This paper studies the effect of an aging labor force on productivity growth at the firm level and for the aggregate economy. In a dynamic model, firms make employment and technology decisions subject to an aging workforce. Firms with a higher share of elderly workers update their technology less often and prefer older, cheaper technologies than firms with a younger workforce. This is due to the shorter expected worklife of elderly workers which lets firms shy away from the investment in these workers. The model is calibrated to the German economy and the expected demographic change of the labor force between 2003–2050 is simulated. The results indicate that labor-force aging reduces the realized annual productivity growth rate by 0.25 percentage points between 2010–2025.
1 Introduction

Demographic change is going to impose major challenges to the industrialized countries in the 21st century. One of its consequences are massive changes in the age composition of the labor force. In many OECD countries, the share of workers aged 55 and above in the labor force will increase by 50–100% between 2000–2030. This paper focuses on the impact of labor-force aging on productivity growth. Several empirical studies have investigated this link. Tang and MacLeod (2006) find that a one-percent increase in the share of workers aged 55 and above reduces productivity growth in Canadian provinces by 0.07%. Two studies by Feyrer (2007) and Werding (2008) analyse the relationship between the age composition of the labor force and TFP for a panel of 27 OECD countries and other countries for the period 1960-2000. They find a positive effect on productivity for workers aged 40–49 but negative effects for workers aged 50 and above. A study by Grönqvist (2009) using industry-level for Finland between 1971–2005 finds that an increase of the share of workers aged 55 and above by 1% lowers annual labor productivity growth by about 0.22 percentage points.

The aim of this paper is to provide a model that allows to explain these effects of the age composition of the labor force on productivity and to quantify the impact of labor-force aging on productivity growth. To do this, I develop a dynamic general equilibrium model in which firms are able to adjust their workforce and to adapt new technologies. The economy is populated with overlapping generations of workers that undergo a stochastic aging process. Due to this aging process, the workforce composition in each firm changes constantly over time. It is shown that firms that employ a higher share of elderly workers update their technology less frequently than firms with younger employees because they fear that investment in training workers for a new technology can not be recuperated as the expected remaining worklife is too short. Also firms with a high share of older workers prefer to adopt non-state-of-the-art technologies at a lower adoption cost to reduce the investment in their elderly workers.

The model is calibrated to match current German data to simulate the projected changes in the age composition of the labor force for the period 2003–2050. An important factor in the analysis of labor-force aging is the fact that demographic change is accompanied
by an increase in the average retirement age in Germany. The simulation allows to disentangle the two effects. It is shown that demographic change for itself lowers annual productivity growth between 2010–2025 by 0.24 percentage points on average. In monetary units, this implies a loss of GDP of approximately 156bn in constant 2000 US$ for this period.

Taking the change in the average retirement age into account does not change the results significantly as the resulting positive effect at the micro-level, shorter updating intervals, and the negative effect at the macro-level, more firms with older workers, outbalance each other. As a policy experiment, the average retirement age in the simulation is increased by three additional years from 2015 onward. It turns out that the cost of demographic change are somewhat lowered, however, the impact is very small. A comparison of the simulation results to other studies indicate that the model’s results are in a plausible range. Since the model focuses only on a single mechanism by which labor force aging effects productivity growth, it is quite possible that the results understate the negative impact of labor-force aging on productivity growth.

There exist only few papers that are directly related to my work. General assessments on the relationship between worker age and productivity argue that older workers are not well prepared for new technologies as economic skill obsolescence reduces their productivity over time (de Grip and van Loo, 2002; Rosen, 1975) and that they are less able to adapt to new technologies (Skirbekk, 2004; Weinberg, 2004). Another possible explanation for reduced productivity of elderly workers that is not based on the assessment of cognitive abilities is the fact that the short remaining worklife duration of older workers makes investment in training for new technologies less attractive. The importance of this link is illustrated by a study by Klös (2000) which shows that only 5% of employees aged 50-55 and only 1% of employees above 55 receive on-the-job-training in Germany.

The latter idea is used in a model by Swanson et al. (1997), which is conceptually close to the model presented in this paper. In their model, individuals decide on adopting new technologies, working and leisure in a life-cycle model with an exogenously moving technological frontier. The authors show that individuals stop to adopt new technologies in the later stages of their lives as the investment could otherwise not be recuperated.
They also suggest that this mechanism leads to lower realized technological progress when the population ages. A very similar paper is Ahituv and Zeira (2000) in which old workers decide between working with an old technology, adopt a new technology and early retirement. My model differs from these papers in that the decision to train workers for a new technology is shifted to the firm which plans its hiring and technology decisions over multiple periods.

Langot and Moreno-Galbis (2008) analyze the technology adoption decision from the view of a firm that employs a single worker who ages stochastically. Their paper is nested in the job-search theory and the focus lies on whether technological progress is beneficial for the employment prospects of old workers or not. Nevertheless, their paper shares some results with the paper presented here: old-worker firms update their technology less often than young-worker firms, a higher training cost for new technologies decreases the updating frequency whereas a higher rate of exogenous technological progress increases it. The model developed here differs from the previous studies in that the focus here is on the technology decision by firms and that firms employ a heterogeneous workforce that changes all the time. With this the model is able to replicate a dynamic environment with rich firm dynamics that allows to quantify the expected impact of labor-force aging and changes in the average retirement age.

