Solving the Puzzle – Hours constraints, Technical Change and Female Labor Supply*

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Abstract

Women with small children have substantially increased their career commitment in terms of labor supply on the intensive margin in recent decades. Concurrently, these women have also entered into more complex jobs. This paper extends the standard theory of labor supply to incorporate an important ingredient in the labor supply decision of today's women; the role of flexibility and time constraints. To describe and formalize this notion, I set up a life cycle model where labor supply depends on a family constraint (child rearing), requirement for minimum hours in different types of jobs and the variation in flexibility of organizational technology. I show that as technology allows jobs to become more flexible, time constrained individuals can supply more hours and may therefor find it attractive to opt for a more demanding career.

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1 Introduction

"Having control over your schedule is the only way that women who want to have a career and a family can make it work"

Anne-Marie Slaughter, The Atlantic

The theory outlined in this paper extends the standard theory of labor supply to incorporate an important ingredient in the labor supply decision of today's women: the role of flexibility and time constraints. There has been a rapid development in the possibilities of a more flexible working life. The emergence and increasing use of email, remote access, video conferences etc. has decreased the need for face time in working life and increased the possibility of being productive in other locations than the actual workplace, thus allowing individuals that are time constrained to supply more hours of work by shifting hours during the day to make room for necessary commitments at home, for instance by doing some work at night, early mornings, weekends etc from home. Hence, as technology makes jobs more flexible, time constrained individuals can supply more hours of work and may therefore find it attractive to opt for a more demanding career.

To describe and formalize this notion, I set up a life cycle model where labor supply depends on a family constraint (child rearing), minimum hours requirements an variation of flexibility in different jobs. The basic intuition is simple: Having children requires parents to drop off and pick up at day care, provide meals, care, help with homework – in short: parenting. Assuming that these household activities can only be partially outsourced, this implies that parents face a binding time constraint. Not only due to the actual time needed for child rearing, but also reflecting the fact that these activities must be carried out at a certain point in time during the day, which infringes on the hours available for work. Moreover, if more complex jobs have a constraint on the hours of work that need to be supplied for the job to be productive, this will imply that individuals that are family constrained, taking the larger responsibly for child rearing, cannot choose these jobs - or can only choose them with large sacrifices as concerns other activities.

Although the role of flexibility has been discussed in the literature (Goldin and Katz, 2011; Flabbi and Moro, 2012), the focus has been on selection into different occupations. The link to labor supply on the intensive margin (and its consequences for occupational choice) has not been considered so far. I formalize the notion that as technology makes jobs more flexible, time constrained individuals can supply more hours and may therefore find it attractive to opt for a more demanding career. The model thus offers a possible mechanism to explain recent changes in female labor supply, where women today both work more on the intensive margin and to a larger extent take on more demanding jobs.

There has been an increase in female labor supply since the beginning of the 1970's and today, women participate in the labor market almost to the same extent as men.[1][2] In Sweden, female labor force participation increased rapidly from below 40 percent in

[1] In 2007, 14 OECD countries had a female labor force participation over 80 percent with the highest participation in the Nordic countries with Sweden at the top at 87.1 percent.

[2] See figure A1 in appendix

the 1960's to over 80 percent already in the 1980's (Gustafsson and Jacobsson, 1985), but it has leveled off since then. The rise in female labor supply is widely studied in the literature, in particular the extensive margin, where the major shifts have occurred.[3] Several papers have documented that this increase was primarily driven by a surge in the labor supply of married women e.g. (Attanasio et al., 2008; Olivetti, 2006; Goldin and Olivetti, 2013). This is the case also in Sweden where by the mid 1980's, there was virtually no difference between the participation rates of married and unmarried women (Selin, 2009).

The intensive labor supply margin, which constitutes the main focus of studies of male labor supply, is much less studied for women. With participation now being so high, it arguably becomes important to also study the conditions forming female behavior at the intensive margin. Clearly, men and women to some extent face different conditions, the most obvious being the consequences of having a family, leading to a natural career break for women when the children are newborns, but also affecting women's working conditions for many years after that. The reason is that there is a time constraint associated with having a family. This family constraint consists of two parts. First, it is the actual time needed for child rearing. The second part is related to timing: some of these activities must be carried out at certain points in time during the day (e.g. picking up from day care) and even if there is no actual activity together with the child, it is still a restriction on *being at home*.

The family constraint may also affect what types of occupations an individual can choose. In particular, many more complex and career oriented jobs are likely to be associated with a requirement on the minimum hours that need to be supplied for the job to be productive and lead to a successful career. This might, for instance, be the case for many management jobs and professional occupations where labor cannot be easily divided. Such jobs will thus be characterized by a tied hours-wage package, i.e. a career constraint on the hours supplied. Altonji and Paxson (1988) study the trade-off between working hours and wages in the labor market where employers place restrictions on the hours required for the job and show that individuals that currently work more than what they find optimal would be willing to forgo some pay in order to adjust their hours. Similarly, individuals who work less than they prefer would be willing to sacrifice wage gains for additional hours. For a family constrained individual, such restrictions on minimum hours could thus mean that it is optimal to choose a job at a lower level than what the individual is qualified for.[4]

Empirically, men work full-time. For women, entering the labor market has arguably

[3]See e.g. Heckman and Macurdy (1980); Altonji and Paxson (1988); Imai and Keane (2004); Killingsworth and Heckman (1986) and, recently, Keane (2011) for an overview.

[4]Relatedly, Albanesi and Olivetti (2009) show that an equilibrium where women have higher home hours and lower earnings can arise even if there are no ex ante gender differences. They employ a principal agent framework, where the cost of effort at work increases with home hours and both home hours and effort are private information and show that it is enough that firms believe that the distribution of home hours differs by gender for a gendered equilibria to arise. Incentive compatible contracts will generate a self-fulfilling prophecy where women will be offered contracts with lower earnings, effort and performance pay relative to men, which in turn gives them a lower relative opportunity cost of home hours. Women will thus allocate more time for home hours which confirms the beliefs of the firm.

created a trade off between family and career.[5] To meet the family constraint, some women may choose less career oriented occupations and/or part time jobs. The different patterns for female and male labor supply may thus be due to time constraints, rather than differences in preferences.

In Sweden, active policy promotion of gender equality through the introduction of individual taxation, subsidized childcare and generous parental leave schemes has aimed at facilitating the family-career trade off.[6] In spite of these policies, the gender wage gap stagnated already in the 1980's (Edin and Richardson, 2002). Studying US top professionals, Bertrand et al. (2010) show that there is still a sharp career penalty to having a family, finding that male MBA earnings outperform those of women by 60 log points a decade after graduation. They identify differences in career interruptions and weekly hours, both largely associated with motherhood, to be two of the main explanatory factors. For Sweden, Angelov et al. (2013) find that the spousal income and wage gaps increase by 35 and 10 percent fifteen years after the birth of the first child.

Goldin and Katz (2011) explore how flexibility affects occupational sorting. They discuss the flexibility of different occupations in a compensating differentials framework and study how the career costs of having a family vary between different high-end occupations and also across time. If women value flexible work hours more than men, they will disproportionally sort into occupations where this flexibility comes at a lower price.[7] When the firms' cost of providing flexibility decreases and with an influx of women demanding this amenity, more employers will adopt such practices. They provide empirical evidence showing that many high-end professions have experienced an increased workplace flexibility and that some, in particular within the corporate and financial sector, have lagged behind. The contribution of this paper is to analyze how the the ability to flexibly shift hours between activities during the day may loosen the career constraint and thereby affect not only occupational choice, but also womens' labor supply on the intensive margin in a given occupation.

The remainder of the paper is organized as follows: In the next section, I motivate the paper by looking at patterns for labor supply and occupational choice using micro data for Sweden. Then, in section 3, I first present a simplified version of the model, using a static set up with standard Cobb-Douglas utility. It delivers straightforward analytical solutions and illustrates the mechanism of the model in a transparent way. Labor supply is affected by the flexibility of the job and time constraint associated with the family constraint. This implies that time constrained individuals will choose the career oriented

[5] Goldin (2004) studies the career-family choices for five generations of American women during the twentieth century. She describes how the conditions have changed from "family or career" for the cohort that graduated from college at the beginning of the twentieth century, via "job then family", "family then job", "career then family", for the following generations, to "career *and* family" being the goal of the most recent generation that graduated between 1980 to 1990.

[6] Selin argues that the tax reform in 1971, when individual taxation was introduced, played a major role in this rapid increase in the labor force participation of married women. The tax reform implied large cuts in marginal taxes. For example, if both spouses worked in an average blue collar job, the marginal tax rate fell from 55 percent to 32.5 percent in 1971 (Sundström, 1991).

[7] Flabbi and Moro (2012) find that college graduates place a higher value on having flexible jobs compared to high school graduates. They also find that jobs requiring a college education can provide flexibility at a lower cost.

job only if they are willing to deplete leisure, or if the job is flexible enough. Hence, I show that i) as technology allows more flexibility in jobs, it becomes optimal for family constrained individuals to pursue more career oriented occupations, and ii) when pursuing the more demanding career, there is a large reallocation of time, where both leisure and home production decrease and flexible working hours and market hours increase.

Next, in section 4, I reformulate the model in a life cycle setting to include two additional important features: First, working life is long and the period when individuals are constrained by having small children is relatively short. Thus, the family constraint does not bind in every period. Second, as shown in e.g. Angelov et al. (2013); Bertrand et al. (2010); Datta Gupta and Smith (2002), there is evidence of penalties associated with taking time off the labor market or working shorter hours. Thus, it is important to take the possible effects of returns to market experience into account. Following Imai and Keane (2004); Olivetti (2006); Wallenius (2011), this is done by making the wage (skill) in the next period dependent on the hours worked (and the type of job) in the previous period. The results show that when flexibility is low, the labor supply profile of family constrained individuals exhibits the double peaked profile documented by Olivetti (2006) for women in the 1970's. As technology improves, the lifecycle profile gradually becomes more similar to the single peaked profile of the unconstrained individuals. Taking the lifecycle into account also further strengthens the incentives to trade off; As the stakes are higher in the sense that current labor supply choices carry over into future periods in terms of wages, it will be optimal to pursue the more demanding career at lower levels of flexible technology. As a consequence, leisure is further depleted to make the more demanding career feasible. Comparing period utility, this trade-off results in family constrained individuals having lower utility in the more demanding career as compared to the simpler career in the period when the family constraint binds. Although not modeled, this has implications for women opting out or shifting to a less demanding job when the requirements from home are most demanding. Relaxing the career constraint in the life cycle model gives some additional insights. When the more demanding job is characterized by accumulation and depreciation of skill, the career cost of less working hours during child bearing years is substantial. In the model, the decrease in hours worked translates into a wage difference where the wage of the family constrained individual is about 70 percent of that of the unconstrained individual at the end of working life. This wage difference decreases as technology improves and when technology is fully flexible, the gap to the unconstrained individual closes (almost) completely as the total efficient labor supply increases. Section 5 concludes the paper.

2 Patterns in data

An empirical observation among Swedish working women constitutes the starting point: Although the participation levels in Sweden flattened out, female labor supply has continued to increase along the intensive margin. For women with small children, the career commitment in terms of the share of full-time has increased substantially over the last fifteen years. Moreover, this movement on the intensive margin came in parallel with these women entering into more complex jobs.

