Competing for College by Human Capital Investment

Tiantian Dai[*], Chao He[†], Xiangting Hu[‡], Xiangbo Liu[§]

May 15, 2014

Abstract

Students face competition. They need to invest in pre-college human capital to compete for better opportunities to further invest in their human capital. We build a two-sector search and matching model where students, before going to the labor market, choose their pre-college human capital investment to increase their chances of being admitted by colleges, which further increases their productivity and allows them to search for skilled jobs. We then study how educational policies, such as subsidy, scholarship, and capacity expansion of colleges affect individual's human capital investment and unemployment rate. We find that scholarship and subsidy always increase pre-college education investment and lower unemployment for college graduates. Capacity expansion, when there is no heterogeneity in innate ability, increases pre-college human capital investment and lowers educational unemployment. This is an example of more human capital investment caused by less competition. Once heterogeneous innate ability is incorporated, its overall effects could reverse.

JEL classification: I20; J21; J64

Keywords: College Admission; Competition; Human Capital; Educational Unemployment

[*]Email: dait@ruc.edu.cn, Hanqing Advanced Institute of Economics and Finance, Renmin University of China.
[†]Email: he.chao@ruc.edu.cn, Hanqing Advanced Institute of Economics and Finance, Renmin University of China.
[‡]Email: xiangthu@ruc.edu.cn, Hanqing Advanced Institute of Economics and Finance, Renmin University of China.
[§]Email: xiangbo.liu@ruc.edu.cn, Hanqing Advanced Institute of Economics and Finance, Renmin University of China.

Abstract

Students face competition. They need to invest in pre-college human capital to compete for better opportunities to further invest in their human capital. We build a two-sector search and matching model where students, before going to the labor market, choose their pre-college human capital investment to increase their chances of being admitted by colleges, which further increases their productivity and allows them to search for skilled jobs. We then study how educational policies, such as subsidy, scholarship, and capacity expansion of colleges affect individual's human capital investment and unemployment rate. We find that scholarship and subsidy always increase pre-college education investment and lower unemployment for college graduates. Capacity expansion, when there is no heterogeneity in innate ability, increases pre-college human capital investment and lowers educational unemployment. This is an example of more human capital investment caused by less competition. Once heterogeneous innate ability is incorporated, its overall effects could reverse.

JEL classification: I20; J21; J64

Keywords: College Admission; Competition; Human Capital; Educational Unemployment

1 Introduction

Should a government make access to higher education easier, either by providing subsidies, scholarship or expanding the capacity of public colleges? How will these policies affect the labor market(s)? Specifically, will the unemployment rate of college graduates go up or go down? These are some fundamental yet not adequately answered questions regarding higher education reforms. These are also particularly relevant questions due to rapid changes in the development of higher education all over the world. Shofer and Meyer (2005) document that the number of college graduates increased globally from 29 million to over 141 million between 1970 and 2006. Over the same period, the number of college students in the United States increased from 3.3 million to 17.5 million. The dramatic increase in tertiary enrollments did not only take place in developed countries. Many developing countries have also embarked on large education reforms aimed at expanding their tertiary education, at astonishing rates. China, for instance, started its reforms on higher education in 1999 and raised her college enrollments from 1.08 to 6.85 million in only 13 years.[1] These changes can undoubtedly have important implications for the supply and demand in the labor market.

Economists have little disagreement on whether educational policies can have a big impact on college enrollment. As Kane (2006, 2007) summarizes, a common finding in the literature is that a $1,000 tuition subsidy can increase college enrollment by 5 percent in the US. However, there are insufficient studies that theoretically investigate the economic consequences of these policies, especially their labor market implications. In their theoretical work, Charlot and Decreuse (2005) show that granting schooling subsidy can lead the less able to acquire education, reducing the average quality of college graduates. This in turn lowers firms' incentive to post job vacancies. Therefore, the rise in the supply of educated workers would not be matched by an increase in demand, which gives rise to higher unemployment rate for educated workers.

[1] For a more detailed review on these discussions, see Freeman (2009).

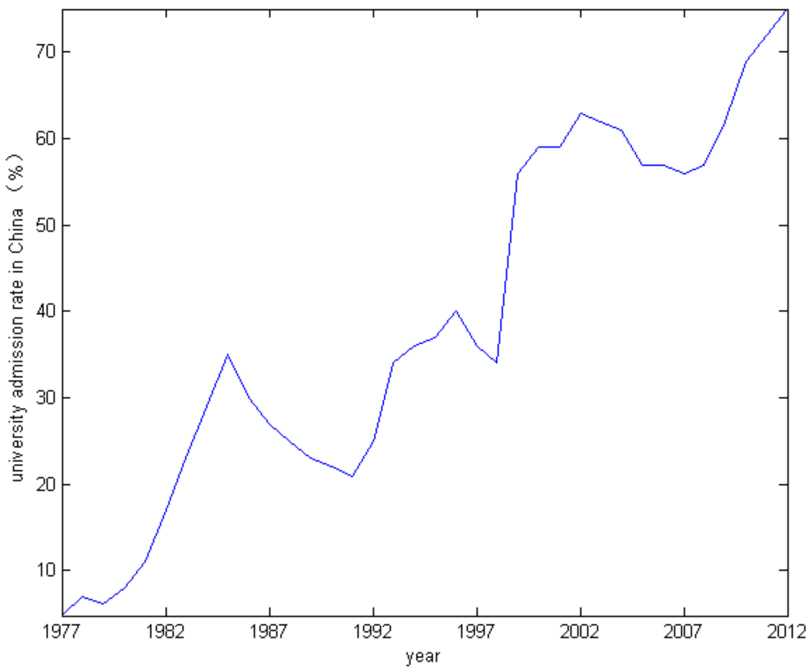
However, one common assumption made by previous studies, such as Saint-Paul (1996), Charlot and Decreuse (2005), and Charlot et al. (2005), is that, if more students want to attend college, the schooling sector can at the same time freely expand admission and allow for more enrollments. While this might be the case for some countries, it is an unsatisfying assumption in at least two aspects. First, in countries where there is excess demand for college education, an increase in subsidy itself may increase applications but is unlikely to affect enrollment, while the capacity of higher education becomes an important and separate policy instrument. Second, even in countries like United States where the supply of higher education is rather elastic, the admission in top universities could be more selective if applications increase. With these two observations, one thing becomes clear: students need to work hard, i.e. invest in their pre-college human capital, in order to get (better) opportunities to further increase their human capital. Any policy that could affect such competition could also change the pre-college human capital investment decisions, and may in turn have an effect on the labor market outcomes.[2] This intensive margin caused by competition for college admission has not been considered by previous studies.

