# Divorce, Remarriage and Child Support\*

# Pierre-Andre Chiappori<sup>†</sup>and Yoram Weiss<sup>‡</sup> March 2005

#### Abstract

Modern marriage markets display increasing turnover, with less marriage but more divorce and remarriage. As a consequence, a large number of children live in single parent and step parent households. There is substantial evidence that children of divorced parents do not perform as well as comparable children in intact families. However, there is also some evidence that this gap declines with the aggregate divorce rate. We develop a model in which the higher expectations for remarriage associated with higher divorce rates can trigger an equilibrium in which divorced fathers make more generous transfers that benefit the children and mother in the aftermath of divorce. As a result, the welfare loss of children from the separation of their parents can be lower when divorce and remarriage rates rise.

## 1 Introduction

The last century has been characterized by changes in family structure, including a reduction in marriage and fertility and increased marital turnover. Divorce has been rising throughout the century and more men and women are now divorced and unmarried. Interestingly, however, the rise in divorce rates is associated with an increase in remarriage rates (relative to first marriage rates); divorce and remarriage rates are substantially higher among recent cohorts (see Figure B1 to B5 in Appendix

<sup>\*</sup>Paper presented at the Conference on Labor and Social Interaction in Economics, tel Aviv, December 2004. An earlier version was presented at seminars in Chicago, London, Paris and Saint Louis. We thank the participants and Gary Becker, Leonardo Felli, James Heckman, Casey Mulligan, Kristina Orfali, Bob Pollak, Barak Weiss and Michael Whinston for their comments, and the editor and two anonymous referees for their suggestions on an earlier draft. Support by a NSF grant (award # SBR9729559) is gratefully aknowledged.

<sup>&</sup>lt;sup>†</sup>University of Chicago and Columbia University, pc2167@columbia.edu

<sup>&</sup>lt;sup>‡</sup>Tel Aviv University, weiss@post.tau.ac.il

B). About a quarter of men and women aged 40 to 49 in 1966 report that they have been married twice or more (see Table B1 in Appendix B); moreover, the remarriage rate among the young is similar to first marriage rate and exceeds the divorce rate, suggesting that, despite the large turnover, marriage is the "natural" state.

One consequence of higher turnover is the large number of children who live in single parent and step parent households. In the US, 2002, 69 percent of children less than 18 years old lived with two parents (including step parents), 23 percent lived only with their mother and 5 percent lived only with their father (the rest lived in households with neither parent present). There is substantial evidence that children of divorced parents do not perform as well as comparable children in intact families.<sup>2</sup> However, the impact of the aggregate divorce rate on transfers and the welfare of children is more difficult to assess. Picketty (2003) shows for instance that the increase in the divorce rate in France have reduced the gap in school performance between children of divorced parents and children from intact families, a result which suggests that comparing the welfare of children in high and low divorce environments is a difficult task.<sup>3</sup>

In this paper, we propose a theoretical discussion of these issues. Specifically, we investigate the conceptual links between the divorce rate and the welfare of children under various living arrangements. We explore how the level of divorce transfers and expenditures on children determine and are determined by the divorce rate in the population, giving rise to social interactions of the type described by Scheinkman and Glaeser (2003). In particular, the higher expectations for remarriage associated with higher divorce rates can trigger an equilibrium in which divorced fathers make more generous transfers that benefit the children and mother in the aftermath of divorce. As a result, the welfare loss of children from the separation of their parents can be *lower* when divorce and remarriage rates rise.

**Divorce and children** The production of children is a major reason for marriage, but an important aspect of the investment in children is the ex-post differences that are created between otherwise identical men and women. A basic reason for such differences is biological in nature. The mother is the one who gives birth and she

<sup>&</sup>lt;sup>1</sup>The especially quick rise in divorce during the seventies in many different countries was probably triggered by the oral contraceptive pill, and can be regarded as exogenous to a certain extent (see Michael, 1988, and Goldin-Katz, 2002).

<sup>&</sup>lt;sup>2</sup>The literature on this topic is large and covers psychological and economic outcomes. See, for instance, Argys et al.(1998), Lamb et al.(1999), Hetherington and Stanley-Hagan (1999), Gruber (2004) and Lerman (2002).

<sup>&</sup>lt;sup>3</sup>Piketty shows that the school completion rates of children of divorced parents are lower than those of children in intact families. These gaps are similar whether the divorced parents remain single or remarry. However, the gaps in school completion declines as the proportion in the population of children who live in intact families declines over time and across regions. Piketty brings further evidence that shows that gaps are created in school completion even before the marriage breaks, suggesting that bad marriages (that end in divorce) also harm the children. A similar finding is reported by Bjorklund and Sundstrom (2002).

is more capable of taking care of the children at least initially. This basic difference may have large economic consequences. If the couple produces children, the mother typically reduces her work in the market and, as a consequence, her future earning capacity is reduced. Thus, wage differences between men and women are created endogenously as a consequence of having children. The ex-post asymmetry between parents has strong implications for the divorce decision, the options for remarriage and the incentive to produce children (see Becker et al, 1997, and Becker, 1991. Ch.2). In this paper, we simplify the problem substantially by assuming that fertility is exogenous and all couples choose to have children even in the absence of any transfers. We further assume that the mother always has full custody of the children in the event of separation. We focus our attention on the agency problems that arise in caring for children and their relation to the aggregate conditions in the marriage market.

Children are a collective good for their natural parents and both care about their welfare. This remains true whether the parents are married or separated. However, marital status can affect the expenditures on children, and the welfare of parents and children. Separation may entail an inefficient level of expenditures on children for several reasons. If the custodial parent remains single, not only does she lose the gains from joint consumption, but she may also determine child expenditures without regard to the interest of the ex-spouse. If she remarries, the presence of a new spouse who cares less about step children reduces the incentives to spend on children from previous marriages. Finally, parents that live apart from their children can contribute less time and goods to their children and may derive less satisfaction from them. These problems are amplified if the partners differ in income and cannot share custody to overcome the indivisibility of children. The custodial parent is usually the mother who has some comparative advantage in caring for children but has lower income. The father has often limited access to the children and lower incentive to provide for them. The outcome is that the level of child expenditures following separation is generally below the level that would be attained in an intact family, reducing the welfare of the children and possibly of their parents.

To mitigate these problems, the partners have an incentive to sign binding contracts that will determine some transfers between the spouses. The purpose of the transfers is to induce an efficient level of child expenditures following divorce. We focus here on contingent contracts, in which the father commits to pay the mother some payment if and only if she remains single. Such a commitment has two effects. First, and quite obviously, it may induce higher expenditures on the children if the mother remains single, provided that the income elasticity of child expenditures is positive. Secondly, it increases the mother's bargaining power if she remarries, which again entails higher children expenditures in this case. We show, in particular, that the incentive for such commitments is stronger when the prospects of remarriage are higher, because then the father is less likely to pay but more likely to benefit from his commitment. In addition, the incentive of each father to make such commitment

depends on the commitments made by other father to their ex-wives, because such commitments also influence the bargaining outcome upon remarriage.

The model To explore these general issues we use a very stylized model in which the gains from marriage depend on economic considerations such as sharing consumption goods and on non monetary benefits such as companionship and love. Marriage is an "experience good" and the quality of match is discovered after some lag. Negative surprises about the quality of the match trigger divorce. However the probability of separation conditioned on a bad realization depends on the prospects of remarriage, the post divorce transfers made by the couple and also on the transfers made by potential mates to their ex-wives. In the absence of adequate transfers, remarriage may have a negative effect on the children because the new husband of the custodial mother may be less interested in the child's welfare. We may refer to this problem as the "Cinderella effect". This effect reduces the incentive of the non custodial father to support the children, because part of the transfer is "eaten" by the new husband.

We analyze post divorce transfers that are signed "in the shadow of the law" (see Mnookin and Kornhauser, 1979). In particular, we assume that some child support payments are mandatory; to sharpen our point, we assume that the mandatory level is large enough to allow the single mother to sustain the same level of children expenditure as under marriage. However, the non custodial father may still augment the transfer if he wishes to influence the expenditures of the custodial mother on the children. Payments made to the custodial mother are fungible and the amount that actually reaches the children depends on whether the mother is single or remarries and on the commitments of prospective mates for remarriage to their-ex-wives. Thus, the commitment that a particular father wishes to make to his ex-wife upon separation depends on the commitments made by others and the prospects of remarriage. The model determines an equilibrium level of transfers and an equilibrium divorce and remarriage rates that are tied to each other. We identify two equilibria. Namely, there exists a low divorce (remarriage) equilibrium in which all fathers transfer nothing to their ex-wives, above the minimal support mandated by law. In this equilibrium, the level of child expenditures falls short of the amount spent in an intact family. However, a high divorce (remarriage) equilibrium also exists, in which all fathers commit to transfer to their ex-wives a substantial amount if they remain single. This amount is sufficient to make the mother indifferent between remarriage and remaining single, so that the influence of the new husband on child expenditures is reduced and the level of child expenditures upon remarriage is the same as it would be if the parents did not separate.

Diamond and Maskin (1979) were the first to examine contracting and commitments in general equilibrium matching models. However, they did not discuss issues connected with children. The presence of children means that parents continue to be connected even if the marriage relationship breaks. This special but important feature is absent from the usual matching models between employers and workers, in which partnerships break without a trace. A more closely related paper is Aiyagari, Greenwood and Guner (2000) who construct and simulate a model of the marriage market which includes individual shocks, divorce, remarriage and child support payments among other things. They show that, at their chosen parameters, an increase in mandated child support raises welfare. Our model is substantially simpler than theirs, allowing us to discuss more explicitly the circumstances under which such an outcome is likely to occur. However, we achieve this added transparency at a substantial cost. In our model, individuals are assumed to be ex-ante identical so that all issues of assortative mating are set aside, and there is no role for ex ante redistribution. Similarly, there are no unexpected changes in earnings that can trigger divorce and create ex-post heterogeneity and we do not discuss wealth accumulation and the intergenerational implications of marriage and divorce.<sup>4</sup>

# 2 The basic ingredients

#### 2.1 Incomes

All men are assumed to be identical and have a fixed income, y. Similarly, all women are identical and assumed to have the same fixed income z. However, women earn less than men (z < y). The basic reason for this asymmetry is the presence of children, which, by assumption, requires that the mother who gives birth to the children and spends time caring for them foregoes some of her earning capacity. Otherwise, we assume that labor supply is fixed and that incomes do not vary over time.

#### 2.2 Preferences

A family spends its income on two goods an adult good a and a child good c. The adult good a is a public good for all members of the same household and the child good c is private to the children.

The (aggregate) utility of the children is

$$u_c = g(c), (1)$$

where g(c) is increasing and strictly concave with g'(0) > 1.

Children are viewed as public good for their natural parents even if the children and parents live apart, with a correction for proximity by a discount factor,  $\delta$ , that captures the idea that "far from sight is far from heart". In addition, each married couple derives utility from companionship that we denote by  $\theta$ . The quality of match,  $\theta$ , is an independent draw from a given symmetric distribution with a non negative mean,  $\bar{\theta}$ .

<sup>&</sup>lt;sup>4</sup>Recent papers that touch on these issues are Burdett and Coles (1997 and 1999), Coles, Mailath and Postlewaite (1998), Burdett and Wright (1998) and Ishida (2003).

The utility of a single parent j is

$$u_j = a_j + u_c, \tag{2a}$$

if the parent and children live together and

$$u_i = a_i + \delta u_c, \tag{2b}$$

if the parent and children live apart, where j = m indicates the mother and j = f indicates the father. Similarly, the utility of a married parent i is

$$u_i = a_i + u_c + \theta_i, \tag{3a}$$

if the parent and children live together and

$$u_i = a_i + \delta u_c + \theta_i, \tag{3b}$$

if the parent and children live apart.

Adult consumption and the quality of match are viewed as household public goods. Any two married individuals who live in the same household share the same value of a and  $\theta$ . Thus, parents who live together in an intact family have the same value of a and  $\theta$  and enjoy equally the utility from their children  $u_c$ . However, if the parents divorce and live apart in different households, they will have different value of a and  $\theta$ , and the custodial parent who lives with the children will have a higher utility from the child.<sup>5</sup>

## 2.3 Matching

There are equal numbers of males and females in each cohort. To keep things simple, we assume that, after separation, each partner can remarry only with a divorced person from the same cohort, provided that a "suitable match" who also wants to remarry is found. However, the search process involves frictions and remarriage is neither immediate nor certain. Consequently, following divorce, agents may fail to meet an eligible new mate and hence remain single. A key ingredient of the model is that the probability of this event decreases with the average divorce rate in the population: remarriage is easier, the larger the number of singles around.<sup>6</sup>

There are several reasons why such increasing returns should be present in our context. One is that although the two sexes meet in a variety of occasions (work,

<sup>&</sup>lt;sup>5</sup>It is easy to generalize the model to allow the child to be affected (linearly) by the amount of the adult good consumed by the parents and by the quality of the match,  $\theta$ .

<sup>&</sup>lt;sup>6</sup>This contrasts with most search models of the labor market that assume constant returns, whereby the probability of meeting would depend on the *ratio* of single individuals of each sex. Matching models with increasing returns have been analyzed by Diamond and Maskin (1979) and Diamond (1982).

sport, social life, etc.),<sup>7</sup> many of these meetings are "wasted", in the sense that one of the individuals is already attached and not willing to divorce. Obviously, non wasted meeting are more frequent when the proportion of divorcees in the population is larger. Another reason is that the establishment of more focused channels, where singles meet only singles, is costly and they will be created only if the "size of the market" is large enough. Thirdly, as noted by Mortensen (1988), the search intensity of the unattached decrease with the proportion of attached people in the population. The reason for that is that attached individuals are less likely to respond to an offer, which lowers the return for search. Empirical support for increasing returns is given by the geographic patterns of matching, which show that the degree of assortative mating into a given group tends to rise with the relative size of the group within the total population.<sup>8</sup> There is also a tendency of singles, of either sex, to congregate in large cities, especially if they have special marital needs.<sup>9</sup>

We do not fully specify the matching process and summarize it by a reduced form  $matching\ function,\ m=\phi(d)$ , where d is the common proportion of divorced men and women, and m is the probability that divorcees of opposite sex meet. In general, one expects each divorcee can meet with several potential mates - in which case  $\phi(d)$  typically exceeds d. However, because the parties initiate contacts independently, it is also possible that some singles will be contacted by no one while others are contacted by more than one person - which might cause  $\phi(d)$  to be smaller than d. In this paper, we assume that  $\phi(d) > d$ . The probability of remarriage is denoted by p, where  $p \leq m$ , with p = m only if each meeting with a potential mate ends up in remarriage.

