Optimal Taxation and Monopsonistic Labor Market: Does Monopsony justify the Minimum Wage?

Pierre Cahuc[1]
Ecole polytechnique, CREST-INSEE, IZA, CEPR

Guy Laroque
CREST-INSEE, UCL, IZA

May 22, 2009

[1]CREST-INSEE, Timbre J 360, 15, Boulevard Gabriel-Péri, 92245, Malakoff, France. Email: cahuc@ensae.fr. We thank Etienne Lehmann, Alan Manning and participants in seminars at CREST, at the University of Oslo and in the Conference *Labor Markets: a transatlantic perspective* (Paris, January 2008) for comments.

Abstract

Does monopsony on the labor market in itself justify the implementation of a minimum wage when it would not be used in a competitive economy? This issue is studied in a model of optimal taxation. We adopt a definition most favorable to the minimum wage: the minimum wage is useful whenever it can replace a non negligible part of the tax schedule. The minimum wage is useful to correct the inefficiencies associated with the monopsony when there is a single skill. But the minimum wage is not useful any more when there are a continuum of skills.

KEYWORDS: Minimum wage, Optimal taxation, Monopsony.

JEL CODES: H31, J30, J42.

1 Introduction

A popular justification of the minimum wage is that it strengthens the hand of the low skilled workers who are exploited by monopsonist employers. As stressed by Dolado et al. (2000), proponents of the minimum wage take the competitive working of the labor market as the exception, rather than the rule, arguing that in many reasonable instances "monopsony" corresponds to the rule. Then, the minimum wage seems to be useful because it increases both employment and the income of low wage workers. This view had a strong influence on economic policy in the last fifteen years. For instance, in 1994, the *OECD Jobs Study* was arguing that there was a need to "reassess the role of statutory minimum wages as an instrument to achieve redistributive goals, and switch to more direct instruments" (OECD, 1994). Four years later, after the publication of a set of papers and a book arguing that minimum wage increases could benefit low skilled employment according to the predictions of the monopsony model of the labor market (Card and Krueger, 1995), the perspective was quite different: the *OECD Employment Outlook* stressed that "a well-designed policy package of economic measures, with an appropriately set minimum wage in tandem with in-work benefits, is likely, on balance, to be beneficial in moving towards an employment-centered social policy" (OECD, 1998).

While the recent minimum wage research finds a wide range of estimates on the overall effects on low-wage employment of an increase in the minimum wage (Neumark and Wascher, 2006), the monopsony model remains influential. For instance, in year 2005, it is still argued by the OECD that "the main impact of downward wage flexibility may be to worsen inactivity, unemployment and low-pay traps." (OECD, 2005, p 142). As a matter of fact, today, statutory or quasi-statutory minimum wages are in place in 21 OECD countries (Immervoll, 2007).

An interpretation of the debate is that the practical justification of a minimum wage is the existence of a large sector of monopsonist employers, while under perfect competition the scope for a minimum wage would be minimal. The purpose of our study is to address this precise question: does monopsony on the labor market in itself justify the implementation of a minimum wage? Our approach is theoretical and considers a situation where the government can use non linear taxes. Of course, if one does not put any restriction on the shape of taxes, a minimum wage $\underline{W}$ can be mimicked by imposing very high taxes on wages below $\underline{W}$, so that the minimum wage would be redundant. To deal with this issue we choose a view point a priori favorable to the minimum wage: we consider that the minimum wage is a valid instrument whenever it can replace a non negligible part of the tax schedule. For instance, we consider that the minimum wage is useful if taxes prevent hirings below a wage threshold. This approach implies that the minimum wage is useful in the monopsony model with a single skill, a result known since the contributions of Robinson (1933) and Stigler (1946). Indeed, when there is a single skill, the government can reach the competitive equilibrium with the minimum wage without using taxes. However, when the heterogeneity of skills is accounted for, the minimum wage cannot play this role any more. The government always needs taxes to correct the inefficiencies associated with the monopsony. Strikingly, in that case, we find that there is no scope for the minimum wage. This result is not limited to the pure monopsony model. It also applies to monopsonistic competition with free entry.

The papers which look at the minimum wage in labor markets with *imperfect competition* typically ask whether increasing the minimum wage can improve employment or welfare in the absence of other policy tools.[1] The efficiency of the minimum wage when there are taxes has mostly been considered in labor markets with *perfect competition*.[2] Most models have adopted the standard Mirrlees (1971) model on optimal taxation with labor supply at the intensive margin where individuals choose the number of hours they work and where the government observes earnings but neither hourly wages nor hours of work. It turns out that the minimum wage can be welfare improving when tax schemes are constrained or when there are specific assumptions made to allow the government to observe skills at the bottom of the income distribution (Allen, 1984, Guesnerie and Roberts, 1987, Boadway and Cuff, 2001). However, as stressed by Guesnerie and Roberts (1987) and more recently by Lee and Saez (2007), informational inconsistencies arise when a minimum wage is introduced in the Mirrlees model because the minimum wage implementation requires observing hourly wages. But if hourly wages were directly observable, then the government could achieve any first best allocation by conditioning taxes and transfers on hourly wage and the minimum wage would obviously not be useful.

This informational inconsistency leads us to focus on labor supply at the extensive margin where the agents' decision is zero-one, to work or not to work, as in the studies of Diamond (1980), Beaudry and Blackorby (1997), Saez (2002), Choné and Laroque (2005, 2008) and Laroque (2005). Lee and Saez (2007) have studied the consequence of the minimum wage in this type of model assuming perfect competition on the labor market. They derive precise conditions on the shape of the rationing scheme on the labor market under which a minimum wage can be a useful complement to taxes in a competitive environment. In particular, they find that the minimum wage is useful under the assumption that workers who involuntary lose their job because of the minimum wage are those with the highest opportunity cost of work.

In our paper, we analyze the scope for minimum wages when labor markets are monopson-

[1] See, among others Robinson (1933), Stigler (1946), Drazen (1986), Jones (1987), Manning, (1995, 2003), Rebitzer and Taylor (1995), Bhaskar and To (1999), Masters (1999), Cahuc et al. (2001), Flinn (2006).

[2] The recent paper of Hungerbühler and Lehmann (2007), where the usefulness of the minimum wage is analyzed in a search and matching model, is an exception.

istic. We consider the standard optimum tax environment of model of optimal taxation with labor supply at the extensive margin. There is a large population of workers who have heterogeneous productivities and opportunity costs of work which are independently distributed in the population. The government is all powerful but is limited by his lack of information on the characteristics of the private agents. Given its preferences for redistribution, it chooses the optimal tax scheme implementing the second best allocation.