The results of the model are in line with the findings of empirical studies on technology adoption at the firm level: for French private sector firms between 1998–2000, Aubert et al. (2006) find that innovative firms have a lower wage bill-share of older workers. Meyer (2007, 2009) shows that an older workforce in firms leads to less technology adoption in German small and medium-sized firms in 2005 and this effect increases with worker age beginning from the age of 30. A study by Malmberg et al. (2008) analyses the productivity of Swedish manufacturing and mining enterprises between 1985–96 finds that firms with a higher share of employees aged 50 and above are generally less productive though more productive when technology is controlled for. This implies that firms with an older workforce use on average less productive technologies.
2 The Model

The model analyses firm dynamics in a competitive economy in a similar framework as Hopenhayn and Rogerson (1993), where firms expand or contract, make technology choices and enter or exit the market. In their decisions, firms take into account workforce aging with overlapping generations of workers.

Workers and Firms

The economy is populated by a continuum of firms and workers. New workers enter every period as young workers, turn into old workers and finally exit the labor market, determined by a stochastic aging process. All variables referring to young workers will be denoted by subindex \( y \) whereas \( o \) is used to indicate old workers. The exogenous probability of becoming old for a young worker is given by \( \lambda_y \) and the exogenous probability of retirement for an old worker is \( \lambda_o \). Consequently, workers are young for \( 1/\lambda_y \) periods on average whereas the expected worklife duration of an old worker is \( 1/\lambda_o \) periods. Employed workers separate from firms with exogenous probabilities \( q_y \) and \( q_o \) or when exiting the labor market. Apart from the expected remaining time in the labor market and the exogenous separation probabilities, old and young workers are equal in all respects. In particular, this means that there are no experience effects or learning on the job on the one hand and no loss of human capital or falling individual productivity on the other hand. This assumption is made to isolate the impact of the expected remaining worklife on training and its effect on technology adoption.

The evolution of the economy’s workforce is given by:

\[
P_{y,t+1} = (1 - \lambda_y) \cdot P_{y,t} + P^N_{y,t}, \tag{1}
\]

\[
P_{o,t+1} = \lambda_y \cdot P_{y,t} + (1 - \lambda_o) \cdot P_{o,t}, \tag{2}
\]

where \( P_{y,t} \) and \( P_{o,t} \) denote the mass of young and old workers in the economy respectively and \( P^N_{y,t} \) denotes the inflow of new young workers in a period.

Workers are employed by firms for which labor is the only input. Each single firm employs a mass of young and old workers that are hired in frictionless, competitive
markets at wage rates \( w_{y,t} \) and \( w_{o,t} \) and hiring cost \( c_{N,t} \) per worker. All firms produce an identical good which is taken as the economy’s numeraire and discount profits at a common, exogenous discount rate \( r \). Firms can enter the market at a positive entry cost \( c_{E,t} \) and exit with exogenous probability \( \delta \). The free entry condition ensures zero expected profits prior to entering.

A firm’s production function is given by

\[
Y_i^t = A_i^t F(y_i^t + o_i^t). \tag{3}
\]

where \( y_i^t \) and \( o_i^t \) denote the total number of young and old workers employed by the respective firm at time \( t \) and \( A_i^t \) is a productivity parameter that depends on the technology that the firm currently uses. The function \( F(x) \) exhibits decreasing returns to scale to restrict the firm’s size.

**Technological Progress**

The economy features exogenous technological progress so that a new technology arrives in every period. New technologies increase the productivity frontier by a constant factor \( g \) so that the productivity parameter evolves according to

\[
A_{t+1} = (1 + g) \cdot A_t. \tag{4}
\]

A firm has two options for adopting new technologies. It can either adopt the newest technology \( A_t \) or a technology that is \( B \) steps away from the technological frontier \( A_{t-B} \).

To adopt a production technology, a firm has to train its workers to use it. This implies that technology is embodied in the workers of a firm. All workers within a firm need to use the same technology and a firm cannot split itself into two entities that use different technologies. If a firm hires new workers, it has to train them for the technology that is currently used in the firm.

Training cost is a fixed cost per worker and depends on the type of technology that is adopted. The training cost for the newest technology \( A_t \) is denoted by \( c_{T,t} \) while adopting the older technology \( A_{t-B} \) involves the costs \( \beta \cdot c_{T,t} \) where \( 0 < \beta < 1 \). If a firm hires new workers without upgrading its technology, the training cost for new workers
is \( c_{T,t} \) if the firm’s technology is \( A_i^t \in [A_t, A_{t-B}] \) and \( \beta \cdot c_{T,t} \) if \( A_i^t < A_{t-B} \). The training costs comprise all direct and indirect costs of adopting a new technology. This includes monetary training costs, lost production for the time of training, and lower production for the time that workers need until they are experienced enough with the new technology to become more productive than with the old technology (see e.g. Helpman and Rangel (1999)). When a firm decides to use a different technology, all of its workers have to be trained for it. Worker training is firm specific, hence a worker who changes his employer has to be trained anew, irrespective of the previous trainings he received. This implies that training costs are borne by the employing firm, not by the worker (Becker, 1962).

Since technology progresses every period, all cost constants and wages are expressed in efficiency units relative to the technological frontier and written without time index, i.e. the actual costs are multiplied by \( p(t) = (1 + g)^t \), e.g. \( c_{T,t} = (1 + g)^t c_T \). This ensures the existence of a steady state equilibrium on the growth path.

**Timing of Events**

Each period starts with production where output is determined by the technology that a firm decided to adopt in the preceding period. Thereafter, the existing firms, together with new firms that enter the market, hire young and old workers or lay off part of their workforce and decide whether to upgrade to a new technology in the next period. At the end of the period, firm exit and worker aging and separation take place.

### 3 Equilibrium

In this section I focus on a stationary equilibrium along a balanced growth path. The simulation of demographic change does of course conflict with a stationary equilibrium but it will be explained later in the paper how both can be combined.

