Wages, Rents and Heterogeneous Moving Costs.

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Abstract: The model of compensating differentials in regional labor and land markets was formalized by Roback (1982). The model interprets regional differences in constant quality wages and rents as compensating firms and residents for inter-regional differences in amenities. While the model assumes that the costs of relocating to a new city are zero, the results hold in the presence of moving costs for the marginal migrant. This paper extends the Roback model to allow for moving costs which vary among a city's residents and businesses. This modification of the model generates new interpretations of regional differences in rents and wages. The theoretical results suggests that the interpretation of inter-city rent and wage differentials as compensating is misguided, that such differentials are inappropriate as weights in *QOL* comparisons and stresses the importance of local housing and labor market parameters in the determination of these differentials. The importance of amenities is retained, but housing supply becomes the main other determinant of regional rents. Housing supply was ignored in the literature following on Roback's initial insight. The new perspective also provides a bridge between the neo-classical perspective implicit in Roback's approach and the newer literature on agglomeration economies.

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Introduction

The formalization of the idea that area differences in wages and rents compensate people and businesses for local differences in desirable local amenities is attributable to Jennifer Roback (1982). This important formalization built on Sherwin Rosen's (1974) seminal analysis of markets for bundled characteristics. An important difference between the two models is the use of a representative consumer (and firm) in Roback's model, as opposed to a focus on heterogeneity in preference and cost functions among firms and people in Rosen's paper. The Roback model also assumed for simplicity that moving costs are well approximated by zero. The addition of migration costs and heterogeneity was not seen as a major problem, as all the results of the model should still hold for the marginal migrant, defined as a person with zero migration costs.

In this paper, we loosen these two assumptions simultaneously, and show this change in the model modifies the interpretation of inter-city differences in wages and rents considerably. Intuitively, the presence of heterogeneous moving costs implies that at any combination of rents and wages, *some* people and businesses will be willing to locate in a city, given its amenity level. For the local labor market to be in equilibrium, an appropriate *amount* of residents and businesses must wish to locate in a city.[1] The quantity side of regional equilibrium is something mostly ignored by Roback and the literature that followed her. For any given amenity level, there are a set of points in rent-wage-population space that will satisfy local labor market equilibrium. However, this set of points includes a continuum of possible rents, populations and possibly wages. We

[1] Throughout the paper, we will use the term "city" in the urban economic sense. This term is approximated in the census data with urban areas or metropolitan statistical areas. We choose to use city because this is the concept the other terms approximate.

show that for any given amenity level, rents – and thus wages and population – are actually set by the local supply of housing.

This paper thus makes two significant contributions. First, the importance of housing supply and other housing market factors in the presence of heterogeneous costs to migration substantially changes the interpretation of coefficients in regressions of rents on local area characteristics. This undermines the use of such coefficients as "market price" weights in measuring local attractiveness. For instance, we show that in such a context amenities that restrict housing supply will appear more "valuable" than amenities that do not. Thus, in terms of computing local quality of life (*QOL*) indices, areas with amenities that also restrict supply (such as coasts, steep mountains, national parks, *etc.*) will appear more attractive than areas with amenities that do not (such as climate and cultural amenities).

A second contribution of the paper is that it allows the Rosen-Roback model to be related more closely with the newer literature on agglomeration. The model implies that a simple exogenous, productive amenity story cannot account for the positive correlation amongst rents, wages and city population we observe in the cross section of cities, with the link between population and the other two quantities being especially tenuous. Since the neoclassical assumptions do not imply that more productive cities will be more populated, it implies that more populated areas become more productive. That is, the causal relationship runs from population to productivity, not in the other direction. While this result is not too surprising, the incorporation of population in the analysis (which was missing in the *QOL* literature) allows for analysis of how population, rents,

wages and amenities interact when greater populations of firms and workers leads to higher productivity, as has been hypothesized in many papers for a variety of reasons.

The rest of the paper is organized as follows. Section I outlines the Rosen-Roback model and its core results. Section II incorporates heterogeneous moving costs, and shows that the results change substantially. The implied effects of amenities change both quantitatively and qualitatively, with housing market factors playing a significant role in the setting of regional rents, wages and populations. Section III makes some extensions of the model. We discuss the effects of more realistic labor market assumptions, the effects of housing supply constraints on the results, and the relationship between productivity, rents, wages and city population A final section concludes with some comments on the empirical content of the model, and some further rumination on the Rosen-Roback model.

I. The Rosen-Roback model.

Roback (1982) envisions a world where all people and business are the same. A firm is characterized by a cost function, which through profit maximization becomes a profit function that depends on local wages, rents and local attributes, or amenities. Costless migration implies that firms will relocate to new cities if they can earn higher profits there. In equilibrium, then, it must be the case that economic profits (profits in the city in question minus profits available elsewhere) must equal zero:

1) $$\pi(w, r; A) - \underline{\pi} = 0.$$

In equation 1, $\underline{\pi}$ represents the profits available to the firm in every other location.

Similarly, people in the Roback model have preferences, which after utility maximization generate an indirect utility function which also depends on wages, rents and amenities. Amenities are defined to be goods for people, although the same assumption is not made for firms. Amenities can either be productive (profit enhancing) or unproductive (profit reducing). Costless migration across cities implies that the utility available in any city be identical to the utility available in any other location:

2) $$v(r, w; A) - \underline{v} = 0,$$

where $\underline{v}$ is the utility available to a resident in every other location. As the discussion above makes clear $\pi_r < 0, \pi_w < 0$ and $v_r < 0, v_w > 0, v_A > 0$ while π_A is indeterminate in sign. Equations 1 and 2 implicitly define Π and V, which are indifference curves for firms and people in rent-wage-amenity space. Figure 1 shows the equilibrium condition as usually represented. For a given level of amenities, rents and wages in a city are determined by the condition that firms and residents are indifferent between the city and all other cities, and that rents and wages are the same for firms and residents.