Just how fierce is such competition? Figure 1 describes the evolution of China's university admission rate[3] between 1977-2012. The competition is substantial. In 1977, 5.7 million high school students sat through the national college entrance exam, when the enrollment was set to be 270 thousand, resulting in an overall admission rate as low as 4.8 percent. In 1999, China implemented a number of reforms designed to increase higher education opportunities for the population at large. The overall admission rate increased dramatically but we could still see the excess demand and the competition to get into higher-ranked universities remains extremely fierce. Such intense competition has also led an increasing number of Chinese students to study overseas. In fact, China is not the only country where students must face fierce competition to go to universities. Students in countries like India, South Korea, Japan and so on also need to take some brutal university entrance exam. The stress has even increased the suicide rate among young people in South Korea significantly, as reported by a New York Times article "Asia's College Exam Mania"[4].

[2]Two commonly used instruments of education policy reform include: (1) directly expanding the supply of higher education, taking the form of an increase of enrollments in existing universities and of an increase in the number of universities; (2) providing substantial subsidy or scholarship to ensure that university education is accessible.

[3]Source: China Ministry of Education. Note: this admission rate is defined as the total college enrollment divided by the total number of students taking the college entrance exam. After taking the yearly exam, students may apply for colleges. Admission decision is based almost entirely on students' scores in the college entrance exam.

[4]Asia's College Exam Mania, The New York Times, November 6, 2013. http://www.nytimes.com/2013/11/07/opinion/asias-college-exam-mania.html?_r=0.



Unless we take the idea in Spence (1973) literally and think that the efforts students put before college (in order to get into college or better colleges) makes no difference for their ultimate labor market productivity, we should consider it as a genuine part of the total human capital investment. The main objective of this paper is to take this pre-college human capital investment caused by competition into account and separately evaluate two types of education policies, subsidy and capacity expansion. The labor market implications are of special interest. To this end, we develop a search and matching model and add an important feature: an education sector that makes college admission decisions based on the signals students send. The quality of the signal sent by an individual depends on her pre-college human capital investment. As a starting point, we fix enrollment when studying the effects of subsidy and treat enrollment as a separate policy instrument.

Using this new framework, we show that the two types of policy reform can deliver very different labor market implications. Granting more subsidies now leads to lower unemployment for the educated. This is because given a fixed number of college admissions, education subsidy induces students to put more efforts to raise their chance to be selected into college. This in turn will translate into higher level of productivity and thus induce firms to post more vacancies. This result can be seen as isolating the intensive margin while excluding the extensive margin discussed in Charlot and Decreuse (2005). That is why we obtain the opposite results regarding education subsidy.

Then will the capacity expansion by colleges decrease the unemployment rate of college graduates? The answer is subtle. When there is no heterogeneity in the innate ability, we prove that capacity expansion always encourages pre-college human capital investment and lowers the unemployment rate for college graduates. It is because the pre-college human capital is assumed to be useful for college graduate (white collar workers) but not for high school graduates (blue collar workers), so when the capacity expands, there are less chances that such investment is wasted, which encourages investment.

The increase in human capital level, in turn, increases firm entry and lowers the unemployment of college graduates. This is an example of more human capital investment caused by less competition. Of course, once heterogeneous innate ability is introduced, capacity expansion would allow less able individuals to get higher education and eventually look for jobs as a college graduates. This channel is similar to Charlot and Decreuse (2005). Due to this extensive margin, pre-college human capital investment and employment of college graduates could theoretically fall, depending on the degree of heterogeneity in the ability endowment.

Our theoretical results have rich empirical implications and of great policy interest. First, if one were to examine the empirical relationship between subsidy and labor market outcomes, our model suggests that the supply elasticity of college education should be taken into account. In the extreme cases, the supply that with zero elasticity and that with perfect elasticity will generate exactly opposite predictions for subsidy on labor market outcomes. Second, many people have argued that capacity expansion will deteriorate the quality of students, and we offers a clear counter example by our version of the model with no heterogeneity. Our study imply that its overall impact on labor market may well depend on the degree of heterogeneity of innate abilities.

The paper is organized as follows. Section 2 provides a search and matching model and characterize the equilibrium. Section 3 discusses the impacts of the two types of education policy reforms. Concluding remarks are given in section 4.

2 The Model

2.1 Model Setup

There is a unit measure of risk-neutral heterogeneous individuals in continuous time. Individuals in the model economy differ in their initial ability e, which is assumed to be uniformly distributed on $[\underline{e}, \bar{e}]$, with $\underline{e} > 0$. At each instant, all individuals are facing a constant risk, γ, of dying, while measure γ are born into unemployment. Hence, the size of the population remains constant. The newborns get i.i.d. draws of initial ability from the same distribution, so that the distribution of initial ability is stable across time. Let r denote the individual discount rate as well as the interest rate of the economy.

Before going to the labor markets, each individual chooses her pre-college human capital investment before college I so that her pre-college ability is $a = e + I$. We assume that the cost to make such investment is $C(I)$, with $C(0) = 0$, $C'(\cdot) > 0$, and $C''(\cdot) > 0$. Then everyone applies for college, but due to the capacity constraint, only a fraction of them can be admitted. The admission process will be explained later. Once an individual is admitted, college education will improve her ability from a to ρa, with $\rho > 1$. For simplicity, we assume that pre-college and college education do not take time.

On the labor markets, we assume that only college graduates will be able to work in the high-skilled market, where the productivity of a college graduate will be $\rho a y_H$. Similarly, individuals without a college degree will go to the low-skilled market where a job that needs no special skill. Therefore, their productivity does not depend on one's ability and human capital investment and is assumed to be y_L.

We index the two labor market by $i = H \ or \ L$. In the following two subsections we discuss the college admission rule and how labor markets work.

