Job Search and the Age-Inequality Profile

Petra Marotzke*

March 4, 2013

Abstract

Empirical studies find that the age-variance profile of wages is U-shaped. The objective of this paper is to explore the driving forces of the U-shape in a model with search frictions. I introduce endogenous search effort and a fixed retirement age into Cahuc, Postel-Vinay, and Robin's (2006) strategic wage bargaining model with counteroffers and heterogeneous firm-worker matches. Three factors shape the age-inequality profile of wages in the model economy: the time horizon before retirement, match heterogeneity, and the worker's bargaining power. Because the working life is finite, the optimal search effort decreases with age. Furthermore, the probability of meeting an outside firm with a higher match quality decreases in the quality of the current match. The bargaining power parameter influences the worker's reservation wage. The model can reproduce the U-shape of the age-inequality profile of wages if the bargaining power of workers is sufficiently high. Furthermore, the model captures the shapes of the empirically observed age profiles of average wages, the unemployment rate, the unemployment to employment transition rate, and the employment to employment transition rate.

JEL classification: J31; J41; J64

Keywords: Search Frictions, Wage Dispersion, Life Cycle, Wage Bargaining

---

*Department of Economics, University of Konstanz, 78457 Konstanz - Germany, Email: petra.marotzke@uni-konstanz.de. I would like to thank Carlos Carrillo-Tudela, Matthias Hertweck, Leo Kaas, Georgi Kocharkov, Bertrand Koebel, Fabian Krueger, Ruben Seiberlich, and Ludo Visschers for valuable comments and suggestions. I am also grateful to seminar participants at the University of Konstanz as well as participants at the ACDD 2012 conference (Strasbourg), and the Annual Congress 2012 of the European Economic Association (Malaga) for helpful comments.

1 Introduction

Much empirical evidence shows that the variance of log wages across workers decreases with age when workers are young, and then increases with age (Mincer, 1974; Heckman, Lochner, and Todd, 2003). The objective of this paper is to explore the driving forces of the U-shape of the age-inequality profile in a search model of the labor market. Understanding the sources of lifetime wage inequality is of importance for the design of welfare policies and insurance programs. Furthermore, the age structure of the population might be an important factor behind differences in income inequality between countries or changes in the wage structure across time.

An alternative explanatory approach to the U-shaped age-inequality profile is provided by Mincer's (1974) human capital investment model.[1] For the design of appropriate labor market policies, it makes a difference whether the high residual wage dispersion of young and old workers is attributed to search frictions or investment in human capital accumulation. Rubinstein and Weiss (2006) explore the implications of the human capital investment model and a search model of the labor market for life cycle wages. They find empirical support for both theories. While they argue that a search model alone cannot give rise to a U-shaped age-inequality profile, the present paper shows that search theory can indeed provide an explanation for the U-shape.

I introduce endogenous search effort and a finite working life into the bargaining model of Cahuc, Postel-Vinay, and Robin (2006). Firm-worker matches have different productivities, workers search on and off the job, and incumbent employers can counter outside wage offers. The optimal search behavior of employed workers depends on the worker's age, the current wage, and the quality of the firm-worker match. On average, this produces intensive search of young workers, moderate search of middle-aged workers, and only minor search of workers close to retirement. I show that the present model captures the shapes of the empirically observed age profiles of average wages, the unemployment rate, the unemployment to employment transition rate, and the employment to employment transition rate.

A large fraction of life cycle wage inequality in the model is driven by match het-

[1] Workers who invest in human capital on-the-job early in their career earn initially a low wage. They are compensated by a high future wage. In such a model, the standard deviation is the lowest for middle-aged workers when the profiles of investors and non-investors cross.

erogeneity. Indeed, the endogenous age-variance profile of match qualities is U-shaped. Young workers, who just obtained their first job offer, have a high standard deviation of match qualities. The employment to employment transition rate of young workers is high. This leads to a sharp decrease in the standard deviation. However, the probability of meeting an outside firm with a higher match quality decreases in the quality of the current match. The employment to employment transition rate of middle-aged workers is only moderate. Furthermore, workers are differently successful in finding good job offers. Hence, the standard deviation of match qualities for middle-aged workers increases. The decrease in the search effort of workers close to retirement is reflected in a strong increase in the standard deviation of match qualities.

In order to reproduce a U-shaped age-variance profile of wages, the bargaining power of workers must be high to reflect the age-variance profile of match qualities. If the bargaining power of workers is too low, the standard deviation of wages declines for workers close to retirement. As in Cahuc, Postel-Vinay, and Robin (2006), the option value of on-the-job search makes workers accept a low starting wage. However, in the present model with a finite working life, it can occur that workers negotiate wage rises from the current employer without any outside job offer, as the decreasing time horizon before retirement reduces the option value of on-the-job search. The option value effect is more important, the lower the worker's bargaining power. Hence, the option value of on-the-job search makes young workers accept a wage far below the productivity of the firm-worker match if the worker's bargaining power is low. The shorter the remaining time horizon before retirement, the lower is the option value of on-the-job search. Hence, the reservation wage increases for older workers. This has a negative effect on the standard deviation of wages for workers close to retirement.

The age-inequality profile of wages is U-shaped if the bargaining power of workers is sufficiently high. In this case, there is a modest increase in the reservation wage only for low quality matches prior to retirement. Apart from that, the reservation wage decreases for old workers. Workers with a short time horizon before retirement accept lower wages since the probability of obtaining a better job offer by waiting decreases.

This paper relates to Bagger, Fontaine, Postel-Vinay, and Robin (2011) and Yamaguchi (2010), who also explore the driving forces of wage dynamics over the life cycle in a bargaining model with counteroffers. However, while they focus on the importance of job search and human capital accumulation for individual wage growth, I focus on the importance of job search and a finite working life for shaping the age-inequality profile of wages.

A related model with a finite working life and on-the-job search is Menzio, Telyukova, and Visschers (2012). They develop a life cycle model with directed search and human capital accumulation. The model is able to reproduce the empirical age profile of transition rates and the age-wage profile.

Other authors have explored the effect of a finite working life on labor market outcomes within a search theoretic model in which workers can only search when unemployed (Hairault, Sopraseuth, and Langot, 2010; Hahn, 2009; De la Croix, Pierrard, and Sneessens, 2009; Chéron, Hairault, and Langot, 2008). These models can explain the hump-shaped age profile of employment. Without additional assumptions, they imply a decreasing age-wage profile. In order to obtain the empirically observed increasing and concave age-wage profile, Hairault, Sopraseuth, and Langot (2010) calibrate age-specific wage offer distributions. De la Croix, Pierrard, and Sneessens (2009) assume that workers' productivities increase with age and then decrease as workers approach retirement. Chéron, Hairault, and Langot (2008) introduce human capital accumulation into their model.

The paper is organized as follows: Section 2 describes the empirical age profile of wage inequality to be explained using the model framework set out in section 3. In section 4, I simulate the model economy and obtain age profiles of transition rates and age-inequality profiles. Section 5 discusses the mechanisms that shape the age-inequality profile of wages. Section 6 concludes.

2 The empirical age profile of wage inequality

This section discusses the empirical age profile of wage inequality. I use the 1996 panel of the US Census' Survey of Income and Program Participation (SIPP), which spans the time period from December 1995 to February 2000.[2] The SIPP contains monthly data on the worker's employment status, earnings, primary job, and information on whether the

[2] Data source: Center for Economic and Policy Research. 2012. SIPP Uniform Extracts, Version 2.1.7 . Washington, DC.

worker has changed the employer. I restrict the analysis to a subsample of men between the ages of 18 and 66, who have a high school degree, and do not have any income from self-employment. The data set comprises 11,798 individuals and 302,946 observations.

Residual wages are derived from a fixed-effects regression of log-wages on 13 occupational dummies, a dummy for disabled workers, 8 regional dummies, and 5 dummies for marital status. Time fixed effects are included. The estimated model is

$$\ln w_{it} = \alpha_i + \beta X_{it} + \varepsilon_{it},$$

where w_{it} is monthly earnings of worker i in period t, α_i is the unknown intercept for worker i, β is a vector of coefficients, X_{it} is a vector of regressors, and ε_{it} is the error term. A description of the regressors and estimation results are presented in Appendix A.

