Mass consumption, exclusion, and technological unemployment

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Abstract

We present a general equilibrium model of monopolistic competition and study the interaction between firms' market power and the extent of inequality in the income distribution. In contrast to the standard (Dixit-Stiglitz) monopolistic competition model, we assume – realistically – (i) that consumers have non-homothetic preferences and (ii) that a typical firm's revenues are bounded. In this context, non-homothetic preferences imply that the rich consume more goods than the poor and bounded revenues impose an upper limit on employment by firms. We show that the general equilibrium may be characterized by unemployment, even though the labor market is competitive. Unemployment is "technological" in the sense that technical progress reduces the demand for labor. In a more unequal society, such an unemployment outcome is more likely and a redistribution from the rich to the poor not only enhances employment but is also Pareto-improving. We also discuss employment and welfare implications of minimum wages and of policies encouraging entry of new firms.

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"...the opinion entertained by the labouring class, that the employment of machinery is frequently detrimental to their interests, is not founded on prejudice and error, but is conformable to the correct principles of political economy."

David Ricardo (1821), On the Principles of Political Economy and Taxation, London: John Murray, 1821. Third edition.

1 Introduction

Consumption is the ultimate end of all production. When consumers are satiated with existing products, technical progress that replaces workers by machines, may make (parts of) the labor force redundant thus hurting the working class. While the theoretical possibility of such "technological unemployment" was clearly seen by classical economists, in particular David Ricardo and Karl Marx, economic history has impressively disproved such visions as an empirically relevant *long-run* phenomenon.

However, there were frequently *medium-run* episodes in economic history were technological shocks revolutionized production processes and/or where certain products ran suddenly out of fashion making labor in various sectors redundant. When, for some reason, other sectors in the economy were unable to absorb the displaced labor force employment problems emerged. Economic history is full of examples where workers were resisting the introduction of new technologies. Mokyr (1990 p. 256) notes[1]

> "Before and during the industrial revolution there were numerous examples of anti machinery agitation in Britain. (...) The hosiers guild's opposition to William Lee's knitting frame (1589) was so intense that the inventor had to leave Britain. (...) John Kay's flying shuttle (1733) was met by fierce hostility from weavers who feared for their livelihood. In 1768, 500 sawyers assaulted a mechanical sawmill in London. Several riots occurred in Lancashire in 1779, and 1792 a Manchester firm that pioneered Cartwright's power loom was burnt down. Its destruction was said to have inhibited the development of powerloom weaving for several years in this area. In the southwest of England (...) resistance to advances tributed to

[1] Mokyr (1990) notes further that workers' resistance to the introduction of labor-saving technologies was not confined to Britain, but was at least equally strong on the European continent.

> the shift of the center of gravity of the woolen industry to the northern counties. Between 1811 and 1816 the Midlands and the industrial counties were the site of the "Luddite" riots, in which much damage was inflicted on machines. In 1826, hand-loom weavers in a few Lancashire towns rioted for three days, and in 1830 the "Captain Swing" riots, aimed at threshing machines in agriculture took place in the south of England."

Conflict of interest in the introduction of new technologies are not a thing of the past. Today such competing interests occur between "workers" and "human capitalists" or between "production workers" and "knowledge workers" whereas, in the context of technology adoption, the traditional worker/capitalist distinction has become largely obsolete. The "Third Industrial Revolution" has even more potential to adversely affect threatened workers as ICT applications concern almost all sectors of the economy, manufacturing and service sectors alike. In fact, a large body of recent empirical and theoretical work has argued that such technical progress may have harmed the less skilled and may have substantially contributed to the increase in *wage inequality* over the last decades.[2] Moreover, as shown in a recent paper Juhn, Murphy, and Topel (2002), recent decades have note only witnessed an increase in wage inequality but also a dramatic increase in the *joblessness* of the U.S. male labor force. Between the boom periods 1967-1969 and 1999-2000 the extent of joblessness (unemployment *plus* non-participation) among U.S. males rose from 6.3 percent to 11.0 percent. This increase was concentrated among the low wage earners. In the wage percentile group 1-10 the non-employment rate increased by 13.5 percentage points, and the wage percentile group 11-20 increased by 11.3 percentage points. In contrast, no such change took place in the wage percentile group 61-100.

In this paper we argue that the causes for such non-employment may not necessarily be a phenomenon that is rooted in imperfections of the labor market. In fact, the widespread occurrence of joblessness among low-skilled males even on the "laissez-faire" U.S. labor market calls for different explanations. Rather than relying on labor market imperfections, we propose a theory that relies on a combination of firms' market power on the one hand and the extent

[2]For a summary see Katz and Autor (2000). Early papers include are Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993). The theoretical explanations by Acemoglu (1998, 2003) focus on the incentives to adopt particular technologies when the skill structure in the labor force changes. Further theoretical explanations are Galor and Moav (2000) and Lloyd-Ellis (1999).

of inequality on the other hand. In particular, we will show that, under sufficiently high productivity, there may be unemployment which is more severe (i) when firms have more market power and (ii) when the distribution of income across household is more unequal.

Clearly, market power of firms and unequal incomes of household are important characteristics of all moderns economies, it is interesting *per se* to study their macroeconomic implications. The fact that firms have substantial market power is undisputed and has been demonstrated in a large empirical literature in industrial organization. Similarly, incomes are quite unequally distributed between households. According to Gottschalk and Smeeding (2000), U.S. households in the 90th percentile of the income distribution earn almost six times as much as household in the 10th percentile. Moreover, as shown by Piketty and Saez (2003), the U.S. income inequality seems to have dramatically polarized since late 1970s. While by the end of the 1970s the top 1 percent tax unit earned about 8 percent of total national income, this share has increased to more than 14 percent by the turn of the century.

An important mechanism that drives our results are differences in consumption behavior between rich and poor households. Introducing non-homothetic preferences we account for a situation, where the range of consumed products can be larger for the rich than for the poor. We show that when the distribution is rather even, the likely outcome is a symmetric one in which all firms charge the same prices and the economic resources are evenly distributed across sector of production. Hence, the production structure and the markups are unaffected by inequality. However, with a more polarized distribution, the outcome may be completely different. It may pay for some firms to set high prices, and sell their product only to the rich while there are other firms that set low prices and serve the whole customer base. Hence, despite that all sectors are ex ante identical (= have the same demand and cost functions), in equilibrium the economy is divided into a sector producing "*mass consumption goods*" and a sector produces "*exclusive goods*". We show that such an outcome is more likely with a high extent of inequality.

A larger extent of inequality does not only lead to more exclusion but also to higher markups. When inequality is high, rich consumers have a higher willingness to pay, raising mark-ups in exclusive industries. To keep production for mass consumption attractive mark-ups have to increase in these industries. As result, a more unequal *size* distribution of income changes the *factor* income distribution in favor of profits.

We show further that a higher extent of economic inequality increases *technological unemployment*. By this we mean a situation where, due to a very productive technology, not all workers are needed to produce the level of output that firms are willing to supply. This output may be limited when consumers are relatively satiated with the existing goods so that the monopolistic firms' revenues are bounded above. (No firm will ever produce more output and employ than is necessary to maximizes revenues). We show that such technological unemployment can exist even when wages are fully downward flexible. With a higher extent of inequality, there is more exclusion and hence there are more firms that reach their revenue maximum at low levels of employment. Hence, in the aggregate, there will be less employment.

When unemployment is caused by inequality, it can be shown that unemployment-equilibria with lower inequality are pareto-superior to employment-equilibria with higher inequality. Hence a redistribution from rich to poor consumers (by appropriate fiscal policies) does not only increase the demand for labor but is beneficial for the whole population. Furthermore, *minimum wage increases*, may have beneficial effects for employment. This is because minimum wages redistribute income in favor of the poor, causing less exclusion and a higher demand for labor. This counteracts the employment-reducing cost effect of minimum wages. As a result, it is not a priori clear whether increases in the minimum wage will increase of decrease employment.

Apart from exploring interactions between income distribution and (un)employment, our paper also makes a *methodological contribution* to the literature. Much of the recent literature on growth, business cylces, economic geography and international trade has used the set-up of Dixit and Stiglitz (1977) to explore the macroeconomic implications of monopolistic competition.[3] We deviate in only one respect from their assumptions: we explore the implications of "variable-elasticity-of-substitution" (VES) preferences. In our model, consumers have quadratic (rather than CES) preferences and linear (rather than isoelastic) demand curves. As a result, consumer heterogeneity is not neutral to central macroeconomic variables such as aggregate output, employment, real wages, mark ups, and the structure of industry. On the negative side, our analysis shows that many results of the Dixit-Stiglitz (1977) framework are

[3]The original work of Dixit and Stiglitz (1977) was focused on the determinants of excess capacity and product diversity in a monopolistically competitive industry, a question central to industrial organization rather than macroeocnomics. Due to its simplicity and tractability, these tools have become central to macroeconomics (see e.g. the influental textbook by Romer (1996, Chapter 6).

not particularly robust with respect to this seemingly slight variation in assumptions. On the positive side, our analysis represents a simple and tractable tool to study environments where both consumer heterogeneity and market power are important.

Most of our analysis is restricted to a special case - quadratic preferences and two types of consumers. While our results are derived under these special assumptions, it is obvious that equilibria with a similar structure may arise under many different specifications of preferences and distributions. We use this set-up because it is simple and tractable.

There are several strands of the macroeconomic literature to which the present paper is related. Our analysis is related to Saint-Paul (2005) who analyzes distribution and growth when consumers have "limited needs". In particular, also Saint-Paul (2005) studies equilibria with unemployment arising from product market power. However, he implicitly assumes that income distribution is sufficiently even so that the macroeconomic equilibrium is always symmetric and an exclusion regime can never arise. In contrast, we show under which conditions asymmetric outcomes arise and how these outcome are affect by the distribution of income.

Gabszewicz and Thisse (1979) discuss the importance of consumer heterogeneity for the distribution of output across sectors in a vertical differentiation framework. They analyze a situation where firms offer different qualities. When the distribution is sufficiently unequal, we may have a situation where the highest quality is sold to the rich and the lower quality is sold to the poor. (See also Gabszewicz and Thisse, 1980, and Shaked and Sutton, 1982, 1983). Our equilibrium outcome is similar in the sense the income distribution affects the industry structure. However, in our paper products are horizontally (instead of vertically) differentiated. More importantly, our analysis focuses on the general equilibrium, whereas in those papers are interested in issues of competition in a partial equilibrium framework. As a consequence, the possibility of unemployment and implications for aggregate welfare are not addressed in those papers.

A different strand of the related literature deals with the importance of inequality for the industry structure in the context of economic development and growth. Murphy, Shleifer, and Vishny (1989) analyze how income inequality affects the size of markets and determines for how many sectors adopt a modern technology. In Murphy, Shleifer, and Vishny (1989) prices and mark-ups are exogenous and inequality affects the size of the various industries because consumers have asymmetric (hierarchic) preferences. A similar approach is followed in Falkinger

(1994) who studies growth along a hierarchy of wants in which the demand for new products is affected by the distribution of income. In that paper, as in Zweimüller (2000), distribution has only income effects and shapes the industry structure prices via preferences that are asymmetric across products. Prices and mark-ups are exogenously given. In contrast, income distribution in the present paper shapes the industry structure despite the fact that goods are symmetric with respect to preferences. Moreover, prices and mark-ups are endogenously determined by the consumers' willingnesses to pay.

