A Job Ladder Model with Stochastic Employment Opportunities*

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Abstract

We set up a model with on-the-job search in which firms infrequently post vacancies and workers occasionally apply for said vacancies. The model nests the standard job ladder and stock-flow models as special cases while remaining analytically tractable and easy to estimate from standard panel data sets. The model fits moments of the data inconsistent with the standard job ladder model. Structurally estimating the model, the parameters are significantly different from the stock-flow or the job ladder model. Further, the estimated parameters governing workers search behavior are found to be consistent with recent survey evidence documented in Faberman et al. (2016). The search behavior implies that the standard job ladder model significantly understates the search option of the employed (and thus underestimates the replacement ratio). Finally, the standard model is unable to generate the declining job finding rate and starting wage with duration of unemployment, both of which are present in the data. Our model is able to reconcile both of these, both of which have important consequences when considering the earnings losses associated with job displacement.

Keywords: On-the-job search, wage dispersion, wage posting, stock-flow
JEL Classification: J31, J64

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1 Introduction

Due to their effectiveness in replicating labor turnover and wage dynamics, search models are extensively used to evaluate labor market policies.[1] Embedded in the standard job ladder model is *a single* friction that prevents the reallocation of workers into more productive jobs. In this paper, we set up a model in which, for a given worker, suitable vacancies are posted infrequently by firms and applications to these are made intermittently by workers. As special cases, our model nests the standard job ladder model and a version of the stock-flow model. In the estimation we reject the restrictions implied by the standard job ladder model are rejected and we find that imposing these restrictions significantly underestimates the search option associated with employment. Furthermore, the model generates a declining job finding rate with the duration of unemployment and substantial wage losses following displacement that increase in the duration of unemployment.

We set up a search model in which, for a given worker, only some job openings are suitable. This set of job opportunities is treated as a latent variable that follows a stochastic process. Firms create suitable job openings at some Poisson rate similar to McCall (1970). At a Poisson rate, a firm will stop looking for workers. Workers, on the other hand, infrequently send out multiple applications similar to Stigler (1962). The worker subsequently accepts the best offer if it is better than her current job. Firms differ in productivity and workers have an individual skill component. We close the model by assuming that firms post wage schedules in worker productivity prior to meeting the worker.[2] In deciding on the optimal wage, a firm trades off the higher chance of hiring a worker and the longer expected duration of the match against the higher wage cost. The resulting model nests the workhorse empirical labor models of Burdett and Mortensen (1998), and models of stock-flow matching, pioneered by Coles and Smith (1998), as special cases. Like Burdett and Mortensen (1998), the model has an analytical closed form solution and is well identified and empirically tractable, allowing us to estimate all of the parameters using a panel dataset on wage

[1] Recent evaluations include, enforcement policy on informal firms (Meghir et al., 2015), public sector wage and employment policy (Bradley et al., 2017) and tax policy (Yazici et al., 2017).

[2] The main results of the paper are unchanged if worker and firm instead bargain over the wage after the match is formed. The implications of alternative assumptions on the nature of wage setting are discussed in Section 3.6.

and employment dynamics.

We estimate the model using a two steps procedure for different skill groups, assuming the labor market is segmented by the workers' level of education. In the first step, the parameters governing workers' search behavior are identified by the flows between labor markets states as well as the duration dependence of the transition rate from unemployment to employment. These moments are calculated from the Current Population Survey (CPS). In the second step, the parameters governing worker and firm heterogeneity are identified from the distribution of average wages across workers as well as the overall distribution of wages. The productivity parameters are estimated in the second step, using data from Survey of Income and Program Participation (SIPP). The estimated model matches the transition rates from employment to unemployment and out of the labor force as well as the declining job finding rate with the duration of unemployment. The decline in the job finding rate with the duration of unemployment comes from the fact that those newly unemployed have, on average, better prospects than the long term unemployed. In the job ladder model, all unemployed are the same, i.e., unemployment is a single state, which implies that the model is unable to match the falling job finding rate with the duration of unemployment. This means that, within the model, the size of the decline of the job finding rate is informative of the importance of varying employment prospects. In the CPS, the chance that an unemployed worker has a job in a month's time halves in over the first three months of unemployment.

The estimated parameters imply that fewer posted vacancies are suitable for unemployed workers. On the other hand, unemployed workers send out applications more often: twice a month compared to less than twice a year for their employed counterparts. Faberman et al. (2016) document that the unemployed send out a much larger number of applications but number of contacts are similar; we explore the relation between the empirical observation of Faberman et al. (2016) and our estimated search process in greater detail in section 4.8. In the model some unemployment is due to workers not applying for jobs and the remaining unemployment comes from the fact that workers lack suitable prospects. If all workers were to have some prospects, i.e., the limit as their arrival rate goes to infinity, the unemployment rate would fall by more than a half. In con-

trast, in the benchmark model of BM, all unemployment is removed as the Poisson rate governing job contacts increases. The fact that the relative importance of applications versus availability of prospects differ starkly by employment state suggests that the one friction representation of the labor market might be particularly bad. In particular, compared to the standard job ladder model, the estimated baseline model implies that the search option is much greater for employed workers (thus generating much more frictional wage dispersion for a given replacement ratio). Secondly, the model fits the hazard rate out of unemployment and the earnings loss associated with job displacement that increase with the duration of an unemployment spell, without relying on human capital depreciation.

In the canonical job ladder model, the arrival rate of job offers differs by employment status. Taking a job is thus associated with a change in search option. The monetary value of said search option depends on the differences, between employment and unemployment, in arrival rates and the wage offer distribution. Hornstein et al. (2011) suggests that a particularly suitable metric for assessing the option value of search is the ratio of the mean to the minimum wage, hereafter the mean-min ratio (Mm). The probability that an unemployed worker is employed in a month's time is about 40% when calculated in the CPS. In the same dataset, the probability that an employed worker changes employer is about 3%. In an estimated job ladder model, these numbers (together with the separation rate) imply that jobs arrive much more frequently to the unemployed. The difference between lowest wage and the flow benefit must at least offset this loss in search option. Hornstein et al. (2011) find that the monetary value of said search option is very large which implies that the standard job ladder model generates very little frictional wage dispersion with a flow benefit in unemployment consistent with the macro labor literature.

In order to generate a Mm ratio of close to two, our estimation of the Burdett and Mortensen (1998) model requires a negative flow income in unemployment. On the other hand, our estimated baseline model with a replacement ratios in the order of 25-50% matches the same frictional wage distribution. The search option in Burdett and Mortensen (1998) is the difference in the job offer arrival rates multiplied by the expected increase in worker value. This is not the case in

our model since employed workers will on average have more offers to choose from. This enters the worker value function through two channels. Firstly, it is as if the employed workers sample wages from a distribution that stochastically dominates that of their unemployed counterparts. This is consistent with recent evidence from Faberman et al. (2016). Secondly, in our model, after losing their job, a worker will, on average, find a job quicker than the long term unemployed, consistent with our data. Interpreting the data through the lens of the Burdett and Mortensen (1998) model, thus, overestimates the foregone search option and hence underestimates the flow benefit in unemployment. This occurs as the model, only relying on the different transition rates, misses the better wage offers received in employment and the better position the worker is in if he subsequently gets fired. The replacement ratio is consequential for the ability of the model to generate cyclical fluctuations in the unemployment rate (Shimer, 2005; Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2015).[3]

Empirically, workers displaced in mass layoffs experience a substantial and persistent earnings loss (Jacobson et al., 1993; Davis and Wachter, 2011). In addition, the average starting wages and the job finding rate decreases with the duration of unemployment. In a search model without OJS or stochastic match quality, there is no wage loss following displacement as the average outstanding wage is equal to the average starting wage. In a job ladder model, on the other hand, workers gradually select into better paying jobs. The average employed worker will thus receive a higher wage than the average worker coming from unemployment. Employed workers therefore, on average, experience a wage loss following displacement but these losses do not increase with

[3] Shimer (2005) shows that observed labor productivity is not variable enough compared to the empirically observed variations seen in market tightness (level of vacancies divided by unemployment) in the standard calibration of the Mortensen and Pissarides (1994) model. A free entry conditions pins down the level of market tightness and any cyclical fluctuations come through variation in profit. The variation in labor productivity over the business cycle is typically found to be small. In order for this to translate into large changes in profit, it has to be that the level of profit is also small (Ljungqvist and Sargent, 2015). If the flow benefit of unemployment is high, then profit will be low and the model is able to generate sufficient amplification to productivity shocks (Hagedorn and Manovskii, 2008). Elsby and Michaels (2013) consider a model in which firms exhibit decreasing returns to scale. Marginal productivity is then less than average productivity and therefore movements in marginal productivity can be greater than the movements at the aggregate level. In our paper we find that the flow value of unemployment is indeed high compared to the marginal job.

the duration of unemployment, as unemployment is a single state. General human capital depreciation in unemployment generates wages falling with the duration of unemployment and thereby additional losses. For this reason, researchers studying earnings losses using job ladder models have incorporated falling general human capital in unemployment as a key driving mechanism.[4] However, falling general human capital would entail falling reservations wages with the duration of unemployment which is inconsistent with recent evidence from Krueger and Mueller (2016). In addition, since all unemployed search in the same market, the standard assumption of log linear production (and benefits) generates a constant job finding rate with the duration of unemployment. If instead the matching set were to decrease, the continuously falling human capital would imply the same for the job finding rate which is inconsistent with the empirical observation that the job finding rate falls quickly in the first three months but is broadly constant thereafter. Our model, in addition to featuring a positive selection into better jobs, also features an additional state variable - employment prospects. When the model is estimated, we find that the newly unemployed have, on average, more prospects than the long term unemployed. This implies that the job finding rate falls with the duration of unemployment as, via dynamic selection, workers with more prospects exit and those without prospects remain. Similarly, via this mechanism, the average starting wage also falls with the duration of unemployment. Our model thus jointly fits: a falling job finding rate with the duration of unemployment and a large wage loss following displacement that is increasing in the duration of unemployment.

Outline. The rest of this paper is structured as follows. Section 2 reviews the relevant literature we build upon and in section 3 we set up the model and derive analytical solutions. In section 4 we present the estimation of the model and the quantitative results and section 5 concludes.

[4]See, for example, the examples Krolikowski (2017), Jarosch (2015), and Burdett et al. (2017). The papers incorporate stochastic human capital into a job ladder model, although with different wage setting. Depreciation of human capital in unemployment is required to explain the size and persistence of earnings loss following job displacement.

2 Related literature.

Worker search behavior. The way search is modeled in our paper builds on the empirical literature documenting the different search behavior of the employed and unemployed. Blau and Robins (1990) find evidence that employed workers are more efficient in their search, yielding a greater number of job offers per applications made. Further, they find differences in the specific behavior, with the unemployed using a larger number of tools for finding work. Faberman et al. (2016) using a survey regarding the search behavior of workers, find that the employed and unemployed receive a similar number of employment contacts. However, the unemployed apply to jobs far more frequently and wages offered to them are, on average, lower than those offered to the employed. Features that are consistent with our estimated model. Belot et al. (2016) ran a controlled experiment on Scottish job seekers that broadens the subjects' occupation and geographic scope. Those treated had an increased number of interviews and particularly so for the long term unemployed. This result is consistent with our estimated model which suggests that an important driver of unemployment is a lack of labor market opportunities rather than the frequency to which unemployment workers apply.[5]

Mismatch models. In search models, a friction, often interpreted as a lack of information, prevents workers and firms from forming matches, thereby resulting in involuntary unemployment. There are a number of papers emphasizing differences, (e.g. skill, location) between available vacancies and the unemployed, creating thin markets, and thereby resulting in the simultaneous coexistence of unemployed workers and vacancies. Examples of these types of models are stock-flow matching (Coles and Smith, 1998) and mismatch unemployment (Lucas and Prescott, 1974; Shimer, 2007).[6] In our model, the stock of prospects can be interpreted as the local conditions for the worker and a number of the unemployed have no prospects and are therefore "mismatch" unemployed. Furthermore, the mismatch and stock-flow family of models generate similar employment dynamics

[5] A labor market opportunity, in the context of our model, is defined as at point of application, a job that a worker is: (i) aware of; and (ii) suitable for. We thus interpret a policy that increasing the set of jobs workers are aware of as increasing the efficiency to which labor market opportunities arise.

[6] See, also, Alvarez and Shimer (2011) and Carrillo-Tudela and Visschers (2013) for recent examples.

as our model. Both classes of models are able to generate the declining job finding rate with the duration of unemployment.

Multiple Applications. Similar to our paper, there are a number of search papers with multiple meetings and applications. However, the key friction in these models differs from our own. A large number of papers have modeled a thick market where workers make multiple applications and are able to direct their search (Albrecht et al., 2006; Kircher and Galenianos, 2009; Kircher, 2009; Wolthoff, 2017).[7] Closer to our paper are papers where workers are unable to direct their search but meet multiple firms (Elliott, 2014; Wolthoff, 2014; Gautier and Holzner, 2017). In all of the models discussed, either the number of applications a worker can make is exogenous, or each application carries with it an additional cost. Workers are ex ante homogeneous in their market conditions and ex post heterogeneous in their position in a network. Our model takes a complementary approach where instead workers are ex-ante heterogeneous in the thickness of their individual markets, and the number of potential opportunities follows a stochastic process.

Frictional wage dispersion. Following Hornstein et al. (2011), a number of recent studies have examined the ability of search models to generate frictional wage dispersion. Either there has to be a counterweighting effect that offsets the foregone search option, or it must be that the search option is not measured correctly. For example, if human capital depreciates quickly in unemployment, then that can motivate workers to take a low paid job (Ortego-Marti, 2016). Such an explanation would entail reservation wages falling quickly with the duration of unemployment which contradicts recent survey evidence (Krueger and Mueller, 2016). Within a sequential auctions framework, like in Postel-Vinay and Robin (2002) and Cahuc et al. (2006), the bargaining position of the worker increases when the worker takes a new job. These models can then generate more wage dispersion via this foot in the door effect (Papp, 2013). A foot in the door effect is also present in Carrillo-Tudela (2009) where there is no search option in unemployment. Faberman et al. (2016) is the closest to this paper, they consider a job ladder model with exogenously different wage offer distributions for employed and unemployed workers. When a worker is employed, the lower arrival

[7] In these models the worker faces two problems: a portfolio problem in deciding which jobs to apply for; as well as the optimum number of applications to send out.

rate of job offers is partly offset by a better offer distribution. In our model, the offer distribution is a time varying object and the average distribution faced by the employed stochastically dominates the distribution facing the unemployed. Our paper can thus be seen as a micro foundation for the two different offer distributions documented in Faberman et al. (2016).

3 Model

3.1 The Environment

Time is continuous and the labor market is populated by risk-neutral workers and firms. Workers leave the labor force at a Poisson rate μ and are replaced by a doppelgänger in unemployment. Workers are ex-ante heterogeneous in their productivity x, distributed with the cumulative distribution $\Gamma_x(\cdot)$ and ex-post vary in their employment state $s \in \{u, e\}$, their employment opportunities j, and if employed, their wage w. Firms are infinitely lived and are heterogeneous in their productivity y, the cumulative distribution of productivity amongst firms is given by $\Gamma_y(\cdot)$. Both worker and firm productivity distributions are primitives of the model. The total output of a match is the product of worker and firm types, xy. In unemployment workers earn a flow income proportional to their productivity type, bx. Jobs become unprofitable at an exogenous rate δ which results in the worker entering the pool of the unemployed. Finally, we do not allow workers to quit, other than to move to a new job.

The Frictions. The labor market is characterized by search frictions. Individual workers differ in their labor market opportunities which is a latent variable. These opportunities evolve stochastically. A worker amasses job opportunities according to a Poisson process as firms post vacancies. This Poisson rate, denoted by λ_s, differs by the employment state s. A firm stops hiring workers, i.e., the worker loses the job prospect, at a rate υ. υ captures, in a reduced form way, a variety of mechanisms: the job becomes unprofitable; the job is taken by another worker; the vacancy expires.

Unlike the sequential search literature, pioneered by McCall (1970) these opportunities are not continuously sampled from. Instead, there is a stock of outstanding vacancies, which we refer to

as the worker's employment prospects or simply their stock. In the stock-flow model of Coles and Smith (1998) a worker can always match with the stock, whereas we assume, this opportunity arises at a Poisson rate γ_s. If a worker can always match with the stock the model has an "instantaneous" property: after being fired the worker will either immediately match or otherwise wait for the inflow of new vacancies. A finite value for the application rate γ_s implies that workers match with the stock intermittently. Some workers will have no prospects and will wait for the flow to increase the stock of prospects, this is analogous to a worker waiting to match with the flow in a standard discrete time stock-flow matching model. When the worker matches with the stock of vacancies, they choose the most appropriate option. We assume that all rejected vacancies will no longer consider the worker which, we believe, captures a realistic feature of the labor market and is similar to the assumption that workers cannot return to their previous employer.[8] In Appendix A.1.1 we describe the flow equations for the distribution of job opportunities, $j \in \mathbb{N}_+$.