**The firm’s problem**

Along the balanced growth path, the problem of an active firm that employs a mass of
Technology Adoption and Demographic Change
Karsten Wasiluk

A firm’s value is given by its instantaneous profit, that is output net of wage payments, and the discounted future value of the firm which depends on the firm’s optimal policy decisions with regard to the amount of young and old workers it employs in the next period and on the technology it uses in the next period. A firm decides in every period on the optimal employment of young and old workers in the next period, given its technology decision and with respect to equations (6) and (7). Regarding its technology, a firm has three options: First, it can continue with its current production technology, second, it can update to the newest technology at cost $c_T$ per worker, and third, it can update its technology to the level $B$ steps behind the technological frontier at cost $\beta \cdot c_T$ per worker. If a firm decides to continue its actual technology and hires new workers, it

$$v(y, o, k) = (1 + g)^{-k} F(y + o) - y \cdot w_y - o \cdot w_o$$

$$\frac{1}{1 + r} v(y', o', k + 1) \cdot (1 + g), \quad (5)$$

$$- c_T (y - y^F + o - o^F) + (c_N + c_T) \left( y^H + o^H \right) + \frac{1 - \delta}{1 + r} v(y', o', 0) \cdot (1 + g),$$

$$- \beta c_T (y - y^F + o - o^F) + (c_N + \beta c_T) \left( y^H + o^H \right) + \frac{1 - \delta}{1 + r} v(y', o', B) \cdot (1 + g),$$

$$\text{s.t. } y' = (1 - \lambda_y)(1 - q_y) \cdot \left[ y + y^H - y^F \right] \quad (6)$$

$$o' = (1 - \lambda_o)(1 - q_o) \cdot \left[ o + o^H - o^F \right] + \lambda_y (1 - q_y) \cdot \left[ y + y^H - y^F \right], \quad (7)$$

$$c_T(k) = \begin{cases} c_T & \text{for } k < B \\ \beta \cdot c_T & \text{for } k \geq B \end{cases}$$

where $y^H, o^H$ and $y^F, o^F$ denote hired and fired young and old workers respectively.

In a stationary equilibrium, wages in efficiency units $w_y, w_o$ are constant over time and for each period, wages are given by $w_{y/o,t} = (1 + g)^t w_{y/o}$.
training cost per worker are $c_T$ if the firm is less than $B$ steps behind the technological frontier and $\beta \cdot c_T$ if the firm is further behind the frontier.

The state of a firm is completely described by the mass of young and old workers it employs and the technology it uses. The state of the economy is given by the distribution of state variables for all individual firms and is expressed as a measure over triples $\mu(y, o, k)$, which is invariant in the stationary equilibrium. The optimal employment decisions for young and old workers are denoted $N_y(y, o, k)$ and $N_o(y, o, k)$ respectively, the optimal technology decision is denoted $X(y, o, k) \in \{0, B, k + 1\}$.

**Entry of new firms and wages**

Entry is free, therefore entering firms must expect zero profits. For entering firms, two technology choices are possible: they can either adopt the newest technology $A_{(0)}$ or the vintage technology $A_{(B)}$ at their respective costs. The zero-profit conditions for the two options are given by:

\[
\frac{1 - \delta}{1 + \tau} v(y', o', 0) - c_E - (c_N + c_T) \left( y^H + o^H \right) \leq 0 \quad \forall y^H, o^H \geq 0, \\
\frac{1 - \delta}{1 + \tau} v(y', o', B) - c_E - (c_N + \beta c_T) \left( y^H + o^H \right) \leq 0 \quad \forall y^H, o^H \geq 0,
\]

where $y^H, o^H$ denote the masses of young and old workers that an entrant hires and $y'$ and $o'$ give the labor force in the next period according to (6) and (7) respectively. Since the model features exogenous firm exit, a stationary equilibrium necessarily requires positive entry of firms. Therefore at least one of the two zero-profit conditions must be binding, that is one combination $y^H, o^H$ exists for which a zero-profit condition is binding. Depending on the model’s parameters, especially $\beta$ and $B$, it is possible that two types of entrants exist, where one type chooses the newest technology $A_{(0)}$ and one type chooses the vintage technology $A_{(B)}$. Otherwise it can turn out that only one technology choice is attractive for entering firms, so that only one entrant type exists and either all entering firms adopt the newest technology $A_{(0)}$ or all entrants adopt the vintage technology $A_{(B)}$. Together with simultaneous labor market clearing for both

\[\text{In the numerical solution where } y \text{ and } o \text{ are restricted to a finite number of values, } \mu(y, o, k) \text{ can be represented as a three-dimensional matrix where each element gives the mass of firms in a particular state.}\]
types of workers, equations (8 and (9) determine the wage rates \(w_y, w_o\). The fact that
the entrants’ technologies move with the technological frontier ensures that wages grow
with the rate of technological progress and are constant in efficiency units.

Entering firms are completely described by their employment decision and the chosen
technology. The distribution of entrants is therefore expressed as a measure over triples
which is denoted as \(\mu^N(y^H, o^H, k)\). With this, enough information has been collected
to trace the evolution of the economy. At the beginning of period \(t\), let the incumbents
be summarized by the measure \(\mu\). Incumbents make optimal employment decisions,
using \(N_y(y, o, k)\) and \(N_o(y, o, k)\) and decide on technology upgrading using \(X(y, o, k)\).
At the same time, new firms summarized by the measure \(\mu^N\) enter the economy and
hire the remaining workers that are not employed by incumbent firms. After workers
have separated from firms with probabilities \(q_y\) and \(q_o\) and aged according to aging
probabilities \(\lambda_y\) and \(\lambda_o\) and firms have exited with probability \(\delta\), the aggregate state of
the economy for period \((t + 1)\) is given by the measure \(\mu'\). The transition from \(\mu\) to \(\mu'\)
is written as \(\mu' = T(\mu, \mu^N, w_y, w_o)\). The wage rates for young and old workers appear in
the operator \(T\) as they determine the decision rules of incumbent and entering firms.