The family constraint can be expected be most severe when the children are relatively small. I use the presence of children aged below seven to identify potentially family constrained individuals. In figure 1, I use data from the Swedish structural wage statistics and plot the average labor supply of women with children of preschool age over the years 1996-2009 for the private and the public sector, respectively.[8] Individual labor supply is measured as fractions of full-time. Each employee has a supply of between 1 percent and 100 percent (full-time), i.e. it only measures supply on the intensive margin. Data is aggregated into four groups of occupations: Higher-level managers and professionals, lower-level managers/professionals and higher-level supervisors and technicians, intermediate and lower level sales, service and routine.[9] Figure 1 shows that women with small children working as higher-level or lower-level managers or professionals in the private sector (the solid black line) have, on average, increased their labor supply by about four percentage points between 1996 and 2009. Of these women, higher-level managers and professionals work the most, on average 95.4 percent of full-time in 2009, and are now closing the gap to men. Comparing the public to the private sector, labor supply is lower in the public sector. The increase in labor supply on the intensive margin is qualitatively similar, but the increase is smaller among higher-level managers/professionals.[10] The labor supply for lower-level sales occupations , service and routine jobs (the dash-dotted line in figure 1), on the other hand, has remained relatively unchanged during the period with an average value of 84 percent.

At the same time as there has been an increase in labor supply, there seems to have been a remarkable change in the work of these women. Figure 2 shows how the employment shares for the four different occupational groups have evolved over time. An interesting pattern emerges where the share of women with small children in employment being higher-level managers and professionals doubled and lower-level managers and professionals increased by 70 percent within the private sector. The public sector experienced a similar change in employment shares, but women with small children are less represented among higher-level managers and professionals and more among lower-level managers and professionals as compared to the private sector. During the same period, the employment share in lower-level sales occupations, service and routine jobs has decreased by 13 and 15 percentage points in the private and the public sector, respectively.

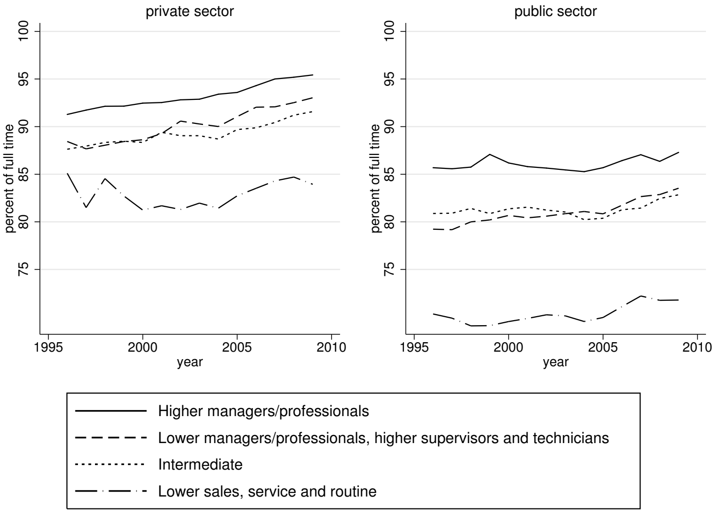
Tables 1 and 2 summarize the corresponding figures for men and women, with and without children of preschool age, in the private and public sector, respectively. It is evident from table 1 that almost all men in the private sector work full-time, regardless of whether they have small children at home or not. Men in the public sector work less, but once more, there is little difference in labor supply based on having children of preschool

[8]The Swedish structural wage statistics covers about 50 percent of the employees in the private sector and all employees in the public sector. To get information about the number of children, the structural wage statistics is matched to population-wide registers rich in demographic variables; see appendix A for details.

[9]The groups are formed aggregating ISCO88 occupational codes to an aggregated version of European Socio-economic Classification (ESeC). See appendix A for details.

[10]This picture is confirmed in the latest Swedish Time Use Survey (Sweden, 2012) where the difference in hours worked for men and women with children of preschool age has declined from 17 in 1990 to just under 10 in 2011, i.e. a decline by 42 percent. Almost all of this change has occurred since the beginning of the 2000's.

Figure 1: Intensive margin labor supply, women with children of preschool age, by sector.



age or not. It is interesting to note, though, that, if anything, the trend among men is to work slightly less, comparing 2009 to 1996, in particular within the public sector.

The increase in the skill content of women's work is studied in Black and Spitz-Oener (2010) using data from Germany 1979-1999. They document that women have witnessed an increase in non-routine analytical tasks and an even more pronounced decrease in routine tasks, relative to men. Moreover, they find that the task content has changed most rapidly in occupations where computers have made major headway. In that perspective, it is interesting to see how the trends in employment shares differ between different industries with different IT profiles. For this purpose, I use the classification of industries in van Ark et al. (2003) grouping industries according to ICT intensity.[11] In figure 3, the employment shares for women with small children are plotted for the four different occupation groups. Interestingly, the large increase in the employment share of higher managers and professionals is driven by changes within ICT-producing manufacturing and service industries supporting the results in Black and Spitz-Oener (2010).

The patterns described in this section are just unconditional trends. Over the period studied, there have been significant compositional changes in the group of women with small children. In particular, women are becoming increasingly more educated than men and also become mothers later in life (Boschini et al., 2011). This can probably explain part of the trends. Fully sorting these patterns out is beyond the scope of this paper. However, as a first-order check, I do the same tabulation holding education constant and restricting the ages observed. Looking only at women with a university degree (minimum 15 years of education) of ages 30-45, the share working as higher-level managers/professional increased by 5 percentage points between 1996 and 2009 among women with children aged

[11] Seven groups are formed: ICT producing manufacturing, ICT using manufacturing, ICT producing service, ICT using service, Non-ICT service, Non-ICT manufacturing and Non-ICT other. Details are given in appendix A.

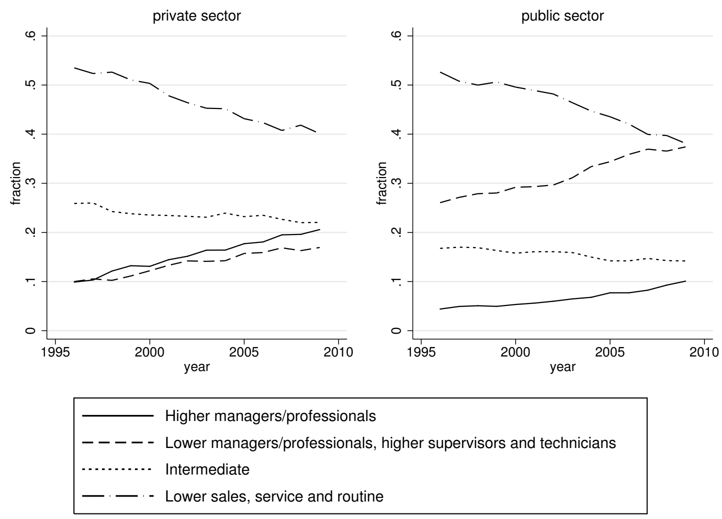
Table 1: Employment shares and labor supply 1996 and 2009, private sector

Type of job	**Fraction of fulltime %**			**Empl. share %**		
	1996	**2009**	**Δ**	**1996**	**2009**	**Δ**
		Women with children aged below 7				
Higher-level managers/professionals	91.3	95.4	4.1	9.9	20.6	10.7
Lower-level managers/professionals, higher-level supervisors and technicians	88.4	93.0	4.6	10.0	16.9	6.9
Intermediate	87.6	91.6	3.9	25.9	22.0	-3.9
Lower sales, service and routine	85.1	83.9	-1.2	53.5	40.1	-13.4
		Women without children aged below 7				
Higher-level managers/professionals	97.3	97.4	0.0	7.4	12.4	5.0
Lower-level managers/professionals, higher-level supervisors and technicians	92.7	94.2	1.4	9.5	13.4	4.0
Intermediate	92.6	93.7	1.1	27.6	21.7	-5.9
Lower sales, service and routine	84.9	82.6	-2.3	54.7	52.3	-2.4
		Men with children aged below 7				
Higher-level managers/professionals	99.4	99.2	-0.2	13.9	24.8	10.9
Lower-level managers/professionals, higher-level supervisors and technicians	99.3	98.8	-0.5	13.8	19.6	5.8
Intermediate	99.4	98.8	-0.7	9.4	10.5	1.1
Lower sales, service and routine	97.0	97.0	0.1	59.5	44.4	-15.1
		Men without children aged below 7				
Higher-level managers/professionals	98.9	99.0	0.1	12.0	16.7	4.7
Lower-level managers/professionals, higher-level supervisors and technicians	98.7	98.6	-0.1	14.2	16.6	2.3
Intermediate	98.0	97.2	-0.8	8.7	8.9	0.3
Lower sales, service and routine	95.3	94.7	-0.6	60.7	57.0	-3.6

Table 2: Employment shares and labor supply 1996 and 2009, public sector

Type of job	Fraction of fulltime %			Empl. share %		
	1996	**2009**	**Δ**	**1996**	**2009**	**Δ**
		Women with children aged below 7				
Higher-level managers/professionals	85.7	87.3	1.6	4.4	10.1	5.7
Lower-level managers/professionals, higher-level supervisors and technicians	79.2	83.5	4.3	26.1	37.4	11.4
Intermediate	80.9	82.8	2.0	16.8	14.2	-2.6
Lower sales, service and routine	70.3	71.8	1.5	52.6	38.1	-14.5
		Women without children aged below 7				
Higher-level managers/professionals	91.9	91.4	-0.5	4.3	7.4	3.1
Lower-level managers/professionals, higher-level supervisors and technicians	89.3	90.2	1.0	29.7	31.9	2.2
Intermediate	88.2	88.8	0.6	16.9	15.5	-1.4
Lower sales, service and routine	75.0	77.6	2.6	48.9	45.1	-3.8
		Men with children aged below 7				
Higher-level managers/professionals	95.0	92.8	-2.2	20.7	23.1	2.4
Lower-level managers/professionals, higher-level supervisors and technicians	94.5	93.5	-1.0	38.9	40.9	2.0
Intermediate	91.6	92.7	1.1	7.8	9.8	2.0
Lower sales, service and routine	90.3	86.3	-4.1	32.0	25.0	-7.0
		Men without children aged below 7				
Higher-level managers/professionals	95.8	93.3	-2.5	18.8	18.8	-0.1
Lower-level managers/professionals, higher-level supervisors and technicians	94.9	94.0	-0.9	41.5	37.7	-3.8
Intermediate	90.0	90.7	0.7	6.9	9.1	2.2
Lower sales, service and routine	88.0	83.8	-4.1	31.6	33.1	1.5

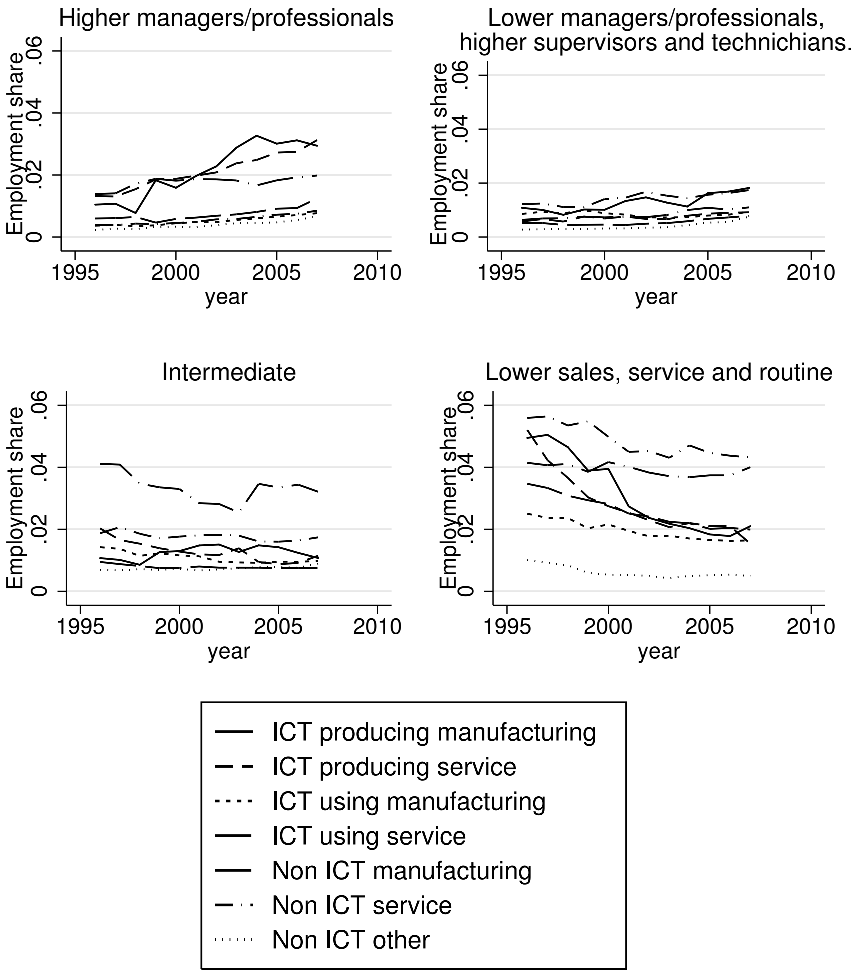
Figure 2: Employment shares, women with children of preschool age, by sector.



below seven and labor supply increased on the intensive margin by 2.5 percentage points. A similar figure is found by restricting the sample to women with at least some tertiary education and shrinking the age span to ages 30-35. For this group, the share of managers/professionals increases by 4 percentage points and labor supply by 5.2 percentage points.[12]

[12]The results are available upon request.