Each employment prospect arrives with a wage that is set optimally by profit maximizing firms. Firms can post wages conditional on worker type x. In this environment a firm's optimal strategy will be to post piece rate contracts in worker type, as in Barlevy (2008). Workers draw piece rate wage offers from a cumulative distribution function $F(w)$ - an endogenous object of the model.

Relation to existing models. Notice that a number of commonly used search models are nested in our framework. First, in the absence of dynamic market thickness, i.e., as $\gamma_s \to \infty$ $\forall s \in \{e, u\}$, the models converges to the standard job ladder model of Burdett and Mortensen (1998). As workers continuously apply, the number of prospects j, is either one or zero and hence there are no dynamics in the thickness of markets. Second, with no OJS, $\gamma_e = 0$, and with applications made continuously by the unemployed, $\gamma_u \to \infty$, the model nests a version of the stock-flow matching function in continuous time, see Coles and Smith (1998). In this case, after separating from a firm, the worker will immediately match with the stock of available prospects. If the stock is non-empty the worker will directly transition to a new job whereas if the stock is empty the worker has to wait for the inflow of new vacancies. Lastly, if $\gamma_e = 0$, and the unemployed

[8] Recall of previous jobs has recently been explored by Fujita and Moscarini (2013) and Carillo-Tudela and Smith (2016) in a Mortensen and Pissarides (1994) and sequential auctions model of the labor market, respectively.

infrequently apply for jobs, $\gamma_u \in (0, \infty)$, then the model shares the feature of stock-flow matching but with search frictions. This case corresponds to a wage setting environment similar to a dynamic version of the Burdett and Judd (1983) model. We estimate the baseline model as well as the model without dynamics of market thickness (NDT), and without OJS (NOJS).

3.2 Worker problem

An individual's utility is given by the present expected discounted value of their future income stream, this will depend on their employment status, if employed their wage, and the opportunity stock. The number of vacancies and the wage of a given offer will only become clear to a worker when they match with the stock. The value function for an unemployed worker of type x with j offers in hand is given by (1). $F \in [0, 1]$ is the rank of wages from the job offer sampling distribution.[9]

The value function for unemployed and employed worker is given by

$$U(x, h_t) = \sum_{j=0}^{\infty} p(j; h_t)\tilde{U}(x, j) \quad \text{and} \quad W(x, w, h_t) = \sum_{j=0}^{\infty} p(j; h_t)\tilde{W}(x, w, j),$$

where $p(j; h_t)$ denotes the probability that the latent variable is equal to j given the employment history h_t of the worker. We specify the model in this way to be agnostic on the exact information set of the agents. It is useful to illustrate the behavior of the model using the value functions conditional on the latent variable j. For the unemployed, the value $\tilde{U}(x, j)$, is defined by

$$\begin{aligned}\mu\tilde{U}(x, j) = bx + \gamma_u \int_0^1 (\tilde{W}(x, w(x, F), 0) - \tilde{U}(x, j))dF^j \\ + \lambda_u(\tilde{U}(x, j+1) - \tilde{U}(x, j)) + \upsilon j(\tilde{U}(x, j-1) - \tilde{U}(x, j)).\end{aligned} \tag{1}$$

The value function of an unemployed worker, discounted by the rate at which they leave the market, is the sum of the flow value of unemployment bx. The search option of the worker is given by the rate at which they access the market, γ_u, multiplied by the expected returns to matching with the stock. $\tilde{W}(\cdot)$ is the value of employment and defined in (2). Notice, when an unemployed worker

[9] Notice, this corresponds perfectly with a firm's rank in the productivity distribution. This is because, as will be seen, firms pay all workers they employ the same piece rate wage and this is a monotonically increasing function in a firm's productivity type.

takes up a job offer they begin their employment spell with no opportunities. This is because the worker has rejected all other offers and we assume there is no recall of offers previously turned down. While in unemployment, the number of employment opportunities a worker has follows a stochastic process with suitable jobs arriving at a rate λ_u and losing a given opportunity at a rate υ.

Given the underlying latent parameter, the value function associated with employment can be written as the sum of the flow wage, the option value of the j opportunities in the worker's stock, the option value of the stochastic process that governs the evolution of j and the option value of becoming unemployed which occurs with probability δ

$$\begin{aligned}\mu\tilde{W}\left(x, w(x,F), j\right) &= w(x,F) + \gamma_e \int_F^1 (\tilde{W}(x, w(x,\tilde{F}), 0) - \tilde{W}(x, w(x,F), 0))d\tilde{F}^j \\ &+ \gamma_e(\tilde{W}(x, w(x,F), 0) - \tilde{W}(x, w(x,F), j)) + \lambda_e(\tilde{W}(x, w(x,F), j+1) - \tilde{W}(x, w(x,F), j)) \\ &+ \upsilon j(\tilde{W}(x, w(x,F), j-1) - \tilde{W}(x, w(x,F), j)) + \delta(\tilde{U}(j) - \tilde{W}(x, w(x,F), j)). \end{aligned} \tag{2}$$

As in a standard sequential search model, a worker's decision is whether to accept or reject a given offer.[10] Once matching with the stock, a worker has potentially more than one offer to contend with. Since wage lasts forever and all jobs are otherwise homogeneous, a worker will always prefer the highest wage job available to them, be it in the stock of opportunities or the job they are currently employed in. An unemployed worker accepts a wage if it yields a higher present value than continuing in unemployment. Since firms post wages optimally, assuming at least a negligible cost in wage posting no firm would post a wage less than this value, therefore we are solving for the infimum of the wage support, $\phi(x) = w(x, 0)$. This is found by solving the equality

$$\tilde{U}(x, 0) = \tilde{W}(x, \phi(x), 0). \tag{3}$$

To solve for this wage level is slightly more difficult than usual due to the evolution of the additional state variable - the number of employment opportunities. Appendix A.2 explains how one can

[10] One could imagine a more sophisticated set of strategies depending on what the worker is aware of (e.g., the number of vacancies in the stock, their job tenure or the wages of individual job opportunities). In such an environment, employed workers would under some conditions optimally quit to unemployment. However, this is beyond the scope of this paper.

compute the value functions and derives an expression for $\phi(x)$.

3.3 Steady-state distribution of match quality

In order to solve for the distribution of wages and outstanding matches we proceed in two steps. First, we define the probability generating function Σ_s for each employment state s as

$$\Sigma_s(F) \quad = \quad \sum_{j=0}^{\infty} p_s(j) F^j.$$

Where $p_s(j)$ is a probability mass function that gives the probability a worker in state s has exactly j employment opportunities. The function $\Sigma_s(F)$ evaluates the probability that when a random worker, in state s matches with the stock, they have no vacancy above rank F. The function $\Sigma_s(F)$ has the steady-state solution

$$\Sigma_e(F) = \frac{1}{1-F} \int_F^1 \exp\left[-\lambda_e/\delta\left(\tilde{F}-F\right)\right] \left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_e+\mu}{\delta}} \frac{\gamma_e+\mu+\delta}{\delta} d\tilde{F},$$

$$\Sigma_u(0) = \frac{\int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u-\delta}{\delta}} \left[\frac{\gamma_u \Sigma_e(\tilde{F})}{\delta}\right] d\tilde{F}}{1-\int_0^1 \exp\left[-\lambda_u/\delta\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u-\delta}{\delta}} \left[\frac{\gamma_u}{\delta}\left(1-\Sigma_e(\tilde{F})\right)\right] d\tilde{F}},$$

$$\Sigma_u(F) = \frac{1}{1-F} \int_F^1 \exp\left[-\lambda_u/\delta(\tilde{F}-F)\right] \left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u-\delta}{\delta}} \left[\frac{\gamma_u \Sigma_u(0)}{\delta} + \frac{(\delta+\mu)(1-u)/u\Sigma_e(\tilde{F})}{\delta}\right] d\tilde{F}.$$

The derivation of this function is in Appendix A.1.2. The rate of inflow into unemployment from employment is given by $\delta + \mu$. Similarly, the rate of outflow from unemployment is given by $\gamma_u\,(1 - p_u(0))$. In steady-state the inflow is equal to the outflow, which using the definition of $\Sigma_u(F)$ gives an expression for the unemployment rate

$$u \quad = \quad \frac{(\delta+\mu)}{(\delta+\mu+\gamma_u(1-\Sigma_u(0)))}. \tag{4}$$

The total unemployment rate contains both the friction by which workers qualify for jobs and the frequency by which they apply. In the hypothetical case in which, $\lambda_u \to \infty$, all workers have some employment prospects, that is $p_u(0) = 0$, then the unemployment rate would be purely due to workers not sending out enough applications and given by

$$\tilde{u} \quad = \quad \frac{(\delta+\mu)}{(\delta+\mu+\gamma_u)}. \tag{5}$$

Comparing the *true* unemployment rate with the hypothetical reveals the relative importance of the two frictions for the unemployment rate. Using the function $\Sigma_s(F)$ we can further solve for the distribution of outstanding matches $G(F)$. Note, the inflow of matches below F is $\gamma_u \sum_{j=1}^{\infty} F^j p_u(j)$, i.e. the probability that an unemployed worker matches with an offer less than F. Similarly the outflow of matches below F is the exogenous separation $\delta + \mu$ plus the endogenous quit of $\gamma_e \left(1 - \sum_{j=0}^{\infty} F^j p_e(j)\right)$. In steady-state the inflow has to equal the out flow which gives

$$(1-u)G(F)\left(\delta + \mu + \gamma_e\left(1 - \sum_{j=0}^{\infty} F^j p_e(j)\right)\right) = u\gamma_u\left(\gamma_u \sum_{j=1}^{\infty} F^j p_u(j)\right). \tag{6}$$

Using the definition of Σ_s we get

$$G(F) = \frac{u\gamma_u\left(\Sigma_u(F) - \Sigma_u(0)\right)}{(1-u)\left(\delta + \mu + \gamma_e\left(1 - \Sigma_e(F)\right)\right)}. \tag{7}$$

The associated density function and its derivative are given in Appendix A.1.3.

3.4 Firm problem

The firm commits to a wage schedule in worker productivity at the time of vacancy creation (section 3.6 discussed the assumptions on wage setting). The firm then sets the wage to optimally trade off the increased retention and hiring with the increased cost associated with a higher wage. The expected profits per vacancy for a firm with match quality rank F posting a wage w is made up by three terms: the probability that a worker is hired; the expected duration; and the markup. Combining these gives the expression for the expected profits at the time of vacancy creation

$$\Pi^e(x, w, F) = \Pr(\text{hire}|x, w)\text{E}(\text{duration}|x, w)(y(F) - w). \tag{8}$$

Hiring. Search is random, a firm posting a vacancy can either meet an employed or unemployed worker. For the worker to accept the offer the wage has to be higher than any other offer the worker holds and, if employed, their current wage. Absent of market thickness dynamics, the Burdett and Mortensen (1998) model, workers match instantaneously which means that the offer is always the best amongst the new job offers. The wage is acceptable if it is above the current wage or the reservation wage for the unemployed. In contrast, without OJS all agents are unmatched but

potentially receive more than one offer as in Burdett and Judd (1983). Either of these mechanisms will generate equilibrium wage dispersion. Our model combines both aspects as there is both search on the job and workers match with the stock. Define m as the probability that a vacancy meets a worker. The probability that a worker is hired can then be calculated using

$$\begin{aligned}\Pr(\text{hire}|w \geq \phi) &= m \Pr(\text{meet an unemployed worker}) \Pr(\text{unemployed worker accepts}|w) \\ &\quad + m \Pr(\text{meet an employed worker}) \Pr(\text{employed worker accepts}|w).\end{aligned}$$

The probability that a vacancy meets an unemployed worker, conditional on a meeting, is the flow rate of meetings with unemployed workers divided by the total flow rate of all meetings. The flow rate of meetings for unemployed workers comprises the product of three terms: the rate at which the worker engages in active search; the stock of unemployed; and the expected number of opportunities. The expected number of job opportunities is given by $\sum_{j=1}^{\infty} jp_s(j)$.

The probability that the worker accepts the offer, conditional on meeting with the vacancy, can be broken up into two parts. The probability that the offer is better than his current offer (1 for the unemployed and $G(w)$ for the employed) times the probability that the offer is the highest among all the offers the worker has received. The probability that the offer F is the highest offer among all offers for the worker in state s is the probability that the vacancy meets a worker with j offers $\frac{jp_s(j)}{\sum_{j=1}^{\infty} jp_s(j)}$, multiplied by the probability that the offer is higher than the $j-1$ alternative offers (F^{j-1}). This gives

$$\frac{\sum_{j=1}^{\infty} jp_s(j) F^{j-1}}{\sum_{j=1}^{\infty} jp_s(j)}.$$

Combining the expressions above and using the definition of Σ_s we get

$$\begin{aligned}\Pr(\text{hire}|w \geq \phi) &= m \frac{\gamma_u u \sum_{j=1}^{\infty} jp_u(j)}{\gamma_u u \sum_{j=1}^{\infty} jp_u(j) + \gamma_e (1-u) \sum_{j=1}^{\infty} jp_e(j)} \frac{\sum_{j=1}^{\infty} jpu(j) F^{j-1}}{\sum_{j=1}^{\infty} jp_u(j)} \\ &\quad + m \frac{\gamma_e (1-u) \sum_{j=1}^{\infty} jp_e(j)}{\gamma_u u \sum_{j=1}^{\infty} jp_u(j) + \gamma_e (1-u) \sum_{j=1}^{\infty} jp_e(j)} (1-u) G(F) \frac{\sum_{j=1}^{\infty} jp_e(j) F^{j-1}}{\sum_{j=1}^{\infty} jp_e(j)} \\ &= m \frac{(\gamma_u u \Sigma_u'(F) + \gamma_e (1-u) G(F) \Sigma_e'(F))}{(\gamma_u u \Sigma_u'(1) + \gamma_e (1-u) \Sigma_e'(1))}.\end{aligned}$$

Duration of a job. Unlike Burdett and Mortensen (1998), the duration of a job is not exponentially distributed. Instead, the quit rate is a time varying object. At the beginning of a

job, the worker has matched with the stock and is therefore unlikely to leave right away, as time progresses the expected number of offers and hence their quit rate increases. It turns out that even though the leaving rate is not constant, the average leaving rate is a sufficient statistic for the expected duration at the time of hiring. Using Little's law from queuing theory we can calculate the expected duration at the time of hiring. The average rate at which a worker working in a firm of productivity rank F leaves the job is given by $\delta + \mu + \gamma_e(1 - \Sigma_e(F))$. The average duration in a job F is therefore just $1/(\delta+\mu+\gamma_e(1-\Sigma_e(F)))$.

3.5 Equilibrium

An equilibrium in this economy is characterized by the function $\{\tilde{W}(x, w, j), \tilde{U}(x, j), w(F), \Pi(x, w, F)\}$ such that the firm profits are given by (8) and the present value for workers by (27) in the Appendix. The wage function $w(F)$ solves the firm problem, such that (8) is maximized, and the worker is indifferent between the lowest wage and unemployment, both absent of opportunities $\tilde{W}(x, \phi(x), 0) = \tilde{U}(x, 0)$ with $\phi(x) = xw(0)$.

3.6 Discussion of assumptions

Within this subsection we discuss two modeling assumptions that we believe warrant further discussion, they are the wage setting mechanism and the recall of previously turned down job offers.

Wage setting. We opt for an environment in which wages are posted ex ante of a worker and firm meeting as it commonly used in the literature and thus proves convenient in comparing our model to the existing literature. However, most results in the paper are analogous if the nature of wage setting was changed as they operate through the worker value functions and not the specificities of wage setting. If wages are negotiated after the match is formed, as in Shimer (2006), and firms can only commit to a wage for some period of time as in Gottfries (2017), the main results of the paper, in terms of the differences in replacement rates and income risk, are unchanged. The incentive for a firm to set a higher wage is then less as hiring does not increase with the wage and, similarly, with shorter contracts, the incentive to pay a higher wage to retain the worker is also lower. However, since in the estimation to come, we match the same wage distributions, there is

no change to the worker's value function (except for the replacement ratio which might be lower as the worker is not necessarily indifferent between unemployment). However, since the wage setting motive by the firm changes, the estimated firm productivity parameters will change.

In contrast, there are some additional mechanisms at play when firms are able to make counteroffers as in Postel-Vinay and Robin (2002) and Cahuc et al. (2006). With this wage setting, the wage gets raised as counteroffers arrive which means the worker initially accept lower wages via this foot-in-the-door effect. In our model, this gets exacerbated as the expected number of counteroffers received at the same time is higher if the worker takes a job. With the posting of wages, the exact timing of the offers is not important whereas wit counteroffers it is. In particular, the matching set will increase. For example, without dynamic thickness and when firms have all the bargaining power as in Postel-Vinay and Robin (2002), the lowest productive firm that is able to hire workers has a productivity equal to the unemployment benefits. However, in our model, firms with lower productivity would also be able to hire workers as they increase the chance that the worker subsequently has multiple offers to contemplate. However, this mechanism is very sensitive to the assumption that the worker has access to the offers at exactly the same time.