**Labor markets**

In a stationary equilibrium without changes in the age composition of the population,
the inflow of young workers must equal the outflow of old workers in every period, so
\(P_y^N = \lambda_o \cdot P_o\). The mass of the total workforce is normalized to one, so the supply of old
and young workers in the labor market is given by:

\[
L_y^s = P_y = \frac{\lambda_o}{\lambda_y + \lambda_o},
\]

\[
L_o^s = P_o = \frac{\lambda_y}{\lambda_y + \lambda_o}.
\]

Total demand for young and old workers by incumbents and entrants is given by:

\[
L_y^d(\mu, \mu^N, w_y, w_o) = \int N_y(y, o, k, w_y, w_o) \, d\mu(y, o, k) + \int y^H \, d\mu^N(y^H, o^H, k),
\]

\[
L_o^d(\mu, \mu^N, w_y, w_o) = \int N_o(y, o, k, w_y, w_o) \, d\mu(y, o, k) + \int o^H \, d\mu^N(y^H, o^H, k).
\]
Definition of equilibrium

A stationary equilibrium is given by constant wages for young and old workers in efficiency units \( w^*_y, w^*_o \geq 0 \), a measure of entering firms \( \mu^*N \) and a measure of incumbent firms \( \mu^* \) such that (i) \( L^d_y(\mu^*, \mu^*N, w^*_y, w^*_o) = L^s_y \) and \( L^d_o(\mu^*, \mu^*N, w^*_y, w^*_o) = L^s_o \), (ii) \( T(\mu^*, \mu^*N, w^*_y, w^*_o) = \mu^* \), and (iii) \( 1-\delta \left( \frac{1}{1+r} v(y', o', 0) - c_E - (c_N + c_T) \left( y^H + o^H \right) \right) \leq 0 \) and \( 1-\delta \left( \frac{1}{1+r} v(y', o', B) - c_E - (c_N + \beta c_T) \left( y^H + o^H \right) \right) \leq 0 \) \( \forall y^H, o^H \geq 0 \), with equality for those pairs \( (y^H, o^H) \) where \( \mu^*N(y^H, o^H, k) > 0 \).

The conditions need not much explanation: Condition (i) demands that labor markets for young and old workers are cleared, condition (ii) states that the state of the economy must replicate itself in each period in a stationary equilibrium, given optimal decision by incumbent firms and entrants, and condition (iii) states that entry in the economy is possible with zero expected profits for entrants. The model is solved by numerical methods as described in more detail in the appendix.

Wages, firm policies, and demographic change

A nice feature of the case with two types of entrants is that typically the two entrant types will pursue different hiring strategies with respect to the age distribution of the hired workforce. This happens due to the fact that the updating decisions of firms strongly depend on the age structure of their workforces, as will be shown in the next section. Since the two entrant types pursue different technology strategies, this implies the age structure of the optimal workforce that entrants hire is different for the two types. In this case, simultaneous labor market clearing for young and old workers is achieved by the adjustment of the masses of entrant types. This implies that when the masses of young and old workers in the economy change, the distribution of entrant types changes, but not the hiring policies and profits of the entering firms itself. Therefore wages are independent of the masses of young and old workers in the economy. From this follows, that firm policies are also independent of the distribution of workers and, if the relation of young and old workers changes, firm policies do not change. This feature allows to simulate demographic change by adjusting the inflow of new workers into the economy while firm policy functions remain constant as long as all other parameters including \( \lambda_y \) and \( \lambda_o \) are left unchanged and the economy stays in the equilibrium with two different entrant types.
Figure 1: Distance from technological frontier at updating

4 Stationary Equilibrium Results

Figure 1 shows a numerical example of the firms’ equilibrium technology decision with respect to their workforce and current technology. The graphic depicts the distance from the technological frontier at which firms update their technology:

\[ k^*(y, o) = \min(k | X(y, o, k) \in \{0, B\}) \forall y, o \geq 0. \]

As a general result, it can be directly seen that the distance to the technological frontier at which a firm decides to update its technology depends primarily on the age structure of a firm’s workforce and to a lower extent on the firm’s size. Adding old workers strongly increases the distance to the technological frontier at which a firm decides to

---

3The parameters used here are chosen for a clear representation of the results and are not the ones used in section 5. In this example and in general for the whole paper, it is assumed that the exogenous separation probability of young workers is not dramatically higher than that of old workers and so the expected remaining time in a firm is higher for young workers than for old workers.
update. Adding young workers on the other does not increase the updating distance, except for very small firms. For heterogeneous firms, increasing the number of young workers can even lower the updating distance as the average age of the workforce in the firm becomes lower. The reason for this is that firms with old workers prefer to delay the investment in technology-training because they expect their workers to retire soon, making the investment unprofitable. A higher number of old workers increases the updating distance irrespective of the age-structure of the firm. This happens because firms with many old workers wait with technology updating to give old workers a chance to drop out of the labor market first. If these firms finally update, they lay-off some of their old workers in the process, as it is unprofitable to invest training cost for all of them.

For very small firms, the distance from the technological frontier at which they decide to upgrade becomes dramatically smaller. This is due to the fact that these firms want to increase their workforce to the optimal level. When a firm hires new workers, it has to invest in training cost for the new hires. However, if a firm has to pay training cost for the new hires anyway, it prefers to train them for the newest technology and train its few already existing employees as well, instead of training the new hires for the vintage technology that the firm currently uses and having to train them again some periods later when the firm finally updates its technology. So, hiring new workers complements technology renewal and the smaller a firm is, the greater are the incentives to hire new workers and to update the firm’s technology at the same time.