We first present the model with heterogeneous skills and heterogeneous opportunity costs of work (section 2). Then, we give the definition of the usefulness of the minimum wage that we apply to the model (section 3). We recall that this definition implies that the minimum wage is useful in the textbook model of the monopsony, which corresponds to the case of a single skill (section 4). The rest of the paper shows that the minimum wage is not useful any more when there are a continuum of skills. To obtain this result, we study the optimal tax scheme when the labor market is monopsonistic instead of perfectly competitive. Section 5 describes the optimal tax schemes when the labor market is perfectly competitive. Section 6 defines second best allocations with monopsonistic labor markets. Technically, the contribution of the paper is to adapt the standard principal agent setup where the principal is the government and the agents are the tax payers to a situation where there is a third party, the monopsonist, who sets wages. Compared with the laissez-faire, apart from the allocative distortions (wage and employment are lower than under perfect competition), the monopsonist' profits create a specific redistributive issue which we analyze in two stages. First, in section 7, we suppose that the government has the information and power to fully tax profits. We prove that there is a labor tax schedule that implements the second best allocation of the competitive model. Taxing profits and giving employment subsidies yield the optimal allocation. This allows us to show that there is no room for minimum wage. Second, in section 8, we introduce a simple model of monopsonistic competition with free entry to investigate what happens when the taxation of profits is limited by information constraints. We consider an economy with a large number of identical islands, which only differ by i.i.d. entry costs. The government implements corporate subsidies (or taxes) on top of the labor tax schedule. Strikingly, the previous result still holds: the tax tools are enough to undo the wrongs of the monopsonist and get the same allocations as in a competitive economy. Accordingly, in our framework, monopsonistic competition does not justify the introduction of a minimum wage. Finally, section 9 provides some concluding comments.

2 The model

We consider an economy made of a continuum of agents of measure 1. A typical agent is described by a couple of exogenous characteristics, denoted by $\theta = (\omega, \alpha)$. The first component ω denotes her productivity when working full time in market activities, producing an undifferentiated desirable commodity. The second component, α, is a fixed cost of participating in the labor market, also measured in commodity units. In the economy there are profit maximizing firms that allow the transformation of the agents' labor into commodity, and a benevolent government with a redistributive social aim, who can raise taxes or distribute subsidies and set a minimum wage. The general structure of the economy and the distribution of agents' characteristics are common knowledge.

The labor market works as follows. The government cannot observe the individual characteristics: it only sees whether an agent works or not, and in the former case, the wage paid by her employer. An employer observes the productivities y's of his employees, but not their opportunity costs of work. When she works, the type-(ω, α) agent produces a quantity y, at most equal to ω (the opportunity cost of work α is *fixed*: it does not depend on the difference $(\omega - y)$). When working and producing y, an employee gets a net wage $W(y)$, possibly subject to a minimum wage constraint $W(y) \geq \underline{W}$. The tax schedule, denoted by $T(W)$ (if negative, the absolute value of T is a subsidy to work), yields a labor cost $C(W) = W + T(W)$.

Production may generate profits. We assume that after tax profits, if any, are dissipated by the owners of the firms with no contribution to social welfare. Under full information, we consider both the normal case where profits are taxed away in a lump sum fashion and the situation where they are not taxed at all. In section 8 we introduce an information based model of profit taxation.

The tax receipts are then used to give a subsistence income r to the unemployed agents.

We assume that ω and α are independently distributed. The cumulative distributions of α and ω are denoted $F(\alpha)$ and $G(\omega)$ respectively. F has support $[\underline{\alpha}, \bar{\alpha}]$ while G has support $[\underline{\omega}, \bar{\omega}]$. We suppose

$$0 \leq \underline{\alpha} < \overline{\alpha} \leq \infty \text{ and } 0 \leq \underline{\omega} < \overline{\omega} \leq \infty.$$

F and G have continuous derivatives, denoted by f and g respectively, which are strictly positive everywhere on their support. A part of the analysis carries through with an unrestricted distribution for the couple (α, ω), involving mass points and correlation between the two characteristics: we shall point out specifically when the independence assumption is needed.

The agents have a simple choice criterion, linear in income. They decide to produce an output y rather than stay on the dole whenever their financial incentive to work, $W(y) - r$, is larger than their work opportunity cost α. Their choice follows from:[3]

$$u(W, r; \alpha, \omega) = \max_{0 \le y \le \omega} [r, W(y) - \alpha]. \tag{1}$$

Let $y(\omega)$ be the production of an agent of productivity ω who works. The proportion of agents of productivity ω that are employed is $F\left[W(y(\omega)) - r\right]$.

The preferences of the government are represented by a social welfare function

$$\int\int \Psi(u(W, r; \alpha, \omega))\, \mathrm{d}F(\alpha)\, \mathrm{d}G(\omega),$$

where Ψ is a non decreasing concave function.

To be feasible, the quadruple (W, T, r, y) must satisfy the budget constraint of the government. When the profits of the firms are not taxed, the budget constraint takes the form

$$\int_{\underline{\omega}}^{\bar{\omega}} T\left\{W(y(\omega))\right\} F[W(y(\omega)) - r]\, \mathrm{d}G(\omega) = r \int_{\underline{\omega}}^{\bar{\omega}} \left\{[1 - F[W(y(\omega)) - r]\right\}\, \mathrm{d}G(\omega). \tag{2}$$

The left hand side represents the collected taxes, while the right hand side measures the unemployment benefits. When the firms profits are taxed away, the government collects taxes T and profits $y - T - W$ on each job, so that the budget constraint becomes:

$$\int_{\underline{\omega}}^{\bar{\omega}} \left\{y(\omega) - W(y(\omega))\right\} F[W(y(\omega)) - r]\, \mathrm{d}G(\omega) = r \int_{\underline{\omega}}^{\bar{\omega}} \left\{[1 - F[W(y(\omega)) - r]\right\}\, \mathrm{d}G(\omega). \tag{3}$$

The sequence of decisions is such that the government announces its policy, the tax function, the subsistence income and possibly the minimum wage at the beginning of the period, while anticipating the budget constraint. Then the firms choose the net wage function which relates productivity to net wage. Finally the workers decide on their labor supply.

3 On taxation and the minimum wage

Through the tax function T, the tax on profits and the subsistence income r, the government has powerful policy tools at his disposal. Is a minimum wage useful in this circumstance? To address this issue we take the most favorable stance toward the minimum wage: we admit that the minimum wage is useful whenever it can replace a non negligible part of the tax schedule.