No recall of rejected offers. In the model, after an offer has been turned down, be it for an alternative job prospect or staying with a current employer, a worker cannot return to said offer. This is analogous to the standard assumption in the job ladder models that previous jobs cannot be recalled. An alternative modeling choice would be to assume that workers can hold on to rejected offers and retain them. This would not complicate the exposition much, instead an unemployed worker's reservation wage would be a function of the number of prospects they have, j. However, quantitatively, the modeling assumption should be fairly innocuous as vacancies and therefore prospects have a fairly short shelf life. We calibrate the rate of expiry υ later based on the duration of a vacancy. We find that prospects last for approximately one month. This is an order of magnitude larger than the rate employed workers, for example, switch jobs and would likely therefore have little effect on our results.

3.7 Identification

Appendix A.3 provides a proof that the transition parameters of our model are identified. Identification of worker and firm productivities are by standard arguments non-parametrically identified. Although, the exact moments discussed in the Appendix A.3 are not practically implemented in estimation we use similar moments for the purpose of estimation. The aim of our estimation is to minimize a criterion defined as a distance between simulated and empirically observable moments. While our proof does not guarantee that the estimated parameters to be presented are a global minimum of the criterion it does imply that the parameters are identifiable. However, since our model is inexpensive computationally we make a global search over the parameters space to make sure the estimates correspond to a global minimum.

4 Estimation

Our estimation will focus on estimating the model presented in the previous sections. In addition, we estimate the special cases of no dynamics of market thickness (NDT) and without OJS (NOJS).

4.1 Data

The data used in this estimation are taken from the Current Population Survey (CPS) and the Survey of Income and Program Participation (SIPP). Moments relating to labor mobility are taken from the CPS due to it having a larger cross-sectional component and wage moments are taken from the SIPP as we rely on panel data for dynamic wage moments. We stratify the sample according to skill level into three different stratum, consistent across data source. They are: the college educated; those whose highest academic achievement is a high school diploma; and those who have not completed high school education. We restrict attention to male workers aged between 25 and 45 and since both models rely on steady-state assumptions and to mitigate issues associated with early retirement decisions. We trim the bottom and top percentile of the wage distribution. We also restrict attention to the relatively short and stable period between the years 1996 and 1999, inclusive. As will be seen, key parameters will be identified from labor mobility by duration and

we do not want cohort effects to play any role. In Appendix A.6 we plot the separation and job finding rates by age and highlight our estimation window in shaded light blue. Separation rates exhibit a clear downward trend and the pattern of the finding rate is less clear. Since our model assumes a constant separation rate we choose a window where this seems a fair approximation of the data.

Identification will rely on employment dynamics and the cross-sectional wage distribution. Table 6 reports moments on hourly earnings and hours worked per week for each stratum. These are computed by dividing the self-reported weekly earnings by self-reported hours worker per week. Since in estimation, wage data is taken from the SIPP and employment dynamics from the CPS we want to show that data in both look quantitatively similar. The two datasets are broadly consistent. The SIPP implies greater number of hours worked for more pay. There are large systematic differences in hourly earnings across skill. These differences are the motivation for stratification. Comparing hourly earnings across strata seems sensible as there is little cross strata variation in hours, with all subgroups working, on average, between 40 and 45 hours per week. Finally, it is worth bearing in mind that the medium-skilled, those with a high school diploma but without a College degree, account for about half of the labor force.

Table 2 presents employment dynamics, estimated from a three state Markov process. The rows represent a worker's employment status at period t and the columns in $t+1$, all changes are conditional on a change in employer. All of the moments presented in Table 2 will be exactly identified in the estimation to come. Inspection of these matrices reveals the large flow from inactivity to employment and differences across strata. Particularly pronounced differences relate to the duration of jobs, with a worker less likely to switch from employment to inactivity, unemployment or to another employer if they are of a higher skill.

4.2 Parameterization

The set of parameters to be estimated is given by the vector θ

$$\theta = \left(\mu, \delta, \upsilon, \lambda_u, \lambda_e, \gamma_u, \gamma_e, b, \Gamma_x\left(x\right), \Gamma_y\left(y\right)\right)'. \tag{9}$$

Table 1: Descriptive Statistics

	All	Low-skill	Medium-skill	High-skill
proportions:				
CPS	100%	12%	52%	37%
SIPP	100%	10%	51%	39%
mean earnings ($/hour)				
CPS	15.8	9.8	13.9	20.2
SIPP	16.5	10.2	13.9	21.4
mean weekly hours worked:				
CPS	43.1	41.4	42.5	44.2
SIPP	43.7	41.7	43.1	45.0

Note: Data comes from the CPS and SIPP, moments are based on male workers aged between 25 and 45 between 1996 and 1999, inclusive.

Notice, (9) contains the entire distributions of $\Gamma_x(y)$ and $\Gamma_y(y)$. We make further parametric assumptions on the primitive initial distribution of worker and firm types. We assume that they follow transformed log-normal and Beta distribution, respectively. The specific distributions were chosen as we found they performed better than alternative parameterizations in fitting the data. With a slight abuse of notation, we define a worker's rank in the distribution as F_x and recall a firm's rank is F. Φ denotes a standard normal distribution and B the Beta distribution. We include location and shape parameters μ_x and σ_x. A firm draws productivity from a generalized log-beta distribution. α_y and β_y are the underlying distributional parameters and for additional flexibility we include a shape parameter σ_y.

$$x(F_x) = \exp(\mu_x) + \exp\left[\sigma_x \Phi^{-1}(F_x)\right]$$

$$y(F) = \exp\left[\sigma_y \mathrm{B}^{-1}(F; \alpha_y, \beta_y)\right]$$

The rate at which workers lose employment opportunities, υ, is the only parameter not directly estimated and calibrated to match the mean duration of a vacancy. Vacancy duration is estimated using the "The Conference Board Help Wanted Online Data Series" (HWOL). Details of exactly

Table 2: Transition Matrices

All

	Inact.	Unemp.	Emp.
Inact.	—	—	0.098
Unemp.	—	—	0.312
Emp.	0.010	0.012	0.024

Low-skill

	Inact.	Unemp.	Emp.
Inact.	—	—	0.078
Unemp.	—	—	0.297
Emp.	0.019	0.026	0.032

Medium-skill

	Inact.	Unemp.	Emp.
Inact.	—	—	0.096
Unemp.	—	—	0.316
Emp.	0.010	0.014	0.025

High-skill

	Inact.	Unemp.	Emp.
Inact.	—	—	0.131
Unemp.	—	—	0.319
Emp.	0.006	0.006	0.022

Note: Transition rates are monthly. Rows do not add up to one. The emp-emp entries represent the fractions of individuals *changing jobs* while remaining employed. **Source:** Data are taken from the CPS and relate to 25-45 year old males between 1996 and 1999, inclusive.

how this parameter is calibrated are provided in Appendix A.5. It should be noted that this data does not cover our estimation window nor can we look at the vacancy duration by skill requirement of the job opening. The implied value of the mean duration ($1/\upsilon$) is approximately one month.

After these assumptions equation (9) can be reduced to the following vector of scalars. The focus of the rest of this section is the estimation of the vector θ

$$\theta = (\mu, \delta, \lambda_u, \lambda_e, \gamma_u, \gamma_e, b, \mu_x, \sigma_x, \mu_y, \sigma_y, \alpha_y, \beta_y)' .$$

4.3 Estimation Protocol

The model is estimated by indirect inference in two steps. In a first step employment transitions are matched, based on CPS data. The second step matches auxiliary wage moments computed from the SIPP to uncover the underlying primitive productivity distributions of workers and firms and the value of home production. To estimate the models of no OJS search and no dynamic market thickness we use an identical first step. In order to match the same degree of frictional wage dispersion we compute the distribution predicted by our baseline model and target this directly.

For the two alternative specifications we therefore do not estimate the distribution of worker types and use an even more flexible Beta distribution to guarantee a satisfactory fit.[11]

Step one. The first step matches aggregate job to job and employment to unemployment transition rates. The rate at which worker's leave the labor market and finally the job finding rate of the unemployed is computed by duration, matching the monthly probability at a weekly frequency for 52 weeks. We thus match 55 moments, which we weight by the precision to which they are estimated in the data. This step is matched varying $\theta^t = (\mu, \delta, \lambda_u, \lambda_e, \gamma_u, \gamma_e)$ and can be done independently of all other parameters. Formally, θ^t is the solution to the following, where $m^t(\theta_t)$ and m^t are the 55 targeted moments, from the model and data, respectively and $\hat{V}$ is the diagonal of the variance-covariance matrix of m^t. Note, for transition calculations based on fewer than 20 observations we replace the diagonal with zeros.

$$\hat{\theta}^t := \arg\max_{\theta^t \in \Theta^t} \left(m^t(\theta^t) - m^t\right)' \hat{V}^{-1} \left(m^t(\theta^t) - m^t\right).$$

Step two. This step estimates the value of home production and worker and firm productivity parameters, $\theta^p := (b, \mu_x, \sigma_x, \alpha_y, \beta_y, \sigma_y)$. We simulate data generated from our model as in the SIPP. That is, we simulate a monthly panel with the same number of individuals, over the same time frame, with the same rate of attrition. Since we only rely on the seam of the SIPP, where wages are not based on recall, we treat the simulated data in the same way. In order to distinguish between the relative contribution of the worker and firm productivities we match each (1st to 99th) percentile of the mean wage of a worker over our time horizon. Further, we match the same percentiles of the overall wage distribution including the infimum of the support. Given all other elements in θ the value of home production pins down the lowest wage. This leaves a total of 199 empirical moments to fit, which we denote by the vector m^p. To review, m^p consists of the 100 quantiles of the wage support $\{w_q\}_{q=0,\ldots 99}$ and the 99 quantiles of the mean worker wage $\{\overline{w}_q\}_{q=1,\ldots 99}$.

Since, unlike step one, we do not have analytical expressions for our moments, but instead rely on Monte Carlo simulations. We simulate the model M times and take the mean of each model

[11] We include additional location parameter such that $y(\Gamma_y) = \xi_y + \zeta_y \exp\left[\sigma_y \mathrm{B}^{-1}(\Gamma_y; \alpha_y, \beta_y)\right]$. Notice, for $\xi_y = 0$ and $\zeta_y = 1$ it becomes identical to the distribution of firm types in the baseline.

predicted moment condition, given by $M^{-1}\sum_{i=1}^{M} m_i^p(\theta^p)$. Further, since the empirical distribution of wages contains many mass points and there is simulation error, a bootstrapped weighting matrix does not seem appropriate. Instead we implement a two-step GMM estimation with the intention of the first step to estimate the asymptotically efficient weighting matrix, $W(\theta)^{-1}$. In the first step take an initial guess at $W(\theta)^{-1}$ as the identity matrix and estimate the model and set $M = 1$. We then simulate $m_i^p(\theta^p)$ numerous times and compute a variance-covariance matrix as our estimate of $W(\theta)$. The second step estimates θ^p as the solution to

$$\hat{\theta}^p := \arg\max_{\theta^p \in \Theta^p} \left(\frac{\sum_{i=1}^{M} m_i^p(\theta^p;\theta^t)}{M} - m^p \right)' \hat{W}(\theta)^{-1} \left(\frac{\sum_{i=1}^{M} m_i^p(\theta^p;\theta^t)}{M} - m^p \right).$$

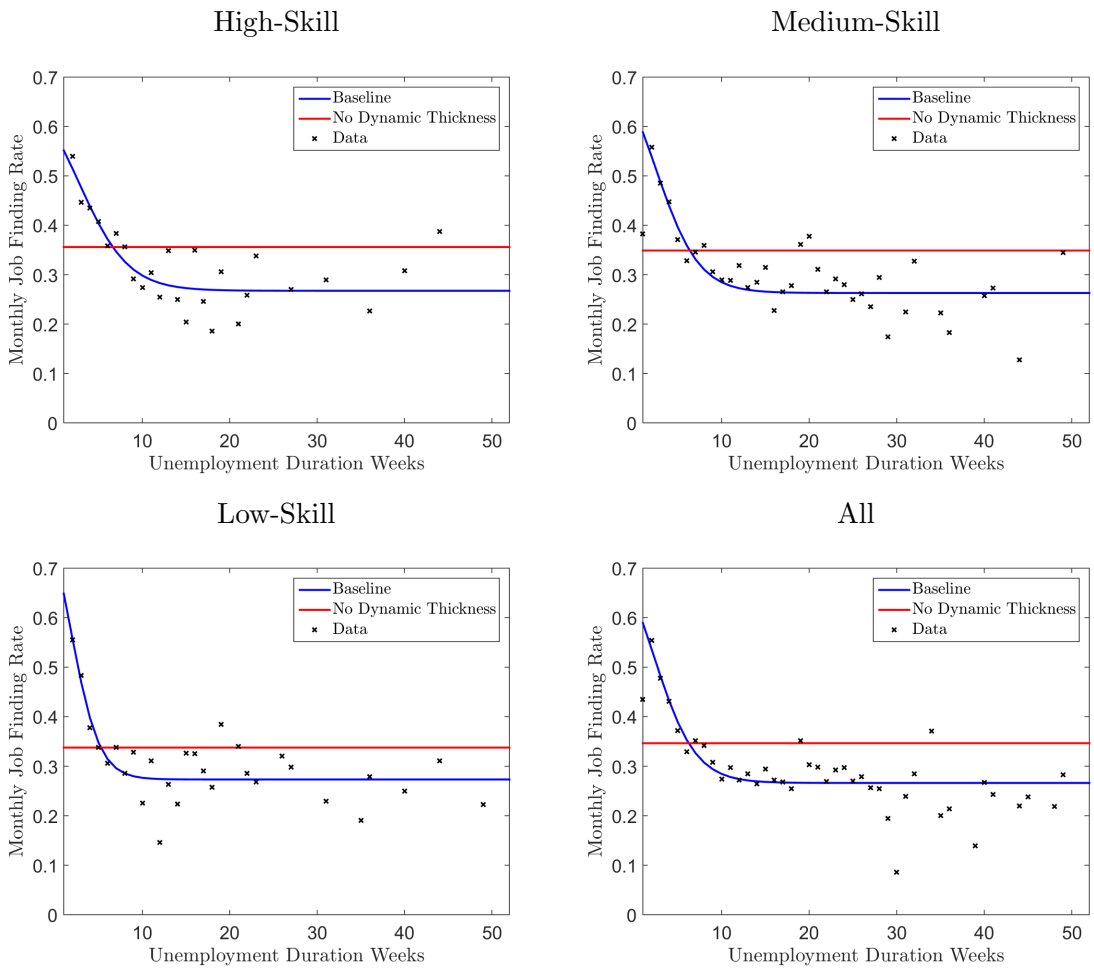
Practicalities. In the second step of estimation we re-simulate our model twenty times, $M = 20$. In order to isolate differences across specification we first estimate our model. Then in the two special cases fix the distribution of worker productivity to be identical to our model. When estimating the two special cases (no market thickness and no OJS), in the second step to ensure a tight fit there are a two parameters further parameters to be estimated (ξ_y, ζ_y). In all three specifications the first step is identical, but for the two nested cases the second step matches the distribution of wages simulated from our baseline model. That is, target 99 percentiles of the $G(w)$ distribution predicted by the baseline model as above, using the identity matrix as a weighting matrix. Finally, the data are re-sampled and the estimation repeated on each re-sample. The bootstrap procedure is implemented in order to make inference on our parameters and results. For computational expediency bootstrapped standard errors of the second step are performed assuming perfect precision in step one. This saves us from resimulating large datasets in every re-estimation, which would become quite cumbersome. Since the two alternative specifications are estimated using frictional wages generated by our baseline model it did not seem informative to compute standard errors for the productivity parameters.

4.4 Fit

Step one. Figure 1 shows the probability an unemployed agent moves to employment, by the

duration of their unemployment spell. The horizontal red line represents that predicted with dynamic thickness (NDT). The declining blue line is our baseline model and the black crosses are the targeted estimates from the data. We omit the special case of no OJS as it is indistinguishable from the baseline. All models match the aggregate transition rates almost exactly.