Firms do not only differ in their distance to the technological frontier at which they decide to update but also choose different technologies when updating, depending on the age structure of their workforce. Figure 2 illustrates which kind of firms choose to update to the newest technology $A(0)$ and which firms prefer to update only to the non-state-of-the-art technology $A(B)$ at a lower updating cost. As expected, firms with a larger share of old workers prefer to upgrade to the older technology to reduce investments in their old workers who may otherwise retire before the training cost for the high technology are recovered. An exception are very small firms that are close to the technological frontier. These firms, that use a in-between technology $A^i \in (A(0), A(B))$ update to the highest level in order to hire new workers in the process even if they have only old workers.
Nevertheless, if such small old-worker firms are further away from the frontier, they would update to the lagged technology instead. However, as the graphic shows only the first time a firm updates for a given workforce, this is not depicted in the figure.\footnote{Typically, with two entrant types, entrants that choose the high technology mainly employ young workers whereas entrants that start with the lower technology hire mainly old workers. This implies that old-worker firms are never close to the technological frontier.}

With regard to the aggregate level, the distribution of firms in the economy shows that firms with older-than-average workforce lag further behind the technological frontier. This replicates the results of the empirical studies mentioned in the introduction. In addition, another well-known empirical result in terms of technology utilization by firms is evident in the firm distribution: Firms that are larger than the average use newer technologies than small firms. This may come as a surprise as the analysis of the optimal firm policy above indicated, that small firms updated their technology earlier than large firms. However, firms that update use this opportunity to hire new workers and hence firms that use the newest technology always have the largest workforce.

**Comparative Statics**

Even though the model can only be solved numerically, the equilibrium variables behave monotonously when confronted with different parameters so that some statements about
the effects of parameter changes can be made. An increase of the training cost for workers $c_T$ increases the distance to the technological frontier at which firms decide to update their technology. This is true for all types of firms; however, the effect is stronger for firms with an older workforce. If the training cost becomes very large, firms stop hiring or retraining old workers since the cost becomes higher than the expected profit that can be obtained within the old workers’ expected duration in the labor market. For smaller training costs, the updating policies converge until all firms update their technology in every period when $c_T$ becomes very low. As a side effect, a higher training cost lowers wages for young and old workers and increases the wage differential $\frac{w_y}{w_o}$.

Increasing the difference $B$ between the newest technology and the non-state-of-the-art technology to which firms can update increases the distance from the technological frontier of old-worker firms that typically use the latter technology. For the firm distribution, this increases not only the total average technology lag of the economy but also strongly increases the dispersion of productivity in the economy. If $B$ becomes too large for a given $\beta$, firms start to update always to the technological frontier. The other way round, it is possible that no firm uses the high technology and all firms start with and upgrade to the lower technology. The interplay of $c_T$, $B$ and $\beta$ together also affects the total updating frequency of the firms or the share of workers receiving training in each period respectively for a given average lag of the economy and a given productivity dispersion.

A change in the exogenous rate of technological progress $g$ has the opposite effect to a change of the training cost $c_T$. An increase of $g$ reduces the distance to the technological frontier at which firms decide to update their technology since the gains from updating increase and the pressure from increasing wages is higher. Increasing the entry cost of firms $c_E$ on the other hand has little effect on the technology decision and mainly affects wages, which become smaller with increasing entry cost. Also, the wage differential $\frac{w_y}{w_o}$ increases. Similar to the training cost $c_T$, if $c_E$ becomes very high, firms stop to hire or retrain old workers.

An increase of the expected worklife duration of an old worker, that is a lower $\lambda_o$, has two opposing effects. At the firm level, it reduces the distance to the technological frontier
at which firms with older workers update. On the other hand, a lower $\lambda_o$ increases the share of old workers in the economy. At the aggregate level, this effect moves the entire economy away from the technological frontier as more firms with old workers exist, which update later than firms that employ a younger workforce. Therefore, no general statement can be made here. A decrease of $\lambda_y$ on the other hand, increasing the expected time span a worker is young, reduces the updating distance on the firm level for young worker firms and moves the whole economy closer to the technological frontier as the share of young workers increases. With respect to wages, a longer expected worklife for old workers, that is a lower $\lambda_o$, makes elderly workers more attractive for firms and thus increases the old workers’ wage $w_o$. At the same time, the wage for young workers $w_y$ decreases because young workers and old workers are substitutes. This implies that the wage differential $\frac{w_y}{w_o}$ decreases. A decrease of $\lambda_y$ has the opposite effects to a decrease of $\lambda_o$.

Changes in the exogenous separation probabilities $q_y, q_o$ affect young workers stronger than old workers. As the expected worklife of old workers is short in any way because of their upcoming retirement, an additional increase of the separation probability does not have a great effect. For young workers on the other hand, who have a long worklife horizon, an increase in the separation probability reduces their profitability for firms and also the distance at which young-worker firms decide to update their technology. This reduces the wage for young workers $w_y$ and the wage differential $\frac{w_y}{w_o}$ decreases. If $q_y$ becomes very large, the expected job duration becomes shorter than that of old workers. This extreme case would reverse most results with regard to the updating decision with respect to the workforce.