2.1.1 College Admission Rule and Education Policies

We assume that there is one centralized college that makes admission decisions. The process is as follows. After making the pre-college human capital investment decision on I, each agent applies for college by sending a signal s to the college. The signal depends on her pre-college human capital level, a. The signal s satisfies a conditional distribution on $(0, \infty)$, with cumulative distribution function $F(s|a)$, probability density function $f(s|a)$. We assume that $E[s|a] = a$ and that the conditional distribution satisfies Monotonic Likelihood Ratio Property (MLRP), i.e. $\frac{f(s|a)}{f(s|a')}$ increases in s when $a > a'$. The idea is that conditional on any signal s (for example, a higher SAT score), the higher s is, the greater probability that the agent's pre-college ability a is large. The college determines a threshold $\bar{s}$ for admission according to its capacity σ, that is, those who send signals higher than $\bar{s}$ will be admitted by the college[5]. Formally, $\bar{s}$ is determined by

$$1 - E_a[F(\bar{s}|a)] = \sigma, \tag{1}$$

where the subscript "a" implies that the expectation is taken with respect to the distribution of a. Apparently, the threshold $\bar{s}$ is based on the distribution of pre-college ability a, which is chosen by the agents.

We would consider three educational policies: scholarship, tuition subsidy and capacity expansion. Note that when awarding scholarship, the government can only observe individual's signal s, not ability a. Let $\xi(s)$ be the government transfer to an individual with signal s. Obviously, $\xi(\cdot)$ is a non-decreasing function. We assume $\xi(s) = \phi_0 + \phi s$, where ϕ_0 can be regarded as a tuition subsidy that does not depend on s, and ϕs can be regarded as the (expected) scholarship award. Our scholarship policy is consistent with reality in the sense that top students receive reward with higher probabilities[6]. In addition, the government can enlarge the capacity of college, that is, σ. After we introduce the labor markets, we will discuss how these three policies affect individual's human capital investment and unemployment rate.

2.1.2 Firm Entry and Labor Markets

As described above, college graduates look for jobs in the high-skilled market while the others look for jobs in the low-skilled sector. There are an unlimited number of potential firms. In each point of time, potential firms can pay a fixed cost d to be active, i.e., to enter either the high-skilled market or the low-skilled market. Each new firm holds one vacancy, which needs a worker to occupy and produce. The labor markets for both sectors are subject to searching-matching frictions. Specifically, let $M(\tilde{u}_i, \tilde{v}_i)$ be the total number of employer-worker contacts in market i ($i = H,\ L$), where u_i and v_i denote the

[5] Suppose the target of college is to admit students with higher as. With MLRP, it is easy to show that the optimal strategy for the college is to conduct the cutoff strategy.

[6] More generally, one can assume both the size of each scholarship and the chance of getting these awards are separately two functions of s (the latter may also be a function of others' signals). Here we assume the expected award of the scholarship is a linear function of (only) one's own signal. This simplification greatly reduce analytical complications and should not change the qualitative results. But most importantly, it still captures the main difference between tuition subsidy and scholarship: one is guaranteed once a student is admitted, the other depends on one's performance before college.

numbers of unemployed and vacancy in market i, respectively. We assume that $M(\tilde{u}_i, \tilde{v}_i)$ is continuous, strictly increasing, strictly concave with respect to each of its argument and exhibits constant returns to scale. Denote $\theta_i = \frac{\tilde{v}_i}{\tilde{u}_i}$ the labor market tightness for market i. Hence, each vacancy is filled with the rate $m(\theta_i) = \frac{M(\tilde{u}_i,\tilde{v}_i)}{\tilde{v}_i}$, and each unemployed worker finds a job according to the rate $\frac{M(\tilde{u}_i,\tilde{v}_i)}{\tilde{u}_i} = \theta_i m(\theta_i)$. In order to ensure that the matching function is well-behaved, we assume that

$$\lim_{\theta_i \to 0} m(\theta_i) = \infty, \lim_{\theta_i \to \infty} m(\theta_i) = 0, \lim_{\theta_i \to 0} \theta_i m(\theta_i) = 0, \lim_{\theta_i \to \infty} \theta_i m(\theta_i) = \infty.$$

In addition, $m(\theta_i)$ decreases in θ_i and $\theta_i m(\theta_i)$ increases in θ_i.

At each point, an individual is in either of two states: unemployed (U) or employed (E). Each active firm can be at one of the two states: it can either hold a vacancy (V) or a filled job (F). Once a worker is matched with a vacancy, production commences immediately. For the low-skilled market, the productivity of a match does not depend on the ability of worker. We directly apply the commonly used Nash bargaining to determine wage, with β being the bargaining power of the workers. We denote J_L^κ (L for low-skilled market), $\kappa = U,\ E$, as the present discounted value of workers in either of their states, and J_L^κ, $\kappa = V,\ F$ as the value function for firms in either of their two states. Specifically, we have the following equations for the low-skilled market:

$$\begin{aligned} rJ_L^V &= -d + m(\theta_L)[J_L^F - J_L^V] && (2) \\ rJ_L^F &= y_L - w_L - (\zeta + \gamma)[J_L^F - J_L^V] && (3) \\ (\gamma + r)J_L^U &= m(\theta_L)\theta_L[J_L^E - J_L^U] && (4) \\ (\gamma + r)J_L^E &= w_L - \zeta[J_L^E - J_L^U] && (5) \end{aligned}$$

where w_L is the wage rate in the low-skilled market. Free entry for firms and Nash bargaining require the following two conditions:

$$\begin{aligned} J_L^V &= 0. && (6) \\ \beta[J_L^F - J_L^V] &= (1-\beta)[J_L^E - J_L^U] && (7) \end{aligned}$$

Equations (2)-(7) can be used to pin down the wage rate w_L and value J_L^U and thus we obtains the equation that describes the market tightness θ_L:

$$\begin{aligned} w_L &= \frac{\beta[\delta + m(\theta_L)\theta_L]y_L}{\delta + \beta m(\theta_L)\theta_L} && (8) \\ J_L^U &= \frac{\beta m(\theta_L)\theta_L y_L}{(\gamma + r)[\delta + \beta m(\theta_L)\theta_L]} && (9) \\ 1 &= \frac{d}{(1-\beta)\cdot y_L} \cdot \frac{\delta + \beta m(\theta_L)\theta_L}{m(\theta_L)} && (10) \end{aligned}$$

where $\delta = \gamma + r + \zeta$.

