setup costs have to be low enough to allow firms to enter the market. Hence low barriers to entry complement stringent EPL.

Table 7 illustrates this complementarity. Reducing entry costs from 0.15 to 0.1 implies a 3.1% decrease of the unemployment rate when firing costs are low, compared with 5.3% when firing costs are high. Moreover, the incidence of bankruptcy is more than two times higher in the economy with stringent EPL (1.8% in column (3) and 0.8% in column (1)). Therefore, the model suggest that lowering the barriers to entry helps neutralizing the negative effect of EPL on job creation.

**The impact of Turbulence.** Many papers have argued that the volatility of the economic environment is substantially higher today than it used to be in the 1960 and 1970s (see Ljungqvist and Sargent, 1998, and their references). Whereas Ljungqvist and Sargent argue that the size of the shock has increased, we augment the frequency of the turbulence by setting  $\delta = 0.7$  and v = 1.4 so that created jobs have 50% chance to persist more than a year whereas destroyed jobs only persist more than a year with 25% probability. These parameter changes leave the steady-state probability mass in the bad state unchanged at  $\delta/(\delta+v) = 1/3$ , but decrease the persistence of each state.

Let us first comment on the general changes before we discuss differences in the changes across columns in Table 8. Not surprisingly, frictional unemployment increases but the duration decreases since more vacancies are posted. Output and welfare decrease because of higher steady-state mobility costs. More interestingly, higher turbulence implies that  $a^{**}$ and  $a^*$  increase, so that only firms with higher permanent productivity continue to produce. Since firms that only operate in the good state produce for a shorter expected duration, loss increases.

Note also that if we rebate firing cost at the aggregate level, output still decreases with higher firing costs since bankrupt firms do not produce in the bad state (the selection effect on  $a^{**}$  and  $a^{*}$ ) and labor hoarding implies lower efficiency for all operating firms. The welfare gain because of lower steady-state vacancy costs is too small to offset these effects.

Equilibrium Variables	US			
Turbulence:	F = 0.04	F = 0.09	F = 0.04	F = 0.09
$\delta=0.7,\upsilon=1.4$	f = 0.1	f = 0.1	f = 0.15	f = 0.15
	(1)	(2)	(3)	(4)
Unempl. rate $U$ (in %)	9.332	8.274	10.36	9.424
Vacancies $V$	0.426	0.381	0.385	0.345
Labor market tightness $\theta$	4.566	4.600	3.713	3.659
Output $Y$	0.365	0.340	0.362	0.339
Welfare $\Omega$	0.310	0.283	0.282	0.258
Productivity (average)	0.906	0.908	0.933	0.934
Prod. margins $(a^{**}, a^*)$	(0.515, 0.516)	(0.508, 0.536)	(0.558, 0.589)	(0.552, 0.610)
Average firm size	1.503	1.526	1.593	1.617
Av. labor hoarded	0.323	0.483	0.334	0.494
Average wage	0.115	0.107	0.105	0.099

Table 8: Equilibrium values for higher turbulence.

 $a^{**}$  increases more than  $a^*$ , and less firms go bankrupt in the bad state. Average firm size slightly decreases because firms in the good state target smaller employment levels  $l_i(\tau = \infty)$ . Finally, more labor is hoarded in the bad state so that average wages are lower.

The effect of turbulence on labor hoarding is very intuitive: if a bad shock is less persistent, firms will find it less attractive to lay off workers even if firing costs are low. Firms hoard labor to avoid labor market frictions whereas firing costs are much less relevant for labor hoarding in an economy with high turbulence: comparing the differences between columns (1) and (2) in Tables 6 and 8, the implied percentage change of labor hoarding with respect to firing costs falls from 64% to 50%. Higher turbulence also increases the welfare loss associated with fixed and firing costs. The total welfare losses of regulation (comparing columns (1) and (4)) increase to 16.8% compared with 15.7% in Table 6. Fixed costs imply less additional labor hoarding (see Table 8, columns (1) and (3) or columns (2) and (4)) so that higher fixed costs now decrease the steady-state mobility costs for the US by 2.6% (compared with 3.2% above). Moreover, higher firing costs increase the welfare losses due to fixed costs by 1.3% (compared with 0.5% for the economy with lower turbulence). Thus, higher turbulence makes regulation less attractive but this cannot be clearly attributed to less complementarity of both types of regulation.

### 7 Conclusions

The model analyzed in this paper extends the framework proposed in BC by considering that, besides idiosyncratic fluctuations in business conditions, firms also differ with respect to their permanent technological productivity. These transitory and permanent differences explain why some firms decide to enter the market while others prefer to remain inactive. Hence, the equilibrium exhibits both firm and job turnover. We have shown that the intensive margin (hiring-firing) and the extensive margin (entry-exit) can be characterized applying rational expectations and steady state requirements.

The distinction between the extensive and intensive margin has allowed us to generate some novel results compared with the literature, especially models based on the "onefirm-one-worker" assumption. Most importantly, the model illustrates how the interactions between labor and product market regulation crucially depend on the link between both margins of adjustment. We find that firing costs are most important for the hiring and firing margin whereas fixed and set-up cost matter more for the entry and exit margin. However, both policies also matter for the respective other margin.