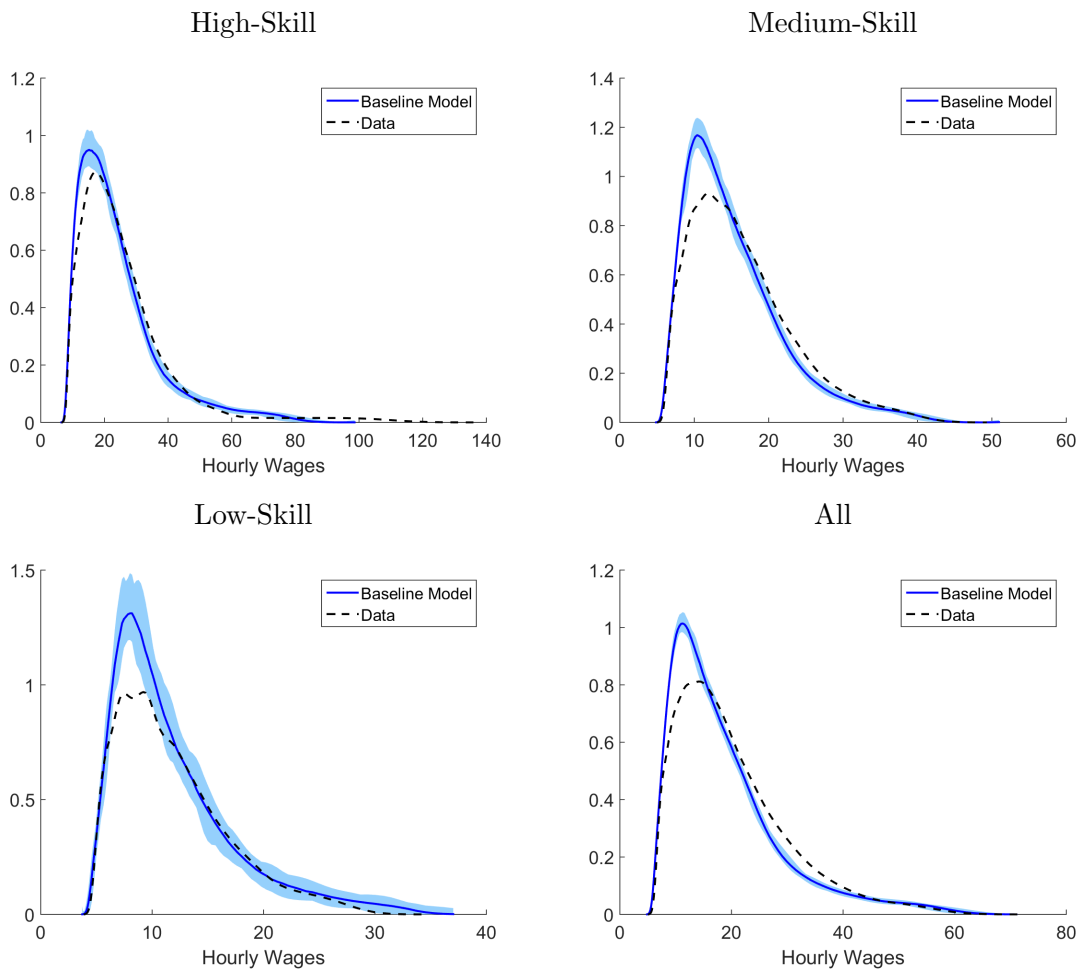
Figure 1: Job finding rate by duration of unemployment



Note: Empirical job finding rate, black 'x' are computed for each week, conditional on there being at least 20 individuals with an appropriate length observed unemployment duration.

Step two. Figure 2 shows the fit of the overall wage distribution for the baseline model. Figure 3 displays the fit of the distribution of mean worker wages. Both distributions are skewed to the right for all worker types. Appendix A.7 presents the fit of the frictional wage dispersion for the two alternative specifications of the model (without market thickness and OJS, respectively).

Figure 2: Fit of the Wage Distributions

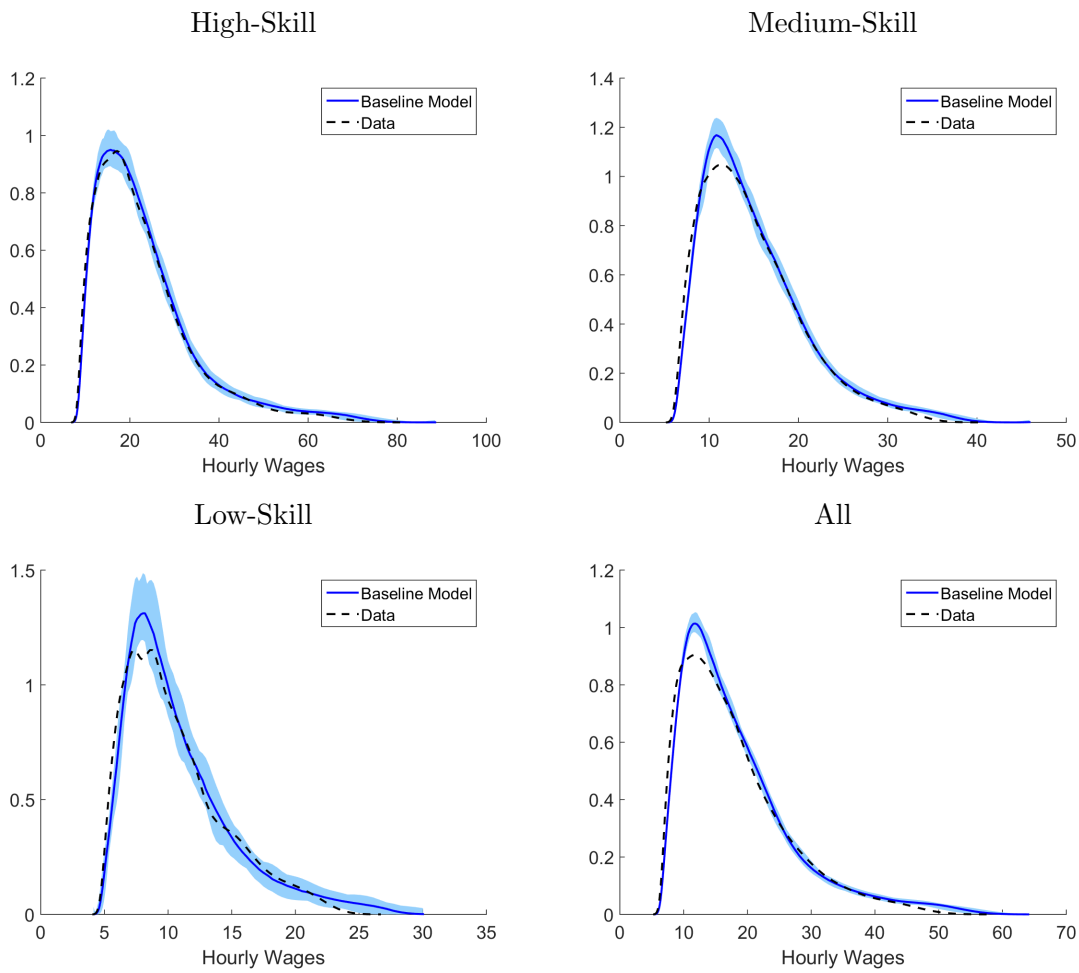


Note: Distributions are kernel density plots of the simulated and empirical data. The shaded blue areas represent 99% confidence intervals based on repeated resimulation of the model.

4.5 Results

Running the multi-step estimation procedure as described yields the parameter estimates presented in Table 3. Bootstrapped standard errors are given in the parentheses and all parameters are statistically significant to any conventional significance level. The rest of this section will discuss each cell of the table in turn and then some implications. Finally, we discuss the importance of the model's mechanisms for the measured search option associated with taking a job.

Figure 3: Fit of the Distribution of Mean Wages



Note: Distributions are kernel density plots of the simulated and empirical data. The shaded blue areas represent 99% confidence intervals based on repeated resimulation of the model.

Transitional Parameters. Immediately apparent from inspection of the transitional parameters, the upper cell of the table, is that each model has a very different interpretation of the functioning of the labor market. Across all skill groups, in the model without market thickness (NDT) $\lambda_u > \lambda_e$ meaning workers are exposed to a greater number of job offers in unemployment than in employment; see Table 2. The higher rate at which the unemployed find new jobs must be rationalized by a lower contact rate in NDT. In our model employment prospects arrive at a similar rate in both states. Instead, the higher finding rate among the unemployed is due to more frequent active search, $\gamma_u > \gamma_e$. This large disparity between γ_u and γ_e is what allows the model to replicate

Table 3: Parameter Estimates

	All			High-skill			Medium-skill			Low-skill		
	NDT	NOJS	Baseline	NDT	NOJS	Baseline	NDT	NOJS	Baseline	NDT	NOJS	Baseline
μ	0.01 (1.346e−4)	0.01 (1.302e−4)	0.01 (1.305e−4)	0.006 (1.663e−4)	0.006 (1.573e−4)	0.006 (1.687e−4)	0.011 (1.91e−4)	0.011 (1.757e−4)	0.011 (1.968e−4)	0.02 (6.274e−4)	0.02 (6.112e−4)	0.02 (5.925e−4)
δ	0.015 (1.904e−4)	0.02 (6.689e−4)	0.02 (7.026e−4)	0.007 (2.031e−4)	0.009 (5.367e−4)	0.009 (7.963e−4)	0.017 (2.659e−4)	0.022 (9.699e−4)	0.022 (9.45e−4)	0.033 (8.952e−4)	0.055 (0.007)	0.055 (0.007)
λ_u	0.434 (0.005)	0.54 (0.02)	0.537 (0.02)	0.445 (0.012)	0.613 (0.056)	0.607 (0.054)	0.439 (0.007)	0.544 (0.026)	0.541 (0.025)	0.431 (0.011)	0.474 (0.028)	0.473 (0.028)
λ_e	0.114 (0.002)	2.119 (0.128)	2.397 (0.162)	0.16 (0.007)	2.43 (0.586)	2.832 (0.675)	0.11 (0.002)	2.273 (0.23)	2.557 (0.237)	0.114 (0.004)	1.968 (0.164)	2.253 (0.181)
γ_u	—	1.445 (0.14)	1.463 (0.155)	—	1.116 (0.243)	1.137 (0.328)	—	1.38 (0.181)	1.398 (0.179)	—	2.39 (0.527)	2.408 (0.55)
γ_e	—	—	0.095 (0.002)	—	—	0.122 (0.008)	—	—	0.091 (0.003)	—	—	0.104 (0.005)
$b/\mathbb{E}[w]$	−78.1% (−−−)	−341.7% (−−−)	19.9% (0.762)	−43% (−−−)	−677.1% (−−−)	44% (0.898)	−197.8% (−−−)	−230.7% (−−−)	27.1% (1.64)	−321.9% (−−−)	−50% (−−−)	41% (1.891)
μ_x	—	—	1.768 (0.01)	—	—	2.056 (0.015)	—	—	1.727 (0.021)	—	—	1.442 (0.024)
σ_x	—	—	1.253 (0.022)	—	—	1.405 (0.042)	—	—	1.079 (0.03)	—	—	1.014 (0.047)
α_y	1.402 (−−−)	0.309 (−−−)	0.679 (0.018)	0.638 (−−−)	0.547 (−−−)	0.371 (0.023)	1.737 (−−−)	1.289 (−−−)	0.784 (0.048)	1.558 (−−−)	0.925 (−−−)	0.811 (0.048)
β_y	4.362 (−−−)	0.288 (−−−)	3.993 (0.083)	3.685 (−−−)	0.477 (−−−)	1.866e4 (6.408e2)	5.148 (−−−)	0.56 (−−−)	3.918 (0.201)	3.853 (−−−)	0.315 (−−−)	1.35 (0.077)
σ_y	5.713 (−−−)	2.258 (−−−)	2.447 (0.037)	4.749 (−−−)	2.563 (−−−)	9.443e3 (2.719e2)	5.769 (−−−)	2.625 (−−−)	2.084 (0.057)	5.076 (−−−)	2.302 (−−−)	1.119 (0.035)
ξ_y	0.879 (−−−)	1.452 (−−−)	—	0.742 (−−−)	1.255 (−−−)	—	0.897 (−−−)	0.76 (−−−)	—	0.977 (−−−)	0.76 (−−−)	—
ζ_y	0.16 (−−−)	0.412 (−−−)	—	0.254 (−−−)	0.351 (−−−)	—	0.151 (−−−)	0.268 (−−−)	—	0.171 (−−−)	0.256 (−−−)	—

the declining hazard rate with unemployment duration. This also implies that the employed and the newly unemployed have, on average, better employment prospects than the long-term unemployed and thus have a higher job finding rate. Finally, across all skill groups the job destruction rate δ is higher in the model with stochastic market thickness. In that version, the newly unemployed find jobs more quickly than they would do in the NDT framework and consequently more workers lose their jobs and find employment with a month.

Unemployment decomposition. In Burdett and Mortensen (1998), there is only one source of unemployment due to the infrequent arrival of job offers. However, in our model there are two sources of labor market frictions. Not only must a worker apply for jobs, they must also have positive employment prospects. The total unemployment rate and hypothetical unemployment rate, assuming $\lambda_u \to \infty$ so that all workers have labor market opportunities are

$$u = \frac{\delta + \mu}{\delta + \mu + \gamma_u \left(1 - \Sigma_0 \left(0\right)\right)} \quad \text{and} \quad \tilde{u} = \frac{\delta + \mu}{\delta + \mu + \gamma_u}. \tag{10}$$

The exit rate from unemployment occurs after a γ_u shock and on top of this, the worker must also have at least one potential job, which occurs with probability $(1 - \Sigma_0(0))$. The frictional rate is governed by only the primitive γ_u which prevents the worker seeing their current opportunities. The relative importance of the two fictions, by skill group, is reported in Table 4. The quantitative importance of a lack of opportunity is apparent, with this mechanism being responsible for approximately half of the unemployment rate.

Table 4: Unemployment Decomposition

Unemployment Rate	All	High-skill	Medium-skill	Low-skill
Hypothetical: $\tilde{u}$	2%	1.3%	2.3%	3.0%
Total: u	4.5%	2.6%	4.9%	8.3%

Note: This table computes and compares u and $\tilde{u}$ as defined in equation (10).

To further understand the relative importance of the frictions on unemployment we compute the elasticity of unemployment with respect to the four frictional parameters.[12] Consistent with Table 4 the parameters governing the frequency opportunities arrive, the λ's, have a greater (absolute) combined elasticity than the frequency in which workers apply to jobs, the γ's. The single most consequential parameter in determining unemployment is γ_u. The effectiveness of reducing unemployment through an increase in applications increases as one moves up the skill distribution. However, encouraging the employed to apply more frequently will have the opposite effect, as by applying for jobs in employment a worker exhausts their opportunities that they could rely on after a layoff in the future. Finally, it is interesting to note that the arrival rate of offers to the employed have a similar impact on the overall unemployment rate as offers to the unemployed.

Figures 4 and 5 show the impact of the two most consequential parameters γ_u and λ_u on the unemployment rate and the long term unemployment rate. The latter is defined as the proportion of people in the economy unemployed for more than three months. These figures are produced by simulating the model on a grid, varying the parameter values of γ_u and λ_u. For the unemployment

[12] For λ_u for example, this is computed as $\partial \log(u) / \partial \log(\lambda_u)$. The derivative is approximated using two sided differences.

Table 5: Elasticity of Unemployment

Parameter	All	High-skill	Medium-skill	Low-skill
γ_u	-0.874	-0.964	-0.898	-0.691
γ_e	0.069	0.074	0.066	0.089
λ_u	-0.608	-0.564	-0.589	-0.644
λ_e	-0.519	-0.428	-0.505	-0.647

Note: This presents elasticities of unemployment with respect to transition parameters of the model, in the case of λ_u the elasticity is $\partial \log(u)/\partial \log(\lambda_u)$. In practice, this is approximated by two sided differences, where the elasticity of λ_u, for example, is given by $\frac{\lambda_u(\log(u(\theta^{\sim}))-\log(u(\theta_{\sim})))}{2\lambda_u \epsilon}$. Where $\theta^{\sim}$ is identical to $\hat{\theta}$ but $\lambda_u^{\sim} := \lambda_u(1+\epsilon)$, where ϵ is arbitrarily small and for $\theta_{\sim}$ $\lambda_{u\sim} := \lambda_u(1-\epsilon)$.

rate as a whole both parameters appear important, with contour lines slightly shallower than the 45-degree line implying γ_u is moderately more important in determining unemployment. Turning to the long term unemployed, Figure 5, the picture changes. As discussed, the long-term unemployed have fewer opportunities on average than their short-term counterparts. Consequently, it is the lack of opportunities which arrive at rate λ_u that prevents them finding work. We see that the contours are much more closely together for the low skilled reflecting the fact that unemployment, being higher, is more responsive to a change in the parameters. Furthermore, the contour lines are steeper for the long-term unemployed, reflecting the higher relative importance of λ_u. The lesser importance of λ_u for the short term unemployed comes as many already have a number of employment prospects.