For the high-skilled market, we use similar notations. The differences here are (i) we use subscript

H for high-skilled market; and (ii) the value functions depend on a, because the output of a match now depends on the productivity of the worker. Since there is heterogeneity in the ability of workers, worker's true productivity could potentially be private information before match. However, we abstract from this informational problem. For one thing, we use random search so that the formation of a match does not depend on worker's ability. Second, once matched, the worker and the firm then enter a long-term relationship. To simplify our analysis, we assume that firms can observe the true productivity of workers once they are matched[7]. Then the wage can again be determined by Nash bargaining. So we can write

$$\begin{aligned}
rJ_H^V &= -d + m(\theta_H)\big[E_a[J_H^F(a)] - J_H^V\big] && (11)\\
rJ_H^F(a) &= a\rho y_H - w_H(a) - (\zeta+\gamma)[J_H^F(a) - J_H^V] && (12)\\
(\gamma+r)J_H^U(a) &= m(\theta_H)\theta_H[J_H^E(a) - J_H^U(a)] && (13)\\
(\gamma+r)J_H^E(a) &= w_H(a) - \zeta[J_H^E(a) - J_H^U(a)] && (14)
\end{aligned}$$

Note that in (11) firm will make expectation $E_a[J_H^F(a)]$ based on the distribution of pre-college ability a that is determined by the optimal choices of the agents. Again, free entry on the firms side and Nash bargaining require:

$$\begin{aligned}
J_H^V &= 0. && (15)\\
\beta[J_H^F(a) - J_H^V] &= (1-\beta)[J_H^E(a) - J_H^U(a)]. && (16)
\end{aligned}$$

As in the case of low-skilled market, from (11)-(16) we can get

$$\begin{aligned}
w_H(a) &= \frac{\beta[\delta + m(\theta_H)\theta_H]a\rho y_H}{\delta + \beta m(\theta_H)\theta_H} && (17)\\
J_H^U(a) &= \frac{\beta m(\theta_H)\theta_H \cdot a\rho y_H}{(\gamma+r)[\delta+\beta m(\theta_H)\theta_H]} && (18)\\
\rho E_a[a|s \geq \bar{s}] &= \frac{d}{(1-\beta)y_H} \cdot \frac{\delta + \beta m(\theta_H)\theta_H}{m(\theta_H)} && (19)
\end{aligned}$$

To simplify our notation later, let $\psi(\theta_H) = \frac{1}{\rho} \cdot \frac{d}{(1-\beta)y_H} \cdot \frac{\delta+\beta m(\theta_H)\theta_H}{m(\theta_H)}$ so (19) can be written as

$$\psi(\theta_H) = E_a[a|s \geq \bar{s}]. \quad (20)$$

2.1.3 Unemployment Rate

Let n_i^κ denote the number of workers in market i in state κ, $i = H, L$, $\kappa = U, E$. then the dynamics of employment ($\dot{n}_i^E$) are given by the following two equations:

$$\begin{aligned}
\dot{n}_L^E &= \theta_L m(\theta_L)n_L^U - (\zeta+\gamma)n_L^E\\
\dot{n}_H^E &= \theta_H m(\theta_H)n_H^U - (\zeta+\gamma)n_H^E
\end{aligned}$$

[7]Without this assumption, standard Nash bargaining cannot be used to determine the wages.

In the steady state, $\dot{n_i^E} = 0$. Then from the above equations we can get the unemployment rate in each market as:

$$u_i = \frac{n_i^U}{n_i^U + n_i^E} = \frac{\zeta + \gamma}{\theta_i m(\theta_i) + \zeta + \gamma} \tag{21}$$

Note that u_i is decreasing in θ_i. Intuitively, a lower θ_i suggests that there are relatively abundant unemployed workers searching for jobs and relatively scarce vacancies posted by the firms, so u_i will be greater.

2.2 Individual's Choice

Given market tightness (θ_L, θ_H) and threshold $\bar{s}$, each agent maximizes his utility by choosing a. Formally, the problem for an agent with initial ability e is given by:

$$\begin{aligned} \max_a \quad & \left\{[1 - F(\bar{s}|a)] \cdot \left[J_H^U(a) + E_s[\xi(s)|a]\right] + F(\bar{s}|a) \cdot J_L^U(a) - C(a-e)\right\} \\ s.t. \quad & a \geq e \end{aligned} \tag{22}$$

For simplicity, in this paper we assume that $\xi(s) = \phi_0 + \phi s$ and thus, $E_s[\xi(s)|a] = \phi_0 + \phi a$. For notation simplicity, let $\pi(\theta_L) = \frac{\beta m(\theta_L)\theta_L}{(\gamma+r)[\delta+\beta m(\theta_L)\theta_L]}$ and $\pi(\theta_H) = \frac{\beta m(\theta_H)\theta_H}{(\gamma+r)[\delta+\beta m(\theta_H)\theta_H]}$, where $\pi(\theta)$ is increasing in θ, so that $J_L^U = \pi(\theta_L)y_L$ and $J_H^U(a) = \pi(\theta_H) \cdot \rho a y_H$. The above problem then becomes

$$\begin{aligned} \max_a \quad & \left\{[1 - F(\bar{s}|a)] \cdot \left[\pi(\theta_H) \cdot \rho a y_H + \phi_0 + \phi a\right] + F(\bar{s}|a) \cdot \pi(\theta_L) y_L - C(a-e)\right\} \\ s.t. \quad & a \geq e \end{aligned}$$

and the first order condition is

$$(\pi(\theta_H)\rho y_H + \phi)[(1 - F(\bar{s}|a)) - a \cdot F_a(\bar{s}|a)] - \phi_0 \cdot F_a(\bar{s}|a) + \pi(\theta_L)y_L \cdot F_a(\bar{s}|a) - C'(a-e) = 0 \tag{23}$$

From (23) we will get $a^* = \eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)$, the optimal pre-college ability of an individual with initial ability e. The second order condition is that

$$SOC = (\pi(\theta_H)\rho y_H + \phi)[-2F_a(\bar{s}|a) - a \cdot F_{aa}(\bar{s}|a)] - \phi_0 \cdot F_{aa}(\bar{s}|a) + \pi(\theta_L)y_L \cdot F_{aa}(\bar{s}|a) - C''(a-e) < 0.$$

We assume $\eta(\underline{e})\rho y_H > y_L$ for the rest of the paper. This implies that the productivity of a matching in the high-skilled market is always greater than the productivity of a matching in the low-skilled market.

We then summarize the comparative statics in the following lemma.

Lemma 1. *From* (23)*, we have* $\frac{\partial \eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial e} > 0$, $\frac{\partial \eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial \theta_H} > 0$, $\frac{\partial \eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial \phi_0} > 0$, *and* $\frac{\partial \eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial \phi} > 0$.

