## Changes in the Transitory Variance of Income Components and their Impact on Family Income Instability

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August 22, 2010

#### Abstract

The well-documented increase in family income instability reflects changes in the underlying joint distribution of individual income components –head's earnings, spouse's earnings, and other income. Existing studies have examined changes in instability of some of these income sources. We go beyond these studies by estimating changes in the joint distribution of these income sources. In addition to examining changes in instability in the marginal distributions, we also estimate changes in the correlation between these income sources. We also add to the literature by modeling changes at the extensive margin. We allow the discrete outcome of receiving positive income from each income source to include a permanent and a transitory component, which can be correlated with the permanent and transitory components of other income sources... Estimation using the Panel Study of Income Dynamics 1970-2004 provides potential explanations for rising family income fluctuations: (1) Increase in instability of male head's earnings, unearned family income, covariance between male head's earnings and unearned family income; (2) Negative covariance between head's and spouse's earnings shocks, and the decrease in married women's earnings instability help reduce family income instability; (3) Although male earnings instability is strongly correlated with business cycle, family income is less influenced by recessions due to household smoothing through spouse's earnings and unearned income.

**JEL Codes**: D31, D63.

**Keywords**: Family Income Instability

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#### 1 Introduction

The increase in earnings instability, as measured by increases in the variance of transitory earnings, is by now well documented. All studies show a sharp increase in the variance of the transitory component of earnings during the 1980s. This is followed by a leveling off during the 1990s. In contrast, Gottschalk and Moffitt (2009) find that when the same methodology is applied to study instability of family income that this broader measure of income shows a steady increase in from the early 1980s to 2002. What accounts for these very different trends in recent years?

While studies of earnings instability in family income instability have documented the basic trends in these two earnings concepts, the conceptual and empirical links between the two is not well developed. This study attempts to fill this gap by exploiting the fact that the increase in the transitory variance of family income reflects changes in the underlying joint distribution of individual income components –head's earnings, spouse's earnings, and other income. Existing studies have examined changes in the transitory variance in the marginal distributions of some of these income sources. We go beyond these studies by estimating changes in the joint distribution of multiple income sources.

In addition to examining changes in the transitory variances in the marginal distributions of each income source, we also estimate changes in the covariance between the transitory components of these income sources. Changes in these covariances of transitory income may reflect changes in income smoothing.

Finally, we contribute to the literature by explicitly modeling the permanent and transitory factors leading to positive income in each of the sources. Previous work has focused almost solely on male heads with positive earnings and families with positive total income. The restriction on male heads is fairly innocuous, since most

male heads have positive incomes. Expanding the analysis to other income sources, such as spouses' earnings, which may themselves go to zero in response to transitory changes and other income sources raises a new set of issues that we address in this paper.

We use the quadrature integration to estimate the parameters of process determining the receipt of each income source and Method of Moments to estimate the parameters of the process determining the permanent transitory components of positive income in each income source. These models are estimated using the Panel Study of Income Dynamics 1970-2004.

#### 2 Model

The methodology used in this paper to estimate changes in the permanent and transitory components of each income source is well developed and reviewed briefly below. This methodology, however, only models changes in the covariance structure of positive income. In essence it models changes at the extensive margin in a single source of income. This ignores changes in the permanent and transitory components at the extensive margin. Namely changes in the permanent and transitory components of the latent propensity to receive income from each source. This margin may be important for income sources other than heads earnings.

# 2.1 Method for Estimating Permanent and Transitory Variances

#### 2.1.1 Family Income and Its Components

Our analysis of the relationship between changes in the variances of the permanent and transitory components of the K subcomponents of income and changes in the variance total family income is based on the following identity

$$Y_{it} = \sum_{k=1}^{K} Y_{it}^k \tag{1}$$

where  $Y_{it}^k$  is the income received by family i from income source k.in year t. and  $Y_{it}$  is total family income from the K sources. The variance of total income is, therefore, given by<sup>1</sup>

$$var(Y_t) = \sum_{k=1}^{K} \left[ var(Y_t^k) + \sum_{l=1}^{K} cov(Y_t^k, Y_t^l) \right]$$
 (2)

The income generating process for each of the K sources of income is given by,

$$Y_{it}^k = \tilde{Y}_t^k + \varepsilon_{it}^k \tag{3}$$

$$\varepsilon_{it}^k = \alpha_t^k \mu_i^k + \beta_t^k \nu_{it}^k \tag{4}$$

where  $\tilde{Y}_t^k$  is the conditional mean log income from source k across all families, controlling for lifecycle changes in income from source  $k^2$ . Family i's income deviates

<sup>&</sup>lt;sup>1</sup>This relationship holds exactly when  $Y_{it}$  is an absolute measure of income but holds only approximately when  $Y_{it}$  is a measure of log income. Using Taylor series approximations it is straightforward to show that the variance of log total income is approximately equal to the weighted sum of the variances and covariances of log income of each source with the weights given by the ratio of the means of each income source relative to the mean of total income  $\sigma^2(Y_t) = \sum_{k=1}^K \left[ \left( \frac{\mu_k}{\mu_T} \right)^2 \sigma^2(Y_t^k) + \sum_{l=1}^K \left( \frac{\mu_k \mu_l}{\mu_T^2} \right) 2\rho(\exp Y_t^k, \exp Y_t^l) \sigma_{Y_t^k} \sigma_{Y_t^l} \right], \text{ where } Y_t^k \text{ should now be interpreted as the ln of income source } k \text{ and } \mu_k = E(\exp Y_t^k)$ 

 $<sup>^2</sup> ilde{Y}_t^k$  is the conditional mean from a first stage regression of  $Y_{it}^k$  on a polynomial in age to take

from the mean by  $\varepsilon_{it}^k$ .