## 2.4 Timing

Agents live two periods. In the beginning of each period, they can marry if they find a match. We assume that in the first period each agent finds a match with probability one. All matches end up in marriage, because individuals are identical and the expected gains from marriage are positive.

We think of marriage as a binding commitment to stay together for one period,

<sup>&</sup>lt;sup>7</sup>Lauman et al. (1994, Table 6.1) report that about half of the marriages arise from meeting in school, work, and private party and only 12 percent originate in specialized channels such as social clubs or bars.

<sup>&</sup>lt;sup>8</sup>For instance, Bisin et al. (2001, Figure 2) report that individuals of a given religion are more likely to marry within their group than one would predict by the share of each religious group in the population, which suggests positive assortative marriage. They also report that the *difference* between within group marriage rates and population shares *rises* with the share of the religious group in the population, which suggests increasing returns. That is, a Jew, who presumably wants to marry a Jew, is more likely to do so if there are many Jews around.

<sup>&</sup>lt;sup>9</sup>Costa and Kahn (2000) bring evidence that shows that singles, especially with high schooling, are more likely to reside in large metropolitan areas. Black et al.(2000) and Lauman et al. (1994, Table 8.1) report that gays are more likely to live in large cities.

with no search "on the job". The quality of the match  $\theta$  is revealed with a lag at the end of each period, after having experienced the marriage. When the partners observe the common value of their match quality,  $\theta$ , each partner chooses whether to continue the marriage or walk away and seek an alternative match.

All married couples produce the same fixed number of children at the beginning of the first period, at some cost, and receive the benefits from the children in the subsequent period. If the parents separate, the mother obtains the custody over the children and the father may make transfers to his ex-wife to induce her to maintain the welfare of the children, about whom he continues to care.

If two divorced man and woman meet at the beginning of the second period, they can choose whether to remarry. Hence, following separation, the parents can be in four different states, depending on the new marital status of the ex- spouses:

- Both parents are single, which we denote by ss.
- The father remains single while the mother is remarried, which we denote by sr.
- The mother is single but the father is remarried, which we denote by rs.
- Both parents are remarried, which we denote by rr.

## 2.5 Legal framework

We assume that the mother is always the custodial parent<sup>11</sup> and discuss two types of payment: a child support payment s that the wife receives if a separation occurs, independently of the subsequent marital status of the parents and an alimony payment  $\sigma$  that may depend on the mother's subsequent marital status. The child support payment s is determined by law, while the additional payment  $\sigma$  is freely contracted upon by the parties<sup>12</sup>; however, it has to be non negative, so that it cannot undo the children support payment mandated by law. We further assume that the payments that the mother receives cannot be earmarked and hence can be freely reallocated in the new household which she forms.

The voluntary payment  $\sigma$  can be determined either ex-post (i.e., after the ex spouses' marital status has been determined), interim (i.e., after divorce but before remarriage) or ex-ante (i.e., at the time of marriage). The-ex ante and interim agreements can be made contingent on future events. Moreover, they generally are binding

<sup>&</sup>lt;sup>10</sup>Relaxing this assumption would not significantly affect the qualitative conclusions of the model, provided that the increasing return property is maintained.

<sup>&</sup>lt;sup>11</sup>Although other custody arrangements are possible, this is still the prevalent arrangement. Mother custody can be justified by the economic comparative advantage of women in child care.

<sup>&</sup>lt;sup>12</sup>While the amount of child support determined by law could (and should in principle) depend on the new marital status of both parents, in practice this feature is not observed.

contracts that require legal enforcement, which may raise renegotiation proofness issues. Ex- post payments, however, are voluntary and self-enforcing.

The focus of this paper is on transfers that are determined at the interim stage following divorce, which is the most common form of transfers. We shall analyze the case in which, ex post, fathers have no incentive to make voluntary transfers to their ex-wives, yet, interim, they may voluntarily commit on contingent payments to the custodial mother that depend on whether she remarries or remains single. In principle, such binding contracts should also depend on the marital status of the father. However, observed divorce settlements are rarely contingent on the husband's marital situation, although they may depend on his income. As we have already simplified by assuming that incomes are constant, we shall further assume that  $\sigma$  is contingent only on the marital status of the mother.<sup>13</sup>

A common legal practice is to set children support at a level that would guarantee a standard of living similar to that obtained under marriage. In the framework presented here, this idea is captured by a payment to the custodial wife that is large enough to restore the same level of children expenditures as under marriage. However, because children support is fungible and children expenditures (especially time spent with the child) are not easily verifiable, it remains to determine what will be actually spent on the children.

### 3 The allocation of household resources

We begin by describing the allocation of household income between the adult and child goods under different household structures.

## 3.1 Intact family

If the parents remain married, they maximize their common utility

$$\max_{a,c} \ a + g(c) + \theta \tag{4}$$

s.t.

$$a+c=y+z,$$

implying that

$$g'(c) = 1. (5)$$

<sup>&</sup>lt;sup>13</sup>The limited scope of ex-ante marriage contracts and interim divorce contracts in modern societies is puzzling, especially in the light of the presence of such contracts in traditional societies. The rarity of ex-ante contracts can probably ascribed to a larger reliance, relative to the past, on emotional enforcement of commitments, and the presumption that too much contracting can "kill love". It is less clear why interim contracts, signed at the time of divorce, are not fully contingent.

We denote by  $c^*$  be the unique solution to (5) and assume that

$$z < c^* < y + z, \tag{6}$$

which means that the income of the mother, z, is not sufficient to support the optimal level of child expenditures, while the pooled income of the two parents y + z is large enough to support the children and still leave some income for adult consumption.

## 3.2 Mother remains single

In this case, the mother solves

$$\max_{a,c} \ a + g(c) \tag{7}$$

s.t.

$$a+c=z+s+\sigma$$
.

where  $\sigma$  denote the transfer that the mother receives from the father if she remains single in addition to the compulsory payment, s.

Given the quasi linear structure of preferences, the choice between adult consumption and children goods follows a very simple rule:

- if  $z + s + \sigma \le c^*$  then a = 0 and  $c = z + s + \sigma$
- if  $z + s + \sigma > c^*$  then  $a = z + s + \sigma c^*$  and  $c = c^*$

That is, the mother spends all her income,  $z + s + \sigma$ , on the children if her income is lower than the children's "needs", as represented by  $c^*$ . If her total income exceeds  $c^*$  then the mother will spend  $c^*$  on the children and the rest on herself.

In particular, a regulation imposing a minimum level of child support payment of  $c^* - z$  or more guarantees that, no matter what voluntary transfers may be, child expenditures in single parent (here mother) households will be as high as in intact households.

#### 3.3 Mother remarries

If the custodial mother remarries, the problem becomes more complicated because of the involvement of a *new* agent, namely the new husband of the mother. The new husband receives little or no benefits from spending on the child good. To sharpen our results, we assume that the new husband derives no utility at all from the step children, which means that the child good is a *private* good for the wife in the new household. It follows that if  $c > c^*$ , both partners agree that the marginal dollar should be spent on the adult good. If, however,  $c < c^*$ , then an increase in the amount spent on the child good raises the utility of the mother, because she values this expenditure more than the forgone adult good, while the benefit for the new

husband is nil. In this range, there is a *conflict* between the mother and her new husband.

One can distinguish two different mechanisms that determine the expenditures on children in newly formed households, depending upon whether binding commitments on child expenditures can be made prior to remarriage. Without any commitment, the custodial mother will decide how much to spend on the children, taking as given the amount she receives from her former husband and the amount that her new husband gives to his former wife. We make the alternative assumption that the matched partners can bargain prior to remarriage on the division of the gains from remarriage and reach some binding agreement (or an 'understanding') that will determine the expenditure on children.

We use a symmetric Nash-Bargaining solution to determine the bargaining outcome. The Nash axioms imply that the bargaining outcome must maximize the product of the gains from remarriage, relative to remaining single, of the two partners. The gain of the remarried mother depends on the transfers that she *expects* to receive from her ex-husband, when remarried or single. Similarly, the gain from marriage of the new husband depend on the *expected* payments that he is going to pay his ex-wife, when married or single. At the time of meeting between the two separated individuals, neither of them knows what the marital status of their ex-spouses will be. Because agents are assumed to be risk neutral, we can use the expected payments in calculating the gains from remarriage.

An important simplifying assumption of the model is that transfers made by the father do not depend on his own marital status. With this assumption we only need to keep track of whether or not the mother is remarried. We shall denote the payments by a given father to his ex-wife by  $\sigma$  and payment made by other men by  $\sigma^{-.14}$  With this notation, the voluntary payments of the new husband to his ex-wife, is  $s + \sigma_s^-$  if his ex-wife remains single and  $s + \sigma_r^-$  if she remarries. The realized value of the transfer is not known at the time of the bargaining and we shall denote its expected value by  $\sigma_e^- = (1-p)\sigma_s^- + p\sigma_r^-$ , where p is the probability of remarriage. We denote by  $y_e^-$  the expected net income that the new husband brings into the marriage, that is  $y_e^- = y - s - \sigma_e^-$ .

Since the new husband cares only about the adult good that he receives in the new household and because, by assumption, his payments to the ex-wife and thus the utility of his children are independent of his marital status, his gain from marriage depend only on the additional adult good and the value of companionship that he expects and given by

$$z + s + \sigma_r - c + \bar{\theta}. \tag{8}$$

The utility gain of the mother upon remarriage consists of the additional adult consumption and the change in her utility from child expenditures. For  $s \ge c^* - z$ ,

<sup>&</sup>lt;sup>14</sup>It is possible that identical agents will select different commitments to their identical ex-wives. However, because we are looking for symmetric equilibria, there is no loss of generality in assuming that all other fathers pay the same amounts to their ex-wives.

these amount to

$$\gamma(c) + y_e^- + \sigma_r - \sigma_s + \bar{\theta},\tag{9}$$

where

$$\gamma(c) \equiv g(c) - c - (g(c^*) - c^*). \tag{10}$$

Note that  $\gamma(c)$  is non positive and concave with a maximum at  $c^*$ , where  $\gamma(c^*) = \gamma'(c^*) = 0$ .

The Nash bargaining solution can be written in the form

$$\gamma'(c) = \frac{\gamma(c) + y_e^- + \sigma_r - \sigma_s + \bar{\theta}}{z + s + \sigma_r - c + \bar{\theta}},\tag{11}$$

where,  $\gamma'(c)$  is the slope of the Pareto frontier (in absolute value) and  $\frac{\gamma(c)+y_e^-+\sigma_r-\sigma_s+\bar{\theta}}{z+s+\sigma_r-c+\bar{\theta}}$  is the ratio of the utility gains of the two partners. Let  $\hat{c}$  be the solution to (11) then, because remarriage occurs only if both partners have a non negative gain from marriage,  $\gamma'(\hat{c}) \geq 0$  and  $\hat{c} \leq c^*$ . That is, the step family generally spends less on child goods, which we may call the "Cinderella effect".

From (11), we obtain that

$$\frac{\partial \hat{c}}{\partial \sigma_{s}} = -\frac{\partial \hat{c}}{\partial y_{e}^{-}} = \frac{-1}{\left(z + s + \sigma_{r} - \hat{c} + \bar{\theta}\right) \gamma''(\hat{c}) - 2\gamma'(\hat{c})} > 0,$$

$$\frac{\partial \hat{c}}{\partial z} = \frac{\partial \hat{c}}{\partial s} = \frac{\gamma'(c)}{\left(z + s + \sigma_{r} - \hat{c} + \bar{\theta}\right) \gamma''(\hat{c}) - 2\gamma'(\hat{c})} > 0,$$

$$\frac{\partial \hat{c}}{\partial \sigma_{r}} = \frac{\partial \hat{c}}{\partial \bar{\theta}} = \frac{-1}{\left(z + s + \sigma_{r} - \hat{c} + \bar{\theta}\right) \gamma''(\hat{c}) - 2\gamma'(\hat{c})} (\gamma'(\hat{c}) - 1).$$
(12)

Clearly,  $\partial \hat{c}/\partial y_e^- < 0$  and  $\partial \hat{c}/\partial z = \partial \hat{c}/\partial s > 0$ , reflecting the impact of each member's resources when single on his\her bargaining strength. In particular,  $\partial \hat{c}/\partial \sigma_s > 0$  implies that an increase in the payment to the wife as single,  $\sigma_s$ , is always beneficial to the children if the mother remarries, because it does not change the total resources of the new household but increases her bargaining power, hence allowing her to control a larger fraction of these resources. The payment given to her directly when she is remarried,  $\sigma_r$ , has an exactly opposite effect: it raises total resources of the new household but not the mother's threat point. Since any money given to the remarried mother in the form of fungible funds is partially "eaten" by the new husband,  $\sigma_r$  entails an implicit tax on the father, with ambiguous effects on the children's welfare. An increase in  $\sigma_r$  might actually decrease child expenditures if the marginal utility derived by the parents from the children's consumption is not too large.<sup>15</sup>

Note also that the average quality of the match,  $\bar{\theta}$ , has exactly the same impact as a monetary transfer  $\sigma_r$ ; it also benefits to both members of the new household,

Then by concavity  $g'(\hat{c}) \leq 2$  and  $\gamma'(\hat{c}) \leq 1$ , implying  $\partial \hat{c}/\partial \sigma_r \leq 0$ .

with no impact on the reservation utilities. Finally, an important feature of the Nash bargaining solution is that the amount of child expenditures in a remarried couple depends only on the difference  $y_e^- - \sigma_s$ . Thus, if the mother remarries a new husband with less commitment and, therefore, a higher bargaining power that induces lower child expenditures, the father can fully offset this effect by raising the payment that the mother receives as single.

# 4 Equilibrium: characterization

## 4.1 Optimal Ex-Post Transfers

We begin our analysis with a case in which all transfers are voluntary. Such transfers are motivated by the father's continued interest in his children and he may give money to the custodial mother, aiming to influence her child expenditures. At the last stage of the game, when the marital status of *all* agents have been determined, each father may consider what is the transfer that he wishes to make, unilaterally. If prior commitment on child expenditures have been made, the father cannot influence the utility of the children, as all subsequent payments will be spent on the adult good. If, however, no commitments have been made then the father can influence the expenditure on the children.