Proof. Apply the Implicit Function Theorem with respect to (23). Hence,

$$\begin{aligned}\frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial e} &= -\frac{C''(a-e)}{SOC}>0;\\ \frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial\theta_H} &= -\frac{\pi'(\theta_H)[1-F(\bar{s}|a))-aF_a(\bar{s}|a)]\rho y_H}{SOC}>0\end{aligned}$$

since $\pi'_H(\theta_H)>0$, $1-F(\bar{s}|a)>0$, and $F_a(\bar{s}|a)<0$; and

$$\begin{aligned}\frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial\phi_0} &= -\frac{-F_a(\bar{s}|a)}{SOC}>0\\ \frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial\phi} &= -\frac{(1-F(\bar{s}|a))-a\cdot F_a(\bar{s}|a)}{SOC}>0.\end{aligned}$$

Note that $F_a(\bar{s}|a)<0$ holds because $f(\bar{s}|a)$ satisfies MLRP. □

Intuitively, the agent with a higher initial ability will have a higher pre-college ability after his optimization. In addition, when the high-skilled market tightness increases, the individuals will have a better prospective for the high-skilled market and will have more incentive to invest on pre-college ability in order to enter college. Both education subsidy and fellowship increase the benefit of going to the college, and, thus, result in more pre-college investments on human capital.

Since the productivity of a match in the high-skilled market is always greater than that in the low-skilled market, we can show that, in equilibrium, the market tightness in the former market is also higher. The economic intuition behind this is that free entry makes firms indifferent between creating vacancies in the two markets. The vacancies created in the low market will be matched with lower productivity workers, but they are compensated by a higher rate of being filled.

Lemma 2. *$\theta_H>\theta_L$ in equilibrium.*

Proof. Note that $\frac{\delta+\beta m(\theta)\theta}{m(\theta)}$ increases in θ. Then from (10) and $\eta(e)\rho y_H\geq\eta(\underline{e})\rho y_H>y_L$ for any $e\in[\underline{e},\bar{e}]$, we know that $\theta_H>\theta_L$. □

Then we can show that a higher admission threshold discourages the investment on human capital and hence decreases the agent's pre-college ability.

Lemma 3. *Suppose $f_a(\bar{s}|a)>0$ in equilibrium. Then we have $\frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial\bar{s}}<0$.*

Proof. Note that $\pi(\theta)$ increases in θ, so when $\theta_H>\theta_L$, we have $\pi(\theta_H)>\pi(\theta_L)$. As $\bar{s}$ changes,

$$\frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial\bar{s}}=\frac{(\pi(\theta_H)\rho y_H+\phi)f(\bar{s}|a)+f_a(\bar{s}|a)[a\phi+\pi(\theta_H)a\rho y_H+\phi_0-\pi(\theta_L)y_L]}{SOC}.$$

Lemma 2 implies $\pi(\theta_H)>\pi(\theta_L)$. When $\eta(\underline{e})\rho y_H>y_L$, we have $\pi(\theta_H)a\rho y_H\geq\pi(\theta_H)\eta(\underline{e})\rho y_H>\pi(\theta_L)y_L$, so that $\pi(\theta_H)a\rho y_H-\pi(\theta_L)y_L>0$. Since $f_a(\bar{s}|a)>0$, $[\pi(\theta_H)a\rho y_H-\pi(\theta_L)y_L]\cdot f_a(\bar{s}|a)>0$. As a result, the numerator is positive such that $\frac{\partial\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)}{\partial\bar{s}}<0$. □

This discussion of Lemma 3 involves a classic question: if the standard increases, do you work harder or do you work less? Here under our condition, $f_a(\bar{s}|a)>0$, we have a negative sign for $\partial\eta/\partial\bar{s}$, which means as the standard increases, agents work less harder. The basic intuition is as follows. First note an agent will choose the pre-college investment level at which the marginal benefit equals the marginal cost of making efforts. As $\bar{s}$ increases, the marginal benefit of making pre-college effort changes. Our condition, $f_a(\bar{s}|a) > 0$, implies $\frac{\partial^2(1-F(\bar{s}|a))}{\partial a \partial \bar{s}} < 0$; that is, as $\bar{s}$ increases, the marginal effect of a on the probability of going to the high-skilled market—i.e.—$\frac{\partial(1-F(\bar{s}|a))}{\partial a}$, decreases. As a result, the marginal benefit, and, thus, the incentive of making pre-college investments decreases in $\bar{s}$. [8] Note that the marginal cost is upward sloping. As a result, the optimal pre-college ability decreases.

2.3 Equilibrium

Recall that e satisfies a uniform distribution on $[\underline{e}, \bar{e}]$. Given $a^* = \eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)$, the capacity of college (1) implies

$$\frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} F(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0))de = 1 - \sigma. \tag{24}$$

In addition,

$$E_a[a|s \geq \bar{s}] = \frac{\frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} \eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)[1 - F(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0))]de}{\sigma}. \tag{25}$$

Plug (25) into (20), we will get

$$\sigma\psi(\theta_H) = \frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} \eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)[1 - F(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0))]de. \tag{26}$$

(24) and (26) pin down θ_H and $\bar{s}$ in equilibrium.

From (10) we know θ_L. Plug the expression of θ_L into (24), we can get rid of θ_L and will get a relationship between θ_H and $\bar{s}$. Note from (24),

$$\frac{\partial \bar{s}}{\partial \theta_H} = -\frac{\int_{\underline{e}}^{\bar{e}} F_a(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)) \cdot \frac{\partial \eta}{\partial \theta_H} de}{\int_{\underline{e}}^{\bar{e}} [F_a(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)) \cdot \frac{\partial \eta}{\partial \bar{s}} + f(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0))]de};$$

and since $F_a(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)) < 0$, $\frac{\partial \eta}{\partial \theta_H} > 0$, $\frac{\partial \eta}{\partial \bar{s}} < 0$, we have $\frac{\partial \bar{s}}{\partial \theta_H} > 0$. Hence, $\bar{s}$ can be written as $\bar{s} = \lambda(\theta_H; \sigma)$ by (24). Plug $\bar{s} = \lambda(\theta_H; \sigma)$ into (26), we will get:

$$\sigma\psi(\theta_H) = \frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} \eta(e; \theta_L, \theta_H, \lambda(\theta_H; \sigma); \phi, \phi_0)[1 - F(\bar{s}|\eta(e; \theta_L, \theta_H, \lambda(\theta_H; \sigma); \phi, \phi_0))]de. \tag{27}$$

Equation (27) will pin down θ_H^*. Then, by plugging θ_H^* into (24), we find the equilibrium $\bar{s}^*$.