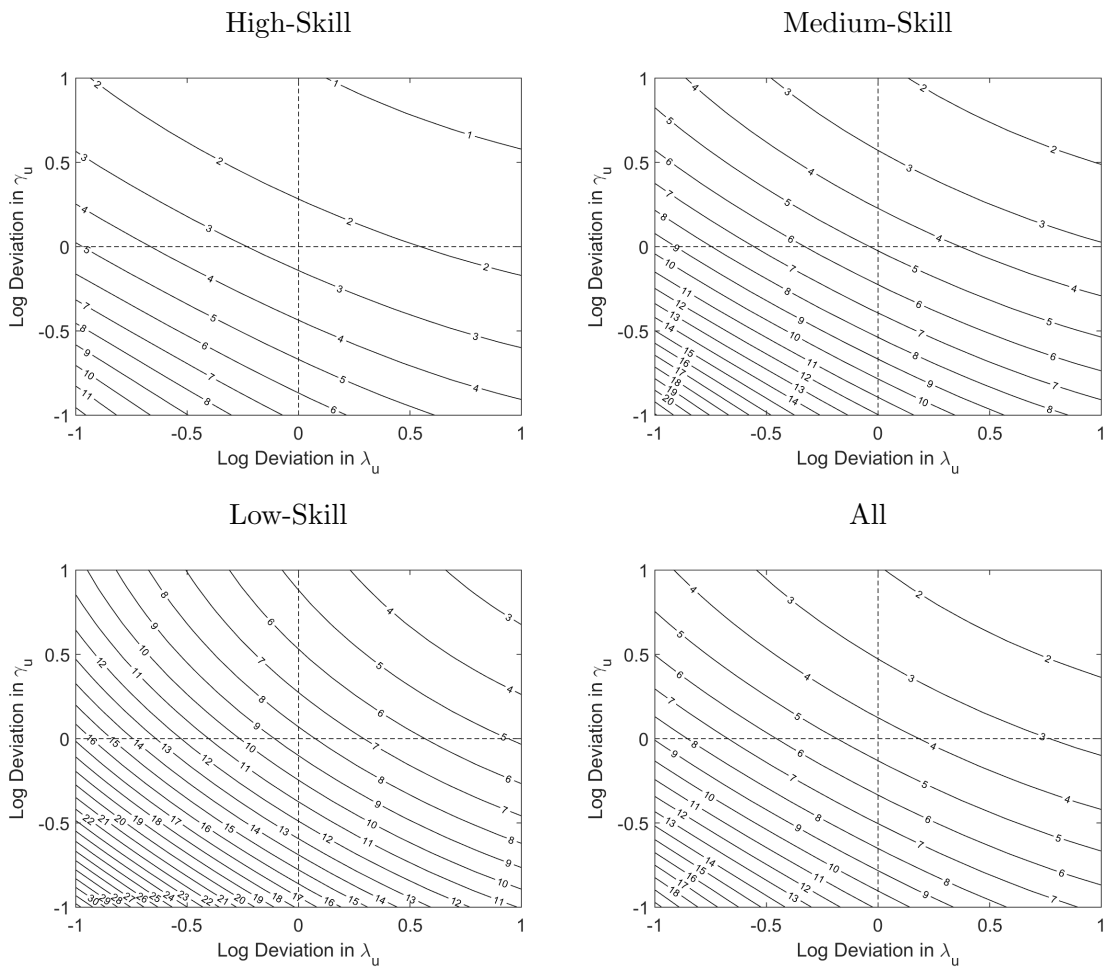
Productivity Parameters. The final cell of Table 3 presents the parameters from the underlying distributions of worker and firm productivity. On their own, they are not easy to interpret so, instead, we discuss the degree of wage competition (i.e., the amount of competition in the different models). (The worker value functions are presented in Appendix A.8.)

The percentage increase in firm type, $\ell'(F)/\ell(F)$, is just $\ell'(F)/\ell(F) = G''(F)/G'(F) = h(F) + r(F)$ we can rewrite the first order condition as

$$\Pi_w(w, F) \quad = \quad \frac{\ell'(F)}{\ell(F)}(y(F) - w(F)) - w'(F) = 0. \tag{11}$$

We refer to $G''(F)/G'(F)$ as the degree of competition. If $G''(F)/G'(F)$ is low then the firm size is

Figure 4: Impact of γ_u and λ_u on Unemployment Rate



Note: For a given rate of unemployment, solid lines connect the values of γ_u and λ_u where that rate occurs. These contour plots are computed by resimulating model, varying the size of the parameters γ_u and λ_u over a fine grid. The numbers on the plot represent the unemployment rate and the dotted line the values of our estimated parameters.

unresponsive to the wage and there is little reason to increase pay. The degree of competition, using our formula, can be written as

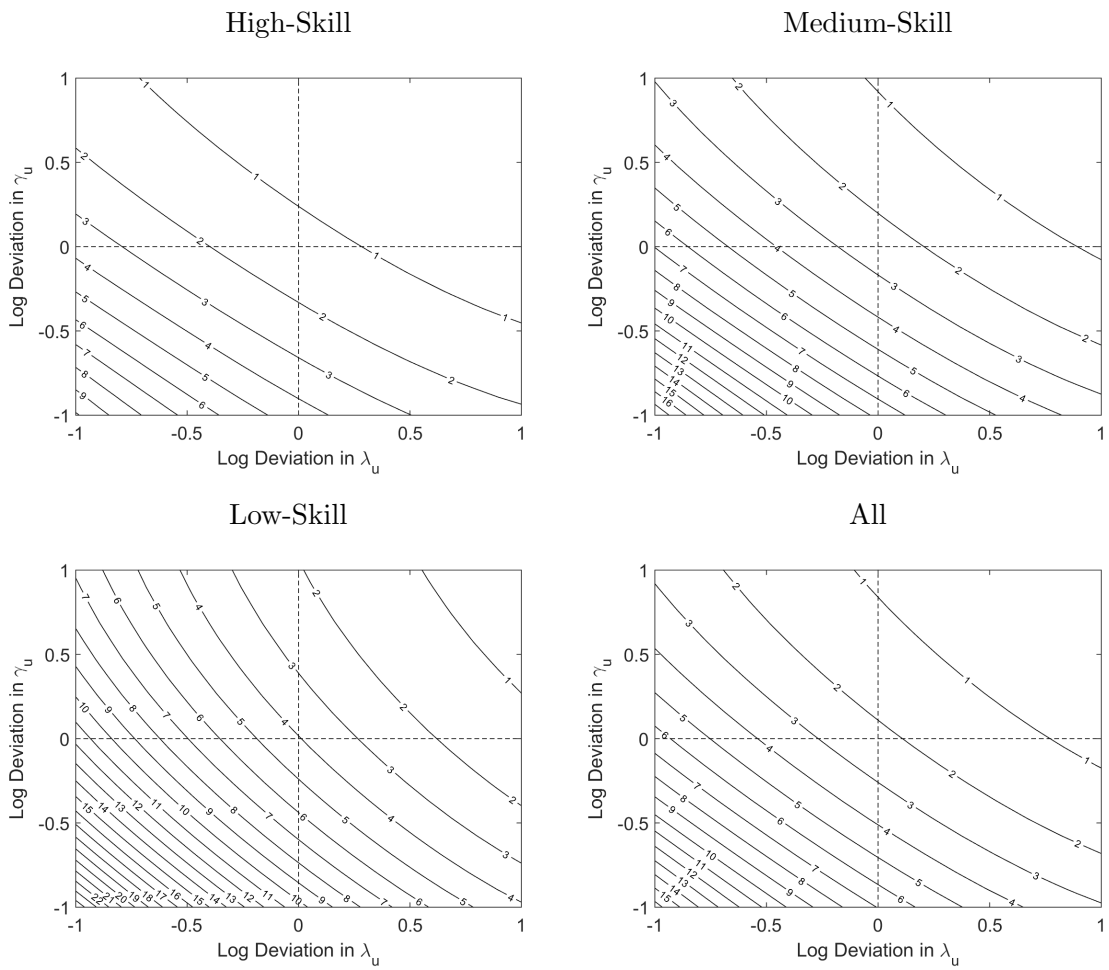
$$\frac{\ell'(F)}{\ell(F)} = \frac{\left[\Sigma_u''(F) + \frac{\gamma_e(\Sigma_u(F)-\Sigma_u(0))}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}\Sigma_e''(F)\right]}{\Sigma_u'(F) + \frac{\gamma_e(\Sigma_u(F)-\Sigma_u(0))}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}\Sigma_e'(F)} + 2\gamma_e \frac{\Sigma_e'(F)}{(\delta+\mu+\gamma_e(1-\Sigma_e(F)))}.$$

Whereas the competition term for the normal Burdett and Mortensen (1998) model is

$$\frac{\ell'(F)}{\ell(F)} = 2\lambda_e \frac{1}{(\delta+\mu+\lambda_e(1-F))}.$$

Note that $\Sigma_s(F)$ is a convex function. When the firm considers the hiring margin in the normal

Figure 5: Impact of γ_u and λ_u on the Long Term Unemployment Rate



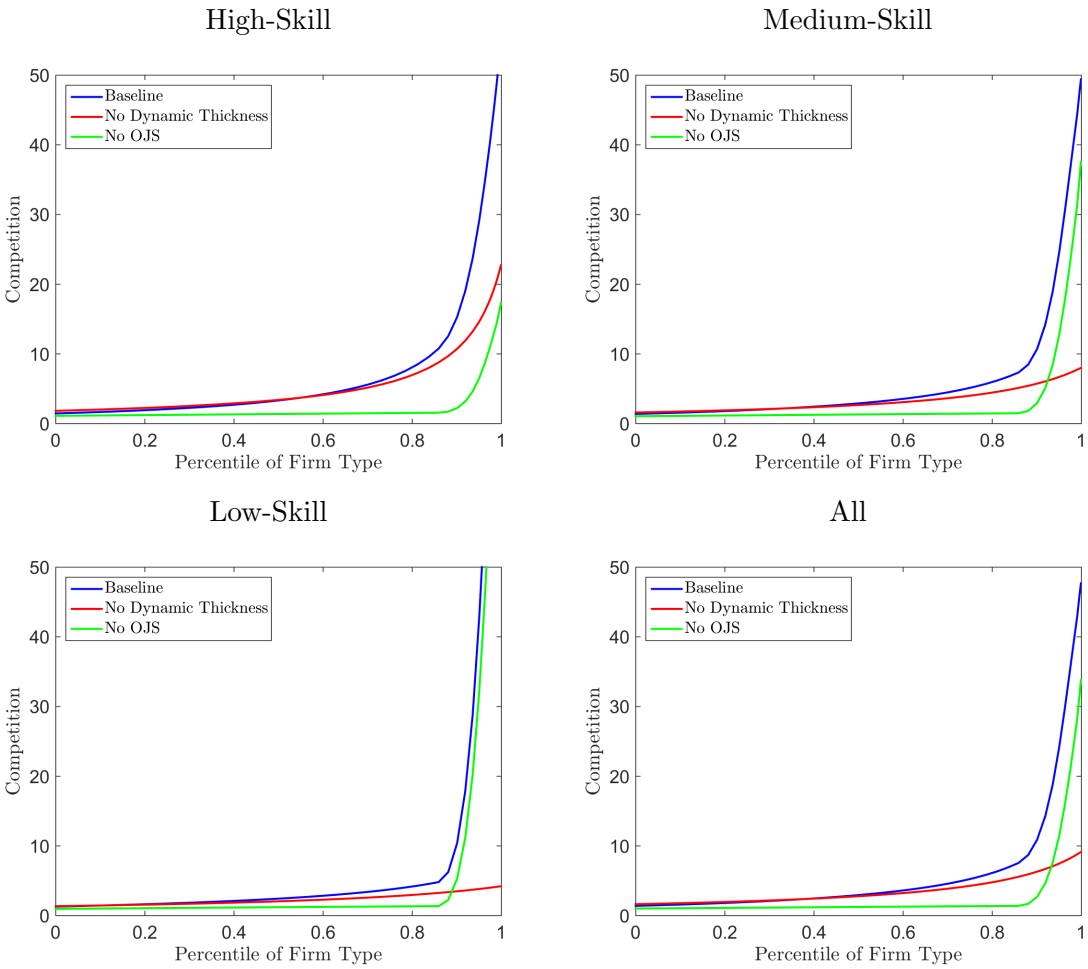
Note: Contour plots are as in Figure 4. However, instead of total unemployment this represents long term unemployment, defined as the proportion of people in the economy who have been unemployed for three months or longer.

Burdett and Mortensen (1998) model, it need only consider the probability that the worker is working at a lower paying firm. In our setup the firm also needs to consider the probability that the worker has a better offer in hand. The competition in our model therefore increases more as we move to the tail of the distribution.

The Burdett and Mortensen (1998) fails to generate as much wage competition in the upper tail of the distribution. Our model includes a further competition term via a Burdett and Judd (1983) mechanism. In order to show the difference, we plot expression 11 to calculate the competition

for different firm types for Burdett and Mortensen (1998) and our model. Figure 6 reveals, that for all skill groups, firms at the upper support exhibit stronger wage competition in our model compared to Burdett and Mortensen (1998). The intuition for this is that since some workers have many offers it is relatively more likely that the highest offer is in the upper part of the distribution. The competition at lower type firm is on the other hand similar in our model and Burdett and Mortensen (1998). The introduction of thin markets into the Burdett and Mortensen (1998) model thus shifts competition from the lower to the upper part of the distribution and thereby increasing the dispersion of wages for a given primitive firm productivity distribution.

Figure 6: Wage Competition



Note: This figure plots the degree of competition in each model, as defined by $\ell'(F)/\ell(F)$.

Wage Posting Motivative. In Burdett and Mortensen (1998) and Bontemps et al. (2000), like in our model, the firm trades off the hiring and retention of the worker against a higher wage, equation (8). The first order condition for the logarithm of expected profits gives a differential equation for the optimal wage. Defining $h(F) = {}^{\partial \log \Pr(\text{hire})}/_{\partial F}$ and $r(F) = {}^{\partial \log E(\text{duration})}/_{\partial F}$ and $w^m(F) = \int_0^F m(F)(y(F) - w(F))dF$ where, $m \in \{h, r\}$ we can decompose the wage $w(F)$ into three terms, the wage increase from the retention motive ($w^r(F)$) and hiring motive ($w^h(F)$) and the wage that satisfies the participation constraint for the worker $w(0)$,

$$w(F) \quad = \quad w^r(F) + w^h(F) + w(0).$$

In the Burdett and Mortensen (1998) model, the motive to pay for retention and hiring are

$$r(F) = h(F) = \frac{\lambda_e}{\delta + \mu + \lambda_e(1 - F)}.$$

In our model, the incentive is

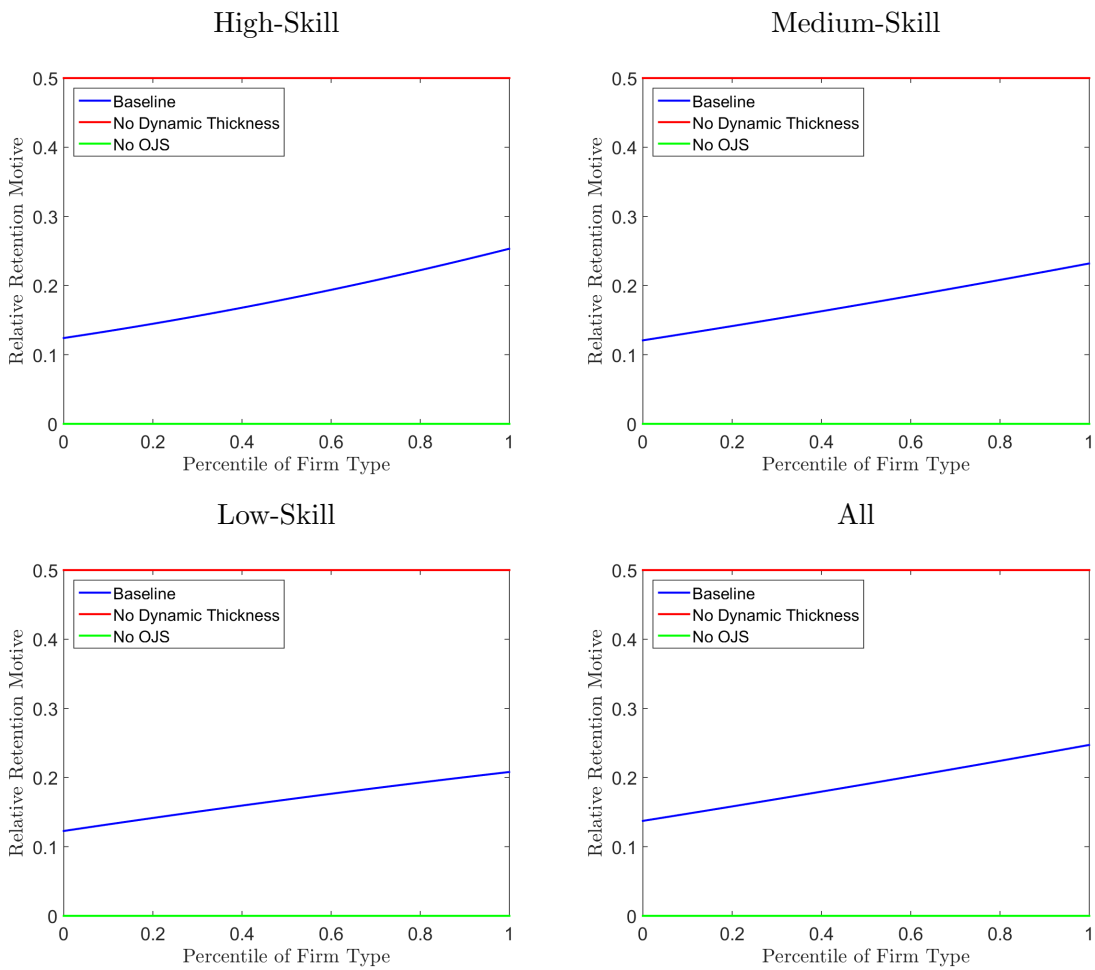
$$\begin{aligned} h(F) &= \frac{(\gamma_u u \Sigma_u''(F) + \gamma_e(1-u)G'(F)\Sigma_e'(F) + \gamma_e(1-u)G(F)\Sigma_e''(F))}{(\gamma_u u \Sigma_u'(F) + \gamma_e(1-u)G(F)\Sigma_e'(F))}, \\ r(F) &= \frac{\gamma_e \Sigma_e'(F)}{(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}. \end{aligned}$$

In Figure 7 we show the fraction of the wage that is paid due to the incentive to retention workers. The results suggest that the hiring motive is quantitatively more important, but relatively less so higher up in the upper support of the wage distribution.