The age-inequality profile is defined by the standard deviation of the residual, $\hat{\varepsilon}_{it}$, within each age group. The residual is determined by

$$\hat{\varepsilon}_{it} = \ln w_{it} - \widehat{\ln w_{it}},$$

where $\widehat{\ln w_{it}}$ denotes the prediction of $\alpha_i + \beta X_{it}$. Figure 1 shows that the age-inequality profile is U-shaped. This result is robust to several alternative model specifications.[3] The main objective of the model developed in the next section is to provide an explanation of the U-shape.

3 A life cycle model with on-the-job search

The labor market is populated by a continuum of competitive firms and a unit mass of risk-neutral workers of different ages $k = 1, 2, ..., K$. Time is discrete and the economy is in steady-state. Firms produce a unique multipurpose good, maximize profits, and live forever. Each worker lives a finite life of K periods. In steady-state, all workers that leave the labor market at age $K+1$ are replaced by unemployed workers of age 1. Hence, the fraction of the population aged k is given by l for all $k < K$.

Firm-worker matches differ in their productivities denoted as a_i with $i = 1, ..., n$ and $a_{j-1} < a_j$, $j = 2, ..., n$. The probability that a potential match has productivity a_i is given

[3] A similar age-inequality profile is obtained if age, age squared, and/or interaction between occupation and age are included.

by p_i. The cumulative distribution of potential match qualities is denoted by P_i. When a firm and a worker meet, the quality of the potential match is revealed. For convenience, I describe a firm that offers a worker a match of quality a_i as a type i firm. Output per period in a firm-worker match does not depend on the worker's age and equals the marginal productivity of labor a_i. Unemployed workers receive an income flow of b_U. Workers derive utility from consumption and discount future utility at the factor $\beta \in (0,1)$.

Workers search on and off the job. Searching for a job is costly for the worker. The cost of spending an effort e on searching is given by a cost function $c(e)$, with $c(0) = 0$. The cost function is increasing and strictly convex. The offer arrival rate per search effort is $\lambda > 0$. The search effort is derived endogenously by the worker's optimizing behavior. The timing of events is as follows. In the beginning of a period, $g(k, a_i)$ workers aged k are employed at a firm a_i. Each of these firm-worker matches is hit by an exogenous separation shock with probability $\delta \in [0,1]$. Workers who become unemployed can immediately search for a new job that starts in the next period. The mass of unemployed workers of age k is then

$$u(k) = l - (1-\delta) \sum_{j=1}^{n} g(k, a_j). \tag{1}$$

All workers that enter the labor market are unemployed, hence $u(1) = l$.

3.1 Wage bargaining

The wage formation rules are based on the bargaining model of Cahuc, Postel-Vinay, and Robin (2006). If an employed worker obtains an outside wage offer, the incumbent employer can counter the outside offer. Workers and employers have complete information over each other's type and over the worker's wage and job offers. Wage contracts specify a wage that can only be renegotiated by mutual agreement. Wage cuts within an employment are not possible. Consider a worker of age k employed at a type i firm earning wage w. When the worker contacts a type h firm, the incumbent and the poaching employer compete for the worker. The maximum wage a firm is able to offer equals the match productivity. The worker chooses the firm that offers the highest lifetime utility. The outcome of the bargaining process depends on the productivity of both firms and on the current wage. Three cases can occur. If $h > i$, the worker switches to the poaching employer since the type h firm will offer the worker a wage that has a higher value than the highest wage the type i firm can offer. Note that the wage from the new employer can be smaller than w as the worker takes into account possible future wage rises. Such a wage cut is possible because of the option value of on-the-job search. An employment within a high productivity match gives the worker a better position for future wage negotiations. If $h < i$, the worker stays with the incumbent employer. The worker obtains a wage rise from the incumbent employer if and only if $i \geq h \geq q(k,w,a_i)$. If h is smaller than the threshold marginal productivity index $q(k,w,a_i)$, nothing changes for the worker. Table 1 gives an overview of the bargaining game. $\phi(k,a_i,a_h)$ denotes the wage that is the outcome of a bargaining game between a type i firm and a type h firm, with $h > i$, and a worker of age k.

Table 1: Outcome of the wage bargaining game between a worker earning wage w, the incumbent employer of type i, and a poaching employer of type h

	negotiation outcome
$h > i$	new employer h and a wage $\phi(k,a_i,a_h)$
$i \geq h \geq q(k,w,a_i)$	wage rise $\phi(k,a_h,a_i) - w$ from current employer
$h < q(k,w,a_i)$	no change

The mechanisms of wage bargaining discussed so far are the same as in Cahuc, Postel-Vinay, and Robin (2006). However, while they assume that workers have an infinite life, workers leave the labor market at a given age in the present model. A young worker's wage bargain outcome is different than that of a worker close to retirement. The option value of on-the-job search makes workers accept a low starting wage. The shorter the time horizon before retirement, the lower is the option value of on-the-job search. Hence, it can occur that workers negotiate wage rises from the current employer without any outside job offer when they have a credible threat to quit.

Let $\mathscr{W}(k,w,a_i)$ denote the value of a job to a worker of age k earning wage w in a match with productivity a_i. When the two competing firms have productivities i and h with $i < h$, type-h firm wins the bargain by offering a wage $\phi(k,a_i,a_h)$ that is determined by

$$\mathscr{W}(k,\phi(k,a_i,a_h),a_h) = \mathscr{W}(k,a_i,a_i) + \gamma[\mathscr{W}(k,a_h,a_h) - \mathscr{W}(k,a_i,a_i)], \quad (2)$$

where the parameter $\gamma \in [0,1]$ is the worker's bargaining power. The worker obtains a value $\mathscr{W}(k,\phi(k,a_i,a_h),a_h)$ that equals his outside option $\mathscr{W}(k,a_i,a_i)$ - the highest value the lower productivity firm can offer - plus a share γ of the match surplus.

Consider a worker of age k earning wage w in a type i firm. The productivity index of the poaching firm must be at least equal to $q(k,w,a_i)$ such that the worker obtains a higher lifetime utility in the bargaining game. Hence, the threshold productivity index $q(k,w,a_i)$ is the lowest index for which

$$\begin{aligned} \mathscr{W}(k,w,a_i) < \ & \mathscr{W}(k,a_{q(k,w,a_i)},a_{q(k,w,a_i)}) + \\ & \gamma\left[\mathscr{W}(k,a_i,a_i) - \mathscr{W}(k,a_{q(k,w,a_i)},a_{q(k,w,a_i)})\right] \end{aligned} \tag{3}$$

is fulfilled. It follows that $q(k,a_i,a_i) = i+1$.

The outside option of an unemployed worker aged k is the value of unemployment denoted by $\mathscr{U}(k)$. A match between an unemployed worker and a type i firm is formed if and only if $\mathscr{W}(k,a_i,a_i) \geq \mathscr{U}(k)$. Provided this condition is satisfied, the firm offers a wage $\phi_0(k,a_i)$ that solves

$$\mathscr{W}(k,\phi_0(k,a_i),a_i) = \mathscr{U}(k) + \gamma[\mathscr{W}(k,a_i,a_i) - \mathscr{U}(k)]\,. \tag{4}$$

A higher match quality offers the worker more opportunities to obtain wage rises because of possible outside job offers. This option value effect makes wages decrease in match quality. However, the higher the productivity of the firm that wins the bargain, the higher is the match surplus. The higher the worker's bargaining power, the more does the wage reflect the match surplus. The bargaining power effect makes wages increase in match quality. In Cahuc, Postel-Vinay, and Robin (2006), wages decrease in the productivity of the firm that wins the bargain if γ is sufficiently small such that the option value effect dominates. If γ is large enough, the bargaining power effect dominates and wages increase in productivity. There are additional implications in a model with a finite time horizon. The shorter the remaining time horizon before retirement, the lower is the option value of on-the-job search.