Our analysis is also related to a literature that addresses the question whether imperfections in the product market *per se* may cause unemployment (Hart, 1982, Dehez 1985, D'Aspremont et al., 1990, Silvestre, 1990, and others). When there is upper bound on the firms' revenues, the maximum level of output (= the maximum level of employment) that a firm is willing to produce is also finite, even if the costs of production (= the wage rate) fall to zero. In other words, downward flexibility of wages does not necessarily eliminate the unemployment problem. This previous literature has been concerned with the existence of unemployment equilibria in a representative-agent environment. In contrast, our model shows that such an unemployment regime is more likely, the more uneven is the distribution of income and that higher inequality may aggravate the unemployment problem.

Finally, our paper contributes to the recent literature on inequality and macroeconomic outcomes. The literature has either focused on the role of capital market imperfections (for seminal papers see Galor and Zeira, 1993, Banerjee and Newman, 1993) or on political mechanisms (see, e.g., Persson and Tabellini, 1994, and Bénabou, 1996, 2005). In contrast, our model emphasizes the interaction between market power and employment decisions of firms.

The paper is organized as follows. In the next section we present our basic model. Section 3 solves the model for the special case when the equilibrium is symmetric. In section 4 we consider the an exclusion equilibrium with full employment and in section 5 we study the determination of unemployment in an symmetric equilibrium. Section 6 discusses our assumptions on income distribution and points to the possibility that the model may exhibit multiple equilibria. In section 7 we show how our results change once we allow for entry. In section 8 we discuss the implications of minimum wage legislation and show that our results are very similar once we allow for a non-zero wage floor. In section 9 we summarize our results and draw conclusions.

2 Monopolistic competition with quadratic preferences

Preferences and individual demand curves. There is a population of consumers of mass 1. All consumers have identical preferences. There is a continuous range of differentiated products $j \in [0, N]$, and the utility gain from consuming c units of a certain good j is $v(c) = -(1/2)(s-c)^2$. Assuming *symmetry* and *separability* across products total utility is

$$u(\{c(j)\}) = \int_0^N v(c(j))dj = -\int_0^N \frac{[s-c(j)]^2}{2} dj. \tag{1}$$

Consumer maximize this utility subject to the budget constraint $\int_0^N p(j)c(j)dj \leq y$, which yields the first order conditions

$$\begin{aligned} c(j) &= s - \lambda p(j) \qquad \text{if } p(j) \leq s/\lambda, \text{ and} \\ c(j) &= 0 \qquad\qquad\quad \text{if } p(j) > s/\lambda. \end{aligned} \tag{2}$$

The specification of a quadratic subutility function has two implication that will be crucial for our analysis below. First, it implies that the marginal utility from consuming the first unit is finite, $v'(0) = s < \infty$. This implies that a poor consumer may not be able to afford goods which prices are very high – the non-negativity constraint may become binding. Second, the quadratic specification implies that individual demand curves of the various consumers are linear and that the price elasticity of demand decreases when consumption is increased. Denoting by $\eta(c)$ the price elasticity of demand we have $\eta(c) = (s-c)/c$ which is decreasing in c.[4] While the quadratic utility may seem special is captures those properties of consumer demand in which we are interested. (i) Consumers get increasingly saturated with certain goods. (ii) Poor consumers may not only consume the goods in smaller quantities but may also consume a smaller range of goods than the rich. Furthermore, we adopt this assumption because it keeps the analysis simple and yields closed form solutions. In section 4 below we will discuss in more detail that our results do not hinge upon this particular quadratic specification.

Technology. All goods are produced with the same technology. Production takes place with labor as the only production factor. We assume a simple linear technology $x(j) = al(j)$ where

[4] Note that the properties of a quadratic subutility function are quite different from those of the standard Dixit-Stiglitz formulation. In that case, $v'(0) = \infty$, so that even the poorest consumers purchases all goods that are supplied; and the elasticity of demand $\eta(c)$ is the same for all consumers, i.e. does not depend on consumed quantities.

$x(j)$ is output of good j and $l(j)$ is the labor input. The productivity parameter $a > 0$ is an exogenously given constant.

Endowments. Consumers are heterogenous with respect to their incomes. As the income level is endogenously determined in the model, the distribution we take as given is that of labor endowments, and that of shares in monopolistic profits. In most of our analysis we will assume that the *composition* of income is identical across households. In other words, a household that earns twice the wage also earns twice the dividends as a poorer consumer. This implies that distribution of firm shares and the distribution of labor endowments is identical. To keep things simple, we will further assume that there are only two types of consumers, rich and poor. The poor, indexed by P, have population size β, and the rich, indexed by R, have size $1-\beta$. The income of the poor household, $y_P = \theta_P Y$ and the income of the rich is $y_R = \theta_R Y$ where Y is per-capita income in the economy. The wealth shares of both groups must sum up to unity so we must have $\beta\theta_P + (1-\beta)\theta_R = 1$. For ease of notation we take $\theta_P \equiv \vartheta$ as an exogenous constant. The group share of the rich is then implicitly given by $\theta_R = (1-\beta\vartheta)/(1-\beta)$.[5]

The labor market. The labor market is competitive. This implies all households earn the same wage w and all firms have the same marginal production cost w/a. The labor supply in the economy is normalized to unity. The labor demand depends on output of the various goods. Denote total production of good j by $x(j)$, then labor demand is $x(j)/a$. The economy's resource constraint can then be written as

$$\int_0^N x(j)dj \leq a. \tag{3}$$

Pricing Decisions of Firms. The market for each good is monopolistic. There is a mass of N monopolists who are unique suppliers for their respective product and who set prices to maximize profits. Each firm is negligible relative to the aggregate and takes wages and the prices for all other goods as given. The level of market demand faced by firm j is simply the sum of individual demands. Using first order conditions (2) for the respective types of consumers (noting that their λ's are different), the market demand function of this firm, $x(j,p(j))$, can

[5] The resulting Lorenz-curve is piecewise linear, with slope ϑ over the range $(0,\beta)$ and with slope $(1-\beta\vartheta)/(1-\beta)$ over the range $(\beta,1)$.

be expressed as

$$x(j,p(j)) = \begin{cases} 0 & \text{if} \quad p(j) \in [s/\lambda_R, \infty), \\ (1-\beta)\,[s - \lambda_R p(j)] & \text{if} \quad p(j) \in [s/\lambda_P, s/\lambda_R), \\ s - [\beta\lambda_P + (1-\beta)\lambda_R]\;\, p(j) & \text{if} \quad p(j) \in [0, s/\lambda_P)\,. \end{cases} \tag{4}$$

When the price exceeds the reservation price of the rich, $p(j) \geq s/\lambda_R$, market demand is zero; when the price is between the reservation prices of rich and poor, $p(j) \in (s/\lambda_P, s/\lambda_R]$, only rich consumers purchase; when the price falls short of the reservation price of the poor, $p(j) < s/\lambda_P$, both rich and poor consumers purchase (Figure 1).

Figure 1

Now consider a monopolist's profit maximizing price taking wages and the prices of all other goods as given. (Note that wages and goods prices do not show up directly in the market demand function (4), but only indirectly via the consumers' marginal utilities of income, λ_R and λ_P. In setting its own price, firm j takes the λ's as given.)

Firms choose the price $p(j)$ that maximizes the profit function $[p(j) - w/a]\, x(j, p(j))$. As the market demand function (4) is piecewise linear, there are two candidates for that price. Either only the rich buy, in which case the upper (steeper) segment is relevant; or both groups of consumer buy, in which case the lower (flatter) segment is relevant. Taking each of the two segments separately, it is straightforward to calculate the respective monopoly prices for these two demand curves as

$$p(j) = \begin{cases} \frac{1}{2}\,[w/a + s/\lambda_R] & \text{if only the rich buy,} \\ \frac{1}{2}\,[w/a + s/\left(\beta\lambda_P + (1-\beta)\lambda_R\right)] & \text{if all consumers buy.} \end{cases} \tag{5}$$

Obviously, selling only to the rich is only a relevant option for the monopolist when the reservation price of the poor s/λ_P is smaller than $[w/a + s/\lambda_R]\,/2$. This is the case if λ_R is sufficiently smaller than λ_P. In other words, setting high prices and selling only to the rich is only profitable, if the income difference between rich and poor is sufficiently high. More generally, firms face a trade off between market size and price. When they charge the high price, their profit margin is high but the level of demand is low. When they charge a low price, they are able to attract the whole customer base but their profit margin is low.

Symmetric and asymmetric equilibria. Recall that all firms face the same demand and cost functions. This implies that, in equilibrium, all firms must earn the same profit. Hence there are two possible outcomes: (i) a symmetric equilibrium where all firms charge the low price and all consumers purchase all goods; (ii) an asymmetric equilibrium where firms are indifferent between the high and the low price. In the latter case some firms charge high prices and sell only to the rich; and some firms charge a low price and serve the whole customer base. In other words, pricing decisions lead to a particular industry structure that divides the economy into an "mass consumption sector" and an "exclusive goods sector". The poor are "excluded" because firms set prices that the poor cannot afford (although they have a willingness to pay above the marginal cost of production). Note further that this outcome is entirely due to the fact that households' endowments with economic resources in unequal.

In what follows we will first focus on the simple case when the equilibrium is symmetric. As this case has been studied previously (including a recent paper by Saint-Paul, 2005) we will be brief. However, we will comment on the question of how endowment inequality may affect outcomes in such equilibria, a question about which previous work has been silent. We then will discuss in more detail the asymmetric case which previous work has so far not addressed at all.

3 Symmetric equilibria

A symmetric equilibrium will only prevail in equilibrium if the distribution of income is sufficiently even. In that case, each firm sets the same price $p(j) = p$. This price is along the lower (flat) segment of the demand curve and strictly dominates all prices along the upper (steep) segment of the demand curve. Given that all goods have the same price each consumer allocates expenditures equally across products. This implies that, in equilibrium, each sector produces the same quantity, so we have $x(j) = x$.

A full employment equilibrium. When the labor force is fully employed, the resource constraint (3) holds with equality equilibrium output per firm is $x = a/N$.

It is interesting to study how these parameters affects prices and wages, and hence the distribution of income in a full employment equilibrium. A useful indicator is the "Lerner index" which relates the profit margin (price minus marginal cost) to the price level. To calculate this index we first note that, in a symmetric equilibrium, firms operate on the lower segment of the demand curve. Hence, from (5), the firms' profit maximizing price is $p = (1/2)\left[w/a + s/\left(\beta\lambda_P + (1-\beta)\lambda_R\right)\right]$ where p, w, λ_P, and λ_R are endogenous variables. To get rid of the λ's, we use the consumers' first order conditions $s - c_i = \lambda_i p$ and the households' budget constraints $pc_iN = \theta_i Y$, where we note that aggregate income $Y = pxN = pa$. We then rewrite the budget constraint of consumer i as $pN\left(s - \lambda_i p\right) = \theta_i ap$ which gives us λ_i as a function of p. This allows us to replace the λ_i's and, using $\beta\theta_P + (1-\beta)\theta_R = 1$, we get $p = (w/a)\left(sN - a\right)/\left(sN - 2a\right)$. Hence the "Lerner index" is

$$\frac{p - w/a}{p} = \frac{a}{sN - a}, \tag{6}$$

and the real wage is

$$\frac{w}{p} = a\frac{sN - 2a}{sN - a}. \tag{7}$$

An equilibrium with unemployment. Full employment is not the only possible outcome. Under certain parameter values, it may be that the labor force is not fully utilized – despite the fact that there is a perfectly competitive labor market with fully flexible wages. To see this, notice that a linear market demand function has the property that revenues $px(p)$ do not monotonically increase but are hump-shaped in x with maximum revenue at $s/2$. This implies that, whatever the cost of production, no firm will ever sell more than $s/2$. Our above discussion has implicitly assumed that equilibrium firm output $x = a/N < s/2$. From equation (7) we see that this condition is required to guarantee that labor demand and labor supply intersect at a positive real wage. In the knife-edge case $a/N = s/2$ the intersection occurs when $w/p = 0$; and when $a/N > s/2$, no intersection of labor demand and labor supply exists. In other words, there is unemployment at any wage. In equilibrium, the real wage falls to zero and employment e is given by

$$e = \frac{sN}{2a}. \tag{8}$$

While one can argue that a situation where wages fall to zero and all income is appropriated by capitalists lacks any realism, it is also clear, that qualitatively similar arguments go through, once we would allow for a minimum wage, unemployment benefits, or other social transfers that put a limit to further wage cuts. The important point here is that the product market power *per se* may cause unemployment. Clearly, bringing labor market imperfections into the picture is necessary to bring the model closer to reality. We also note that the argument according to which product market power alone may cause unemployment is not new and has been brought forward by Dehez (1985), D'Aspremont, Dos Santos Fereira, and Gerard-Varet (1989, 1990), and Silvestre (1990). (For a survey of this literature, see Silvestre 1993).