4.6 Frictional wage dispersion

As has been discussed, in order to generate the level of frictional wage dispersion observed in the data as measured by the mean-min ratio, the typical search model requires an implausibly low or even negative flow benefit associated with unemployment. Our results show that the implied replacement ratio, the ratio of the flow benefit to mean wage in the economy needed to justify the observed wage distribution is much higher under the baseline model see Table 3. Across all skill groups only the baseline model predicts a positive replacement ratio and depending on the skill the other two specifications require enormous costs associated with unemployment.

Figure 7: Proportion of Wages Driven by the Retention Motive



Note: The relative retention motive is bounded on $[0, 1]$ and defined as $r(F)/(r(F)+h(F))$.

The replacement ratio in the BM model can be decomposed into the min-mean ratio and the search option,

$$\underbrace{\frac{b}{\mathrm{E}[w]}}_{\text{Rep. ratio}} = \underbrace{\frac{w(0)}{\mathrm{E}[w]}}_{Mm} + \underbrace{\int_0^1 \frac{w'(\tilde{F})}{\mathrm{E}[w]} \frac{(\lambda_e - \lambda_u)(1-\tilde{F})}{\delta + \mu + \lambda_e(1-\tilde{F})} d\tilde{F}}_{\text{Search}}. \tag{12}$$

The flow value in our model is instead

$$\underbrace{\frac{b}{\mathrm{E}[w]}}_{\text{Rep. ratio}} = \underbrace{\frac{w(0)}{\mathrm{E}[w]}}_{Mm} + \underbrace{\int_0^1 \frac{w'(\tilde{F})}{\mathrm{E}[w]} \frac{\gamma_e(+1-\Sigma_e(\tilde{F})) - \gamma_u(1-\Sigma_{uu}(\tilde{F}))}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))} d\tilde{F}}_{\text{Search}}$$
$$+\underbrace{\delta\left(\frac{\gamma_u \int_0^1 \frac{w'(\tilde{F})}{\mathrm{E}[w]} \frac{\Sigma_{uu}(\tilde{F})-\Sigma_u(\tilde{F})}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))} d(\tilde{F})}{(\mu+\gamma_u(1-\Sigma_u(0)))}\right)}_{\text{Insurance}}. \tag{13}$$

The terms are intuitive. There are differences in the flow value and in the search option, captured by how often search occurs, the sampling distribution of wages and finally because, in our model, workers who separate from a job are in a different position compared to the average unemployed. The second term, $\gamma_i(1-\Sigma_i(\tilde{F}))$, differs because the unemployed and employed, on average, sample from different distributions, $\Sigma_i(\tilde{F})$, and at different rates, γ_i. Faberman et al. (2016) provide evidence that the employed on average sample from a better distribution. Similarly, the third term, $\Sigma_{uu}(\tilde{F})-\Sigma_u(\tilde{F})$, captures the effect that workers moving from employment to unemployment have on average a different number of prospect than the average unemployed, consistent with the declining job finding rate with duration of unemployment observed in the data. These effects are missed in the standard job ladder model. Table 6 provides a thorough decomposition of the of the replacement ratios for the different specifications and skill groups. A consistent finding across all skill groups is that only the baseline specification can accommodate the degree of frictional wage dispersion with a positive replacement ratio. Inspection of Table 6 reveals that while the insurance option helps it is the reduction in the search option which is more important quantitatively. While still negative, the better prospects the employed are exposed to reduces the value of waiting in unemployment significantly and consequently unemployed workers for the same value of b are prepared to accept much lower wages.

4.7 Earnings loss

Our baseline model and the two alternative specifications provide very different predictions regarding the average wage an unemployed worker receives in employment as a function of the duration of their unemployment spell. As is displayed in Figure 8, with no dynamic market thickness, an

Table 6: Decomposition of the Flow Value of Unemployment

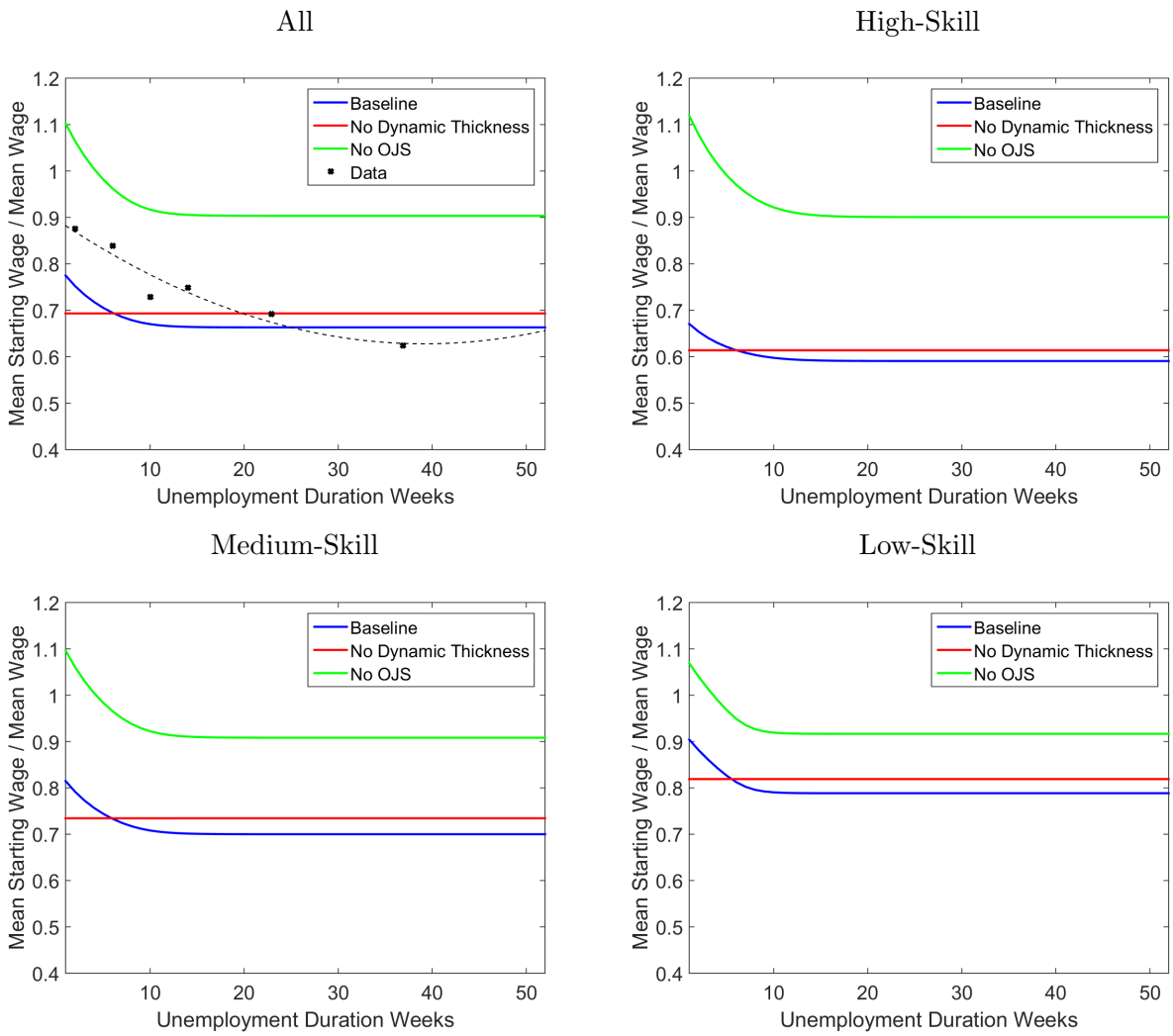
	Replacement Ratio	Min-mean Ratio	Search Option	Insurance Option	Replacement Ratio	Min-mean Ratio	Search Option	Insurance Option
	All				**High-skill**			
NDT	-78.1%	49.7%	-127.8%	0%	-43.0%	50.6%	-93.6%	0%
NOJS	-341.7%	47.6%	-412.6%	23.3%	-677.1%	48.6%	-745.3%	19.7%
Baseline	19.9%	50.7%	-36.4%	5.5%	44.0%	50.7%	-9.4%	2.3%
	Medium-skill				**Low-skill**			
NDT	-197.8%	53.2%	-251.0%	0%	-321.9%	57.9%	-379.8%	0%
NOJS	-230.7%	55.7%	-307.2%	20.7%	-50.0%	60.7%	-131.7%	21.0%
Baseline	27.1%	54.2%	-32.9%	5.8%	41.0%	59.7%	-28.3%	9.6%

Note: This table provides results from decomposing the replacement ratio into its three constituent parts derived in equations (12) and (13).

unemployed worker samples from the same distribution of wages independent of the duration of the unemployment spell to date. However, because of selection into better jobs, the job ladder, this wage is lower than the mean wage amongst employed workers. Without OJS, there is no selection into better jobs for the employed. Thus, the average wage taken by an unemployed worker equals the average wage amongst the employed workers. However, because of dynamic selection, a worker with longer duration of unemployment will on average have fewer prospects and thus samples from a distribution with a lower mean wage. This results in a decline in the average starting wage within the first couple of months. Our baseline model has both these features and thus generates both an average earnings loss, via selection in employment, and increasing losses with the duration of unemployment via dynamic selection.

While a more thorough empirical analysis is required, we plot the starting wages relative to the mean wage for the SIPP sample, following individuals reporting consecutive spells of unemployment. There appears to be a consistent story between model and data, one of an initial fall from the job ladder followed by a further, more gradual decline, with the duration of unemployment.

Figure 8: Wage Loss after Job Loss



Note: Data are taken from the SIPP, presented for the 'all' group, where we have sufficient observations to plot credibly. The points represent a comparison of the mean wage in the economy and the mean wage following reporting being unemployed, plotted at the mean duration of their bin. Note, this is not quite consistent with the models as workers could have had intermediate employment/unemployment spells between observation dates. The dashed black line is a fitted quadratic function of the data.

4.8 Search process

To demonstrate that our model replicates worker's search behavior in a realistic manner we compare the underlying theoretical mechanism with direct evidence on workers' search behavior. For this exercise, we rely on two data sources not used in estimation. They are a supplement of the Survey

of Consumer Expectations (SCE) provided by the New York Fed from 2013 and 2014 and is a subset of that used in Faberman et al. (2016). The survey is a repeated cross-section, nationally representative and has approximately 1,200 individuals per year. In addition, we use the Survey of Unemployed Workers in New Jersey. The data, and its construction, is detailed in Krueger and Mueller (2011). 6,025 unemployed workers in the New Jersey area are surveyed at a weekly frequency for up to 24 weeks. A feature of the data is that it asks workers about the job offers they receive (not necessarily take) and their contemporaneous reservation wage.[13] Results from both datasets are weighted by the weights provided and described in Faberman et al. (2016) and Krueger and Mueller (2016), respectively.

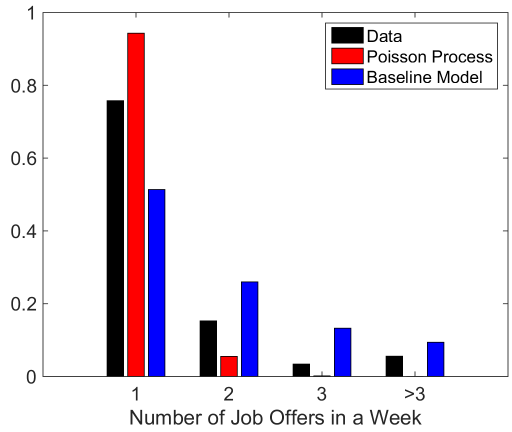
Firstly we exploit data from the Survey of Unemployed Workers in New Jersey. In Figure 9 we present the number of offers received by unemployed workers in a month and compare this to what is predicted by our model and a memoryless Poisson process. The memoryless Poisson process is computed, given the proportion of people in the data with no offers. Our baseline model is a representation of the distribution $p_u(j)$, the solution of the flow equations (16) and (17). Since the data covers a different time period and only focuses on New Jersey there is no reason to assume that the model will fit the data well. However, what is clear from panel (b) is that a memoryless Poisson process cannot generate the number of people with large numbers of offers that is observed in the data. A feature that our baseline model has little problems in replicating.

Turning to the supplement of the Survey of Consumer Expectations (SCE). It is worth noting that the statistics presented here are merely to demonstrate that the underlying search process reported in the survey is quite different from what is assumed in a standard search model and bears some resemblance to the mechanism in our model. Any further inference is difficult to make as although representative, the sample has a fairly small cross-section meaning inference about the unemployed is based on 61 (26) individuals (males). Table 7 shows by employment status, over a four-week period, the mean number of applications, the proportion of those making at least one application and the mean number of contacts received. We present these from model and

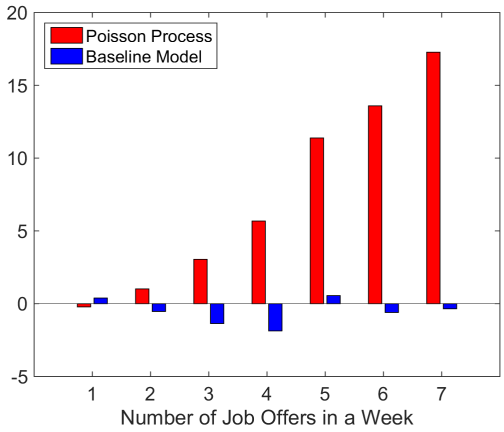
[13]The data for this analysis is available for public download at http://opr.princeton.edu/archive/njui/.

Figure 9: Number of Job Offers

(a) Offer Distribution



(b) Log Ratio



The data comes from the survey of unemployed job seekers in New Jersey. We restrict our attention to male workers between the age of 25 and 45. Panel (a) shows the distribution of reported job offers received in a week, conditional on receiving at least one offer: as observed empirically (weighted by sampling weights); implied by a Poisson process and as implied by the baseline model. Panel (b) is the log ratio of the implied distributions with the data.

data, and we further distinguish between the unemployed and long term unemployed in our model. The unemployed, on average, send out more applications and more frequently engage in active search compared to their employed counterparts. A fact that our model successfully reconciles. The number of applications sent out in the data is an order of magnitude larger as a few people are observed sending out hundreds of applications, something not present in our model. The unemployed being more active is robust to trimming said distributions and the levels begin to look somewhat similar between model and data. The number of contacts received in a month on the other hand is similar across the two groups. However, since we have relatively few unemployed in the sample it is hard to establish this clearly. What is certain is that as implied by a standard job ladder model the unemployed do not receive an order of magnitude larger contacts than their employed counterparts. Finally, we have included in the table the model's predictions for the long term employed to further inform the reader. However, with so few unemployed in the data we could not credibly include the same moments in the table.

Table 7: Mean Job Prospects by Employment Status

	# of applications		prop. who apply		# of contacts	
	Model	Data	Model	Data	Model	Data
Unemployed	0.52	7.23	64%	88%	0.54	0.77
Employed	0.18	1.31	8%	23%	2.4	0.74
L.T. Unemployed	0.08	—	31%	—	0.54	—

Data are taken from the Survey of Consumer Expectations and attention is restricted to male workers. Applications are calculated based on the question "*How many potential employers, if any, did you apply to for employment within the LAST 4 WEEKS? Please include all applications made in person, online, or through other direct methods. Do not include inquiries that did not lead to a job application.*". Similarly, the number of contacts are computed based on the question "*In the LAST 4 WEEKS, how many potential employers contacted you about a job opening? Please include all contacts, even those that were not solicited by you.*". All moments are computed based on appropriate sampling weights. Long term unemployed is defined as having reached the ergodic distribution of prospects, in practice this occurs in under three months.

5 Conclusion

This paper sets up a model which extends the standard job ladder model to incorporate thin markets. The model is solved analytically and estimated on U.S. survey data. The estimated model delivers declining job finding rates by the duration of unemployment as observed in the data. Further, the flow value associated with unemployment required to match the wage distribution does not need to be large and negative. Our estimates of the replacement ratio, in the order of a quarter to a half of the worker's average wage, are consistent with the numbers used in the macro labor literature. On the other hand, the estimation of the Burdett and Mortensen (1998) model or a model without on the job search require large and negative replacement ratios. Further, to generate a wage penalty associated with the duration of an unemployment spell the standard job ladder model requires decreasing general human capital in unemployment. Our model generates this via the stochastic process for employment prospects. This has implications for the persistence in earnings losses following job displacement. Whether this mechanism can generate sufficient persistence in earnings remains an open question and could prove fruitful for future research.