3.2 Value functions

Each period, a worker decides how much effort e to spend on job search. The problem of an unemployed worker of age $k < K-1$ is summarized by

$$\mathscr{U}(k) = \max_{e\geq 0}\Big\{ b_U - c(e) + \beta\Big[\mathscr{U}(k') + (1-\delta)e\lambda \sum_{j=r(k')}^{n} \left[\mathscr{W}(k', \phi_0(k', a_j), a_j) - \mathscr{U}(k')\right] p_j\Big]\Big\},$$

where $k' = k+1$ and $r(k')$ is the minimum productivity index of a match that a worker of age k' accepts. The reservation productivity $a_{r(k')}$ is the lowest productivity level for which

$$\mathscr{W}(k, a_{r(k')}, a_{r(k')}) \geq \mathscr{U}(k')$$

holds. Since unemployed and employed workers face the same search cost function and the same offer arrival rate per search effort, the lowest acceptable match productivity for a worker equals the flow income when unemployed, b_U. In the remainder of the paper, I set

$$a_1 = a_{r(k)} = b_U.$$

Using equation (4), the value of unemployment becomes

$$\mathscr{U}(k) = \max_{e\geq 0}\Big\{ b_U - c(e) + \beta\Big[\mathscr{U}(k') + (1-\delta)e\lambda\gamma \sum_{j=1}^{n} \left[\mathscr{W}(k', a_j, a_j) - \mathscr{U}(k')\right] p_j\Big]\Big\}. \tag{5}$$

The optimal search effort of an unemployed worker aged k, $e_U(k)$, is the solution to the first order condition (FOC) of the maximization problem

$$c'\left[e_U(k)\right] = \beta(1-\delta)\lambda\gamma \sum_{j=1}^{n} \left[\mathscr{W}(k', a_j, a_j) - \mathscr{U}(k')\right] p_j. \tag{6}$$

The value of a job to a worker of age $k < K-1$ earning wage w in a match with productivity a_i is derived as follows:

$$
\begin{aligned}
&\mathscr{W}(k,w,a_i) = \max_{e\geq 0}\Bigg\{ w - c(e) + \beta \Bigg[\delta \mathscr{U}(k') + (1-\delta)\Bigg[\\
&\Big(1 - e\lambda [1 - P_{q(k',w,a_i)-1}]\Big) \\
&\qquad\qquad \max\Big\{ \mathscr{W}(k',w,a_i), \mathscr{U}(k') + \gamma\left[\mathscr{W}(k',a_i,a_i) - \mathscr{U}(k')\right] \Big\} \\
&+ e\lambda \sum_{j=q(k',w,a_i)}^{i} \Big(\mathscr{W}(k',a_j,a_j) + \gamma\left[\mathscr{W}(k',a_i,a_i) - \mathscr{W}(k',a_j,a_j)\right] \Big) p_j \\
&+ e\lambda \sum_{j=i+1}^{n} \Big(\mathscr{W}(k',a_i,a_i) + \gamma\left[\mathscr{W}(k',a_j,a_j) - \mathscr{W}(k',a_i,a_i)\right] \Big) p_j \quad \Bigg]\Bigg]\Bigg\}. \qquad (7)
\end{aligned}
$$

The worker's value is the current wage minus search costs plus the discounted continuation value. The worker becomes unemployed and earns a value $\mathscr{U}(k')$ with probability δ. The employed worker does not meet an outside firm that has a productivity larger than $a_{q(k',w,a_i)-1}$ with probability $1 - e\lambda(1 - P_{q(k',w,a_i)-1})$. In this case the worker stays in his current match. As the option value of on-the-job search decreases with age, the worker renegotiates the wage if $\mathscr{W}(k',w,a_i)$ becomes smaller than $\mathscr{U}(k') + \gamma[\mathscr{W}(k',a_i,a_i) - \mathscr{U}(k')]$. If the worker meets an outside firm with lower productivity than a_i but above $a_{q(k',w,a_i)-1}$, she expects a wage rise from the incumbent employer and a bargain outcome with value $\sum_{j=q(k',w,a_i)}^{i} (\mathscr{W}(k',a_j,a_j) + \gamma\left[\mathscr{W}(k',a_i,a_i) - \mathscr{W}(k',a_j,a_j)\right]) p_j$. If the worker meets an outside firm with match productivity larger than a_i, he switches to the poaching firm and expects a value $\sum_{j=i+1}^{n} (\mathscr{W}(k',a_i,a_i) + \gamma[\mathscr{W}(k',a_j,a_j) - \mathscr{W}(k',a_i,a_i)]) p_j$. Let $e_W(k,w,a_i)$ denote the optimal search effort of a worker earning wage w at a type-i firm. The optimal search effort is the solution to the FOC

$$
\begin{aligned}
c'\left[e_W(k,w,a_i)\right] = \beta(1-\delta)\lambda \Bigg(& \sum_{j=q(k',w,a_i)}^{i} \Big[(1-\gamma)\mathscr{W}(k',a_j,a_j) + \gamma\mathscr{W}(k',a_i,a_i)\Big] p_j \\
& + \sum_{j=i+1}^{n} \Big[(1-\gamma)\mathscr{W}(k',a_i,a_i) + \gamma\mathscr{W}(k',a_j,a_j)\Big] p_j \\
& - (1-P_{q(k',w,a_i)-1}) \max\Big\{ \mathscr{W}(k',w,a_i), \mathscr{U}(k') + \gamma\left[\mathscr{W}(k',a_i,a_i) - \mathscr{U}(k')\right] \Big\} \Bigg).
\end{aligned} \tag{8}
$$

Let $\mathscr{R}(K)$ be the value of retirement.[4] A worker aged $K-1$ faces the following values:

$$
\mathscr{U}(K-1) = b_U + \beta\mathscr{R}(K) \tag{9}
$$

$$
\mathscr{W}(K-1,w,a_i) = w + \beta\mathscr{R}(K) \tag{10}
$$

3.3 Steady-state labor market flows

The labor market dynamics lead to the following distribution of workers across employment states. Let $g(k,w,a_i)$ be the fraction of the population aged k, earning wage w, and being employed at a type i firm. The fraction of the population aged k being employed at a type i firm is given by $g(k,a_i) = \int g(k,w,a_i)dw$. The fraction of the population aged k' being employed at a type i firm is made up of the pool of unemployed workers that form a match with a type i firm, the workers that are recruited out of lower productivity jobs, and the workers that have stayed in a type i match:

$$
\begin{aligned}
g(k',a_i) = {} & e_U(k)\lambda u(k) p_i \\
& + (1-\delta)\lambda p_i \sum_{j=1}^{i-1} \int e_W(k,w,a_j) g(k,w,a_j) dw \\
& + (1-\delta) \int \Big[1 - e_W(k,w,a_i)\lambda(1-P_i)\Big] g(k,w,a_i) dw.
\end{aligned} \tag{11}
$$

[4]The value of $\mathscr{R}(K)$ has an effect only on the scale of the value functions but not on equilibrium wages or search efforts.

3.4 Wage distribution

Let $G(w|k,a_i)$ be the cumulative distribution of wages conditional on age and productivity. The maximum wage a type i firm can offer is a_i. Hence

$$G(a_i|k,a_i) = 1.$$

All newborns are unemployed. Employed workers aged $k = 2$ earn a wage $\phi_0(2,a_i)$ since they were hired out of unemployment and have not yet searched on-the-job. The cumulative distribution of wages conditional on age and productivity for workers of age $k' \geq 3$ is determined by

$$\begin{aligned}
G(w|k',a_i) = I_{w\geq\phi_0(k',a_i)} \Big\{ & e_U(k)\lambda u(k) p_i \\
& + (1-\delta)\lambda p_i \sum_{j=1}^{q(k',w,a_i)-2} \int e_W(k,\tilde{w},a_j) g(k,\tilde{w},a_j) d\tilde{w} \\
& + (1-\delta) \int^{w} \left[1 - e_W(k,\tilde{w},a_i)\lambda(1-P_{q(k',w,a_i)-1})\right] g(k,\tilde{w},a_i) d\tilde{w} \Big\} \\
& / g(k',a_i),
\end{aligned} \tag{12}$$

where $I_{w\geq\phi_0(k',a_i)}$ is a dummy variable equal to 1 if $w \geq \phi_0(k',a_i)$ and 0 otherwise. The conditional cumulative distribution of wages $G(w|k',a_i)$ is the sum of unemployed workers with reservation wage $\phi_0(k',a_i) \leq w$ who meet a type i firm, workers that switch from a lower productivity firm to a type i firm for a wage $\leq w$, and workers that stay in their current match of type i who do not earn a wage larger than w. Workers are only willing to switch to a type i firm for a wage $\leq w$ if the match productivity of the current employment is smaller than $q(k',w,a_i)$.[5] The cumulative distribution of wages conditional on age is determined by

$$G(w|k) = \frac{\sum_{j=1}^{n} G(w|k,a_j) g(k,a_j)}{g(k)}. \tag{13}$$

3.5 Equilibrium

A *stationary equilibrium* consists of

[5]The index above the summation sign in equation (12) is set equal to $q(k',w,a_i)-2$ such that the equation fulfills $G(a_i|k',a_i) = 1$ and equation (11).