Fixed and set-up costs interact negatively by reducing both the incentives to enter the market and the capacity of potential entrants to overcome existing entry barriers. High firing costs and low set-up costs complement each other because the bankruptcy option is a profitable alternative to paying the firing costs for firms with low permanent productivity. This motivates why EPL is not applied to small firms in countries with strict employment protection legislation. Finally, fixed and firing costs are slightly complementary. More importantly, we find that they have countervailing effects on job turnover. This prediction of the model provides a potential explanation for why empirical studies using cross-country flow data have failed to document the strong negative relationship between EPL and job

flows predicted by the theory. Given that both regulations are strongly positively correlated, if PMR stimulates job reallocation, the negative impact of EPL needs not be evident in cross-country data.

Our framework lends itself naturally to study many interesting issues. In ongoing research we investigate the implications of our model on the wage and employment distributions across firms with different size. From a theoretical perspective it is worth analyzing optimal regulation if the social planner can condition this regulation on firm size. Finally, we have taken the market power of firms as given in our analysis. In other terms, we have interpreted the decreasing marginal revenue schedule of firms as technological and not as reflecting market power. Further research could extend the model to endogenize market power by making it an explicit function of the number of operating firms. This would certainly introduce additional channels of interaction between both policies.

### Appendices

### Appendix A: Solution of the model

### Workers.

Given that workers are homogenous, the asset value of employment solely depends on the firm characteristics. Workers receive a utility flow b if unemployed and an endogenous wage  $w_i^{g,n}(\tau)$  if employed in a good firm or  $w_i^{b,1}$  if employed in a bad firm.

The asset value of the representative unemployed worker in steady state is

$$rW^{u} = b + \xi \theta^{1+\gamma} (\overline{W}^{e} - W^{u}), \tag{A1}$$

where the Poisson hazard of finding a job is  $q(\theta)V/U = \xi \theta^{1+\gamma}$ ; and  $\overline{W}^e$  is the expected asset value of being matched to one of the posted vacancies. This expected value depends on the realized distribution of posted vacancies. Note that  $W^u$  by definition is independent of the type of firm *i*. The value of being in a bad job is

$$rW^{b,1} = w_i^b + \upsilon(W_i^{g,1}(0) - W^b)$$
(A2)

where n has been set to one since only "labor-hoarding" firms employ workers in the bad state. The asset value of employment in a good firm which has been  $\tau$  periods in the good state is

$$rW_i^{g,n}(\tau) = w_i^{g,n}(\tau) + \delta(W^u - W_i^{g,n}(\tau)) + \dot{W}_i^{g,n}(\tau) , \qquad (A3)$$

where  $\delta$  is the exogenous Poisson hazard of a bad shock. As shown in BC, p. 441-442, nonenforceability of long-term contracts implies that the asset value of a worker in a firm with low productivity  $\varepsilon_b$  is equal to the outside option  $W^u$  (firms can credibly threaten workers to fire them otherwise). Thus,  $W^b$  is also independent of firm-specific productivity  $a_i$  since unemployed workers and workers in firms with temporarily low productivity have the same expected discounted utility. Equation (A2) implies that the wage in the bad state  $w_i^b$  will absorb differences in  $W_i^{g,1}(0)$ .

#### Wage determination.

Wages are determined by Nash bargaining between the worker and the firm. Nonenforceability of contracts implies that all workers in a given firm earn the same wage. However, wages between firms differ as long as workers have some bargaining power,  $\beta > 0$ . Wages differ for firms in the good state depending on the time they have spend in the good state and the number of workers they have hired in this time. As is standard the Nash bargain implies that

$$\beta(S_i^{g,n}(.) - S^o) = (1 - \beta)(W_i^{g,n}(\tau) - W^u) , \qquad (A4)$$

where the shadow value of posting a vacancy  $S^{o}$  is zero (the shadow value of hiring a worker equals the flow cost of posting the vacancy discounted by the probability that the vacancy is matched to a worker).

Plugging the shadow value of hiring a worker (6) into the optimality condition of the Nash bargain (A4), we get

$$W_i^{g,n}(\tau) = W^u + \frac{\beta}{1-\beta} \frac{cv_i^n(\tau)}{q(\theta)}$$
(A5)

and thus

$$\dot{W}_i^{g,n}(\tau) = \frac{\beta}{1-\beta} \frac{c \dot{v}_i^n(\tau)}{q(\theta)}$$

where dots denote time derivatives. The outside option of workers does not change as firms experience good times, but the number of posted vacancies does. Inserting these two expressions into (A3), we get

$$w_i^{g,n}(\tau) = rW^u + \frac{\beta c}{1-\beta} \frac{(r+\delta)v_i^n(\tau) - \dot{v}_i^n(\tau)}{q(\theta)} .$$
(A6)

Wages of "bankrupt" firms. Reinserting the explicit expression for the shadow value of a hired worker (7) into (A6) yields

$$w_i^{g,0}(\tau) = rW^u + \frac{\beta}{1-\beta} \left( \eta_i^{g,0} - \sigma l_i^0(\tau) - \omega_i^{g,0}(\tau) \right) \; .$$