Figure 3: Employment shares by industry and occupation, women with children aged below 7.



3 A model of labor supply and hours constraints

The key trade off in this model comes from two assumptions regarding the labor market. First, more complex, better paid jobs require a minimum number of hours of labor supply to be productive. Second, workplace flexibility determines the degree to which labor can be shifted during the day so that some hours of labor can be supplied from home. This will create the basic mechanism in the model where time constrained individuals will only find it attractive to access the career oriented job if there is a high degree of workplace flexibility.

The model is presented in two steps. First, to build the intuition, I formulate a simple static version of the model with a standard Cobb-Douglas utility function. It delivers simple analytical solutions and gives a transparent view of the mechanism. Second, I reformulate the model into a life-cycle model taking the basic structure of Olivetti (2006) who analyzes the role of returns to experience in explaining women's labor supply. By adding time restrictions to occupations and individuals and the possibility of workplace flexibility, I provide a mechanism to help explain recent changes in female intensive margin labor supply and also why women today take on more demanding jobs to a larger extent.

Consider a standard Cobb Douglas utility function for an individual working in occupation s:

$$ln(U) = a_1 \ln(c) + a_2 \ln(h) + a_3 \ln(1 - n - h) \tag{1}$$

where c is individual consumption, h is hours spent in home production (child rearing) and $l = 1 - n - h$ is leisure. It is assumed that consumption comes from the individual salary alone; adding an exogenous spousal transfer (positive or negative) would somewhat complicate the analysis, but not add much essence. This model adds three features that create the mechanism in the model; a career constraint (CC), a family constraint (FC) and the possibility of flexible work, f.

Suppose that there are two jobs; one analytic, more complex job, A, and one simpler, routine job, R. The analytical job pays more, $w_A > w_R$ but in return, has a minimum requirement, N, for how many hours of labor supply that are needed for the job to be productive. Assuming that the minimum hours required are higher than what an individual would choose under unconstrained optimization, I will call this the career constraint, CC. In order to hold the complex job, the individual will have to make a sacrifice in terms of household production and/or leisure. The routine job, on the other hand, pays less but carries no restriction on the number of hours. An individual is equally productive working part time as full time.[13]

Suppose further that individuals are heterogeneous with respect to the time constraint they face at home, the family constraint (FC). The family constraint does not only reflect the actual time needed for child rearing, but also that these hours must be carried out at a certain point in time during the day, which might infringe on the hours available for work. To formalize the family constraint, let there be a time cost $x = X(children)$ associated with having children. $x_j \in (0, 1)$ and X is increasing concave ($X' > 0, X'' < 0$) with a

[13] This implies that the routine job is fully dividable. Two individuals working at 50 percent are equivalent to one full-time employee.

population average $\bar{x}$. The time constraint for mothers is assumed to be $H^m = \xi x$ and $H^f = (1 - \xi)x$ for fathers, where ξ is a distribution parameter. The size of an individuals' H thus captures family size, spouse characteristics etc., but may also reflect institutional features such as the cost and availability of childcare, which arguably is important for participation and potentially also for labor supply on the intensive margin.[14]

Let h^{UR} represent the unconstrained choice of home production. Then, FC binds for all individuals with $H > h^{UR}$ and all individuals with $H < h^{UR}$ are unconstrained. For $\xi = 1$, this implies that fathers are unconstrained and mothers on average have a time constraint $H = \bar{x}$. A perfectly equal division of the time cost, $\xi = 0.5$, would imply that both mothers and fathers on average have a time constraint $H = 0.5\bar{x}$, i.e more individuals may be constrained, but to a lesser extent.

The focus of this paper is how the consequences of the family constraint are affected by improvements to technology enabling flexibility in work. I capture this idea by introducing the possibility of performing some work from home – flexible work. Then, I assume that the family constraint can be met in two ways; either by home production only, or by a combination of home production and flexible work (from home), f, such that:

$$h + f \geq H \qquad (FC)$$

In this way, as flexibility increases, individuals can supply more hours of labor and still meet the family constraint. The intuition is simple; many activities with children are tied to specific points in time during the day, i.e dropping of at and picking up from day care, providing food and parenting etc. This imposes a restriction on the actual hours spent at work - unless the job is flexible in the sense that individuals are able to continue working once the family requirements are met. It is the possibility of distributing hours across time and location that is going to be the key for family constrained individuals.

I assume that only the A job has some degree of flexibility $\theta \in [0, 1)$. θ thus measures productivity working from home relative to working at the work place. Let n denote the share of time spent *in* the workplace (market hours) and f work supplied from home, then the individuals' total effective labor supply is $n + \theta f$. For simplicity, the R job is assumed to have zero flexibility. This will produce a clear cut off for which different careers are optimal.[15] Note that since $\theta \in [0, 1)$, an individual unconstrained by FC in the A job will also choose zero flexible hours as these are less productive. Once more, this

[14] An increasing public provision of childcare can be seen as affecting the entire distribution of X: for every family size, the time cost decreases. Domeij and Klein (2012) find that subsided child care can have a large impact on female labor force participation. Calibrating their model to Germany, they find that subsidizing childcare to a level of 50 percent almost doubles the labor supply for women with small children. In Sweden, most women work, have children and use childcare. In the mid 1970's, about 20 percent of the children aged 1-5 were enrolled in subsidized childcare. The share has increased steadily to reach 73 percent in 1998 and 86 percent by 2010. Lundin et al. (2008) study the impact of a nation-wide reform of the price of childcare in Sweden. They find no or economically insignificant effects on mothers' labor supply. One explanation put forth by the authors is that in countries with a well-developed and highly subsidized childcare system, further reductions in the price of childcare seem to have small effects on mothers' labor supply. Cortes and Tessada (2011) however, find positive effects on female labor supply, especially on highly educated mothers who worked longer hours.

[15] It is straightforward to allow flexibility to vary also in the R job. The main differences are that there is a set of pairs $\{\theta_R, \theta_A\}$ for which the preferred choice of career changes.

is a simplification; one can easily make the argument for situations where work carried out from home is *more* productive, e.g. by shutting down distractions, interruptions etc. Allowing for this possibility would complicate the analysis, but also make it less focused so it is left for future examination.

3.1 The individual problem

Individuals choose career $\{A, R\}$ so as to maximize utility. Individuals are endowed with one unit of time that is allocated to market work, n, flexible work from home, f, home production, child rearing, h and leisure, l. Incorporating the possibility of flexible working hours, the individuals' decision problem becomes:

$$\max_{\{c,h,n,f\}} U = a_1 \ln(c) + a_2 \ln(h) + a_3 \ln(1 - n - h - f) \qquad s \in \{A, R\}$$

subject to:

$$\begin{aligned} &(BC) \quad w_s(n + \theta f) - c = 0 \\ &(FC) \quad h + f - H \geq 0 \\ &(CC) \quad n + \theta f - N \geq 0 \\ &\qquad\quad 1 - h - n - f \geq 0 \qquad n, h, f \in [0, 1] \end{aligned}$$

in any A job; in an R job the wage is w_R, N=0 and $f = 0$. The individual then chooses between A and R to maximize utility. Given the two different jobs and the different time constraints, you can identify four different cases; i) the unrestricted case where an individual works at the R job and is unrestricted by family, ii) having the R job and being restricted by the family constraint, iii) working at the A job, being subject to the career requirements, but unrestricted by family, and finally iv) working the A job and being restricted by both the career requirements and the family constraint. Which of the cases an individual ends up in depends on the parameter values. In what follows, I solve for the four different cases separately and then discuss the implications of changes in technology.[16]

i) Unrestricted case

The unrestricted allocation delivers the standard Cobb Douglas weights. The individual optimizes over n and h:

$$\max_{\{n,h\}} a_1 \ln(w_R n) + a_2 \ln(h) + a_3 \ln(1 - n - h)$$

The first-order conditions are:

$$\begin{aligned} h: \quad & \frac{1}{h(h+n-1)}(ha_2 - a_2 + ha_3 + na_2) = 0 \\ n: \quad & \frac{1}{n(h+n-1)}(ha_1 - a_1 + na_1 + na_3) = 0 \end{aligned}$$

[16]Here it is assumed that CC always binds in the A job. Section 4.5 relaxes this assumption and explores the case where there are no formal requirements for minimum hours in the A job.

The optimal solution is:

$$\begin{aligned} h &= \frac{a_2}{a_1 + a_2 + a_3} \\ n &= \frac{a_1}{a_1 + a_2 + a_3} \end{aligned}$$

ii) R job when only the family constraint binds
Once more, there is only one decision variable. Since there is no flexibility in the R job and the FC binds such that $h = H$, the individual can only decide over n. The objective function now reads:

$$\max_{\{n\}} a_1 \ln(w_R n) + a_2 \ln(H) + a_3 \ln(1 - n - H)$$

The first-order condition with respect to n:

$$\frac{1}{n\,(H + n - 1)}\,(Ha_1 - a_1 + na_1 + na_3) = 0$$

The optimal solution is:

$$\begin{aligned} n &= \frac{a_1(1-H)}{a_1 + a_3} \\ h &= H \\ l &= \frac{a_3(1-H)}{a_1 + a_3} \end{aligned}$$

Thus, with logarithmic utility, we obtain a time allocation which is independent of the wage; the substitution effect from a higher wage (working more) cancels the income effect (enjoying more leisure and home production).

iii) A job when only the career constraint binds
Since $\theta < 1$, it is assumed that one hour in f can never be as productive as one hour of market work, n. Thus, an individual unconstrained by family will under optimization always choose $f = 0$. Once more, substituting the remainder of the constraints into the objective function:

$$\max_{\{f\}} a_1 \ln(w_A N) + a_2 \ln(h) + a_3 \ln(1 - N - h)$$

The first-order condition with respect to h:

$$\frac{a_2(N - 1 + h) + a_3 h}{h\,(N + h - 1)} = 0$$

and the optimal allocation of time is:

$$\begin{aligned} h &= \frac{a_2(1-N)}{a_2 + a_3} \\ n &= N \\ l &= \frac{a_3(1-N)}{a_2 + a_3} \end{aligned}$$

Thus, as the career constraint is tightened, individuals substitute away leisure and home production according to their relative weights ($\frac{\partial h^*}{\partial N}, \frac{\partial l^*}{\partial N} < 0$).

iv) A job when both family and career constraint bind

For the family constrained individual working in the A job, there is only one decision variable, the choice of the amount of flexible work, f. To see this, substitute the constraints BC, FC and CC into the objective function. The problem reduces to:

$$\max_{\{f\}}\{a_1 \ln(w_A(N)) + a_2 \ln(H - f) + a_3 \ln(1 - N - H + \theta f)\}$$

The first-order condition with respect to f delivers:

$$f = \max\left\{\frac{(a_2(H + N - 1) + H\theta a_3)}{\theta(a_2 + a_3)}, 0\right\}$$

For sufficiently low levels of θ, the amount of flexible hours will be zero. The utility cost from the lower productivity is just too large, so all adjustments to the allocation of time will occur on leisure. Taking the partial derivatives with respect to H and N, f is increasing in both arguments. The response to an increase in H is larger.[17]

3.2 Static solution - an illustration

To illustrate the mechanism, the weights on consumption (a_1), home production (a_2) and leisure (a_3), are set to 0.3, 0.1 and 0.6, respectively. The family constraint H is set to 0.3 and the career constraint N to 0.45. The parameter values are chosen so as to allow an easy illustration and are not an attempt at calibration. w_A is set to 1 and w_R to 0.7 to reflect the earnings difference between the two jobs. The model is then solved over a grid of different θ going from zero flexibility to (near) full flexibility.