If both parents are single, the father will voluntarily augment the mother's total income up to some minimal level  $c_{\delta}$ , given by

$$\delta g'(c_{\delta}) = 1. \tag{13}$$

That is,

$$\sigma = \begin{cases} c_{\delta} - (z+s) & if \quad z+s \le c_{\delta} \\ 0 & if \quad z+s > c_{\delta} \end{cases}$$
 (14)

In words: if the mother is relatively poor and  $z + s < c_{\delta}$ , the father will transfer whatever amount is needed to guarantee that child expenditures reach the optimal level (from his perspective), i.e.  $c_{\delta}$ . Clearly,  $c_{\delta} \leq c^*$  if  $\delta < 1$ , implying a reduction in the children's welfare relative to continued marriage.

If the father or mother are remarried the father's incentive to transfer to the custodial mother is further reduced, because of the involvement of the new husband of the mother and the new wife of the father.

Given these weakened incentives, there is a role for legal intervention, in the form of setting minimal children support standards and enforcing binding contracts. A common consideration in determining the size of the compulsory child support payments is the "accustomed standard of living" of the children, should the marriage continue. For this reason, we shall assume that s is set at the level which can support the level of child expenditures  $c^*$ , that is,  $s = c^* - z$ . Such a policy clearly "crowds out" all voluntary ex-post payments. In addition, it guarantees that the children

receive exactly  $c^*$  if the mother remains single. However, it is generally insufficient to guarantee that the children receive  $c^*$  if the mother remarries.

## 4.2 Optimal interim Contracts

Our previous analysis reveals an interesting dilemma; if the custodial mother remains single, the father is unwilling to give her any transfer beyond the minimum set by law. However, if she remarries, he would like her to have more money as single, because this would boost her bargaining power vis a vis her new spouse, hence benefit the children at no cost for the father.

This situation calls for voluntary binding contract, whereby the father commits to pay a certain amount to his ex-wife if she remains single. Our focus in this paper is on such payments. In order to simplify our analysis of such contingent contracts we shall assume that the father is unwilling to give the custodial mother any additional payment if she remarries, given that the mother is already protected by a compulsory payment  $s = z - c^*$ . As mentioned above, a sufficient (but not necessary) condition for such an outcome is  $g'(0) \leq 2$ . Indeed, it is then the case that  $\partial \hat{c}/\partial \sigma_r < 0$ ; clearly, the father will refrain from promising a transfer in the contingency that the wife remarries, if such a transfer in fact reduces the expenditure on the children.

However, the father may still want to commit *interim* to a payment conditioned on the mother being single, because such a commitment will influence her bargaining position (hence child expenditures) if she remarries. We can then omit the subscript and refer to  $\sigma$  as an *alimony* payment that the mother receives if she remains single and is stopped if she remarries. We define the expected net incomes of the father and new husband of the mother as as  $y_e = y - s - (1 - p)\sigma$  and  $y_e^- = y - s - (1 - p)\sigma_e^-$ , respectively.

Setting  $s = z - c^*$  and letting  $x = \sigma - y_e^-$ , condition (12) can be rewritten as

$$\gamma'(c) = \frac{\gamma(c) - x + \bar{\theta}}{c^* - c + \bar{\theta}},\tag{15}$$

or equivalently as

$$\gamma'(c)(c^* - c + \bar{\theta}) - \gamma(c) = -x + \bar{\theta}. \tag{15'}$$

We denote by  $\hat{c} = h(x; \bar{\theta})$ , the unique solution of (15) for  $\hat{c}$  as a function of x, for a given parameter  $\bar{\theta} \geq 0$ , where  $0 \leq h(x; \bar{\theta}) \leq c^*$ . The level of child expenditure upon

$$f(c) = \gamma'(c)(c^* - c + \bar{\theta}) - \gamma(c).$$

Because of the assumed properties of  $\gamma(c)$ , f'(c) < 0. Therefore, if a solution of (15') exists in the region  $0 \le c \le c^*$  then it must be unique. Because  $f(c^*) = 0$ , the solution for  $\hat{c}$  is positive if

$$\gamma'(0)(c^* + \bar{\theta}) - \gamma(0) > \bar{\theta} - x$$

and equals zero otherwise. Sufficient conditions for the solution to be positive are g'(0) = 2 and

 $<sup>\</sup>overline{^{16}}$ Let

remarriage cannot exceeds  $c^*$  because the remarried mother and the new husband have a common interest not to exceed this level of expenditure. The function  $h(x; \bar{\theta})$  is defined only for values of x in the range  $-(y-s) \leq x \leq \bar{\theta}$ . The lower bound is a consequence of the requirement that  $\sigma$  and  $\sigma^-$  cannot be negative and the upper bound arises from the requirement that both partners must have non negative gains from marriage, otherwise a remarriage would not occur. From (15) and the properties of  $\gamma(c)$ , we see that if  $x = \bar{\theta}$  then  $h(x; \bar{\theta}) = c^*$ , because  $\gamma(c^*) = \gamma'(c^*) = 0$ . At this point, the mother is just indifferent between remarriage and remaining single, while the new husband has a positive gain if  $\bar{\theta} > 0$  and is indifferent towards remarriage if  $\bar{\theta} = 0$ .

When  $0 < h(x; \bar{\theta}) < c^*$ ,

$$h'(x; \bar{\theta}) = \frac{1}{2\gamma'(\hat{c}) - (c^* - \hat{c} + \bar{\theta})\gamma''(\hat{c})} > 0,$$

$$h''(x; \bar{\theta}) = [h'(x; \bar{\theta})]^3 ((c^* - \hat{c} + \bar{\theta})\gamma'''(\hat{c}) - 3\gamma''(\hat{c})).$$
(16)

An interesting feature of the Nash bargaining solution is that the bargaining outcome  $h(x; \bar{\theta})$  can be convex in x. As seen in (16), a sufficient condition for that is that the marginal utility from children expenditures g'(x) is convex, implying  $\gamma''(c) = g''(\hat{c}) < 0$  and  $\gamma'''(c) = g'''(\hat{c}) \geq 0$ . That is, the mother gains less from a marginal increase in c at higher levels of child expenditures while the marginal cost for her new husband (in terms of adult good) remains the same. The remarried mother is, therefore, more willing to give up her children's consumption and the bargaining outcome becomes more responsive (in terms of the child good) to transfers from the father or the new husband. This situation is illustrated in Figure 1.

Recalling that  $x = \sigma - y_e^- = \sigma + (1 - p)\sigma^- + s - y$ , we see that the convexity or concavity of  $h(x; \bar{\theta})$  creates strategic interactions among different agents, in the sense that the marginal impact of the commitment made by the father to his exwife,  $\sigma$ , is affected by the commitments made by others,  $\sigma^-$ . These interactions have different consequences at different marital states. If  $h(x; \bar{\theta})$  is convex (concave) then, if the mother remarries, a larger  $\sigma^-$  will increase (decrease) the marginal impact of  $\sigma$ . However, if the father remarries, a higher commitment by others raises the bargaining power of the new wife and the marginal cost of the commitment made by the father will be higher (lower). Thus, to fully describe the strategic interactions we need to look at the effects of commitments on the expected utilities of the fathers.

The expected utility of a particular father upon separation consists of several parts and can be written as  $E(u_f) = \delta E(u_c) + E(a_f) + p\bar{\theta}$ . Where,

$$E(a_f) = y_e + p(c^* - h(\sigma^- - y_e; \bar{\theta}))$$
(17)

 $g(c^*) > y + z - c^*$ . That is, the utility from child expenditures must be sufficiently large relative to the utility from the adult good.

<sup>&</sup>lt;sup>17</sup>Note that  $h(x; \bar{\theta})$  is usually not differentiable at  $x = \bar{\theta}$ , because behavior changes if the boundary is crossed and the mother prefers to stay single, but the left derivative exists.

is the father's expected adult consumption,

$$E(u_c) = pg(h(\sigma - y_e^-; \bar{\theta})) + (1 - p)g(c^*)$$
(18)

is the expected utility of the children upon separation, and  $\bar{\theta}$  is the father's expected value of companionship upon remarriage.

Taking the derivative of  $E(u_f)$  with respect to  $\sigma$ , holding  $\sigma^-$  constant, we obtain

$$\frac{\partial E(u_f)}{\partial \sigma} = -(1-p) - p(1-p)h'(\sigma^- - y_e; \bar{\theta}) 
+ p\delta g'(h(\sigma - y_e^-; \bar{\theta}))h'(\sigma - y_e^-; \bar{\theta}).$$
(19)

The father pays  $\sigma$  only if the mother remains single, which occurs with probability (1-p). At the margin, this commitment would cost him 1 dollar if he remains single and  $h'(\sigma^- - y_e; \bar{\theta}) + 1$  dollars if he remarries, where the added term  $h'(\sigma^- - y_e; \bar{\theta})$  represents the additional expenditures on the children of the *new* wife, resulting from the decline in the father's bargaining power when he increases the commitment to his ex-wife. The father gets benefits from  $\sigma$  only if the mother remarries, which occurs with probability p. In this case, the payment raises the mother expenditures on the children because her bargaining power is stronger. The increase in child expenditures is  $h'(\sigma - y_e^-; \bar{\theta})$  and the father gain from this increase is  $\delta g'(h(\sigma - y_e^-; \bar{\theta}))h'(\sigma - y_e^-; \bar{\theta})$ .

The expression in (19) is valid only in the range in which the commitments are consistent with remarriage of the mother and the new wife of the father, that is

$$\sigma \leq \bar{\theta} + y_e^- = \bar{\theta} + y - s - \sigma^- (1 - p), 
\sigma^- \leq \bar{\theta} + y_e = \bar{\theta} + y - s - \sigma (1 - p).$$
(20)

We shall refer to condition (20) as the incentive compatibility constraint. In addition, the commitments must be feasible and satisfy

$$0 \leq \sigma \leq y - s, 
0 \leq \sigma^{-} \leq y - s.$$
(21)

An interior optimal solution for  $\sigma$  given  $\sigma^-$  must satisfy these two constraints and the necessary conditions for individual optimum  $\frac{\partial E(u_f)}{\partial \sigma} = 0$  and  $\frac{\partial^2 E(u_f)}{\partial \sigma^2} < 0.^{18}$  However, as we shall show shortly, *corner solutions* in which agents select either  $\sigma = 0$  or the maximal level permitted by constraints (20) and (21) will play an important role in the analysis.

$$\frac{\partial^{2} E(u_{f})}{\partial \sigma^{2}} = p \delta g''(h(\sigma - y_{e}^{-}; \bar{\theta}))[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2} + p \delta g'(h(\sigma - y_{e}^{-}; \bar{\theta})h''(\sigma - y_{e}^{-}; \bar{\theta}))^{2} + p \delta g'(h(\sigma - y_{e}^{-}; \bar{\theta})h''(\sigma - y_{e}^{-}; \bar{\theta}))^{2}$$
$$-p(1-p)^{2}h''(\sigma^{-} - y_{e}; \bar{\theta}).$$

<sup>&</sup>lt;sup>18</sup>The second order derivative is

A salient feature of the model is that the probability of remarriage, p, has a systematic influence on the willingness of each father to commit. The reason is quite simple. The father commits to pay only if the mother remains single and gets the benefits only if she remarries. Thus, if p is low he is more likely to pay and less likely to benefit. Conversely, if p is high, the father is less likely to pay and more likely to benefit.

We can now see how the commitments of others affect the expected utility of each father and his incentives to commit. These interactions between agents are summarized by

$$\frac{\partial E(u_f)}{\partial \sigma^-} = -ph'(\sigma^- - y_e; \bar{\theta}) 
+ p(1-p)\delta g'(h(\sigma - y_e^-; \bar{\theta}))h'(\sigma - y_e^-; \bar{\theta}),$$
(22)

and

$$\frac{\partial^2 E(u_f)}{\partial \sigma \partial \sigma^-} = p(1-p) [\delta g''(h(\sigma - y_e^-; \bar{\theta})) [h'(\sigma - y_e^-; \bar{\theta})]^2 
+ \delta g'(h(\sigma - y_e^-; \bar{\theta})h''(\sigma - y_e^-; \bar{\theta}) - h''(\sigma^- - y_e; \bar{\theta})].$$
(23)

We see in (22) that an increase in  $\sigma^-$  reduces the gains of the father if he remarries by  $h'(\sigma^- - y_e; \bar{\theta})$ , because his new wife will have a higher bargaining power. On the other hand, if the mother remarries she will have a higher bargaining power if her prospective new husband has higher commitments to his ex-wife, which raises the utility of the father by  $(1 - p)\delta g'(h(\sigma - y_e^-; \bar{\theta}))h'(\sigma - y_e^-; \bar{\theta})$ . Generally, it is not clear which of these two effects is stronger. However, it is seen from (22) and (23) that both  $\frac{\partial E(u_f)}{\partial \sigma^-}$  and  $\frac{\partial^2 E(u_f)}{\partial \sigma \partial \sigma^-}$  are negative if  $\sigma - y_e^- = \bar{\theta}$ . This happens because the father already sets  $\sigma$  at a high level and the father's gain from further increase in the children's consumption is small (or non existent) while an increases in  $\sigma^-$  can still reduce his own consumption upon remarriage. In contrast, when  $\sigma$  is small and child expenditures upon remarriage are set at a low level, then an increase in  $\sigma^-$  can be beneficial to the father and increase his incentive to commit. The impact of others also depends on the probability of remarriage, either directly or because  $\sigma$  depends on p. The upshot is that although we can easily determine the impact of others ex-post, this impact is generally ambiguous when marital status is still unknown.

# 4.3 Partial Equilibrium

A symmetric partial (or conditional) equilibrium exists when, given the probability of remarriage p, all agents choose the same level of  $\sigma$ , taking the choices of others as given. The term partial is used here because the remarriage rate is endogenous in our model and must be determined too.

We first consider an interior equilibrium with a common  $\sigma$  that satisfies the feasibility and incentive compatibility constraints as strict inequalities. For such an

equilibrium to exist, it is necessary that the first order condition,

$$\frac{\partial E(u_f)}{\partial \sigma} = -(1-p)(1+ph'(h(\sigma-y_e;\bar{\theta})) + p\delta g'(h(\sigma-y_e;\bar{\theta}))h'(\sigma-y_e;\bar{\theta}) = 0, (24)$$

and the second second order condition,

$$\frac{\partial^2 E(u_f)}{\partial \sigma^2} = p\delta g''(h(\sigma - y_e; \bar{\theta}))[h'(\sigma - y_e; \bar{\theta})]^2 + p\delta g'(h(\sigma - y_e; \bar{\theta})h''(\sigma - y_e; \bar{\theta}) - p(1 - p)^2 h''(\sigma - y_e; \bar{\theta}) < 0,$$
(25)

be satisfied, together with the incentive compatibility constraint

$$0 < \sigma < \frac{y - s + \bar{\theta}}{2 - p},\tag{26}$$

and feasibility constraint

$$0 < \sigma < y - s. \tag{27}$$

In the appendix we prove that

**Proposition 1** If the marginal utility from children expenditures is convex (i.e.,  $g'''(c) \geq 0$ ),  $\frac{\partial E(u_f)}{\partial \sigma} = 0$  entails  $\frac{\partial^2 E(u_f)}{\partial \sigma^2} > 0$  for all  $p \in [0, \frac{2}{3}]$ , implying that an interior symmetric equilibrium does not exist in this range.

