

# Skill Wage Premia, Employment, and Cohort Effects in a Model of German Labor Demand

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**Abstract:** Operationalizing the relationship between employment and wage structures, Card and Lemieux (2001) analyze skill wage differentials in a supply and demand framework that includes age as an additional important dimension of heterogeneity. Our paper extends this analytical framework and confronts it with the IAB employment subsample (IABS) data for Germany. After having identified cohort effects in skill wage premia and in the evolution of relative employment measures, we estimate elasticities of substitution between employees in three different skill groups and between those of different age taking account of the endogeneity of employment. Compared to estimates in the related literature, our estimated elasticities are rather high. Drawing on the estimated parameters, we simulate the magnitude of wage changes within the respective skill groups that would have been necessary to halve skill-specific unemployment rates in 1997. The required nominal wage reductions range from 8.7 to 12.3% and are the higher the lower the employees' skill level.

**Keywords:** Labor Demand, Heterogeneity, Age, Skill, Wage Structure, Cohort Effects, Unemployment.

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# 1 Introduction

Heterogeneity in the labor market has become a major issue in the literature in recent years. The growing availability of large micro data sets has given rise to numerous empirical studies that record descriptive evidence on the evolution of wages and employment measures; cf. the overview article of Katz and Autor (1999). To structure the inhomogeneous factor “labor”, authors usually undertake a grouping into different classes based on observed covariates like age and sex of employees or on the basis of job characteristics. Heterogeneity is further reflected in varying wages. Available studies generally report considerable wage dispersion both between and within adequately defined classes.

In this context, particular attention is turned to skill wage premia and the evolution of skill-specific employment. For decades, unemployment rates have proven the higher the lower the (formal) qualifcational level of the employees. In West Germany the respective rates for employees without a vocational degree, for those with, and for workers with a university degree were 19.4%, 5.7%, and 2.6% in the year 2000.<sup>1</sup>

Rigidity of the wage structure is often referred to as a major cause for the different degrees of affection by unemployment; compare, e. g., Fitzenberger and Franz (2001). As elaborated in the discussion about employment impacts of skill-biased technical change (SBTC; cf. Katz and Autor, 1999, Acemoglu, 2002), relative demand for low-skilled labor decreases faster over time than does relative supply. In line with neoclassical demand theory (Hamermesh, 1993), market clearing would in this case require an increase of qualifcational wage differentials.

Despite the popularity and plausibility of this hypothesis an empirical operationalization of the interrelation between wage structure and employment that goes beyond mere descriptive evidence turns out to be difficult. Conventional empirical analyses of qualifcational labor demand typically take into account only a small number of homogeneous skill groups—mostly not more than three; cf. the surveys in Hamermesh (1993) and Katz and Autor (1999) and for Germany the studies of Fitzenberger (1999), Steiner and Wagner (1998b), or Falk and Koebel (1999, 2002), for example. This proceeding is rationalized in light of the fact that satisfying solutions to the resulting problem of aggregation do not exist.<sup>2</sup> Besides, implementations based on cost-minimizing behavior which allow for a larger number of factors quickly become impracticable.

Based on US data, Katz and Murphy (1992) analyze wage differentials between high school and college graduates in the context of supply and demand effects. A CES model proves consistent with the developments of wage premia and employment over time. These come along with the labor market entry of young and the exit of older birth cohorts on the one hand and an increase in average educational attainment on the other. The literature interprets these trends as a race between changes in the skill structure of labor supply and that of labor demand; cf., for example, Johnson (1997) and Topel (1997), Machin (2002). However, in addition to the variation of skills between different cohorts, human capital endowments also change with age. Whereas increasing labor market experience

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<sup>1</sup>Cf. Reinberg and Hummel (2002), p. 27.

<sup>2</sup>For discussions of the problem of aggregation in the context of labor demand estimations see, e. g., Koebel (2003) and Katz and Autor (1999).

and job tenure augment human capital stocks with age, skill-biased and accelerating structural change might invalidate individual endowments of older workers. Card and Lemieux's (2001) investigation into US, UK, and Canadian data extends the model of Katz and Murphy (1992). In a set-up using the nested CES model developed by Sato (1967), the inclusion of age as an additional dimension of heterogeneity not only enables the separation of age, time, and cohort effects, but also facilitates the estimation of a specification with a relatively large number of different input factors. The estimation strategy undertaken in particular yields elasticities of substitution both between high school and college graduates and between workers belonging to different age classes.

The paper at hand broadens the scope of this analytical framework and confronts it with the IAB employment sample (IABS) data for Germany. Our treatment goes beyond Card and Lemieux (2001) in several directions: First, we let three skill groups account for heterogeneity within the qualification dimension. Though this extension necessitates systems estimation techniques, it is considered adequate in light of the coexistence of vocational training and university education in Germany. Second, we treat the identification of cohort effects more rigorously. Tests for separability of age, time, and cohort effects in the tradition of MaCurdy and Mroz (1995) and Fitzenberger, Hujer, MaCurdy, and Schnabel (2001) are applied to check the validity of the specification. Third, we take a closer look at the notions of observed employment and let instrumental variable techniques account for the endogeneity of both wages and employment. Finally, we draw on the estimated substitution parameters to conduct two simulation experiments: We calculate the magnitude of wage changes in the three skill groups that would have been necessary to halve skill-specific unemployment rates in 1997 (the latest period available). While allowing for relative changes between skill groups, this would have left the wage structure within skill groups unaffected. Alternatively, one might be interested in changes of the wage structure within skill groups, holding the structure across the respective groups constant. Here, the model set-up may provide an answer to the question how wages for employees of different age would have had to change to reduce all age-specific unemployment rates by one half.

The remainder of the paper is organized as follows: Section 2 outlines the trends in skill wage premia and skill-specific employment in the IABS between 1975 and 1997. Following an investigation into the nature of cohort effects in section 3, section 4 discusses the nested CES model which allows for the reconciliation of the stylized empirical facts and estimates elasticities of substitution across and within skill groups. Based on the resulting parameters, the simulation experiments are presented in section 5. Section 6 concludes.

## 2 Descriptive Evidence

A number of recent empirical studies provides descriptive evidence for skill wage differentials in the German labor market. Among the analyses—comprising, e. g., Christensen (2003), Christensen and Schimmelpfennig (1998), Fitzenberger (1999), Möller (1999), Steiner and Mohr (2000), and Steiner and Wagner (1998a)—there is some consensus that, by and large, the earnings distribution across skill groups stayed relatively stable during the 1980's and 1990's.

A closer look calls for detailed investigations which take into consideration further aspects

of heterogeneity. In the tradition of Mincer (1974) work experience has proven an important additional determinant of individual earnings, and the effects of age—often used as a proxy for experience—are of interest themselves. Studies explicitly accounting for the age dimension of wage distributions examine single age profiles, like Fitzenberger and Reize (2003), or focus specifically on cohort analyses, as Fitzenberger, Hujer, MaCurdy, and Schnabel (2001), for example. Beißinger and Möller (1998) account for the age dimension in the distribution of (un)employment for discrete years between 1980 and 1990.

Our study scrutinizes both wages and employment across the two dimensions skill and age for the time span 1975–1997. It is based on the IAB employment subsample (IABS), a 1% random draw of German employment spells subject to social insurance contributions. The IABS covers about 80% of all employed persons in the cross section, and it provides detailed information on daily wages for blue and white collar workers as well as the exact timing of employment spells. We classify employees into three skill groups and consider six age classes. An extensive description of the data and classifications used is given in the appendix.

## 2.1 Stylized Facts I: The Evolution of Wage Differentials

Skill wage differentials or skill wage premia  $r_{s,at}$  among workers of age  $a$  at time  $t$  are defined as the difference in mean *log* wage of high-skilled ( $s = h$ , employees with a university degree) or low-skilled workers ( $s = l$ , employees with neither university nor vocational degree) and that of medium-skilled workers ( $s = m$ , employees with a vocational degree). Using dummy variables  $d_{s,at}$  for the different skill groups and possibly controlling for further influences,<sup>3</sup> they can be derived from regressions

$$\ln(w_{at}) = \text{constant}_{at} + r_{l,at} \cdot d_{l,at} + r_{h,at} \cdot d_{h,at} + \text{controls}_{at} + \epsilon_{at} \quad (1)$$

in the respective age-time cells. Due to the social security threshold, wage data in the IABS are censored from above. Thus (1) is estimated by means of Tobit regressions. Observations are weighted by the respective length of the employment spell. Results are provided in table 4 in the appendix.

Figure 1 illustrates the evolution of age-specific wage differentials for males over time. All wage differentials or qualification premia are the higher the older the employee. This result is well in line with classical human capital theory (Becker, 1993), and it does support the interpretation of age as a proxy for experience. Premia have evolved quite differently, though.

The education premium for high-skilled employees compared to the medium-skilled stayed roughly constant for the oldest age class until 1987 and declined by about 8% thereafter. The relative position of 30- to 35-year-old high-skilled, on the other hand, deteriorated by about 8% during the late 1970's, partly rose again in the first half of the 80's, and stayed constant from 1986 on.

The differential between older medium- and low-skilled workers exhibited a decline of about 5% during the eighties and recovered to an overall decline of about 2% during

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<sup>3</sup>Cf. the appendix for details on implemented specifications.

the nineties. In the youngest age class this wage premium exhibited even more volatility: Between 1975 and 1986, low-skilled workers on average gained around 6% compared to the medium-skilled. Later on the differential increased again and even exceeded the 1975-level in 1997.

To infer the evolution of age profiles across time, we plot the wage differentials for three years against the age dimension in figure 2. Average wage differentials between high- and medium-skilled generally increase rather steeply with age: The premium grows by up to 24%. However, the shape of the profiles changes over time.

In 1975 the profile is considerably curved, showing especially a profound rise for young individuals. In transition to the mid-1980's, the curvature declines whilst the profile still shows a similarly high increase over the entire age span: It is in particular middle-aged workers whose premium for higher education declines compared to 1975. Starting in the second half of the eighties, one observes a twist of the profile. Whereas the upsurge for workers up to their mid-thirties is much the same in 1997 as in 1986, the difference profile has gotten flatter for employees in their later years: The relative position of older high-skilled has deteriorated in comparison to the situation in 1986.

The profile of the wage differential between low- and medium-skilled workers generally turns out much flatter, especially for older workers. But even though the maximum decrease—roughly 8% in 1975—is found to be small relative to the one experienced by the high-to-medium-skilled differential, the picture of the developments over time is still striking. In 1975 the average education premium moderately rose with age, showing increments declining with age. Up to 1986, the profile shifted downward by about 2–6%, becoming steeper for younger age classes. In 1997 however, the profile shows a twisted shape: Whilst the differential for older workers partly revived in a parallel kind of manner, the youngest workers now face a premium increased by 6% which renders the entire profile nearly flat.

Taking the above results together, we assert a first stylized fact:

Between the mid-1970's and the mid-1990's, age profiles of skill wage premia have not moved in parallel fashion over time, but rather experienced a twist.

The developments thus are not likely to be the result of pure age and time effects alone. Cohort effects, i. e., systematic differences across birth cohorts, supposedly play an additional important role. The subsequent theoretical and empirical investigation into the development of skill wage premia hence takes account of age, time, and cohort effects.

## 2.2 Stylized Facts II: Trends in Relative Employment

Based on the individual spell data, a weighted headcount provides a measure of employment: In each age-time cell, the number of skill-specific employed is summed up, weighted by the duration of the respective employment spell.

Inferred time trends in relative employment for the different age classes, i. e., respective employment counts of the high- and the low-skilled relative to the employment in the

medium-skill group, are shown in figure 3. The measures give account of the skill upgrading that took place over the past decades: For the biggest part of the sample period, both the ratio of high-skilled to medium-skilled and that of medium-skilled to low-skilled employment were the higher the younger the respective age class. Furthermore, the skill-intensity of employment has increased over time. Starting from a situation of uniform skill upgrading in all age classes, however, the increase of relative employment of the skilled slows down considerably or even comes to an end at some point in time. Beginning in the mid 1980's, this break occurs first for the youngest age group. It then works through the older classes during the following years until it affects the oldest employees in the second half of the 1990's.<sup>4</sup>

We keep hold of a second stylized fact:

There is a break in the inter-cohort trend of relative employment such that younger birth cohorts do not follow the older ones towards further skill upgrading.

The empirical evidence thus suggests the existence of cohort effects in the employment dimension, too.