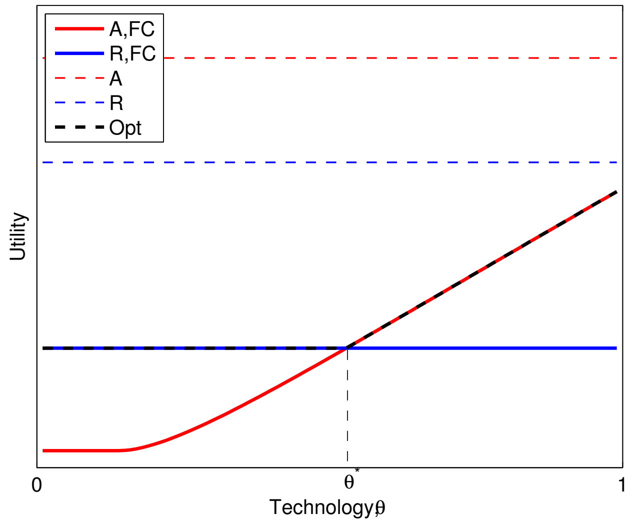
Throughout the remainder of the paper, in the figures the results for unrestricted individuals in the two different jobs are labeled A and R respectively. Results for family constrained individuals are instead labeled FC, A and FC, R. Results for the A job are in red lines and the R job is drawn in blue. Optimal paths are in dashed black.

3.2.1 Optimal career and labor supply

Figure 4 shows the utility for the two different careers as a function of the key object of interest; the technology parameter θ. The dashed red and blue lines show the level of utility for the case of having an A and R job, respectively, when FC does not bind. The solid thick blue and red lines show the utility of the corresponding family constrained individual. When flexibility is low, i.e. when work can be performed from home only with very low productivity, choosing the R job clearly dominates choosing the A job. The penalty from sacrificing family and leisure if choosing the career oriented job is simply too high. When firms adopt a workplace organization and technologies that enable flexible work such that productivity at home becomes high enough, $U^{A,FC} > U^{R,FC}$, and it

[17] $\frac{\partial f}{\partial H} = \frac{a_2}{\theta(a_2+a_3)} + \frac{a_3}{a_2+a_3} > \frac{\partial f}{\partial N} = \frac{a_2}{\theta(a_2+a_3)} > 0$.

Figure 4: Utility



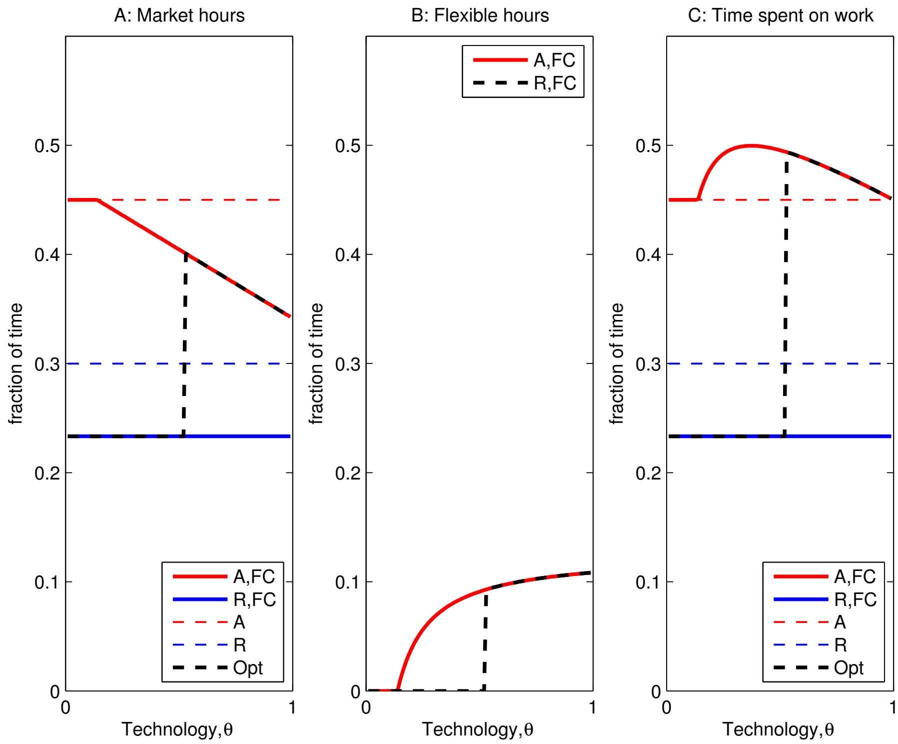
becomes optimal to instead pursue the more demanding career (indicated by θ^* in the figure).[18]

Market hours are plotted in panel A of figure 5. The optimal supply of hours if there is no restriction on minimum time is just the Cobb-Douglas weight on consumption, 0.3. FC individuals in the routine job work less than unrestricted individuals in the same job (0.23 vs 0.30). For low levels of flexible technology, holding the A job, the CC implies that labor supply increases by 50 percent for unrestricted individuals and almost double for family constrained individuals. When $\theta \geq \theta^*$ the optimal career shifts to the A job.[19] To accomplish the shift to the A job, the FC individual increases the market hours dramatically from 0.23 to 0.40 and increases the flexible hours from 0 to 0.09. Hence, total time devoted to work $(n + f)$ is larger than for unconstrained individuals to compensate for the lower productivity at home, as seen i panel C. In case of further increases to flexibility, FC individuals in the A job can decrease their market hours and substitute using more flexible hours. Near full efficiency working from home implies that time constrained individuals supply almost a quarter of their total labor supply from home (0.11 of 0.45). The magnitudes described are, of course, a product of the parameters chosen but the broad picture remain also for different weights in the utility function. A higher level on the family or career constraint will increase the use of flexible hours. Sections 3.2.3 and 3.2.4 explore how the optimal choice of career and allocation of time varies with the magnitude of the time constraint.

[18] If you allow θ to vary also for the R job, a substitution of hours into flexible hours will also occur in the R job, which will improve the utility from holding the R job.

[19] If θ_R is allowed to vary, improvements to the flexible technology will decrease the market hours but increase total labor supply as flexible hours are used. However, compared to the A job, the use of flexible hours starts at higher levels of technology.

Figure 5: Labor supply



3.2.2 Solving the puzzle - home production and leisure

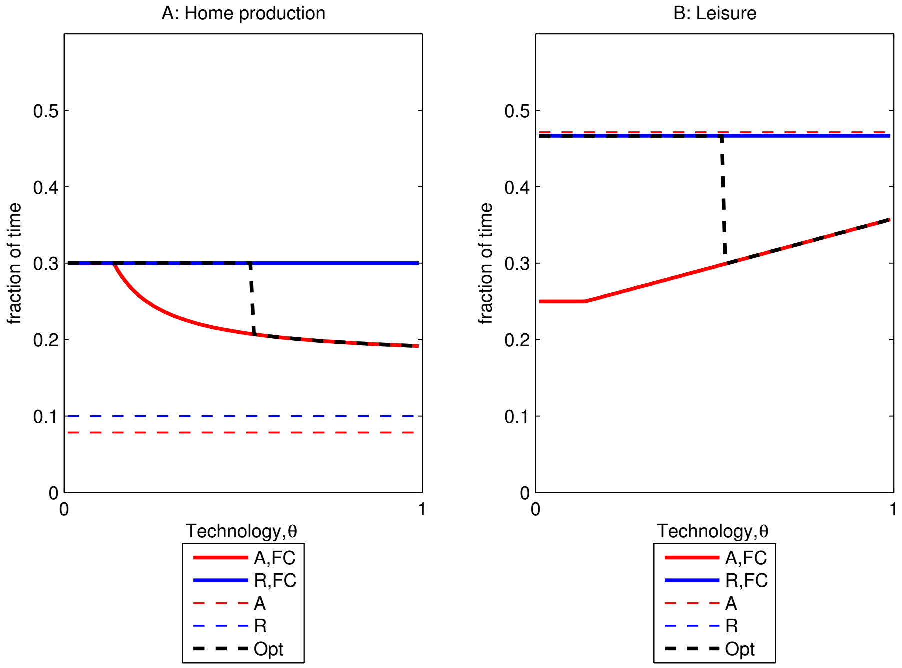
The implications for home production and leisure are shown in figure 6. The dashed black line once more represents the optimal choice between the two jobs for different levels of technology. From figure 6, the trade-off between career, family and leisure becomes very clear. The consequences for an FC individual of pursuing the more demanding career is large for low levels of flexible technology. To meet the requirements of the A job, leisure would be depleted by 45 percent (from 0.46 to 0.25). This sacrifice is simply too high in terms of utility and it is optimal to instead choose the less demanding R job.

The optimal career choice changes at θ^*. This involves a large reallocation of time for the FC-individual. Home production drops from 0.30 to 0.21 and leisure from 0.47 to 0.30 to meet family and career constraints. By comparison, individuals unconstrained by family decrease home production to 0.08 (0.10) and leisure to 0.47 (0.60) to hold the A job.

3.2.3 Changing the career requirements, N.

How does the optimal allocation vary with the magnitude of the career requirements? Start at the labor supply chosen by the family constrained individual in the R job ($n_{FC,R} = \frac{a_1(1-H)}{(a_1+a_3)}$) and increase the constraint step by step. For each level of the career requirement, the minimum level of flexible technology needed to still pursue the A job is solved for and then, given that level of technology, also the optimal allocation of time. This process is

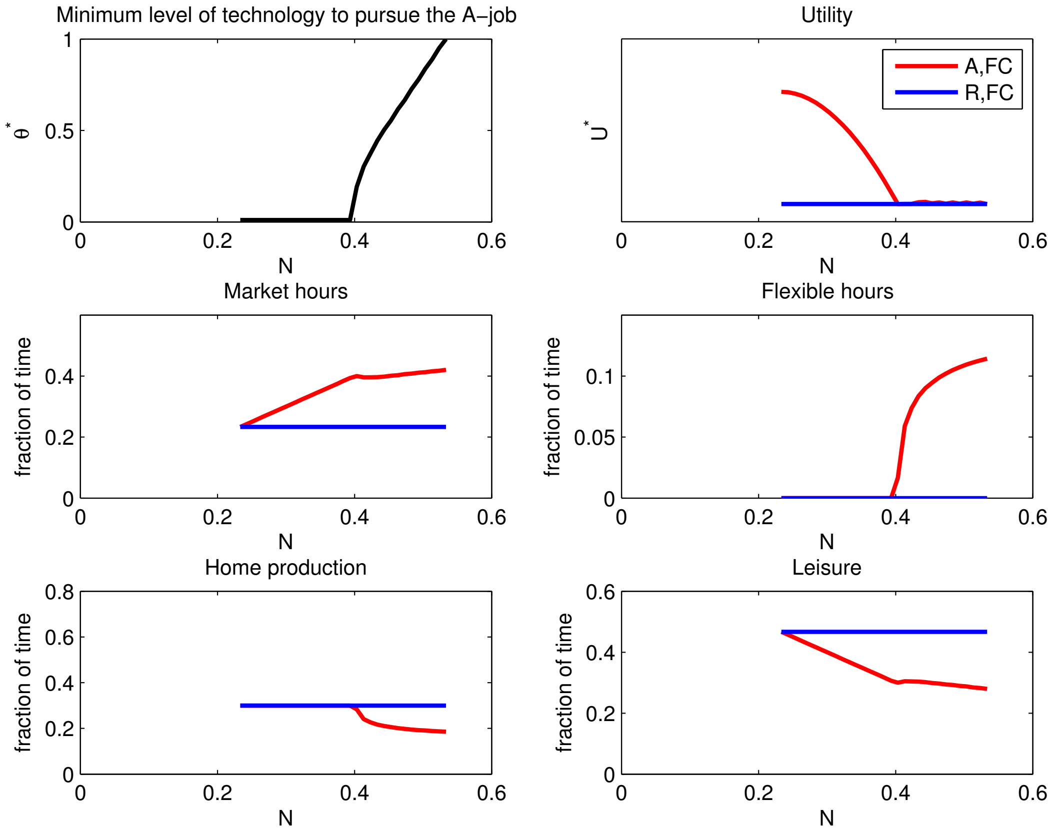
Figure 6: Home production and leisure



continued until there is no value for θ, for which it is still optimal to choose the A job. The results are presented in figure 7, with career requirement N on the x axis.