Let us define the usefulness of a policy instrument when the government has multiple, possibly redundant, instruments at its disposal. Here, the set of instruments is made of open wage intervals $[W_n, W_{n+1})$ with non zero associated tax functions, $T_n(W)$, and a positive minimum wage $\underline{W}$. When an instrument is not used, *laissez-faire* is the benchmark: $T(W) = 0$ on the relevant interval or $\underline{W} = 0$, keeping the subsistence income r unchanged. We define an instrument

[3] In case of indifference between several maxima, we suppose that the worker chooses the largest production.

as *useful* if: first, it enters a combination of instruments that supports the optimal allocation; second, among all such combinations, there exists at least one from which the instrument cannot be dropped out without reducing welfare. Consider a situation where the optimum allocation is supported by two instruments: a tax function equal to infinity on $[0, \underline{W})$, and another non zero function $T(W)$ above $\underline{W}$. Suppose that both instruments are useful: the allocation is not implemented with a zero tax on $[0, \underline{W})$. This allocation is also supported by a minimum wage $\underline{W}$ and the function $T(W)$ above $\underline{W}$. Dropping the minimum wage, as the infinite tax before, would be welfare reducing. Thus, the minimum wage is useful.[4]

This definition has the following implication:

Proposition 1 *Consider an allocation where W_0 is the lower bound of the wage distribution supported by taxes. Then the minimum wage is useful if and only if laissez-faire over the interval $[0, W_0)$ does not support the optimum.*

Proof: Consider an allocation where W_0 is the lower bound of the wage distribution supported by taxes $T(W)$. A minimum wage can be substituted for the tax $T(W)$ over the interval $[0, W_0)$. The question is whether it is useful. There are two cases to consider.

First, suppose that *laissez-faire* does not support the optimum over the interval $[0, W_0)$. This means that either $T(W) > 0$ or the minimum wage prevent individuals from working over some parts of $[0, W_0)$. Thus, the minimum wage is useful.

Second, suppose that the minimum wage is useful over the interval $[0, W_0)$. This means that setting the minimum wage to zero and $T(W) = 0$ over $[0, W_0)$ induces individuals to work at wages below W_0. Thus, *laissez-faire* does not support the optimum over the interval $[0, W_0)$. QED.

This proposition implies that the minimum wage is not useful when the lowest value of productivity of employed workers, ω_{inf}, is equal to the sum of the subsistence income, r, and the lowest work opportunity cost, $\underline{\alpha}$. In this case, under *laissez-faire* over the interval $[0, W_0)$, the potential wage of persons of productivity smaller than ω_{inf} is below $r + \underline{\alpha}$, the minimum income required to participate in the labor market.

With this definition we recall in the next section that the minimum wage is useful to correct the inefficiencies associated with the monopsony when there is a single skill. However, the definition also implies that there is no need for a minimum wage when skills are heterogeneous.

[4] From an institutional viewpoint, it may make sense to separate the enforcement of the tax scheme at the bottom of the income distribution, when it presents features of a minimum wage, from the general accounting rules associated with a tax schedule. For instance, in the US, the minimum wage can be enforced through monetary, civil or criminal penalties. Employers who willfully or repeatedly violate the minimum wage requirements are subject to a civil money penalty of up to $1,000 for each violation. Such considerations are absent from our theoretical model.

4 The case with a single skill

It is worth briefly recalling the justification of the minimum wage in the textbook model of monopsony introduced by Robinson (1933). All workers are homogeneous with respect to productivity: there is a single productivity level ω. Moreover, taxes and transfers on labor are usually not considered, so that we can set r and T equal to zero for now.

The monopsonist is assumed to face a labor supply curve that relates the wage, W, to the level of employment, equal to $F(W)$. The monopsonist maximizes his profit, knowing the shape of the supply curve, i.e. the quantity

$$\Pi = (\omega - W)F(W).$$

This leads to the first order condition[5]

$$\omega = W + \frac{F(W)}{F'(W)}. \tag{4}$$

The left-hand side of this equation is the marginal productivity of labor and the right-hand side is the marginal cost of hiring an extra worker. The marginal cost is higher than the wage W because the employer computes the overall effect of the increase on his wage bill, knowing the labor supply schedule: the derivative of the wage bill $WF(W)$ with respect to W is $WF'(W) + F(W)$, implying a cost per worker of $W + F(W)/F'(W)$ (there are $F'(W)$ extra workers). The solution is represented graphically on Figure 1, where the marginal cost of labor (MCL) is represented as a function of the employment level. The equilibrium employment level is at the intersection of (MCL) with the horizontal line of intercept ω. The wage chosen by the monopsonist, denoted by W_M, is read on the labor supply curve: it is lower than the marginal productivity ω. Both wage and employment are lower than under perfect competition. The employer is making positive profits on workers who are all paid below their productivity.

As Robinson (1933, p 295) argued, "monopsonistic exploitation of this type can be removed by the imposition of a minimum wage". It suffices to set a minimum wage up to the competitive wage ω to force the monopsonist to the competitive level of employment, equal to $F(\omega)$. Notice that the minimum wage eats away the profits of the monopsonist. A government who does not care about redistribution (whose preferences are represented by a linear function Ψ) would systematically choose this minimum wage level in order to maximize production. But the government can also use taxes to implement this optimum. The profit of the firm, when wages are taxed (the typical net wage W bears a tax t), is

$$(\omega - t - W)\, F(W).$$

[5] When the function F is log concave, an assumption which is often made, the logarithm of profit is concave so that there is a single maximum characterized by the first order condition.

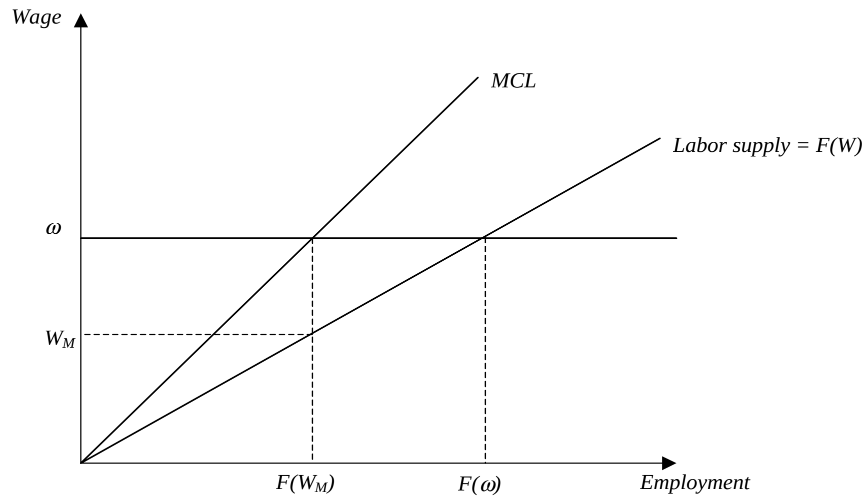

Figure 1: The textbook model of monopsony

Profit maximization yields the same first order condition as (4) where $\omega - t$ is substituted for ω. Accordingly, the employer is induced to choose the competitive wage $W = \omega$ and the competitive employment level if $t = -F(\omega)/F'(\omega)$, i.e. the tax is a subsidy. Assume that profits can be taxed away in a lump sum way: this allows the government to finance the subsidies. Therefore, a complete system of taxes is equivalent to the minimum wage in this model. This is reminiscent of the subsidy proposed by A. Robinson, (Robinson, 1933, p. 163), to lead a monopolist to produce the competitive output.