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A Appendix

A.1 Distribution of match quality

A.1.1 Steady-state distribution of vacancy stock

We denote the number of vacancies in the stock by $j \in \mathbb{N}_+$. The probability that a worker in employment state s has j vacancies in the stock is denoted by $p_s(j)$. The inflow of employed workers with $j \geq 1$ offers comes from two sources: (i) those with $j-1$ offers who received an additional offer and (ii) those with $j+1$ offers who lose an offer. The inflow for employed workers with no offers $j = 0$ is from two sources: employed workers with one offer which they lose and workers who just matched with stock, independent of employment state

$$\begin{aligned} \text{inflow} &= \lambda_e p_e(j-1) + \upsilon(j+1)p_e(j+1) \quad \forall j \geq 1, \\ \text{inflow} &= \upsilon p_e(1) + \gamma_e + \gamma_u(1 - p_u(0))\frac{u}{1-u} \quad j = 0. \end{aligned}$$

Similarly, the outflow can be due to separation from a job, losing or losing an offer in hand or because the worker was matched with the stock. The outflow is then given below

$$\text{outflow} = (\lambda_e + \gamma_e + \mu + \delta + \upsilon j)p_e(j).$$

The steady-state distributions are given by equalizing the outflow and inflow of a given number of job offers j. The number of outstanding offers is then

$$(\lambda_e + \gamma_e + \mu + \delta + \upsilon j)p_e(j) = \lambda_e p_e(j-1) + \upsilon(j+1)p_e(j+1) \quad \forall j \geq 1, \tag{14}$$

$$(\lambda_e + \gamma_e + \mu + \delta)p_e(0) = \upsilon p_e(1) + \gamma_e + \gamma_u(1 - p_u(0))\frac{u}{1-u} \quad j = 0. \tag{15}$$

The inflow of unemployed workers with stock of $j \geq 1$ vacancies can either be because a worker had a stock of $j-1$ vacancies and accrues one more, or because a worker with a stock $j+1$ loses one or an employed worker with that number of opportunities is hit by a job destruction shock. The inflow for $j = 0$ is from unemployed workers who lose an offer and employed workers with no offers that are hit by a destruction shock

$$\begin{aligned} \text{inflow} &= \lambda_u p_u(j-1) + \upsilon(j+1)p_u(j+1) + (\delta + \mu)\frac{u}{1-u}p_e(j) \quad \forall j \geq 1, \\ \text{inflow} &= \upsilon p_u(1) + (\delta + \mu)\frac{u}{1-u}p_e(0). \end{aligned}$$

For the unemployed, the outflow can be due to workers taking job offers, which when they match at a rate γ_u. In addition, they also acquire new offers at a rate λ_u and lose offers at rate δ

$$\begin{aligned} \text{outflow} &= (\lambda_u + \gamma_u + \upsilon j)p_u(j) \quad \forall j \geq 1, \\ \text{outflow} &= \lambda_u p_u(0) \quad j = 0. \end{aligned}$$

The steady state distribution solves the equations

$$(\lambda_u + \gamma_u + \upsilon j)p_u(j) = \lambda_u p_e(j-1) + \upsilon(j+1)p_u(j+1) + (\delta + \mu)\frac{u}{1-u}p_e(j) \quad \forall j \geq 1, \tag{16}$$

$$\lambda_u p_u(0) = \upsilon p_u(1) + (\delta + \mu)\frac{u}{1-u}p_e(0) \quad j = 0. \tag{17}$$

A.1.2 Derivations of Σ

Employed Σ_e. Define the probability generating function (pgf) of the stationary distribution as

$$\Sigma_e(F) = \sum_{j=0}^{\infty} F^j p_e(j). \tag{18}$$

Summing equation (14) and (15) over j and using the definition of $\Sigma_e(F)$ gives

$$0 = -(\lambda_e(1-F) + \gamma_e + \mu + \delta)\Sigma_e(F) + \upsilon(1-F)\Sigma_e'(F) + \gamma_e + \mu + \delta.$$

Solving the differential equation gives

$$\Sigma_e(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_e/\upsilon\left(\tilde{F} - F\right)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_e+\mu+\delta}{\upsilon}-1}\frac{\gamma_e+\mu+\delta}{\upsilon}d\tilde{F}.$$

The limits are

$$\Sigma_e(1) = 1, \tag{19}$$

$$\frac{\partial \Sigma_e(F)}{\partial F}|_{F=1} = \frac{\frac{\lambda_e}{\upsilon}}{1 + \frac{\gamma_e+\mu+\delta}{\upsilon}} = \frac{\lambda_e}{(\gamma_e + \mu + \delta + \upsilon)}, \tag{20}$$

$$\frac{\partial^2 \Sigma_e(F)}{\partial F^2}|_{F=1} = \frac{2\lambda_e^2}{(\gamma_e + \mu + \delta + \upsilon)(\gamma_e + \mu + \delta + 2\upsilon)}. \tag{21}$$

Unemployed Σ_u. Define the pgf for the average unemployed as

$$\Sigma_u(F, t) = \sum_{j=0}^{\infty} F^j p_u(j, t). \tag{22}$$

Summing equation (16) and (17) over j and using the definition of $\Sigma_u(F)$ gives

$$0 = -(\lambda_u(1-F)+\gamma_u)\Sigma_u(F)+\upsilon(1-F)\Sigma_u'(F)+(\delta+\mu)(1-u)/u\Sigma_e(F)+\gamma_u\Sigma_u(0).$$

Solving the differential equation using $\Sigma_u(1)=1$ gives

$$\Sigma_u(0) = \frac{\int_0^1 \exp\left[-\lambda_u/\upsilon\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u}{\upsilon}-1}\left[\frac{\gamma_u\Sigma_e(\tilde{F})}{\upsilon}\right]d\tilde{F}}{1-\int_0^1 \exp\left[-\lambda_u/\upsilon\tilde{F}\right]\left(1-\tilde{F}\right)^{\frac{\gamma_u}{\upsilon}-1}\left[\frac{\gamma_u}{\upsilon}\left(1-\Sigma_e(\tilde{F})\right)\right]d\tilde{F}},$$

$$\Sigma_u(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\upsilon(\tilde{F}-F)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u}{\upsilon}-1}\left[\frac{\gamma_u\Sigma_u(0)}{\upsilon}+\frac{(\delta+\mu)(1-u)/u\Sigma_e(\tilde{F})}{\upsilon}\right].$$

Unemployed Σ_{uu}. Lastly, we derive the distribution of wages that a worker expects that starts in unemployment with no prospects. The flow equations are then given by

$$0 = -(\lambda_u+\gamma_u+\upsilon j)p_{uu}(j)+\lambda_u p_{uu}(j-1)+\upsilon(j+1)p_e(j+1)\forall j\geq 1,$$

$$0 = -(\lambda_u+\gamma_u)p_{uu}(0)+\upsilon p_{uu}(1)+\gamma_u.$$

Rewrite in terms of the probability generating function gives

$$0 = -(\lambda_u(1-F)+\gamma_u)\Sigma_{uu}(F)+\upsilon(1-F)\Sigma_{uu}'(F)+\gamma_u.$$

Again, solving the differential equation gives

$$\Sigma_{uu}(F) = \frac{1}{1-F}\int_F^1 \exp\left[-\lambda_u/\upsilon\left(\tilde{F}-F\right)\right]\left(\frac{1-\tilde{F}}{1-F}\right)^{\frac{\gamma_u}{\upsilon}-1}\frac{\gamma_u}{\upsilon}d\tilde{F}.$$

A.1.3 Distribution of outstanding matches G

The first and second derivative of $G(\cdot)$ is given by

$$u = \frac{(\delta+\mu)}{(\delta+\mu+\gamma_u(1-\Sigma_u(0)))}, \tag{23}$$

$$G(F) = \frac{(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)}, \tag{24}$$

$$G'(F) = \frac{(\delta+\mu)\Sigma_u'(F)}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)}+\frac{\gamma_e(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)\Sigma_e'(F)}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^2}, \tag{25}$$

$$\begin{aligned} G''(F) &= \frac{(\delta+\mu)\Sigma_u''(F)}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)}+\frac{\gamma_e(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)\Sigma_e''(F)}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^2} \\ &+ 2\gamma_e\Sigma_e'(F)\frac{(\delta+\mu)\Sigma_u'(F)}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^2} \\ &+ 2\frac{\gamma_e^2(\delta+\mu)\left(\Sigma_u(F)-\Sigma_u(0)\right)\Sigma_e'(F)^2}{(1-\Sigma_u(0))\left(\delta+\mu+\gamma_e(1-\Sigma_e(F))\right)^3}. \end{aligned} \tag{26}$$

A.2 Value Functions

The value function can be calculated using the (expected) average flow value and the (expected) average duration using the formula

$$(\text{Avg. Duration})W(w(F),0) \quad = \quad \text{Avg. Flow benefit.}$$

The average duration in a job with quality F is $\delta+\mu+\gamma_e(1-\Sigma_e(F))$. The average flow benefits are given by the wage $w(F)$ plus the search option $\gamma_e\int_F^1 W(w(\tilde{F}),0)d\Sigma_e(\tilde{F})$ and separation value δU_{eu}. Defining $W_F(w(F),j)=W_w(w(F),j)w'(F)$. The value function at the time of hiring is then given by[14]

$$W(w(F),0) = \frac{w(F)+\gamma_e\int_F^1 W(w(\tilde{F}),0)d\Sigma_e(\tilde{F})+\delta U_{eu}}{\delta+\mu+\gamma_e(1-\Sigma_e(F))}, \tag{27}$$

$$W_F(w(F),0) = \frac{w'(F)}{\delta+\mu+\gamma_e(1-\Sigma_e(F))}, \tag{28}$$

$$W(w(F),0)-W(0,0) = \int_u^F \frac{w'(\tilde{F})}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))}d\tilde{F}. \tag{29}$$

[14]Note that differentiating (2) with respect to F gives

$$(\gamma_u(1-F^j)+\mu+\lambda_e+\delta+j\delta)W_F(w(F),j)=w'(F)+\lambda_e W_F(F,j+1)+\delta j W_F(F,j-1)+\gamma_u(1-F^j)W_F(F,0).$$

One can write the above expression in the form below, where the row and column of the matrix corresponds to the number of job offers, starting with zero

$$W_F = \begin{bmatrix} (\rho+\lambda_e+\delta+\mu) & -\lambda_e & 0 & 0 & \dots \\ -\delta-\gamma_u(1-F) & (\gamma_u(1-F)+\mu+\lambda_e+\delta+\upsilon) & -\lambda_e & 0 & \dots \\ -\gamma_u(1-F^2) & -2\upsilon & (\gamma_u(1-F^2)+\mu+\lambda_e+\delta+2\upsilon) & -\lambda_e & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}^{-1} w'(F).$$

Inverting the matrix and using the first element we get the following

$$W_F(w(F),0) \quad = \quad \frac{w'(F)}{\delta+\mu+\gamma_e(1-\Sigma_e(F))},$$

Similarly, for the unemployed we have

$$\begin{aligned}
U(0) &= \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{uu}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{uu}(0))}, \\
U_{ue} &= \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{u}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{u}(0))}, \\
U_{ue}-U(0) &= \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{u}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{u}(0))} - \frac{b+\gamma_u \int_0^1 W(w(\tilde{F}),0)d\Sigma_{uu}(\tilde{F})}{\mu+\gamma_u(1-\Sigma_{uu}(0))}.
\end{aligned}$$

Evaluating the value function for an employed worker at the worst match ($F = 0$) and using $W(w(0), 0) = U(0)$ gives

$$\begin{aligned}
b &= w(0) + \gamma_e \int_0^1 \frac{w'(\tilde{F})(1-\Sigma_e(\tilde{F}))}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))} d\tilde{F} - \gamma_u \int_0^1 \frac{w'(\tilde{F})(1-\Sigma_{uu}(\tilde{F}))}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))} d\tilde{F} \\
&\quad +\delta \left(\frac{\gamma_u \int_0^1 \frac{w'(\tilde{F})}{\delta+\mu+\gamma_e(1-\Sigma_e(\tilde{F}))} \left(\Sigma_{uu}(\tilde{F}) - \Sigma_u(\tilde{F})\right) d(\tilde{F})}{(\mu+\gamma_u(1-\Sigma_u(0)))} \right).
\end{aligned}$$

A.3 Proof of Identification

The lowest wage worker, $w(0) = \phi$ is identified from the data. The transition rates μ and δ can be estimated using the rate at which the employed workers leave employment for unemployment and out of the labor force, respectively. υ is calibrated outside the estimation. From the data, we can estimate the job finding rate of someone employed in the lowest job $F = 0$ as a function of tenure. We denote quit rate in the lowest match quality by $\gamma_e(1 - \Sigma_{e0}(0, t))$. The differential equation for $\gamma_e(1 - \Sigma_{e0}(F, t))$ is

$$\frac{\partial \gamma_e(1-\Sigma_{e0}(F,t))}{\partial t} = -\gamma_e \left(\upsilon(1-F)\frac{\partial \Sigma_{e0}(F,t)}{\partial F} - \lambda_e(1-F)\Sigma_{e0}(F,t) + \gamma_e \Sigma_{e0}(0,t)(1-\Sigma_{e0}(F,t)) \right). \tag{30}$$

Replacing $\gamma_e(1 - \Sigma_{e0}(F, t))$ by $H_0(t, F)$ and $\gamma_e \frac{\partial \Sigma_{e0}(F,t)}{\partial F}$ by $-\frac{\partial H_0(t,F)}{\partial F}$, $\Sigma_{e0}(F, t) = 1 - \frac{H_0(t,F)}{\gamma_e}$ gives

$$\frac{\partial H(t,F)}{\partial t} = \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) - (\gamma_e - H(t,0))\, H(t,F) - \lambda_e(1-F)\,(\gamma_e - H(t,F)). \tag{31}$$

Note that $H(t, F)$ is identified in the date from the rate at which worker take job that pays $w(F)$. Evaluating this expression at at $t = 0$ we have $H(0, F) = \gamma_e(1 - \Sigma_e(F, 0)) = 0$ and $\frac{\partial H(0,F)}{\partial F} = 0$

which implies

$$\lambda_e = \frac{-\frac{\partial H(t,F)}{\partial t}|_{t=0}}{\gamma_e(1-F)}. \tag{32}$$

Using this in the "original" expression we get

$$\frac{\partial H(t,F)}{\partial t} = \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) - (\gamma_e - H(t,0))\,H(t,F) + \frac{1}{\gamma_e}\frac{\partial H(t,F)}{\partial t}|_{t=0}\,(\gamma_e - H(t,F))\,. \tag{33}$$

This gives the quadratic equation above in γ_e with the solution

$$\gamma_e = \frac{-\left[-\frac{\partial H(t,F)}{\partial t} + \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) + H(t,0)H(t,F) + \frac{\partial H(t,F)}{\partial t}|_{t=0}\right]}{-2H(t,F)}$$
$$\frac{\pm\sqrt{\left[-\frac{\partial H(t,F)}{\partial t} + \frac{\partial H(t,F)}{\partial F}\upsilon(1-F) + H(t,0)H(t,F) + \frac{\partial H(t,F)}{\partial t}|_{t=0}\right]^2 - 4H(t,F)\frac{\partial H(t,F)}{\partial t}|_{t=0}H(t,F)}}{-2H(t,F)}.$$

Noting that $H(t,F) \geq 0$ and $\frac{\partial H(t,F)}{\partial t}|_{t=0} < 0$ implies there is a unique positive solution. The equation therefore solves for γ_e. Using the previous equation, we get λ_e. Having identified λ_e, γ_e it is straightforward to identify γ_u. Note that we can calculate $\Sigma_e(F)$, the instantaneous job finding rate following separation is

$$\gamma_u(1 - \Sigma_e(F)), \tag{34}$$

which gives γ_u. We can then identify λ_u by the job finding rate of the long term unemployed.

A.4 Sample Selection

We make a special effort to ensure the variables circumscribing the samples are consistent across surveys. That is, the following filters are passed through each survey.

(i) Attention is restricted to a sample of only male workers. The sex of a worker is defined in the SIPP by the variable *esex* and *pesex* in the CPS.

(ii) We use the full-time window in the 1996 SIPP, including early observations based on recall of previous employment. That corresponds to observations from December 1995 until February 2000, inclusive. The identical window is used in the CPS.

(iii) Motivated by differential mobility rates by age, see Appendix A.6, attention is restricted to only workers between 25 and 45. Where age is defined as a respondents age as of last

birthday in the variable *tage* in the SIPP and age as of the end of the survey week in the CPS by the variable *peage*. Note, this will introduce negligible difference across samples when a respondent's birthday occurs in a CPS surveying week.