The top left panel shows the minimum level of flexible technology needed to still pursue the A job. For quite a large range of the career requirement ($N \leq 0.4$), the FC-individual will choose the A job regardless of the level of θ. To meet the increasing career constraint, leisure is substituted for market hours (mid left and bottom right panel) and the utility decreases towards that in the R job. As shown in the mid-right panel, it is only beyond that the FC-individual begins utilizing flexible work. Since productivity is lower in flexible work, that option is only executed when further sacrifice in terms of leisure is costly enough. With the parameters used, for $N > 0.53$ it will no longer be optimal to choose the A job, even if technology is fully flexible. At the maximum level of the career requirement, the FC-individual supplies 25 percent of total labor supply by flexible working hours.

Figure 7: Minimum technology and optimal allocation for different levels of the career constraint



3.2.4 Changing the family requirement, H

Next, holding N fixed at 0.45, H is gradually increased, starting at the level of home production chosen by the family constrained individual in the R job ($h_{FC,R} = \frac{a_2(1-N)}{(a_2+a_3)}$) and again solving for the minimum level of technology needed to still prefer the A job. As in the previous experiment, given that level of technology, I solve for the optimal allocation of time. The first thing to note is that, as seen in the top left panel of figure 8, technology almost immediately becomes an important factor. At H=0.22, the minimum level of flexible technology θ^* jumps up and increases rather steeply to reach its maximum. To be able to meet the requirements of both constraints, in principle all work carried out from home (see the two mid panels). This level of the family requirements would reflect a situation with no limited access to child care and a very unequal division of the requirement for family time between spouses. Consequently, almost all of the time is spent around the children (actively or passively).

Consider again the basic case with $H = 0.30$ and $N = 0.45$. The minimum level of technology needed to pursue the more demanding career is θ^*. Now, suppose that technology improves further. As the model abstracts from the fertility choice and H is taken as exogenous (and fixed), the response of the FC-constrained individuals is to improve on their time allocation by using more flexible hours, spending more time on leisure and less time on market hours and home production (see figure 5 and 6). In reality, however, it could be that you can see other kinds of behavior. In particular, if you endogenize H, it may well be that the "slack" created by improved technology is used to instead choose a higher H. If having a demanding career previously meant that you had to abstain from having a family, or were limited to one child only, an improvement in flexibility might open up the opportunity to also have a second child. Boschini et al. (2011) find that it becomes more and more common for professional women in Sweden to have a second child. They find that the share of professional women having two children has increased from about 38 percent in the cohort born in 1945 to almost 50 percent in the cohorts born at the beginning of the 1960's. Moreover, for these women, there was a decrease in childlessness over time. One interpretation of this development is that professional women have traded off some of the increased possibilities in terms of technology, i.e. the possibility of using more flexible hours and getting more leisure to instead having a larger family and thus "choosing" a higher H. This is a natural extension which I intend to explore further in future work.

Figure 8: Minimum technology and optimal allocation for different levels of the family constraint

graphics[,width=14cm,keepaspectratio=true]/Users/chk/Dropbox/MATLAB/HC/analytic/theta$_s tar_H.eps$

4 Returns to experience - the lifecycle model

Next, following Olivetti (2006), the model is reformulated and extended into four periods, each representing ten years of working life. The family constraint FC is assumed to bind only in the second period (and, as before, not for everyone). There is no uncertainty, individuals have perfect foresight. The key difference from the static model is the returns to experience. In particular, the wage in the next period is determined by the current wage but also total labor supply in the current period, to be specified in greater detail below. This makes the stakes higher in the family constrained period since choosing less hours and/or a less demanding job has consequences for the entire span of working life.[20] In this section, the utility function is slightly modified compared to the static version. In particular, I use a similar functional form to that of Olivetti, but introduce the same time restrictions and different career paths as those presented in the static case. In addition, home production is simplified compared to Olivetti's treatment: she models h as a CES aggregate of parents' time and purchased services. I abstract from explicitly modeling the purchase of childcare services. However, as discussed in section 3, the price and availability of childcare are reflected in the family constraint, H. As before, individuals have preferences over consumption, home production and leisure and are assumed to have the following utility function:

$$U(c_t, h_t, n_t, f_t) = ln(c_t) + b\,ln(h_t) - B\frac{(n_t + h_t + f_t)^{\alpha}}{\alpha}$$

where c is consumption and b represents the weight that individuals put on home production (family). $B\frac{(n_t+h_t+f_t)^{\alpha}}{\alpha}$ is the disutility of work, B and $b > 0$. Notice that this is a slightly different formulation than in the static case, where $-B\frac{(n_t+h_t+f_t)^{\alpha}}{\alpha}$ represents $ln(1 - n - h - f)$. As in the static model, it is assumed that the disutility of work is equally large for home production, market work, and flexible work from home. This can be relaxed but does not qualitatively change the implications of the model.

Many studies suggest that there are (large) career penalties to having a family. For example, Bertrand et al. (2010) find that male MBA earnings outperform those of women by 60 log points a decade after graduation and identify differences in career interruptions and weekly hours, both largely associated with motherhood, to be two of the main explanatory factors. The cost of a career interruption is larger the larger the depreciation of human capital and the larger the forgone skill accumulation. Hence, the cost of a career interruption would be higher the more skill intensive the job is. For that reason, family constrained individuals may self-select into occupations where the penalties are lower (Goldin and Katz, 2011). To incorporate this feature, I introduce skill accumulation into the A job. In particular, assuming that the wage equals the individual's productivity and modeling the wage in the next period, w^A_{t+1} is a function of a depreciation rate, δ, labor supply in the current period, $n_t + \theta f_t$ and the current wage:[21]

$$w^A_{t+1} = G^A(n_t, f_t, w^A_t) = (1 - \delta)w^A_t + \varphi w^A_t(n_t + \theta f_t)^{\gamma} \quad (2)$$

[20] The model abstracts from saving.

[21] The functional form is similar to that used in Olivetti (2006) and Wallenius (2011).

This way, the labor supply decision in one period does not only affect current earnings, but also determines future wages.

In the R job, skills are assumed not to depreciate, but there is also no potential for accumulating skill. Wages are instead assumed to grow exogenously at a rate g. Hence, the career cost of reducing the working hours during intensive child rearing years will be lower in this job, which is why family constrained individuals might find it preferable and self-select. Thus, for the R job, we have:

$$w_{t+1}^R = G^R(g^R) = w_t^R(1+g)^{10} \tag{3}$$

The occupational choice between A and R is assumed to be permanent; thus the individual chooses careers in period one only. This is a simplification although arguably many careers are difficult, if not impossible, to pursue if started later in life. However, this is an area which it is interesting to explore further. There is evidence of women opting out and shifting down, i.e leaving the workforce or choosing a less demanding job, once family enters the picture (Boushey, 2008), but also of resuming a career after the intensive childbearing years. Allowing for a career shift is especially interesting in view of the accumulation and depreciation of skills. However, this is left to future work.

Adding the wage(skill) accumulation, the individual's decision problem in the A job becomes:

$$\max_{\{c_t,h_t,n_t,f_t,w_{it+1}^A\}} \sum_{t=0}^{T} \beta^t\{U(c_t,h_t,n_t,f_t)\}$$

subject to:

$$\begin{array}{llrl}
(BC) & w_t^A(n_t+\theta f_t)-c_t=0 & & (\lambda_t)\\
 & w_{t+1}^A = G_t^A & & (\eta_t)\\
(FC) & h_t+f_t-H \geq 0 & \text{if } t=2 & (\mu_{1,t=2})\\
(CC) & n_t+\theta_t^s f_t - N \geq 0 & \text{if } s=A & (\mu_{2,t})
\end{array} \tag{4}$$

and for the R job, the individual's decision problem is:

$$\max_{\{c_t,h_t,n_t,f_t,w_{it+1}^R\}} \sum_{t=0}^{T} \beta^t\{U(c_t,h_t,n_t)\}$$

subject to:

$$\begin{array}{llrl}
(BC) & w_t^A n_t-c_t=0 & & (\lambda_t)\\
 & w_{t+1}^R = G_t^R & & (\eta_t)\\
(FC) & h_t+f_t-H \geq 0 & \text{if } t=2 & (\mu_{1,t=2})
\end{array} \tag{5}$$

Let λ_t be the multiplier on the time t budget constraint (BC), η_t the multiplier on the human capital accumulation constraint, $\mu_{1,t=2}$ the multiplier on the family constraint (FC) and $\mu_{2,t}$ the multiplier on the career constraint (CC) if holding the A job. In what follows, the different cases are solved for, starting with the unrestricted case in the R job and ending in the A job where both time restrictions bind. In the cases where the family constraint binds, the second period is solved for separately to make clear the key components of the model.

4.1 The R job

i) Unrestricted case

In this case, none of the time restrictions bind. The individual optimizes over consumption, home production and hours worked using no flexible hours. Wages grow exogenously at a rate g. The first-order conditions are:

$$c_t: \quad \beta^{t-1}U_{c_t} - \lambda_t = 0 \tag{6}$$

$$h_t: \quad \beta^{t-1}U_{h_t} = 0 \tag{7}$$

$$n_t: \quad \beta^{t-1}U_{n_t} = \lambda_t w_t^R \tag{8}$$

$$\lambda_t: \quad w_t^R n_t - c_t = 0 \tag{9}$$

Dividing equation 8 by 6, we get the usual relationship between the marginal utility of work and consumption, $\frac{U_{n_t}}{U_{c_t}} = c_t B(n_t + h_t)^{\alpha-1} = w_t^R$. By equation 7 we have that the marginal utility of home production is equal to the disutility of work, $\frac{b}{h_t} = B(n+h)^{\alpha-1}$. Combining these, the optimal relationship between market hours and home production is $n_t = \frac{1}{b}h_t$.

ii) Family constrained

The problem of the family constrained individual in the R job is very similar. The first, third and fourth period are the same as the problem described above. In the second period, $h_2 = H$ since there is no flexibility in the R job . Substituting H into the expression of the MRS between work and consumption, n is now the solution to $B(n_t + H_t)^{\alpha-1} = \frac{1}{n}$.