### 3 Testing for Cohort Effects

To separate age, cohort, and time effects Card and Lemieux (2001) undertake a decomposition of wage premia by the following regression:

$$r_{at} = b_a + c_{t-a} + d_t + \epsilon_{at} \tag{2}$$

where  $b_a$ ,  $c_{t-a}$ , and  $d_t$  denote age, cohort, and time dummies, respectively. However, one should be cautious with respect to the identification of wage premia. When separating cohort effects from pure time and age effects an identification issue arises because the cohort an individual belongs to—be it defined by the individual's year of birth—is an exact linear combination of the individual's age and the point of time being.

As a first identification approach, we follow Card and Lemieux by estimating equations (2), setting the effects for the oldest birth-cohorts—those up to 1928—equal to zero. The model is formally “identified” based on annual data by using five-year age intervals and implicitly assuming age and cohort effects to be constant within each interval. A test for the existence of cohort effects is then conducted by testing for joint significance of all other cohort terms. This approach is suggestive from an economic point of view. However, it resolves the identification problem in an ad hoc way. We employ an alternative approach introduced by MaCurdy and Mroz (1995) and also used in Fitzenberger, Hujer, MaCurdy, and Schnabel (2001) which deals with the identification issue explicitly.

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<sup>4</sup>Note that the approximate zero-growth of the relative employment of high-skilled in the first age class should not be over-interpreted in our context, because it likely reflects the extension of education durations and the corresponding deferments of labor market entries during the last decades; cf., for example, Reinberg and Hummel (1999).

Following this approach, we formalize the interpretation of cohort effects as the outcome of interaction between age and time by allowing for interaction terms of different order. For identification, the linear cohort effect is explicitly set to zero.<sup>5</sup> In light of our aim to test for the existence of cohort effects, we apply the following basic specifications:

In the first instance, we again employ age and time dummy variables and estimate

$$r_{s,at} = b_{s,a} + d_{s,t} + \sum_{i=1}^4 \gamma_{is} R_{i,at} + \xi_{at} K_{s,\text{after}}(c_{at}) + (1 - \xi_{at}) K_{s,\text{before}}(c_{at}) + \epsilon_{s,at}, \quad s \in \{l, h\}; \quad \xi_{at} = \begin{cases} 1 & : c_{at} \geq 0 \\ 0 & : \text{else} \end{cases} . \quad (3)$$

The pure cohort effects for those entering the labor market after and before 1975, respectively, write

$$K_{s,k}(c_{at}) = \delta_{k,1s} c_{at}^2 + \delta_{k,2s} c_{at}^3 + \delta_{k,3s} c_{at}^4, \quad k \in \{\text{after}, \text{before}\}; \quad s \in \{l, h\}, \quad (4)$$

with  $c_{at}$  denoting normalized birth cohorts. The terms  $R_{i,at}$  capture polynomial interaction terms between age and cohorts in the time derivative of  $r_{s,at}$  as defined in MaCurdy and Mroz (1995).<sup>6</sup>

As a second specification, we use polynomials of order four in time instead of time dummies. In both specifications separability of age and time effects holds if  $\gamma_i = 0$  for all  $i$ . Only under this assumption an additive model structure as the one presented in section 4.1 is valid at all. Uniform wage growth holds if additionally the pure effects for the cohorts after 1975 are equal to zero:  $\gamma_i = \delta_{\text{after},js} = 0$  for all  $i, j$ . In this case, the existence of cohort effects is denied for those whose entire working life cycle falls into the observation period. Finally, one may test whether even older cohorts do not face any cohort effects:  $\gamma_i = \delta_{\text{after},js} = \delta_{\text{before},hs} = 0$  for all  $h, i, j$ .

The detailed decomposition estimates to identify cohort effects in the skill wage differentials and affiliated tests for cohort effects can be found in table 5 in the appendix. Our two major findings are that there is evidence for cohort effects in both specifications but that additive separability of age, time, and cohort effects is not rejected.<sup>7</sup> Given these results, the estimation of the structural model introduced in the subsequent section is in fact justified.

## 4 An Economic Reconciliation

Building on the stylized facts, we follow Card and Lemieux (2001) in applying a model based on the two-level CES production function developed by Sato (1967). However,

<sup>5</sup>See Heckman and Robb (1985) for a note on the identification issue in this particular context.

<sup>6</sup>Adapted to our notation, the interaction terms up to second order write  $R_{1,at} = c_{at} a_{at}^2/2 + a_{at}^3/3$ ,  $R_{2,at} = c_{at}^2 a_{at}^2/2 + 2a_{at}^3 c_{at}/3 + a_{at}^4/4$ ,  $R_{3,at} = c_{at} a_{at}^3/3 + a_{at}^4/4$ , and  $R_{4,at} = c_{at}^2 a_{at}^3/3 + a_{at}^4 c_{at}/2 + a_{at}^5/5$ .

<sup>7</sup>Since the restrictive decomposition of cohort and age effects in equation (2) following Card and Lemieux (2001) is rejected, we do not discuss the associated results. Though, if accepted as such, this approach indicates the existence of cohort effects as well.

the following treatment extends their analysis to the three skill group-case. The model treats not only workers with different educational attainment, but—well in line with the conjecture of Freeman (1979)—also similarly educated workers of different age as imperfect substitutes. Given factor remunerations according to their respective marginal products, it can be transformed into relative wage equations which permit to separate age, time, and cohort effects on the wage gaps—and therefore provides an analytical framework to link the stylized facts outlined above.

## 4.1 The Two-Level CES Model

The Sato (1967) framework suggests a CES model of aggregate production  $y_t$ :

$$y_t = (\theta_{l,t}L_{l,t}^\rho + \theta_{m,t}L_{m,t}^\rho + \theta_{h,t}L_{h,t}^\rho)^{\frac{1}{\rho}}, \quad (5)$$

where  $L_{s,t}$ , the measures of employment in skill group  $s$  and period  $t$ , themselves are CES subaggregates of the skill- and time-specific employment quantities  $L_{s,at}$  of individuals in age groups  $a$ :

$$L_{s,t} = \left[ \sum_a (\phi_{s,a} L_{s,at}^\pi) \right]^{\frac{1}{\pi}}, \quad s \in \{l, m, h\}. \quad (6)$$

The productivity parameters  $\theta_{s,t}$  covering the usual CES distribution parameters as well as the (relative) efficiency terms of the different skill groups are allowed to vary over time to capture (skill biased) technical change, and  $\phi_{s,a}$  map the productivities of the different age classes within the skill classes.  $\sigma_S = 1/(1 - \rho)$  and  $\sigma_A = 1/(1 - \pi)$  denote the elasticity of substitution between two skill groups and the elasticity of substitution between different age groups within the same skill group, respectively.

Let wages be determined by the respective marginal products:

$$\frac{w_{s,at}}{w_{m,at}} = \frac{\frac{\partial y_t}{\partial L_{s,at}}}{\frac{\partial y_t}{\partial L_{m,at}}} = \frac{\theta_{s,t} \cdot L_{s,t}^{\rho-\pi} \cdot y_t^{1-\rho} \cdot \phi_{s,a} \cdot L_{s,at}^{\pi-1}}{\theta_{m,t} \cdot L_{m,t}^{\rho-\pi} \cdot y_t^{1-\rho} \cdot \phi_{m,a} \cdot L_{m,at}^{\pi-1}}, \quad s \in \{l, h\}. \quad (7)$$

Then age specific skill premia  $r_{s,at} = \ln(w_{s,at}/w_{m,at})$  result as

$$\begin{aligned} r_{s,at} = & \ln \left( \frac{\theta_{s,t}}{\theta_{m,t}} \right) + \ln \left( \frac{\phi_{s,a}}{\phi_{m,a}} \right) - \left( \frac{1}{\sigma_A} \right) \ln \left( \frac{L_{s,at}}{L_{m,at}} \right) \\ & + \left[ \left( \frac{1}{\sigma_A} \right) - \left( \frac{1}{\sigma_S} \right) \right] \ln \left( \frac{L_{s,t}}{L_{m,t}} \right), \quad s \in \{l, h\}. \end{aligned} \quad (8)$$

The occurrence  $\sigma_A \rightarrow \infty$ , i. e., different age groups as perfect substitutes, nests the standard case of a CES with skill groups homogeneous in the age dimension.

Typically, one would expect substitutability to be higher within skill groups than across, i. e.,  $\sigma_A > \sigma_S$ . In this case both age group-specific relative employment  $\ln(L_{s,at}/L_{m,at})$  and



aggregate relative employment  $\ln(L_{s,t}/L_{m,t})$  exert a negative impact on the skill premia in (8).

Rewriting equation (8) as

$$r_{s,at} = \ln\left(\frac{\theta_{s,t}}{\theta_{m,t}}\right) + \ln\left(\frac{\phi_{s,a}}{\phi_{m,a}}\right) - \frac{1}{\sigma_S} \ln\left(\frac{L_{s,t}}{L_{m,t}}\right) - \frac{1}{\sigma_A} \left[ \ln\left(\frac{L_{s,at}}{L_{m,at}}\right) - \ln\left(\frac{L_{s,t}}{L_{m,t}}\right) \right], \quad s \in \{l, h\} \quad (9)$$

allows to discuss cohort effects. If  $\ln(L_{s,at}/L_{m,at}) - \ln(L_{s,t}/L_{m,t})$  varies over time, i. e., age-specific relative employment evolves differently from the aggregate measure, and if, in addition,  $\sigma_A$  is finite, then differences in cohort size affect  $r_{s,at}$  through the term in brackets. Assume

$$\ln\left(\frac{L_{s,at}}{L_{m,at}}\right) = \psi_{s,t-a} + \mu_{s,a}, \quad s \in \{l, h\}, \quad (10)$$

such that the model involves year- (index  $t$ ), age- (index  $a$ ), and cohort-specific (index  $t - a$ ) effects:<sup>8</sup>

$$r_{s,at} = \ln\left(\frac{\theta_{s,t}}{\theta_{m,t}}\right) + \ln\left(\frac{\phi_{s,a}}{\phi_{m,a}}\right) - \frac{1}{\sigma_A} \mu_{s,a} - \left[ \frac{1}{\sigma_A} - \frac{1}{\sigma_S} \right] \ln\left(\frac{L_{s,t}}{L_{m,t}}\right) - \frac{1}{\sigma_A} \psi_{s,t-a}, \quad s \in \{l, h\}. \quad (11)$$

Besides the simplicity of implementation (cf. section 4.2), theoretical consistency in light of the neoclassical production theory is a great merit of the CES framework. The two-step CES offers the additional advantage that it accounts for some heterogeneity within the skill groups: Workers of different age are allowed to be imperfect substitutes.<sup>9</sup>

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<sup>8</sup>A further aspect of cohort effects would arise by allowing age-specific productivity  $\phi_{s,a}$  to vary with time. This case might match trends in the returns (price) to experience over time which subsist beside the evolution of the educational skill measure  $\theta_{s,t}$ , as indicated, e.g., by Juhn, Murphy, and Pierce (1993). However, this particular productivity component would not be separable from the time effects captured by  $\theta_{s,t}$ ; cf. the discussion about the identification of cohort effects in section 3. By disregarding interactions between the productivity terms, any cohort effects found in wage premia are implicitly attributed to changes in labor quantities—an assumption suited in light of our main focus to operationalize the relationship between relative wages and employment; see also Welch (1979).

<sup>9</sup>Still, one might judge the model's functional form restrictive. In particular, the elasticities of substitution between (identically skilled) workers of different age are restricted to be all equal, so that, say, a 55-year-old executive can be replaced by an experienced 50-year-old as well as by a 25-year-young entrant. However, the model is well-suited in light of our major aim to tell apart the effects of the two dimensions age and time. Compared to feasible translog systems, for example, its age×time dimensioning allows to incorporate a relatively large number of input factors. For discussions on functional specification and aggregation see, e.g., Katz and Autor (1999) and Koebel (2003).

## 4.2 Empirical Implementation

Estimation of the nested CES structure can be achieved by simply estimating linear models in three steps.<sup>10</sup> At the first stage, the two-equation system

$$r_{s,at} = b_{s,a} + d_{s,t} - \frac{1}{\sigma_A} \ln \left( \frac{L_{s,at}}{L_{m,at}} \right) + \epsilon_{s,at}, \quad s \in \{l, h\} \quad (12)$$

can be estimated by 3SLS, yielding an estimate for  $\frac{1}{\sigma_A}$ , which is equal across the two equations.