- a constant mass of employed workers $g(k, a_i)$
- a constant mass of unemployed workers $u(k)$
- the optimal search efforts $e_U(k)$ and $e_W(k, w, a_i)$
- wages $\phi(k, a_i, a_h)$ and $\phi_0(k, a_i)$
- the cumulative distribution functions of wages conditional on age and productivity $G(w|k, a_i)$,

for all combinations of age $k < K$ and match productivities a_i, given an exogenous productivity distribution, a constant mass of new workers of age $k = 1$, and the value of retirement $\mathscr{R}(K)$.

4 A quantitative analysis

In this section, I derive the equilibrium life cycle profiles of the unemployment to employment transition rate, the employment to employment transition rate and the wage distribution by simulating the model economy. In order to match the model economy to the data, I take into account labor market transitions that are exogenous to the model.

The definition of employment states and transition rates derived from the SIPP data follows Menzio, Telyukova, and Visschers (2012). A worker is assigned an employer based on his primary job where he worked the most hours. A worker is not in the labor force (N) if she reports having no job, not looking for work, and not being on layoff. A worker is unemployed (U) if she reports having no job and looking for work or being on layoff. A worker is employed (E) if she reports having a job and being either on layoff or not and absent without pay or not. A worker is in the labor force (L) if he is either employed or unemployed. The unemployment to employment transition (UE) rate is defined as the number of workers that experience a transition from unemployment to employment in a given month divided by the number of unemployed workers at the beginning of the month. The other transition rates are defined analogously.[6]

[6] I use a similar data selection as Menzio, Telyukova, and Visschers (2012). Their transition rates differ slightly from those that I obtain because they do not consider any individuals who flow in and out of retirement before the measured age.

Up to now, I have assumed that the job destruction rate is the same for each age group. However, the empirical rate at which employed workers become unemployed is decreasing with age (see Figure 2). In the following, the now age dependent job destruction rate is given by the empirical smoothed EU rate. The data further shows that there are workers who flow in and out of the labor market across all age groups (see Figures 3 to 5). These transitions influence the wage distribution and are therefore introduced into the model. Figure 6 shows how these labor market dynamics enter the model simulation. Simulation details are provided in Appendix B.

4.1 Calibration

One period in the simulation refers to one month. I therefore set the discount factor $\beta = 0.9967$. Workers retire after 49 years in the labor market, i.e. $K = 588$. The distribution of match qualities is Weibull with scale parameter ϕ and shape parameter τ. The number of grid points is $n = 40$, and $a_n = 4.5$. The cost of spending an effort e on searching is given by the quadratic cost function[7]

$$c(e) = ce^2.$$

I set $c = 0.5$.[8] The parameters γ, λ, ϕ, τ, and $a_1 (= b_U)$ are chosen such that they minimize the distance between simulated moments and corresponding moments obtained from the SIPP 1996 panel. I use 100 estimation targets: the standard deviation of residual wages within each age group (49 targets), the EE rate within each age group (49 targets), the average UE rate (1 target), and the ratio of the 5^{th} percentile to the 95^{th} percentile of the wage distribution (1 target). I do not use the life cycle profile of the UE rate as a target but only the average UE rate as the UE rate does not correspond exactly to the UE rate of the model economy. That is because the data contains individuals who directly flow from non-employment to employment, which cannot happen in the model environment. The estimate of the vector of structural parameters $\theta = (\gamma, \lambda, \phi, \tau, a_1)$ minimizes

$$(G^d - G^s(\theta))' \times I \times (G^d - G^s(\theta)), \tag{14}$$

[7] Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) estimate a model with on-the-job search in which the search effort is endogenous and the offer arrival rate per search effort is the same for employed and unemployed workers. Their results support a quadratic cost of search function.

[8] One can either set an arbitrary value to c and then calibrate the parameter λ or the other way around.

where G^d is a 100×1 vector containing the target moments from the data, G^s is a 100×1 vector of moments produced from model simulation, and I is a 100×100 identity matrix. Table 2 contains the calibration targets and table 3 the estimated parameters.

Table 2: Calibration

Calibration target	Data	Model
St.d. of residual wages within each age group	Figure 9	
EE rate within each age group	Figure 8	
Average UE rate	0.1998	0.2315
5^{th} percentile / 95^{th} percentile	0.0958	0.0887

Table 3: Point Estimates

Description	Parameter	Estimate
Workers' bargaining power	γ	0.6635
Offer arrival rate per search effort	λ	0.1495
Shape parameter	ϕ	1.8775
Scale parameter	τ	1.0955
Lowest productivity	a_1	0.0507

4.2 Life cycle profiles

Figure 7 displays the life cycle profile of the model UE rate. The UE rate remains relatively constant until a few years before retirement, then declines dramatically as workers reduce their search effort substantially when they approach the retirement age. Searching for a job is costly and the expected value of a job offer is small for workers close to retirement. The UE rate slightly increases for young and middle-aged workers because the job destruction rate decreases with age and this has a positive effect on the value of a job. The UE rate seems to be too high compared to the empirical rate. Because there exist NE transitions in the data, the pool of job searchers without a job is different than the number of workers that are counted as unemployed workers in the dataset.

The life cycle profile of the EE rate decreases with age (see Figure 8). The decreasing EE age profile can be explained by the longer search history of older workers who are on average employed in higher quality matches. The probability of obtaining a higher quality outside job offer decreases in the quality of the current match. The EE rate approaches zero for workers close to retirement because these workers reduce the search effort substantially. The simulated EE rate matches the empirical one well. Compared with the data, the EE rate obtained from the simulation declines sharply for old workers as all workers retire at the same age in the model economy.

Let us turn to the standard deviation of wages illustrated in Figure 9. The age-inequality profile of wages is U-shaped. The age-inequality profile falls sharply for young workers because the EE rate is high for this age group and workers are gradually matched to better jobs. However, better job offers become less frequent for workers in a high quality match. The EE rate is only moderate for middle-aged and old workers. Hence, workers in low productivity jobs move more slowly to higher productivity jobs. Furthermore, workers are differently successful in finding good job offers. The standard deviation of match qualities increases. The sharp decrease in the EE rate for old workers is reflected in the sharp increase in the standard deviation of this age group.

Figure 10 shows that the age profile of the unemployment rate is U-shaped in the model. It is decreasing for young workers, since workers are initially unemployed and gradually matched to their first jobs. It is increasing for older workers because workers close to retirement reduce their search effort substantially and therefore have a higher probability of remaining unemployed.

5 Discussion

The age-inequality profile of wages is U-shaped if the bargaining power of workers is sufficiently high. If the bargaining power of workers is smaller than $\gamma = 0.6$, the standard deviation of wages sharply decreases for old workers (see Figures 12 and 11). The bargaining power parameter has an effect on the age-inequality profile of wages through the worker's reservation wage. When γ is smaller or equal to 0.6, the reservation wage increases with age when the worker approaches retirement. Hence, the lowest bound of the wage distribution is increased. This has a negative effect on the standard deviation of wages. The reservation wage reflects the role of the option value of on the job search when the worker's life is finite. The importance of the option value effect is lower the higher the worker's bargaining power. Hence, the reservation wage increases in the worker's bargaining power. Because of the option value of on-the-job search, workers accept a wage far below the productivity of the firm-worker match. The shorter the time horizon before retirement, the lower is the option value of on-the-job search. Hence, if the bargaining power parameter γ is sufficiently small, the reservation wage increases for old workers. When the bargaining power parameter is larger than 0.6 and the match quality is relatively high, the decreasing time horizon has the opposite effect on wages. Old workers accept lower wages since the probability of obtaining a better job offer by waiting decreases.