To summarize, the textbook model of monopsony shows that efficiency can be reached either with the minimum wage or with taxes on profits whose proceeds are used to finance employment subsidies. In this setup, the minimum wage is useful according to our definition. Moreover, it seems easier to use the minimum wage, rather than levying taxes on profits and redistributing the proceeds of these taxes under the form of employment subsidies. In what follows, we show that the minimum wage does not perform so well when there is a diversity of skills.

5 Second best tax schemes with perfectly competitive labor markets

Before studying imperfect competition, it is useful to recall the properties of optimal taxation and of the second best allocations in a perfectly competitive economy with heterogeneous skills.

A second best optimum is a triple (W, r, y), such that

1. the workers choose whether to work or not, and the amount they produce $y(\omega)$, taking as

given (W, r), according to

$$\max_{0 \le y \le \omega} [r, W(y) - \alpha], \tag{5}$$

2. anticipating the behavior of the agents, the government chooses a tax schedule (W, r) which maximizes the social welfare function:

$$\int_{\underline{\omega}}^{\bar{\omega}} \left\{ \int_{\underline{\alpha}}^{W[y(\omega)]-r} \Psi\left[W(y(\omega)) - \alpha\right] \mathrm{d}F(\alpha) + \{1 - F[W(y(\omega)) - r]\} \, \Psi(r) \right\} \mathrm{d}G(\omega), \tag{6}$$

subject to the budget constraint:[6]

$$\int_{\underline{\omega}}^{\bar{\omega}} \{y(\omega) - W[y(\omega)] + r\} \, F[W(y(\omega)) - r] \, \mathrm{d}G(\omega) = r. \tag{7}$$

The optima, as defined by 1. and 2., are studied in some details in Choné and Laroque (2008). Following the definition, second best optima are found as follows. First (5) implies that without loss of generality one can restrict the attention to non decreasing functions W and that workers work at their full productivity. Second, one solves the optimization problem of the government, maximizing (6) with respect to (W, r) subject to the constraint (7) on the set of non decreasing functions W. The tax wedge is then merely the difference between the productivity ω and the net wage $W(\omega)$.

At the optimum the wage function is uniquely defined on an interval $[\omega_{\text{inf}}, \overline{\omega}]$ of productivities, for which there are working agents. It can be given any arbitrary value, at most equal to $r + \underline{\alpha}$ for smaller productivities. We can also show that the lowest productivity among the employees, denoted by ω_{inf}, is at most equal to $\underline{\alpha}$.[7] Accordingly, ω_{inf} is lower than $r + \underline{\alpha}$ and following proposition 1, the minimum wage is of no use at the competitive equilibrium in our setup. This would not be necessarily the case when the elasticity of labor demand is finite as in the contribution of Lee and Saez (2007).

6 Second best tax schemes with monopsonistic labor markets

We consider now the case where the government faces a large firm, which is the sole buyer on the labor market. We define second best allocations. Formally, a second best optimum is a

[6] Under perfect competition, profits are equal to zero, so that (2) coincides with (3).

[7] Let us write the Lagrangian of the program of the government for productivity ω :

$$\int_{\underline{\alpha}}^{W-r} \Psi(W - \alpha) \mathrm{d}F(\alpha) + [1 - F(W - r)]\Psi(r) + \lambda \left[(\omega - W + r)F(W - r) - r\right]$$

where λ stands for the marginal cost of public funds. Suppose that $W(\omega) = r + \underline{\alpha}$ for $\omega \in (\underline{\alpha}, \omega_{\text{inf}}]$. On this interval, the derivative of the Lagrangian would be

$$\lambda(\omega - W + r)f(W - r) = \lambda(\omega - \underline{\alpha})f(\underline{\alpha}) > 0.$$

Therefore, a small increase in W would increase the value of the Lagrangian. Thus $\omega_{\text{inf}} \le \underline{\alpha}$ at the optimum.

quadruple (W, T, r, y), such that

1. the workers choose whether to work or not, and the amount they produce $y(\omega)$, taking as given (W, r), according to
$$\max_{0 \leq y \leq \omega} [r, W(y) - \alpha]; \tag{8}$$

2. the monopsonist chooses the net wage, taking as given (T, r), and anticipating the reactions of the workers to its choice, by maximizing
$$\max \int_{\underline{\omega}}^{\bar{\omega}} [y(\omega) - W(y(\omega)) - T(W(y(\omega)))] \, F[W(y(\omega)) - r] \mathrm{d}G(\omega); \tag{9}$$

3. the government chooses (T, r), anticipating the behavior of the monopsonist and of the workers, maximizing the social welfare function:
$$\int_{\underline{\omega}}^{\bar{\omega}} \left\{ \int_{\underline{\alpha}}^{W(y(\omega))-r} \Psi(W(y(\omega)) - \alpha) \mathrm{d}F(\alpha) + \{1 - F[W(y(\omega)) - r]\} \, \Psi(r) \right\} \mathrm{d}G(\omega), \tag{10}$$
subject to the feasibility constraint
$$\int_{\underline{\omega}}^{\bar{\omega}} \{y(\omega) - W(y(\omega)) + r\} \, F(W(y(\omega)) - r) \, \mathrm{d}G(\omega) = r, \tag{11}$$
when profits are taxed, or
$$\int_{\underline{\omega}}^{\bar{\omega}} \{T(W(y(\omega))) + r\} \, F(W(y(\omega)) - r) \, \mathrm{d}G(\omega) = r, \tag{12}$$
when they are not. We assume that the government chooses a continuous and bounded below tax function T.

In order to simplify the resolution of this problem, it is useful to show that one can restrict the analysis to net wages that increase with productivity.

Lemma 1 *At a second best optimum, without loss of generality:*

1. *the monopsonist can choose a net wage function W that is non decreasing with respect to productivity;*

2. *the government can choose a tax schedule T such that the function $x \to x + T(x)$ is everywhere non decreasing;*

3. *all the employees work at their full productivity ω.*

Proof: see appendix.

This lemma shows that any second best allocation can be reached with non decreasing net wage and cost schedules. At a second best optimum individuals work at their full time productivity and the net wage is a non decreasing function of productivity in the competitive environment. These properties also hold when the labor market is dominated by a monopsonist.

7 Monopsony with full taxation of profits

We consider a monopsony whose profits can be fully taxed by the governement. We prove that the government can bypass the monopsonist and implement the second best allocation of the competitive economy. Then we present two simple examples that illustrate the shapes of the tax schedules.

7.1 The optimal tax schedule

The description of the programs of the government (compare 2. in the definition of second best optima in section 5 with 3. in the definition of second best optima in section 6) makes it clear that, when profits can be fully taxed, the only difference between the competitive and monopsony problems comes from the (possible) restrictions imposed by the behavior of the monopsonist (9). As a consequence, the second best optima of the monopsonistic economy cannot Pareto dominate those of the competitive economy. They can at best coincide with them if the government manages to undo the wrongs caused by the monopsonist. We are going to show that this is indeed the case.