Making explicit the dependence of wages on employment  $w_i^{g,0} = g^0(l_i^0(\tau))$ . Since  $\omega_i^{g,n}(\tau) = g^n(l_i^n(\tau)) + g_l^n(l_i^n(\tau)) l_i^n(\tau)$ , we find that the following condition must hold

$$g^{0}(l_{i}^{0}(\tau)) = rW^{u} + \frac{\beta}{1-\beta} \left( \eta_{i}^{g,0} - \sigma l_{i}^{0}(\tau) - g^{0}(l_{i}^{0}(\tau)) - g_{l}^{0}(l_{i}^{0}(\tau)) l_{i}^{0}(\tau) \right)$$

This first-order differential equation has the linear solution

$$w_i^{g,0} = (1-\beta) \, r W^u + \beta \eta_i^{g,0} - \frac{\beta \sigma}{1+\beta} l_i^0(\tau) \,. \tag{A7}$$

Wages of "labor-hoarding" firms. Solving the explicit expression for the shadow value of a hired worker (9) and plugging this into (A6) results in

$$w_i^{g,1}(\tau) = rW^u + \frac{\beta}{1-\beta} \left( \eta_i^{g,1} - \sigma l_i^1(\tau) - \omega_i^{g,1}(\tau) - \delta F \right) \; .$$

Solving for  $w_i^{g,1}$  in terms of  $l_i^1(\tau)$  as before finally yields

$$w_i^{g,1}(\tau) = (1-\beta) \, r W^u + \beta \left( \eta_i^{g,1} - \delta F \right) - \frac{\beta \sigma}{1+\beta} l_i^1(\tau) \;. \tag{A8}$$

Wages in good firms are a weighted average of the workers outside option and the firm's surplus net of expected firing costs.

### Optimal labor demand schedules.

By definition

$$\omega_i^{g,n}(\tau) = w_i^{g,n}(\tau) - \frac{\beta\sigma}{1+\beta} l_i^n(\tau)$$

Note the incentive of firms to reduce the surplus appropriated by workers by increasing employment. This incentive is stronger the larger is  $\beta$  and  $\sigma$ .

Reinserting this expression into (7) and (9), differentiating with respect to time, we get (notice that  $\theta$  does not change in the steady state)

$$-\dot{w}_i^{g,n}(\tau) - \frac{\sigma}{1+\beta}\dot{l}_i^n(\tau) + \frac{c\ddot{v}_i^n(\tau)}{q(\theta)} = (r+\delta)\frac{c\dot{v}_i^n(\tau)}{q(\theta)}$$

Differentiating equation (A6) with respect to time we have

$$\dot{w}_i^{g,n}(\tau) = rW^u + \frac{\beta c}{1-\beta} \frac{(r+\delta)\dot{v}_i^n(\tau) - \ddot{v}_i^n(\tau)}{q(\theta)}$$

Using the two equations to substitute out  $\dot{w}_i^{g,n}(\tau)$ ,

$$-\frac{\beta c}{1-\beta}\frac{(r+\delta)\dot{v}_i^n(\tau)-\ddot{v}_i^n(\tau)}{q(\theta)}+\frac{c\ddot{v}_i^n(\tau)}{q(\theta)}-(r+\delta)\frac{c\dot{v}_i^n(\tau)}{q(\theta)}-\frac{\sigma}{1+\beta}\dot{l}^n(\tau)=0.$$

Using (5) and rearranging, results in

$$\ddot{v}_i^n(\tau) - (r+\delta)\dot{v}_i^n(\tau) - \sigma \frac{1-\beta}{1+\beta} \frac{\xi^2 \theta^{2\gamma}}{c} v_i = 0$$

The solution of this second-order differential equation that satisfies  $\lim_{\tau\to\infty} v_i^n(\tau) = 0$  is

$$v_i^n(\tau) = v_i^n(0)e^{-\lambda\tau} \text{ with } \lambda = 1/2 \left( -(r+\delta) + \sqrt{(r+\delta)^2 + 4\sigma \frac{1-\beta}{1+\beta} \frac{\xi^2 \theta^{2\gamma}}{c}} \right) .$$
(A9)

Permanent differences between firms,  $a_i$ , matter only for the absolute number of posted vacancies but not for the behavior of the vacancy policy over time ( $\lambda$  would depend on *i* if we allowed  $\sigma$  to differ across firms). The rate of convergence is also independent of firm entry and exit (n = 0, 1).

Given the exogenous destruction rate  $\delta$ , open vacancies are distributed exponentially over  $\tau$  with parameter  $\delta + \lambda$  independently of the value of  $a_i$ . Equation (A5) then implies that the expected gain from finding a job in a good firm is