Within this framework, the effect of amenities on rents and wages are derived by taking the derivative of the equilibrium price and wages, as implicitly defined by the equality of Π and V. Equations 3 and 4 show these derivatives and sign them for a productive amenity.

3) $$\frac{dr}{dA} = \frac{\pi_w v_A - \pi_A v_w}{\pi_r v_w - \pi_w v_r} > 0.$$

4) $$\frac{dw}{dA} = \frac{\pi_A v_r - \pi_r v_A}{\pi_r v_w - \pi_w v_r} \gtreqless 0.$$

The signs of these derivatives are also available from the manipulation of the indifference curves in Figure 1, as shown in Figure 2.

These results are important, because they provide a theoretical grounding for the use of the partial correlation of rents and wages with city characteristics as weights in constructing *QOL* indices, as was done by Roback and many later authors.[2] Under the model's assumptions, the regression coefficients that these equations represent are appropriate measures of the value of the amenities because they incorporate only the effects on utility and profits, and because they incorporate the preferences of both firms and residents. Combining the information from partial correlations of several amenities (indexed with *k*) with wages and rents across several local labor markets (controlling for differences in housing and worker quality) the residents' revealed willingness to pay for area Quality of Life (*QOL*) can be computed and compared across cities (indexed by *c*), as in equation 5.

$$5) \qquad QOL_c = \sum_k \left(\frac{dr}{dA_k} - \frac{dw}{dA_k} \right) A_{kc}$$

Rankings of cities based on such indices have been an important part of the amenity literature since Roback (1982), and their sensitivity to various changes in specification have been examined thoroughly. More important than the rankings themselves, however, is the underlying view of regional equilibrium the rankings represent. This view allows for the interpretation of inter-city differences in rents and wages as compensating differentials. While the rankings of cities may be of little import (in this model, after all, utility is the same in all cities), the view of regional price differences as equilibrium compensation for differences in quality of life is more fundamental, as it gives inter-city price differentials informational content.

[2] See Gyourko *et al.* (1999) for a review of this literature.

II. Heterogeneous moving costs

While Roback's model offers powerful insight into the processes that set regional wages and housing costs, it is perhaps too persuasive. There has been little research into the effects on the model of firm and worker heterogeneity and moving costs. Heterogeneity has generally been dealt with in a footnote noting that in its presence, the results hold for the marginal migrant. This marks a major departure from the original hedonic model of Rosen (1974), where the entire point is arguably to show the possibility of efficient sorting of buyers and sellers. The possibility of migration costs preventing the equilibrium described above was seen as making small deviations from equilibrium values possible, but has not been seen as either theoretically or empirically interesting.

We make one minor modification to the Roback model. Firms and residents are still assumed to be identical in their preference or cost functions, except for an idiosyncratic component representing costs of moving away from their current location. Equations 1 and 2 are thus rewritten as equations 6 and 7:

6) $\pi(w,r;A) - \underline{\pi} \geq \psi_j \sim F(\cdot)$

7) $v(r,w;A) - \underline{v} \geq \varphi_i \sim G(\cdot)$.

$F(.)$ and $G(.)$ represents the CDF of the gains to migration for firms and individuals, respectively. This change means that, for any given location (or, equivalently, any given level of amenities), firms and residents have identical preferences for wages, rents and amenities in general, but have idiosyncratic attachment to the location (moving costs). Because their preferences for wages, rents and amenities are the same, the idea of the reservation profits and utility ($\underline{\pi}$ and $\underline{v}$) is still valid. This set of assumptions makes it

impossible for us to address the sorting aspect of location choice as well as Rosen (1974) does. However, it simplifies the analysis.

The idiosyncratic attachments to a city could arise for a number of reasons. For people, investments in social ties, location-specific human capital investments (as in Krupka (2007)), a sentimental and unreasoning fear of change, uncertainty about other cities or a difficult-to-replace job in the current city (such as an academic position) could all increase migration costs. From the perspective of the firm, large investments in fixed capital, adaptation to local business norms or the use of locally concentrated distribution networks, as well as the personal interests of the firm's decision makers would generate similar attachment to the current location. In both cases, a purely idiosyncratic taste for the area could also exist. These attachments will vary across individuals. We find these assumptions so descriptively obvious as not to warrant further elaboration.[3]

Equations 6 and 7 imply that the indifference curves used in figures 1 and 2 represent the preferences (or profits) of only one person (or firm). For any given amenity level, rent-wage space will be characterized by a field of V and Π curves, each representing a different person or firm. City population of households and firms will be set by the proportion of firms and households for which inequalities 6 and 7 hold, which is determined by plugging the left hand side of each inequality into the CDF of the idiosyncratic attachments (F and G, respectively).[4] In general, for any given amenity level, indifference curves more to the left will be consistent with higher populations of firms and residents (because rents are lower for any given wage level), while indifference

[3] Tabuchi and Thisse (2002) embed such heterogeneity in a core-periphery model and show that the heterogeneity is a strong force for dispersion.

[4] In the following, we will normalize the utility and profit functions so that reservation utility and profits ($\underline{v}$ and $\underline{\pi}$) are zero.

curves to the right will be associated with fewer residents willing to live in the city at those rent-wage combinations. With population modeled as continuous, this set of indifferences curves could be thought of as two surfaces in rent-wage-population space sloping down as one moves away from the wage-axis.

While any rent-wage combination will be an equilibrium for *some* assumed combination of residential and commercial populations, only a subset of them will satisfy local labor market equilibrium, where the quantity of jobs equals the number of workers. That is, the city's economy will not be in equilibrium without an additional condition:

8) $$G\left(v(r,w;A)\right) = F\left(\pi(r,w;A)\right).$$

Equation 8 requires that the number of jobs be equal to the number of workers.[5] Figure 4 highlights the set of points in rent-wage space where labor market equilibrium is achieved. This set of points could also be thought of as the intersection of the two surfaces described above.