(iv) Skill groups are defined by the variables *eeducate* in the SIPP and *peeduca* in the CPS. The two variables are defined identically with one exception. The CPS variable differentiates between having a '*diploma or certificate from a voc, tech, trade or bus school beyond high school*' and having an '*associate degree in college - occupational/vocational program*' while the SIPP variable agglomerates the two. We treat these two groupings as college educated and include them as high-skill workers. All other groupings are non-controversial.

A.5 Vacancy Duration

The data are taken from the "The Conference Board Help Wanted Online Data Series" (HWOL). The HWOL aims at exhaustive coverage of all job vacancies advertised online. Data are thus collected from over 16,000 online job boards. The data contain two time series, starting in May 2005 and updated contemporaneously. The first is 'new ads', that is, the number of unduplicated ads that did not appear in the previous reference period. An ad is only counted as 'new' in the first reference point it appears in. The second variable is 'total ads'. This is the total number of unduplicated ads appearing in the reference period. This is the sum of 'new ads' and reposted ads from previous periods. Finally, it is worth noting that a reference periods is centered on the first of the months. For example, the 'total ads' for October is the sum of all posted ads from September 14th until October 13th.

Expiry Rate of a Vacancy. We use this data to infer the rate at which vacancies expire. A steady-state approximation implies that the inflow of new vacancies in month t (n_t) is equal to the total amount of vacancies expiring, the product of the stock (υ_t) and the expiry rate (σ_t).

$$n_t \approx \sigma_t \upsilon_t$$

Unfortunately, we do not observe a snapshot of the stock of vacancies. Instead, we observe the total vacancies that have accumulated over that reference period, which we call V_t. Since given

our steady-state assumption the stock of vacancies is constant over a reference period, we can approximate υ_t as

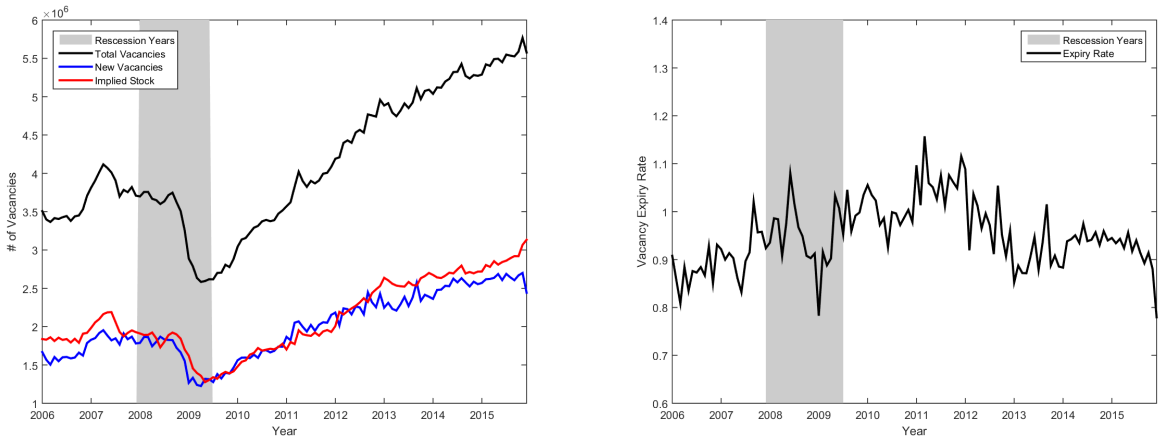
$$\upsilon_t \approx V_t - n_t.$$

Combining the above gives a straightforward approximation of the monthly rate at which vacancies expire for a reference period t.

$$\sigma_t \approx \frac{n_t}{V_t - n_t}$$

We restrict attention to the decade commencing January 2006 until December 2016. Changing the time horizon does little to change the mean monthly expiration date which is computed as 0.95 implying vacancies last a little longer than a month. The series are presented in Figure 10. The first panel shows the raw series of total and new vacancies as well as the implied number of vacancies in the stock at that point in time. The second panel shows the implied expiry rate of vacancies over the period.

Figure 10: Vacancy Series



A.6 Transition Rates by Age

A.7 Fit of Frictional Wage Distribution

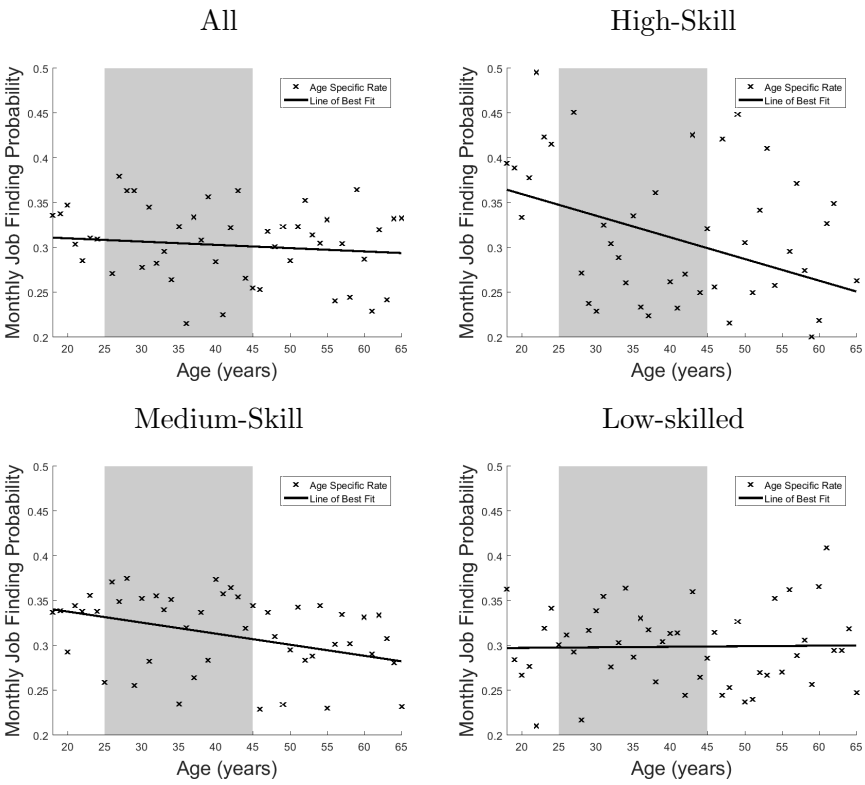
A.8 Worker value

The value functions for employed and unemployed workers, (1) and (2) depends on the employment prospects of the individual, and for an employed worker, their wage. Figure 11 shows how the value of employment changes with these two state variables. For low wages there is a higher return in increasing one's employment prospects than there is from increases in wages. Since there is a long tail to the wage distribution, high returns come from obtaining these top wages and one can increase the probability of this occurrence by gaining in employment prospects. Consequently, particularly for lower wages there is substantial heterogeneity in welfare for a given wage.

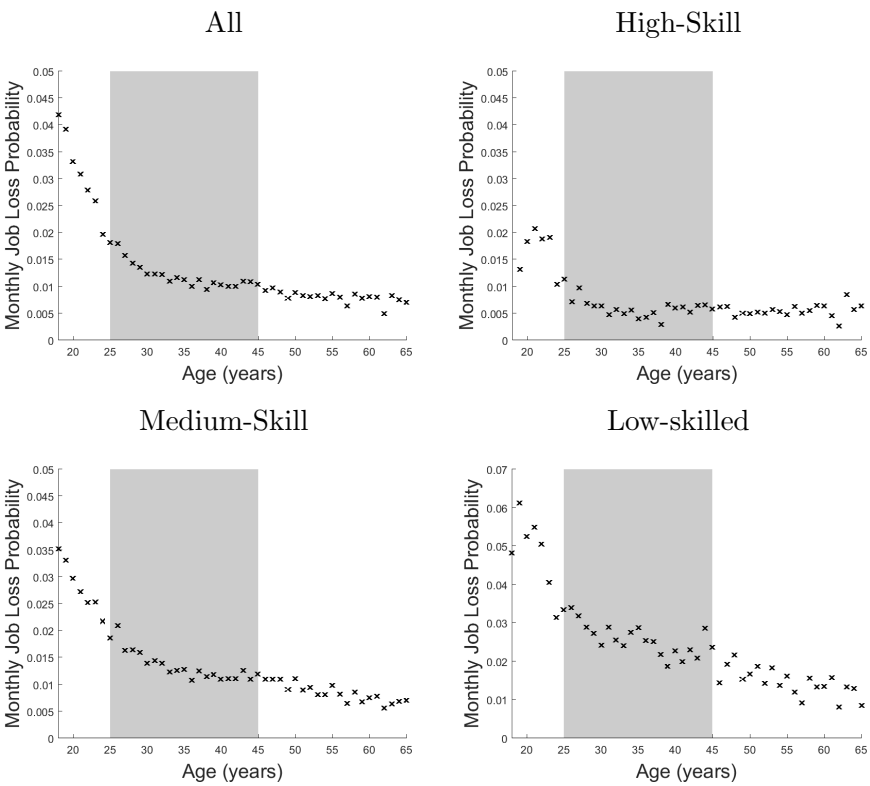
Figure 12 presents the ergodic distributions of worker value across employment state. The two distributions are overlapping. Implying that unlike standard search models, not all employed workers are better off than all unemployed ones. The worst position one can be in is on the lowest wage with no prospects, for which there are relatively few people, or unemployed with no prospects, for which there are many people. Across all skill groups, more than 60% of the unemployed have no prospects. However, inspection of the distributions shows that as soon as the unemployed gain positive prospects. They are better off than approximately 20% of the employed workers, depending on the skill group.

A.6: Transition Rates by Age

Job Finding Rate

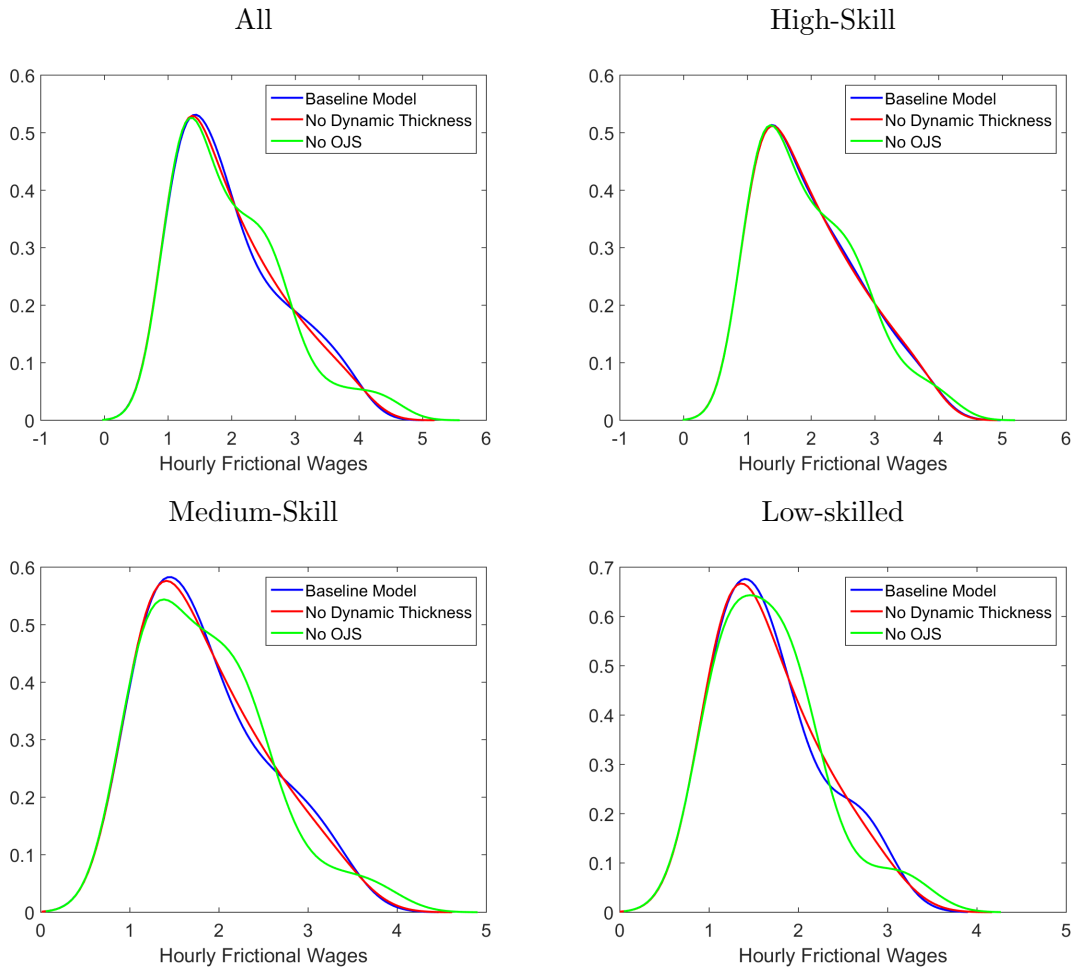


Separation Rate



Note: the 'x's represent the appropriate monthly transition probability for a male of that age. The shaded region represents the specific age we will focus on in our analysis. Data comes from the CPS, 1996-1999 inclusive.

A.7: Fit of Wage Moments



Note: Kernel density plot showing the fit of the two nest models, no dynamic thickness and no on the job search to the distributional distribution of frictional wages as predicted by the baseline.

Figure 11: Employed Worker's Value by Wage and Opportunity

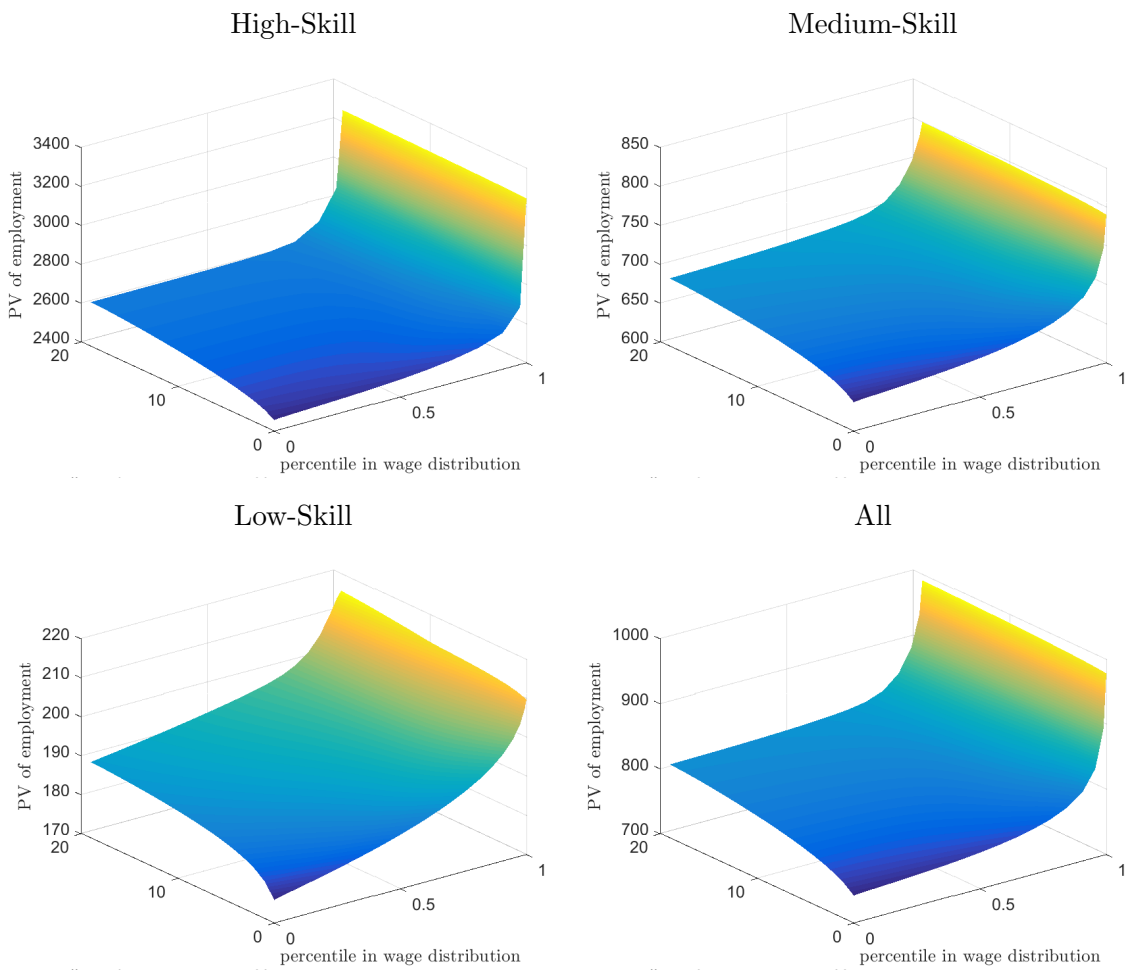


Figure 12: Pdf (Cdf) of Unemployed (Employed) Worker Value