$$\overline{W}^{e} - W^{u} = \frac{\beta}{1-\beta} \frac{c}{q(\theta)} (\delta+\lambda) \left( \frac{1}{1-U(a^{**}\wedge a^{*}))} \right) \begin{bmatrix} \int_{a^{**}\wedge a^{*}}^{a^{*}} \left( \int_{0}^{\infty} v_{i}^{0}(\tau) e^{-(\delta+\lambda)\tau} d\tau \right) u(a) da \\ + \int_{a^{*}}^{\overline{a}} \left( \int_{0}^{\infty} v_{i}^{1}(\tau) e^{-(\delta+\lambda)\tau} d\tau \right) u(a) da \\ = \frac{\beta}{1-\beta} \frac{c}{q(\theta)} \frac{\delta+\lambda}{\delta+2\lambda} \left( \frac{\int_{a^{**}\wedge a^{*}}^{a^{*}} v_{i}^{0}(0)u(a) da + \int_{a^{*}}^{\overline{a}} v_{i}^{1}(0)u(a) da}{1-U(a^{**}\wedge a^{*}))} \right)$$

where  $a^{**} \wedge a^* \equiv \min\{a^{**}, a^*\}$ , u(a) and U(a) respectively denotes the PDF and CDF of a. The second equality follows from  $v_i^n(\tau) = v_i^n(0)e^{-\lambda\tau}$  and the fact that  $a_i$  and  $\tau$  are independently distributed. Notice that the distribution is normalized by the actual mass of operating firms  $1 - U(a^{**} \wedge a^*)$  since the expected asset value is conditioned on the formation of a match.

Moreover, equation (A1) implies that

$$rW^{u} = b + c\theta \frac{\beta}{1-\beta} \frac{\delta+\lambda}{\delta+2\lambda} \left( \frac{\int_{a^{**}\wedge a^{*}}^{a^{*}} v_{i}^{0}(0)u(a)da + \int_{a^{*}}^{\overline{a}} v_{i}^{1}(0)u(a)da}{1-U\left(a^{**}\wedge a^{*}\right)\right)} \right)$$

and equation (A2) implies

$$\begin{aligned} rW^{b,1} &= w_i^b + v(W_i^{g,1}(0) - W^{b,1}) \\ &= w_i^b + v \frac{\beta}{1 - \beta} \frac{cv_i^1(0)}{q(\theta)} \end{aligned}$$

where the second equality follows from equation (A5). In equilibrium  $W^{u} = W^{b}$  and thus

$$w_{i}^{b} = b + \frac{\beta}{1-\beta} c \left( \theta \frac{\delta + \lambda}{\delta + 2\lambda} \left( \frac{\int_{a^{**} \wedge a^{*}}^{a^{*}} v_{i}^{0}(0)u(a)da + \int_{a^{*}}^{\overline{a}} v_{i}^{1}(0)u(a)da}{1 - U\left(a^{**} \wedge a^{*}\right)} \right) - \frac{v}{\xi \theta^{\gamma}} v_{i}^{1}(0) \right) .$$
(A10)

The wage in the bad state depends positively on the total number of posted vacancies which increase the outside option; but negatively on the expected number of vacancies posted in the own firm i if good times arrive. Workers are willing to take larger wage cuts in bad times if this is compensated in good times. Equations (A6) and (A9) imply

$$w_i^{g,n}(\tau) = rW^u + \frac{\beta c}{1-\beta} \frac{(r+\delta+\lambda)v_i^n(0)e^{-\lambda\tau}}{q(\theta)}$$
(A11)

Plugging in  $W^u$ , we get

$$w_i^{g,n}(\tau) = b + \frac{\beta}{1-\beta} c \left( \theta \frac{\delta + \lambda}{\delta + 2\lambda} \left( \frac{\int_{a^{**} \wedge a^*}^{a^*} v_i^0(0)u(a)da + \int_{a^*}^{\overline{a}} v_i^1(0)u(a)da}{1 - U\left(a^{**} \wedge a^*\right)} \right) + \frac{(r+\delta+\lambda)e^{-\lambda\tau}}{\xi\theta^{\gamma}} v_i^n(0) \right)$$
(A12)

Note that the wage in good times depends positively on  $v_i^n(0)$ . As  $\tau \to \infty$  all workers earn the same wage as firms exploit their monopsony power and hire until

$$w^{g,n}(\infty) \equiv \lim_{\tau \to \infty} w_i^{g,n} = b + \frac{\beta}{1-\beta} c\theta \frac{\delta+\lambda}{\delta+2\lambda} \left( \frac{\int_{a^{**} \wedge a^*}^{a^*} v_i^0(0)u(a)da + \int_{a^*}^{\overline{a}} v_i^1(0)u(a)da}{1 - U\left(a^{**} \wedge a^*\right)\right)} \right).$$

To sum up: workers in firms with high permanent productivity  $a_i > a^{**}$  earn lower wages in bad times, higher wages upon arrival of good times and the same wage as  $\tau \to \infty$ .