Denote $\Delta(\theta_H) = \sigma\psi(\theta_H) - \frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} \eta(e; \theta_L, \theta_H, \lambda(\theta_H; \sigma); \phi, \phi_0)[1 - F(\bar{s}|\eta(e; \theta_L, \theta_H, \lambda(\theta_H; \sigma); \phi, \phi_0))]de$. We have the following result.

Proposition 1. *Suppose $\Delta'(\theta_H) > 0$. Then there exists a unique equilibrium $(\theta_L^*, \theta_H^*, \bar{s}^*)$ satisfying* (10),

[8] Note that the condition $f_a(\bar{s}|a) > 0$ is a local condition instead of a global condition. From Lemma 3, it is a sufficient condition under which for some $\bar{s}$, agents work harder as $\bar{s}$ decreases. Here, we further argue that there always exists a domain of $\bar{s}$ on which agents work harder as $\bar{s}$ decreases. To see this, first note that as $\bar{s} \to \infty$, agents will have no incentive to make pre-college investment since the probability of going to the high market is zero. Hence, as $\bar{s} \to \infty$, $\eta(e; \bar{s}) \to 0$ for any e. When there exists some domain of $\bar{s}$ on which agents make positive pre-college investment, since $\eta(e; \bar{s})$ is continuous, there must exist a domain of $\bar{s}$ on which agents work harder as the admission threshold decreases.

(20), *and* (24).

Proof. First, (10) determines the unique θ_L^*. In addition, it is straightforward to check that

$$\lim_{\theta_H \to 0} \sigma\psi(\theta_H) \to 0,$$

and

$$\int_{\underline{e}}^{\bar{e}} \eta(e;\theta_L,\theta_H,\lambda(\theta_H;\sigma);\phi,\phi_0)[1-F(\bar{s}|\eta(e;\theta_L,\theta_H,\lambda(\theta_H;\sigma);\phi,\phi_0))]de > 0,$$

when $\theta_H \to 0$; so $\lim_{\theta_H \to 0} \Delta(\theta_H) < 0$. Further, $\lim_{\theta_H \to \infty} \sigma\psi(\theta_H) = \infty$ and the rest expression of $\Delta(\theta_H)$ is bounded; so $\Delta(\theta_H) > 0$ when θ_H is sufficiently large. Therefore, a θ_H^* that satisfying (27) must exist. Since $\Delta'(\theta_H) > 0$, the θ_H^* that satisfies $\Delta(\theta_H) = 0$ must be unique. $\bar{s}^*$ is also unique because $\lambda(\theta_H)$ increases in θ_H. □

In the rest of the paper, we assume that $f_a(\bar{s}|a) > 0$ and $\Delta'(\theta_H) > 0$ hold such that the equilibrium exists and is unique.

3 Impacts of Educational Policies

In this section, we will study the impacts of the three different educational policies, namely providing subsidies, scholarship or expanding the capacity, on labor market and college competition.

3.1 Subsidies and Scholarship

We first have the following lemma.

Lemma 4. *In equilibrium, θ_H^* increases in both ϕ and ϕ_0.*

Proof. Consider equation (27). By Implicit Function Theorem,

$$\frac{\partial \theta_H^*}{\partial \phi} = -\frac{-\frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} \frac{\partial \eta(e;\theta_L,\theta_H,\lambda(\theta_H;\sigma);\phi,\phi_0)}{\partial \phi}[1-F(\lambda|\eta) - a \cdot F_a(\lambda|\eta)]de}{\frac{\partial \Delta}{\partial \theta_H}} > 0.$$

Following a similar proof, we can show that $\frac{\partial \theta_H^*}{\partial \phi_0} > 0$. □

As ϕ or ϕ_0 increases, individuals will have more incentive to make investment on human capital; as a result, the average productivity from a matching in the high-skilled market increases. Hence, firms are more willing to post vacancies in the high-skilled market so that market tightness rises.

With (21), we are in a position to give:

Proposition 2. *In equilibrium, as the subsidy ϕ_0 or the scholarship ϕ increases, the unemployment rate on the high market decreases.*

It is worth mentioning that Charlot and Decreuse (2005) show that a higher education subsidy induces agents with less human capital endowment to go to college, which lowers the average productivity of college graduates and hence reduces the market tightness and raises the unemployment rate. In their model, college admission is determined only by the demand side of college education: those who want to go to college are all enrolled by college, and the supply of college education is implicitly regarded as being perfectly elastic.

In contrast to their work, we take the supply of college education into consideration. In our model, because the size of college is restricted by college capacity and an agent with a higher human capital level has a higher probability of being admitted by college, we partly shut down the channel examined by Charlot and Decreuse (2005). In our model, what affects the average productivity of college graduates is individual's pre-college investment in human capital. Subsidy induces more competition among students: they would like to work harder to increase the chances of receiving this prize. This resonates the notion of "one has to invest on human capital in order to compete for better opportunities to invest in human capital".

In addition, as ϕ or ϕ_0 increases, since individuals have higher levels of pre-college ability, the threshold for college admission also increases for any given capacity. As ϕ or ϕ_0 increases, competition during the college admission process becomes more severe in equilibrium, since now each agent needs to make more efforts to be admitted.

Proposition 3. *In equilibrium,* $\bar{s}^*$ also increases in both ϕ and ϕ_0.

Proof. Note that

$$\frac{\partial \bar{s}^*}{\partial \phi} = \frac{\partial \lambda}{\partial \theta_H} \cdot \frac{\partial \theta_H^*}{\partial \phi}.$$

The result is immediately known from Lemma 4 and the fact that $\frac{\partial \lambda}{\partial \theta_H} > 0$. Following a similar proof, we can show that $\frac{\partial \bar{s}^*}{\partial \phi_0} > 0$. □

3.2 Capacity Expansion

The discussion of capacity expansion is subtle. So we start with a special case: when there is no heterogeneity in e, or to say, $\underline{e} = \bar{e}$. This allows us to remove the extensive margin.

3.2.1 Homogeneous Innate Ability

Proposition 4. *In equilibrium, when agents are homogeneous in e, an increase in college capacity σ, raises investment in pre-college human capital of the representative agent, the labor market tightness in the high market, and lowers unemployment rate on the high market.*

The proof is shown in Appendix B.