Least squares regressions of

$$\ln(w_{s,at}) + \frac{1}{\hat{\sigma}_A} \ln(L_{s,at}) = d_{s,t} + \ln(\phi_{s,a}) + \epsilon_{s,at}, \quad s \in \{l, m, h\} \quad (13)$$

provide estimates of  $\phi_{s,a}$  at the second stage and allows to calculate the skill group aggregates  $L_{s,t}$  defined in (6).

Finally, at the third stage, equation (9) is extended by an additive error term and estimated for  $s \in \{l, h\}$ . Again, 3SLS takes account of the cross-equations restrictions concerning  $1/\sigma_A$  and  $1/\sigma_S$ . Following the literature, the evolution of the relative productivity of workers over time,  $\ln(\theta_{s,t}/\theta_{m,t})$ , is assumed to follow a linear time trend. This approach captures the steady demand hypothesis in the notation of Acemoglu (2002): The steady shift of the relative demand for higher-skilled labor mirrors a constant rate of SBTC.<sup>11</sup> Concerning the age-productivity within skill groups,  $\phi_{s,a}$ , two specifications are possible: First,  $\phi_{s,a}$  may be treated as predetermined by the estimate from the second stage (model variants (a)). Alternatively,  $\ln(\phi_{s,a}/\phi_{m,a})$  can be freely estimated using age dummies in analogy to the first stage (model variants (b)).

To address the imposed rigidity of substitutability discussed in section 4.1, we consider three types of model relaxations. First, we allow for elasticities of substitution between age groups that are different across skill groups by replacing (6) with

$$L_{s,t} = \left[ \sum_a (\phi_{s,a} L_{s,at}^{\pi_s}) \right]^{\frac{1}{\pi_s}}, \quad s \in \{l, m, h\}. \quad (14)$$

The first stage now estimates

$$r_{s,at} = b_{s,a} + d_{s,t} - \frac{1}{\sigma_{As}} \ln(L_{s,at}) + \frac{1}{\sigma_{Am}} \ln(L_{m,at}) + \epsilon_{s,at}, \quad s \in \{l, h\}. \quad (15)$$

While this relaxation appears intuitively plausible, the hypothesis  $\sigma_{As} = \sigma_A$  for all  $s \in \{l, m, h\}$  is easily tested.

<sup>10</sup>Obviously, the model can be estimated in one step using nonlinear techniques. Following Card and Lemieux (2001), we proceed in three steps to avoid numerical difficulties. This is crucial since we apply bootstrapping to obtain standard errors.

<sup>11</sup>For an optimistic appraisal of this hypothesis see also Murphy and Welch (1992, 2001), for a more pessimistic review Card and DiNardo (2002).

A second relaxation, regarding the uniformity of the elasticity of substitution between the skill groups, can be implemented at the third stage by estimating

$$r_{s,at} = b_{s,a} + \beta_s t - \frac{1}{\sigma_{Ss}} \ln(L_{s,t}) + \frac{1}{\sigma_{Sm}} \ln(L_{m,t}) - \frac{1}{\sigma_{As}} \ln\left(\frac{L_{s,at}}{L_{s,t}}\right) + \frac{1}{\sigma_{Am}} \ln\left(\frac{L_{m,at}}{L_{m,t}}\right) + \epsilon_{s,at}, \quad s \in \{l, h\} \quad (16)$$

and testing whether  $\sigma_{Ss} = \sigma_S$  for all  $s \in \{l, m, h\}$ . Note however that this ad hoc-type relaxation comes at the price of abandoning the theoretical consistency of the model. In particular, the parameters  $\sigma_{Ss}$  are no longer elasticities of substitution. So the relaxation can be viewed as a specification test for the model.

This caveat also holds for our third specification which estimates parameters  $\sigma_{Ss\tilde{s}}$  freely across the set of equations:

$$\ln\left(\frac{w_{s,at}}{w_{\tilde{s},at}}\right) = b_{s\tilde{s},a} + \beta_{s\tilde{s}} t - \frac{1}{\sigma_{Ss\tilde{s}}} \ln\left(\frac{L_{s,t}}{L_{\tilde{s},t}}\right) - \frac{1}{\sigma_{As}} \ln\left(\frac{L_{s,at}}{L_{s,t}}\right) + \frac{1}{\sigma_{A\tilde{s}}} \ln\left(\frac{L_{\tilde{s},at}}{L_{\tilde{s},t}}\right) + \epsilon_{s\tilde{s},at} \quad (17)$$

for  $s, \tilde{s} \in \{l, m, h\}$  and  $s \neq \tilde{s}$ . In this case, we can test the hypothesis  $\sigma_{Ss\tilde{s}} = \sigma_S$  for all  $s, \tilde{s}$ .

To illustrate the adequacy of the nested model, a comparison with traditional CES models is in order. Thus, we also estimate models (9), (16), and (17), but restrict  $\sigma_A$  to infinity. While this procedure concentrates on the elasticity of substitution between skill groups,  $\sigma_S$ , it still allows for productivity differences across age.

Alternatively, we estimate a traditional CES model

$$r_{s,t} = \text{constant}_s + \beta_s t - \frac{1}{\sigma_S} \ln\left(\frac{L_{s,t}}{L_{m,t}}\right) + \epsilon_{s,t}, \quad s \in \{l, h\}, \quad (18)$$

again questioning the uniqueness of  $\sigma_S$ . Here, time-specific mean wage differences  $r_{s,t} = \ln(w_{s,t}/w_{m,t})$  are calculated as a weighted average

$$r_{s,t} = \frac{1}{L_{s,t} + L_{m,t}} \sum_a ((L_{s,at} + L_{m,at})(\omega_{s,at} - \omega_{m,at})), \quad s \in \{l, h\} \quad (19)$$

of time- and age-specific differences  $\omega_{s,at}$  estimated by pre-stage Tobit estimations

$$\ln(w_t) = \sum_s \sum_a \omega_{s,at} \cdot d_{s,at} + \text{controls}_t + \epsilon_t, \quad s \in \{l, m, h\} \quad (20)$$

for all periods  $t$ . As in equation (1),  $d_{s,at}$  indicate dummies for the different skill groups.

In contrast to the nested model, the latter procedure averages out the age dimension already at the pre-stage. Besides,  $L_{s,t}$  measures aggregate employment by an additive headcount rather than in efficiency units. Resulting elasticities should hence be comparable to those found in the literature.

### 4.3 Notions of Employment and Wages: An Estimation Issue

The different quantity concepts of employment are crucial to the analysis. The basic set-up constitutes a demand framework. However, studies for the US—including Card and Lemieux (2001)—often treat imputed quantities as inelastic (short-run) supply, implicitly assuming equality of supply and demand. Even if one considers the market clearing assumption to be reasonable for the US, it is highly questionable in the case of Germany, since it disregards unemployment driving a wedge between effective demand for and the supply of labor.

Moreover, both observed wage premia, i. e., the relative price of skilled labor, as well as observed relative employment generally result as outcomes of all labor market processes—and should therefore be treated as endogenous in the empirical implementation. Even if a classical interaction of labor supply and demand is regarded as inadequate, endogeneity of employment is implied, e. g., by wage-setting models with right-to-manage (RTM) assumption or efficient bargaining, in which wages are negotiated in consideration of the repercussions to the firms’ employment decisions (McDonald and Solow, 1981, Nickell and Andrews, 1983, or Arnsperger and de la Croix, 1990).

Consider an RTM framework, which—in contrast to efficient bargaining—does not abandon the assumption that observations on wages and employment lie on a demand curve. Then, the coefficient on relative employment  $-1/\sigma$  in any of the models above gages the (negative) relationship between wage premia  $r_s = \ln(w_s/w_m)$  and relative employment  $\ln(L_s/L_m)$  on the demand schedule. Not unlikely, however, actors additionally face unobserved shocks such as unexpectedly good or particularly bad business conditions which affect wages and employment in the same direction. Such shocks render relative employment endogenous and dilute the negative interrelation of interest. OLS (SUR) estimation yields (in absolute terms) downward-biased estimates of the true relationship or, put differently, upward-biased estimates for the elasticity of substitution  $\sigma$ .

As a remedy, we implement an instrumental variable (IV) approach by means of inelastic (short-run) labor supplies, which may reasonably be assumed as predetermined by past human capital investment decisions (Katz and Autor, 1999).<sup>12</sup> We compile measures of skill- and age-specific labor force numbers from German Microcensus data available at the Federal Statistical Office. Our set of instruments at the first stage contains the logs of age- and skill-specific labor supplies  $L_{s,at}^{\text{supply}}$ . The third stage of the procedure outlined above additionally incorporates the logs of aggregate supplies  $L_{s,t}^{\text{supply}} = \sum_a L_{s,at}^{\text{supply}}$ , interacted with age dummies. See the appendix for details on how to construct the instruments.

### 4.4 Estimation Results

Table 6 in the appendix summarizes the substitution parameters estimated at the first and at the third stage of various model variants, all of which assume a constant rate of SBTC, and reports the results of related specification tests. Our most preferred specifications are presented here in table 1.

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<sup>12</sup>Accounting for the endogeneity of relative employment may be considered even more important in face of traditional demand systems which often specify quantities as left hand side variables and treat prices/wages as exogenous; see, e. g., Hamermesh (1993).

Table 1: Elasticities of Substitution, Preferred Specifications of the Nested CES

Model Estimation	Version 3.1(a)		Version 3.1(b)		
	SUR	IV	SUR	IV	
$\sigma_A$	l	<b>15.87*</b> (3.46)	<b>11.77*</b> (2.98)	<b>21.88</b> (2.15)	<b>22.74</b> (2.95)
	m	<b>21.01*</b> (12.44)	<b>12.92*</b> (6.55)	<b>11.06</b> (1.16)	<b>11.17</b> (1.29)
	h	<b>20.19*</b> (2.70)	<b>16.64*</b> (3.20)	<b>17.60</b> (2.39)	<b>15.59</b> (3.86)
$\sigma_S$		<b>7.04</b> (3.86)	<b>4.69</b> (3.56)	<b>9.13</b> (3.15)	<b>6.73</b> (163.27)

Model versions: See table 6. (a) Age specific relative productivities predetermined by the calculations at the second stage. (b) Age specific relative productivities estimated by means of age dummies at the third stage. IV: Employment instrumented by labor force. Standard errors in parentheses estimated by 500 bootstrap replications. Bold numbers: Elasticities significantly finite (reciprocals significantly different from zero) at 0.95 level. \* Respective parameters identical at 0.99 level.

Data sources: IABS 1975–1997. German Microcensus.

Specification 3.1 (stage 3, relaxation 1) lets the elasticity between age groups  $\sigma_A$  vary across skill classes but sticks to a single elasticity of substitution across skill classes  $\sigma_S$ . As can be inferred from the tests in table 6, the assumption of identical  $\sigma_{As}$  would be overly restrictive. Variation of  $\sigma_S$  across skill groups (relaxation 2) on the other hand is not statistically required. The same holds for relaxation 3: According to table 8 in the appendix, variation of  $\sigma_S$  across equations does not seem necessary, either.

A comparison of results from the first and from the third stages in tables 1 and 6 reveals that the free estimation of age-specific relative productivities in the model variants (b) may be superior to versions (a) with respect to the assessment of  $\sigma_A$ . However, for stability of the  $\sigma_S$ -estimates the higher degree of preset structure in variants (a) appears preferable. The high standard errors in some cases may be attributed to two issues: First, the third stage includes aggregate employment measures as pre-generated regressors, the variation of which the bootstrap procedure takes account of. Second, the labor force numbers taken to instrument employment closely resemble linear time trends such that especially  $\sigma_S$ , the coefficient of predicted aggregate employment, is difficult to estimate.

Yet we obtain very plausible results. As expected, IV estimation yields lower estimates for  $\sigma_S$ , in particular. Along the reasoning of section 4.3, unobserved shocks affect particularly aggregate relative employment, rendering this measure endogenous and SUR estimates of  $\sigma_S$  inconsistent. Still, our IV estimates of  $\sigma_S$ , ranging from 4.7 to 6.7, imply a rather high degree of substitutability compared to findings in the related literature; cf. the synopses in Hamermesh (1993) and Katz and Autor (1999). Card and Lemieux (2001) report elasticities of substitution between college graduates and high school alumni for

Canada, the UK, and the US between 2 and 2.5.<sup>13</sup> In international comparison, our high elasticities may reflect the fairly small amount of wage dispersion in Germany or the more compressed distribution of skills; cf. Nickel and Bell (1996) and Freeman and Schettkat (2001).

Comparable studies for Germany also take account of three skill types, but they find elasticities not higher than 3.6.<sup>14</sup> Differences might in part be attributed to the selection of data, since we restrict our attention to prime age males in the IABS only, who may be relatively homogeneous.