### Employment and boundary conditions for $v_i^n(0)$ and $l_i^n(0)$ .

The employment and vacancy schedules are fully characterized by the initial conditions  $v_i^n(0)$  and  $l_i^n(0)$  since  $v_i^n(\tau) = v_i^n(0)e^{-\lambda\tau}$  and

$$l_i^n(\tau) = l_i^n(0) + \frac{q(\theta)}{\lambda} (1 - e^{-\lambda\tau}) v_i^n(0)$$

Initial vacancy posting of "bankrupt" firms. Since  $l_i^0(0) = 0$ , we only need one boundary condition to characterize the optimal labor demand schedule. Technically speaking, there is no need to ensure that workers are indifferent between employment and unemployment. The value of  $v_i^0(0)$  can be determined noticing that (A7) and (A11) must be equal, so that

$$\frac{c}{1-\beta}\frac{(r+\delta+\lambda)}{q(\theta)}v_i^0(0) = \eta_i^{g,0} - b - rW^u \tag{A13}$$

Initial vacancy posting of "labor hoarding" firms. For these firms,  $l_i(0)$  differs from zero so we need to determine two boundary conditions. Equation (8) together with the result that workers are indifferent between employment and unemployment in the bad state and  $\omega_i^{b,1}(0) = w_i^{b,1}$  implies

$$vv_i^1(0) = \frac{q(\theta)}{c} \left( \sigma l_i(0) - \left( \eta_i^{b,1} + (v+r)F - w_i^{b,1} \right) \right)$$
(A14)

As before, the second boundary condition follows from equating (A8) and (A12) using (A10):

$$(1-\beta)rW^u + \beta\left(\eta_i^{g,1} - \delta F\right) - \frac{\beta\sigma}{1+\beta}l_i^1(0) = w_i^b + \frac{\beta c}{1-\beta}\frac{v + (r+\delta+\lambda)}{q(\theta)}v_i^1(0)$$

Since  $W^u = W^b$ ,

$$\frac{c}{1-\beta}\frac{\beta v + r + \delta + \lambda}{q(\theta)}v_i^1(0) = -\frac{\sigma}{1+\beta}l_i^1(0) + \left(\eta_i^{g,1} - \delta F - w_i^{b,1}\right) .$$
(A15)

Inserting,  $w_i^b$  from (A10), the two boundary conditions can be used to solve for  $v_i^1(0)$  and  $l_i^1(0)$  (for given  $a^*$ ), and average vacancies and employment (integrating over  $a \in [a^{**} \wedge a^*; \overline{a}]$ ). This completes the characterization of firm *i*'s optimal policies. It remains to close the model by determining the aggregate stock of vacancies V and employment L and thus  $\theta$ .

#### Equilibrium.

In steady state the number of firms turning good has to equal the number of firms turning bad (for each  $a_i$ ). Thus,

$$v\phi_b = \delta\phi_g$$

and

$$\phi_b + \phi_q = 1$$

so that

$$\phi_b = \frac{\delta}{\delta + \upsilon}$$
 and  $\phi_g = \frac{\upsilon}{\delta + \upsilon}$ .

Given that the density of  $\tau$  is exponentially distributed, we get

$$V = \frac{v\delta}{\delta + v} \int_{a^{**} \wedge a^*}^{\overline{a}} \int_0^\infty v_i^n(\tau) e^{-\delta\tau} d\tau u(a) da = \frac{v}{\delta + v} \frac{\delta}{\delta + \lambda} \int_{a^{**} \wedge a^*}^{\overline{a}} v_i^n(0) u(a) da$$

and

$$L = \frac{\upsilon\delta}{\delta + \upsilon} \int_{a^{**} \wedge a^*}^{\overline{a}} \int_0^\infty l_i^n(\tau) e^{-\delta\tau} d\tau dU(a) + \frac{\delta}{\delta + \upsilon} \int_{a^*}^{\overline{a}} l_i^1(0) u(a) da$$

Plugging in  $l_i^n(\tau)$  we get

$$L = \frac{vq(\theta)}{(\delta+v)(\delta+\lambda)} \int_{a^{**}\wedge a^*}^{\overline{a}} v_i^n(0)u(a)da + \int_{a^*}^{\overline{a}} l_i^1(0)u(a)da$$

where the aggregate employment level depends negatively on  $a^{**}$  and  $a^*$ . We now mention how output and welfare are computed in the model.

### Output and Welfare.

Each firm has a "production-equivalent" flow

$$y_i^{g,n} \equiv \eta_i^g l_i^n(\tau) - \frac{\sigma}{2} l_i^n(\tau)^2 - \frac{c}{2} v_i^n(\tau)^2 - \delta F(l_i(\tau) - l_i(0))n$$

Firms in the good state bear a steady-state mobility cost  $\delta F(l_i(\tau) - l_i(0))$  if n = 1, and costs of vacancy posting  $cv(\tau)^2/2$  (below we add the fixed cost f which all firms have to pay). Instead each firm in the bad state has a "production-equivalent" flow

$$y_i^{b,n} \equiv n \left( \eta_i^b l_i^n(0) - \frac{\sigma}{2} l_i^n(0)^2 \right) \; .$$