The reason for non existence can be traced back to the convexity of the Nash bargaining outcome in the commitment made by each father to the custodial mother. This convexity implies that each father can individually gain from a unilateral departure from the interior equilibrium. However, a symmetric equilibrium can still occur at the *boundaries* of either the feasibility constraints or the incentive compatibility constraint, whichever is binding.

To simplify the analysis of equilibria at the boundary, we shall now assume that  $\bar{\theta} = 0$  and  $\delta = 1$ . The restriction on the non monetary factor,  $\bar{\theta}$ , guarantees that the incentive compatibility constraint is the binding constraint for all p. The assumption that  $\delta = 1$  guarantees that the father cares sufficiently about the children to support an equilibrium in which everyone is willing to commit. Finally, we shall assume that g(c) is quadratic, which is the borderline case for the class of functions for which  $g'''(c) \geq 0$  and also the easiest one to calculate.

We can provide only a partial characterization, for the case in which the best response functions are always at the boundary. That is, if all individuals set  $\sigma^- = 0$  then each father who deviates makes the maximal feasible transfer  $\sigma = y - s$ , and if all other individual selects  $\sigma^- = \frac{y-s}{2-p}$  then any father who deviates chooses the minimal transfer,  $\sigma = 0$ . This extreme behavior can be supported by sufficient convexity of h(x; 0). In the appendix we prove that such a pattern must hold when others set

 $\sigma^- = 0$ . For the case in which others set  $\sigma^- = \frac{y-s}{2-p}$ , we cannot prove that the optimal deviation is necessarily to 0, but shall assume that this is the case.<sup>19</sup>

We can then characterize the strategic interaction globally and say that  $\sigma$  and  $\sigma^-$  are complements if, for all p

$$V_f(\frac{y-s}{2-p}, \frac{y-s}{2-p}) - V_f(0, \frac{y-s}{2-p}) > V_f(y-s, 0) - V_f(0, 0)$$
(28)

and substitutes if the inequality is reversed, where  $V_f(\sigma, \sigma^-)$  stands for the expected utility of the father at given  $\sigma$  and  $\sigma^-$ . We can also define critical points for the probability of remarriage,  $p_0$  and  $p_1$  such that each father is indifferent between deviating and conforming, given that others choose,  $\sigma^- = \frac{y-s}{2-p}$  and  $\sigma^- = 0$ , respectively.

We can then prove (see Appendix)

**Proposition 2** Suppose that the utility of children, g(c), is quadratic with g'(c) < 2 and that the implied best response functions are always at the boundary. Assume no expected gain from companionship,  $\bar{\theta} = 0$ , and no discounting if the parent and children live apart,  $\delta = 1$ . Then  $p_0$  and  $p_1$  are uniquely determined. If  $\sigma$  and  $\sigma^-$  are complements then  $p_1 > p_0$  and for  $p < p_0$  all fathers set  $\sigma = 0$ , while for  $p > p_1$  all fathers voluntarily commit to pay their ex-wife the maximal  $\sigma$  that is incentive compatible  $\frac{y-s}{2-p}$ . For  $p_1 > p > p_0$  both types of equilibrium can exist. If  $\sigma$  and  $\sigma^-$  are substitutes then  $p_0 > p_1$  and for  $p < p_1$  all fathers set  $\sigma = 0$ , while for  $p > p_0$  all fathers voluntarily commit to pay their ex-wife the maximal  $\sigma$  that is incentive compatible  $\frac{y-s}{2-p}$ . If  $p_0 > p > p_1$  there is no symmetric equilibrium. In either case, equilibria at the upper, incentive compatible, boundary imply that children expenditures are set at the efficient level  $c^*$ , while equilibria at the lower boundary imply that children expenditures are set at an inefficient level  $c^*$ .

The pattern described in the Proposition 2 is illustrated in Figure 2. We see that, irrespective of the strategic interactions, a higher probability of remarriage is conducive to equilibria in which fathers are willing to commit on a payment that is conditioned on the event that the mother remains single, because such promises are carried out less often and are more likely to yield benefits. In this regard, the probability of remarriage serves a coordination device that induces fathers to behave similarly in terms of their commitments. The strategic interactions can reinforce or mitigate this pattern depending on complementarity or substitution. If  $\sigma$  and  $\sigma^-$  are complements each father is more willing to contribute if others do, and thus the critical value at which all fathers contributes occurs at a lower p than it would if others do not contribute. Conversely, if  $\sigma$  and  $\sigma^-$  are substitutes.

The basic characterization of the equilibrium of Proposition 2 holds also if  $\bar{\theta} > 0$ . However, in this case, the feasibility constraint will bind at high remarriage rates, and it is then impossible to maintain efficiency of children expenditures. The modified

<sup>&</sup>lt;sup>19</sup>In the numerical examples that follow, we do verify that this property holds.

statement is then that equilibria at the upper feasible level,  $\sigma = y - s$ , imply higher levels of child expenditures than equilibria in the lower feasible boundary with  $\sigma = 0$ .

#### 4.4 Divorce

Having observed the realized quality of the current match, each spouse may consider whether or not to continue the marriage. A parent will agree to continue the marriage if, given the observed  $\theta$ , the utility in marriage exceeds his/her expected gains from divorce. Under divorce at will, the marriage breaks if

$$u^* + \theta < \max\{E(u_m), E(u_f)\},\tag{29}$$

where  $E(u_m)$  and  $E(u_f)$  are the expected utility of the mother and father at divorce and

$$u^* = y + z + g(c^*) - c^* (30)$$

is the common utility of the husband and wife if the marriage continues, not incorporating the quality of the match. Note that this divorce rule is different from the more familiar condition

$$2(u^* + \theta) < E(u_m) + E(u_f) \tag{30'}$$

that would apply if utility is transferable within couples. Our assumption that all the goods that are consumed in an intact family are public precludes compensation within couples that would "bribe" the parent with the better outside options to remain in the marriage.

Let us define the critical value of  $\theta$  that triggers divorce as

$$\theta^* = \max\{E(u_m), E(u_f)\} - u^*. \tag{31}$$

Excluding the quality of match  $\theta_i$ , the utility of each parent following separation cannot exceed the common utility that the parents attain if marriage continues, because the allocation between adult and child goods in an intact family is efficient and all the opportunities of sharing consumption are exploited. Therefore, the critical values  $\theta^*$  must be lower than the expected quality of match following remarriage.

The probability that a couple will divorce is

$$\Pr\{\theta \le \theta^*\} = F(\theta^*),\tag{32}$$

where F(.) is the cumulative distribution of  $\theta$ . Assuming independence of the marital shocks across couples and a large population, the proportion of couples that will choose to divorce is the same as the probability that a particular couple divorces. Symmetry implies that  $F(\bar{\theta}) = \frac{1}{2}$  and, therefore, the fact that divorce is costly from an economic point of view implies that less than half of the marriages will end up in divorce as a consequence of "bad" realizations for the quality of match.

An important feature of the model is that the decision of each couple to divorce depends on the probability of remarriage that in turn depends on the decision of others to divorce, because a remarriage is possible only with a divorcee. In addition, the decision to divorce depends on the nature of the commitments that the couples makes, as well as the commitment made by others. Post divorce transfers between the parents can reduce their cost of separation in the event of a bad quality of match. However, commitments made by others imply that prospective matches are less attractive for remarriage, which can increase the cost of divorce.

To analyze these complex issues, we limit our attention to the commitment equilibria that occur at the boundary. We shall also maintain our simplifying assumptions that  $\bar{\theta} = 0$ ,  $\delta = 1$  and that g(c) is quadratic with g'(c) < 2. Based on Proposition 2, we can now examine the expected utilities of the children, husband and wife, evaluated at the time of divorce.

In equilibria without commitment,  $\sigma = 0$ ,

$$E(u_c) = pg(\hat{c}) + (1 - p)g(c^*),$$

$$E(u_f) = y + z - c^* + p(c^* - \hat{c}) + E(u_c),$$

$$E(u_m) = p(y + z - \hat{c}) + E(u_c),$$
(33)

where the Nash Bargaining outcome  $\hat{c}$  is given by the solution to

$$\gamma'(\hat{c}) = \frac{\gamma(\hat{c}) + y + z - c^*}{c^* - \hat{c}}.$$
(34)

The children's expected utility declines with the probability of remarriage, p, because child expenditures if the mother remarries,  $\hat{c}$ , are lower than if the mother remains single,  $c^*$ . The mother must gain from an increase in p because, remarriage is voluntary and she fully internalizes the impact of her remarriage on the child.<sup>20</sup> By the same logic, the father's expected adult consumption must increase with p, or else he would mot remarry. However, taking into account the utility of the child that is controlled by the mother, the expected utility of the father declines in p.<sup>21</sup> Nevertheless, the expected utility of the father, upon separation exceeds that of the mother by  $(1-p)(y+z-c^*)$ , because of his higher consumption of adult goods if he remains single.

$$\frac{\partial E(u_m)}{\partial p} = y + z + g(\hat{c}) - \hat{c} - g(c^*),.$$

which is exactly her own gain from remarriage.

<sup>21</sup>The net effect of p on the father's expected utility is

$$\frac{\partial E(u_f)}{\partial p} = c^* - \hat{c} - (g(\hat{c}) - g(c^*)),$$

which is negative because  $g(c^*) - c^* > g(\hat{c}) - \hat{c}$ .

 $<sup>\</sup>overline{^{20}}$  The effect of p on the mother's expected utility is

In equilibria in which all fathers commit to  $\sigma = \frac{y-s}{2-p}$ ,

$$E(u_c) = g(c^*),$$

$$E(u_m) = \frac{y+z-c^*}{2-p} + g(c^*)$$

$$E(u_f) = \frac{y+z-c^*}{2-p} + g(c^*)$$
(35)

That is, the efficient level of children expenditures is attained whether or not the mother remarries. Both the father and the mother are indifferent between remarriage and remaining single. The expected utility of the mother upon divorce equals that of the father and both rise with the probability of remarriage, p.

We conclude

**Proposition 3** Suppose that proximity does not matter,  $\delta = 1$ , and all gains from remarriage are monetary,  $\bar{\theta} = 0$ . Then, the expected utility of the father upon divorce is at least as large as that of the mother and he determines whether or not the marriage will continue. If no father commits,  $\sigma = 0$ , then  $E(u_f) > E(u_m)$  and the father will break the marriage for all  $\theta$  such that  $\theta < E(u_f) - u^*$ . Inefficient separations occur when the father wants to leave but and the mother wants to maintain the marriage,  $E(u_f) - u^* > \theta > E(u_m) - u^*$ . If all fathers commit to  $\sigma = \frac{y-s}{2-p}$  then  $E(u_f) = E(u_m)$  and separations are efficient.

The assumptions that  $\delta=1$  and  $\bar{\theta}=0$  are crucial for the result that the father and mother have the same expected utility. Clearly, the father is at a disadvantage if proximity is valuable and the mother gains custody. The level of  $\bar{\theta}$  matters because it affects the payment that is required to maintain the mother's indifference between marriage and non marriage. The result that  $E(u_f)=E(u_m)$  when  $\bar{\theta}=0$  and the associated efficiency of child expenditures reflect a knife edge situation in which neither men nor women gain from remarriage. When  $\bar{\theta}$  is raised then, as long as  $\frac{y-s+\bar{\theta}}{2-p} < y+z-c^*$ , it is possible to maintain the level of child expenditure at  $c^*$ , by raising the commitment  $\sigma$  and keeping the mother indifferent between marriage and non marriage. In this case, the mother's expected utility is  $\frac{y-s+\bar{\theta}}{2-p}+g(c^*)$  but the expected utility of the father is now smaller,  $\frac{y-s+\bar{\theta}}{2-p}+g(c^*)-(1-p)\bar{\theta}$ , because a larger transfer is needed to maintain indifference. In this case, the mother will determine whether the couple divorces. Finally, if  $\bar{\theta}$  is such that  $\frac{y-s+\bar{\theta}}{2-p}>y-s$  then both men and women gain from remarriage, but it is impossible to maintain the child expenditures at the efficient level  $c^*$ , which means that the children suffer from the mother's remarriage.

It is also clear that equilibrium outcome in the aftermath of divorce is inferior to the utility that an average couple obtains in marriage, because  $\frac{y-s+\bar{\theta}}{2-p} \leq y-s+\bar{\theta}$ . This difference reflects the lack of companionship and the inability to share in the adult good when one of the partners remains single. It is only when remarriage is certain, that one can expect to recover the average utility in the first marriage.

# 5 Full equilibrium

We can now close the model and determine the equilibrium levels of divorce and remarriage. Equilibrium requires that all agents in the marriage market act optimally, given their expectations, and that expectations are realized. The decision of each couple to divorce depends on the expected remarriage rate, p. Given a matching function  $m = \phi(d)$ , and that all meetings end up in marriage,<sup>22</sup>we must have

$$p = m = \phi[F(\theta^*(p))]. \tag{36}$$

In addition, the contracting choices of all participants in the marriage market must be optimal, given p. Based on our previous analysis, we define  $\theta_0^*(p)$  as the trigger if all fathers set  $\sigma = 0$  and  $\theta_1^*(p)$  as the trigger if all fathers set  $\sigma = \frac{y-s+\bar{\theta}}{2-p}$ . Then, the equilibrium divorce and remarriage rates are determined by  $p = \phi[F(\theta_0^*(p))]$  or  $\phi[F(\theta_1^*(p))]$ , depending upon whether the induced commitment is  $\sigma = 0$  or  $\sigma = \frac{y-s+\bar{\theta}}{2-p}$ .