Discussion. There are several points worth mentioning. The *first* point relates to the question how the extent of inequality in the distribution of economic resources affects the employment level, mark-ups and real wages. From equations (8) and (6) neither the employment levels nor the Lerner index and the real wage are affected by the distribution parameters β and ϑ. To see why notice that the Lerner index equals the inverse of the market demand elasticity $\varepsilon(x)$. It is straightforward to show that the *market* demand elasticity is the weighted sum of *individual* demand elasticities $\eta(c_i)$, the weights being the relative consumption levels c_i/x, hence $\varepsilon(x) = \beta\eta(c_P)c_P/x + (1-\beta)\,\eta(c_R)c_R/x$. With quadratic utility we have $\eta(c_i) = (s - c_i)\,/c_i$ which implies that the weighted elasticities $\eta(c_i)\cdot(c_i/x) = (s - c_i)\,/x$ are linear in c_i. As a consequence, variations in the distribution of c_i have no effect on $\varepsilon(x)$. Obviously, this logic holds for any level of x, so it also holds both for the full employment case (when $\varepsilon(x = a/N) > 1$) and for the unemployment case (when $\varepsilon(x = s/2) = 1$).[6] We will see in the next section that the neutrality of distribution does not longer hold in asymmetric equilibria. In that case, we have $c_P/x = 0$ in some sectors which destroys the linearity of $\eta(c_i)\cdot(c_i/x)$ in c_i and generates distributional implications for employment or factor income shares.

A *second* point refers to our assumption that all households earn wage and profit income in the same proportion (albeit in different levels). This implies that the extent of inequality remains unchanged, irrespective of the particular equilibrium considered. In particular, when there is unemployment and wages have fallen to zero, low income households gain from high profits in the same proportion as rich households. However, a more realistic assumption is that poor households earn predominantly wage income, where rich households earn predom-

[6] Moreover, this result is not specific to quadratic preferences but applies to all utility functions that belong to the HARA-class (see Foellmi and Zweimüller (2004)). Only utility functions that do not belong to the HARA-class feature non-linearity of $\eta(c_i)\cdot(c_i/x)$ in c_i implying distributional effects on the Lerner index and real wage (in a full employment equilibrium) and on employment levels (in an unemployment equilibrium). For instance, higher endowment inequality increases the market demand elasticity if $\eta(c_i)\cdot(c_i/x)$ is convex in c_i – and vice versa if $\eta(c_i)\cdot(c_i/x)$ is concave in c_i. It is hard to say, however, which case is the more plausible one from an empirical point of view. The answer involves higher order-derivatives of the subutility function $v(c_i)$.

inantly profit income. In the extreme case, where there are workers (owning no firm shares) and capitalists (supplying no labor), an equilibrium as described above where all households purchase all goods can no longer be supported. Obviously, when workers have no income, the lower (flat) segment of the market demand curve in Figure 1 disappears and only the upper (steep) segment is relevant. More generally, with sufficiently unequal factor income compositions across households, equilibria of types described in this section may not be supportable. In section 6 below we will come back to this issue.

A *third* point worth mentioning refers to the role of productivity and product variety in the determination of the factor income distribution or the employment levels. Equation (6) reveals that an increase in product variety N decreases the Lerner index (in the full employment equilibrium) or increases employment levels (in the unemployment equilibrium). Exactly the opposite is the case for an increase in productivity a. The reason is the following. In a full employment equilibrium, increases in product variety N lets consumers spread out expenditures across a larger number of goods. In the new equilibrium, each single firm gets less demand and hence operates at a point along the demand curve that is associated with a higher price elasticity of demand – implying a lower Lerner index. This is perfectly in line with intuition: a larger number of firms implies more competition which squeezes profits. Exactly the opposite is the case for increases in productivity a. Increases in productivity allow firms to produce more output per variety. In the new equilibrium, each single firm produces at a point that is associated with a *lower* demand elasticity. Hence the increase in a acts as if there was less competition. In an unemployment equilibrium, increases in a simply displaces workers. To produce revenue maximizing output level, a lower number of worker suffices. In contrast, product innovation increase employment: each additional revenue-maximizing firms will hire $s/2a$ unemployed workers. In sum, our analysis suggests that *product* innovations and *process* innovations may have fundamentally different implications for employment and/or the factor income distribution.

A *fourth,* and related, point has recently been elaborated in an interesting paper by Saint-Paul (2005). He studies the role of technical progress for real wages and shows that, when individual demand elasticities are decreasing, changes in productivity a have two opposing effects. On the one hand, there is the standard mechanism according to which higher productivity pushes up wages. On the other hand, an increase in a unambiguously increases mark-ups thus pushing wages down. These mechanisms can be inferred from differentiating equation (7) with respect to a. We see that for low levels of a the former effect dominates the latter, but beyond a critical level, call it $\bar{a}$, further increases in a reduce the real wage. It is straightforward to see that $\bar{a} = sN(1 - \sqrt{2}/2)$. As a result of technical progress, workers become increasingly less valuable on the market, a situation reminiscent of Marx's vision of technical progress as a cause of exploitation and the pauperization of the proletariat.[7]

4 Mass consumption and exclusion under full employment

The last section has studied symmetric equilibria. Intuitively, such outcomes will occur if consumer heterogeneity is sufficiently small and all firms are better off by selling to the whole customer base rather than serving only rich consumers. Let us now consider situations where the extent of inequality is high. In that case the high purchasing power of the rich generates an incentive for firms to set high prices and sell only to the rich. In other words, monopolists pursuing such a price policy "exclude" the poor from consumption. (Their willingness to pay is above the marginal product but monopolistic prices imply that they cannot afford such goods). In such an equilibrium, the economy is divided into a sector of "exclusive goods" and a sector of "mass consumption goods". Firms are indifferent between the exlusion strategy (= setting a low price and selling to the whole customer base) and the mass consumption strategy (= setting a high price and selling only to the rich).

Existence of an exclusion regime. Let us first consider conditions under which an asymmetric equilibrium with exclusive and mass producers exists. We have to compare profits from selling exclusively to the rich (the upper segment of the demand curve in Figure 1) to the profits from selling on mass markets (the lower segment). Selling exclusively to the rich yields market demand $(1 - \beta)(s - \lambda_R p)$, the profit maximizing price is $(w/a + s/\lambda_R)/2$, and profits are

$$\Pi_R = (1 - \beta)(s - \lambda_R w/a)^2 / (4\lambda_R).$$

[7] Interestingly, various writers including Joan Robinson have defined "exploitation" as a situation where workers are paid below their marginal product - drawing however on firms' monopsony power in the labor market rather than monopoly power in the product market . (For a recent review of these arguments see Boal and Ransom, 1997) An increase in a increases the wedge between real wages and the marginal product and, in this sense, increases "exploitation".

Selling to all customers yields market demand $s - [\beta\lambda_P + (1-\beta)\lambda_R]\, p$, the profit maximizing price is $[w/a + s/(\beta\lambda_P + (1-\beta)\lambda_R)]/2$, and profits are

$$\Pi_{tot} = [s - (\beta\lambda_P + (1-\beta)\lambda_R)w/a]^2 / [4(\beta\lambda_P + (1-\beta)\lambda_R)] .$$

We note first that, in a symmetric equilibrium, the mass consumption strategy strictly dominates the exclusion strategy $\Pi_{tot} > \Pi_R$. In contrast, in an asymmetric equilibrium, mass consumption producers and exclusive producers must earn the same profit $\Pi_{tot} = \Pi_R$. Note also that $\Pi_{tot} < \Pi_R$ cannot be an equilibrium. In such a situation no firm would sell to the poor, which would leave this group with idle purchasing power and a very high willingness to pay for some goods.

To understand under which conditions an asymmetric equilibrium exists, it is instructive to consider first a symmetric Nash equilibrium where all firms are mass producers and look at incentives to deviate from the mass consumption strategy. Let us denote equilibrium profits under symmetry by $\tilde{\Pi}_{tot}$ and the deviation profit (under the exclusion strategy) by $\tilde{\Pi}_R$. In a symmetric equilibrium, all firms have demand $x = a/N$. The above expressions for the Π's contain the endogenous variables λ_P, λ_R, and w. To get rid of the λ's we use $c_i = s - \lambda_i p$ and $c_i = \theta_i a/N$. Using equation (6) to replace p, the marginal utility of income becomes $\lambda_i = (s - \theta_i a/N)(a/w)(sN-2a)/(sN-a)$, and we get the interesting profits levels $\tilde{\Pi}_R$ and $\tilde{\Pi}_{tot}$ in terms of w and exogenous parameters

$$\tilde{\Pi}_R = \frac{(1-\beta)}{4}\frac{wa}{N}\frac{((1+\theta_R)\,sN - 2\theta_R a)^2}{(sN-a)(sN-2a)(sN-\theta_R a)}, \quad \text{and} \quad \tilde{\Pi}_{tot} = \frac{wa}{N(sN-2a)}. \tag{9}$$

The symmetric outcome is a Nash equilibrium if – starting from a situation where *all* firms are mass producers and earn profit $\tilde{\Pi}_{tot}$ – no single firm has an incentive to deviate and adopt the exclusive strategy. In other words, the inequality $\tilde{\Pi}_R < \tilde{\Pi}_{tot}$ must hold strictly. Using equations (9), noting that $\theta_R = (1-\beta\vartheta)/(1-\beta)$, we get

$$\beta < \frac{4\vartheta(1-z)^2}{4\vartheta^2(1-z)^2 + (1+\vartheta)^2 - 4\vartheta(z + \vartheta(1-z))}. \tag{10}$$

where $z \equiv a/sN$.

If condition (10) is violated, an asymmetric equilibrium arises. It is easy to check that the right hand side of this inequality goes to zero if $\vartheta \to 0$, goes to unity if $\vartheta \to 1$, and is monotonically increasing as ϑ increases from 0 to 1. Hence condition (10) is more likely violated with higher inequality, that is, when β is large and/or ϑ is small (see footnote 4).

This confirms our claim that high inequality makes an *asymmetric* outcome with exclusive goods and mass consumption goods more likely.