The imposition of labor market equilibrium line in equation 8 does two things. First, G-F=0 implicitly defines wage as a function of rents and amenities, w=$w(r; A)$. In Figure 4, $w(r; A)$ is drawn as a straight line with the mildest of positive slopes. In general, the slope of $w(r; A)$ will be

$$w_r = \frac{F'\pi_r - G'v_r}{G'v_w - F'\pi_w},$$

which cannot be signed without further assumptions. As higher rents drive away both firms and residents, the effect of higher rents on equilibrium wage will depend on whether these higher rents affect firm location more or less than residential location.

[5] To keep things simple, we assume that each resident works and each firm hires one worker. We discuss the effects of relaxing this assumption below.

The imposition of labor market equilibrium also defines the population of firms and residents in the city for any amenity-rent combination. We define the population of residents in the city as $\Omega(r;A) = G\left(v\left(r, w(r;A);A\right)\right)$. A similar formulation is available for the population of firms in a city. $\Omega(r; A)$ will be important later when we close the model. The effect of rents on city population is easily derived: $\Omega_r = G'\left(v_r + v_w w_r\right)$. Because w_r is ambiguous, the effect of rents on population may appear ambiguous, but the derivative can be shown to be unambiguously negative: higher rents drive down population, amenities constant.

It is the presence of $\Omega(r; A)$ that begins to set the current model apart from the Roback (1982) formulation, which neglected population levels for the most part. While area population is dealt with in Roback (1980), Roback (1988) and Bloomquist *et al.* (1988), population is set by dividing average housing demand into the exogenously determined land area, which is defined either as the land area of a county or the amount of land available with a certain measured level of amenity. None of these models allow for migration in or out of the area based on amenity, rent or wage levels. We consider this addition to be highly desirable, as it certainly says something about the attractiveness of a location if more people live in it. If there is a small town somewhere in Colorado with wages and rents similar to those in San Francisco, the additional information that only hundreds of people live in the small town while millions live in San Francisco is information which to us seems relevant in terms of understanding the nature of the amenities in the two locations.

While the effects of rents on wages and population are relatively easy to derive, the effects of amenities are somewhat muddled by the typology of amenities. We classify

amenities in three categories: productive, nonproductive and unproductive.[6] Productive amenities increase utility and profits; nonproductive amenities increase utility but do not affect profits, and unproductive amenities increase utility but decrease profits. In general, the effects of these kinds of amenities map directly onto the results from the standard Roback model, except that it does not yet make sense to talk about the effect of amenities on rents, since we have not yet derived equilibrium rents. Holding rents constant, $\Omega_A = G'\left(v_A + v_w w_A\right)$ and

$$w_A = \frac{F'\pi_A - G'v_A}{G'v_w - F'\pi_w} \ .$$

The signs of these partial derivatives will depend on the type of amenity. Productive amenities will increase population but have an ambiguous effect on wages. Nonproductive amenities will have a positive effect on population and a negative effect on wages. Unproductive amenities will have a negative effect on wages and an ambiguous effect on population.

For the partial effects above, it was necessary to hold rents constant because assuming labor market equilibrium in equation 8 did not actually close the model. For any distribution of moving costs and any level of amenities, there is a continuum of possible rent-wage-population combinations. To close the model, we must also assume that the local housing market is also in equilibrium. Equation 9 defines this condition:

9) $$S(r;C) = \Omega(r;A)D(r, w(r;A)) \ .$$

In equation 9, we introduce the housing supply function, which depends upon rents (positively, so that $S_r > 0$) and other cost factors (so that $S_C < 0$). The demand for housing depends on Ω, or population, and the per-capita demand for housing, D, which depends

[6] This 2nd term is new, and we are very open to better ones.

on rents and wages.[7] Although w_r is ambiguous of sign and D_w is positive, we will assume that $D_r+D_w w_r < 0$, so that a form of the law of demand holds.

Equation 9 means that S-ΩD=0 implicitly defines rent as a function of amenities and housing cost shifters: $r(A, C)$. Having equilibrium rents defined allows us to determine equilibrium wages, $w(r; A)$. Together, equilibrium rents and wages allow us to determine equilibrium residential (and firm) population, $\Omega(r; A)$. Thus, housing market equilibrium closes the model and we are able to derive the effects of any exogenous factor on rents, wages or population. In particular, we are able to derive the effect of a change in amenities on the equilibrium rental rate:

$$10) \qquad \frac{dr}{dA} = \frac{\Omega_A D + \Omega D_w w_A}{S_r - \Omega_r D - \Omega\left(D_r + D_w w_r\right)}.$$

This amenity effect bears little resemblance to the effect as derived in section I of this paper reproducing the Roback capitalization result (equation 3), despite the fact that it represents the "marginal" migrant. This underscores just how much the addition of heterogeneous moving costs affects the model. The Roback "open city" result can be reproduced by assuming that $G' = F'$ and taking the limit of equation 10 as these quantities approach infinity.[8] This exercise confirms that the Roback (1982) formulization is a special case of the heterogeneous moving costs model, where moving cost heterogeneity is eliminated. One interesting factor in equation 10 is the S_r term in the denominator. As this term approaches infinity (as housing is supplied more

[7] D represents the combined demand for land for each resident. This includes the resident's living space as well as his work space. If land and labor are strong compliments in production, increases in wages will decrease firms' demand for land while increasing residents' demand for land, making the total effect on demand for land of an increase in wage ambiguous. We will assume that D_w>0, but this assumption is not important for our results.

[8] G' and F' enter into equation 10 throught the Ω_A and Ω_r terms, which contain w_r and w_A as well as G'.

elastically) the rent effect of amenities approaches zero. This is a formulization of a result suggested casually in the conclusion of Glaeser *et al.* (2006), which is another paper stressing the importance of housing supply in the setting of city rents.