Figures 13 and 14 compare the age-wage profiles of the model economy with the empirical one. The age-wage profile in the US economy is concave.[9] Average wages increase with age for young and middle-aged workers. They decrease with age a few years before retirement. The worker's bargaining power must be sufficiently high such that the present model reproduces a concave age-wage profile. Because workers are gradually matched to better jobs, the average match quality and the average wage in the model economy increase with age. It increases at a decreasing rate because job offers from higher quality matches become less probable the higher the productivity of a match. All workers recruited out of unemployment who have not obtained any outside offer, have the same distribution of match productivities with a low average match quality independent of age. Because the search effort decreases sharply before retirement, a large fraction of the workers in low quality matches does not move to higher quality matches. Hence, the average match quality decreases some years prior to retirement. When the worker's bargaining power γ is high, the age-wage profile is similar to the age-match productivity profile and depicts the empirically observed concave age-wage profile. When γ is low, the above explained increase of the reservation wage has a positive effect on the average wage prior to retirement.

When the bargaining power parameter is chosen to match the empirically supported U-shape of the age-inequality profile of wages and the concave age-wage profile, it must be rather high (roughly 0.7) in this model environment. This is in contrast to Cahuc, Postel-Vinay, and Robin (2006) who find for French data, that the bargaining power of workers is rather low. In their model in which workers have an infinite working life, γ lies between 0 and 0.35. An exception is the high value of $\gamma = 0.98$ for high skilled workers

[9] A concave age-wage profile can be found in several empirical studies including Kambourov and Manovskii (2009) and Mincer (1974).

in the construction sector. Bagger, Fontaine, Postel-Vinay, and Robin (2011) explore the importance of human capital accumulation and labor market competition for life cycle wage dynamics in a bargaining framework similar to Cahuc, Postel-Vinay, and Robin (2006). They find in their analysis of Danish data that the bargaining power γ lies between 0.2475 and 0.4141 and declines with education.

6 Conclusions

In this paper, I consider a life cycle model of labor market search with strategic wage bargaining, counteroffers, match heterogeneity and endogenous search effort. I show that the model can reproduce the U-shape of the age-inequality profile of wages if the bargaining power of workers is sufficiently high. Furthermore, the present model captures the shapes of the empirically observed age profiles of average wages, the unemployment rate, the unemployment to employment transition rate, and the employment to employment transition rate. The shape of the age-inequality profile of wages is mainly driven by the age profile of reservation wages and by employment to employment transitions. The optimal search effort of employed workers depends on the worker's time horizon before retirement, the current wage, and the quality of the firm-worker match. Furthermore, the probability of meeting an outside firm with a higher match quality decreases in the quality of the current match. This leads to frequent employment to employment transitions of young workers, a moderate employment to employment transition rate of middle aged workers, and a sharp decrease in the employment to employment transition rate of workers close to retirement. The bargaining power parameter plays an important role in the model because the option value of on-the-job search decreases when the time horizon before retirement shortens. A low bargaining power makes young workers accept a wage far below the productivity of the firm-worker match. Since the option value of on-the-job search is low for workers close to retirement, the reservation wage increases for old workers. This leads to a decline in the standard deviation of wages for old workers when the worker's bargaining power parameter is too low.

References

BAGGER, J., F. FONTAINE, F. POSTEL-VINAY, AND J. M. ROBIN (2011): "Tenure, Experience, Human Capital and Wages: A Tractable Equilibrium Search Model of Wage Dynamic," *Mimeo*.

CAHUC, P., F. POSTEL-VINAY, AND J. ROBIN (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74(2), 323–364.

CHRISTENSEN, B., R. LENTZ, D. MORTENSEN, G. NEUMANN, AND A. WERWATZ (2005): "On-the-Job Search and the Wage Distribution," *Journal of Labor Economics*, 23(1), 31–58.

CHÉRON, A., J. HAIRAULT, AND F. LANGOT (2008): "Life-Cycle Equilibrium Unemployment," *IZA Discussion Paper No. 3396*.

DE LA CROIX, D., O. PIERRARD, AND H. SNEESSENS (2009): "Demographics and labour market outcomes in an olg model with labour market frictions," *Mimeo*.

HAHN, V. (2009): "Search, unemployment, and age," *Journal of Economic Dynamics and Control*, 33(6), 1361–1378.

HAIRAULT, J., T. SOPRASEUTH, AND F. LANGOT (2010): "Distance to Retirement and Older Workers‘ Employment: The Case for Delaying the Retirement Age," *Journal of the European Economic Association*, 8(5), 1034–1076.

HECKMAN, J. J., L. J. LOCHNER, AND P. E. TODD (2003): "Fifty Years of Mincer Earnings Regressions," *National Bureau of Economic Research Working Paper Series*, No. 9732.

KAMBOUROV, G., AND I. MANOVSKII (2009): "Accounting for the Changing Life-Cycle Profile of Earnings," *manuscript, University of Pennsylvania, Philadelphia*.

MENZIO, G., I. A. TELYUKOVA, AND L. VISSCHERS (2012): "Directed Search over the Life Cycle," *Mimeo*.

MINCER, J. A. (1974): *Schooling, Experience, and Earnings*. Columbia University Press.

Rubinstein, Y., and Y. Weiss (2006): "Post Schooling Wage Growth: Investment, Search and Learning," in *Handbook of the Economics of Education, Volume 1, Chapter 1*, p. 1–67. Elsevier.

Yamaguchi, S. (2010): "Job Search, Bargaining, and Wage Dynamics," *Journal of Labor Economics*, 28(3), 595–631.

APPENDIX

A Estimation

Table 4: Regressors

Variable	Description
Major occupations (occ14)	1 Executive, Administrative, and Managerial
	2 Professional Speciality
	3 Technicians and Related Support
	4 Sales
	5 Administrative Support , Including Clerical
	6 Private Household Services
	7 Protective Services
	8 Services, except Household and Protective
	9 Farming, Forestry, and Fishing
	10 Precision Production, Craft, and Repair
	11 Machine Operators, Assemblers, and Inspectors
	12 Transportation and Material Moving
	13 Handlers, Equipment Cleaners, Helpers, and Laborers
	14 Armed Forces
Disability that limits work (disabled)	0 not disabled
	1 disabled
Census Region (region)	1 New England
	2 Middle Atlantic
	3 E. North Central
	4 W. North Central
	5 South Atlantic
	6 E. South Central
	7 W. South Central
	8 Mountain
	9 Pacific
Marital Status (ms)	1 Married, spouse present
	2 Married, Spouse absent
	3 Widowed
	4 Divorced
	5 Separated
	6 Never Married

Table 5: Estimation results

Variable	Coefficient	(Robust Std. Err.)
1b.occ14	0.000	(0.000)
2.occ14	-0.044	(0.050)
3.occ14	-0.079†	(0.043)
4.occ14	-0.124**	(0.032)
5.occ14	-0.112**	(0.031)
6.occ14	-0.843**	(0.222)
7.occ14	-0.150**	(0.052)
8.occ14	-0.294**	(0.033)
9.occ14	-0.308**	(0.047)
10.occ14	-0.060*	(0.026)
11.occ14	-0.080**	(0.028)
12.occ14	-0.088**	(0.034)
13.occ14	-0.151**	(0.029)
14.occ14	-0.072	(0.108)
0b.disabled	0.000	(0.000)
1.disabled	-0.111**	(0.020)
1b.region	0.000	(0.000)
2.region	0.083	(0.137)
3.region	0.169	(0.166)
4.region	0.034	(0.178)
5.region	0.056	(0.143)
6.region	0.352*	(0.160)
7.region	0.156	(0.146)
8.region	0.160	(0.164)
9.region	0.225	(0.164)
1b.ms	0.000	(0.000)
2.ms	0.006	(0.046)
3.ms	-0.030	(0.064)
4.ms	-0.009	(0.020)
5.ms	-0.021	(0.027)
6.ms	-0.120**	(0.024)
Intercept	7.509**	(0.137)

A $\dagger / * / **$ next to the coefficient indicates significance at the 10/5/1% level.