Thus, gross output is defined as

$$Y = \frac{\upsilon}{\upsilon + \delta} \begin{bmatrix} \int_{a^{**} \wedge a^{*}}^{a^{*}} \left( \int_{0}^{\infty} \delta e^{-\delta \tau} \left( \eta_{i}^{g} l_{i}^{0}(\tau) - \frac{\sigma}{2} l_{i}^{0}(\tau)^{2} - \frac{c}{2} \upsilon_{i}^{0}(\tau)^{2} \right) d\tau \right) u(a) d(a) + \\ \int_{a^{*}}^{\overline{a}} \left( \int_{0}^{\infty} \delta e^{-\delta \tau} \left( (\eta_{i}^{g} - \delta F) l_{i}^{1}(\tau) - \frac{\sigma}{2} l_{i}^{1}(\tau)^{2} - \frac{c}{2} \upsilon_{i}^{1}(\tau)^{2} \right) d\tau \right) u(a) d(a) + \\ + \frac{\delta}{\upsilon + \delta} \int_{a^{*}}^{\overline{a}} \left( \left( \eta_{i}^{b} + \upsilon F \right) l_{i}^{1}(0) - \frac{\sigma}{2} l_{i}^{1}(0)^{2} \right) u(a) d(a) ,$$

up to a constant of integration that can be neglected if profits are zero for firms that do not use labor. We compute welfare  $\Omega$  adding the production-equivalent flow b for all unemployed workers and subtracting f for all firms in the market, as well as the set-up costs incurred by the firms which enter the market, so that

Plugging in the expression for  $l_i^n(\tau)$  and  $v_i^n(\tau)$ , the first integral in Y reads

$$\int_0^\infty \delta e^{-\delta\tau} \left( \eta_i^g l_i^0(\tau) - \frac{\sigma}{2} l_i^0(\tau)^2 - \frac{c}{2} v_i^0(\tau)^2 \right) d\tau$$
$$= \frac{q(\theta)}{\delta + \lambda} \eta_i^g v_i^0(0) - \frac{\sigma q(\theta)^2}{(\lambda + \delta) (2\lambda + \delta)} v_i^0(0)^2 - \frac{\delta}{\delta + 2\lambda} \frac{c}{2} v_i^0(0)^2.$$

Integrating this expression over  $a \in [a^{**} \wedge a^*; a^*]$  allows us to compute the first term of Y, whereas

$$\int_0^\infty \delta e^{-\delta\tau} \left( \left(\eta_i^g - \delta F\right) l_i^1(\tau) - \frac{\sigma}{2} l_i^1(\tau)^2 - \frac{c}{2} v_i^1(\tau)^2 \right) d\tau$$
  
=  $\left(\eta_i^g - \delta F\right) l_i^1(0) + \frac{q(\theta)}{\delta + \lambda} \left(\eta_i^g - \delta F\right) v_i^1(0) - \frac{\sigma}{2} l_i^1(0)^2$   
 $- \frac{\sigma q(\theta)^2}{(\lambda + \delta) (2\lambda + \delta)} v_i^1(0)^2 - \frac{\sigma}{\delta + \lambda} q(\theta) l_i^1(0) v_i^1(0) - \frac{\delta}{\delta + 2\lambda} \frac{c}{2} v_i^1(0)^2.$ 

which can be integrated over  $a \in [a^*; \overline{a}]$  to compute the second term of Y.

#### Appendix B: Social-planner problem

The social-planner problem without bankruptcy. The social planner maximizes  $\Omega$  subject to the additional constraints to internalize the congestion externalities

$$\Omega^{+} + \mu \left( V - \frac{1}{\overline{a} - \underline{a}} \int_{a^{+}}^{\overline{a}} \frac{\upsilon}{\delta + \upsilon} \frac{\delta}{\delta + \lambda} v_{i}^{+}(0) da \right) + \zeta \left( (1 - U) - \frac{1}{\overline{a} - \underline{a}} \int_{a^{+}}^{\overline{a}} \underbrace{\left\{ \frac{\upsilon\delta}{\delta + \upsilon} \int_{0}^{\infty} l_{i}(\tau) e^{-\delta\tau} d\tau da + \frac{\delta}{\delta + \upsilon} l_{i}(0) \right\}}_{L} da \right)$$

where the shadow prices of the two additional constraints are  $\zeta$  and  $\mu$ , and the superscript  $a^+$  is the cut-off of the permanent productivity which needs to be determined. As in BC, we can rewrite  $\Omega^+$ , incorporating the constraints and using that  $a_i$  is uniformly distributed on the interval  $[0; \overline{a}]$  so that

$$\begin{split} \widetilde{\Omega}^{+} &= \frac{1}{\overline{a}} \frac{\upsilon}{\upsilon + \delta} \int_{a^{+}}^{\overline{a}} \left[ \begin{array}{c} \left( \int_{0}^{\infty} \delta e^{-\delta \tau} \left( \left( \eta_{i}^{g} - \delta F - \zeta - b \right) l_{i}(\tau) - \frac{\sigma}{2} l_{i}(\tau)^{2} - \frac{c}{2} \upsilon_{i}(\tau)^{2} - \mu \upsilon_{i}(\tau) \right) d\tau \right) \\ &+ \frac{\delta}{\upsilon} \left( \left( \eta_{i}^{b} + \upsilon F - \zeta - b \right) l_{i}(0) - \frac{\sigma}{2} l_{i}(0)^{2} \right) \\ &+ b - \frac{\overline{a} - a^{+}}{\overline{a}} f \; . \end{split} \right] da$$

exploiting that bU = b - bL.

The constancy of the shadow prices,  $\zeta$  and  $\mu$ , across  $a_i$  implies that the social planner is solving a series of independent optimization problems for each  $a_i$ . We associate the Hamiltonian shadow prices  $\kappa_i$  (·) to the dynamic constraints

$$\dot{l}_i( au) = q( heta) v_i( au)$$
 .