It is also interesting to see how z affects condition (10). In a full employment equilibrium, we must have $0 < z \leq 1/2$. It is straightforward to check that, for $z = 1/2$, inequality (10) reduces to $\beta < \vartheta$ and, for $z = 0$, the inequality becomes $\beta < 4\vartheta/\left(1+\vartheta\right)^2$. Since $4\vartheta/\left(1+\vartheta\right)^2 > \vartheta$, an increase in z implies there are more (β, ϑ)-combinations for which condition (10) is violated, so that an asymmetric equilibrium becomes more likely.[8] The reason is the following. A higher $z = a/sN$ means higher production per firm and allows an increase in consumption for both groups. This increases mark-ups as both types of consumers purchase at a less elastic point on their individual demand curves. (Recall that, under our specification of preferences, the demand elasticity decreases along the demand curve). However, since rich consumers are closer to their saturation point than the average consumer this causes a disproportionate decrease in their demand elasticity. In other words, when z increases mark-ups increase more strongly when firms sell exclusively to the rich and increase less strongly when they sell on mass markets. As a result, the exclusion strategy becomes more attractive for a degree of inequality.

Exclusion and the extent of inequality. In an exclusion equilibrium mass consumption goods and exclusive goods co-exist. We are free to order the goods in such a way, that $j \in [0, n]$ are mass consumption goods and $j \in (n, N]$ are exclusive goods. Note that the variable n describes the *industry structure* of the economy. Within $[0, n]$ and within $(n, N]$ firms are identical. We are also free to choose a numeraire for which we use the price of mass consumption goods. In what follows, we will denote the price of the exclusive good by p. Inter alia, this implies that w is the real consumption wage of the poor and the real product wage of mass consumption producers. Notice also that $p > 1$.

Let us now solve this model. It turns out convenient to focus on the three endogenous variables p, w, and n. To solve for the respective equilibrium values we need three equations. The first equilibrium condition is the firms' arbitrage equation $\Pi_R = \Pi_{tot}$. The second condition is the resource constraint (3). The third condition follows from the consumers' budget

[8] Note that the right hand side (10) of is monotonically decreasing in z over the relevant range. Taking the derivative of (10) with respect to z gives $-8(1-\vartheta)\vartheta(1-z)/(1+\vartheta(1-2z))^3 < 0$. A different way to interprete these conditions is that $\beta > \vartheta$ is a *necessary* condition for an equilibrium with exclusive and mass consumption goods, whereas $\beta > 4\vartheta/\left(1+\vartheta\right)^2$ is a sufficient condition.

constraints (see below).

To get the *first* equilibrium condition, some calculations are needed. We know from (5) that the price of exclusive goods is $p = [w/a + s/\lambda_R]/2$ and that the price of mass consumption goods is $1 = [w/a + s/(\beta\lambda_P + (1-\beta)\lambda_R)]/2$. We use these two equations to express the marginal utilities of income of both poor and rich, λ_P and λ_R in terms of p and substitute the resulting expression for the λ's in the consumers' first order conditions (2). We get for the consumption levels of the rich and the poor

$$\begin{aligned} c_R(j) &= \begin{cases} s - [s/(2p - w/a)] & j \in [0, n], \\ s - [s/(2p - w/a)]\, p & j \in (n, N]. \end{cases} \\ c_P(j) &= \begin{cases} s - (s/\beta)\,[1/(2 - w/a) - (1-\beta)/(2p - w/a)] & j \in [0, n], \\ 0 & j \in (n, N]. \end{cases} \end{aligned} \tag{11}$$

Finally, the equilibrium quantities of market demand can then be written as

$$\begin{aligned} x(1) &= s\,(1 - w/a)\,/\,(2 - w/a) \quad \text{for} \quad j \in [0, n] \\ x(p) &= (1-\beta)\,s\,(p - w/a)\,/\,(2p - w/a) \quad \text{for} \quad j \in (n, N]. \end{aligned} \tag{12}$$

Using these expression, it is straightforward to calculate the profit Π_{tot} of a firm that serves the entire market; and the profit Π_R for a firm that sells only to the rich. This yields our first equilibrium condition

$$\Pi_R = \Pi_{tot} \quad \Leftrightarrow \quad s(1-\beta)\frac{(p - w/a)^2}{2p - w/a} = s\frac{(1 - w/a)^2}{2 - w/a}. \tag{13}$$

Note that we can solve the latter equation for p and we get the following monotonically decreasing relationship between p and w

$$p = \frac{w}{a} + \frac{(1 - w/a)^2 + (1 - w/a)\sqrt{(1 - w/a)^2 + (1-\beta)\,(2 - w/a)\,w/a}}{(1-\beta)\,(2 - w/a)} \equiv g(w)$$

with $g'(w) < 0$. Note that the monotonicity of the function $g(w)$ is very intuitive. Combinations of p and w that satisfy equation (13) guarantee that profit maximizing firms are indifferent between selling to only to the rich and selling to all customers. If the wage rate is higher mass consumption producers' profits decrease. To prevent mass consumption producers from switching to exclusion, a lower exclusive price p is required for the arbitrage condition (13) to hold.

The *second* equilibrium condition derives from the resource constraint (3). Since there is full employment, the resource constraint has to be satisfied with equality and aggregate production equals a. The resource constraint can be written as $a = nx(1) + (N - n)\, x(p)$, and using the equations in (12) the resource constraint can be rewritten as

$$ns\frac{1 - w/a}{2 - w/a} + (N - n)s(1 - \beta)\frac{p - w/a}{2p - w/a} = a. \tag{14}$$

The *third* equilibrium condition is derived from the consumers' budget constraints. Using equations (11) we can write for the rich consumers' budget constraint as

$$y_R = ns\,[(2p - 1 - w/a)\,/\,(2p - w/a)] + (N - n)s\,[(p - w/a)\,/\,(2p - w/a)]\,p.$$

The budget constraint of poor consumers is

$$y_P = ns\,[1 - (1/\beta)\,(1/\,(2 - w/a) - (1 - \beta)\,/\,(2p - w/a))]\,.$$

We further note that $y_R = \theta_R Y$ and $y_P = \theta_P Y$. Using $\theta_P = \vartheta$ and $\theta_R = (1 - \beta\vartheta)/\,(1 - \beta)$ allows us to divide both sides of the former budget constraint by the latter. This yields a third conditions in n, p, and w

$$\frac{1 - \beta\vartheta}{(1 - \beta)\,\vartheta} = \frac{ns\,(2p - 1 - w/a)\,/\,(2p - w/a) + p(N - n)s\,(p - w/a)\,/\,(2p - w/a)}{ns\,[1 - (1/\beta)\,(1/\,(2 - w/a) - (1 - \beta)\,/\,(2p - w/a))]}. \tag{15}$$

The three equations (13), (14), and (15) have a convenient recursive structure. We first solve the resource constraint (14) for n. We then use the resulting expression to replace n in equation (15), which leaves us with an equation in p and w. Finally, we make use to the fact that equation (13) implies $p = g(w)$ with $g'(w) < 0$. This means we end up with a single equation that determines w . This equation can be written as

$$\frac{1 - \beta\vartheta}{(1 - \beta)\,\vartheta} = h\,(w, g(w); z)\,. \tag{16}$$

where again $z \equiv a/sN$. Once w is known all other endogenous variables of the model can be determined in a straightforward way. We are now ready to state the following

Proposition 1 *a) There exists a unique equilibrium. b) More inequality, in terms of a lower ϑ, increases markups and leads to more exclusion.*

Proof. a) See appendix.

b) Since $h(w, g(w); z)$ monotonically decreases in w, we directly see that a lower ϑ decreases the equilibrium value of w. This in turn implies a higher p, since p is negatively related to w. Hence, mark-ups will unambiguously rise. Further, the production levels $x(1)$ and $x(p)$ both rise as they increase in p and decrease in w. Hence, the resource constraint (3) can only be fulfilled if n, the number of goods sold to all, is lower. ■

Discussion. Part b. of the proposition states a first important result of our model. Higher inequality affects the industry structure and leads to more exclusion. A more unequal distribution also leads to larger price distortions in terms of higher mark-ups and increases the profit share. What is the intuition behind these results? First note that more inequality leads to more exclusion, that is more products will be sold only to the rich. As there is full employment, aggregate output is constant and equal to a. Since more products are sold at the lower quantity, the production levels of the exclusive goods must rise to keep aggregate output at its full employment level. In particular, both $x(1)$ and $x(p)$ must rise to satisfy the arbitrage condition (13). Since the demand becomes more inelastic when quantities are higher, the mark-ups must be higher as well. Hence, the mark-ups on all other markets are higher are higher and/or products are sold at higher prices p because there is more exclusion. Therefore the mark-ups and profits share rise due to more inequality.

The impact of the population share β is more difficult to analyze because it enters directly in $h(w, g(w); z)$. However, by means of simulations we can show that an increase in β has the same effects on markups as a decrease in ϑ. Finally, the function $h(w, g(w); z)$ decreases in z. When the productivity a rises, real wages decrease because - with higher equilibrium production of each good - the monopolists may set higher markups.

The results do not hinge on the particular quadratic utility function adopted here. We would get the same qualitative outcome with a more general subutility function $v(c)$ with properties $v'(0) < \infty$ and $-cv''(c)/v'(c)$ increasing in c. The latter assumption implies that the price elasticity of demand decreases along the demand curve. Intuitively, consumers get increasingly saturated with this product when consumption becomes large.[9]

A final caveat concerns the uniqueness of the equilibrium. This result hinges critical on

[9]When preferences exhibit a bliss point, the elasticity of substitution equals zero at $c = s$, hence the assumption is trivially satisfied for c near to s.

the equal factor income composition of the rich and the poor. We will come to this issue in section 6 below.

5 Exclusion and unemployment

Unemployment due to product market power may arise when firms' revenues are bounded. Such a maximum level of revenues imposes an upper limit on the number of workers that firms are willing to hire. We have seen above that, in a symmetric equilibrium, unemployment (with zero wages) arise when the (economy-wide) feasible output per firm a/N is larger than the level of output that guarantess the revenue maximum $s/2$. In the asymmetric equilibrium, where the poor are excluded from some markets, unemployment will arise more easily in the sense that a weaker condition than $a/N > s/2$ holds. With exclusion, some firms (= exclusive producers) operate on the upper segment of the demand curve and some firms (= mass producers) operate on the lower segment. Along the former, only rich consumers purchase and the revenue maximum is reached when market demand equals $(1-\beta)s/2$. Hence the highest level of employment that firms are willing to hire is smaller than $sN/2a$ and will be the smaller the larger is the degree of exclusion.