Table 5 (continued): Estimation results

Observations	235,489
Groups	9,657
R^2 within	0.031
R^2 between	0.155
R^2 overall	0.103
$F_{(76,9656)}$	42.935

B Simulation details

Let δ_k denote the age dependent job destruction rate. Unemployed workers in the model experience a transition out of the labor market at rate η_k given by the empirical smoothed UN rate. Employed workers leave the labor market at rate ζ_k given by the empirical smoothed EN rate. Workers enter the labor market at rate μ_k becoming unemployed at first. The empirical counterpart is the smoothed NL rate. Let $n(k)$ be the mass of workers aged k who are not in the labor force. The empirical labor force participation rate of 18 year old workers ($k=1$) is given by α.

The wage distribution is discretized for the model simulation. $u(k)$, $n(k)$, $g(k,w_s,a_i)$, $g(k,a_i)$, and $G(w_s|k,a_i)$ with $s=1,...,S$ and $w_{s-1}<w_s$ are determined as follows:[10]

- l is given
- $u(1)=\alpha l$
- $n(1)=(1-\alpha)l$
- $g(2,a_i)=e_U(1)\lambda u(1)p_i$
- $$G(w_s|2,a_i)=\begin{cases} 1 & \text{if} \quad w\geq \phi_0(2,a_i) \\ 0 & \text{if} \quad w<\phi_0(2,a_i) \end{cases}$$
- $$g(2,w_s,a_i)=\begin{cases} g(2,a_i)G(w_1|2,a_i) & \text{if} \quad s=1 \\ g(2,a_i)\left[G(w_s|2,a_i)-G(w_{s-1}|2,a_i)\right] & \text{if} \quad s>1 \end{cases}$$

[10] I obtain the mass of unemployed and employed workers, and the wage distribution of workers aged 18 ($k=1$) by running a simulation covering 13 months ahead of the main simulation.

- $u(2) = (1 - \lambda e_U(1) - \eta_1)u(1) + \mu_1 n(1)$
- $n(2) = (1 - \mu_1)n(1) + \eta_1 u(1)$

For $k \geq 2$ the following steps are repeated:

1.

$$\begin{aligned} g(k+1, a_i) = {} & e_U(k)\lambda u(k) p_i \\ & + (1 - \delta_k - \zeta_k)\lambda p_i \sum_{j=1}^{i-1} \sum_{s=1}^{S} e_W(k, w_s, a_j) g(k, w_s, a_j) \\ & + (1 - \delta_k - \zeta_k) \sum_{s=1}^{S} [1 - e_W(k, w_s, a_i)\lambda(1 - P_i)]\, g(k, w_s, a_i) \end{aligned}$$

2.

$$\begin{aligned} & G(w_s|k+1, a_i) = I_{w_s \geq \phi_0(k+1, a_i)} \Big\{ e_U(k)\lambda u(k) p_i \\ & + (1 - \delta_k - \zeta_k)\lambda p_i \sum_{j=1}^{q(k+1, w_s, a_i)-2} \sum_{r=1}^{S} e_W(k, w_r, a_j) g(k, w_r, a_j) \\ & + (1 - \delta_k - \zeta_k) \sum_{r=1}^{s} \big[1 - e_W(k, w_r, a_i)\lambda(1 - P_{q(k+1, w_s, a_i)-1}) \big] g(k, w_r, a_i) \Big\} \\ & /g(k+1, a_i). \end{aligned}$$

3.

$$g(k+1, w_s, a_i) = \begin{cases} g(k+1, a_i) G(w_1|k+1, a_i) & \text{if} \quad s = 1 \\ g(k+1, a_i) \left[G(w_s|k+1, a_i) - G(w_{s-1}|k+1, a_i) \right] & \text{if} \quad s > 1 \end{cases}$$

4. $u(k+1) = (1 - \lambda e_U(k) - \eta_k)u(k) + \mu_k n(k) + \delta_k \sum_{j=1}^{n} g(k, a_j)$
5. $n(k+1) = (1 - \mu_k)n(k) + \eta_k u(k) + \zeta_k \sum_{j=1}^{n} g(k, a_j)$

It can further be shown that

$$g(k+1) = \sum_{j=1}^{n} g(k+1, a_j) = \lambda e_U(k)u(k) + (1 - \delta_k - \zeta_k)g(k)$$

and

$$\begin{aligned}
&u(k+1) + g(k+1) \\
&= (1 - \lambda e_U(k) - \eta_k)u(k) + \mu_k n(k) + \delta_k g(k) + \lambda e_U(k)u(k) + (1 - \delta_k - \zeta_k)g(k) \\
&= (1 - \eta_k)u(k) + (1 - \zeta_k)g(k) + \mu_k n(k).
\end{aligned}$$