5 Simulation of Demographic Change

Calibration
The model is calibrated to match the German economy and the projected changes in the labor force between 2003 – 2050 are simulated to analyze the resulting changes in the economy’s average distance from the technological frontier. Figure 3 shows the projected
changes in the old-age ratio for the German population and labor force. The threshold age of 55 years that separates young workers from old workers has been chosen because the participation rate in the labor force drops dramatically after the age of 55 whereas it is rather constant between the age 20–54. That implies that the probability that a worker leaves the firm and exits the labor force is strongly increased once he has reached an age of 55. The expected time to be a young worker is therefore 35 years which gives $\lambda_y = 0.0286$.5

The change in the age composition of the labor force is not completely caused by demographic change alone but also by a change in the average retirement age or a change in the participation rate of older employees. Figure 3 illustrates the change of the labor force composition with and without the latter effect. It can be seen that the increase of the share of old workers in the economy is strongly augmented by the expected increase of the retirement age. Table 1 depicts the average retirement age and respective $\lambda_o$ over

---

5 A robustness test has been undertaken to check whether the results depend on the chosen cut-off age for young and old workers. For a series of different combinations of $\lambda_y, \lambda_o$ with equal total expected life-time of a worker, firm policies and the stationary firm distribution have been derived. It turns out, that the variable of interest, the average distance of the economy from the technological frontier, did vary only very slightly for the different young and old worker combinations.
Technology Adoption and Demographic Change

Karsten Wasiluk

<table>
<thead>
<tr>
<th></th>
<th>2003</th>
<th>2010</th>
<th>2015</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Exit Age</strong></td>
<td>60.1</td>
<td>61.8</td>
<td>63.3</td>
<td>63.5</td>
<td>62.1</td>
<td>62.6</td>
<td>62.6</td>
</tr>
<tr>
<td><strong>λ₀</strong></td>
<td>0.164</td>
<td>0.128</td>
<td>0.108</td>
<td>0.106</td>
<td>0.124</td>
<td>0.116</td>
<td>0.116</td>
</tr>
</tbody>
</table>

Table 1: Expected Retirement Age and Resulting λ₀

Carone (2005); OECD (2011), Own calculations

the simulation period. Different retirement ages imply different firm policies, whereby firms have to adopt their policies dynamically over time to adjust to the new exit age of old workers. To avoid this, the simulation is restricted to a one-time change of the retirement age and firms are assumed to be surprised by this change of the environment. I take the averages of the retirement ages for 2003–2010 and 2015–2050, which gives λ₀,(2000–2010) = 0.146 and λ₀,(2015–2050) = 0.114. For each value, optimal equilibrium policies are derived and demographic change is simulated for both retirement ages. The simulation of demographic change is implemented by varying the number of new workers that enter the economy such that the relation of old to young workers mimics exactly the movement in Figure 3.²

Table 2 provides an overview of the calibration for the model’s parameters. The period length is one year. The interest rate and rate of exogenous technological progress are standard values. The probability of a firm destruction shock is set to 9% to match the average annual firm exit rate in Germany between 2005–2007, taken from Eurostat (2012). The exogenous job separation probability of young workers is set to qᵧ = 0.02 to match an average job-duration of 9.2 years for workers aged 20–54, taken from OECD (2012). The average job-duration of workers aged 55 and older is longer than the model is able to reproduce, therefore the exogenous separation probability is set to qₒ = 0. The recruitment cost for new workers is set to 70% of the monthly average wage, which is taken from a recent study for skilled workers in Germany by Muehlemann and Pfeifer (2012). The entry-cost is calibrated to present a capital share of 30% in the model which has no capital otherwise. The three technology parameters c_T, B and β are matched to

²It turns out that for the chosen parameters, two types of new firms with different technology choices exist in equilibrium. As described in section 3, this implies that firm policies do not change when the amount of young and old workers in the economy changes as long as λᵧ and λₒ remain constant.
the average lag to the technological frontier at the beginning of the simulation, taken from Comin and Hobija (2010), the productivity dispersion of the firms given in Pfeifer and Wagner (2012), and the share of workers that receive training per year, taken from Kuwan et al. (2006). The calibration procedure for the entry cost and the technology-cost parameters is explained in more detail in the appendix.

The production function takes the form

\[ F(y, o) = (y + o)^\alpha, \]

where \( \alpha \) is calibrated to give an average firm size of 12.5 employees (OECD, 2010).

**Results**

Figure 4a illustrates how the economy’s average distance from the technological frontier\(^7\) is affected by demographic change for a constant average retirement age. The solid line presents the evolution of the economy with an average retirement age of 60.8 years which

\(^7\)The distance from the technological frontier is measured in output terms, that is the average output of the simulated economy is compared to an economy where all firms produce at the technological frontier.
matches the retirement age in Germany for the beginning of the simulation period and the dashed line presents the effects of demographic change for an average retirement age of 62.8 years to represent the projected development of old-age labor-force participation in Germany from 2015 onward. Both curves follow the pattern of demographic change given in Fig. 3. As the share of old workers in the labor force increases, the economy moves away from the technological frontier. This is the expected result as more old workers imply that firms delay their updating decision as shown in Figure 1.

Between 2010–2025, when the extent of demographic change is greatest, the economy moves two steps away from the technological frontier for the early retirement case. As the projected share of old workers in the economy decreases after 2025, the economy moves closer to the frontier again until 2040 from when the distance increases as the share of older workers rises again. For the case with higher average retirement age, it can be seen that the curve starts with a greater distance from the technological frontier at the beginning. As described in Section 4 there are two effects at work. First, a greater expected worklife induces firms with old workers to update their technology earlier which moves the economy closer to the frontier. Second, a larger share of old workers (due to their longer worklife) implies that the economy moves further away from the technological frontier because more firms employ old workers. For the calibrated model, it can be seen
that the latter effect outweighs the former at the beginning of the simulation. However, it turns out that the effect of demographic change on the average technology lag of the economy is less pronounced for the version with higher retirement age. During 2010–2025, the increase of the technology lag is about 20% lower than for the economy with the lower retirement age. This effect is nearly strong enough to compensate for the higher starting lag, so that the two curves move roughly in unison from 2015 onward.