We solve the model with exclusion and unemployment in the same way as for the case with exclusion and full employment. Consider *first* the firms' arbitrage condition. Since a firm that serves the entire market reaches its maximum revenue at $s/2$ and a firm that sells exclusively to the rich reaches its maximum revenue at $(1-\beta)s/2$, the conditions which guarantees that no firm has an incentive to deviate form its current pricing policy is $s/2 = p(1-\beta)s/2$ or

$$p = \frac{1}{1-\beta}. \tag{17}$$

The *second* condition derives from the economy's resource constraint. In the unemployment equilibrium (with real wages equal to zero), there are n mass producers and $N-n$ exclusive firms, with respective output levels $s/2$ and $(1-\beta)s/2$. Such a situation arises if in such a situation the labor resources are not fully utilitzed, that is when $a > ns/2 + (N-n)(1-\beta)s/2$. We denote the degree of resource utilization by e (the employment rate) which replaces the real wage w as the endogenous variable. The labor market equilibrium condition is then

$$e = ns/(2a) + (N-n)(1-\beta)s/(2a). \tag{18}$$

The *third* equilibrium condition derives from consumers' budget constraints. The consumption level of a poor household is $c_P = s - \lambda_P$ for goods $j \in [0,n]$ and $c_P = 0$ for goods $j \in (n,N]$. The consumption level of a rich household is $c_R = s - \lambda_R$ for goods $j \in [0,n]$ and $c_R = s - \lambda_R p$ for goods $j \in (n,N]$. We know from (5) and $w = 0$ that the marginal utilities of, respectively, a rich and a poor consumer are $\lambda_P = (s/\beta)\left[1/2 - (1-\beta)/(2p)\right]$ and $\lambda_R = s/(2p)$. Using $p = 1/(1-\beta)$, from our first equilibrium condition, we can calculate the respective consumption levels as $c_P(j) = \beta s/2$ and $c_R(j) = (1+\beta)s/2$ for $j \in [0,n]$. As we know that $c_P(j) = 0$ and $c_R(j) = s/2$ for $j \in (n,N]$ we can write the budget constraint for a rich consumer as

$$\frac{1-\beta\vartheta}{(1-\beta)\,\vartheta} = \frac{n(1+\beta)s/2 + (N-n)\,ps/2}{n\beta s/2}. \tag{19}$$

Conditions (17), (18), and (19) contain the three unknowns e, p, and n. For later use, it will be convenient to express the equilibrium condition in terms of the employment level e. We solve the resource constraint for n, which yields n as a function of e and we the arbitrage condition $p = 1/(1-\beta)$. This allows us to replace n and p in (19) and we end up with an expression in the endogenous variable e

$$\frac{1-\beta\vartheta}{(1-\beta)\,\vartheta} = h(0, g(0), ez) = \frac{1-\beta\left[2ez - (1-\beta)\right]}{(1-\beta)\left[2ez - (1-\beta)\right]} \equiv \tilde{h}(e). \tag{20}$$

We note that the function $\tilde{h}(e)$ is decreasing in e. It is now easy to calculate the equilibrium employment level

$$e = \frac{sN}{2a}\left(1 + \vartheta - \beta\right), \tag{21}$$

and the equilibrium amount of exclusion is

$$\frac{n}{N} = \frac{\vartheta}{\beta}. \tag{22}$$

As $\vartheta < \beta$, which is the necessary and sufficient condition for the existence of an exclusion regime, the share n/N is smaller than one. The result is very intuitive. If inequality increases, either because relative income of the poor ϑ goes down, or because the group size of the poor β increases, the fraction of goods purchased by both groups of consumers decreases and the fraction of exclusive goods increases.

Proposition 2 *If there is exclusion, more inequality leads to higher unemployment. Also in the unemployment equilibrium, more inequality raises exclusion.*

Proof. If ϑ falls or β rises, inequality rises in a Lorenz sense. The proposition follows directly from (21) and (22). ■

In the exclusion regime, the unemployment rate depends on the extent of inequality. It is easy to see that the employment rate e is higher in an asymmetric equilibrium than in the symmetric regime. If inequality rises, because ϑ falls or β rises, the poor will be excluded from more markets and n/N falls. More monopolists choose to set the high price $p = 1/(1-\beta)$ and produce only $(1-\beta)s/2$ (for the rich), instead of setting the low price (equal to unity) and producing $s/2$ (for both rich and poor). As a result, more inequality reduces aggregate real output and labor demand.

This result has striking welfare implications.

Proposition 3 *In the unemployment equilibrium, a redistribution of income from the rich to the poor is Pareto improving.*

Proof. Evaluating utility function (1) for both types of consumers, respectively, at $c_P = \beta s/2$ and $c_R = (1+\beta)\,s/2$ for $j \in [0,n]$ and $c_P = 0$ and $c_R = s/2$ for $j \in (n,N]$ yields $u_P = -\left(4 - 4\vartheta + \vartheta\beta\right) s^2 N/8$ and $u_R = -\left(1 - 2\vartheta + \vartheta\beta\right) s^2 N/8$. Obviously, both u_P and u_R increase in ϑ and decrease in β. ■

To understand the intuition behind this result let us consider an increase in ϑ. From equation (22) we know that this reduces the number of exclusive producers and increases the number of mass producers whereas the consumption levels are not affected. The poor consume a larger range of goods in quantity $c_P = \beta s/2$ which obviously raises their welfare. The rich consume a larger range of goods in quantity $c_R = (1+\beta)\,s/2$ and a smaller range of goods in quantity $c_R = s/2$ so that also their welfare is increased. Less inequality due to a reduction β also increases the range of mass consumption goods. While a reduction of β also reduces the mass consumption levels for both rich and poor households, the net effect is an unambiguous increase in welfare for both groups.

An alternative way to phrase this result is that redistribution has a demand effect. It increases the level of demand by the poor which raises not only employment but also aggregate profits. Obviously, this increases the level of real income of the poor: they have a larger share in a larger pie. It also increases the level of income by the rich: their smaller share is overcompensated by the larger size of the pie. So, on net, also the rich have a higher real income.

Figure 2 summarizes how the distribution parameters affect equilibrium outcomes. Panel a. is depicted for the case $z \equiv a/(sN) < 1/2$. The separation between the symmetric and the asymmetric regime is given by equation (10). In that case, unemployment is only possible in the exclusion regime and it occurs if and only if $z > (1 + \vartheta - \beta)/2$. If β rises and/or ϑ falls, unemployment rises. If $\beta = 1$ and $\vartheta = 0$, aggregate output diverges to zero and the unemployment rate equals one. In Panel b. the parameter z is higher than 1/2. Then, unemployment will arise already in the symmetric case. However, in the exclusion regime (which arises if $\beta > \vartheta$) unemployment is higher and depends on distribution.

Figure 2a and 2b

6 Variable income composition and multiple equilibria

So far, we have assumed identical income composition of rich and poor households. This is clearly a very unrealistic assumption. It implies that the share of capital income in total individual income is as high for the poor as it is for the rich. In reality, the incomes of the poor consist manly of labor income whereas richer household typically own a disproportionate share in aggregate wealth so their income typically consists to a large extent of returns on those assets. This has the implication that labor income is typically less unequally distributed than capital income; or that total income is more equally distribution than wealth.

To capture these empirical facts, we now allow for situations where the factor bundle owned by the poor are different from those owned by the rich. Assume that a poor household owns Δ_P units of labor and the rich own Δ_R units. Since we have normalized aggregate labor supply to unity we have $\beta\Delta_P + (1-\beta)\Delta_R = 1$. Again this leave us with one degree of freedom and we take $\Delta_P \equiv \delta$ as exogenous from which $\Delta_R = (1-\beta\delta)/(1-\beta)$ is determined. Similarly, we assume that profits distributed to a poor household amount to a fraction $\Gamma_P < 1$ of profits per capita where the rich equals $\Gamma_R > 1$. Again, we must have $\beta\Gamma_P + \Gamma_R(1-\beta) = 1$ and set $\Gamma_P \equiv \gamma$ as exogenous from which $\Gamma_R = (1-\beta\gamma)/(1-\beta)$ is determined. So far, we have studied the case $\delta = \gamma \equiv \vartheta$. Now we concentrate on the more realistic case where $\gamma < \delta$. In the special case when workers own no firms and firm owners do not work, that is a "worker-capitalist" economy, we have $\gamma = 0$ and $\delta = 1/\beta$.

We can use our analysis from the last two sections to see how this affects the equilibrium in a situation with exclusion. Notice that the first two equilibrium conditions (the firms' arbitrage conditions (13) and (17) and the resource constraints (14) and (18)) and also the right hand side of relative budget constraints (15) and (19) hold for any arbitrary distribution of factor incomes and hence the right hand sides of equations (16) and (20) are the same as before. However, the left hand sides are now different. In the full employment equilibrium, the income ratio y_R/y_P can be expressed as

$$\frac{y_R}{y_P} = \frac{w\,(1-\beta\delta)\,/\,(1-\beta) + N\Pi(w)\,(1-\beta\gamma)\,/\,(1-\beta)}{w\delta + N\Pi(w)\gamma} \equiv \phi(w)$$

where, from (13), we have $\Pi(w) = s\,(1-w/a)^2\,/\,(2-w/a)$. Notice that $\phi'(w) < 0$. The negative slope of the function ϕ is intuitive. If real wages increase, the income ratio falls because poor households benefit disproportionately from an increase in the compensation of labor. When there is unemployment with zero wages, as in the last section, aggregate income consists only of profits and the income ratio is determined solely by profit distribution parameters β and γ of profit income whereas the labor income distribution parameter δ does not play any role. The general equilibrium condition can then be rewritten as

$$\begin{aligned} \phi(w) &= h\,(w, g(w); z) \qquad \text{with full employment} \\ \frac{1-\beta\gamma}{(1-\beta)\,\gamma} &= h\,(0, g(0); ez) \quad \text{with unemployment} \end{aligned} \tag{23}$$

To establish the equilibrium, we draw the left hand side and the right hand side of the equations (23) against unemployment $u = 1-e$ (left part of figure 3; u measured from right to left) and against w (right part of figure). We have already checked above that the right hand sides coincide at $w = 0$ and $e = 1$ or $u = 0$ (the knife-edge case where labor demand and labor supply cut exactly at $w = 0$). Moreover, we have seen that both $h\,(w, g(w))$ decreases in w and $h\,(0, g(0); (1-u)z)$ decreases in $e = 1-u$.

Similarly, it is straightforward to check that also the income ratios (left hand side of (23)) does is continuous when going from full employment to unemployment as $\phi(0) = (1-\beta\gamma)\,/\,[(1-\beta)\,\gamma]$. Furthermore, we know that the expenditure ratio is decreasing under full employment ($\phi(w)$ decreasing in w) but constant under unemployment.

Proposition 4 *With an unequal income composition, the general equilibrium need no longer be unique. In case of multiplicity, there is at most one equilibrium with unemployment.*

Figures 3a and 3b

Using simulations, it is straightforward to show that the above proposition holds. Obviously, several outcomes are possible. Panel 3a shows a unique equilibrium. Depending on parameters, the intersection point may lie in the right part (full employment) or in the left part (unemployment). Exactly the same mechanisms are at work as discussed in the last sections. We have already proved in these sections that for the special case where $\delta = \gamma$ the equilibrium is unique. Intuitively, if δ and γ do not differ much, multiple equilibria are less likely.

However, when the factor income composition is very different there may be multiple equilibria, see figure 3b. It can be shown that full employment and unemployment equilibria may co-exist.[10] It may also be that more than one full employment equilibrium exist. Note however, that one and only one unemployment equilibrium can exist. The reason is that, under unemployment, the income ratio is independent of the level of employment whereas the consumption expenditure ratio is decreasing in the level of employment.

The reason why multiple equilibria may occur is a that firms' employment decisions are strategic complements. When all firms employ many workers and pay high wages, inequality and hence exclusion is low. But this implies that labor demand is high so that a full employment equilibrium with high wages can be sustained. When firms do not employ all workers and pay zero wages, income inequality is high as workers have only little (or no) income and hence only little (or no) demand for products. In that case, labor demand is low and hence an equilibrium with unemployment and zero wages is sustained.

7 Minimum wages

One reason that we do not observe in practice the extreme case of zero real wages is that policy would prevent mark-ups to rise excessively. A possible measure to retain the purchasing power of wages is the introduction of a minimum wage $\bar{w}$, measured in terms of the numeraire. In other words, we consider a minimum wage that is indexed to the costs of living of the poor – which is given by the price of mass consumption goods.