At large, employees from different formal skill levels are more difficult to substitute than those with identical qualificational backgrounds. The substitutability across different age groups  $\sigma_{As}$  with values between 11.2 and 22.7 (version (b)) is lowest among the medium-skilled. This finding supports the view that low-skilled employees, mainly in positions which do not require any vocational training, can be substituted relatively easily. Contrary to the hypothesis that substitutability between young and old workers diminishes (monotonically) with educational attainment (Welch, 1979),<sup>15</sup> an analogous reasoning applies to university graduates of different age, whose education is often said to provide them with a high competence in general problem solving. Workers with a vocational degree, however, qualify for specific tasks such that, say, younger colleagues can take the place of older coworkers less easily.

In effect, all estimated elasticities of substitution and in particular the estimates for  $\sigma_A$  prove finite: Employees of different age are imperfect substitutes. The structural model consistently mirrors the dimensions of cohort effects uncovered by the descriptive inspection in section 2. Nevertheless, to put our results into further perspective, table 7 additionally reports the outcomes of models which assume perfect substitution across age classes. Third stage estimations restricting  $\sigma_A$  to infinity as well as the results of traditional CES models by and large draw a similar picture to the one stated above: Again, standard errors that account for the pre-estimation of the aggregate employment measure are very high. It is for this reason that we also estimated a specification 3.1(c) which—in analogy to the non-nested CES—uses the sums of all age-specific employment accounts rather than efficiency measures as aggregate quantities  $L_{s,t}$ . The results are very

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<sup>13</sup>Other studies quantifying elasticities for the US report  $\sigma$ -estimates within a similar range: Bound and Johnson (1992), Katz and Murphy (1992), and Krusell, Ohanian, Rios-Rull, and Violante (2000) acquaint 1.8, 1.4, and 1.7, respectively. Ciccone and Peri (2003) prefer a span between 1.2 and 2.2, and Stapleton and Young (1988) note a value of 3.0.

<sup>14</sup>Fitzenberger and Franz (2001) estimate elasticities of substitution between medium- and low-skilled of 0.6–1.4 for manufacturing and of 3.0–3.6 for non-manufacturing industries, while Steiner and Wagner (1998b) and Steiner and Mohr (2000) report values for all three classes of merely 0.3–0.5 for manufacturing and 1.4 for construction and transportation. Falk and Koebel (1999, 2002) find at most substitutability between medium- and low-skilled employees, whereas Koebel, Falk, and Laisney (2003) bilaterally classify high- and medium-skilled as well as medium- and low-skilled as substitutes, but they find complementarity between low- and high-skilled employees. Entorf (1996) finds elasticities between 0.5 and 1.5 for blue and white collar workers and Beißinger and Möller (1998) of 1.8 for males and 3.3 for females.

<sup>15</sup>Studies for the US report a much higher degree of substitutability between age classes within the group of high school graduates than among those with a college degree: Freeman (1979) finds elasticities of 14 and 2, respectively (even if the estimated reciprocals of both values show insignificant). Stapleton and Young (1988) note amounts of 73.6 (reciprocal insignificant) and 2.5. Card and Lemieux (2001) do not find any significant differences, though. They report significantly finite values of  $\sigma_A$  in the range of 4–6.

similar. Yet again, instrumentation proves relevant and reduces estimated elasticities from values around 8 to values between 5.5 and 6.1. Our results thus are consistent with employees in the sample indeed forming a fairly homogenous group of workers.

## 5 Two Simulation Experiments

In light of the ongoing policy debate about cures for unemployment, estimates from the above model can be used to assess the effect of wage changes on employment by means of simulation experiments similar to those conducted by Fitzenberger and Franz (2001).

First, we estimate the magnitude of wage changes in the three skill groups that would be necessary to induce, say, a reduction of unemployment rates<sup>16</sup> by one half in all three skill groups. The relative wage changes are assumed to be equal for all age groups within the respective skill groups:  $\Delta \ln(w_{s,a}) = \Delta \ln(\bar{w}_s)$  for all  $a$ . While allowing for relative changes between skill groups, this leaves the wage structure within skill groups unaffected.

We undertake the calculations for the base period 1997, the latest period available. In what follows, the time index  $t$  is omitted for notational simplicity. We use a first order Taylor approximation of overall employment in each skill group  $s$  as the sum of employment in the respective age groups  $a$ :

$$L_s^* = \sum_a L_{s,a}^* = \sum_a \left( L_{s,a} + \sum_{\tilde{s}} \sum_{\tilde{a}} \frac{\partial L_{s,a}}{\partial \ln(w_{\tilde{s},\tilde{a}})} \Delta \ln(w_{\tilde{s},\tilde{a}}) \right), \quad s \in \{l, m, h\}, \quad (21)$$

where  $L_s^*$ ,  $L_{s,a}^*$  are the employment targets consistent with the goal to reduce unemployment rates by one half. Drawing on the wage elasticity of labor demand

$$\eta_{s\tilde{s},a\tilde{a}} = \frac{\partial L_{s,a}}{\partial w_{\tilde{s},\tilde{a}}} \frac{w_{\tilde{s},\tilde{a}}}{L_{s,a}} = \frac{\partial \ln(L_{s,a})}{\partial \ln(w_{\tilde{s},\tilde{a}})} = \frac{\partial L_{s,a}}{\partial \ln(w_{\tilde{s},\tilde{a}})} \frac{1}{L_{s,a}}, \quad (22)$$

equation (21) can be written in terms of relative changes:

$$\frac{\Delta L_s}{L_s} = \frac{L_s^* - L_s}{L_s} = \sum_a \frac{L_{s,a}}{L_s} \sum_{\tilde{s}} \sum_{\tilde{a}} \eta_{s\tilde{s},a\tilde{a}} \Delta \ln(w_{\tilde{s},\tilde{a}}), \quad s \in \{l, m, h\}. \quad (23)$$

The relationship between wage elasticities  $\eta_{s\tilde{s},a\tilde{a}}$ , Allen-Uzawa elasticities of substitution  $\sigma_{s\tilde{s},a\tilde{a}}$ , and cost shares  $S_{s,a}$  implied by cost minimizing behavior of employers is given by

$$\eta_{s\tilde{s},a\tilde{a}} = S_{\tilde{s},\tilde{a}} \sigma_{s\tilde{s},a\tilde{a}} + S_{\tilde{s},\tilde{a}} \eta, \quad a \neq \tilde{a} \quad \vee \quad s \neq \tilde{s}, \quad (24)$$

where  $\eta$  denotes the price elasticity of product demand and

$$\eta_{ss,aa} = \eta - \sum_{\tilde{s}} \sum_{\tilde{a} \neq a} \eta_{s\tilde{s},a\tilde{a}} - \sum_{\tilde{s} \neq s} \eta_{s\tilde{s},aa} = S_{s,a} \eta - \sum_{\tilde{s}} \sum_{\tilde{a} \neq a} S_{\tilde{s},\tilde{a}} \sigma_{s\tilde{s},a\tilde{a}} - \sum_{\tilde{s} \neq s} S_{\tilde{s},a} \sigma_{s\tilde{s},aa}; \quad (25)$$

<sup>16</sup>The skill-specific and age-specific rates of unemployment in West Germany our simulations make use of are displayed in the appendix.

see, e. g., Hamermesh (1993). Based on the nested CES production function, inter-class Allen-Uzawa partial elasticities of substitution and intra-class elasticities,<sup>17</sup> respectively, write

$$\sigma_{s\tilde{s},a\tilde{a}} = \sigma_S, \quad s \neq \tilde{s} \quad \text{and} \quad \sigma_{ss,a\tilde{a}} = \sigma_S + \frac{1}{S_s}(\sigma_A - \sigma_S), \quad a \neq \tilde{a}. \quad (26)$$

In principle, cost shares for the nested CES model can be derived directly from the model via Shepard's Lemma as functions of the productivity parameters  $\theta_s$  and  $\phi_{s,a}$  and wages  $w_{s,a}$ ; cf., for example, Chung (1994). Yet the actual calculation fails this way due to the underidentification of the productivity parameters. Hence, we employ observed cost shares

$$S_{s,a} = \frac{w_{s,a}L_{s,a}}{\sum_{\tilde{s}} \sum_{\tilde{a}} w_{\tilde{s},\tilde{a}}L_{\tilde{s},\tilde{a}}} \quad \text{and} \quad S_s = \sum_a S_{s,a}. \quad (27)$$

The targeted relative change of employment can be inferred from the unemployment rates  $ur_s = U_s/WF_s = 1 - L_s/WF_s$ , where  $U_s$  and  $WF_s$  denote unemployment and work force in skill group  $s$ , respectively:

$$\frac{\Delta L_s}{L_s} = \frac{L_s^* - L_s}{L_s} = \frac{(0.5WF_s + 0.5L_s) - L_s}{L_s} = 0.5 \frac{ur_s}{1 - ur_s}. \quad (28)$$

As  $\eta$  we take a weighted average of the elasticities estimated by Fitzenberger and Franz (2001) separately for the manufacturing and the non-manufacturing sector, with employment ratios in the respective sectors as weights.

Since we set  $\Delta \ln(w_{s,a}) = \Delta \ln(\bar{w}_s)$  for all  $a$ , the system (23) yields unique solutions for the necessary wage changes based on our estimation results. The calculation of standard errors is based on the errors of the estimated parameters.

Alternatively, one might be interested in changes of the wage structure within the skill groups, holding the structure across the respective groups constant. In this context, the model set-up allows us to answer the question how the wages for employees of different age would have to change—identically in all skill groups—to reduce all age-specific unemployment rates  $ur_a = U_a/WF_a = 1 - L_a/WF_a$  by one half.

In analogy to (21), we write

$$L_a^* = \sum_s L_{s,a}^* = \sum_s \left( L_{s,a} + \sum_{\tilde{s}} \sum_{\tilde{a}} \frac{\partial L_{s,a}}{\partial \ln(w_{\tilde{s},\tilde{a}})} \Delta \ln(w_{\tilde{s},\tilde{a}}) \right) \quad \text{for all } a. \quad (29)$$

Now assuming  $\Delta \ln(w_{s,a}) = \Delta \ln(\bar{w}_a)$  for all  $s$ , the system

$$\frac{\Delta L_a}{L_a} = \frac{L_a^* - L_a}{L_a} = \sum_s \frac{L_{s,a}}{L_a} \sum_{\tilde{s}} \sum_{\tilde{a}} \eta_{s\tilde{s},a\tilde{a}} \Delta \ln(\bar{w}_{\tilde{a}}) \quad (30)$$

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<sup>17</sup>Given the model relaxation (14), the expression  $\sigma_A$  in equation (26) has to be replaced by  $\sigma_{A_s}$ .



can be solved for the necessary wage changes within the skill groups.