In our setup, there is one positive effect of capacity expansion on pre-college human capital investment: the pre-college human capital is assumed to be useful for college graduate (white collar workers) but not for high school graduates (blue collar workers), so when the capacity expands, there is more chances that investment on pre-college human capital is not wasted, which encourages investment. In addition, since there is no heterogeneity, the (pre-college) human capital level chosen by a *representative agent* determines the *average* productivity of the high market and then the tightness θ_H. As a result, an increase in college capacity σ, raises the labor market tightness and lowers unemployment rate in the high-skilled market.

How about the effects of capacity expansion on $\bar{s}$? Note

$$\frac{\partial \bar{s}^*}{\partial \sigma} = \frac{\partial \lambda(\theta_H^*;\sigma)}{\partial \sigma} + \frac{\partial \lambda(\theta_H^*;\sigma)}{\partial \theta_H} \cdot \frac{\partial \theta_H^*}{\partial \sigma}.$$

When there is no heterogeneity, since $\frac{\partial \lambda(\theta_H^*;\sigma)}{\partial \sigma} < 0$ and $\frac{\partial \lambda(\theta_H^*;\sigma)}{\partial \theta_H} \cdot \frac{\partial \theta_H^*}{\partial \sigma} > 0$ from the proof of Proposition 4, we have an ambiguous result.

This is in fact very intuitive. On the one hand, Proposition 4 clearly predicts that the pre-college human capital investment increases for the representative agent. How could $\bar{s}$ decrease when everyone works harder? The answer is that the schools are now admitting more students so we are using a lower ranked signal as our threshold. If the density function $f(s|a)$ is very "flat", the fact that we are now picking a lower ranked signal could be dominated by the fact that every signal is higher because every student invests more in pre-college human capital. If the conditional distribution is very "steep", the resulting $\bar{s}$ could be lower.

There are different ways to characterize the degree of competition. If we use the admission ratio (it is the inverse of σ in our model) then the capacity expansion means less competition. This version of the model is an example where less competition result in higher investment in human capital (pre-college). Note it is in sharp contrast to the our analysis of scholarship and subsidy, where the government intervention raises the prize for competition thus increase the effort by students. On the other hand, if one use $\bar{s}$ as an indicator of the degree of competition. Then it is possible that "competition" is more intense or less intense, despite that every student works harder than before.

3.2.2 Heterogeneous Innate Ability

Next consider the version of the model with heterogeneity. Consider equation (27), by Implicit Function Theorem,

$$\frac{\partial \theta_H}{\partial \sigma} = -\frac{\psi(\theta_H) - \frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} \left\{ \frac{\partial \eta(e;\theta_L,\theta_H,\lambda(\theta_H;\sigma);\phi)}{\partial \lambda} \cdot [1 - F(\lambda|\eta) - \eta \cdot F_a(\lambda|\eta)] - \eta \cdot f(\lambda|\eta) \right\} \cdot \frac{\partial \lambda}{\partial \sigma} de}{\frac{\partial \Delta}{\partial \theta_H}}.$$

Note that $\frac{\partial \eta(e;\theta_L,\theta_H,\lambda(\theta_H;\sigma);\phi)}{\partial \lambda} < 0$, $1 - F(\lambda|\eta) - \eta \cdot F_a(\lambda|\eta) > 0$, $\eta \cdot f(\lambda|\eta) > 0$ and

$$\frac{\partial \lambda}{\partial \sigma} = -\frac{1}{\frac{1}{\bar{e}-\underline{e}} \cdot \int_{\underline{e}}^{\bar{e}} [F_a(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi)) \cdot \frac{\partial \eta}{\partial \bar{s}} + f(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi))] de} < 0;$$

but $\psi(\theta_H) > 0$. Hence, the sign of the numerator is ambiguous. As a result, the sign of $\frac{\partial \theta_H}{\partial \sigma}$ is ambiguous.

Now capacity expansion has some different implications compared with the case without heterogeneity: As σ increases, those individuals who have lower initial abilities are also enrolled in the college, and thus lowers the average human capital on the high market. Firms thus create relatively fewer vacancies and the labor market tightness tends to decrease and unemployment rate tends to increase. On the other hand, as discussed in Proposition 4, an increase in σ increases the probability of an individual going to the high-skilled market, and thus his incentive to make pre-college investment. This leads to a greater average productivity on the high market, and firms are willing to post more vacancies, so unemployment rate tends to decrease. As a consequence, the net effect is ambiguous.

If an increase in unemployment rate among the educated population is accompanied with both an increase of educational subsidy and an increase of college enrollment, then it is possible that unemployment on the high-skilled market is caused by rising labor supply instead of educational subsidy. The results in the present paper and that in Charlot and Decreuse (2005) imply that, the characteristics on the supply side of high education sector may play a crucial role.

4 Concluding Remarks

Taking competition for college admission into account, this paper examines the effects of different types of educational policies, such as subsidy, scholarship and capacity expansion on individual's human capital investment, and unemployment. To achieve this, we build a two-sector search model where students, before going to the labor market, choose their pre-college human capital level to increase their chances of being admitted by colleges, which further increases their human capital level and allows them to search for skilled jobs. We find that scholarship and subsidy always lower unemployment for college graduates. In the version of the model where we have homogeneous agents, capacity expansion increases investment in pre-college human capital for every agent and lowers the unemployment for college graduates. Of course, once heterogeneous innate ability is introduce, the overall effect of higher education expansion will depend on the degree of heterogeneity of innate abilities, since it leads the less able to look for jobs as college graduates.

It is worth discussing the applicability of the model. The most important assumption is that students need to invest in pre-college human capital to compete for better opportunities to further invest in their human capital. We assume the higher the pre-college human capital level, the better the signal sent to colleges. Notice that such signals could include test scores, but can also include extracurricular activities and performances. What we are saying in our analysis of college admission is that both innate ability and effort matter for the pre-college human capital level which determines the quality of the signals. Another assumption worth discussing is that the pre-college human capital increases the college graduates productivity but has no effect on the low-skilled workers. This seemingly extreme assumption is to capture the observation that what one learns preparing for college (to increase the quality of signals) is more valuable to someone who actually attend college than to someone who do not.

Of course, the model made some simplifying assumptions. For example, we focus on the case where there is excess demand for college education and the supply of education is set by the government. These assumptions are realistic for countries like China, but are not so for some other countries, such as United States. But our framework of college admission and pre-college human capital investment could well be incorporated into a model where the supply of higher education is elastic, and there is endogenous demand for college (i.e. not everyone would like to have college education). Lastly, based on our theoretical analysis, future research on capacity expansion could definitely bring more quantitative results given more data on the heterogeneity of innate abilities in countries that are considering or have experienced such reforms.