The socially optimal  $l_i^+(\tau)$  and  $v_i^+(\tau)$  schedules satisfy the following Hamiltonian conditions

$$\kappa_i(\tau) = \frac{cv_i^+(\tau) + \mu}{q(\theta)} ,$$
  
$$\delta\kappa_i(\tau) = \frac{d\kappa_i(\tau)}{d\tau} + (\eta_i^g - \delta F - \zeta - b) - \sigma l_i^+(\tau) .$$

Combining the two conditions yields

$$\delta \frac{cv_i^+(\tau) + \mu}{q(\theta)} = \frac{c(v_i^+)(\tau)}{q(\theta)} + (\eta_i^g - \delta F - \zeta - b) - \sigma l_i^+(\tau) ,$$

which can be again differentiated to obtain

$$(v_i^+)(\tau) - \delta(v_i^+)(\tau) - \frac{\sigma}{c}q(\theta)^2 v_i^+(\tau) = 0 .$$

The solution of this second-order differential equation that satisfies  $\lim_{\tau\to\infty} v_i^+(\tau) = 0$  is

$$v_i^+(\tau) = v_i^+(0)e^{-\lambda^+\tau} \text{ with } \lambda^+ = 1/2\left(-\delta + \sqrt{\delta^2 + 4\sigma\frac{\xi^2\theta^{2\gamma}}{c}}\right) .$$
(A16)

As in BC, compared with equation (A9) for the decentralized equilibrium, the social planner seeks to maximize the total surplus and not the revenues net of wages. Hence, the bargaining parameter does no longer determine the convergence rate of vacancy posting and firms converge more quickly to their targeted employment rate. Notice that the rate of convergence is identical across firms.

Equation (A16) implies that

$$l_i^+(\tau) = l_i^+(0) + q(\theta) \int_0^\tau v_i^+(0) e^{-\lambda^+ \tau} d\tau = l_i^+(0) + q(\theta) v_i^+(0) \left(\frac{1 - e^{-\lambda^+ \tau}}{\lambda^+}\right) \,.$$

To determine  $v_i^+(0)$  we evaluate the first-order condition of the Hamiltonian derived above at  $\tau = 0$ . That is

$$\delta \frac{cv_i^+(0) + \mu}{q(\theta)} = \frac{c(v_i^+)(0)}{q(\theta)} + (\eta_i^g - \delta F - \zeta - b) - \sigma l_i^+(0)$$

which, given that

$$\frac{d}{d\tau}\frac{cv_i^+(\tau)}{q(\theta)} = -\lambda^+ \frac{ce^{-\lambda^+\tau}v_i^+(0)}{q(\theta)} ,$$

can be rewritten for  $\tau = 0$  as

$$\mu = \frac{q(\theta)}{\delta} \left[ \eta_i^g - \delta F - \zeta - b - \sigma l_i^+(0) - (\delta + \lambda^+) \frac{cv_i^+(0)}{q(\theta)} \right] .$$

The second boundary condition is given by the firing condition for  $\tau = 0$  (keeping r = 0)

$$\eta_i^b - \sigma l_i^+(0) - b - \zeta + \upsilon \left(\frac{cv_i^+(0)}{q(\theta)} - (-F)\right) = 0$$

which can be rearranged to

$$\zeta = \eta_i^b + vF - b - \sigma l_i^+(0) + v \frac{cv_i^+(0)}{q(\theta)} \,.$$

Plugging this into the expression for  $\mu$  above and solving for  $v_i^+(0)$ , we find that

$$v_i^+(0) = \frac{q(\theta)}{c(\upsilon+\delta+\lambda^+)} \left[ \eta_i^g - \eta_i^b - (\upsilon+\delta)F - \delta \frac{\mu}{q(\theta)} \right] .$$

Solving the second boundary condition for  $l_i^+(0)$  and plugging in the solution for  $v_i^+(0)$ , we find that

$$l_i^+(0) = \frac{1}{\sigma} \left[ \eta_i^b + \frac{\upsilon \lambda^+}{\upsilon + \delta + \lambda^+} F - b - \zeta + \frac{\upsilon}{\upsilon + \delta + \lambda^+} \left( \eta_i^g - \eta_i^b - \delta \frac{\mu}{q(\theta)} \right) \right] .$$

### The social-planner problem with bankruptcy.

We assume that the social planner cannot enforce payment of F if firms find it optimal to default. Furthermore, the social planner cannot set different firing taxes for firms with different  $a_i$ . Having these caveats in mind, we characterize the social planner problem as

$$\Omega^{++} + \mu \left( V - \frac{1}{\overline{a} - \underline{a}} \frac{\upsilon}{\upsilon + \delta} \left( \int_{a^+}^{\overline{a}} \frac{\delta}{\delta + \lambda} v_i^+(0) da + \int_{a^{++} \wedge a^+}^{a^+} \frac{\delta}{\delta + \lambda} v_i^{++}(0) da \right) \right) + \zeta \left( (1 - U) - \frac{1}{\overline{a} - \underline{a}} \int_{a^+}^{\overline{a}} \underbrace{\left\{ \frac{\upsilon \delta}{\delta + \upsilon} \int_0^{\infty} l_i^+(\tau) e^{-\delta \tau} d\tau da + \frac{\delta}{\delta + \upsilon} l_i^+(0) \right\}}_L da \right) da$$

where firms default if  $a_i \in [a^{++}; a^+]$ .