The Lagrangian of the program of the government in a competitive economy is

$$\mathcal{L} \;=\; \int_{\underline{\omega}}^{\bar{\omega}} \left\{ \int_{\underline{\alpha}}^{W(\omega)-r} \Psi(W(\omega)-\alpha))\,\mathrm{d}F(\alpha) + \Psi(r)\{1-F[W(\omega)-r]\} \right.$$

$$\left. + \lambda\left\{[\omega - W(\omega)+r]F[W(\omega)-r]-r\right\} \right\} \mathrm{d}G(\omega), \tag{13}$$

to be maximized over $(W(.), \lambda, r)$, for non decreasing W's. Let $[\omega_{\inf}, \bar{\omega}]$ be the endogenous set of productivities for which there are a positive number of employees at the optimum. The Lagrangian can be rewritten equivalently as

$$\frac{\mathcal{L}}{\lambda} = -r + \frac{\Psi(r)}{\lambda} + \tag{14}$$
$$\int_{\omega_{\inf}}^{\overline{\omega}} \left\{ \omega - W(\omega) + r + \int_{\underline{\alpha}}^{W(\omega)-r} \frac{\Psi(W(\omega)-\alpha)-\Psi(r)}{\lambda F[W(\omega)-r]}\,\mathrm{d}F(\alpha) \right\} F[W(\omega)-r]\,\mathrm{d}G(\omega).$$

Now, the objectives of the government (14) and of the monopsonist (9) are aligned provided that to any value W, $W > r + \underline{\alpha}$, of the net wage corresponds a value $\bar{C}(W) = W + \bar{T}(W)$ of the labor cost such that

$$\bar{C}(W) = W - r - \int_{\underline{\alpha}}^{W-r} \frac{\Psi(W-\alpha) - \Psi(r)}{\lambda F[W-r]} \mathrm{d}F(\alpha). \tag{15}$$

Note that $\bar{C}$ can take any value when wages are smaller than $r + \underline{\alpha}$ since nobody works for such wages. Therefore

Theorem 1 *The second best optimal allocations in a monopsonistic economy where the profits can be fully taxed are identical to that of a competitive economy. An optimum in the monopsonistic economy can be implemented through a tax wedge $\bar{T}(W)$ which satisfies*

$$\bar{T}(W) = -r^c - \frac{1}{\lambda^c}\left\{\int_{\underline{\alpha}}^{W-r^c} \frac{\Psi(W-\alpha)}{F[W-r^c]} \mathrm{d}F(\alpha) - \Psi(r^c)\right\} \text{ for } W \geq r^c + \underline{\alpha},$$

where r^c and λ^c are respectively the optimal subsistence income and marginal cost of public funds of the competitive economy.

Theorem 1 states that all allocations that can be reached in a competitive economy can also be reached, with different tax schedules, when the labor market is monopsonistic. It is a striking result which means that the government can systematically undo the wrongs caused by the monopsonist at no cost. Actually, this is already the case in the textbook example, discussed above, where the government can use either employment subsidies financed by taxes on profits or a minimum wage to reach the desired allocation. However, when skills are heterogeneous, the minimum wage is not useful any more. Indeed, since the allocation is the same as in the competitive economy, the lowest productivity of the employees is smaller than the lowest bound of the opportunity costs of work, $\underline{\alpha}$. This implies that nobody with productivity below ω_{inf} would want to work under *laissez-faire*, i.e. when $T(W) = 0$ for all wages below $r^c + \underline{\alpha}$. Then, Proposition 1 entails that the minimum wage is not useful.

Also it is easily checked that the tax function $\bar{T}$ is negative for $W \geq r^c + \underline{\alpha}$. Employment subsidies are efficient because they counteract the natural inclination of the monopsonist to reduce its demand for labor[8].

It should be stressed that the proof of Theorem 1 relies on the independence of the distributions of productivities and work opportunity costs. Indeed, in case of dependence, the argument

[8]Note that the function $\bar{C}(W)$, defined in (15), can have decreasing parts. The construction of Lemma 1 allows us to define an equivalent non decreasing cost function $C(W) = \min_{x \geq W} \bar{C}(x)$. It is easy to see that the associated tax schedule, T, is also negative, as the original function $\bar{T}$, for all $W > r^c + \underline{\alpha}$.

does not go through: the number of workers, that is equal to $F(W(\omega) - r)$ in equation (14), becomes a function $F(W(\omega) - r|\omega)$ that depends on ω. Then, the expression of the cost that aligns the objective of the government and the monopsonist, defined in equation (15), has no economic meaning since it depends on the unobserved ω.

It would be of interest to know whether Theorem 1 extends to situations where productivity is correlated with work opportunity cost. We do not have a general answer to this question. However, there is an extreme polar case which is easily dealt with: this is the situation where the work opportunity cost is a continuous increasing function of productivity, say $a(\omega)$. It can be shown that the first best optimum consists in putting to work all individuals of productivity ω greater than or equal to $a(\omega)$, while distributing welfare equally with a utility level equal to r for everyone where

$$r = \int_{\omega \geq a(\omega)} [\omega - a(\omega)] G(\omega).$$

The government can implement this optimum in a monopsonistic economy, when the monopsony profits are taxed away. Let $C(W) = W - r$ (compare with (15)). Then the monopsonist pays the workers at the lowest acceptable net wage, $W(\omega) = a(\omega) + r$, and employs all individuals who bring a non negative profit, i.e. such that $\omega - C(W(\omega)) = \omega - a(\omega) \geq 0$, the same individuals as in the first best optimum.[9]

Let us now look more precisely at the shape of the second best tax schedules in the monopsonistic model.

7.2 The shape of optimal tax schedules: two examples

The tax policy is not the same in the competitive and the monopsonistic case. The properties of the tax schedule in these two cases can be illustrated by looking at two polar assumptions about the objective of the government: first an output maximizing government, whose preferences are represented by the social welfare function $\Psi(x) = x$; then a Rawlsian government which maximizes the welfare of the most disadvantaged agents.

Output maximizing government

When the labor market is competitive, the equilibrium without tax and subsistence income, $r^c = T(W) = 0$, yields a first best allocation. It is easy to check, from (13), that the marginal cost of public funds λ^c is equal to 1. An agent with characteristics (ω, α) gets a utility level

[9] In the case where labor supply is infinitely elastic one might think that minimum wages are justified because monopsonistic employers take subsidies in their pocket instead of giving wage increases. Actually, in that case, the monopsonist always sets the net wage at the reservation level $a(\omega) + r$, whatever the labor tax or subsidy. Employment subsidies serve to implement the optimal level of employment, and the minimum wage is not needed. Indeed the objectives of the monopsonist and of the government are aligned when the elasticity of labor supply is infinite as far as the determination of the net wage is concerned.