To separate the economic considerations embedded in  $\theta^*(p)$  from the exogenous distribution of match quality  $F(\theta)$  and matching function  $\phi(d)$ , it is useful to rewrite condition (36) in the form

$$F^{-1}[\phi^{-1}(p)] = \theta^*(p). \tag{36'}$$

The function  $F^{-1}[\phi^{-1}(p)]$  is always increasing in p, while  $\theta^*(p)$  depends on the commitments that individual fathers wish to make, given their evaluation of the remarriage prospects of their ex-wife. Our previous analysis shows that  $\theta^*(p)$  first declines and then rises in p, with a discontinuity when all fathers switch from no commitment to full commitment. Therefore, depending on the parameters, the model is capable of generating multiple equilibria and large responses to relatively small exogenous changes.

To better understand these issues, let us summarize the main feedbacks that are present in our model.

• The increasing returns in matching, whereby it is easier to remarry if there are more divorces around, creates a *positive* feedback from the expected remarriage rate to the realized divorce rates. As is well known, this force alone can create multiple equilibria, because a higher divorce rate generates a higher remarriage

<sup>&</sup>lt;sup>22</sup>Strictly speaking, all agents are indifferent toward marriage if.  $\bar{\theta} = 0$ . However, for any positive  $\bar{\theta}$ , the father gains and the mother is either indifferent or gains too. Thus, we interpret the case with  $\bar{\theta} = 0$  as a limit in which the expected gains from companionship approach zero.

<sup>&</sup>lt;sup>23</sup>These equilibrium requirements implicitly assume symmetric equilibria in which all agents behave in the same manner. Such equilibria are a natural choice given that all agents are initially identical, but other equilibrium may exist. In a more general analysis, one can incorporate also mixed equilibria such that some couples choose to have a child, some choose to remain childless and all couples are *indifferent* between having and not having a child.

rate and a higher expected remarriage rate creates stronger private incentives to divorce.<sup>24</sup>

- However, in our model, a higher remarriage rate encourages divorce only if there are adequate transfers that ensure that the children do not suffer if the custodial mother remarries. In the absence of such transfers, the father who determines the divorce decision, will in fact be *less* likely to divorce if the remarriage rate is high.
- Our assumption that all goods are public for first married couples weakens the positive feed backs and reduces the likelihood of multiple equilibria. If it would be possible to transfer utilities within marriage on to one basis (transferable utility) then the divorce rule would depend on the sum of the expected gains from divorce of the two parents. As we have shown, in the absence of commitments, the father's expected utility upon divorce declines in p, while that of the mother rises with p. Thus, adding the two expected gains, it is more likely for  $\theta^*(p)$  to rise with p in the range in which fathers make no commitments and  $\sigma = 0$ .
- An important feature of our model that creates *positive* feedbacks is that stronger commitments on payments that are contingent on the mother remaining single are made when the expected remarriage rate is high, because then the father is less likely to pay and more likely to reap the benefits.<sup>26</sup>
- Finally, our model allows for *strategic* interactions in the commitments made by different parents. These interactions arise because the bargaining outcome for remarried couples depends, in a non linear manner, on both the transfer that the father is committed to make and the transfer that the new husband of the mother commits to his ex-wife. Strategic complementarity stengthens the positive feedback, because in the intermediate range of remarriage probability, each father is willing to commit if others do, but not if they do not.

<sup>&</sup>lt;sup>24</sup>Diamond (1982) discusses the connections between increasing returns and multiplicity in a search economy.

<sup>&</sup>lt;sup>25</sup>This result is in contrast to Burdett and Wright (1998) who consider marriage between heterogeneous agents and show that non transferable utility can lead to multiplicity even in the absence of increasing returns. A negative feedback arises in their model, whereby if one side (say men) is more selective then the other side (women) will have fewer options and become less selective. Thus an equilibrium in which men are selective and women are not and an equilibrium in which women are selective but men are not can both exist.

<sup>&</sup>lt;sup>26</sup>Diamond and Maskin (1979) discuss compensation for the damage imparted on the other partner when a separation occurs and show that the willingness to make such commitments *declines* with the probability of rematching. Our model differs in that the incentive to commit is related to the maintenance of child quality by the custodial mother and, consequently, commitments rise with the probability of remarriage.

#### 5.0.1 Example 1

We now present a numerical example that illustrates the general points stated above. The linearly declining line in Figure 3 represents  $\theta_0^*(p)$ , which is the critical remarriage rate that would trigger divorce if no father pays his ex-wife beyond the minimum required by law. The increasing convex line in the figure represents  $\theta_1^*(p)$  which is the critical value of remarriage rate that would trigger divorce if all father commit to an additional payment  $\sigma = \frac{y-s}{2-p}$  that the mother receives if she remains single. The vertical lines at  $p = p_1 = .53$  indicates that for all lower values of the expected remarriage rate, no father is willing to commit if others do not. The vertical line at  $p = p_0 = .59$  indicates that for all higher values of the expected remarriage rates each father is willing to commit if others do. The result that  $p_0 > p_1$  shows that, for the chosen parameters,  $\sigma$  and  $\sigma^-$  are strategic substitutes. The two steep convex lines represent the function  $F^{-1}[\phi^{-1}(p)]$ , where  $F(\theta)$  is uniform over [-u, u] and the matching function is given by  $\phi(d) = 1 - (1 - u)^2$ . The curve to the right represents a higher variability in  $\theta$  than the curve to the left. The increase in the variance holding the mean constant (i.e., an increase in u) shifts the equilibrium from a low divorce- remarriage equilibrium with p=.5 and d=.29 to high divorce-remarriage equilibrium with p = .71 and d = .46. The higher equilibrium occurs above  $p_1$  and is associated with more generous commitments by fathers who pay  $\sigma = \frac{y-s}{2-p}$  to their ex-wives if they remain single. The lower equilibrium is below  $p_0$  and in this case fathers pay nothing to their ex-wives ( $\sigma = 0$ ), beyond the amount stipulated by law,  $s=c^*-z$ . This example illustrates the coordination role of aggregate divorce in inducing stronger commitments.<sup>27</sup>

## 6 Welfare

From a policy perspective, it is meaningful to evaluate welfare ex-ante, before of the realization of the quality of match, because at the time of marriage all men and women are identical and their expected life time represent the average outcome for respective populations, that would arise after each couple draws it idiosyncratic quality of match. The expected utility of a member j in a particular couple, evaluated at the time of marriage, is

$$W_j(p) = u^0 + \bar{\theta} + \int_{\theta^*(p)}^{\infty} (u^* + \theta) f(\theta) d\theta + F(\theta^*(p)) V_j(p), \tag{37}$$

 $<sup>^{27}</sup>$ Although the model is generally capable of generating multiple equilibria, these do not arise with the current specification. Multiplicity requires that  $\sigma$  and  $\sigma^-$  be strategic complements. The specification  $g(c) = 2c - \frac{1}{2}c^2$ , that implies  $c^* = 1$ ,  $\sigma$  and  $\sigma^-$  are strategic complements when z + y is close to 1. However, there are no multiple equilibria at such parameters. It seems that a strict convexity for g'(c) is needed to generate "realistic" examples with multiple equilibria.

where,  $V_j(p)$  denote the expected utility upon divorce, j = f for the father and j = m for the mother. The term  $u^0$  represent the parents average utility in the first period and is given by y + z, while  $u^* = y + z + g(c^*) - c^*$  represents the utility in an intact family (excluding the impact of the non monetary return  $\theta$ ).<sup>28</sup> The expected utility following divorce,  $V_j(p)$ , may be different for the two parents, depending upon the agreement they make about transfers and on the agreement made by others, which determines their value as potential mates for remarriage. As a consequence, the expected life time utility evaluated at the time of marriage,  $W_j(p)$ , may differ for males and females.

The expected life time utility is higher for the partner with the higher gains from divorce, who determines the divorce decision. In fact, the expected life time utility can be rewritten as

$$W_{j}(p) = \begin{cases} u^{0} + u^{*} + 2\bar{\theta} + \int_{-\infty}^{\theta^{*}(p)} (\theta^{*}(p) - \theta) f(\theta) d\theta & if \quad V_{j}(p) \ge V_{i}(p) \\ u^{0} + u^{*} + 2\bar{\theta} + \int_{-\infty}^{\theta^{*}(p)} (\theta^{*}(p) - \theta) f(\theta) d\theta & if \quad V_{j}(p) < V_{i}(p) \\ -F(\theta^{*}(p))(V_{i}(p) - V_{j}(p)) \end{cases}$$
(38)

where the term  $u_{\delta}^{0} + u_{\delta}^{*} + 2\bar{\theta}$  is the expected value of the marriage if it never breaks and the term  $\int_{-\infty}^{\theta_{\delta}^{*}(p)} (\theta_{\delta}^{*}(p) - \theta) f(\theta) d\theta$  is the option value of breaking the marriage if it turns sour because of a bad draw of  $\theta$ . The option to sample from the distribution of  $\theta$  is a motivation for marriage that exists even if marriage provides no other benefits. However, this option is more valuable for the person with higher gains from divorce, who determines the divorce. When the marriage breaks, an event that happens with probability  $F(\theta^{*}(p))$ , the spouse who does not initiate the divorce and is left behind suffers a capital loss given by  $V_{i}(p) - V_{j}(p)$ . The value of the option for the spouse who determines the divorce, increases with the variability in the quality of match, because then the ability to avoid negative shocks becomes more valuable.

Using the expressions in (37) and (38), we can calculate the welfare of each agent in equilibrium. The main result is that exogenous shocks that raise the divorce rates can increase the welfare of the children and the mother, because they provide incentive to fathers to raise their commitments.

**Proposition 4** The welfare of the children declines with the probability of remarriage, p, if fathers make no commitments,  $\sigma = 0$ , and is unaffected by p if fathers make full commitments,  $\sigma = \frac{y-s+\bar{\theta}}{2-p}$ . Starting at a low p, a large change in p is needed

 $<sup>^{28}</sup>$ The economic costs of bearing and raising children are reflected by the assumption that z < y. Because these cost are largely borne by the mother, she may refrain from having children unless the father make further commitments at the time of marriage about post divorce settlements. To avoid these further complications, we assume here that the total gains from having children (including possible benefits in the first period) exceed these economic costs.

to induce fathers to commit to a level of transfer that entails an improvement in the child's welfare.

We can further note

- If fathers make full commitments,  $\sigma = \frac{y-s+\bar{\theta}}{2-p}$ , the life time utility of both parents rises with the remarriage rate. If two equilibrium points exist in this range then welfare will be higher when the equilibrium divorce (remarriage) rate is higher. This is a direct outcome of the assumed increasing returns, whereby a higher divorce rate makes remarriage easier, and the result that no other externalities exist. In particular, the mother is not hurt by the decision of the father to break the marriage (i.e., divorce is efficient) and the children (and thus the father) are not hurt by the remarriage of the mother.
- In contrast, if fathers make no commitments,  $\sigma = 0$ , then all family members can be hurt by a higher divorce (remarriage) rate. The father and children are certainly hurt and the mother may also be hurt because separations are inefficient and the father determines the divorce. However, the mother never loses from remarriage, because she fully internalizes the (negative) impact on the child in her decision to remarry.

#### 6.0.2 Example 2

In Table 1, we show the outcomes for some particular parameter values. The upper panel corresponds to the outcomes shown in Figure 3 and, as seen, the expected utility of the father is lower in the equilibrium with higher divorce and remarriage. In the lower panel, we illustrate the impact of improved matching, which might be an outcome of increased use of the internet in looking for mates. Such exogenous shift raises the welfare of all family members, father mother and children. Notice that, despite the large increase in the remarriage rate, the divorce rate is hardly affected. This happens because in the range without commitment the gains from marriage decline in the probability of remarriage so that easier remarriage reduces divorce. It is only when transfers are operative that the gains from divorce rise with the probability of remarriage and the two move together.

# 7 Conclusion

Broadly viewed, divorce is a corrective mechanism that enables the replacement of bad matches by better ones. The problem, however, is that private decisions may lead to suboptimal social outcomes because of the various externalities that infest search markets. These externalities exist at the level of a single couple and the market at large. At the level of a couple, the spouse who initiates the divorce fails to internalize the interest of the other spouse in continued marriage and the parent that remarries fails to internalize the impact on the children and consequently of the ex-spouse who continues to care about the child. At the market level, a person who chooses to divorce fails to take into account the impact on the remarriage prospects of others, and if commitments are made, on the quality of prospective mates. We have shown that the problems at the couple's level can be resolved by voluntary commitments that entail efficient level of child expenditures and efficient separation. However, such commitments are made only if the expected remarriage rate is sufficiently high.

The willingness to commit at high divorce levels is a consequence of the social interaction between participants. In the marriage market, as in other "search markets", finding a mate takes time and meetings are random; the decision of each couple to terminate its marriage depends not only on the realized quality of the particular match, but also on the prospects of remarriage and, therefore, on the decisions of others to divorce and remarry. We have shown that such feedbacks may improve the welfare of children, because fathers may be more willing to commit on post divorce transfers to their ex-wives in high divorce environments, in order to influence their bargaining power upon remarriage. Of course, a higher divorce rate can induce other mechanisms that affect the welfare of children, in addition to the impact on voluntary commitments discussed here. In fact, the rise in divorce was associated with new guidelines for the courts that facilitate children support agreements and with increased enforcement of children support awards (see Del Boca, 2003, and Lerman and Sorensen, 2003).

The analysis of this paper can be extended to include endogenous fertility. As we have shown, the ex-post asymmetry between parents caused by having children can create problems in caring for them if the marriage breaks up and contracts are incomplete. Because men often have higher expected gains from divorce, they initiate the divorce, at some situations in which the mother would like to maintain the marriage. Such inefficient separations imply that the gains from having children are smaller to the mother than to the father. Because the production of children requires both parents, the mother may avoid birth in some situations in which the husband would like to have a child. The consequence is an inefficient production of children. This suggests another role for contracts, to regulate fertility, which may require some ex-ante contracting at the time of marriage. However, contracts that couples are willing to sign at the time of marriage may be inconsistent with contracts that the partners are willing to sign in the interim stage, after divorce has occurred and the impact of the contract on the divorce and fertility decisions is not relevant any more. With such time inconsistency, the partners may wish to renegotiate, thereby creating a lower level of welfare for both of them from an ex-ante point of view. Assuming that renegotiation takes place, the contracts will be similar to the interim contracts discussed here but they would apply for a broader range of remarriage probabilities. It can then be shown that fertility choice creates further feedbacks that can generate multiple equilibria, with and without children. These are important issues for further research.