4.2 The A job

iii) Unrestricted case

An individual working the A job who is unconstrained by family is only deciding on one variable, i.e. the amount of home production. To see this, remember that $\theta < 1$ so no flexible hours will be used, $f_t = 0$. Moreover, since the career constraint binds, the amount of market work is predetermined, $n_t = N$. Looking at equation 11, the optimal amount of home production is given by the solution to $\frac{b}{h_t} = B(N + h_t)^{\alpha-1}$.

The first-order conditions for the individual problem with respect to c, h, n and w_{t+1} are:

$$c_t: \quad \beta^{t-1}U_{c_t} - \lambda_t = 0 \tag{10}$$

$$h_t: \quad \beta^{t-1}U_{h_t} = 0 \tag{11}$$

$$n_t: \quad \beta^{t-1}U_{n_t} = \lambda_t w_t^A + \eta_t G^A_{n_t} + \mu_{2,t} \tag{12}$$

$$w_{t+1}: \quad \eta_t = \lambda_{t+1}n_{t+1} + \eta_{t+1}G^A_{w^A_{t+1}} \tag{13}$$

In period II, the family constraint binds and the individual may find it optimal to also use flexible hours, which is why the first-order condition changes to:

$$Period\, II:$$

$$c_t: \quad \beta^{t-1}U_{c_t} - \lambda_t = 0 \tag{14}$$

$$h_t: \quad \beta^{t-1}U_{h_t} + \mu_{1,t} = 0 \tag{15}$$

$$n_t: \quad \beta^{t-1}U_{n_t} = \lambda_t w_t^A + \eta_t G_{n_t} + \mu_{2,t} \tag{16}$$

$$f_t: \quad \beta^{t-1}U_{f_t} = \lambda_t w_t^A \theta + \eta_t G_{n_t} + \mu_{1,t} + \mu_{2,t}\theta \tag{17}$$

$$w_{t+1}: \quad \eta_t = \lambda_{t+1}(n_{t+1} + \theta f_{t+1}) + \eta_{t+1}G_{w^A_{t+1}} \tag{18}$$

Equations 14 and 15 are typical first-order conditions for consumption and home production, respectively. In the second period, when the family constraint binds, the (net) marginal utility of home production is equal to the shadow cost of the family constraint. Dividing 15 by 14, we get:

$$\frac{U_{h_t}}{U_{c_t}} = \frac{\mu_{1,t}}{\lambda_t} \tag{19}$$

So the marginal rate of substitution between home production and consumption equals the ratio of the shadow cost of the constraint to the marginal utility of wealth. In periods where the family constraint does not bind, the right-hand side is equal to zero, and the net marginal utility of home production equals the marginal utility of consumption.

Equation 16 is the first-order condition with respect to market hours. The first term on the right-hand side is the wage multiplied by the marginal utility of wealth. The second term captures the effect of human capital accumulation from working. It is clear from 16 that additional hours of work do not only result in the benefit of additional income but also in the benefit of higher wages in the future. Both these effects must be weighed against the disutility of working another hour and the shadow value of the career constraint. As in Wallenius (2011), this implies that the opportunity cost of time is flatter than the observed wage schedule, (in this case even more due to $\frac{\mu_{2,t}}{\lambda_t}$). The first-order condition with respect to flexible hours is very similar to that of market hours. The key difference is that both the benefit from income and from higher future wages is scaled down by θ as the flexible hours are less productive. Moreover, the shadow cost of the family constraint and the shadow cost of the career constraint (scaled by θ) enter on the right-hand side. Equation 18 is the law of motion for wage (skill).

Dividing 16 and 17 by 14 gives the MRS between market work and consumption and flexible work and consumption, respectively:

$$\frac{U_{n_t}}{U_{c_t}} = w_t^A + \frac{\eta_t}{\lambda_t}G^A_{n}{}_t + \frac{\mu_{2,t}}{\lambda_t} \tag{20}$$

$$\frac{U_{f_t}}{U_{c_t}} = w_t^A \theta_t + \frac{\eta_t}{\lambda_t} G_{f_t}^A + \frac{\mu_{1,t}}{\lambda_t} + \frac{\mu_{2,t}}{\lambda_t} \tag{21}$$

Combining 20 and 21, we see that the shadow cost of the family constraint is equal to the difference in income generated by supplying f instead of n plus the difference in marginal human capital accumulation for n compared to f, multiplied by the shadow value of skill. The last term $\mu_{2,t}(1-\theta)$ reflects the shadow cost of the career constraint, weighted by the productivity loss of using flexible hours:

$$\mu_{1,t} = \lambda_t w_t^A (1-\theta) + \eta_t (G_{n_t}^A - G_{f_t}^A) + \mu_{2,t}(1-\theta) \tag{22}$$

4.3 Parameters

The parameters $\{\alpha, b, B, w_{A,0}, w_{R,0}, D_t, \gamma, \beta, \varphi_t, g,\ N\, and\, H\}$ need to be calibrated. Taking the model to the data and doing a full quantitative assessment is beyond the scope of this paper. The aim here is to develop a framework that in a structured way incorporates an important feature of the conditions forming labor supply for women today. To this end, I therefore calibrate the model drawing on the results of other authors. The parameters of the utility function are from Olivetti (2006), unless otherwise stated.[22] The weight on home production, b, is set to 0.35. The parameters that govern the disutility of work, A and α, are set to 25 and 3.02, respectively. Olivetti estimates α to be 2.88 for women and 3.02 for men. The higher value is chosen as there are no differences in preferences between individuals in the model, only differences in the constraints they face. The discount factor $\beta = 1/(1-0.05)^{10}$ is set to 0.62 based on an annual interest rate of 5 percent. The wage in the initial period for the more complex analytical job, $w_{A,0}$, is normalized to 1. The corresponding wage for the routine job, $w_{R,0}$, is set to 0.8 to reflect the initial wage difference between complex and routine jobs. As we will see, over the life cycle, wage inequality grows as wages increase faster in the A job.

The career constraint N is set to 0.45, as in the static case. Wallenius (2011) created an average supply of life cycle hours using PSID data and found hours to be on average 0.40 over the life cycle, peaking just over 0.41. Choosing 0.45 is thus not too high and results in a binding CC. Moreover, this is very close to the average share of hours men devoted to work in the 1990's as documented by Olivetti (2006) using PSID data.[23] As in the static model, I assume H to be 0.3. This is the same order of magnitude as in Olivetti (2006) who finds that women spend 32 percent of their time in the second ten-year period on childcare.

In the literature, a wide range of values has been documented for the learning parameter γ. Imai and Keane (2004) estimate γ to be 0.23, Huggett et al. (2006) find a value around 0.7. Wallenius (2011) evaluates her model at $\gamma = 0.5$ which is close to Olivetti (2006) who sets γ to be 0.4. Once more, I here follow the parameter values of Olivetti. Finally, the parameter values for φ_t are set using the estimated parameters of the human capital production function for men in 1970 in Olivetti.[24] The parameter values are summarized in table 3.

[22] As a robustness check I use the parameters in Wallenius (2011). The qualitative results remain unchanged.

[23] Assuming a total endowment of 5000 hours, the average share devoted to work is 0.44 in ages 20-29 and 0.46 for the remaining three 10 year periods.

[24] Since these parameter estimates are obtained using yearly data, I follow the procedure in Olivetti (2006) and iterate the law of motion for wages (skill) over the ten-year period assuming constant hours of work.

Table 3: Parameter values

Parameter	Value	Comment
b	0.35	Weight on home production
B	25	Disutility of work
$\varphi_{t=1}$	0.0136	Returns to experience
$\varphi_{t=2}$	0.0118	Returns to experience
$\varphi_{t=3}$	0.0100	Returns to experience
α	3.02	Frisch elasticity= $\frac{1}{\alpha-1}$
γ	0.4	Learning parameter
β	0.62 (over 10 years)	Discounting
δ	0.2	Depreciation
w_1^A	1	Initial wage A job
w_1^R	0.8	Initial wage R job
H	0.3	Family constraint FC
N	0.45	Career constraint CC
g	0.03	Exogenous wage growth

4.4 Results

As in the static version, the model is solved over a grid of technology for the average constrained individual with $\xi = 1$ and $\xi = 0$, respectively (translating into a FC-individual with H=0.3 and an unconstrained individual) for the two different careers.[25] In figure 9, the life-time utility for an FC-individual is shown for the different careers. The blue solid line represents the utility in the R job and the red solid line that in the A job. To illustrate the impact of returns to experience, dashed lines represent the utility when returns to experience are shut down. In that case, the wages in the A and R job are assumed to grow exogenously. As seen in figure 9, adding returns to experience makes it optimal to pursue the A job at lower levels of flexible technology. The reason is that more skill is accumulated in the A job compared to the R job as a result of more hours of work. In this setting, since A is career constrained, hours of work do not change as a result of skill accumulation.[26]

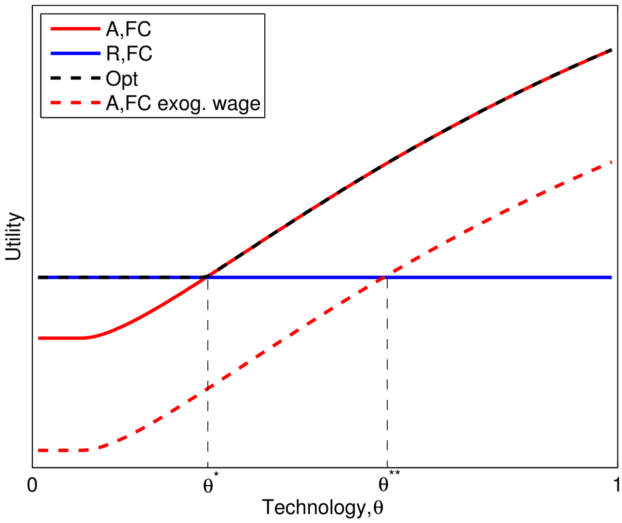
Olivetti (2006) documents that the lifecycle profile of married women in the 1970's was double peaked, i.e that labor supply is high at the beginning of working life, substantially lower during the child bearing years, and then high again as the children grow older. She also documents a distinct difference for married women in the 1990's, who instead had a single peaked profile, more similar to that of men. Comparing the two periods, Olivetti estimates that women's returns to experience had increased by 25 percent and those of men only by 6 percent and finds that the change in returns to experience could explain almost the entire increase in labor supply. However, Olivetti's model is silent on why the returns to experience had increased. The mechanism suggested by the present model is that it does not only involve more hours, but also more complex jobs.

In figure 10, market hours and flexible hours are plotted for low levels of technology, $\theta = 0.01$, near full flexibility, $\theta = 0.99$ and the point $\theta = \theta^*$ where utility in the A job just dominates that in the R job. First, the blue solid line is the lifecycle profile of an FC-individual in the R job. Since there is no flexibility in the R job, market hours drop substantially in the second period when the family

[25] θ is varied from 0.001 in steps of 0.001 to 0.999.

[26] If there is skill accumulation also in the R job, the hours profile will change in that job. Compared to exogenous wage growth, individuals will then find it optimal to work more in the early periods to accumulate a higher wage.

Figure 9: Lifetime utility



constraint binds and the profile is double peaked as in Olivetti. The red solid lines show the market hours (panel A) and flexible hours (panel B) in the A job. For very low levels of technology, the market hours would equal what is demanded by the career requirements N, but since the sacrifice on home production and leisure would be so large, the less demanding job is chosen. Once more, as flexibility improves sufficiently, the A job becomes optimal. The new allocation of time involves substituting market hours for flexible hours and the total time devoted to work is larger than for the unconstrained individual to compensate for lower productivity at home. Interestingly, when flexible hours can be used, the change in choice of careers implies that the dip in job the life cycle profile of labor supply (as seen in the blue solid line of panel A) disappears, the profile effective labor supply $n + \theta f$ of the A is flat to meet the career requirements. A fully flexible technology implies that the FC-individual decreased the total time devoted to work to that of the unconstrained individual, supplying about one third of total labor supply from home.