Appendix A

The effects of different types of educational policies, such as subsidy, scholarship and capacity expansion on individual's human capital investment. To see this, just note that in equilibrium,

$$\frac{\partial \eta(e;\theta_H^*(\phi_0,\phi,\sigma),\theta_L,\bar{s}^*(\phi_0,\phi,\sigma);\phi,\phi_0)}{\partial \phi} = \frac{\partial \eta}{\partial \theta_H}\cdot\frac{\partial \theta_H^*(\phi_0,\phi,\sigma)}{\partial \phi} + \frac{\partial \eta}{\partial \bar{s}}\cdot\frac{\partial \bar{s}^*(\phi_0,\phi,\sigma)}{\partial \phi} + \frac{\partial \eta}{\partial \phi}.$$

According to our previous analysis, $\frac{\partial \eta}{\partial \theta_H} > 0$, $\frac{\partial \theta_H^*(\phi_0,\phi,\sigma)}{\partial \phi} > 0$, $\frac{\partial \eta}{\partial \bar{s}} < 0$, $\frac{\partial \bar{s}^*(\phi_0,\phi,\sigma)}{\partial \phi} > 0$, and $\frac{\partial \eta}{\partial \phi} > 0$, so the total effect is ambiguous. The effect of ϕ_0 follows similar steps. Further, the impact of σ on the equilibrium pre-college human capital is as follows:

$$\frac{\partial \eta(e;\theta_H^*(\phi_0,\phi,\sigma),\theta_L,\bar{s}^*(\phi_0,\phi,\sigma);\phi,\phi_0)}{\partial \sigma} = \frac{\partial \eta}{\partial \theta_H}\cdot\frac{\partial \theta_H^*(\phi_0,\phi,\sigma)}{\partial \sigma} + \frac{\partial \eta}{\partial \bar{s}}\cdot\frac{\partial \bar{s}^*(\phi_0,\phi,\sigma)}{\partial \sigma}.$$

Since $\frac{\partial \eta}{\partial \theta_H} > 0$, $\frac{\partial \eta}{\partial \bar{s}} < 0$ but the other two partial derivatives have ambiguous sign, the total effect is also ambiguous.

Appendix B

Proof for Proposition 4

First, when there is no heterogeneity, all the results in Section 2.2 still hold. Then let us discuss the equilibrium. The capacity constraint implies

$$F(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)) = 1-\sigma. \tag{28}$$

In addition,

$$E_a[a|s \geq \bar{s}] = \eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0). \tag{29}$$

Plug (29) into (20), we will get

$$\psi(\theta_H) = \eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0). \tag{30}$$

(28) and (30) pin down θ_H and $\bar{s}$ in equilibrium.

From (10) we know θ_L. Plug the expression of θ_L into (28), we can get rid of θ_L and will get a relationship between θ_H and $\bar{s}$. Note from (28),

$$\frac{\partial \bar{s}}{\partial \theta_H} = -\frac{F_a(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0))\cdot\frac{\partial \eta}{\partial \theta_H}}{F_a(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0))\cdot\frac{\partial \eta}{\partial \bar{s}} + f(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0))]};$$

and since $F_a(\bar{s}|\eta(e;\theta_L,\theta_H,\bar{s};\phi,\phi_0)) < 0$, $\frac{\partial \eta}{\partial \theta_H} > 0$, $\frac{\partial \eta}{\partial \bar{s}} < 0$, we have $\frac{\partial \bar{s}}{\partial \theta_H} > 0$. Hence, $\bar{s}$ can be written as

$\bar{s} = \lambda(\theta_H; \sigma)$ by (28). Plug $\bar{s} = \lambda(\theta_H; \sigma)$ into (30), we will get:

$$\psi(\theta_H) = \eta(e; \theta_L, \theta_H, \lambda(\theta_H; \sigma); \phi, \phi_0). \tag{31}$$

Equation (31) will pin down θ_H^*. Then, by plugging θ_H^* into (28), we find the equilibrium $\bar{s}^*$.

As before, let

$$\tilde{\Delta}(\theta_H) = \psi(\theta_H) - \eta(e; \theta_L, \theta_H, \lambda(\theta_H; \sigma); \phi, \phi_0)$$

and suppose $\tilde{\Delta}'(\theta_H) > 0$. Following the similar discussion in Proposition 3, the equilibrium exists and is unique. Then

$$\frac{\partial \theta_H^*}{\partial \sigma} = -\frac{-\frac{\partial \eta}{\partial \bar{s}} \cdot \frac{\partial \lambda(\theta_H; \sigma)}{\partial \sigma}}{\tilde{\Delta}'(\theta_H)}.$$

Since

$$\frac{\partial \lambda(\theta_H; \sigma)}{\partial \sigma} = -\frac{1}{F_a(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0)) \cdot \frac{\partial \eta}{\partial \bar{s}} + f(\bar{s}|\eta(e; \theta_L, \theta_H, \bar{s}; \phi, \phi_0))]} < 0,$$

$\frac{\partial \eta}{\partial \bar{s}} < 0$, and $\tilde{\Delta}'(\theta_H) > 0$, we have $\frac{\partial \theta_H^*}{\partial \sigma} > 0$.

References

[1] Charlot, O., Decreuse, B., 2005. Self-selection in education with matching frictions. Labour Economics 12, 251-267.

[2] Charlot, O., Decreuse, B., Granier, P., 2005. Adaptability, productivity and educational incentives in a matching model. European Economic Review 49, 1007-1032.

[3] Freeman. R., 2009. What does global expansion of higher education mean for the US? NBER working paper 14962.

[4] Kane, J., 2006. Public intervention in post-secondary education. In Hanushek, E. and Welch, F., editors, Handbook of the Economics of Education, vol (2), 1369-1401. Elsevier Science, North-Holland, Amsterdam, New York and Oxford.

[5] Kane, J., 2007. Evaluating the impact of the D.C. tuition assistance grant program. Journal of Human Resources, XLII(3), 555-582.

[6] Saint-Paul, G., 1996. Dual labor market: A macroeconomic perspective. MIT Press, London, England and Cambridge, Massachussetts.

[7] Shofer, E., Meyer. J., 2005. Worldwide expansion of higher education. Center on democracy, development, and the rule of law, Standford institute of international studies, working paper 32.

[8] Spence, M., 1973. Job market signaling. The Quarterly Journal of Economics 87, 355-374.