Let $w \geq 0$ be the equilibrium wage in the laissez-faire equilibrium. Obviously, when $\bar{w} \leq w$ the minimum wage is not binding and firms pay the wage w. Instead, when $\bar{w} > w$, the

[10]This can be shown by means of an example. When the exogenous parameters take the following values: $a = 3.72$, $\beta = 0.65$, $N = 10$, $s = 2$, $\delta = 0.485$, $\gamma = 0$; there are two equilibria with exclusion but full employment: $w_1 = 2.857$ and $w_2 = 0.752$ and one equilibrium with unemployment where $u = 0.0591$.

minimum wage becomes a constraint for firms. In this case the model is readily solved. We have to replace w by $\bar{w}$ in the full employment relations (13), (14), and (15). Moreover, we have to take account of the fact that not all resources are fully utilitzed $e < 1$. This implies that a on the right hand side of (15) has to be replaced by $ea < a$. These are three equations in the three unknowns e, n, and p. Just like in the full employment case (where we could reduce the general equilibrium conditions to the single reduced form equation (16) in w), we can now reduce our three equilibrium conditions to a single equation in the employment rate e.

Identical income composition. It is instructive to consider first the case when rich and poor households have an identical income composition so that $\gamma = \delta \equiv \vartheta$. In that case, the general equilibrium condition can be written as

$$\frac{1 - \beta\vartheta}{(1-\beta)\,\vartheta} = h\left(\bar{w}, g(\bar{w}); ez\right). \tag{24}$$

It is straightforward to verify that the right hand side of equation (24) is falling in e. As the left hand side is independent of e, an unemployment equilibrium, if it exists, is unique. Just like in an unemployment equilibrium with zero wages, the amount of consumption expenditures of the rich relative to those of the poor, as described by $h\left(\bar{w}, g(\bar{w}); ez\right)$, is the lower the higher the level of employment.

What is the effect of an increase in the minimum wage? We know from our above discussion that the function $h\left(\bar{w}, g(\bar{w}); ez\right)$ is decreasing $\bar{w}$. As the function h is decreasing in e, this implies a negative relationship between e and $\bar{w}$. In other words, an increase the minimum wage leads to a reduction in employment. The reason is very intuitive. A minimum wage raises the cost of production and reduces mark-ups. This induces firms to operate on a point along the demand curve that is associated with less production (and a higher demand elasticity), firms hire less workers and unemployment increases. The increase in the minimum wage reduces profits more strongly in the mass consumption sector. Hence in the new equilibrium there are more exclusive and less mass consumption producers.

Above we have shown that a redistribution of income from the rich to the poor may enhance employment and welfare, starting from an unemployment equilibrium with zero wages. Does this result still hold in the in the presence of positive minimum wages? Equation (24) makes it clear that this result remains unchanged even with positive minimum wages. To see this, notice that an increase in ϑ reduces the left hand side of equation (24) but leaves the right hand side unaffected. To establish equilibrium, employment e has to increase for equation (24) to be satisfied. Intuitively, a reduction in inequality due to a rise in ϑ leads to less exclusion and more products are mass consumption goods. This increases aggregate production and employment. Notice that, as long as the minimum wage $\bar{w}$ remains unchanged prices remain unchanged. Hence all adjustment is made by former exclusive firms now adopting the mass consumption strategy. This implies there are more sectors that supply goods at low prices and less sectors that supply goods at high prices. As this is beneficial for both groups of consumers, such redistribution is Pareto-improving.

Unequal income composition. Under identical income compositions the introduction of minimum wages has no impact on inequality and relative incomes $y_R/y_P = (1-\beta\vartheta)\,/\,[(1-\beta)\,\vartheta]$ are independent of $\bar{w}$. This is clearly very unrealistic. In fact, minimum wage policies are often explicitely adopted to reduce poverty and inequality. Let us therefore study the relevant case, $\delta > \gamma$, when the poor draw mainly labor income and the rich predominantly profit income. In that situation, relative income y_R/y_P are no longer constant, but depend on the level of employment e and on the minimum wage $\bar{w}$. Formally, we have

$$\frac{y_R}{y_P} = \frac{e \cdot \bar{w}\,(1-\beta\delta)\,/\,(1-\beta) + N\Pi(\bar{w})\,(1-\beta\gamma)\,/\,(1-\beta)}{e \cdot \bar{w}\delta + N\Pi(\bar{w})\gamma} \equiv \tilde{\phi}(\bar{w}, e).$$

(Notice that $\Pi(\bar{w})$ depends only on $\bar{w}$ but not on e; replacing w by $\bar{w}$ equation (13) is still relevant; furthermore, we assume here that rich and poor are equally harmed by unemployment in the sense that the degree of underutilization of labor same $1-e$ is the same for both types of consumers). It is straightforward to verify that $\partial\tilde{\phi}(\bar{w}, e)/\partial e < 0$.

Now consider the general equilibrium. The right hand side of equation (24) is still relevant, we only have to replace the left hand side by $\tilde{\phi}(\bar{w}, e)$. This yields

$$\tilde{\phi}(\bar{w}, e) = h\left(\bar{w}, g(\bar{w}); ez\right). \tag{25}$$

Both function $\tilde{\phi}$ and h are downward sloping in e and provided that $\tilde{\phi}$ is flatter than h at the point of intersection, we have a stable general equilibrium. Furthermore, it is no longer sure, whether the equilibrium is unique or whether we have multiple equilibria. (In the case of multiplicity low and high unemployment equilibria – at the same minimum wage – may co-exist, the reason for multiplicity being again a labor demand complementarity).

Let us consider again the employment effect of an increase in the minimum wage $\bar{w}$. An increase in the minimum wage now reduces both the left hand side and the right hand side of equation (25). Clearly, if both sides of the equation fall by exactly the same amount, the general equilibrium condition still holds and no adjustments in employment e are necessary. In general, however, it is not clear whether the left hand side falls more strongly than the right hand side. Intuitively, there is still a *cost effect,* which induces firms to hire less labor. This cost effect dominates when minimum wages do not have an impact on relative incomes (the case discussed above when $\delta = \gamma \equiv \vartheta$). However, when wages increase relative incomes and hence reduce inequality, there is also a *purchasing power effect.* This purchasing power effect leads to less exclusion and has positive effects on the demand for products and hence on employment. The former effect is captured by the right hand side of equation whereas the latter effect is captured by the left hand side of (25). It can be shown by simulations that, under some paramter constellations, the purchasing power effect dominates the cost effect, whereas under other parameter values, the opposite is the case.

Notice further that, also under differing income compositions between household types, redistributions from the rich to the poor do not only enhance employment and but also improve welfare for both groups. The following proposition summarizes the role of minimum wages and of redistribution of income in an unemployment equilibrium.

Proposition 5 *Consider an unemployment equilibrium characterized by a positive minimum wage $\bar{w} > 0$. a) When the composition of income is identical across households, an increase in the minimum wage reduces employment. b) When the poor draw mainly labor income and the rich draw mainly profit income, an increase in the minimum wage has an ambiguous impact on employment. c) A redistribution of income from the rich to the poor (for instance, by appropriate fiscal policies) raises employment.*

8 Entry

We have shown that unemployment may be an equilibrium under the assumption that the number of firms N is fixed and entry is prohibited. Since firm's revenues are maximized at a finite quantity of output, there is an upper limit on the firms' demand for labor. Furthermore, in such an equilibrium, an increase in productivity a must eventually lead to decline in labor demand. However, an obvious possibility that may let the economy escape this demand constraint, is to allow for entry.

We proceed in two steps. We first assume as before that all workers are homogenous and that they can work either in production or in a sector that introduces new goods. In all other respects, the model is the same as before. We will show that under such conditions, unemployment disappears. Second, we analyze a situation where the design of products requires skills. When these skills are scarce unemployment may still prevail even when we allow for entry. We show conditions under which such technological unemployment is still a theoretical possibility and the results we reached in previous sections hold even if we allow for entry.

Homogenous workers. Assume that G workers are needed to develop a new product. We assume free entry, hence the setup costs wG must equal profits. Profits are given by equation (13) and the free entry condition reads

$$wG = s\frac{(1-w/a)^2}{2-w/a}. \tag{26}$$

Note that - as before - marginal costs in (final) goods production equal w/a and act as numeraire. The number of products is then determined by the following relationship

$$N = G\,(1 - L_Y) \tag{27}$$

where L_Y denote the number of workers employed in production. Labor force equals 1 and thus $1 - L_Y$ workers work in the design sector.

Consequently, output in final goods production equals aL_Y. Taking this into account we insert the labor market equilibrium condition (14) to get the rewritten general equilibrium condition (16)

$$\frac{1-\beta\vartheta}{(1-\beta)\,\vartheta} = h\left(w, g(w); \frac{aL_Y}{sN}\right). \tag{28}$$

where $g(w)$ is defined as above and z is replaced by $aL_Y/\,(sN)$. We have two new endogenous variables, N and L_Y, and two new equations: (26) and (27). The arbitrage condition (13) is of the same form as in the baseline model.

The solution of this extended model is straightforward. Wages w are determined by (26). The price of the exclusive good p can be calculated from (13). We may insert the prices and (27) into (28) which yields a unique solution for $aL_Y/(sN)$.

An increase in productivity a triggers entry of new firms because this decreases the costs of production and increases mark-ups. Unemployment cannot arise any more. Put differently, real wages cannot fall to zero as becomes clear from equation (26). As workers may be employed both in production and design sectors, a wage of zero would imply zero entry cost. This would trigger entry of new firms and prevents aggregate labor demand from falling short of labor supply.

High and low skilled workers. The assumption that labor is homogeneous and employed both in the design and final goods production sector prevented long run unemployment to arise. However, when there are low skilled workers who are not employed (or to a lesser extent) in the design sector they may not profit from increases in the productivity and / or stay unemployed even in the long run where the number of products is endogenous.

Assume that there exist two types of workers: high-skilled and low-skilled. In the design sector, only high-skilled workers are employed. In analogy to the model just analyzed we assume that G high-skilled workers are needed to create a new design. Instead, in goods production both types of workers are employed. The technology is given by

$$y = AF(h_Y, l) \tag{29}$$

where h_Y and l denote the number of high- and low-skilled workers employed in a single sector. The production function has constant returns in factors h_Y and l. The production function has an associated marginal cost function which we denote by

$$m \equiv \frac{w_H}{A} q\left(\frac{w_L}{w_H}\right), \text{ with } c' > 0 \tag{30}$$

where w_L and w_H denote the high- and low-skilled wage, respectively. The other elements of the model are identical to before. Hence, we get the isomorphic equilibrium equations where we simply replace w/a with m.

Firms minimize costs and produce with the same factor intensity due to constant returns to scale. Denote total employment in goods production by $H_Y = \int_0^N h_Y(j)dj$ for high skilled workers and $L = \int_0^N l(j)dj$ for low skilled workers. As all firms have the same production function, all firms choose the same factor intensity H_Y/L. The wage ratio must satisfy the following first order condition

$$\frac{w_L}{w_H} = \frac{F_L(h_Y, l)}{F_H(h_Y, l)} \equiv \varphi\left(\frac{H_Y}{L}\right), \text{ with } \varphi' > 0. \tag{31}$$

To keep the model as close as possible to our baseline treatment let us take the extreme assumption that poor and rich have the same relative endowment of the high- and low-skilled factor. (Profits are zero with free entry). This implies that the expenditure share is exogenous. Using the variety production function $N = G(H - H_Y)$, the resource and the individual budget constraints we find the first equilibrium condition

$$\frac{1-\beta\vartheta}{(1-\beta)\vartheta} = h\left(m, g(m); \frac{AF(H_Y, L)}{sN}\right). \tag{32}$$

The only difference to above is that total goods output equals $AF(H_Y, L)$ instead of aL. As we know from Proposition 1 the right hand side of (32) is decreasing in m and decreasing in $AF(H_Y, L)/(sN)$, hence (32) defines a negatively sloped curve in the (H_Y, m) space with H_Y on the horizontal axis.