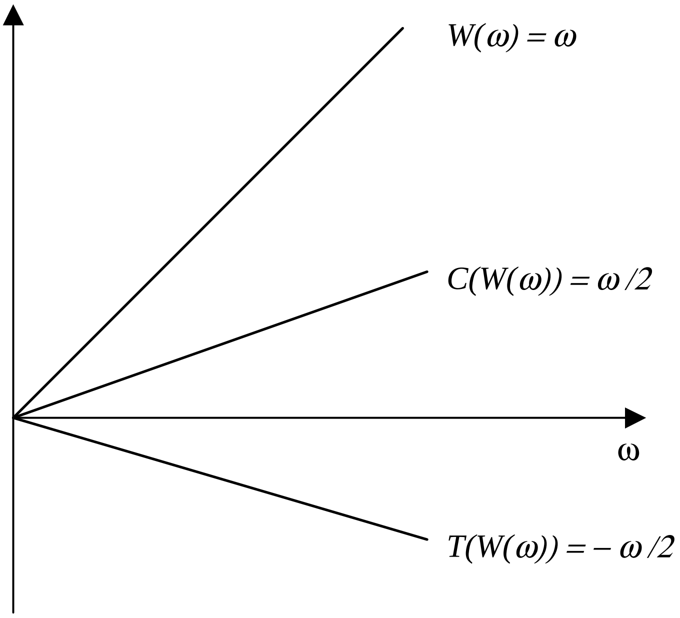

Figure 2: The net wage schedule $W(\omega)$, the tax schedule $T(W(\omega))$, the labor cost schedule $C(W(\omega))$ when there is an output maximizing government in a monopsonistic economy with uniform distribution functions.

$\max(0, \omega - \alpha)$. When the labor market is monopsonistic, the equilibrium without taxes does not yield a first best allocation any more. The previous section has shown that the first best allocation can nevertheless be implemented. The tax system which supports it can be found by rewriting the Lagrangian of the government program as $\int_{\underline{\omega}}^{\bar{\omega}} [\omega - C(W(\omega))] F(W(\omega))\mathrm{d}G(\omega)$, with $C(W) = W - \int_{\underline{\alpha}}^{W} \frac{W-\alpha}{F(W)} \mathrm{d}F(\alpha)$. The tax, equal to the difference between labor cost $C(W)$ and net wage W, is negative, equal to $-\int_{\underline{\alpha}}^{W} (W - \alpha)\mathrm{d}F(\alpha)/F(W)$.

The solution is represented on Figure 2 in the case where F and G are uniform over $[0, \bar{\alpha}]$ and $[0, \bar{\omega}]$ respectively. Then, measured as a function of net wage, the subsidy is equal to $W/2$, so that the labor cost associated with a net wage W is also $W/2$ and the monopsony eventually chooses the net wage schedule $W(\omega) = \omega$.

Rawlsian government

A Rawlsian government maximizes the value of the subsistence income provided to the unemployed agents. Noting $I(\omega) = W(\omega) - r$ the incentive to work, the program of the Rawlsian government can be written as

$$\max_{I(\cdot)} r = \int_{\underline{\omega}}^{\bar{\omega}} \{\omega - I(\omega)\} F[I(\omega)] \,\mathrm{d}G(\omega). \tag{16}$$

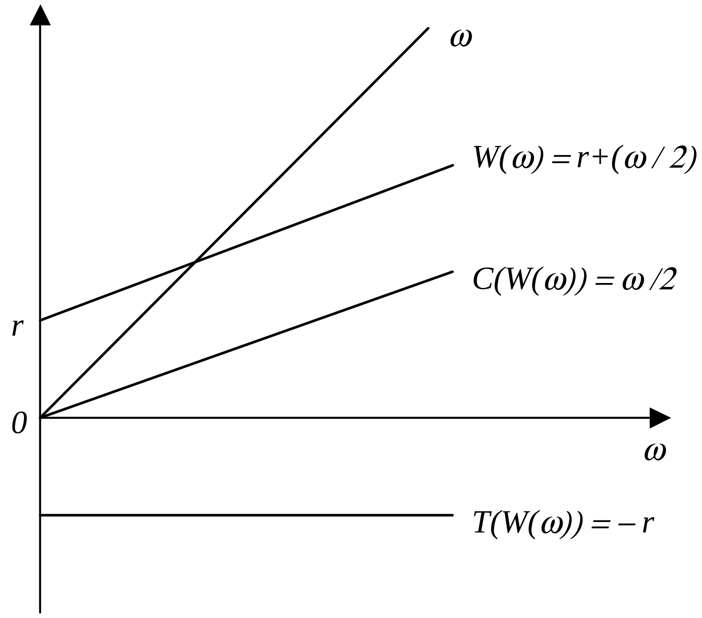

Figure 3: The net wage schedule $W(\omega)$, the tax schedule $T(W(\omega))$, the labor cost schedule $C(W(\omega))$ when there is a Rawlsian government in a monopsonistic economy with uniform distribution functions.

The comparison of this program with the program of the monopsonist (9) immediately shows that the objectives of the Rawlsian government and the monopsonist are aligned if the labor cost schedule $C(\cdot)$ is equal to the incentive to work $W - r$. As shown in figure 3, when F and G are uniform over $[0, \bar{\alpha}]$ and $[0, \bar{\omega}]$ respectively, the resulting allocation provides net wages $W(\omega) = r + (\omega/2)$ and a subsistence income $r = \bar{\omega}/8\bar{\alpha}$.

8 Entry and limits on corporate taxes

We have so far assumed that profits are entirely taxed away, as is consistent with the fact that the government knows the profit level. To put limits to the power of the government to tax firms while keeping with the spirit of optimal taxation models à la Mirrlees, we introduce additional information asymmetries which allow the firms to keep some rents. A simple and illustrative way to proceed is to consider a situation where the government regulates a continuum of separate local labor markets. In all these labor markets, there is the same distribution of workers as the one of the previous section: these markets work independently and there is no labor mobility across markets. Firms have to pay an entry cost[10] h, which is drawn independently across labor markets with the c.d.f. H. The government, which observes net wages as before but not

[10] A positive value of h corresponds to an entry cost; a negative value can be interpreted as a rent of size $|h|$.

the entry costs of the firms, announces the same tax schedule (T,r) over the whole territory, with additionally a lump sum subsidy s ($-s$ is the level of the tax when s is negative) given to the firms to help them pay their costs. Given our informational assumption, the subsidy can only be constant, identical across labor markets. The economy works as follows: first the government announces the full tax and subsidy schedule common to all markets; second the random entry costs and productivity are drawn; the firms decide to operate when the sum of their expected profits and subsidy exceeds the entry cost; finally production is undertaken and wages are paid in the markets which operate; in the other markets, nobody works and everyone gets the subsistence income r.