An important result arising from equation 10 is that its sign is actually ambiguous for any kind of amenity. If the amenity in question is productive, the first term in the numerator is positive but the second term is ambiguous. If the amenity is nonproductive, the first term is positive while the second term is negative. Finally, in the case of an unproductive amenity (which reduces profits but increases utility), the first term in the numerator is ambiguous while the second term is negative. The denominator is always positive. While ambiguous results are generally not considered as important as ones we can sign *a priori*, we think the ambiguity of equation 10 is an important result in its own right.[9] It underscores that in the presence of heterogeneous moving costs – which certainly exist in the world which generates our data – we know much less about the effects of amenities on rents and wages than the Roback open-city formulation suggests.

Furthermore, the importance of housing market factors both in the setting of equilibrium rents (and thus wages), and in equation 10 is new. While Glaeser *et al.* (2005), Glaeser and Gyourko (2005), and Gyourko *et al.* (2006) have been moving towards this conclusion from other directions, the above frames the importance of housing supply directly in a compensating differentials model. What is important about equation 10 is not so much that it is ambiguous of sign, but that it includes several non-preference factors, such as housing supply and housing demand parameters and the homogeneity of residents and firms (through the Ω_A and Ω_r terms). This is an important

[9] Bloomquist *et al.* (1988) generate ambiguous effects by assuming that population (set as described above) has an ambiguous productivity or congestion effect after Tolley (1974). Much of the ambiguity in equation 10 can be resolved if we assume that firms are not heterogeneous in moving costs.

difference from the Roback amenity effect, which depends only on preference and profit parameters. This difference raises questions about the interpretability of the assumed hedonic prices derived from hedonic regressions. If these coefficients reflect elasticities of housing demand and supply and the distribution of moving costs among firms and people as well as the effects of amenities on utility and profits, how appropriate are they as weights in a *QOL* index?

We can also derive the effects of amenities on wages:

$$11) \qquad \frac{dw}{dA} = w_A + w_r \frac{dr}{dA}.$$

Because the last term in equation 11 represents the product of two ambiguous terms, this effect is also technically ambiguous in sign for all types of amenity. Taking the limit of equation 11 as $G' = F'$ approaches infinity confirms (after much tedious algebra) that the Roback wage effect (equation 4) is nested inside the heterogeneous moving cost model. While we think the ambiguity of sign is important in equation 11, more important is the composition of the effect, which includes influences from the housing market as in equation 10 as well as all the influences of firm and resident heterogeneity through G' and F', which appear in both w_r and w_A.

We feel that the heterogeneous moving costs model is an important extension of the Roback (1982) model, and that it offers important insights into the nature of inter-area price differences and the setting of regional equilbirum in area-specific prices and population distributions. The results above suggest that it would take a very clever econometrician to extract appropriate *QOL* weights from cross-city hedonic regression coefficients, which would reproduce empirical estimates of equations 10 and 11. The

following section makes three extensions of the model that, we think, further underscore its significance either empirically or theoretically.

III. Some Extensions.

Driving the sign of a partial derivative to ambiguity is not a constructive contribution. In some sense, the simplification of a model is what allows us to get explicit signs in our theoretical relationships, and is the entire point of theory. We believe the empirical investigation of the importance of moving costs and amenity capitalization is an important next step in our understanding of cities' interaction and the workings of inter-city labor markets. The model also contains several factors not stressed in the original theory (w_A and w_r), but which are in principle observable, so we do not view this model as purely destructive or critical. Instead, we see it as improving our understanding of regional equilibrium in prices and populations. To that end, this section makes a few extensions to the heterogeneous moving costs model.

More realistic labor market assumptions

The imposition of labor market equilibrium in equation 8 assumes that each firm and each household demands or supplies a fixed amount of labor. This unrealistic assumption has not been one that has worried most authors in the *QOL* literature, but we relax it here to see if it has any important effects in the context of heterogeneous moving costs. To do so, we re-write equation 8 as follows:

12) $$\sigma G\left(v(r,w;A)\right) = \delta F\left(\pi(r,w;A)\right).$$

In equation 12, σ represents the household supply of labor and δ represents the per-firm demand for labor. As written, equation 12 has added nothing but two constants to the analysis. Allowing σ and δ to be functions of other variables, however, offers a straightforward way of assessing the effects of labor market adjustments on the intensive margin on the regional equilibrium.

The most obvious extension is to allow σ to depend positively on wages and δ to depend negatively on wages. Such a change has no *qualitative* effect on any of the model's results. All ambiguous relationships remain so, and none of the relationships we had been able to sign become ambiguous or switch sign. Allowing labor supply and demand to depend both on wages and rents also has little effect. If rents affect the supply and demand for labor, the only change in the model is that the effect of rents on wages (w_r) becomes more difficult to sign than before, and dependent on labor market parameters.

The only way in which a more realistic labor market has a qualitative effect on the results of the model is when labor demand and supply depend on wages, rents and on amenities. Such dependence could occur if amenities and leisure are compliments, for instance, so that people who live in high amenity areas tend to work less. Labor demand could depend on amenities if amenities are compliments to labor (say, because scenic mountain views make transport more labor intensive). If either of these two conditions apply ($\sigma_A<0$ or $\delta_A>0$), the partial effect of amenities on wages, holding rents constant (w_A) becomes ambiguous for all types of amenities (productive or unproductive). If neither of these conditions apply (neither $\sigma_A<0$ nor $\delta_A>0$), the model is qualitatively unaffected. These two possibilities ($\sigma_A<0$ or $\delta_A>0$), represent more than half the theoretical

possibilities, although the empirical importance of such labor market effects of amenities is clearly debatable. It does bear emphasis that the more realistic labor market assumptions will quantitatively change the values of the rent and wage capitalization of amenities, even if they have no qualitative effect. We believe that this puts additional burden on econometricians seeking to extract amenity value information from cross-city hedonic analysis.