Figure 4b shows the time path of the distance from the technological frontier for the actual average retirement age over time, derived from the two curves of Figure 4a. Since the two effects of the increased average retirement age nearly outbalance each other during the time of demographic change, the combined curve mimics the curve for the early retirement age very closely, only the peak distance reached in 2025 is slightly higher and the movement towards the technological frontier between 2030–2040 is a bit less pronounced. These results indicate that the projected changes in the average retirement age for Germany have very little impact on the change of the economy’s distance from the technological frontier.

The economy’s movement away from the technological frontier can be translated into lower productivity growth during that period. This is illustrated in Figure 5 where the deviation of the realized productivity growth from the long-run trend is plotted. As indicated above, there are only small differences between the case of constant average retirement age, depicted by the dashed line, and the combined version with the projected change in the retirement age, represented by the solid line. It can be seen that demographic change has a strong negative impact on realized productivity growth. As the share of old workers in the economy increases, realized productivity growth decreases with a negative peak in 2017 where productivity growth is nearly 0.4 percentage points below the long-run trend. As demographic change slows down, productivity growth returns to its trend and increases above it between 2030–2040 when the share of old workers is projected to decrease slightly.

Between 2010–2025, the average rate of realized productivity growth was 0.247 percentage points below the long-run trend when the projected change of the average retirement age is accounted for and 0.244 percentage points when the average retirement age
The quantitative results of the simulation can be directly compared to the results in Werding (2008) who computes forecasts for productivity and output growth for various OECD countries based on regression estimates. The evolution of productivity growth in his forecast for Germany for the same period is very similar to the results presented here, only the magnitude of the effect is higher. For the period 2010–2025, Werding’s estimates indicate an average loss of productivity growth of 0.4 percentage points. The result of the simulation undertaken here is about one third lower than the estimate in this model. Nevertheless, this model focuses on one single channel of how demographic change affects realized productivity growth. Obviously, other effects are at work as well.

For Finland, where the aging of the labor force took place earlier, Grönqvist (2009) finds for the period 1991—2005, demographic change lowered the annual growth of labor productivity by 1.5 percentage points on average. This is a much larger effect than the model here can replicate. Closer to the results of this study are the results of the study by Tang and MacLeod (2006) who forecast that annual reductions of productivity
Figure 6: The economy’s average distance from the technological frontier with increased retirement age

Increasing the retirement age:

As a policy experiment, the effect of an additional increase of the average retirement age by 3 years is simulated. Such an increase can be achieved by raising the statutory retirement age as it is done in Germany and many other European countries at the very moment, or by reducing the number of people who drop out of the labor force early. For the simulation, the three additional years are added to the average retirement age that is projected for 2015 onward. This gives an average retirement age of 65.8 years. The results are compared to the benchmark economy with an average retirement age of 60.9 years, which is the average of the projected retirement age for the period 2003–2010. As before, it is assumed that the increase of the average retirement age comes unexpected for the firms, so transitional dynamics are not regarded.

The effects of the additional increase of the average retirement age are illustrated in Figure 6a, where the dashed line represents the evolution of the economy with increased
Figure 7: Productivity Growth: Deviation from the trend with increased retirement age

retirement age compared to the solid line which stands for the economy with low retirement age. The economy with higher average retirement age starts now with an even greater distance to the technological frontier because the negative effect of an increase in the retirement age predominates. However, the effect of demographic change on the economy’s technology lag is so much lower that this negative effect is completely compensated by 2025 and the economy’s maximum distance from the technological frontier is not higher than that of the economy with the early retirement rate.

Figure 7 compares the deviation of the realized technological progress from the long-run trend for all three scenarios: the solid line represents the evolution of the economy with the experimentally increased retirement age from 2015 onward, the dashed line represents the projected real change of the retirement age and the dash-dot line illustrates the economy when the average retirement age of the period 2003–2010 is held constant. It can be seen the experimental economy is the first to lose technological progress, however, productivity growth does not fall as deep as in the economy with the real projected change in the average retirement age and it recovers faster. The predicted loss of productivity growth for the period 2010–2025 is 0.233 percentage points which translates into a monetary loss of 148.7bn in constant 2000-US$. These numbers imply that an additional increase of the average retirement age would lower the cost of demographic change over the period 2010–2025 compared to the scenario with the real projected change of the
average retirement age and the case with constant average retirement rate. However, the positive effect is very small.

6 Conclusion

The demographic change that is ongoing in the industrialized countries leads to massive changes in the age composition of the labor force. In many OECD countries, the share of workers aged 55 and above in the labor force will increase by 50–100% between 2000–2030. This paper focuses on the consequences of labor force aging for the rate of productivity growth.

A number of empirical studies have shown that firms that employ on average older workers lag further behind the technological frontier than firms with younger workers. (Aubert et al., 2006; Meyer, 2007, 2009; Malmberg et al., 2008) To replicate these results, I develop a model of firm dynamics in a competitive economy where firms expand or contract, make technology choices and enter and exit the market. The economy is populated with overlapping generations of workers that undergo a stochastic aging process, which the firms take into account. The model shows that having older workers in their workforce lets firms delay technology updating and choose older technology.