Again, we can rewrite  $\Omega^{++}$  incorporating the constraints as

$$\begin{split} \widetilde{\Omega}^{++} &= \frac{1}{\overline{a}} \frac{\upsilon}{\upsilon + \delta} \left[ \begin{array}{c} \int_{a^{++} \wedge a^{+}}^{a^{+}} \left( \int_{0}^{\infty} \delta e^{-\delta \tau} \left( (\eta_{i}^{g} - b) \, l_{i}^{++}(\tau) - \frac{\sigma}{2} l_{i}^{++}(\tau)^{2} - \frac{c}{2} \upsilon_{i}^{++}(\tau)^{2} - \mu \upsilon_{i}^{++}(\tau) \right) d\tau \right) d(a) + \\ \int_{a^{+}}^{\overline{a}} \left( \int_{0}^{\infty} \delta e^{-\delta \tau} \left( (\eta_{i}^{g} - \delta F - \zeta - b) \, l_{i}^{+}(\tau) - \frac{\sigma}{2} l_{i}^{+}(\tau)^{2} - \frac{c}{2} \upsilon^{+}(\tau)^{2} - \mu \upsilon_{i}^{+}(\tau) \right) d\tau \right) d(a) \\ &+ \frac{1}{\overline{a}} \frac{\delta}{\upsilon + \delta} \int_{a^{+}}^{\overline{a}} \left( \left( \eta_{i}^{b} + \upsilon F - \zeta - b \right) \, l_{i}^{+}(0) - \frac{\sigma}{2} l_{i}^{+}(0)^{2} \right) d(a) \\ &+ b - \frac{a^{+} - (a^{++} \wedge a^{+})}{\overline{a}} \frac{\upsilon}{\upsilon + \delta} (f + \delta C) - \frac{\overline{a} - a^{+}}{\overline{a}} f \; . \end{split}$$

As before the socially optimal  $l_i^+(\tau)$  and  $v_i^+(\tau)$  schedules satisfy the following Hamiltonian conditions

$$\kappa_i(\tau) = \frac{cv_i^+(\tau) + \mu}{q(\theta)}$$
$$\delta\kappa_i(\tau) = \frac{d\kappa_i(\tau)}{d\tau} + (\eta_i^g - \delta F - \zeta - b) - \sigma l_i^+(\tau) ,$$

which result in the same vacancy dynamics as before for  $v_i^+(\tau)$ . Similarly, the socially optimal  $l_i^{++}(\tau)$  and  $v_i^{++}(\tau)$  schedules satisfy

$$\kappa_i(\tau) = \frac{cv_i^{++}(\tau) + \mu}{q(\theta)}$$
$$\delta\kappa_i(\tau) = \frac{d\kappa_i(\tau)}{d\tau} + (\eta_i^g - b) - \sigma l_i^{++}(\tau)$$

Combining the two conditions yields

$$\delta \frac{cv_i^{++}(\tau) + \mu}{q(\theta)} = \frac{c(v_i^{++})(\tau)}{q(\theta)} + (\eta_i^g - b) - \sigma l_i^{++}(\tau)$$

which can be again differentiated to obtain

$$\delta \frac{c(v_i^{++})(\tau)}{q(\theta)} = \frac{c(v_i^{++})(\tau)}{q(\theta)} - \sigma q(\theta) v_i^{++}(\tau) .$$

The solution is the same as above (for  $\lim_{\tau\to\infty} v_i^{++}(\tau) = 0$ )

$$v_i^{++}(\tau) = v_i^{++}(0)e^{-\lambda^+\tau}$$

Thus, the vacancy dynamics are the same for firms that declare bankruptcy and firms that hoard labor in the bad state. Evaluating the Hamiltonian condition at  $\tau = 0$  we find

$$\mu = \frac{q(\theta)}{\delta} \left[ \eta_i^g - b - \sigma l_i^{++}(0) - (\delta + \lambda^+) \frac{cv_i^{++}(0)}{q(\theta)} \right] .$$

Using that  $l_i^{++}(0) = 0$  and solving the expression for  $v_i^+(0)$ , we get

$$v_i^{++}(0) = \frac{q(\theta)}{c(\delta + \lambda^+)} \left(\eta_i^g - b - \frac{\delta}{q(\theta)}\mu\right)$$

Thus,  $v_i^{++}(0) > v_i^+(0)$  if

$$\frac{q(\theta)}{c(\delta+\lambda^+)} \left(\eta_i^g - b - \frac{\delta}{q(\theta)}\mu\right) > \frac{q(\theta)}{c(\upsilon+\delta+\lambda^+)} \left[\eta_i^g - \eta_i^b - (\upsilon+\delta)F - \delta\frac{\mu}{q(\theta)}\right] ,$$

a sufficient condition for which is (setting v = 0 in the denominator on the right-hand side)

$$\eta_i^b - (\upsilon + \delta)F > b \; .$$

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