Turning to home production, we saw in section 3.2.2 that changing careers implied a large reallocation of time. Here, the fact that FC only binds in one period and that returns to experience increase the opportunity cost of working less is larger make the reallocation even larger in the second period compared to the static case, since it now becomes optimal to pursue the A job at less efficient levels of technology. Consequently, comparing $\theta = \theta^*$ going from R to A implies a drop in home production from 0.20 to 0.08, and in leisure from 0.56 to 0.41. Compared to the unconstrained individuals, a FC-individual in the A job has about 30 percent less time for leisure. The trade off is thus substantial both in terms of home production and leisure.

Although, for $\theta \geq \theta^*$, it is optimal to choose the A job from a lifetime utility perspective, the results are mixed when utility for R and A is compared period by period. The blue dashed line in figure 12 is discounted utility for a family constrained individual in the R job and the red line the corresponding utility in the A job. Utility is evaluated at θ^*, i.e where it just becomes optimal to choose the more demanding career. As seen in figure 12, the trade off in terms of leisure and home production that is

Figure 10: Labor supply

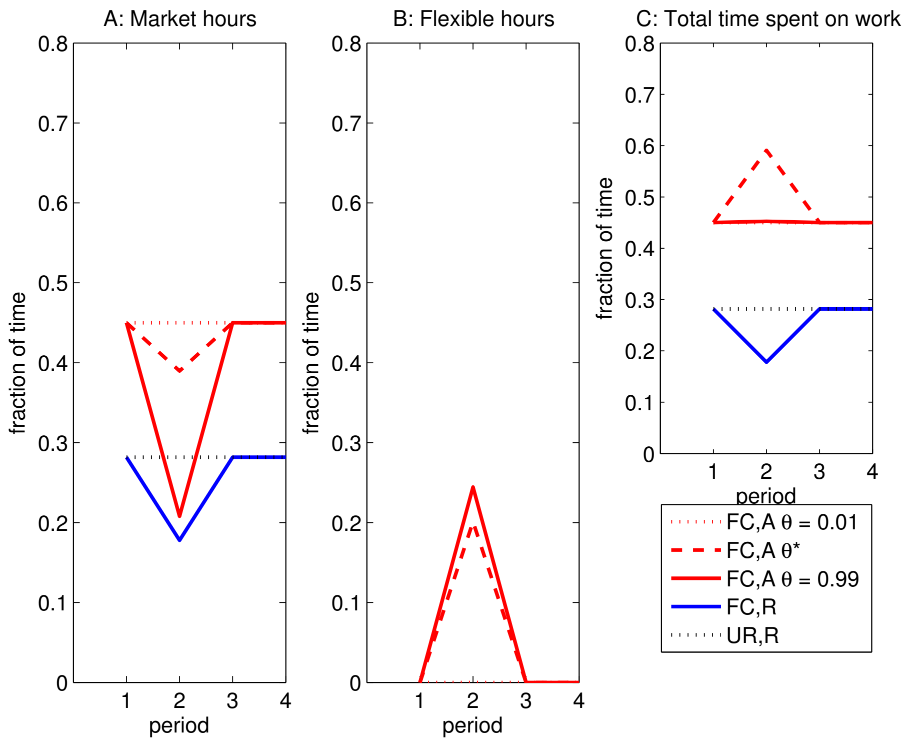


Figure 11: Home production and leisure

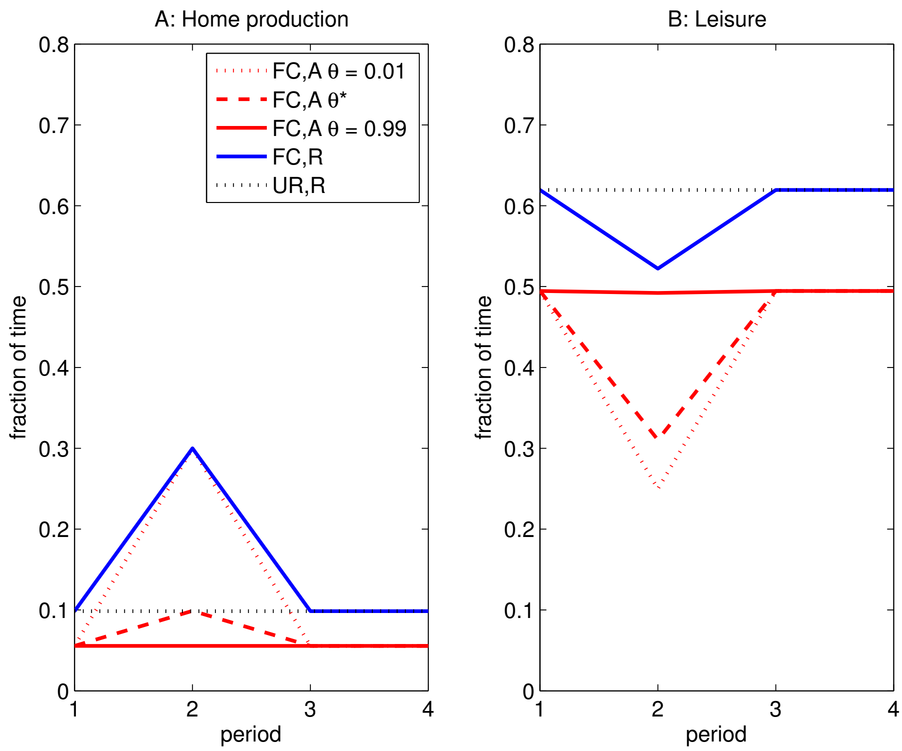
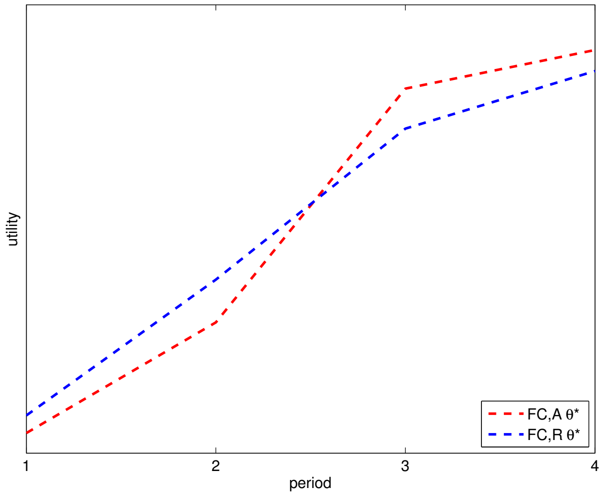


Figure 12: Period utility



involved with choosing the more demanding career makes the period utility for the family constrained individual in the A job utility in the second period lower than that in the R job.

This result is connected to a recent literature studying female happiness. Stevenson and Wolfers (2009) document a decline in female happiness, both in absolute terms and relative to men. A possible hypothesis is that the source of women's declining happiness is the "second shift´´ associated with pursuing both career and family. However, Stevenson and Wolfers (2009) find no evidence that the decline in happiness is more pronounced among working mothers as compared to non working mothers, when comparing single to married mothers, prime age women to other age-groups or highly educated to less educated. Hence, although they document a decline in overall female happiness, they find no evidence that working the second shift is the underlying cause. However, as pointed out by the authors, there are large compositional changes within these groups over the period studied. Women have become more educated, they participate in the labor market to a larger extent and have children later in life. On the other hand Mencarini (2012) do find negative effects on happiness for working women. Using data from the European Social Survey between 2002 and 2008, they find that in particular for women employed more than 30 hours per week, happiness is negatively affected by a large share of housework compared to other groups.

4.5 Relaxing the minimum requirement on an A job

So far, the trade-off between the A and the R job has been created by imposing a minimum requirement with respect to hours worked in the A job. This assumption is relaxed in this section. There is, however, still a 'natural' minimum requirement in place since skills depreciate in the A job. The cost of a career interruption is larger, the larger the depreciation of human capital and the larger the forgone skill accumulation. This means that if labor supply is too low, skills depreciate on net, thus making the A job less attractive than the R job. Remember, the R job neither has skill accumulation nor depreciation and the wage growth is thus unaffected by career interruptions or lower hours during childbearing years.

Returning to the problem in section 4.2 and removing the career requirements, the first-order condition with respect to n becomes:

$$n_t: \quad \beta^{t-1} U_{n_t} = \lambda_t w_t^A + \eta_t G^A_{n_t} + \mu_{2,t} \tag{23}$$

$$w_{t+1}: \quad \eta_t = \lambda_{t+1} n_{t+1} + \eta_{t+1} G^A_{w^A_{t+1}} \tag{24}$$

and in the second period, for the family constrained individual, the first-order condition with respect to f changes to:

$$f_t: \quad \beta^{t-1} U_{f_t} = \lambda_t w_t^A \theta + \eta_t G_{n_t} + \mu_{1,t} + \mu_{2,t}\theta \tag{25}$$

Redoing the manipulations in 4.2, the MRS between market work and consumption and flexible work and consumption looks almost as before, only without the terms in μ_2, t:

$$\frac{U_{n_t}}{U_{c_t}} = w_t^A + \frac{\eta_t}{\lambda_t} G^A_n t \tag{26}$$

$$\frac{U_{f_t}}{U_{c_t}} = w_t^A \theta_t + \frac{\eta_t}{\lambda_t} G^A_{f_t} + \frac{\mu_{1,t}}{\lambda_t} + \frac{\mu_{2,t}}{\lambda_t} \tag{27}$$

Combining 26 and 27, the shadow cost of the family constraint is equal to the difference in income generated by supplying f instead of n plus the difference in marginal human capital accumulation for n compared to f, multiplied by the shadow value of skill.

$$\mu_{1,t} = \lambda_t w_t^A (1-\theta) + \eta_t (G^A_{n_t} - G^A_{f_t}) \tag{28}$$

How are the time allocations affected? Qualitatively, the same patterns remain.[27] As seen in figure 13, the skill accumulation creates an incentive to work more early in working life and less toward the end, compared to the constrained case. At $\theta = \theta^*$, the double peak is still present in the A job, although to a lesser extent than in the R job. This implies that the individual working in the A job will accumulate less skill over the career and the fact that the loss of hours happens early in the lifecycle makes the consequences larger.

The effects of forgone skill accumulation are illustrated in figure 15. First, compare the wage profile of a family constrained individual when it just becomes optimal to pursue the A job (red dashed line) to the wage profile when technology is nearly fully flexible in the same job (the red solid line). The decrease in hours worked in the second period translates into a wage difference where the wage of the family constrained individual is 71 percent of that of the unconstrained individual in the fourth period. This wage difference decreases as technology improves; at $\theta = 0.7$, only a ten percent difference remains and when technology is fully flexible, the gap to the unconstrained individual closes (almost) completely.

[27] Note that when the minimum requirements on the A job are relaxed, the utility from this job increases and the life time utility shifts upwards. To give the R job a fair chance, set the initial wages to be equal in the two jobs and just let the different processes for wage(skill) accumulation play a role.