# 8 Appendix A

# 8.1 Non Existence of a symmetric equilibrium with an interior solution

Suppose that  $\sigma = \sigma^-$  and the solution is interior. Then the first order condition

$$\frac{\partial E(u_f)}{\partial \sigma} = -(1-p)(1+ph'(h(\sigma-y_e;\bar{\theta})) + p\delta g'(h(\sigma-y_e;\bar{\theta}))h'(\sigma-y_e;\bar{\theta}) = 0, \text{ (A1)}$$

and the second second order condition,

$$\frac{\partial^2 E(u_f)}{\partial \sigma^2} = p\delta g''(h(\sigma - y_e; \bar{\theta}))[h'(\sigma - y_e; \bar{\theta})]^2 + p\delta g'(h(\sigma - y_e; \bar{\theta})h''(\sigma - y_e; \bar{\theta}) - p(1 - p)^2 h''(\sigma - y_e; \bar{\theta}) < 0, \tag{A2}$$

are satisfied, together with the incentive compatibility constraint

$$\sigma \le \frac{y - s + \bar{\theta}}{2 - p},\tag{A3}$$

and feasibility constraint

$$0 \le \sigma \le y - s. \tag{A4}$$

Rewriting

$$\frac{\partial^2 E(u_f)}{\partial \sigma^2} = p[h'(\sigma - y_e; \bar{\theta})]^2 \{ \delta g''(\hat{c}) + (\delta g'(\hat{c}) - (1 - p)^2) h'(\sigma - y_e; \bar{\theta}) \frac{h''(\sigma - y_e; \bar{\theta})}{[h'(\sigma - y_e; \bar{\theta})]^3} \}$$
(A5)

and using equations (16) and (A1) we obtain

$$\frac{\partial^{2}E(u_{f})}{\partial\sigma^{2}} = p[h'(\sigma - y_{e}; \bar{\theta})]^{2} \{\delta g''(\hat{c}) - \frac{\delta g'(\hat{c}) - (1 - p)^{2}}{\delta g'(\hat{c}) - (1 - p)} \frac{1 - p}{p} (3g''(\hat{c}) - (c^{*} - \hat{c} + \bar{\theta})g'''(\hat{c}))\} 
= p[h'(\sigma - y_{e}; \bar{\theta})]^{2} \{g''(\hat{c})[\delta - 3\frac{\delta g'(\hat{c}) - (1 - p)^{2}}{\delta g'(\hat{c}) - (1 - p)} \frac{1 - p}{p}] + \frac{\delta g'(\hat{c}) - (1 - p)^{2}}{\delta g'(\hat{c}) - (1 - p)} \frac{1 - p}{p} (c^{*} - \hat{c} + \bar{\theta})g'''(\hat{c})\}.$$
(A6)

Now because  $\delta \leq 1$  and  $\frac{\delta g'(\hat{c}) - (1-p)^2}{\delta g'(\hat{c}) - (1-p)} \geq 1$ , the first term must be positive if  $3\frac{1-p}{p} > 1$  or  $p < \frac{3}{4}$ . The second term is non negative if  $g'''(\hat{c})) \geq 0$ . Therefore, if  $g'''(\hat{c})) \geq 0$  and  $p < \frac{3}{4}$ ,  $\frac{\partial E(u_f)}{\partial \sigma} = 0 \Rightarrow \frac{\partial^2 E(u_f)}{\partial \sigma^2} > 0$  and a symmetric equilibrium with an interior solution for the commitment  $\sigma$  does not exist.

## 8.2 Equilibria at the boundary

The possible symmetric commitment equilibria at the boundary are at  $\sigma = 0$ , at  $\sigma = y - s$  and at  $\sigma = \frac{y - s + \bar{\theta}}{2 - p}$ .

To analyze these potential equilibria we shall assume that  $g(c) = g_1c - \frac{g_2}{2}c^2$  is quadratic, where  $g_1$  and  $g_2$  are fixed parameters such that  $2 \ge g_1 > 0$  and  $g_2 > 0$ . The restriction that  $g_1 \le 2$  ensures that the father never wants to transfer money to the custodial mother if she remarries. The restriction that g'''(c) = 0 implies that

$$h''(x; \bar{\theta}) = 3g_2[h'(x; \bar{\theta})]^3 > 0.$$

We can now prove the following

**Lemma 5** For a quadratic function g(c), there is no interior solution for  $\sigma$  such that  $\sigma \geq \sigma^-$  in the region where  $p < \frac{3}{4}$ . In particular, if all agents set  $\sigma^- = 0$ , then any agent that considers deviation must choose between the lower boundary, i.e.,  $\sigma = 0$  and the upper boundary given by  $\sigma = y - s$ .

**Proof.** Suppose that

$$\frac{\partial E(u_f)}{\partial \sigma} = -(1-p) - p(1-p)h'(\sigma^- - y_e; \bar{\theta}) + p\delta g'(h(\sigma - y_e^-; \bar{\theta}))h'(\sigma - y_e^-; \bar{\theta}) \ge 0.$$

At such a point, the second order derivative is

$$\frac{\partial^{2}E(u_{f})}{\partial\sigma^{2}} = p\delta g''(h(\sigma - y_{e}^{-}; \bar{\theta}))[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2} + p\delta g'(h(\sigma - y_{e}^{-}; \bar{\theta})h''(\sigma - y_{e}^{-}; \bar{\theta})) \\
-p(1-p)^{2}h''(\sigma^{-} - y_{e}; \bar{\theta}) \\
= -pg_{2}[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2} \{\delta - 3\delta h'(\sigma - y_{e}^{-}; \bar{\theta})g'(h(\sigma - y_{e}^{-}; \bar{\theta})) \\
+3(1-p)^{2}h'(\sigma^{-} - y_{e}; \bar{\theta})\frac{[h'(\sigma^{-} - y_{e}; \bar{\theta})]^{2}}{[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2}} \} \\
\geq -pg_{2}[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2} \{\delta - 3(\frac{1-p}{p} + (1-p)h'(\sigma^{-} - y_{e}; \bar{\theta})) \\
+3(1-p)^{2}h'(\sigma^{-} - y_{e}; \bar{\theta})\frac{[h'(\sigma^{-} - y_{e}; \bar{\theta})]^{2}}{[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2}} \} \\
= -pg_{2}[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2} \{(\delta - 3\frac{1-p}{p}) \\
+3(1-p)h'(\sigma^{-} - y_{e}; \bar{\theta})((1-p)\frac{[h'(\sigma^{-} - y_{e}; \bar{\theta})]^{2}}{[h'(\sigma - y_{e}^{-}; \bar{\theta})]^{2}} - 1) \}.$$

Now

$$\sigma^- - y_e = \sigma^- - [(y-s) - (1-p)\sigma],$$
  
 $\sigma - y_e^- = \sigma - [(y-s) - (1-p)\sigma^-],$ 

and

$$\sigma^{-} - y_{e} - (\sigma - y_{e}^{-}) = \sigma^{-} - \sigma + (1 - p)(\sigma - \sigma^{-}) = p(\sigma^{-} - \sigma).$$

Because for a quadratic  $g(.), h''(.; \bar{\theta}) > 0$ , we have

$$\sigma^- \le \sigma \Rightarrow \frac{[h'(\sigma^- - y_e; \bar{\theta})]^2}{[h'(\sigma - y_e^-; \bar{\theta})]^2} \le 1.$$

Thus, for  $\sigma^- \leq \sigma$  and  $1 - 3\frac{1-p}{p} < 0$  (or  $p < \frac{3}{4}$ ) we see that  $\frac{\partial E(u_f)}{\partial \sigma} \geq 0 \Rightarrow \frac{\partial^2 E(u_f)}{\partial \sigma^2} > 0$ . That is, there is no interior solution.

We shall now consider the choice between these two alternatives.

#### 8.2.1 Equilibrium at the lower boundary

If all men set  $\sigma = 0$  then

$$E(u_c) = pg(c_0) + (1 - p)g(c^*), \tag{A7}$$

where  $c_0$  solves

$$\gamma'(c) = \frac{\gamma(c) + y - s + \bar{\theta}}{c^* - c + \bar{\theta}}.$$
 (A8)

The expected utility of each father if no one commits and  $\sigma = 0$  is

$$E(u_f) = (1-p)(y-s) + p(y+z-c_0) + \delta[pg(c_0) + (1-p)g(c^*)] + p\bar{\theta}$$
  
=  $y-s+p(c^*-c_0) + \delta[pg(c_0) + (1-p)g(c^*)] + p\bar{\theta}$ . (A9)

If one father deviates and promises his wife  $\sigma = y - s$ , the amount spent on his children if the mother remarries will be  $c_1$  where  $c_1$  solves

$$\gamma'(c) = \frac{\gamma(c) + \bar{\theta}}{c^* - c + \bar{\theta}}.$$
 (A10)

If the father remarries, the amount spent on the children of his new wife will be  $c_2$ , where  $c_2$  solves

$$\gamma'(c) = \frac{\gamma(c) + p(y-s) + \bar{\theta}}{c^* - c + \bar{\theta}}$$
(A11)

and  $c_0 < c_2 < c_1 < c^*$ . The expected utility of the father is then

$$E_d(u_f) = (1-p)[p(y-s) + (1-p)0] + p[p(y+z-c_2) + (1-p)(s+z-c_2)]$$

$$\delta[pg(c_1) + (1-p)g(c^*)] + p\bar{\theta}$$

$$= p(y-s) + p(c^*-c_2) + \delta[pg(c_1) + (1-p)g(c^*)] + p\bar{\theta}.$$
(A12)

Taking differences, and setting  $\delta = 1$ , we have

$$E(u_f) - E_d(u_f) = (1 - p)(y - s) + p[c_2 - c_0 + g(c_0) - g(c_1)] \equiv D_1(p).$$
 (A13)

We see that  $D_1'(p) < 0$ , because  $\frac{\partial c_2}{\partial p} < 0$ ,  $c_2 - c_0 + g(c_0) - g(c_1) < c_1 - c_0 + g(c_0) - g(c_1) < 0$ , and  $c_0$ ,  $c_1$  are independent of p. That is, the father is more likely to deviate the higher is p. Now  $D_1(p) = y - s > 0$  and  $D_1(1)$  and there must be  $p_1$  such that  $D_1(p_1) = 0$  and the father is indifferent between deviating and conforming. Note that

$$D_1(\frac{1}{2}) = \frac{1}{2}[(y-s) + (c_2 - c_0) + g(c_0) - g(c_1)]$$
(A14)

is negative for  $c^*$  close to y+z, implying that, in this case,  $p_1<\frac{1}{2}$ .

#### 8.2.2 Equilibrium at the upper boundary

A symmetric equilibrium at the upper boundary is either at  $\sigma = y - s$ , or at  $\sigma = \frac{y - s + \bar{\theta}}{2 - p}$ , whichever is larger. The incentive compatibility constraint is the binding one if  $\bar{\theta} < (y - s)(1 - p)$  and the feasibility constraint binds otherwise. We shall assume here that  $\bar{\theta}$  is sufficiently small to guarantee that incentive compatibility is the binding one in the "relevant range". We shall later set  $\bar{\theta} = 0$  to ensure that this always holds.

one in the "relevant range". We shall later set  $\bar{\theta} = 0$  to ensure that this always holds. If all fathers commit to pay  $\sigma = \frac{y-s+\bar{\theta}}{2-p}$ , implying that  $c = c^*$  upon remarriage, then the expected utility of each father is

$$E(u_f) = (1-p)[p(y-s) + (1-p)(y-s - \frac{y-s+\theta}{2-p})] +$$

$$p[p(y+z-c^*) + (1-p)(y+z-c^* - \frac{y-s+\bar{\theta}}{2-p})] +$$

$$+\delta g(c^*) + p\bar{\theta}$$

$$= y-s - \frac{y-s+\bar{\theta}}{2-p}(1-p) + \delta g(c^*) + p\bar{\theta}$$

$$= \frac{y-s+\bar{\theta}}{2-p} + \delta g(c^*) - (1-p)\bar{\theta}.$$
(A15)

We first note that no father wants to deviate up from this pattern and select  $\sigma > \frac{y-s+\bar{\theta}}{2-p}$ . Such deviation would mean that the father cannot remarry, because  $\sigma^- = \frac{y-s+\bar{\theta}}{2-p} > \bar{\theta} + y - s - \sigma(1-p)$  and also that the mother does not remarry because  $\sigma > \frac{y-s+\bar{\theta}}{2-p} = \bar{\theta} + y - s - (1-p)\frac{y-s+\bar{\theta}}{2-p}$ . In this case, the gain from not deviating becomes

$$E(u_{f}) - E_{d}(u_{f}) = \frac{y - s + \bar{\theta}}{2 - p} - (1 - p)\bar{\theta} - (y - s - \sigma)$$

$$> \frac{y - s + \bar{\theta}}{2 - p} - (1 - p)\bar{\theta} - (y - s) + \frac{y - s + \bar{\theta}}{2 - p}$$

$$= p \frac{y - s + \bar{\theta}}{2 - p} + p\bar{\theta} > 0.$$
(A16)

Now suppose that a father that deviates selects some  $\sigma < \frac{y-s+\bar{\theta}}{2-p}$ . Then, if the mother remarries,  $c = \hat{c} = h(\sigma - y_e^-, \bar{\theta})$ . That is,  $\hat{c}$  is determined by

$$\gamma'(c) = \frac{\gamma(c) + y - s - (1 - p)\frac{y - s + \bar{\theta}}{2 - p} - \sigma + \bar{\theta}}{c^* - c + \bar{\theta}}$$

$$= \frac{\gamma(c) + \frac{y - s + \bar{\theta}}{2 - p} - \sigma}{c^* - c + \bar{\theta}}.$$
(A17)

If the father remarries  $\tilde{c} = h(\sigma^- - y_e, \bar{\theta})$ . That is  $\tilde{c}$  is determined by

$$\gamma'(c) = \frac{\gamma(c) + y - s - (1 - p)\sigma - \frac{y - s + \bar{\theta}}{2 - p} + \bar{\theta}}{c^* - c + \bar{\theta}}$$

$$= \frac{\gamma(c) + (\frac{y - s + \bar{\theta}}{2 - p} - \sigma)(1 - p)}{c^* - c + \bar{\theta}}.$$
(A18)

Thus,  $\hat{c} \leq \tilde{c} < c^*$  and, for any fixed  $\sigma$ ,  $\hat{c}$  declines p, and  $\tilde{c}$  rises in p if  $\sigma < \frac{y-s+\bar{\theta}}{(2-p)^2}$  and declines in p if  $\frac{y-s+\bar{\theta}}{(2-p)^2} < \sigma < \frac{y-s+\bar{\theta}}{2-p}$ . The father expected utility upon deviation is now

$$E_{d}(u_{f}) = (1-p)[p(y-s) + (1-p)(y-s-\sigma] + p[p(y+z-\tilde{c}) + (1-p)(y+z-\tilde{c}-\sigma)] + \delta[(1-p)g(c^{*}) + pg(\hat{c})] + p\bar{\theta}$$

$$= (1-p)(y-s-\sigma) + p(y+z-\tilde{c}) + \delta[(1-p)g(c^{*}) + pg(\hat{c})] + p\bar{\theta}.$$
(A19)

Let  $\sigma(p)$  be the optimal deviation given that others choose  $\sigma = \frac{y-s+\bar{\theta}}{2-p} \equiv \sigma^-(p)$ , then using the envelope theorem,

$$\frac{dE_{d}(u_{f})}{dp} = c^{*} - \tilde{c} + \delta(g(\hat{c}) - g(c^{*})) + p\delta g'(\hat{c})h'(\sigma - y_{e}^{-}, \bar{\theta})(\frac{d\sigma^{-}}{dp}(1 - p) - \sigma^{-}(p)) 
-ph'(\sigma^{-} - y_{e}, \bar{\theta})(\frac{d\sigma^{-}}{dp} - \sigma(p)) + \bar{\theta}.$$
(A20)

Setting  $\delta = 1$ , and defining

$$D_0(p) \equiv E(u_f) - E_d(u_f) = \frac{y - s + \bar{\theta}}{2 - p} - (y - s) - p(c^* - \tilde{c}) + (1 - p)\sigma(p) + p[(g(c^*) - g(\hat{c})],$$
(A21)

we see that  $D_0(0) = -(y-s) + \frac{y-s}{2}$  and  $D_0(1) = g(c^*) - c^* - (g(\hat{c}) - \tilde{c}) > g(c^*) - c^* - (g(\hat{c}) - \tilde{c}) > 0$ . Therefore, there must exist  $p_0$  such that  $D_0(p_0) = 0$  and the father is indifferent between conforming to  $\frac{y-s+\bar{\theta}}{2-p}$  and deviating to  $\sigma(p)$ .