To evaluate the respective real magnitudes of the wage changes, we calculate the price adjustments induced by the nominal wage reductions. Here, the assumption of profit maximizing behavior under monopolistic competition takes account of endogenous output effects. We consider the Amoroso-Robinson relation for the output price level  $p$  and a constant elasticity of product demand  $\eta$ ,

$$\left(1 + \frac{1}{\eta}\right) p = MC, \quad \text{such that} \quad d \ln(p) = d \ln(MC), \quad (31)$$

with marginal costs

$$MC = \sum_s \sum_a w_{s,a} \frac{\partial L_{s,a}}{\partial y} = \sum_s \sum_a w_{s,a} \frac{L_{s,a}}{y} \frac{\partial L_{s,a}}{\partial y} \frac{y}{L_{s,a}} = \sum_s \sum_a \frac{w_{s,a} L_{s,a}}{y}. \quad (32)$$

The last step in (32) follows because of the constant returns to scale assumption. Relative price changes then arise from (31) as

$$d \ln(p) = \frac{\sum_s \sum_a \frac{L_{s,a} w_{s,a}}{y} d \ln(w_{s,a})}{\sum_{\tilde{s}} \sum_{\tilde{a}} \frac{L_{\tilde{s},\tilde{a}} w_{\tilde{s},\tilde{a}}}{y}} = \sum_s \sum_a \frac{L_{s,a} w_{s,a}}{\sum_{\tilde{s}} \sum_{\tilde{a}} L_{\tilde{s},\tilde{a}} w_{\tilde{s},\tilde{a}}} d \ln(w_{s,a}). \quad (33)$$

Now let  $\Delta \ln(w_{s,a}) = \Delta \ln(\bar{w}_s)$  for all  $a$  in the first experiment. Then,

$$\Delta \ln(p) = \sum_s \Delta \ln(\bar{w}_s) \sum_a \frac{L_{s,a} w_{s,a}}{\sum_{\tilde{s}} \sum_{\tilde{a}} L_{\tilde{s},\tilde{a}} w_{\tilde{s},\tilde{a}}}. \quad (34)$$

In the second experiment,  $\Delta \ln(w_{s,a}) = \Delta \ln(\bar{w}_a)$  for all  $s$ , and so

$$\Delta \ln(p) = \sum_a \Delta \ln(\bar{w}_a) \sum_s \frac{L_{s,a} w_{s,a}}{\sum_{\tilde{s}} \sum_{\tilde{a}} L_{\tilde{s},\tilde{a}} w_{\tilde{s},\tilde{a}}}. \quad (35)$$

Table 2 displays the outcome of the first simulation experiment and compares it to results obtained in Fitzenberger and Franz (2001). Considering the employment target of reducing skill-specific unemployment rates, wages paid are too high in all skill groups, and the necessary wage reductions—ranging from 8.7 to 12.3%—are the higher the lower the skill level. This result provides evidence for wage compression across skill groups. The fact that estimated wage reductions appear rather modest may be ascribed to at least two reasons: on the one hand to the high wage elasticities resulting from the substantial elasticities of substitution, and to the assumption of constant returns to scale on the other. The latter point becomes evident by the comparison of our results to those of Fitzenberger and Franz (2001): Their specification 4, which likewise postulates constant returns to scale, yields estimates very similar to ours, whilst their unrestricted specification 3 indicates higher (nominal) reductions. The range of dispersion, however, turns out rather similar in all models.<sup>18</sup>

<sup>18</sup>It is not unlikely that all results in table 2 overestimate actual necessary wage changes since neither Fitzenberger and Franz (2001) nor our estimations take into consideration substitution effects with respect to intermediate inputs or capital stocks. For the importance of the latter cf. Krusell, Ohanian, Rios-Rull, and Violante (2000).

Table 2: Wage Changes for Different Skill Groups Necessary to Halve Skill-Specific Unemployment Rates in 1997 and Induced Price Change

Model	$\Delta \ln w_l$	$\Delta \ln w_m$	$\Delta \ln w_h$	$\Delta \ln p$
3.1(a)IV <sup>a</sup>	-0.123 (0.0151)	-0.091 (0.0140)	-0.087 (0.0140)	-0.093 (0.0140)
3.1(b)IV <sup>a</sup>	-0.114 (0.0143)	-0.092 (0.0140)	-0.089 (0.0140)	-0.093 (0.0140)
3.1(a)SUR <sup>a</sup>	-0.113 (0.0142)	-0.092 (0.0140)	-0.089 (0.0140)	-0.093 (0.0140)
3.1(b)SUR <sup>a</sup>	-0.108 (0.0141)	-0.092 (0.0140)	-0.090 (0.0140)	-0.093 (0.0140)
Fitzenberger and Franz (2001) <sup>b</sup>	-0.141 (0.019)	-0.103 (0.020)	- (-)	-0.105 (0.020)
Fitzenberger and Franz (2001) <sup>c</sup>	-0.342 (0.099)	-0.313 (0.020)	- (-)	-0.314 (0.020)

<sup>a</sup> Calculations based on the results displayed in table 1. Standard errors in parentheses estimated by 500 bootstrap replications.

<sup>b</sup> Specification 4 of their model; assumption of constant returns to scale; elasticities of substitution between the high-skilled on the one hand and medium- and low-skilled on the other restricted to equal 1; no changes in wages and employment for the high-skilled; results for 1995.

<sup>c</sup> Specification 3 of their model; elasticities of substitution between the high-skilled on the one hand and medium- and low-skilled on the other restricted to equal 1; no changes in wages and employment for the high-skilled; results for 1995.

The induced relative price changes are a weighted average of the wage reductions; see equation (34). Thus, given our estimates of nominal wage reductions, the high-skilled experience a real wage increase, whereas the low-skilled face real losses ex constructione.

The results of the second experiment, regarding a reduction of age specific unemployment rates, are displayed in table 3. The calculated wage reductions in the different age groups are very similar. However, the small degree of variation comes as no surprise because the differences in unemployment rates across the age classes are rather small. As to the underlying high elasticities of substitution and concerning the interpretation of the induced price changes, the same caveats as for the first experiment apply.

## 6 Conclusions

Investigating descriptively the evolution of age-specific skill wage premia in the German labor market between 1975 and 1997 reveals that the age profiles of skill wage differentials have not moved parallelly over time, but rather experienced a twist. Accordingly, it is unlikely that these developments are associated merely with pure age and time effects. Furthermore, we observe a break in the inter-cohort trend of skill- and age-specific relative

Table 3: Wage Changes for Different Age Groups Necessary to Halve Age-Specific Unemployment Rates in 1997 and Induced Price Change

$\Delta \ln w_{27}$	$\Delta \ln w_{32}$	$\Delta \ln w_{37}$	$\Delta \ln w_{42}$	$\Delta \ln w_{47}$	$\Delta \ln w_{52}$	$\Delta \ln p$
-0.086	-0.086	-0.086	-0.085	-0.085	-0.086	-0.086
(0.0129)	(0.0129)	(0.0129)	(0.0129)	(0.0129)	(0.0129)	(0.0129)

Results identical for model versions 3.1(a) and 3.1(b) from table 1. Standard errors in parentheses estimated by 500 bootstrap replications.

employment such that young birth cohorts do not follow the path of the older ones towards further skill upgrading. The empirical evidence thus suggests the existence of cohort effects affecting the evolution of both skill wage premia and relative employment.

Testing for cohort effects as suggested by MaCurdy and Mroz (1995), we find cohort effects in the development of wage premia, but we can also confirm their separability from age and time effects.

Due to the heterogeneous nature of the input factor “labor”, a coherent operationalization of the above findings in general proves difficult. In light of the trade-off between explanatory power and practicality, however, an extension of the structural approach of Card and Lemieux (2001) based on the nested CES model of Sato (1967) draws a complex picture: On the one hand, it consistently maps rational behavior within the framework of neoclassical production theory. On the other, its age×time dimensioning allows to incorporate a relatively large number of input factors. That way, our extended implementation analyzes wage differences between 23×6 types of labor in 3 different skill classes, respectively.

The results are compatible with the steady demand hypothesis of a constant rate of SBTC in the notion of Acemoglu (2002). Moreover, employees of different age are found to be imperfect substitutes—the model indeed takes account of age, time, and cohort effects and, therefore, approves the intuitive link between the outlined stylized facts. Compared to the literature, our estimated elasticities of substitution are rather high. We reckon that employees in our data set are in fact considerably homogeneous.

On the basis of the estimated parameters, simulation experiments can give rise to policy-relevant implications. Similar to Fitzenberger and Franz (2001), we first simulate the magnitude of wage changes in the three skill groups that would have been necessary to reduce skill-specific unemployment rates in 1997 by one half. With wage changes equal for all age groups within the respective skill classes, this would have left the wage structure within skill groups unaffected. The necessary nominal wage changes range between 8.7 and 12.3% and are the higher the lower the employees’ qualification—a finding which provides evidence for wage compression: Compared to the reference situation of reduced unemployment, there is too little wage dispersion across the different skill groups.

Alternatively, we are interested in changes of the wage structure within the skill groups, holding the structure across the respective groups constant, and answer the question how

the wages for employees of different age would have had to change to cut all age-specific unemployment rates in half. Since age-specific unemployment rates are very similar, the similarity in wage changes for different age groups comes as no surprise.

In general, our analysis substantiates the necessity to integrate different dimensions of heterogeneity into meaningful models of labor demand. Yet our implementations show that, compared to available investigations solely focussing on the skill dimension of heterogeneity, the additional differentiation by age comes at the price of further restrictions on the production technology. In particular, the functional form restricts elasticities of substitution between (identically skilled) workers of different age to be all equal. However, if confronted with real data, the assumption that, say, a 55-year-old executive can be deputized by an experienced 50-year-old as well as by a 25-year-young entrant, is anything but beyond dispute.

Moreover, the neoclassical framework traditionally fails to incorporate residual wage inequality that remains after any grouping adapted from observed covariates; see, e.g., Card, Kramarz, and Lemieux (1999). But within-cell wage dispersion usually amounts to a major part of total observed dispersion—cf. Juhn, Murphy, and Pierce (1993) and Fitzenberger, Garloff, and Kohn (2003)—and therefore should be addressed by future research on the link between wage structure and labor demand.

## Appendix

Throughout the empirical investigation, we make use of the IAB employment subsample (IABS) 1975–1997, a representative 1% random draw of German employees with employment spells subject to social insurance contributions. Excluding civil servants, self-employed, and freelancers, the IABS covers about 80% of all employed persons. For an extensive description of these register-based data see Bender, Hilzendegen, Rohwer, and Rudolph (1996) and Bender, Haas, and Klose (2000). Selected data at first comprise spells of both men and women employed full-time in West-Germany, excluding parallel employment spells.

We restrict attention to prime-age employees between 25 and 55 years to circumvent a number of sample selection problems. Since the IABS contains no information on hours worked, we undertake a headcount to derive an employment measure, weighting each observation with the length of the respective employment spell. This procedure assumes that the number of, say, monthly hours does not change over time nor does it differ by individual, justifying the concentration on full-time employees only.

Concerning the wage data, Steiner and Wagner (1997) report a structural break between 1983 and 1984. In order not to deceptively interpret this as increasing wage inequality across skill groups, we apply the correction procedure suggested by Fitzenberger (1999).

Observations are classified into three skill groups according to the individuals' educational attainment. The group of the low-skilled consists of employees without any vocational training. Those with a vocational training are considered medium-skilled, and individuals with a university or technical college degree form the group of the high-skilled. To deal with measurement error in the education information when defining the skill groups, we correct the skill information such that formal degrees an individual has once obtained are not lost later.

Stage zero of the estimation approach estimates wage differentials by means of Tobit regressions due to the censoring of wage data induced by the social security threshold (“Beitragsbemessungsgrenze”). Observations are weighted by the respective length of the employment spell. As a first approach, equation (1) includes dummies for foreigners and women as control variables and further allows for possible interactions of these with the skill variables. Besides, a linear age term captures variation within the age classes. Cross terms of female and skill dummies prove significant in nearly all cells. Consequently, we base our analysis on males only. Period-specific wage differentials for the traditional CES are similarly estimated by pre-stage Tobit estimations (20), using age-specific skill dummies and a dummy for foreigners.

Estimation equations at the first and at the second stage include a full set of age dummies and time dummies for 1976–1997. The latter are replaced by a linear time trend at the third stage.