Figure 13: Labor supply - no minimum requirements on the A job

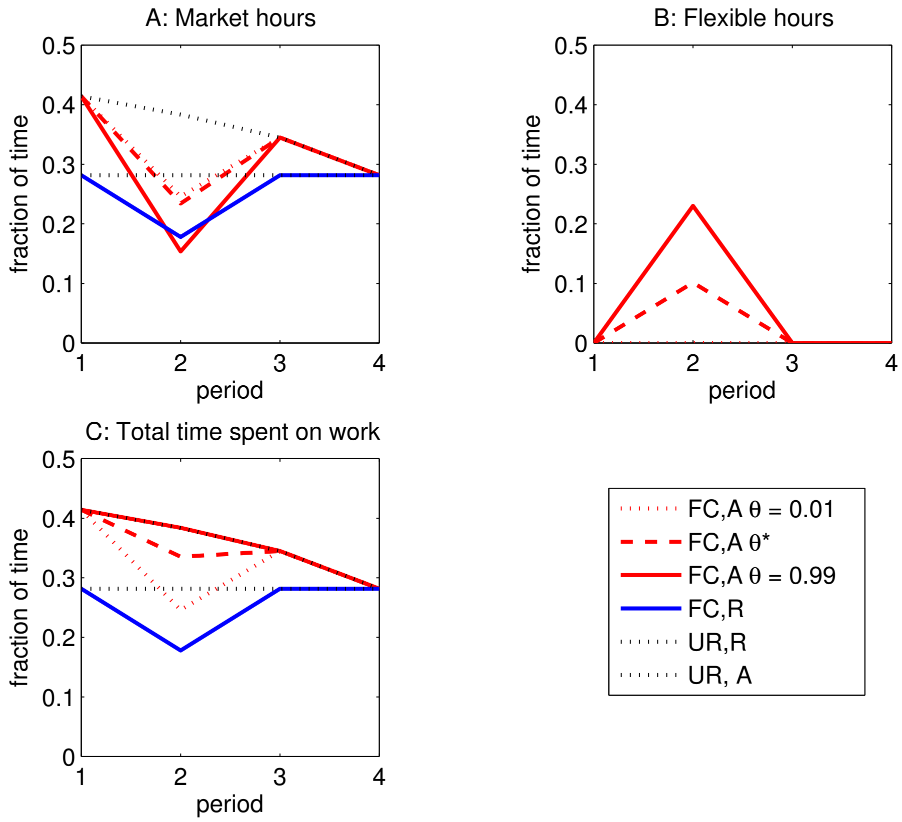


Figure 14: Home production and leisure - no minimum requirements on the A job

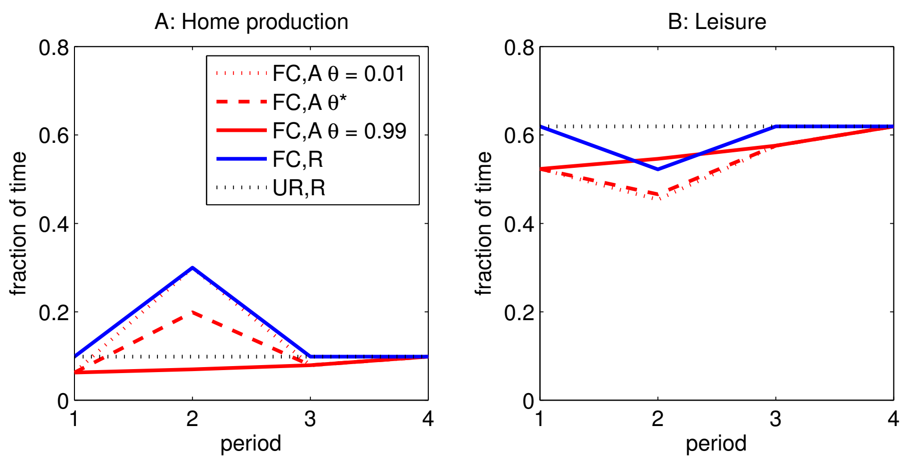
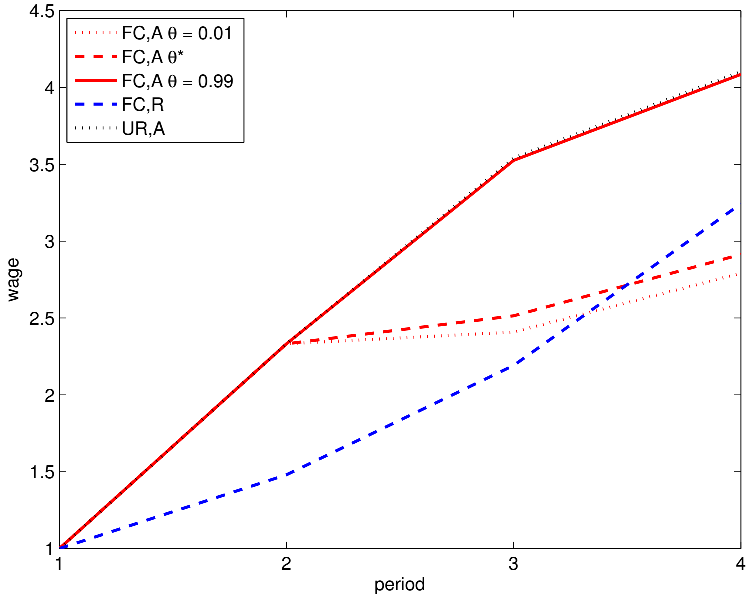


Figure 15: Wage profiles - no minimum requirements on the A job



5 Discussion and concluding remarks

Clearly men and women to some extent face different conditions on the labor market. The most obvious is the consequences of having a family, which does not only imply a career break (short or long), but also affects women's working conditions during the years the children are young. One reason is that there is a time constraint associated with having a family, one that traditionally has largely been imposed on the mothers. Yet, women today do not only work more (Attanasio et al., 2008; Olivetti, 2006), they do so in jobs with a larger skill content (Black and Spitz-Oener, 2010) and have higher returns to experience Olivetti (2006). In Sweden, although the participation levels have flattened out, female labor supply along the intensive margin has continued to increase. For women with small children, the career commitment in terms of share of full-time has increased substantially over the last fifteen years. Moreover, this movement on the intensive margin came in parallel with these women entering more complex jobs.

In this paper, I suggest a mechanism to help explain recent changes. The key components are two assumptions regarding the labor market. First, more complex (and therefor) higher paying jobs require a minimum number of hours of labor supply to be productive and lead to a successful career. Second, workplace flexibility determines the degree to which labor can be shifted during the day so that some hours of labor can be supplied from home. This implies that time constrained individuals will choose the career oriented job only if they are willing to deplete leisure, or if the job is flexible enough.

I find that for low levels of flexibility, the sacrifice of pursuing a more demanding career is too large, and family constrained individuals thus opt for the routine job. The labor supply profile of family constrained individuals exhibits the double peaked profile documented by Olivetti (2006) for women in the 1970's. When flexible technology becomes efficient enough, also family constrained individuals will find it attractive to pursue a more career oriented occupation. However, this implies a large reallocation of time, where both leisure and home production decrease and flexible labor supply and market hours increase.

Adding skill accumulation to the model further strengthens the trade-off. When the stakes are higher in the sense that current labor supply choices carry over into future periods in determining wages, it will be optimal to pursue the more demanding career at lower levels of flexible technology and as a consequence, leisure is further depleted. Comparing period utility, this trade-off results in family constrained individuals having lower utility holding the analytic job, compared to the routine job in the period when the family constraint binds. Although not modeled, this has implications for women opting out or shifting down when the requirements from home are most demanding.

Relaxing the minimum requirements on the career oriented A job, I show that the skill accumulation (and depreciation) by itself sustains the basic patterns found. Family constrained individuals in the A job will work less than those who are unconstrained, but more as compared to what they would in the routine job. Lower labor supply implies that the individual working on the A job will accumulate less skill over her working life. The cost of a career interruption is larger the larger the depreciation of human capital and the larger the forgone skill accumulation. Evaluating at the level of flexible technology where it just becomes optimal to choose the more demanding career, the decrease in hours worked translates into a rather large difference in wage, where the wage of the family constrained individual is 71 percent of that of the unconstrained individual in the last period.

The relatively simple setup in the model presented here leaves some questions open for future work. The next obvious step is to take the model to the data and make a more careful calibration. In doing so, extending the model to endogenize the fertility choice and thus the size of the family constraint would shed some light on recent trends not only in working life, but also on their interaction with trends in family formation. Another aspect that needs to be addressed is how patterns differ by educational groups. Clearly, the concept of flexibility applies to many high-skilled occupations, but may also be relevant in some low-skilled occupations that rely on the use of ICT. In addition, for some occupations, the relevant concept of flexibility is to have a flexible schedule rather than the possibility of supplying work from home.

To conclude, this paper focuses on the supply side of the story. One consequence of women becoming increasingly more educated than men is that also the share of highly skilled labor with a high demand for flexibility increases. To compete for talent, this may increase the incentives for firms to adopt such practices. In an ongoing project, we thus turn to the firm side and aim at modeling the adoption of flexible work practices.

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A Appendix

From the Swedish population wide register data LOUISE, I collect basic demographic variables such as education, sex, age and number of children aged below seven for the years 1996-2009.[28] Employees are then linked to their employers using the RAMS data base which contains information on all workers employed in a firm at some point in time each year. RAMS includes worker annual earnings by employer, the month the employment started and ended, and firm-level information such as ownership and industry. For workers who are recorded as having more than one employer during a given year, only the employer that corresponds to the highest annual earnings is retained. I obtain information on wages and occupations from the Structural Wage Statistics (SWS) which is based on annual surveys covering about 50 percent of the private sector and the entire public sector.

Industries are classified according to intensity ICT in production/usage following van Ark et al. (2003). Table A1 shows the industry codes included in the different groups. The Swedish industry classification SNI92 is equivalent to NACE rev 2 up to the four-digit level.

Table A1: Classification of industries

	NACE Rev 2 Industry codes (SNI92)
ICT producing manufacturing	30,313,321,322,323,300
ICT producing service	64,72
ICT using manufacturing	18,22,29,310,311,312,314,315,316,332,333,334,335,351,352,353,359,36,37
ICT using service	51,52,65,66,67,71,73,741,742,743,744
Non ICT manufacturing	15,16,17,19,20,21,23,24,25,26,27,28,34,354,355
Non ICT service	50,55,60,61,62,63,70,745,746,747,748,749,75,80,85,91,90,91,92,93,95,99
Non ICT other	1,2,3,4,5,10,11,12,13,14,40,41,45

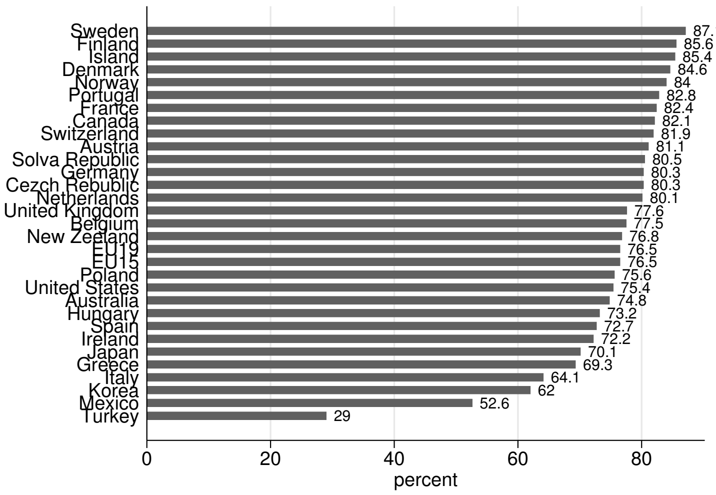
The occupational groups are formed aggregating ISCO88 occupational codes in the SWS to an aggregated version of European Socio-economic Classification (ESeC), see table A2 (see https://www.iser.essex.ac.uk/archi and Harrison and Rose (2006) for details on EsEc). The correspondence code to the ISCO 88 was kindly provided by Erik Bihagen, Swedish Institute for Social Research, Stockholm University.

Table A2: Occupational groups

	EsEc code
Higher-level managers/professionals	1
Lower-level managers/professionals, higher-level supervisors and technicians	2
Intermediate	3
Lower sales, service and routine	5, 7, 8, 9

[28]The year 1996 is is chosen as a starting year as it is the earliest year for which there is a consistent occupation classification.

Figure A1: Female labor force participation in 2007



Source: OECD, rates in 2007 for women aged 25 to 54