Cost shifters and QOL indices

The importance of housing supply in equation 10 is one of its contrasts with equation 3. The traditional Roback formulization of the effect of amenities on rents did not include housing supply factors because with costless migration (at least for the marginal migrant), cities with low amenities could not support higher rents driven by local housing supply (cost) differences. To the extent that such differences increased rents, they would cause out-migration, thus lowering rents back to the level the local amenities made feasible. When migration costs are heterogeneous, however, cost-related rent increases can increase rents locally. While this will cause some people to move away (those with the lowest moving costs), some people will be willing to accept the higher housing costs in order to continue to capitalize on their local attachments. This result is easily shown by taking the derivative of the implicit rent function with respect to the cost term:

13) $$\frac{dr}{dC} = \frac{-S_C}{S_r - \Omega_r \delta - \Omega \delta_r} > 0 .$$

Given the above discussion, equation 13 is not at all surprising, although it bears emphasis that this result was simply unavailable in the Roback framework. The Roback result on local housing costs can be reproduced in this context by allowing Ω_r to

approach negative infinity. This drives the denominator towards positive infinity, and the derivative as a whole to zero. Thus, with regard to the effect of housing costs on rents, the difference between the heterogeneous moving cost model and the Roback model arises from a difference in our assumption on the rent-elasticity of city population. Roback implicitly assumes this elasticity is negative infinity, we assume it is something larger than that.

While equation 13 is not too surprising, a more important result is available if we allow for the existence of some amenities that affect the cost of land or of construction. There are many reasons why amenities may cause land to be more costly. For instance, in the canonical model of the monocentric city, high agricultural yields increase the opportunity cost of land city-wide. Such high agricultural yields could be the result of favorable climate. Rough terrain or large swaths of undevelopable area (such as water or national parks) could also increase the cost of land in a city by making land scarce or forcing longer commutes over or around these obstacles. Gyourko and Saiz (2006) show that topography also appears to have a positive direct effect on construction costs. As such features also offer considerable scenic and recreational value (are amenities) and sometimes increase profits (through shipping on coasts or mining in mountains) these features have two effects on local rents. First, they may increase them because of their value as amenities. Second, they will increase rents through their effect on land or construction costs in the metro area. Other factors that could have similar effects (through both amenity and cost effects) would include the risk of natural disaster or regulations restricting development such as a binding urban growth boundary or reactionary zoning (Glaeser *et al*. (2005)).

The effects on rents of such supply-restricting amenities will be different from those that do not restrict supply, as shown in equation 14.

14) $$\frac{dr}{dA} = -\frac{S_C C_A - \Omega_A \delta - \Omega \delta_A}{S_r - \Omega_r \delta - \Omega \delta_r}.$$

Equation 14 differs from equation 10 in that the term $S_C C_A$ has been added. This term represents the amenity's effect on the supply function through the cost term. What is somewhat troubling is that most natural amenities that leap to mind – coasts, mountains, parks – either restrict developable land or increase the cost of development. On the other hand, most cultural amenities have no supply effect. Equation 14 tells us that such cultural amenities will appear to be less important in the rental equation of a cross-city hedonic model than natural amenities that restrict housing supply. While equation 14 is still ambiguous for every kind of amenity, it is unambiguously greater when the amenity causes greater supply restrictions.[10]

The existence of this change in the effects of amenities on rents (and thus wages) is troubling because the *QOL* literature uses the effects of amenities on rents and wages as weights in the construction of all-encompassing *QOL* indices. Roback's beautifully argued theory leads us to believe that in estimating such weights, we are estimating equation 3 in the rent equation. However, heterogeneous moving costs imply that in fact we are estimating equation 10. While equation 10 may still be defensible as a weight in such an index, the situation is actually worse. For some amenities we are estimating equation 10, while for other, supply-restricting amenities, we are estimating equation 14, which will vary depending on the amenity's effect on construction costs, C_A. The net effect of this is that *QOL* indices will tend to overemphasize the value of supply-

[10] This result could be confirmed by taking the derivative of equation 14 with respect to C_A.

restricting amenities relative to supply-neutral amenities, and thus rank areas with high levels of supply-restricting amenities higher in quality of life indices than areas specializing in more supply-neutral amenities. Even approaches which avoid the direct estimation of equation 14 (as in Kahn (1995) and Cragg and Kahn (1997)) will be affected by this supply-restricting effect because the lower supply of housing in some areas will be pushing rents up, making them look more attractive. It is unlikely that this bias will be cancelled out by the information from the cross-city wage hedonic. Examining equations 5 and 11, we see that this canceling out of the bias could occur if w_r were exactly one. However, we cannot even be sure that w_r is positive, let a lone equal to one. If $w_r < 0$, the wage side of the *QOL* index will actually *exacerbate* the bias introduced in the rent equation.

We are not the first to suggest that coefficients from the cross-city hedonic might be biased. Gyourko *et al* (1991) make a similar point with regard to local public finance issues. However, the bias we highlight here is perhaps more vexing because, empirically, correcting for this tendency will be extremely difficult. As we do not observe the value people and businesses place on certain characteristics, or the level and patterns of development that would have occurred in the absence of the supply-restricting features, it will be very difficult to determine how much supply has been restricted in a given urban area and how much rents have responded to that supply restriction, as opposed to the utility- and/or profit-enhancing aspects of the features restricting supply. However, without making such a distinction, it is hard to imagine how the coefficients in a cross-city hedonic would be appropriate in assigning weights to area characteristics, even laying aside the concerns raised in section II. Such coefficients may be reflecting the

high average moving costs of an area's population as much as the great value the citizens place on their local characteristics.

Compensating differentials and agglomeration.

An objection to the model above is that it derives a negative relationship between rents and city population, and an indeterminate relationship between wages and rents. These results do not square with the strong correlation amongst wages, rents and city population as noted by Glaeser and Maré (2001). This apparent inconsistency is resolved by recalling that the negative relationship between rents and population and the ambiguous relationship between wages and rents holds only when the levels of amenities are held constant.