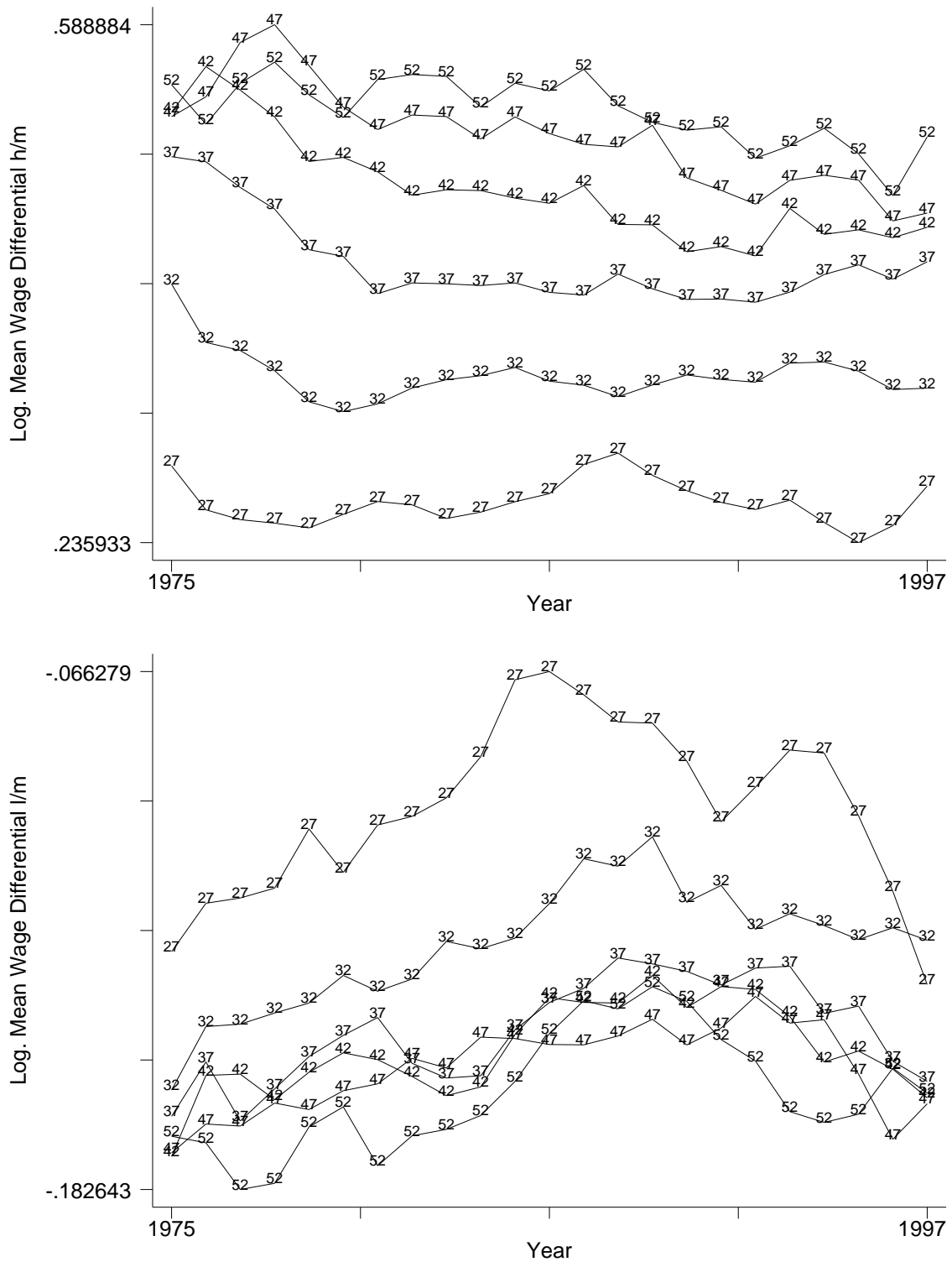
At stages one and three we instrument observed employment measures by means of the size of the labor force obtained from the German Microcensus, a representative 1% population sample collected annually, typically via face-to-face interviews. We use representative subsamples available through the Federal Statistical Office (“Statistisches Bundesamt”). The

cell-specific labor force is imputed as the sum of (male) employed and unemployed workers within the skill $\times$ age groups. For several years within our sampling period, however, individual records of educational attainment were voluntary, leading to sizable shares of missing values. We apply the procedure developed in Fitzenberger, Schnabel, and Wunderlich (2004) to assign the shares of missings to the three skill groups in each cell. For the years without any skill information in the German Microcensus, we interpolate; see also Fitzenberger (1999).

For the first simulation experiment, skill-specific unemployment rates are taken from Reinberg and Hummel (2002). Rates for the low-, medium-, and high-skilled in 1997 read 27.1%, 6.8%, and 3.0%, respectively. Age group-specific unemployment rates for the second experiment are calculated based on Statistisches Bundesamt (1998). For the six age groups (from young to old) the rates are 8.5%, 7.5%, 7.4%, 7.1%, 7.0%, and 8.1%.

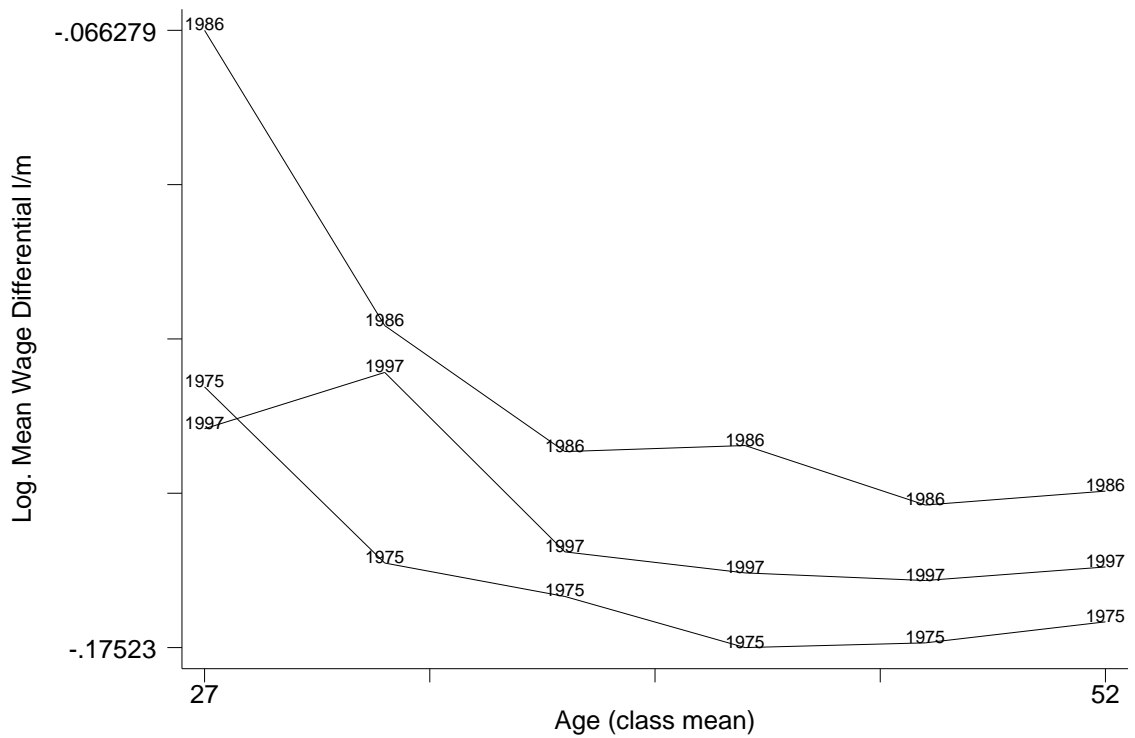
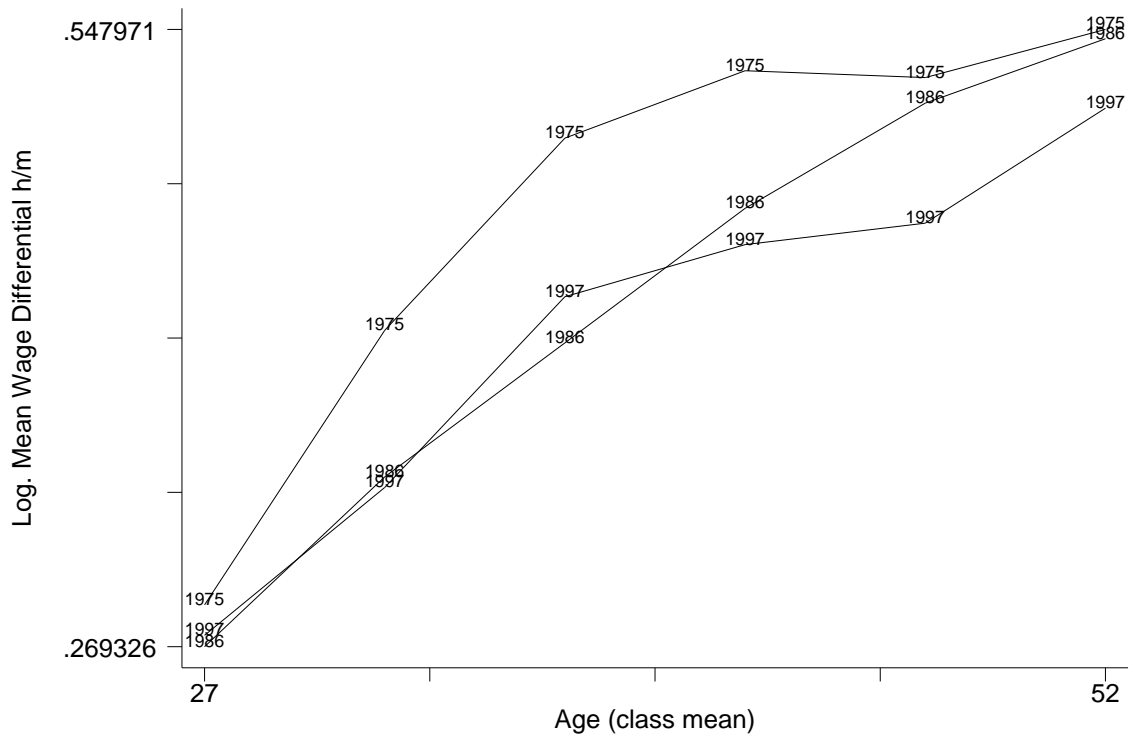
To obtain employment weights for the manufacturing and the non-manufacturing sector, we assign the IABS sector codes to the two categories as done in Fitzenberger (1999). Using the 1997-weights (0.4412 for manufacturing and 0.4746 for non-manufacturing), we calculate the price elasticity of demand,  $\eta$ , as a weighted average of the elasticities  $\eta_{\text{man}} = -0.7994$  and  $\eta_{\text{non-man}} = -0.1762$  estimated by Fitzenberger and Franz (2001). To draw inference on the estimated wage changes, we assume these elasticities to be independently normally distributed.

Figure 1: Evolution of Wage Differentials over Time



Calculations based on IABS 1975–1997. Digits within the graphs indicate the middle points of the respective age classes.

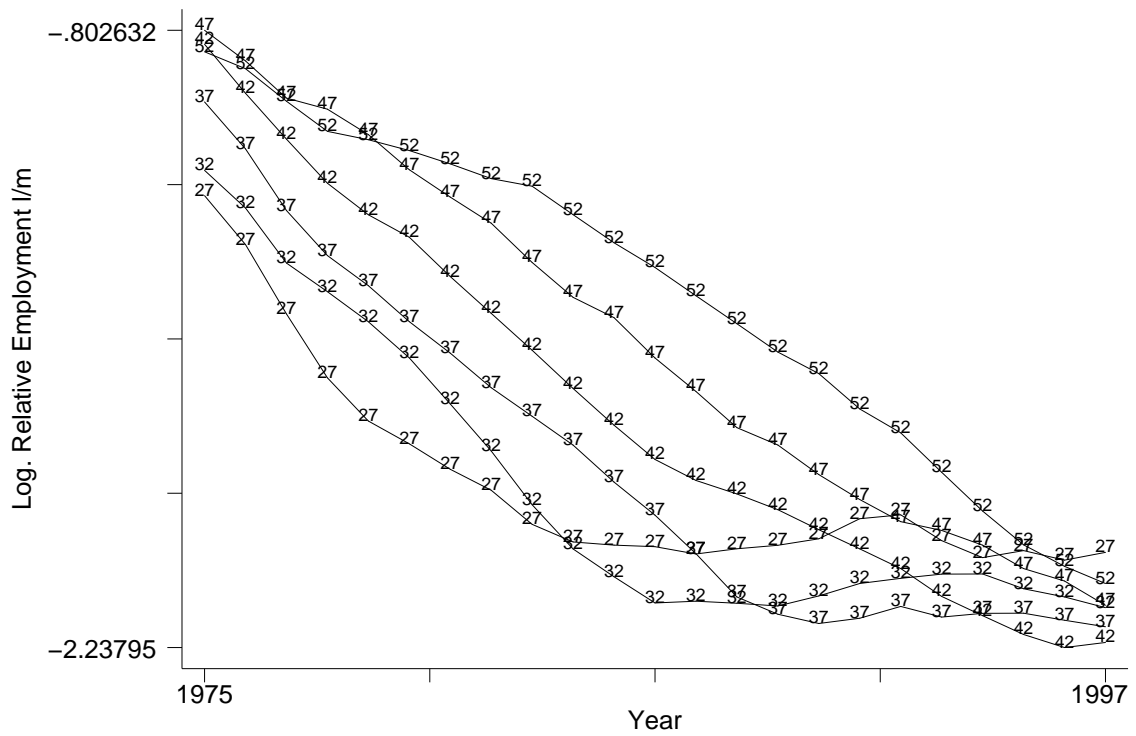
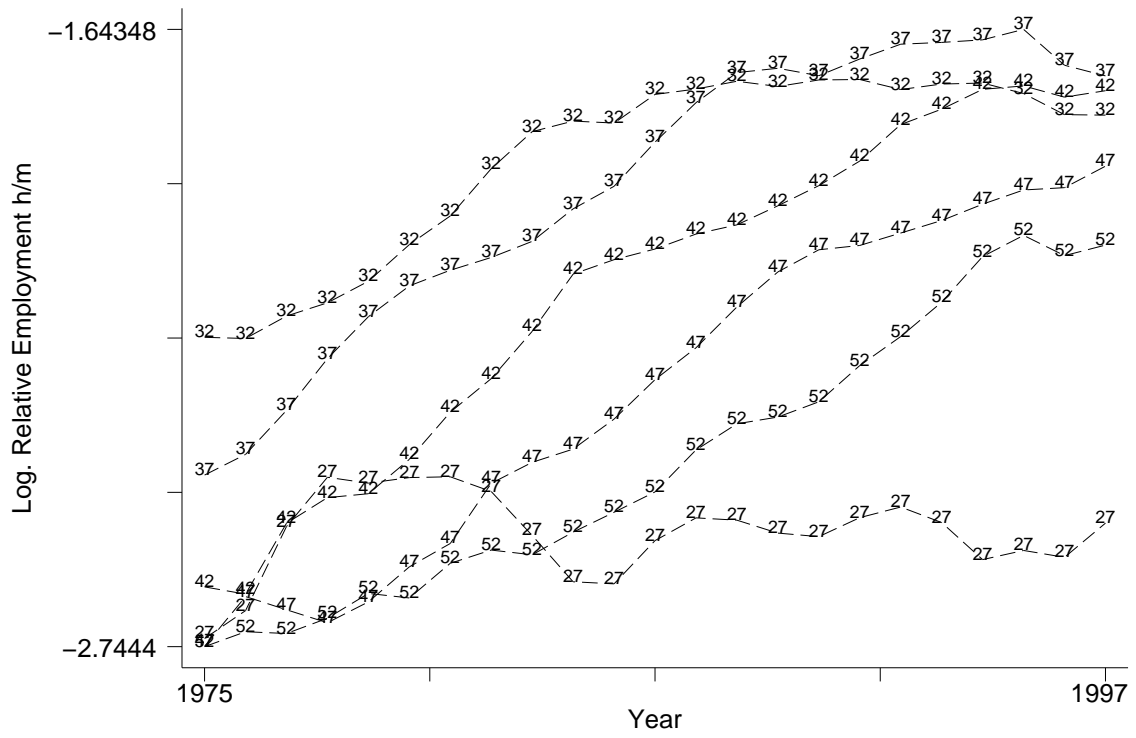
Figure 2: Age Profiles of Wage Differentials