When labor markets are competitive operating profits are equal to zero under constant returns to scale and firms decide to operate when $s-h$ is non negative. The government program then is

$$\max_{[T(\cdot),r,s]} H(s)\int_{\underline{\omega}}^{\bar{\omega}}\left\{\int_{\underline{\alpha}}^{W(\omega)-r}\Psi(W(\omega)-\alpha)\,\mathrm{d}F(\alpha)+\{1-F[W(\omega))-r]\}\,\Psi(r)\right\}\mathrm{d}G(\omega)$$
$$+[1-H(s)]\Psi(r),$$

subject to the feasibility constraint

$$H(s)\int_{\underline{\omega}}^{\bar{\omega}}\{\omega-W(\omega)+r\}\,F(W(\omega)-r)\,\mathrm{d}G(\omega)=r+sH(s).$$

Let λ be the multiplier of the feasibility constraint, and for further reference denote (T^c,r^c,s^c,λ^c) a solution of the above program for the competitive economy.

When the labor markets are monopsonistic, the firms' operating profits allow them to finance some of the entry costs and the subsidy may be negative. The before tax operating profit π of the typical firm is

$$\pi=\int_{\underline{\omega}}^{\bar{\omega}}\{\omega-W(\omega)-T(W(\omega))\}\,F(W(\omega)-r)\,\mathrm{d}G(\omega),$$

under (T,r). The firm decides to operate when $\pi+s-h$ is non negative, i.e. with probability $H(\pi+s)$.

The program of the government becomes

$$\max_{[T(\cdot),r,s]} H(\pi+s)\int_{\underline{\omega}}^{\bar{\omega}}\left\{\int_{\underline{\alpha}}^{W(\omega)-r}\Psi(W(\omega)-\alpha)\,\mathrm{d}F(\alpha)+\{1-F[W(\omega))-r]\}\,\Psi(r)\right\}\mathrm{d}G(\omega)$$
$$+[1-H(\pi+s)]\Psi(r),$$

subject to the feasibility constraint

$$H(\pi+s)\int_{\underline{\omega}}^{\bar{\omega}}\{\omega-W(\omega)+r\}\,F(W(\omega)-r)\,\mathrm{d}G(\omega)=r+(\pi+s)H(\pi+s),$$

while the monopsonist sets a wage schedule according to

$$W(\cdot) = \arg\max_{S(\cdot)} \int_{\underline{\omega}}^{\bar{\omega}} \{\omega - S(\omega) - T(S(\omega))\} F(S(\omega) - r) \,\mathrm{d}G(\omega).$$

Although the profit π, which depends on T, shows up in the objective and the feasibility constraint of the government, this does not create difficulties since it only appears through the after tax profit $\pi + s$. Indeed put $\pi + s = s^c$ in the program of the government: ignoring the behavior of the monopsony the solution of the program is then a competitive optimum (the government manages to tax away all monopsony profits!). This allows us to transpose here the argument of Theorem 1. The objective of the government and of the monopsonist are aligned and yield the competitive net wage W^c if one defines

$$T^m(W) = -r^c - \int_{\underline{\alpha}}^{W(\omega)-r^c} \frac{\Psi(W-\alpha) - \Psi(r^c)}{\lambda^c F[W - r^*]} \,\mathrm{d}F(\alpha).$$

The profit of the monopsonistic firm follows, and the value of s^m is computed from $s^m = s^c - \pi^m$. This implies that the government can reach the same allocation as in the competitive economy.

It is worth noting that the function T^m has the same analytical expression as in Theorem 1, but different values of the marginal costs of public funds and the subsistence income. Therefore, as discussed in section 7, the government does not need the minimum wage to undo the wrongs of the monopsony even when profits cannot be fully taxed. With the instruments adapted to its information structure, the government has all the necessary fiscal tools to reach the second best optimum of the competitive economy without using the minimum wage. It could be worth analyzing the degree of generality of this result.

9 Conclusion

In this paper we compare income redistribution in two economies which only differ by the functioning of their labor markets, competitive or monopsonistic. The government has a large range of instruments at its disposal, which includes wages and profits taxes and the minimum wage. By defining the minimum wage as useful whenever it can replace a non negligible part of the tax schedule, we take the view point the most favorable to the minimum wage. Even with this view point, the minimum wage appears to be of no use to correct the transfers or inefficiencies associated with the monopsony when the heterogeneity of skills is accounted for.

This result holds when abilities are distributed independently of work opportunity costs. It is also satisfied when work opportunity cost is a deterministic increasing function of ability. We do not know how to handle the intermediate cases.

The conclusion that monopsony does not justify the minimum wage does not mean that the minimum wage is useless as a part of efficient redistributive schemes in all circumstances (indeed see Lee and Saez (2007) for a counter example). What we claim here is that in the absence of restrictions on the set of available tax instruments, a monopsony on the labor market can be dealt with without using a minimum wage in a linear technology setup. Whether the minimum wage somehow can be useful in other circumstances is an open question. More research is needed in this area.

References

[1] Allen, S. (1987), "Taxes, Redistribution, and the Minimum Wage: A Theoretical Analysis", *Quarterly Journal of Economics*, 102, 477-489

[2] Bhaskar, V. and To, T. (1999), "Minimum Wages for Ronald McDonald Monopsonies: A Theory of Monopsonistic Competition", *Economic Journal*, 109, 190-203.

[3] Beaudry, P., and Blackorby, C. (1997), "Taxes and Employment Subsidies in Optimal Redistribution Programs," Discussion Paper 22, University of British Columbia.

[4] Boadway, R. and Cuff, K. (2001) "A Minimum Wage can be Welfare Improving and Employment-Enhancing", *European Economic Review*, 45, 553-576.

[5] Cahuc, P., Zylberberg, A. and Saint-Martin, A. (2001), "The consequences of the minimum wage when other wages are bargained over", *European Economic Review*, 45, 337-352.

[6] Card, D. and Krueger, A. (1995), *Myth and Measurement: The new economics of minimum wage,* Princeton University Press.

[7] Choné, P. and Laroque, G. (2005),"Optimal Incentives for Labor Force Participation", *Journal of Public Economics*, 89, 395-425.

[8] Choné, P. and Laroque, G. (2008), "Optimal Taxation in the Extensive Model", working paper n°8, Institute for Fiscal Studies.

[9] Diamond, P. (1980), "Income Taxation with Fixed Hours of Work", *Journal of Public Economics,* 13, 101-110.

[10] Dolado, J., Felgueroso, F. and Jimeno, J. (2000), "The Role of the Minimum Wage in the Welfare State: An Appraisal", IZA Discussion paper 152, published in *Swiss Journal of Economics and Statistics*, 136 (2000), 1-33.

[11] Drazen, A. (1986), "Optimal Minimum Wage Legislation", *Economic Journal*, 96, 774-784.

[12] Flinn, C. (2006) "Minimum Wage Effects on Labor Market Outcomes under Search, Bargaining, and Endogenous Contact Rates", *Econometrica* 74, 1013-1062.

[13] Guesnerie, R., Roberts, R. (1987), "Minimum Wage Legislation as a Second-Best Policy", *European Economic Review,* 31, 490-498.

[14] Hungerbühler, M. and Lehmann, E. (2007), “On the optimality of a minimum wage: New insights from optimal tax theory”, mimeo CREST.