Of course, amenities differ across cities, so the partial relationship among population, rents and wages with amenities held constant – while important for our analytical results – is not of much use empirically. The positive correlation between rents, wages and population suggests the importance of productive amenities with much greater effects on firm costs than on individuals' utility. One might call these firm or business amenities.[11] In the model above, such non-utility enhancing amenities increase the attractiveness of a location for businesses but not for residents. In symbols, $\pi_A >> 0$, but $v_A = 0$. Plugging these values into the appropriate formulas generates $w_A > 0$ and $\Omega_A > 0$. The higher population and higher wages imply higher demand for housing in the city in question, and thus higher rents, as can be seen by plugging the derived partials into

[11] We make the distinction between firm amenities and productive amenities because amenities as usually treated are assumed to increase utility, while amenities are described as productive or unproductive depending on their effect on profits. A firm amenity would be assumed to benefit profits and could be characterized further by distinguishing between firm amenities that enhance or detract from utility.

equation 10. Thus if sites differ in the amount of this firm amenity, even if housing supply is the same across sites, the higher firm amenity sites could have higher populations, rents and wages in equilibrium.

However, because higher rents then have indirect effects on population and wages, the situation is not so simple. Equation 11 states that the effect on wages of amenities consists of a direct effect of amenities on wages (w_A which we know is positive) and an indirect effect through the amenity's effect on rents. This indirect effect depends on w_r, which is of ambiguous sign. If w_r is positive or near zero (or the rental effect is small), then the firm amenity's effect on wages will be positive. Strong negative relationships between rents and wages (amenities held constant) will imply ambiguous equilibrium relationships between wages and firm amenities. While technically ambiguous in sign, one might be willing to hazard a guess that this effect is in fact positive.

The effects of firm amenities on area population are quite a bit more ambiguous. The equilibrium effect of a firm amenity on city population is represented in equation 15:

15) $$\frac{d\Omega}{dA} = \Omega_A + \Omega_r \frac{dr}{dA}.$$

The first term of equation 15 is unambiguously positive, while the second term is unambiguously negative. Stacking the deck in favor of a positive sign is also difficult. For instance, assuming a very large direct effect ($\Omega_A >> 0$), will also increase the negative indirect effect (because *dr/dA* is a positive function of Ω_A). Decreasing the population effect of rent (Ω_r) will tend to increase the rent effect of the firm amenitiy (because *dr/dA*

is a negative function of Ω_r). Thus, the heterogeneous moving costs model does not generate a cross-sectional positive correlation amongst rents, wages and city population.

If the firm amenities discussed above exist as features of the landscape, then we call them "natural advantage." The results here suggest that a natural advantage story based on an exogenous increase in some factor which increases profits but has little effect on utility does not explain an important aspect of economic geography. On the other hand, if these firm amenities arise endogenously as a product of larger population, we call them agglomeration or urbanization economies, and we employ a very different set of theoretical tools in understanding them. An interpretation of the ambiguous results above is that exogenous features of the landscape cannot explain the patterns of rents, wages and city population we observe. That is, amenities do not cause large populations. On the other hand, these results suggest that the opposite story might be the case. Large populations must generate amenities. The story which emerges from this perspective is that larger populations generate firm amenities, which increase wages and rents.

Interest in agglomeration economies has been increasing since Krugman's seminal 1991 *Journal of Political Economy* piece synthesized many older strands from the regional science literature to generate a micro-theory general equilibrium model that exhibited strong divergence and several interesting dynamic possibilities. Ottaviano and Thisse (2004) offer a review of the substantial theoretical literature that followed. Other authors have offered other avenues from which similar dynamics might arise.[12] It has been difficult to assess the implications of this literature on the more neoclassical regional literature that follows Roback partly because Roback's model is mute on the

[12] Glaeser (1999), Berry and Glaeser (2005) and Helseley and Strange (1990) are a few. Also see Duranton and Puga (2004) for a review of some of the more current theories.

scale of the city. By bringing city size into the analysis, the assumption of heterogeneous moving costs allows for a connection between the *QOL* literature and the newer agglomeration literature because amenities can be produced endogenously. We believe this extension is extremely important since almost any amenity imaginable (with the exception of climate, terrain and possibly water cover) is endogenously generated by local populations. Cultural amenities, professional sports teams, land cover, architectural beauty, infrastructure, public services and a host of new economic geographic agglomeration economies and human capital externalities *all* arise endogenously from the local population. We see this as an important next step in our understanding of amenities. Tabuchi and Thisse (2002) embed amenities in an agglomeration model, but the amenities are assumed exogenous. The entire new economic geography theoretical literature can be seen as an attempt to build microfoundations for a set of endogenous firm amenities. Here we stress that the set of endogenous amenities that exist in the real world are much broader than those modeled in the recent agglomeration literature, and constitute most of the things that make cities cities, as opposed to overgrown towns.

While obviously lacking in microfoundation for the urbanization economies hypothesized, we believe this framework is attractive in that it is flexible enough to extend to additional analysis, which we do not pursue in this more limited paper. For instance, Roback, most of the agglomeration literature and this paper all assume perfectly functioning labor markets. This model is flexible enough to add labor market imperfections and/or dynamic labor market adjustments with local unemployment as an output. Such a dynamic extension of this static model could shed light on the cyclicality

of employment, wages and rents, and how that cyclicality may (or may not) be conditioned by agglomeration externalities or amenities.

A more dynamic frame for the model could also yield insight into the longer-term fates of economic regions. While most productive amenities will increase rent, some will favor businesses more than others. Highly firm-oriented amenities will tend to increase rent and wage both, while more resident-oriented amenities will tend to increase rent alone. Differences in these kinds of amenities (football stadiums versus stock markets) would thus exhibit themselves in different *relative* changes in rents and wages as population increases. If one views the historical development of a city as the slow drawing out of a curve in rent-wage-population space, the difference in kinds of amenities suggests that at any given point in time, the future course of the curve could depend on the *kinds* of agglomeration economies generated by the city's growth. Over time, these kinds of agglomeration economies could be affected by random chance, cultural or historical factors, or deliberate government decisions. As the national and global economy changes, relative values of different endogenous amenities might change, affecting the capitalization into wages and rents. Some cities might be left on growth paths that become non-optimal under the new prices. As they cannot go back in time, such changes could require painful transitions in the local economy. From this perspective, it becomes possible to begin asking questions about optimal growth paths in a much more nuanced way.