The second equilibrium condition is derived using the free entry condition $w_H G = \Pi_{tot}(m)$. The wage w_H can be expressed as a function of H_Y/L with (30) and (31). We get

$$\frac{AG}{q(\varphi(H_Y/L))} m = s\frac{(1-m)^2}{2-m}. \tag{33}$$

The right hand side, $\Pi_{tot}(m)$, decreases in its argument and the left hand side decreases in H_Y/L. Hence, equation (33) defines a monotonically increasing curve in the (H_Y, m) space.

Proposition 6 *There exists a unique equilibrium with $H_Y^* \geq 0$. a) If the elasticity of substitution ε between production factors is between zero and one, $0 < \varepsilon \leq 1$, $H_Y^* > 0$ and there is no unemployment among the low skilled. b) If $\varepsilon > 1$, there may be unemployment. Unemployment arises if $sGH(1+\vartheta-\beta)/2 < AF(0, L)$.*

Proof. Existence. We argue graphically (Figure 4). The slope of the equilibrium curves is discussed above. The budget constraint curve crosses the H_Y - axis at $AF(H_Y, L)/(sG(H - H_Y)) = (1+\vartheta-\beta)/2$ which must occur at a $\hat{H}_Y < H$. If the curves do not cross, $H_Y^* = 0$ in equilibrium.

a. When the m axis intercept of budget constraint curve exceeds that of the free entry curve, $H_Y > 0$ must hold in equilibrium. Note first, if $\varepsilon \leq 1$, equation (32) only holds true for $H_Y = 0$ when $m = 1$. On the other hand, both factors are necessary in production or $F(0, L) = 0$. In that case $c(\varphi(0)) = c(0) = 0$, hence the value of marginal costs m satisfying the free entry condition (33) goes to zero when H_Y approaches zero.

b. If $\varepsilon > 1$, positive production can be achieved using one factor only $F(0, L) > 0$. In a possible unemployment equilibrium, markups are infinite and aggregate demand for low skilled

labor equals $GH\,(1+\vartheta-\beta)\,s/2$. If this number falls short of $AF(0,L)$, there is unemployment.
$\blacksquare$

Figure 4

Discussion. Unemployment arises if the low skilled workers can produce the final goods output alone which possible because the maximum number of varieties is pinned down by the stock of high skilled workers: $N = GH$. Intuitively, the elasticity of substitution is low, the productivity of an additional low skilled worker decreases strongly. Exactly this pattern helps the low skilled to escape from unemployment. This is an analogous result to the impact of productivity a discussed in our baseline model. Further note that an increase in the stock of high skilled reduces the unemployment problem of the low skilled. With more high skilled workers the potential range of products is higher which increases the demand for low skilled workers in final goods production.

A further interesting point of this model is the fact that an increase in productivity A - although not itself skill-biased - leads to an increase in the skill-premium w_H/w_L. Because the preferences exhibit satiation, an increase in A raises markups since all monopolists face a more inelastic demand curve with higher production per good. This raises the value of product design hence the size of the skill-intense design sector increases what drives up high-skilled wages.

9 Summary and conclusions

We have studied the macroeconomic equilibrium in a model of monopolistic competition where consumers have linear (rather than isoelastic) demand curves. We have seen that this apparently slight change in assumptions compared to the standard monopolistic competition model has major consequences for macroeconomic outcomes. As a result, our analysis raises strong doubts about the innocence of the representative agent assumption of the monopolistic competition model.

First, we have seen that the extent of inequality has implications for the structure of industry. With sufficiently high inequality, there may be *mass consumption* sectors and sectors producing *exclusive goods*. Such an outcome – which arises despite ex ante identical cost and demand conditions across sectors – is more likely when incomes are more unequally distributed. Hence our model supports the intuition that mass consumption sectors are less prevalent in less egalitarian societies.

A second main implication is that the macroeconomic equilibrium may feature *technological unemployment.* This is a situation which arises when productivity is high relative to the potential market size and such unemployment is more likely to occur with a higher degree of income inequality. Hence firms' market power and inequality in the household income distribution may be important factors explaining why resistance to the introduction of new technologies is more important in certain environments and less important in others.

A third main results concerns the implications of *redistributive fiscal policies* in an unemployment equilibrium. We have shown that a policy that redistributes income from the rich to the poor may not only help reducing unemployment but may benefit all groups in the population. The employment effect is due to a purchasing power effect on the part of the poor. The increase in product demand supports higher employment. This leads to less exclusion, that a larger number of sectors that charge low prices for their products. This redistribution obviously benefits the poor, because they can afford more products. This redistribution also benefits the rich, because they benefit from the lower price in those sectors that swith from exclusion to mass production.

A fourth result is that there may be *multiple (Pareto-rankable) equilibria* due to a complementary in the demand for labor. On the one hand, there may by an equilibrium where all firms hire many workers and pay them high wages. This leads to a high demand for consumption goods and supports a high-wage high-employment equilibrium. On the other hand, there may an equilibrium where, under the same parameter values, firms hire few workers and pay them low wages. This leads to a low demand for consumption goods and supports a low-wage equilibrium with unemployment.

A fifth main results of our analysis concerns the *role of the minimum wage* for the employment level. Increasing the minimum wage has two opposing effects. On the one hand, it raises costs and induces firms to hire less labor. On the other hand, increases in the minimum wage reduce income inequality which increases the demand for firms' output. It is not a priori clear which effect dominates and it has been shown by simulations that the latter effect may dominate the former. Note that such positive employment effects may arise despite a perfectly competitive labor market. Hence our analysis shows that product market imperfections may be another reason (in addition to monopsonistic labor markets) why many empirical studies fail to find significantly negative employment effects of minimum wage increases.

Finally, we explore the consequences of *entry of new firms* in unemployment equilibria. When entry is costly and requires a fixed homogenous labor input, unemployment will be eliminated on a perfect labor market. However, when entry requires a scarce resource (skills such as "entrepreneurial talent") unemployment is not necessarily eliminated since scarcity of resources puts an upper limit on the number of firms that enter. Moreover, in reality, unemployment equilibria are associated with positive wages and, in such a situation, firms have not only an incentive to introduce new products but also to reduce costs of production. We have shown that technological unemployment results from a "race" between product and process innovations. Unemployment will be eliminated only if the employment gains by additionally entering firms outweighs the employment losses that from increases in productivity. Hence our model features the intuition that product innovations are necessary for employment and economic growth because cost-saving technical progress alone makes workers in existing industries obsolete.

How general are our results? We have assumed quadratic preferences and have restricted the distribution to two types of consumers. (With respect to the assumption of quadratic preferences we note that this specific utility function belongs to the HARA class. This class has nice aggregation properties but these properties disappear as soon as non-negativity constraints do become binding.) However, our results do not hinge on the specific formulation of quadratic utility. Two important assumptions on preferences are needed to generate our results. The first assumption is that $v'(0)$ is finite, hence the prohibitive price is finite. The second assumption is that the elasticity of demand is falling in the consumption level. Hence richer individuals will also have the more inelastic demand, and monopolists are tempted to sell exclusively to the rich as mark-ups from such a strategy are higher. It is in this sense that the quadratic utility function is an interesting example that highlights potentially important mechanisms relating inequality and market power.

Our focus on the simple case with only two groups of consumers is less essential. It is easy to imagine (though somewhat tedious to calculate) an equilibrium in which there are three (or more) different groups. If these groups are sufficiently different from each other, the asymmetric equilibrium will be characterized by a situation where a certain range of product is purchased only by the rich, another range also by the middle class, and the remaining goods will be mass consumption goods. Our result are robust to different assumptions about the distribution of income. What is essential to get an asymmetric equilibrium, however, is a sufficiently polarized income distribution.

Finally, let us emphasize that our results on technological unemployment refer to contexts where increases in productivity tend to make (certain groups of) workers redundant without a corresponding increase in the potential market size. This is not to say that, over the long-run, the introduction of new technologies has harmed workers. In historical perspective exactly the opposite has been the case. Nevertheless, our analysis sheds light on mechanisms that may be potentially important to understand medium-run episodes in which (groups of) workers are in low demand and face severe employment problems over extended time intervals. The problems that low-skilled workers currently face on labor markets in most industrial countries is a recent example for such times.

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Appendix

Proof of Proposition 1 To show that the equilibrium exists it suffices to show that $h(w, g(w))$ is larger than $\frac{1-\beta\vartheta}{(1-\beta)\vartheta}$ for small values of w and smaller than $\frac{1-\beta\vartheta}{(1-\beta)\vartheta}$ for high w because $h(\cdot)$ is a continuous function. The maximum wage w for which equation (16) is relevant, is the wage $\hat{w}$ which just implies $n = N$ in the resource constraint (14). For $w < \hat{w}$ the resource constraint can only be fulfilled for $n < N$. Inserting $n = N$ into (14) and solving for $\hat{w}$ yields $\hat{w} = a\frac{1-2z}{1-z}$ where $z = a/(sN)$. We calculate $h(\hat{w}, g(\hat{w}))$ (note that the formula greatly simplifies since $n = N$) and get after rearranging

$$\begin{aligned} h(\hat{w}, g(\hat{w})) &= \frac{2g(\hat{w}) - 1 - w/a}{2g(\hat{w}) - w/a - \frac{1}{\beta}\left(\frac{2g(\hat{w})-w/a}{2-w/a} - (1-\beta)\right)} \\ &= \frac{\beta}{1-\beta}\frac{(2z-1)^2 - \beta(2z-1) - 2(z-1)\left(z-1+\sqrt{(z-1)^2+\beta(2z-1)}\right)}{\beta(2z-1) + 2(z-1)\left(z-1+\sqrt{(z-1)^2+\beta(2z-1)}\right)} \end{aligned}$$

Hence, as is easy to see, $h(\hat{w}, g(\hat{w})) < \frac{1-\beta\vartheta}{(1-\beta)\vartheta}$ iff $\theta < \frac{\beta(2z-1)+2(z-1)\left(z-1+\sqrt{(z-1)^2+\beta(2z-1)}\right)}{\beta(2z-1)^2}$. This condition is equivalent to equation (10) which is necessary and sufficient that the exclusion regime exists.

We now show that $h(w, g(w)) > \frac{1-\beta\vartheta}{(1-\beta)\vartheta}$ for $w \to 0$. First note that $h(\cdot)$ contains the term $\left[\frac{1-w/a}{2-w/a} - z\right] / \left[z - (1-\beta)\frac{\bar{p}-w/a}{2\bar{p}-w/a}\right]$ which goes to $+\infty$ when $\bar{p} = \frac{w}{a}\frac{1-z/(1-\beta)}{1-2z/(1-\beta)}$ (remember that $\frac{1-w/a}{2-w/a} > (1-\beta)\frac{\bar{p}-w/a}{2\bar{p}-w/a}$ because (13) holds). Hence, if $z < (1-\beta)/2$, $h(w, g(w))$ goes to infinity as w declines and the claim is trivially satisfied. It remains to consider the case $z \geq (1-\beta)/2$. In that case we have to calculate $\lim_{w\to 0} h(w, g(w))$. Applying de l'Hôpital's Rule and noting that $\lim_{w\to 0} g'(w) = -\frac{1}{a}\frac{3}{2}\frac{\beta}{1-\beta}$, we get

$$\lim_{w\to 0} h(w, g(w)) = \frac{1 + \beta - \beta^2 - 2\beta z}{(1-\beta)(2z + \beta - 1)}.$$

Remember that $z \leq 1/2$. Since the expression above is decreasing in z, we get a lower bound if it is evaluated at $z = 1/2$

$$\lim_{w\to 0} h(w, g(w)) \geq \frac{1-\beta^2}{(1-\beta)\beta} > \frac{1-\beta\vartheta}{(1-\beta)\vartheta}$$

where the latter inequality follows from $\beta > \frac{4\vartheta(1-z)^2}{(1+\vartheta)^2 + 4\vartheta z(\vartheta z - (1+\vartheta))} > \vartheta$.