Calculations based on IABS 1975–1997. Digits within the graphs indicate the calendar years of the respective age-time cells.



Figure 3: Trends in Relative Employment



Calculations based on IABS 1975–1997. Digits within the graphs indicate the middle points of the respective age classes.

Table 4: Estimated Wage Differentials by Age and Time

Age Time	25–29		30–34		35–39	
	$r_{l,at}$	$r_{h,at}$	$r_{l,at}$	$r_{h,at}$	$r_{l,at}$	$r_{h,at}$
1975	-0.1379 (0.0059)	0.2817 (0.0100)	-0.1796 (0.0058)	0.4062 (0.0102)	-0.1781 (0.0047)	0.4940 (0.0130)
1976	-0.1349 (0.0056)	0.2517 (0.0088)	-0.1756 (0.0063)	0.3668 (0.0092)	-0.1736 (0.0049)	0.4946 (0.0111)
1977	-0.1285 (0.0059)	0.2442 (0.0081)	-0.1766 (0.0068)	0.3628 (0.0087)	-0.1915 (0.0055)	0.4747 (0.0104)
1978	-0.1304 (0.0063)	0.2440 (0.0078)	-0.1765 (0.0071)	0.3462 (0.0084)	-0.1898 (0.0060)	0.4623 (0.0096)
1979	-0.1131 (0.0064)	0.2417 (0.0077)	-0.1775 (0.0070)	0.3273 (0.0076)	-0.1875 (0.0064)	0.4354 (0.0090)
1980	-0.1219 (0.0066)	0.2490 (0.0076)	-0.1668 (0.0071)	0.3164 (0.0074)	-0.1882 (0.0072)	0.4319 (0.0093)
1981	-0.1114 (0.0069)	0.2576 (0.0077)	-0.1714 (0.0073)	0.3218 (0.0071)	-0.1824 (0.0080)	0.4078 (0.0093)
1982	-0.1118 (0.0071)	0.2594 (0.0079)	-0.1630 (0.0076)	0.3328 (0.0069)	-0.1926 (0.0085)	0.4109 (0.0093)
1983	-0.1077 (0.0071)	0.2512 (0.0080)	-0.1474 (0.0082)	0.3394 (0.0069)	-0.1979 (0.0089)	0.4059 (0.0091)
1984	-0.1006 (0.0071)	0.2540 (0.0083)	-0.1478 (0.0086)	0.3435 (0.0070)	-0.2005 (0.0091)	0.4006 (0.0087)
1985	-0.0793 (0.0070)	0.2611 (0.0083)	-0.1446 (0.0088)	0.3494 (0.0070)	-0.1901 (0.0092)	0.4045 (0.0085)
1986	-0.0762 (0.0069)	0.2695 (0.0078)	-0.1385 (0.0090)	0.3432 (0.0068)	-0.1832 (0.0093)	0.4020 (0.0082)
1987	-0.0845 (0.0068)	0.2869 (0.0076)	-0.1283 (0.0089)	0.3439 (0.0067)	-0.1734 (0.0096)	0.3986 (0.0079)
1988	-0.0955 (0.0069)	0.2914 (0.0077)	-0.1273 (0.0085)	0.3348 (0.0062)	-0.1552 (0.0099)	0.4146 (0.0076)
1989	-0.0955 (0.0066)	0.2764 (0.0074)	-0.1250 (0.0084)	0.3420 (0.0062)	-0.1638 (0.0099)	0.4078 (0.0076)
1990	-0.1074 (0.0065)	0.2679 (0.0073)	-0.1434 (0.0079)	0.3488 (0.0060)	-0.1687 (0.0094)	0.3992 (0.0073)
1991	-0.1155 (0.0066)	0.2566 (0.0072)	-0.1353 (0.0078)	0.3442 (0.0060)	-0.1708 (0.0095)	0.4000 (0.0072)
1992	-0.1080 (0.0067)	0.2501 (0.0071)	-0.1439 (0.0077)	0.3390 (0.0058)	-0.1603 (0.0093)	0.3990 (0.0070)
1993	-0.0914 (0.0070)	0.2549 (0.0072)	-0.1448 (0.0078)	0.3523 (0.0058)	-0.1531 (0.0094)	0.4064 (0.0069)
1994	-0.0903 (0.0075)	0.2393 (0.0078)	-0.1489 (0.0078)	0.3526 (0.0057)	-0.1652 (0.0095)	0.4187 (0.0069)
1995	-0.1017 (0.0077)	0.2308 (0.0080)	-0.1583 (0.0081)	0.3468 (0.0058)	-0.1672 (0.0094)	0.4193 (0.0069)
1996	-0.1124 (0.0082)	0.2428 (0.0084)	-0.1565 (0.0083)	0.3322 (0.0059)	-0.1705 (0.0094)	0.4101 (0.0069)
1997	-0.1466 (0.0091)	0.2695 (0.0091)	-0.1467 (0.0086)	0.3347 (0.0060)	-0.1758 (0.0095)	0.4207 (0.0069)

Tobit estimations, standard errors in parentheses.  
Data source: IABS 1975–1997.

Table 4: Estimated Wage Differentials by Age and Time (Continued)

Age Time	40–44		45–49		50–54	
	$r_{l,at}$	$r_{h,at}$	$r_{l,at}$	$r_{h,at}$	$r_{l,at}$	$r_{h,at}$
1975	-0.1851 (0.0050)	0.5246 (0.0175)	-0.1795 (0.0052)	0.5215 (0.0196)	-0.1734 (0.0065)	0.5472 (0.0236)
1976	-0.1704 (0.0050)	0.5636 (0.0162)	-0.1775 (0.0053)	0.5540 (0.0171)	-0.1754 (0.0061)	0.5149 (0.0189)
1977	-0.1722 (0.0052)	0.5466 (0.0136)	-0.1812 (0.0054)	0.5939 (0.0175)	-0.1883 (0.0063)	0.0063 (0.0190)
1978	-0.1766 (0.0054)	0.5259 (0.0121)	-0.1785 (0.0057)	0.6039 (0.0173)	-0.1866 (0.0062)	0.5702 (0.0179)
1979	-0.1751 (0.0053)	0.5036 (0.0110)	-0.1802 (0.0056)	0.5755 (0.0153)	-0.1765 (0.0060)	0.5425 (0.0157)
1980	-0.1745 (0.0055)	0.5029 (0.0107)	-0.1773 (0.0056)	0.5548 (0.0144)	-0.1735 (0.0058)	0.5324 (0.0153)
1981	-0.1769 (0.0058)	0.4950 (0.0105)	-0.1721 (0.0057)	0.5383 (0.0137)	-0.1881 (0.0059)	0.5553 (0.0154)
1982	-0.1837 (0.0063)	0.4808 (0.0100)	-0.1682 (0.0057)	0.5349 (0.0122)	-0.1837 (0.0060)	0.5578 (0.0147)
1983	-0.1887 (0.0070)	0.4808 (0.0097)	-0.1740 (0.0060)	0.5362 (0.0116)	-0.1826 (0.0061)	0.5602 (0.0143)
1984	-0.1897 (0.0077)	0.4763 (0.0096)	-0.1677 (0.0061)	0.5192 (0.0109)	-0.1776 (0.0064)	0.5434 (0.0138)
1985	-0.1791 (0.0088)	0.4664 (0.0102)	-0.1676 (0.0064)	0.5328 (0.0109)	-0.1704 (0.0067)	0.5620 (0.0137)
1986	-0.1777 (0.0099)	0.4604 (0.0105)	-0.1714 (0.0068)	0.5181 (0.0104)	-0.1592 (0.0068)	0.5553 (0.0132)
1987	-0.1845 (0.0105)	0.4739 (0.0109)	-0.1766 (0.0074)	0.5113 (0.0106)	-0.1530 (0.0068)	0.5683 (0.0129)
1988	-0.1857 (0.0107)	0.4416 (0.0102)	-0.1780 (0.0080)	0.5050 (0.0102)	-0.1593 (0.0071)	0.5359 (0.0117)
1989	-0.1803 (0.0109)	0.4447 (0.0102)	-0.1821 (0.0088)	0.5138 (0.0107)	-0.1600 (0.0073)	0.5266 (0.0117)
1990	-0.1810 (0.0104)	0.4262 (0.0095)	-0.1909 (0.0098)	0.4781 (0.0109)	-0.1645 (0.0073)	0.5170 (0.0114)
1991	-0.1782 (0.0102)	0.4323 (0.0094)	-0.1892 (0.0112)	0.4656 (0.0117)	-0.1750 (0.0078)	0.5242 (0.0117)
1992	-0.1775 (0.0102)	0.4280 (0.0088)	-0.1834 (0.0114)	0.4534 (0.0113)	-0.1830 (0.0082)	0.4979 (0.0113)
1993	-0.1803 (0.0104)	0.4588 (0.0088)	-0.1876 (0.0118)	0.4739 (0.0114)	-0.1923 (0.0091)	0.5056 (0.0114)
1994	-0.1854 (0.0106)	0.4421 (0.0085)	-0.1841 (0.0119)	0.4769 (0.0107)	-0.1964 (0.0104)	0.5130 (0.0116)
1995	-0.1864 (0.0107)	0.4434 (0.0084)	-0.1980 (0.0118)	0.4737 (0.0107)	-0.2008 (0.0119)	0.4926 (0.0122)
1996	-0.1900 (0.0109)	0.4405 (0.0084)	-0.2122 (0.0114)	0.4479 (0.0099)	-0.1908 (0.0131)	0.4660 (0.0124)
1997	-0.1940 (0.0108)	0.4474 (0.0084)	-0.2041 (0.0116)	0.4562 (0.0098)	-0.2020 (0.0141)	0.5000 (0.0131)

Tobit estimations, standard errors in parentheses.  
Data source: IABS 1975–1997.

Table 5: Existence of Cohort Effects?