Whether these broadly drawn suggestions bear more formal fruit remains to be seen. Here we have merely drawn the connection between the line of research using a neoclassical form of regional equilibrium following Roback and the agglomeration

literature that has blossomed over the last decade and a half. As the *QOL* literature rests heavily on the assumption of a neoclassical regional equilibrium, and considering the recent importance of the agglomeration literature in both general interest and regional journals, we believe such a connection is crucial in the understanding of local quality of life comparisons and their relevance in a world of increasing returns. Hopefully the agglomeration literature will also benefit from the bridge between the two branches of regional economics.

IV. Conclusion

This paper has made a minor modification to the compensating differential model formalized by Roback (1982). By allowing for heterogeneous moving costs, the model's output changed considerably. First, the solution method used by Roback and those following her does not close the model: wages and rents do not depend only on amenities and preference parameters, but on a host of other factors. Second, implications for city population, which had been missing, are drawn out. Third, the vital importance of local housing market conditions comes to the fore. The model is not closed without local land supply, and the interaction between land supply and amenities can be seen as completely determining both rents and wages. Fourth, the theoretical partial derivatives of amenities on rents and wages become ambiguous in sign for all amenity types, and contain many extraneous terms which lead us to question their appropriateness as weights in *QOL* indices when they are derived from hedonic regressions. These results lead us to wonder whether such indices are even possible. Fifth, we show that if there is a relationship (either causal or coincidental) between certain amenities and local land supply, *QOL*

indices that use regression coefficients as weights will be biased in favor of areas with high concentrations of supply-reducing amenities. Finally, the inclusion of city size as an output of the model offers a bridge between the amenities or quality of life literature and the agglomeration literature that has been so important over the past decade. We consider these all to be significant contributions.

More generally, we consider it an important step in the literature in terms of generating testable hypotheses from the Roback model. The logic behind Roback's original paper was so forceful, and her exposition so clear that we believe that it has been easy to be blinded to the fact that the theory does not actually offer any testable hypotheses, except perhaps that wages and rents differ systematically across cities. Given that, however, any combination of rent and wage coefficients could be rationalized with the model by assuming that the characteristic in question was either productive or unproductive; an amenity or a disamenity. Roback thus provides us with more of a perspective for interpreting inter-city wage and rent differentials than a theory explaining them.

This paper suffers from many of the same limitations as the Roback paper. For instance, we assume that the city and system of cities is in equilibrium. We assume good information flows exits. The model is not dynamic. Foremost of these shared deficiencies is the fact that it is difficult to imagine a data set that could confirm the existence, for instance, of the supply restriction effect on prices. Any cross-sectional or time-series evidence can be explained by assuming a feature constricting supply is a productive amenity. Nonetheless, we hope that by beginning to force our gaze on alternative explanations and alternative interpretations of the data we can continue the

process of assessing the validity of Roback's assumptions. This is an important process because those assumptions determine the validity of the *QOL* indices based on them. These assumptions also imply a view of the important determinants of metropolitan prices. We have shown here that heterogeneous moving costs, if important, radically change the interpretation of inter-city price differentials and cast into doubt the possibility of a valid *QOL* index. We feel our assumptions are more realistic. What remains to be seen is whether the practical significance of the differences in results substantially changes our attitudes about such indices or about inter-area price differentials more generally.

References

Berry, Christopher and Ed Glaeser (2005). "The Divergence of Human Capital Levels Across Cities." *Papers in Regional Science,* vol. 84(3): 407-444.

Bloomquist, G.C., M.C. Berger and J.P. Hoehn (1988). "New Estimates of Quality of Life in Urban Areas." *The American Economic Review* vol. 78().

Cragg, Michael and Matthew Kahn (1997). "New Estimates of Climate Demand; Evidence from Migration." *Journal of Urban Economics,* vol. 42: 261-284.

Duranton, Gilles and Diego Puga (2004). "Micro-foundations of Urban Agglomeration Economies." chp. 48 in *Handbook of Regional and Urban Economics,* vol. 4, V. Henderson and J.F. Thisse, eds. Amsterdam: North-Holland: 2063-2117.

Glaeser, Edward (1999). "Learning in Cities." *Journal of Urban Economics,* vol. 46: 254-277.

Glaeser, Edward and Joseph Gyourko (2005). "Urban Decline and Durable Housing." *Journal of Political Economy*, vol. 113: 345-375.

Glaeser, Edward, Joseph Gyourko and Ravan Saks (2005). "Why is Manhattan so Expensive?: Regulation and the Rise in House Prices." *Journal of Law and Economics*, vol. 48(2): 331-370.

Glaeser, Edward, Joseph Gyourko and Ravan Saks (2006). "Urban Growth and Housing Supply." *Journal of Economic Geography* vol. 6(1): 71-90.

Glaeser, Edward and David Maré (2001). "Cities and Skills." *Journal of Labor Economics,* vol. 19(2): 316-342.

Gyourko, Joseph, Christopher Mayer and Todd Sinai (2006). "Superstar cities." Wharton Real Estate Center Working Paper, June 2006.

Gyourko, Joseph and Albert Saiz (2006). "Construction costs and the supply of housing structure." *Journal of Regional Science*, vol 46(4): 661-680.

Gyourko, Joseph, Matthew Kahn and Joseph Tracy (1999). "Quality of Life and Environmental Comparisons," chp. 37 in vol.3 of in *Handbook of Regional and Urban Economics,* Edwin S. Mills and Paul Cheshire, eds. Amsterdam: North-Holland.

Gyourko, Joseph and Joseph Tracy (1991) "The structure of local public finance and the quality of life." *The Journal of Political Economy,* vol. 99(4): 774-806.

Helseley, R.W. and W.C. Strange (1990). "Matching and agglomeration economies in a system of cities." *Regional Science and Urban Economics,* vol. 20: 189-212.