C Figures

Figure 1: Age-inequality profile of residual log wages

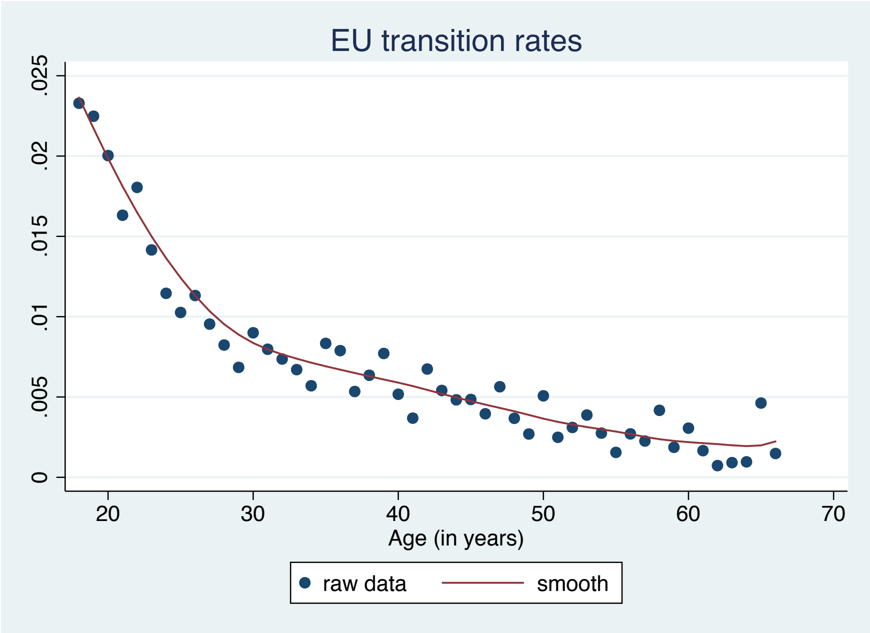

Figure 2: Life cycle profile of the job destruction rate

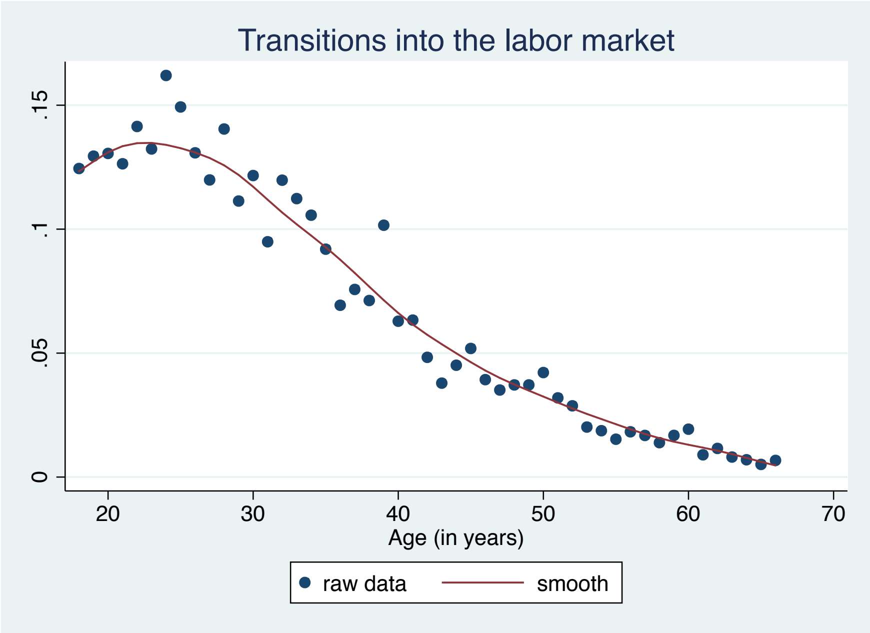

Figure 3: NU rate

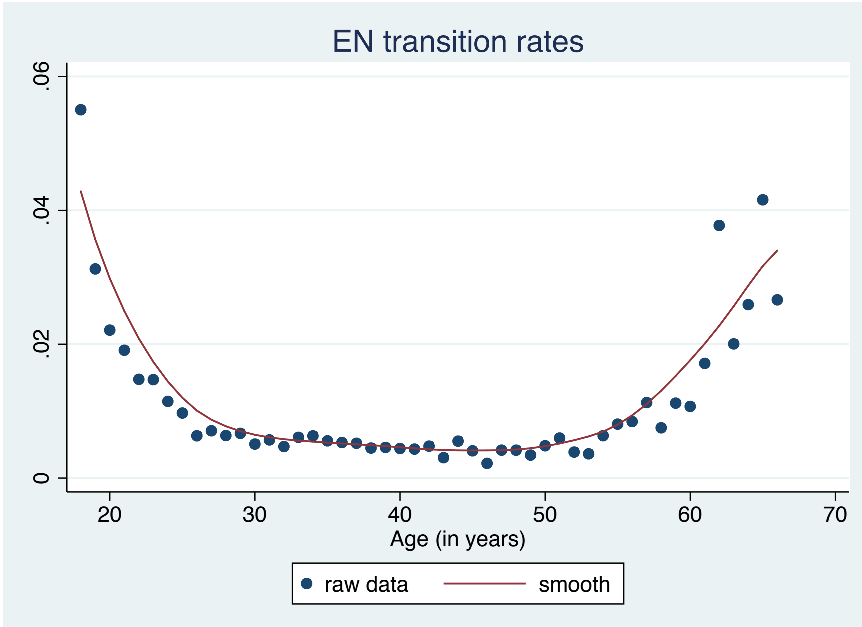

Figure 4: Transitions out of the labor market

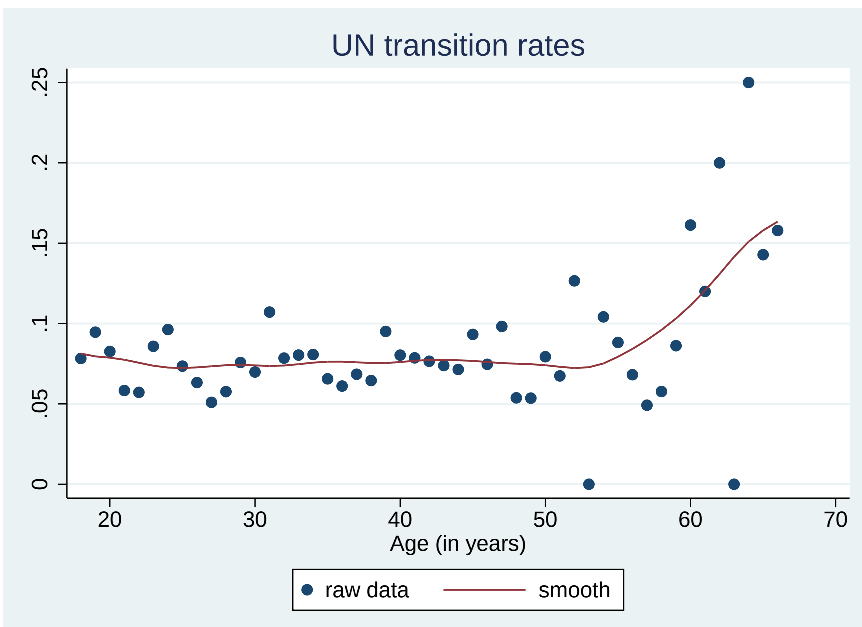

Figure 5: Transitions out of the labor market

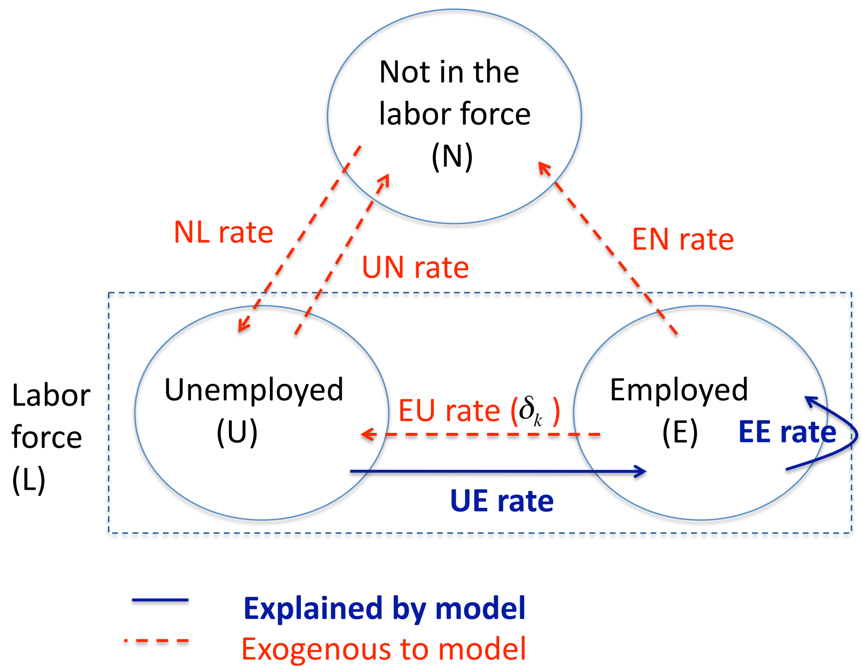

Figure 6: Labor market transitions

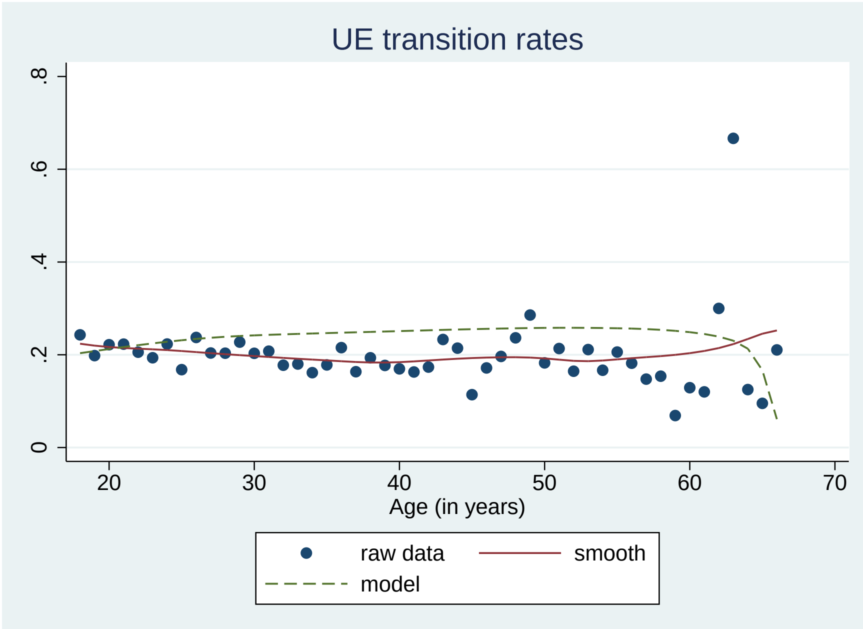

Figure 7: Average unemployment to employment transition rates

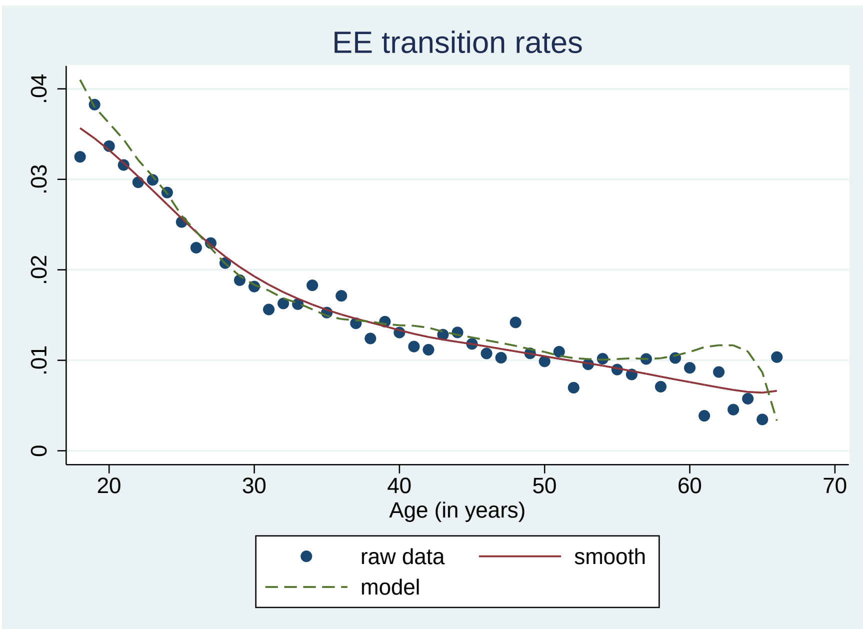

Figure 8: Average employment to employment transition rates