The uniqueness of the equilibrium can be shown that the derivative of $h(w, g(w))$ with respect to w is negative whenever $h(w, g(w)) = \frac{1-\beta\vartheta}{(1-\beta)\vartheta}$ holds (the straightforward but tedious calculations are available upon request from the authors).

Figure 1:
Aggregate Demand and Monopolistic Pricing Decision

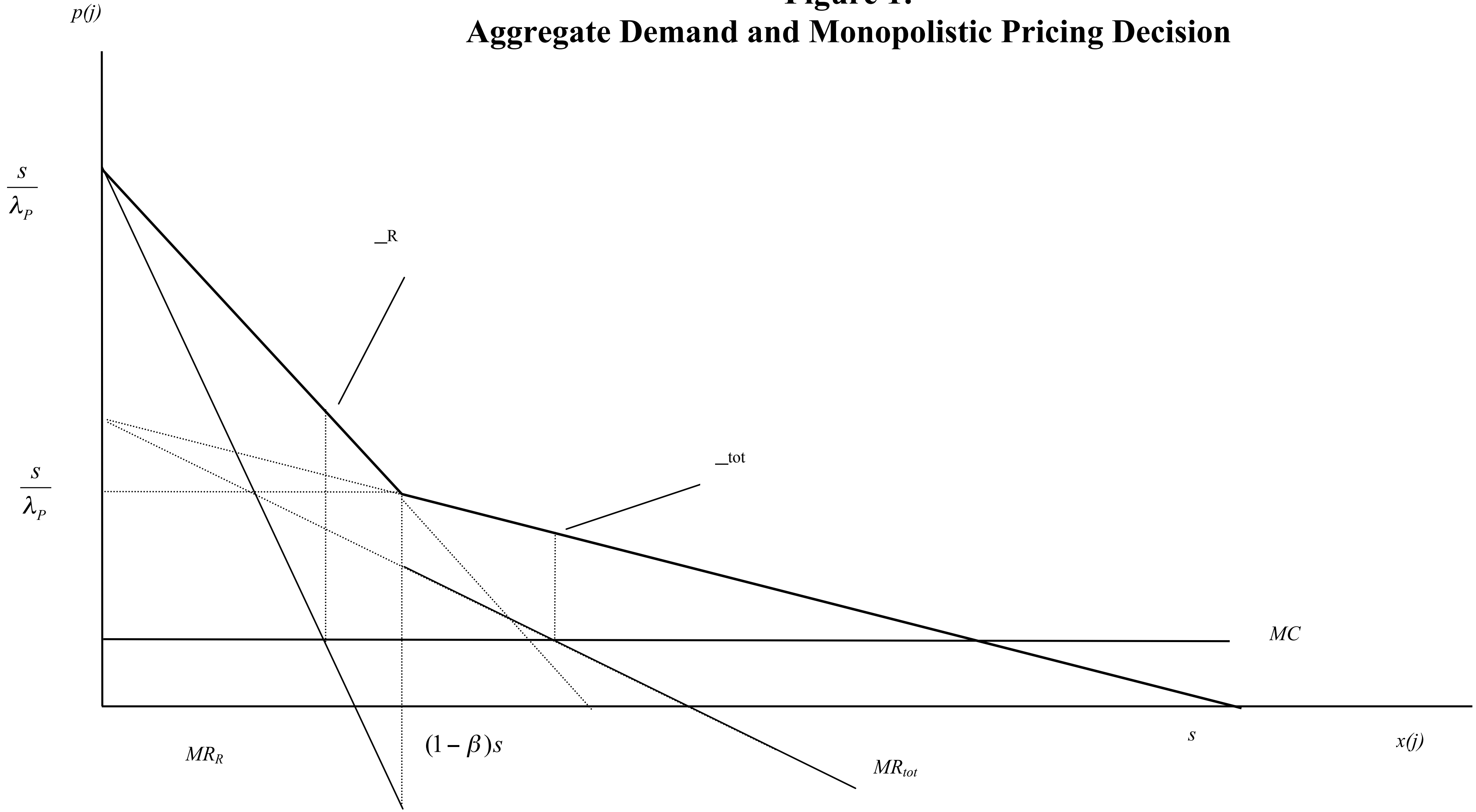


Figure 2:
Exclusion and Unemployment Depending on Inequality Parameters

Panel a

$z > \frac{1}{2}$

Panel b

$z < \frac{1}{2}$

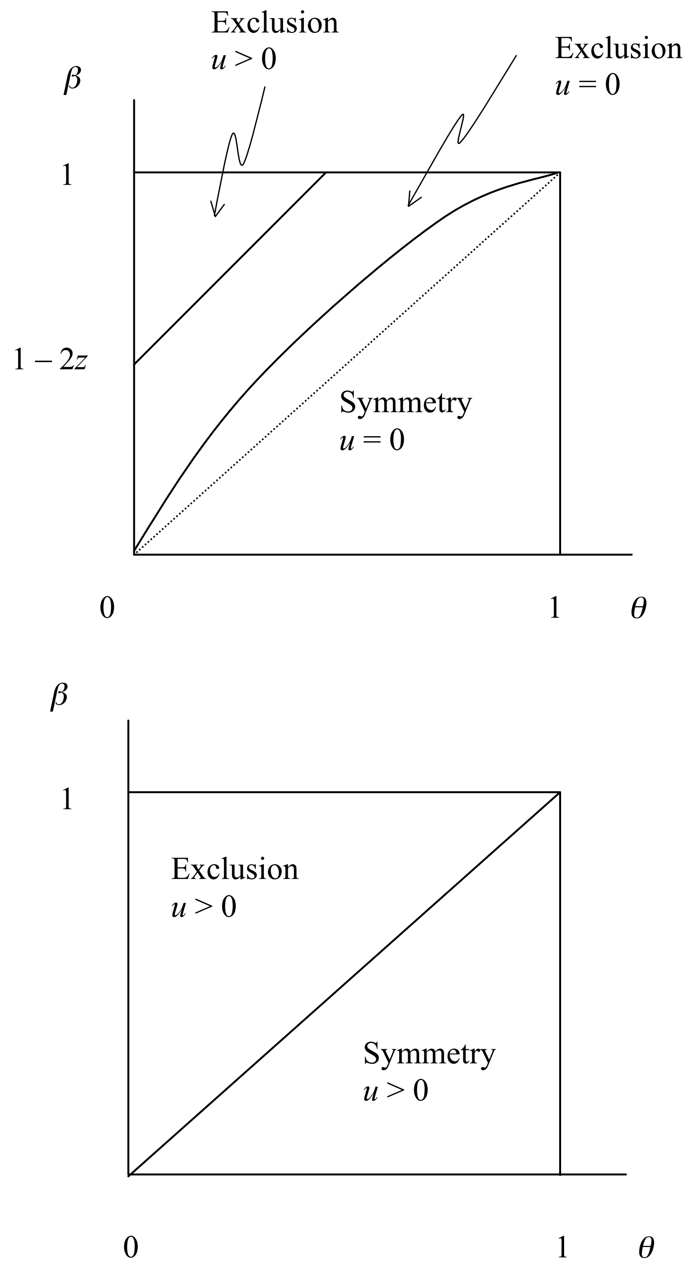
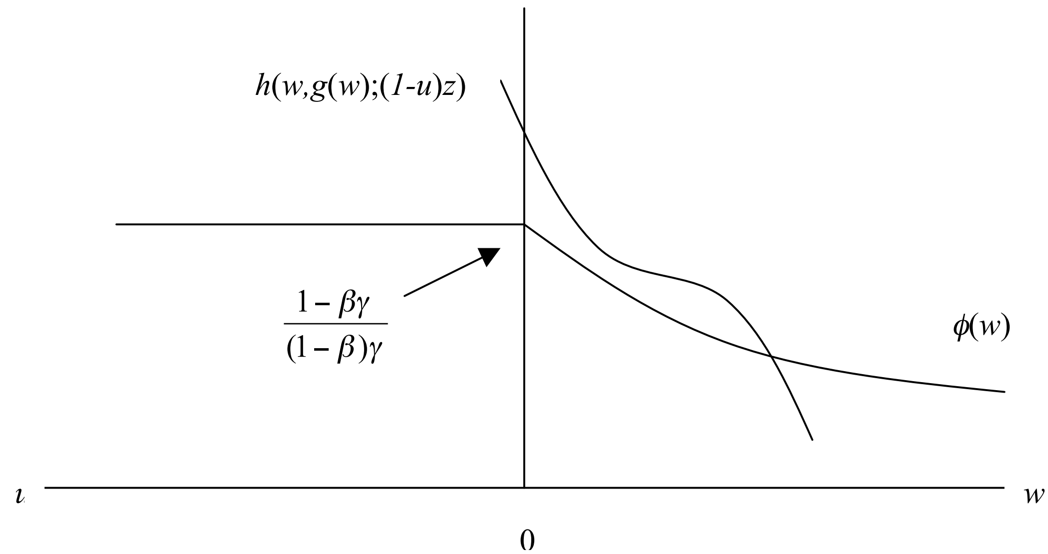


Figure 3: Variable income composition $\phi(w)$ and multiple equilibria

Panel a.



Panel b.

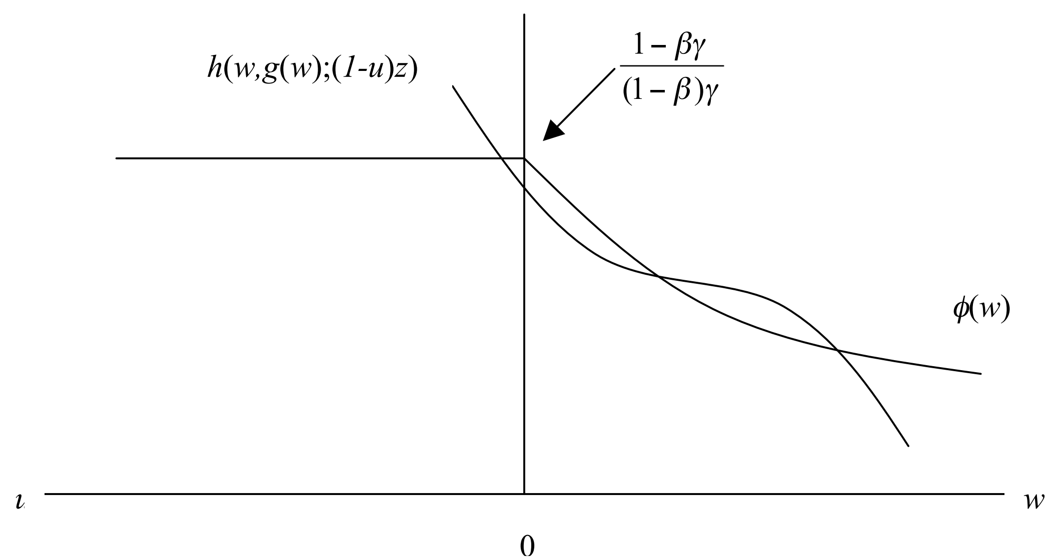


Figure 4: General equilibrium with low-skilled and high-skilled workers