It is seen from (A20) that if  $\sigma(p) < \frac{d\sigma^-}{dp} = \frac{y-s+\bar{\theta}}{(2-p)2}$  then  $\frac{dE_d(u_f)}{dp} - \bar{\theta} < 0$ , because  $\frac{d\sigma^-}{dp}(1-p) - \sigma^-(p) = -\frac{y-s+\bar{\theta}}{2-p}$  and  $c^* - \tilde{c} + g(\hat{c}) - g(c^*)$  are both negative. In contrast,  $\frac{dE(u_f)}{dp} - \bar{\theta} > 0$ , because  $\frac{y-s+\bar{\theta}}{2-p}$  rises in p. It follows that  $D_0'(p) > 0$  if the best response against  $\sigma^- = \frac{y-s+\bar{\theta}}{2-p}$  is to set  $\sigma(p) < \frac{y-s+\bar{\theta}}{(2-p)2}$  and, as a special case, if  $\sigma(p) = 0$ . Such an outcome arises, if  $E_d(u_f)$  is convex globally or locally, in the sense that  $\frac{\partial E_d(u_f)}{\partial \sigma} = 0 \Rightarrow \frac{\partial^2 E_d(u_f)}{\partial \sigma^e} > 0$ , so that there is no interior solution for  $\sigma$  in the range  $(0, \frac{y-s+\bar{\theta}}{2-p})$ . In this case,  $D_0'(p) > 0$  and there is a unique  $p_0$  such that  $D_0(p_0) = 0$  and the father is indifferent between conforming to  $\frac{y-s+\bar{\theta}}{2-p}$  and deviating to  $\sigma = 0$ .

Assuming that either  $\sigma(p) = 0$  or  $\sigma(p) = \frac{y-s+\bar{\theta}}{2-p}$  are the only possible best responses against  $\frac{y-s+\bar{\theta}}{2-p}$ , definition (28) implies that  $D_0(p) > -D_1(p)$  if  $\sigma$  and  $\sigma^-$  are strategic complements and  $D_0(p) < -D_1(p)$  if  $\sigma$  and  $\sigma^-$  are strategic substitutes, as defined in (28). From the results that  $D_0'(p) > 0$  and  $D_1'(p) < 0$ , it then follows that  $p_1 > p_0$  if  $\sigma$  and  $\sigma^-$  are strategic complements  $p_1 > p_0$  if  $\sigma$  and  $\sigma^-$  are strategic complements and  $p_1 < p_0$  if  $\sigma$  and  $\sigma^-$  are strategic substitutes.

### 8.2.3 An example

The functional forms used in the examples are as follows. The utility of the children is

$$g(c) = 2c - \frac{c^2}{2}.$$

This specification satisfies  $g'(\hat{c}) \leq 2$  so that the father does not want to transfer if the mother remarries. In this case,

$$c^* = 1,$$

$$g(c^*) = \frac{3}{2}.$$

The implied  $\gamma(c)$  function used in the solution of the Nash Bargaining model is

$$\gamma(c) = g(c) - c - (g(c^*) - c^*) = c - \frac{c^2}{2} - \frac{1}{2}$$
  
$$\gamma'(c) = 1 - c.$$

Solving

$$\gamma'(c) = \frac{\gamma(c) - x + \bar{\theta}}{c^* - c + \bar{\theta}},$$

we get

$$\hat{c} = 1 + \frac{\bar{\theta}}{3} - \frac{[\bar{\theta}(\bar{\theta} + 6) + 6(y_e^- - \sigma)]^{\frac{1}{2}}}{3},$$

which is reduced to

$$\hat{c} = 1 - (\frac{2}{3}(y_e^- - \sigma))^{\frac{1}{2}}$$

if  $\bar{\theta} = 0$ . Also if  $\bar{\theta} = x$  then  $\hat{c} = c^* = 1$ .

We can use this specification to verify that the Nash bargaining problems defined above have interior solutions, as we have implicitly assumed. Rewriting the Nash requirement as

$$\gamma'(c)(c^* - c + \bar{\theta}) - \gamma(c) = y + z - c^* + \bar{\theta}$$

and noting that  $\gamma'(0) = 1$ , we require that

$$c^* + \bar{\theta} + g(c^*) - c^* > y + z - c^* + \bar{\theta}$$
  
 $g(c^*) + c^* > y + z,$ 

which would guarantee that  $0 < c_0 < c^*$ . Because  $c_0$  is the lowest solution, the other cases values  $c_1, c_2, \hat{c}$ , and  $\tilde{c}$  are also positive.

A convenient choice for the matching function  $p = \phi(d)$  is

$$\phi(x) = 1 - (1 - d)^{\alpha},$$

where  $\alpha \geq 1$ . This function maps from [0,1] to [0,1] and satisfies  $\phi(0) = 0$ ,  $\phi(1) = 1$ ,  $\phi(d) > x$ ,  $\phi'(d) > 0$ .

A higher  $\alpha$  corresponds to a better matching function. With a uniform distribution for  $\theta$  on [-u, u], the equilibrium condition becomes

$$p = 1 - (1 - F(\theta^*(p)))^{\alpha}$$
$$= 1 - (1 - \frac{\theta^*(p) + u}{2u})^{\alpha}$$

or,

$$\theta^*(p) = u(1 - 2(1 - p)^{\frac{1}{\alpha}}).$$

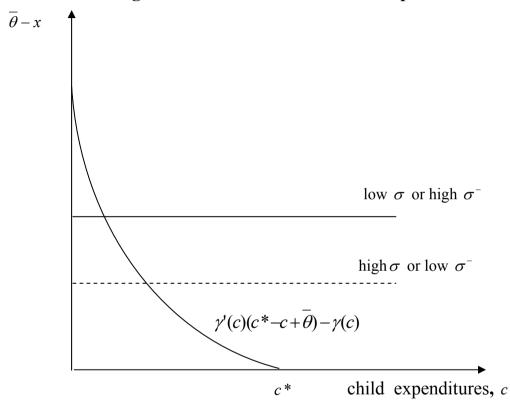
# References

- [1] Aiyagari, S., Greenwood J. and N. Guner (2000), "On the State of the Union," **Journal of Political Economy**, 108, 213-244
- [2] Argys, L., Peters, E., Brooks-Gunn, J. and J. Smith (1998), "The Impact of children Support on Cognitive Outcomes of Young Children," **Demography**, 35, 159-173.
- [3] Becker, G., Landes E. and R. Michael (1977), "An Economic Analysis of Marital Instability," **Journal of Political Economy**, 85, 1141-1188.
- [4] Becker G. (1991), **Treatise on the Family**, Haravard University Press.
- [5] Bisin, A., G. Topa and T. Verdier (2004), "Religious Intermarriage and Socialization in the US," **Journal of Political Economy**, **112**, 612-665.
- [6] Bjorklund, A. and M. Sundstrom (2002), "Parental Separation and Children Educational Attainment: A Sibling Approach," IZA Discussion Paper 643.
- [7] Black, D., Gates, G., Sanders, S. and L. Taylor (2000), "Demographics of the Gay and Lesbian Population in the United States: Evidence from Available Systematic Data Sources," **Demography**, 37, 139-154.
- [8] Burdett, K. and M. Coles, (1997), "Marriage and Class", Quarterly Journal of Economics; 112, 141-68.
- [9] Burdett, K., and M. Coles (1999), "Long-Term Partnership Formation: Marriage and Employment," **The Economic Journal**, 109, F307-F334.
- [10] Burdett, M. and R. Wright (1998), "Two Sided Search," Review of Economic Dynamics, 1, 220-245.
- [11] Clarke, S. (1995), "Advance Report of Final Marriage Statistics, 1989-90," Monthly Vital Statistics Report, 43, No 9, Supplement.
- [12] Cole, H., Mailath, G. and A. Postlewaite (1998), "Class Systems and the Enforcement of Social Norms," **Journal of Public Economics**, 70, 5-35.
- [13] Costa, D. and M. Kahn (2000), Power Couples: Changes in the Locational Choice of College Educated, 1940-1990," Quarterly Journal of Economics, 115, 1287-1315.

- [14] Del-Boca, D. (2003). "Mothers, Fathers and Children after Divorce: The role of Institutions," **Journal of Population Economics**, 16, 399-422.
- [15] Diamond, P. and E. Maskin (1979), "An Equilibrium Analysis of Search and Breach of Contract, I: Steady States," **The Bell Journal of Economics**, 10, 282-316.
- [16] Diamond, P. (1982), "Aggregate Demand Management in Search Equilibrium," **Journal of Political Economy**, 90, 881-894.
- [17] Glaeser, E. and J. Scheinkman (2002), "Non-Market Interactions", in **Advances** in **Economics and Econometrics Theory and Applications, Eight's World Congress**, M. Dewaterpoint, L. Hansen and S. Turnovsky (eds.).
- [18] Goldin, C. and L. Katz (2002), "The Power of the Pill: Oral Contraceptives and Women's Career and Marriage Decisions," **Journal of Political Economy**, 110, 730-770.
- [19] Gruber, J. (2004), "Is making Divorce Easier Bad for Children? The Long Run Implications of Unilateral Divorce," **Journal of Labor Economics**, 22, 768-799.
- [20] Hetherington, M. and M. Stanley-Hagan (1999), "Stepfamilies," in M. Lamb (ed.) Parenting and children Development in "Non-Traditional" Families, Mahwah: Lawrence Erlbaum.
- [21] Ishida, J. (2003), "The role of Social Norms in a Model of Marriage and Divorce," **Journal of Economic Behavior&Organization**, 51, 131-142.
- [22] Lamb, M., Sternberg, K. and R. Thompson (1999), "The Effects of Divorce and Custody Arrangements on Children's Behavior, Development and Adjustment," in M. Lamb (ed.) Parenting and children Development in "Non-Traditional" Families, Mahwah: Lawrence Erlbaum.
- [23] Lauman, O., Gagnon, J., Michael R. and S. Michaels (1994), **The Social Organization of Sexuality: Sexual Practices in the US,** Chicago: The University of Chicago Press.
- [24] Lerman R. (2002), "The impact of Marital Status and Parental Presence on the Material Hardship of Families with Children," The Urban Institute.
- [25] Lerman, R. and E. Sorensen (2003), "children Support: Interactions Between Private and Public Transfers," in R. Moffit (ed.) **Mean Tested Transfer Programs**, University of Chicago Press.
- [26] Norton, A. and L. Miller (1992), "Marriage, Divorce, and Remarriage in the 1990's," Current Population Reports, Special Studies, P23-180.

- [27] Michael, R. (1988), "Why Did the Divorce Rate Double Within a Decade?" Research in Population Economics, 6, 367-399.
- [28] Mnookin, R. and Kornhauser, R. (1979), "Bargaining in the Shadow of the Law," Yale Law Journal, 88, 950-997.
- [29] Mortensen, D. (1988), "Matching: Finding a Partner for Life or Otherwise," **American Journal of Sociology**, 94 (supplement), s215-s240.
- [30] Piketty, T. "The Impact of Divorce on School Performance: Evidence from France, 1968-2002," An unpublished Manuscript, EFESS, Paris- Jourdan.
- [31] Weiss, Y. and R. Willis (1993), "Divorce Settlements: Theory and Evidence," **Journal of Labor Economics**, 11, 629-679.





 $<sup>^{1}</sup>$   $c^{*}$  is the efficient level of child expenditures,  $\overline{\theta}$  is the expected quality of match and x is the difference between the father's commitment,  $\sigma$ , and the new husband's expected income,  $(y-s)-\sigma^{-}(1-p)$ . The function  $\gamma(c)$  equals  $g(c)-c-(g(c^{*})-c^{*})$ . The graph is drawn for the case in which g'(c) is convex.