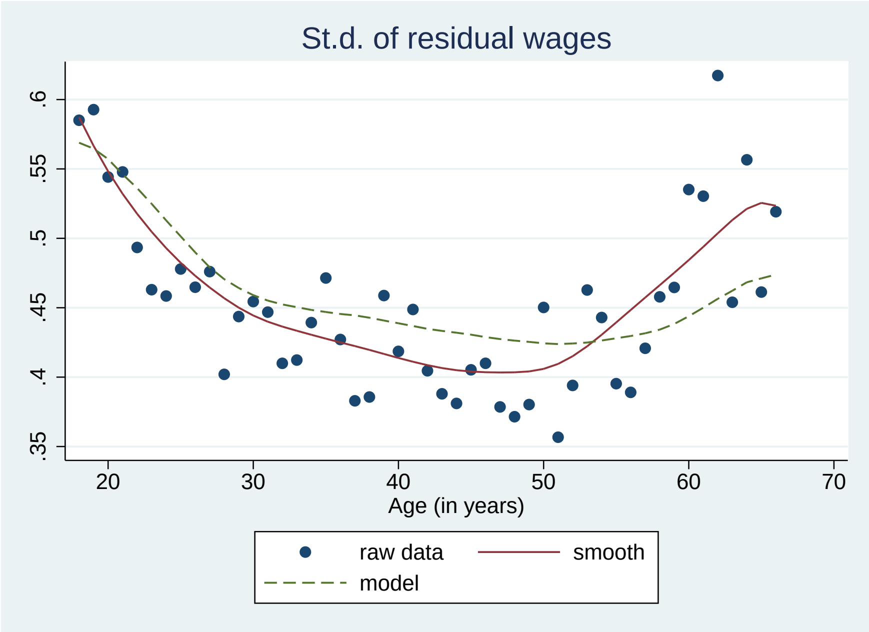

Figure 9: Wage dispersion

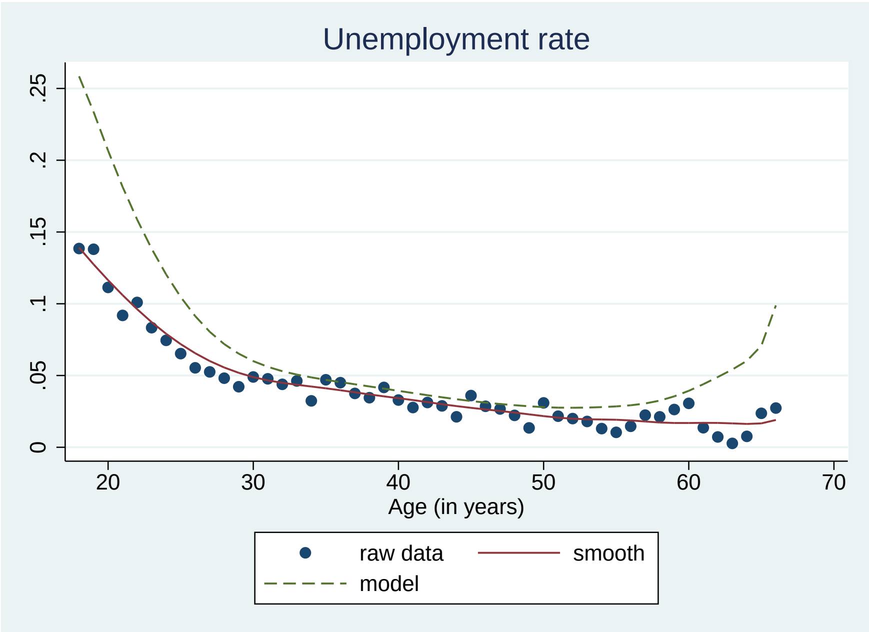

Figure 10: Unemployment rates

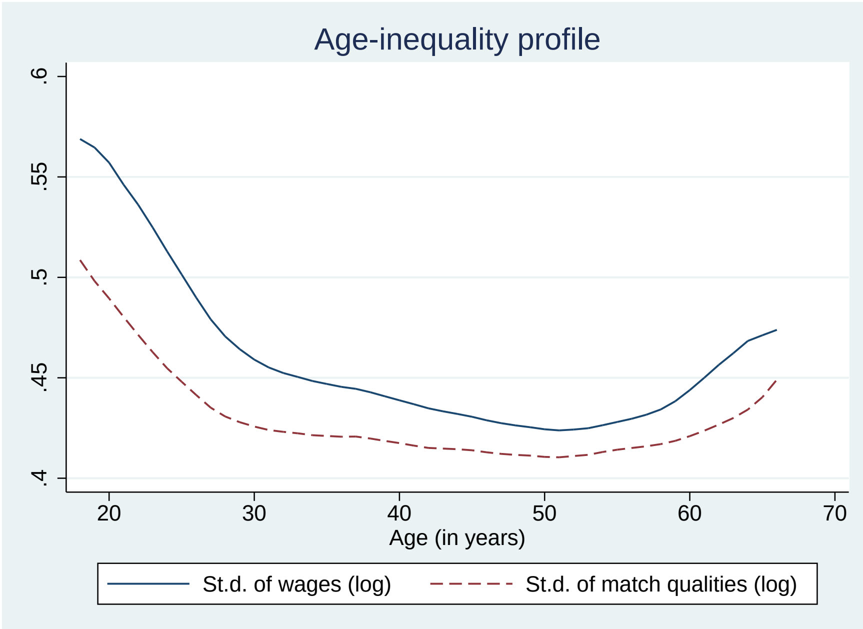

Figure 11: Dispersion of wages and match qualities

Figure 12: Calibrated model if $\gamma = 0.5$

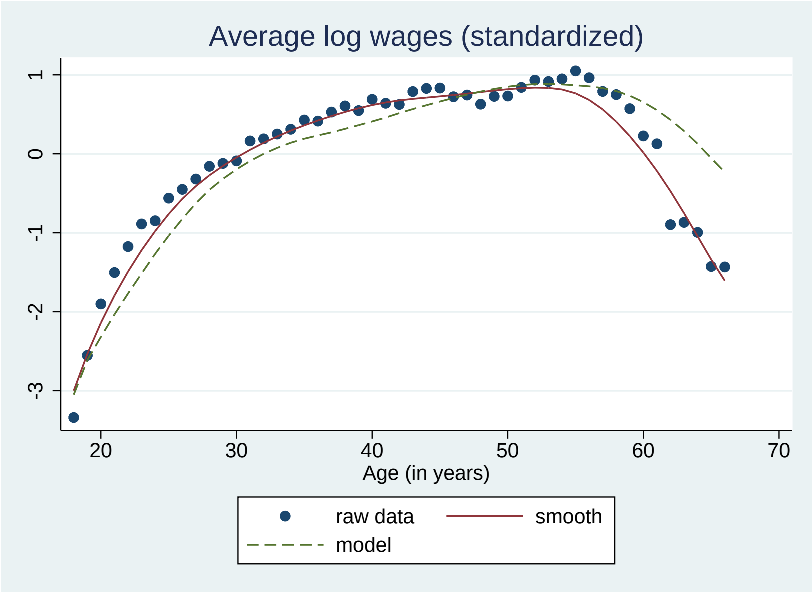

Figure 13: The age profiles of average log wages are standardized for comparability between model and data. The standardized age-specific average wage is derived by dividing the difference between the age-specific average wage and its mean by its standard deviation.

Figure 14: Calibrated model if $\gamma = 0.5$