I then calibrate the model to match the German economy and simulate the predicted changes in the labor force composition for the period 2003–2050. This time period is not only marked by a strong increase in the share of old workers but also by an increase in the average retirement age. In order to disentangle the two effects, I analyze the consequences of demographic change on the macroeconomic performance with and without the increase in the retirement age. The results show that labor force aging has severe consequences on the economy: as the share of workers aged 55 and above increases strongly from 2010 onward, the economy moves away from the technological frontier which results in lower realized growth during this period.
It is found that demographic change lowers the average rate of realized productivity growth by about 0.24 percentage points below the long-run trend over the period 2010–2025. This loss of productivity growth translates into a loss of approximately 156bn in constant 2000-US$. Taking the change in the average retirement age into account changes the results only very little as the resulting effect at the micro-level, shorter updating intervals, and the effect at the macro-level, more firms with older workers, outbalance each other. A comparison of the simulation results to other studies indicate that the model’s results are in a plausible range. Since the model focuses only on a single mechanism by which labor force aging effects productivity growth, it is quite possible that the results understate the negative impact of labor-force aging on productivity growth. Finally, a policy experiment is undertaken and an increase of the average retirement age by three additional years from 2015 onward is simulated. It turns out, that the loss of productivity growth due to demographic change is slightly reduced compared to the other two scenarios, but the difference is very small.
References


Appendix

Numerical Solution Procedure
The numerical solution of the stationary equilibrium is split into two steps: the derivation of firm policies and wages, and the simulation of the stable firm distribution. As it will be explained below, depending on parameters both steps have to be either carried out only once or repeated multiple times until the stationary equilibrium is found.

The first part, the derivation of firm policies and wages is an iterative procedure. First, for given wages $w_y, w_o$, firm policies are derived by value function iteration. Then the free entry conditions (8) and (9) have to be checked in order to adapt the wages. As pointed out in section 3, there are two possibilities for firm entry in equilibrium: either two entrant types exist and both free entry conditions are binding for a certain pair $(\Delta y, \Delta o)$ of hired workers, or only a single entrant type exist, i.e. only one of the free entry conditions is binding, the other is strictly negative for all hiring possibilities. If two entrant types exist and these entrant types hire workforces with different age structures, as described in section 3, the two labor markets can be cleared by adjustment of the entering firms. The wages for young and old workers are adapted until both free entry conditions are binding. For every change in the wages, firm policies have to be derived anew, so the whole process is iterative. Once the wages have been found, the firm distribution can be simulated. This is done by populating the economy with a constant flow of young workers in every period and allow firms to enter that hire these workers. This simulation is run until the firm distribution, represented by the measure $\mu(y, o, k)$ has become stationary.

If it is not possible for both free entry conditions to be binding, then only one entrant type exist. In this case the wages for young and old workers have to be adapted to have one of the free entry conditions binding and to clear both labor markets simultaneously by the single entrant type while the other free entry condition one is strictly negative. The single entrant type must hire exactly the ratio of young and old workers that becomes unemployed in a period and is not directly hired by existing firms in equilibrium. To find this solution, the firm distribution has to be simulated every time a new pair of wages is chosen and policy functions have been derived and it is checked whether labor markets are cleared in equilibrium. In the case of a single entrant type, wages and firm policies are not independent of the share of young and old workers in the economy. This implies that a change in the relation of young and old workers (by demographic change) demands for a different hiring policy of entrants at different wages.
Calibration of entry cost $c_E$:
The model features no variable capital that is needed for production, hence capital appears only indirectly in the fixed cost for firm creation $c_E$. Therefore, the entry cost is interpreted as the capital share in the economy, which is taken to be 30%. The labor share is given by the total amount of wages that a firm expects to pay in its lifetime, calculated at present value at the time of firm entry. With a survival probability of $(1 - \delta)$ for a firm, an average workforce of 12.5 workers, and the average wage in the economy given by $\bar{w} = \frac{\lambda_o \omega_y + \lambda_y \omega_o}{\lambda_o + \lambda_y}$, the free entry cost is given by:

$$c_E = 0.3 \cdot 12.5 \cdot 0.7 \cdot \sum_{t=0}^{\infty} (1 - \delta)^t \cdot \frac{1 + g}{1 + r} \cdot \bar{w}.$$ 

Calibration of technology parameters: $c_T$, $B$, $\beta$:
The training cost is derived by calibrating $c_T$ to achieve an average technology lag of the economy of 6 years at the beginning of the simulation. This implies that it takes firms on average 6 years to adopt a new technology. This value is taken from Comin and Hobija (2010) who estimate the average lag of technology adoption for 15 technologies in 166 countries. These lags vary considerably for different technologies, as some technologies require large investments in capital (e.g. new means of steel production) whereas other are less capital intensive and are mainly based on a change of working procedures and environment. These latter technologies resemble the technologies that this paper focuses on: technologies that require training for workers and time to adapt to new processes, so that the technology is strongly embodied in the workers. The technology in Comin and Hobija (2010) that comes closest to this definition is the use of the internet at workplace, for which an average adoption lag of 6 years for OECD countries has been estimated, which is taken as benchmark for the calibration.

As $B$ defines the lag between the newest technology and the non-state-of-the-art technology that is mainly chosen by old-worker firms, it increases the technology spread over the firms and thus increases the productivity dispersion among firms in the economy. As a target for the productivity dispersion, data from Pfeifer and Wagner (2012) is used, who calculate a normalized average standard deviation of labor productivity over firms within industries over the period 2003–2006 of 0.21, which is taken as target for productivity dispersion in the model. In interplay with the other parameters, $\beta$ determines the total updating frequency or the share of workers receiving training in each period respectively for a given average lag of the economy and a given productivity dispersion. As a target for $\beta$, I use data on the share of workers in the labor force that received on-the-job training over the duration of one year which is provided in Kuwan et al. (2006) and gives an average of 10.4% over the time 1999–2002.