Figure 2a: Incentives to commit  $V_f(\sigma^-, \sigma^-) - V_f(\sigma, \sigma^-)$  in relation to the probability of remarriage, p, and commitments of others,  $\sigma^- = \frac{y-s+\overline{\theta}}{2-p}$  or  $\sigma^- = 0$ , when  $\sigma^-$  and  $\sigma$  are strategic complements

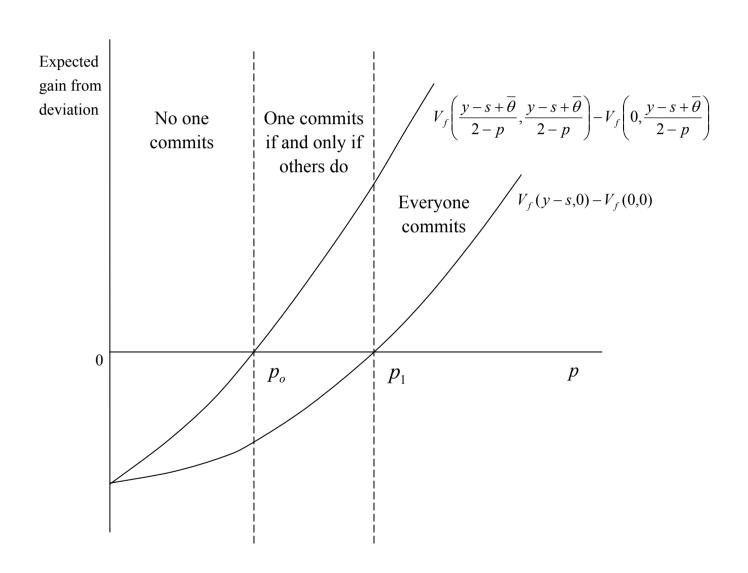


Figure 2b: Incentives to commit  $V_f(\sigma^-, \sigma^-) - V_f(\sigma, \sigma^-)$  in relation to the probability of remarriage, p, and commitments of others,  $\sigma^- = \frac{y-s+\overline{\theta}}{2-p}$  or  $\sigma^- = 0$ , when  $\sigma^-$  and  $\sigma$  are strategic substitutes

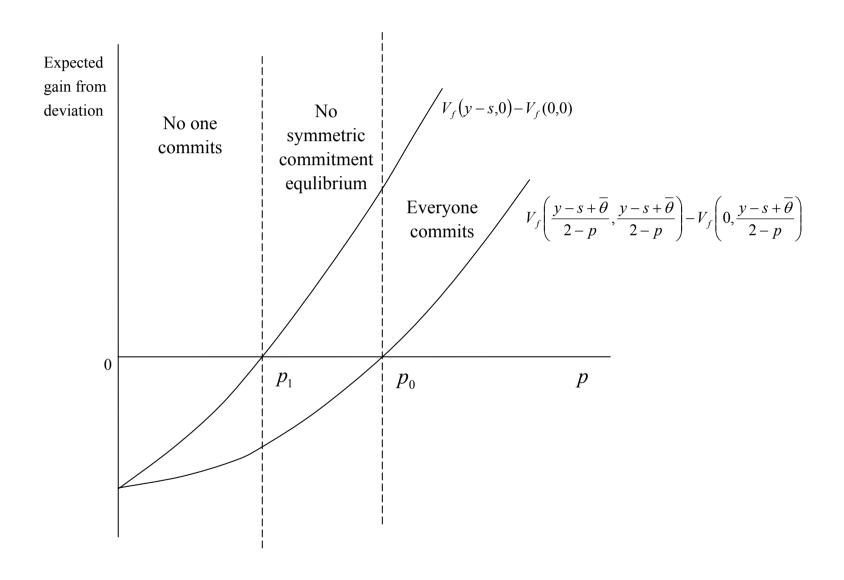


Figure 3: Equlibrium points with high and low variability of shocks

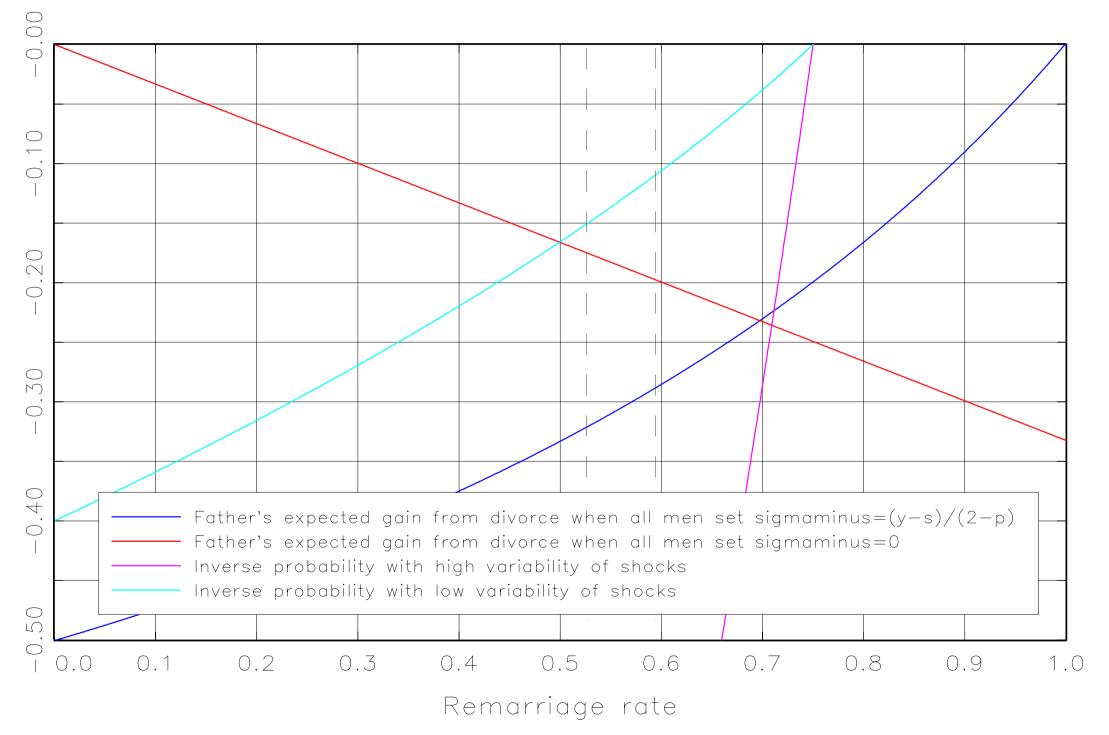


Table 1 : Equlibrium divorce and remarriege rate for different parameters values

Changes in the variance of the shocks, u. Matching parameter,  $\alpha=2$ 

Commitment	Variability parmeter	Divorce rate	Remar.	$\begin{array}{c} \text{Child} \\ c \end{array}$	Child $E(u_c)$	Mother $E(u_m)$	Father $E(u_f)$
$\sigma = \frac{y-s}{2-p}$ $\sigma = 0$	3.0	.46	.71	1.0	1.5	2.28	2.28
	.4	.29	.50	.18	.93	1.83	2.33

## Changes in the matching parameter, u. Variability parameter, u = 1.5

Commitment	Matching parmeter	Divorce rate	Remar.	$\frac{\text{Child}}{c}$	Child $E(u_c)$	Mother $E(u_m)$	Father $E(u_f)$
$\sigma = \frac{y-s}{2-p}$ $\sigma = 0$	3.0	.45	.84	1.0	1.5	2.36	2.36
	1.33	.44	.54	.4	.88	1.86	2.32

The fixed parameters are: Proximity,  $\delta = 1$ . Efficient level of child expenditure,  $c^* = 1$ . Family income, y + z = 2. Critical value when other fathers commit,  $p_0 = .594$ . Critical value when other fathers do not commit,  $p_1 = .526$ .

Figure A1: The feasibility and incentive compatibility constraints on the commitment by the father,  $\sigma$ , and the commitments of others  $\sigma^-$ 

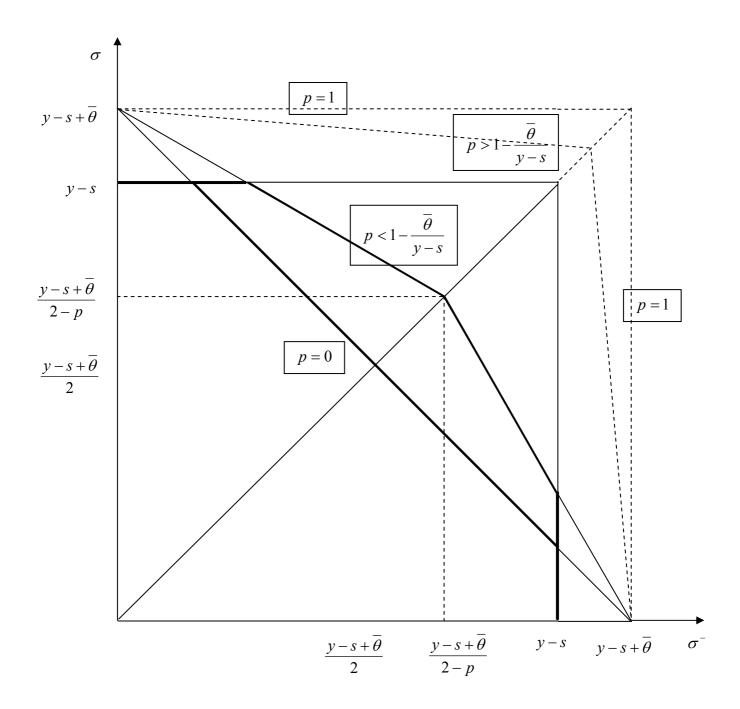


Table B1: Marital history by age and sex US,  $1996^1$ 

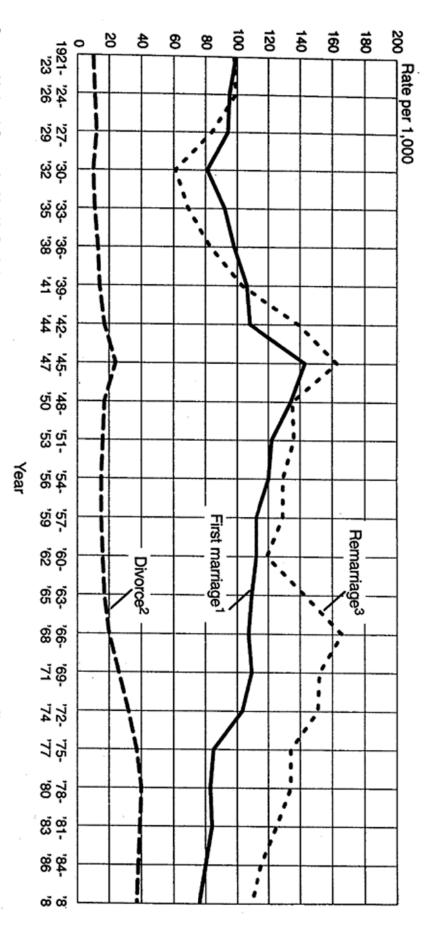
Age:	30-34	35-39	40-49	50-59	60-69		
Men (White, non-Hispanic)							

Never married	25.5	16.9	10.5	5.1	4.6	
Married Once	64.1	66.0	62.9	64.4	70.4	
Married Twice	9.7	14.6	21.2	22.8	19.2	
Married 3 or more times 0.7 2.5 5.4 7.8 5.9						
Women (White, non-Hispanic)						

Never married	14.3	10.9	6.8	4.2	3.2
Married Once	70.5	67.8	65.3	68.7	74.6
Married Twice	13.5	17.4	21.9	20.0	17.7
Married 3 or more times	1.7	3.8	5.9	7.2	4.4

<sup>&</sup>lt;sup>1</sup>Source: U.S. Census Bureau, Survey of Income and Program Participation (SIPP), 1996

(3-Year Averages) Figure 1. Rates of First Marriage, Divorce, and Remarriage: 1921 to 1989



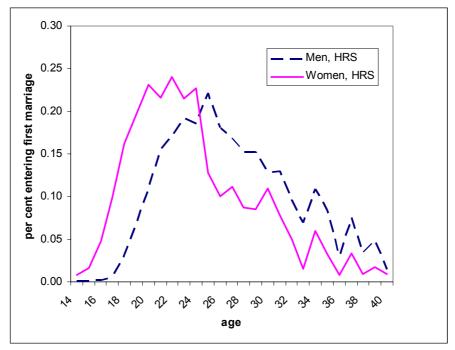
Source: National Center for Health Statistics.

<sup>&</sup>lt;sup>1</sup>First marriages per 1,000 single women, 15 to 44 years old

<sup>&</sup>lt;sup>2</sup>Divorces per 1,000 married women, 15 to 44 years old.

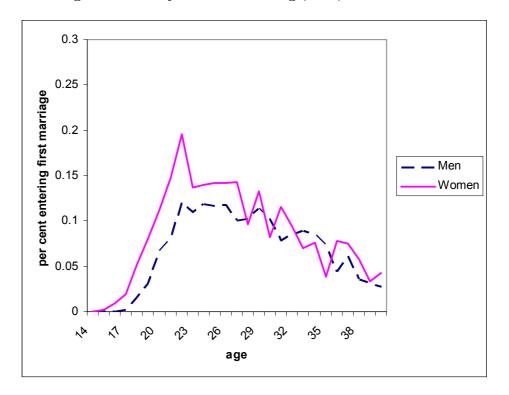
<sup>3</sup>Remarriages per 1,000 widowed and divorced women, 15 to 54 years old.

 ${\bf Figure \, B2 \, Entry \, into \, first \, Imarriage, \, US., \, HRS}$ 



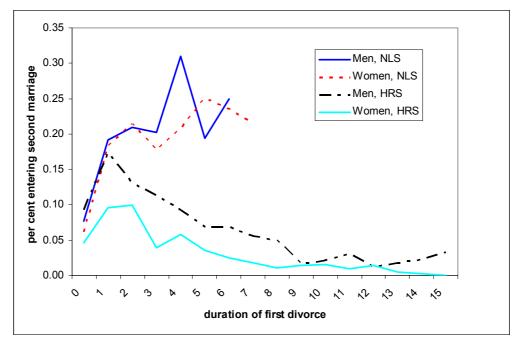
Source:  $\Box$ HRS \( \panel \) data \( \Box 1992 \( \wave \)

 ${\bf Figure B3:} \\ {\bf Entry into first imarriage, ius., NLS}$ 



 $Source \verb| CNLS | \verb| panel | \verb| data | | \verb| Youth | | 1979$ 

 ${\bf Figure B4:} {\bf Entry Cin To Sencond Cmarriage}$ 



 $Source \\ \square NLS \\ \square panel \\ \square data, \\ \square Youth \\ \square 979, \\ \square HRS \\ \square panel \\ \square data, \\ \square 992 \\ \square wave$ 

0.08 0.07 Early marriage, NLS Late marriage, NLS 0.06 divorce cent entering divorce 0.04 0.03 0.02 - Early marriage, HRS Late marriage, HRS 0.01 0.00 0 ტ 6 9 N. ゟ **γ**% g duration of first marriage

Figure B5: Entry into divorce

 $Source \verb| INLS | \verb| panel | \verb| data, | Youth | 1979, | HRS | \verb| panel | | data, | 1992 | | wave$