[15] Immervoll, H. (2007), “Minimum Wages, Minimum Labour Costs and the Tax Treatment of Low-wage Employment”, OECD Social, Employment and Migration Working Papers n°46.

[16] Jones, S. (1987), “Minimum Wage Legislation in a Dual Labor Market”, *European Economic Review,* 33, 1229-1246.

[17] Laroque, G. (2005), “Income Maintenance and Labor Force Participation”, *Econometrica*, 73, 341-376.

[18] Lee, D and Saez, E. (2007) ”Optimal Minimum Wage Policy in Competitive Labor Markets”, Working paper, University of Berkeley.

[19] Manning, A. (1995), “How do We Know that Real Wages are Too High ?”, *Quarterly Journal of Economics,* 110, 1111-1125.

[20] Manning, A. (2003), *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press.

[21] Masters, A. (1999), “Wage posting in two-sided search and the minimum wage”, *International Economic Review,* 40, 809-826.

[22] Mirrlees, J. (1971), “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies*, 38, 175-208.

[23] Neumark, D. and Wascher, W. (2006), “Minimum Wages and Employment: A Review of Evidence from the New Minimum Wage Research”, NBER working paper N°12663.

[24] OECD, (1994), *The OECD jobs study. Facts, Analysis, Strategies,* Paris: OECD.

[25] OECD, (1998), *Employment Outlook*, Paris: OECD.

[26] OECD, (2005), *Employment Outlook*, Paris: OECD.

[27] Rebitzer, J. and Taylor, L. (1995), “The Consequences of Minimum Wage Laws, Some New Theoretical Ideas”, *Journal of Public Economics*, 56, 245-255.

[28] Robinson, J., (1933), *The Economics of Imperfect Competition,* MacMillan, London.

[29] Saez, E. (2002), “Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses,” *Quarterly Journal of Economics*, 117, 1039-1073.

[30] Stigler, G. (1946), "The Economics of Minimum Wage Legislation", *American Economic Review*, 36, 535-543.

A Appendix: Proof of Lemma 1

1) This is a consequence of the behavior of workers. When confronted with a net wage schedule $\overline{W}$, workers' choice of y follows from:

$$\sup_{0 \leq y \leq \omega} [r, \overline{W}(y) - \alpha].$$

Let

$$W(y) = \sup\{\overline{W}(z) | 0 \leq z \leq y\}.$$

By construction W is non decreasing, the utility levels attained by the agents under $\overline{W}$ and W are identical, while the labor supply under $\overline{W}$ is a subset of the labor supply under W. Therefore, the monopsonist can choose a non decreasing net wage schedule without loss of generality.

2) This follows from a study of the behavior of the monopsonist. Consider any bounded below function $\overline{C}$, and let

$$C(w) = \inf_{x \geq w} \overline{C}(x).$$

Define $\mathcal{C} = \{w | C(w) < \overline{C}(w)\}$, or equivalently $\{w|$ there exists $z > w$ such that $\overline{C}(z) < \overline{C}(w)\}$. This set is empty if $\overline{C}$ is non decreasing, and C coincides with $\overline{C}$ outside of $\mathcal{C}$.

We show that a monopsonist facing the tax schedule leading to $\overline{C}$ will never employ a worker at a net wage in $\mathcal{C}$, so that C and $\overline{C}$ lead to the same allocation.

Let $\overline{W}$ be a net wage schedule. The monopsony profits are

$$\int_0^{\overline{\omega}} \left[\overline{y}(\omega) - \overline{C}(\overline{W}(\overline{y}(\omega)))\right] F\left(\overline{W}(\overline{y}(\omega)) - r\right) \, \mathrm{d}G(\omega),$$

where $\overline{y}(\omega)$ is the production of a worker of productivity ω who faces the net wage schedule $\overline{W}$.

Property: *Assume that $\overline{C}$ is a continuous bounded below function on $\mathbb{R}_+$. There is an optimal non decreasing net wage schedule W such that*

$$W(\mathbb{R}_+) \cap \mathcal{C} = \emptyset.$$

Proof of the property: Take a W associated with $\overline{C}$, which is non decreasing by 1), whose range may have a non empty intersection with $\mathcal{C}$. We modify it into a new function W whose range does not intersect $\mathcal{C}$, while weakly increasing the profits of the monopsonist.

By continuity of $\overline{C}$, the set $\mathcal{C}$ is made of the union of disjoint intervals, say (w_0, w_1). One of the two following situations arises:

a) either $\overline{C}(z) > \overline{C}(w_1)$ for all z smaller than w_1;

b) or there exists w_0 such that $\overline{C}(w_0) = \overline{C}(w_1)$, and $\overline{C}(z) > \overline{C}(w_1)$ for all z such that $w_0 > z > w_1$.

We treat the two cases in turn, supposing that there is some z in the interval and some y such that $z = W(y)$.

a) In the first case, let

$$y_1 = \inf\{y|W(y) \geq w_1\}.$$

We modify W for all $y \leq y_1$ through define:

$$\begin{array}{ll} W(y) \leq \underline{\alpha} + r & \text{for } y < \overline{C}(w_1) \\ W(y) = w_1 & \text{for } \overline{C}(w_1) < y < y_1. \end{array}$$

By construction the new W is non decreasing. It leads to profits at least as large as the original W: if there are points such that $y < \overline{C}(w_1)$, the pointwise profit initially equal to $[y - \overline{C}(W(y))]F(W(y))$, is negative, but becomes null because nobody works when $W(y) \leq \underline{\alpha} + r$; when there are points such that $\overline{C}(w_1) < y < y_1$, the pointwise profit is initially positive, but is at least as large after the transformation since $\overline{C}(w_1) \leq \overline{C}(\overline{W}(y))$ and $F(w_1) \geq F(\overline{W}(y))$. The range of the modified W has an empty intersection with $[0, w_1)$ as desired.

b) In the second case, let

$$y_0 = \sup\{y|W(y) \leq w_0\},$$

and

$$y_1 = \inf\{y|W(y) \geq w_1\}.$$

We modify W on the interval (y_0, y_1) through

$$W(y) = \begin{cases} w_0 & \text{for } y_0 < y < \overline{C}(w_1) \\ w_1 & \text{for } \overline{C}(w_1) < y < y_1. \end{cases}$$

By construction, the new W is non decreasing. It leads to profit sat least as large as the original W: for the y's such that $y_0 < y < \overline{C}(w_1)$, if any, this follows from the fact that the pointwise profit $[y - \overline{C}(W(y))]F(W(y) - r)$ is negative, associated with the inequalities $\overline{C}(\overline{W}(y)) > \overline{C}(w_0)$ and $F(w_0 - r) \leq F(W(y) - r)$; similarly for the y's such that $\overline{C}(w_1) < y < y_1$, the pointwise profit is positive and $F(w_1 - r) \geq F(\overline{W}(y) - r)$. The range of the modified W has an empty intersection with (w_0, w_1).

This completes the proof of the Property.