Coefficients	Wage Differential $l/m$				Wage Differential $h/m$			
DJ32	-.0317987	(-10.28)	-.0318108	(-9.17)	.1056535	(12.93)	.1060495	(16.97)
DJ37	-.0496214	(-8.56)	-.0496432	(-7.80)	.1916479	(14.20)	.1923991	(16.30)
DJ42	-.0553868	(-5.07)	-.0554281	(-4.68)	.2462518	(10.18)	.2474574	(11.28)
DJ47	-.0618105	(-3.14)	-.061881	(-2.90)	.2899294	(7.26)	.291567	(7.94)
DJ52	-.0639311	(-1.99)	-.0640378	(-1.86)	.3213904	(5.50)	.3234338	(6.21)
DT76	.0106519	(3.38)			-.0125476	(-1.15)		
DT77	.0074305	(2.18)			-.0121037	(-1.19)		
DT78	.0099738	(3.15)			-.0181724	(-1.50)		
DT79	.0170265	(5.46)			-.0421963	(-3.75)		
DT80	.0197538	(5.02)			-.0510009	(-4.43)		
DT81	.0196692	(4.85)			-.0542491	(-4.34)		
DT82	.0203808	(4.96)			-.0535146	(-4.18)		
DT83	.0212386	(4.14)			-.0553889	(-3.96)		
DT84	.0247125	(4.09)			-.0619131	(-3.94)		
DT85	.0330176	(4.80)			-.0565483	(-3.44)		
DT86	.0386849	(5.08)			-.0631226	(-3.50)		
DT87	.0411417	(4.65)			-.0590222	(-2.84)		
DT88	.0410905	(4.03)			-.0673472	(-2.88)		
DT89	.0445655	(3.82)			-.0716446	(-2.81)		
DT90	.0379228	(2.82)			-.0854641	(-3.01)		
DT91	.0360729	(2.47)			-.090025	(-2.85)		
DT92	.0370359	(2.05)			-.1003289	(-2.86)		
DT93	.0357092	(1.79)			-.0892978	(-2.27)		
DT94	.032124	(1.45)			-.0931998	(-2.17)		
DT95	.0291516	(1.17)			-.101762	(-2.14)		
DT96	.0244662	(0.87)			-.117647	(-2.24)		
DT97	.0205092	(0.64)			-.1065852	(-1.83)		
TIME			.0011869	(0.74)			-.0164975	(0.05)
TIME2			.000383	(1.29)			.0018165	(-0.61)
TIME3			-.0000241	(-1.21)			-.0001028	(0.23)
TIME4			2.58e-07	(0.56)			1.97e-06	(1.19)
R1	-.0005094	(-0.99)	-.000511	(-0.91)	.0000951	(0.06)	.0000929	(1.32)
R2	-5.90e-06	(-0.22)	-5.97e-06	(-0.21)	-.0000652	(-0.63)	-.0000645	(-1.22)
R3	.0001716	(1.87)	.0001719	(1.69)	.0001182	(0.24)	.0001191	(-0.99)
R4	2.95e-06	(0.45)	2.96e-06	(0.43)	.0000248	(1.29)	.0000247	(1.11)
COHORTA2	.0001901	(1.92)	.0001897	(1.84)	.0012808	(3.74)	.0013166	(4.06)
COHORTB2					.000722	(2.98)	.0007122	(2.90)
COHORTA3	-.0000107	(-3.70)	-.0000107	(-3.88)	-.000091	(-3.02)	-.0000944	(-3.16)
COHORTB3					.0000299	(2.03)	.0000298	(2.06)
COHORTA4					1.88e-06	(2.32)	1.97e-06	(2.41)
CONSTANT	-.1246824	(-39.77)	-.1205044	(-33.98)	.2743946	(25.33)	.2791018	(44.29)
Tests <sup>a</sup>								
Separability <sup>b</sup>	9.12*		7.35		7.36		8.60*	
Cohorts after 1975 <sup>c</sup>	34.35**		33.32**		36.18**		38.92**	
Any cohort effects <sup>d</sup>					263.93**		254.87**	

Data source: IABS 1975–1997. White robust t-values in parentheses. Specification of equation (4): Inclusion of additional polynomial cohort terms as long as neither the respective coefficient nor those of lower orders turn insignificant.

<sup>a</sup> Wald tests,  $\chi^2$ -values, \*(\*\*) Hypothesis rejected at 0.90 (0.95) level.

<sup>b</sup>  $H_0 : R_i = 0$  for all  $i$ .

<sup>c</sup>  $H_0 : R_i = \text{COHORTA}_{js} = 0$  for all  $i, j$ .

<sup>d</sup>  $H_0 : R_i = \text{COHORTA}_{js} = \text{COHORTB}_{hs} = 0$  for all  $h, i, j$ .

Table 6: Elasticities of Substitution, Specifications of the Nested CES

Model	1.0	1.1	3.0(a)	3.0(b)	3.1(a)	3.1(b)	3.2(a)	3.2(b)
$\sigma_A^{\text{SUR}}$	l	<b>22.10</b> (4.38)			<b>15.87*</b> (3.46)	<b>21.88</b> (2.15)	<b>22.05</b> (3.20)	<b>21.48</b> (2.12)
	m	<b>26.41</b> (6.50)	<b>11.09</b> (1.68)	<b>33.80</b> (13.77)	<b>26.28</b> (2.34)	<b>21.01*</b> (12.44)	<b>11.06</b> (1.16)	<b>11.31</b> (1.51)
	h	<b>16.80</b> (27.44)				<b>20.19*</b> (2.70)	<b>17.60</b> (2.39)	<b>16.94</b> (11.83)
$\sigma_S^{\text{SUR}}$	l						<b>9.85**</b> (8.62)	<b>10.37**</b> (10.70)
	m			<b>25.85</b> (6.37)	<b>8.80</b> (5.15)	<b>7.04</b> (3.86)	<b>9.13</b> (3.15)	<b>7.17**</b> (2.04)
	h						<b>8.51**</b> (38.13)	<b>7.19**</b> (4.87)
$\sigma_A^{\text{IV}}$	l	<b>23.31</b> (5.73)			<b>11.77*</b> (2.98)	<b>22.74</b> (2.95)	<b>22.35</b> (4.63)	<b>22.84</b> (3.21)
	m	<b>26.00</b> (6.97)	<b>10.59</b> (1.81)	<b>34.36</b> (23.93)	<b>27.12</b> (3.63)	<b>12.92*</b> (6.55)	<b>11.17</b> (1.29)	<b>10.67</b> (1.67)
	h	<b>11.91</b> (5.36)				<b>16.64*</b> (3.20)	<b>15.59</b> (3.86)	<b>12.40</b> (4.62)
$\sigma_S^{\text{IV}}$	l						<b>7.79**</b> (129.07)	<b>6.52**</b> (3048.8)
	m			<b>25.66</b> (6.77)	<b>6.92</b> (247.19)	<b>4.69</b> (3.56)	<b>6.73</b> (163.27)	<b>5.91**</b> (30.09)
	h						<b>6.01**</b> (51.65)	<b>8.06**</b> (172.03)

Model Labeling: ‘stage.relaxation(version)’. Versions (a): Age specific relative productivities predetermined by the calculations at the second stage. Versions (b): Age specific relative productivities estimated by means of age dummies at the third stage. IV: Employment instrumented by labor force. Standard errors in parentheses estimated by 500 bootstrap replications. Bold numbers: Elasticities significantly finite (reciprocals significantly different from zero) at 0.95 level. \*(\*\*) Respective parameters identical at 0.99 (0.95) level.

Data sources: IABS 1975–1997. German Microcensus.

Table 7: Elasticities of Substitution Between Skill Groups, Perfect Substitution between Age Classes Assumed

Model	3.1(a) <sup>#</sup>	3.1(b) <sup>#</sup>	3.1(c) <sup>#</sup>	3.2(a) <sup>#</sup>	3.2(b) <sup>#</sup>	3.2(c) <sup>#</sup>	CES	CES.2	
$\sigma_S^{\text{SUR}}$	l			<b>8.49**</b> (2650.5)	<b>9.19**</b> (1851.7)	<b>9.61**</b> (1.20)		<b>11.88</b> (2.35)	
	m	<b>5.98</b> (0.63)	<b>8.21</b> (2936.7)	<b>8.46</b> (0.91)	<b>7.93**</b> (676.67)	<b>6.66**</b> (683.48)	<b>6.53**</b> (1.04)	<b>8.40</b> (1.08)	<b>6.38</b> (1.20)
	h			<b>8.11**</b> (5412.2)	<b>6.55**</b> (674.22)	<b>6.46**</b> (1.83)		<b>4.88</b> (0.60)	
$\sigma_S^{\text{IV}}$	l			<b>5.60**</b> (268.92)	<b>5.82**</b> (582.81)	<b>6.03**</b> (1.63)		<b>8.47**</b> (2.69)	
	m	<b>6.01</b> (0.73)	<b>5.47</b> (402.95)	<b>5.54</b> (1.05)	<b>5.50**</b> (270.25)	<b>4.96**</b> (479.51)	<b>5.01**</b> (0.99)	<b>6.10</b> (1.23)	<b>5.31**</b> (1.07)
	h			<b>5.28**</b> (40598.1)	<b>6.80**</b> (6251.8)	<b>6.63**</b> (18.65)		<b>5.08**</b> (0.96)	

Model Labeling: ‘stage.relaxation(version)’. Versions (a): Age specific relative productivities predetermined by the calculations at the second stage. Versions (b): Age specific relative productivities estimated by means of age dummies at the third stage. Versions (c): Sum of age-specific employment as aggregate employment. <sup>#</sup> Perfect substitution between age classes assumed at third stage; standard errors in parentheses estimated by 500 bootstrap replications. IV: Employment instrumented by labor force. Bold numbers: Elasticities significantly finite (reciprocals significantly different from zero) at 0.95 level. \*(\*\*) Respective parameters identical at 0.99 (0.95) level.

Data sources: IABS 1975–1997. German Microcensus.

Table 8: Substitution Parameters, Variation Across Equations Allowed

Model	3.3(a)	3.3(b)	3.3(a) <sup>‡</sup>	3.3(b) <sup>‡</sup>	3.3(c) <sup>‡</sup>	CES.3	
$\sigma_A^{\text{SUR}}$	l	<b>16.04*</b> (3.50)	<b>21.62</b> (2.14)				
	m	<b>21.14*</b> (12.91)	<b>11.26</b> (1.19)				
	h	<b>20.37*</b> (2.71)	<b>19.81</b> (2.46)				
$\sigma_S^{\text{SUR}}$	lm	<b>6.95*</b> (3.89)	<b>6.99**</b> (2.23)	<b>5.15**</b> (0.60)	<b>8.19**</b> (3199.0)	<b>8.45**</b> (0.91)	<b>8.18</b> (1.00)
	hm	<b>7.00*</b> (3.89)	<b>6.80**</b> (2.96)	<b>6.69**</b> (1.12)	<b>8.46**</b> (3313.6)	<b>8.62**</b> (1.16)	<b>9.01</b> (2.74)
	hl	<b>7.25*</b> (4.00)	<b>7.31**</b> (2.15)	<b>-32.05**</b> (1090.9)	<b>8.28**</b> (1947.2)	<b>8.50**</b> (0.93)	<b>10.43</b> (2.16)
$\sigma_A^{\text{IV}}$	l	<b>12.17*</b> (3.15)	<b>22.86</b> (2.89)				
	m	<b>13.14*</b> (6.97)	<b>11.53</b> (1.37)				
	h	<b>17.38*</b> (3.31)	<b>16.98</b> (1.37)				
$\sigma_S^{\text{IV}}$	lm	<b>4.55**</b> (3.64)	<b>6.45**</b> (166.96)	<b>4.91**</b> (0.58)	<b>5.62**</b> (489.05)	<b>5.74**</b> (1.14)	<b>6.99**</b> (2.23)
	hm	<b>4.54**</b> (4.80)	<b>10.85**</b> (111.50)	<b>7.99**</b> (7.31)	<b>7.57**</b> (1292.1)	<b>7.36**</b> (5.72)	<b>6.80**</b> (2.96)
	hl	<b>5.27**</b> (6.53)	<b>7.68**</b> (89.04)	<b>1.29**</b> (16.70)	<b>6.24**</b> (356.89)	<b>6.24**</b> (1.29)	<b>7.31**</b> (2.15)

Model Labeling: ‘stage.relaxation(version)’. Versions (a): Age specific relative productivities predetermined by the calculations at the second stage. Versions (b): Age specific relative productivities estimated by means of age dummies at the third stage. Versions (c): Sum of age-specific employment as aggregate employment. <sup>‡</sup> Perfect substitution between age classes assumed at the third stage. IV: Employment instrumented by labor force. Standard errors in parentheses estimated by 500 bootstrap replications. Bold numbers: Elasticities significantly finite (reciprocals significantly different from zero) at 0.95 level. \*(\*\*) Respective parameters identical at 0.99 (0.95) level. Data sources: IABS 1975–1997. German Microcensus.

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