Kahn, Matthew (1995). "A revealed preference approach for ranking city quality of life." *Journal of Urban Economics,* vol. 38: 221-235.

Krugman, Paul (1991). "Increasing Returns and Economic Geography." *The Journal of Political Economy,* vol. 99: 483-499.

Krupka, Douglas (2007). "Location-specific Human Capital, Amenity Valuation and Location Choice." IZA discussion paper number . . . ..

Ottaviana, Gianmarco I.P. and Jacques-Francois Thisse (2004). "Agglomeration and economic geography," chp. 58 in *Handbook of Regional and Urban Economics,* vol. 4, V. Henderson and J.F. Thisse, eds. Amsterdam: North-Holland: 2563-2608.

Roback, Jennifer (1980). *The Value of Local Urban Amenities*: *theory and measurement*. Ph.d. dissertation, University of Rochester.

Roback, Jennifer (1982). "Wages, Rents and the Quality of Life." *The Journal of Political Economy,* vol. 90(6): 1257-78.

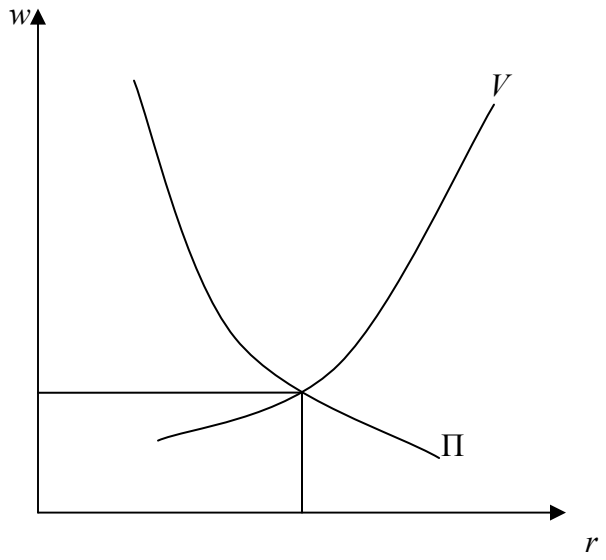
Roback, Jennifer (1988). "Wages, Rents and Amenities: Differences among workers and regions." *Economic Inquiry*, vol 26(1): 23-41.

Rosen, Sherwin (1974). "Hedonic Prices and Implicit Markets: Product differentiation in pure competition." *The Journal of Political Economy,* vol. 82(1): 34-55.

Tabuchi, Takatoshi and Jacques-Francois Thisse (2002). "Taste heterogeneity, labor mobility and economic geography." *Journal of Development Economics*, vol. 69: 155-177.

Tolley, George S. (1974). "The Welfare Economics of City Bigness." *Journal of Urban Economics*, vol 1: 324-345.

Figure 1: equilibrium in the Roback Model



Firgure 2: Effect on wages and rents of a productive amenity

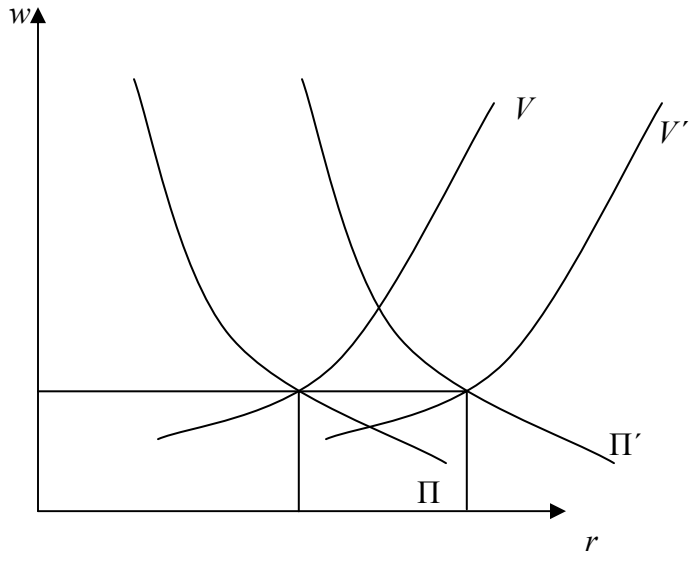


Figure 3: Π and V with heterogeneous moving costs

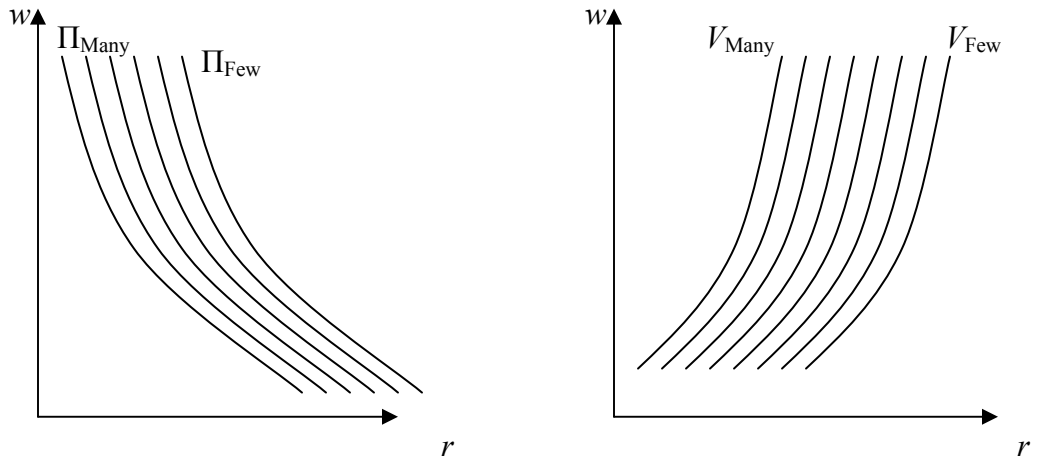


Figure 4: Local labor market equilibirum