The deviation of family i's income from the mean income from source k is composed of a permanent component,  $\alpha_t^k \mu_i^k$ , and a transitory component  $\beta_t^k \nu_{it}^k$ . Both the permanent and transitory components of  $\varepsilon_{it}^k$  are allowed to shift via the parameters  $\alpha_t^k$  and  $\beta_t^k$ 

The variances and covariances of  $\varepsilon_{it}$  are given by

$$\sigma_{\varepsilon_{it}}^2 = \alpha_t^2 \sigma_\mu^2 + \beta_t^2 \sigma_\nu^2 \tag{5}$$

$$\sigma_{\varepsilon_{it},\varepsilon_{is}} = \alpha_t \alpha_s \sigma_\mu^2 + \beta_t \beta_s \sigma_{\nu_{it}\nu_{is}} \tag{6}$$

The relative magnitudes of  $\alpha_t^k$  and  $\beta_t^k$  determine how much of a change in the variance of income arises from a change on the variance of permanent rather than the transitory income. For example, if  $\beta_t^k$  is constant across time, then any change in the total variance of income must have arisen from a change in the variance of the permanent component.

# 2.1.2 Method for Estimating the Permanent and Transitory Variances for a Single Income Source

The main object of interest for present purposes is  $\beta_t$ . We would like to know if it has changed with calendar time and, if so, how much of the increase in the variance of log earnings or income reflects an increase in the transitory variance. Several methods, described in Moffitt and Gottschalk (2008), have been proposed to estimate changes in the transitory variance.

The method we use in this paper is based on equation 5 which imposes relatively little structure on the distributions of  $\mu_i$  or  $\nu_{it}$ . Parameters to be estimated are out lifecycle changes in  $Y_{it}^k$ .

<sup>&</sup>lt;sup>3</sup>An alternative is to place functional form assumptions on the time series properties of  $\mu_i$  and

 $\alpha_t, \beta_t, \sigma_{\mu}^2$  and  $\sigma_{\nu}^2$ . If the lag between s and t is sufficiently long to insure that  $\sigma_{\nu_{it}\nu_{is}} = 0$  then the covariance between income in t and s reflect only the permanent component:

$$\sigma_{\varepsilon_t,\varepsilon_s} = \alpha_t \alpha_s \sigma_u^2 \tag{7}$$

GMM can, therefore be used to estimate  $\alpha_t$ ,  $\alpha_s$  and  $\sigma_{\mu}^2$  off of long lags.

This can be implemented by using OLS if the identity matrix is used as the weighting matrix. Define  $\epsilon$  as the difference between the logs of the empirical and theoretical covariances

$$\tilde{\epsilon} = \ln\left(s_{\hat{\varepsilon}_t,\hat{\varepsilon}_s}\right) - \ln\left(\sigma_{\varepsilon_t,\varepsilon_s}\right) \tag{8}$$

Substituting the empirical covariances,  $s_{\hat{\varepsilon}_t,\hat{\varepsilon}_s}$ , into (7) and taking logs yields

$$\ln(s_{\hat{\varepsilon}_t,\hat{\varepsilon}_s}) = \ln \sigma_u^2 + \ln \alpha_t + \ln \alpha_s + \epsilon \tag{9}$$

As in GMM the objective is to minimize the weighted sum of squared deviations,  $\sum_t \sum_s w_{ts} \epsilon_{ts}^2$ , between the empirical and theoretical covariances. If the identity matrix is used as the weighting matrix, this is equivalent to estimating:

$$\ln(s_{\hat{\varepsilon}_t,\hat{\varepsilon}_s}) = \gamma_0 + \sum_{j=1}^T \gamma_s D_s + \epsilon \tag{10}$$

by OLS, where the D's are time dummies. Since  $E(\hat{\gamma}_0) = \ln \sigma_{\mu}^2$  and  $E(\hat{\gamma}_j) = \ln \alpha_j$ , the estimated  $\gamma's$  can be used to recover the parameters of interest. Using these

 $<sup>\</sup>nu_{it}$  and to derive the implied covariance structure. This procedure, followed by Haider (2001); Moffitt and Gottschalk (2008) Moffitt and Gottschalk (2002); Baker and Solon (2003); Moffitt and Gottschalk (2008); can be estimated by GMM using all T(T+1)/2 covariances. A third, and simpler method developed in Gottschalk and Moffitt (1994) is to calculate the mean of income for each individual over a fixed window and use the variance of these means as an estimate of the permanent variance. The transitory variance is obtained as the variance of deviations from each individual's means. While this more intuitive method has been used often, it has substantial limitations (see Moffitt and Gottschalk (2008))

results we estimate the permanent variances as  $\hat{\alpha}_t^2 \hat{\sigma}_{\mu}^2$ . Transitory variances in each year are then estimated as the difference between the observed total variance and and the estimated permanent variance.

This model can be modified to allow different age cohorts to have different permanent and transitory variances. In the OLS regression, we allow intercept  $(ln\sigma_{\mu})$  to be age specific by adding age dummies take the value of 25, 35, 45, 55. We also condition on a quadratic in lag length.

## 2.1.3 Method for Estimating the Joint Distribution of Permanent and Transitory Variances for Two Income Sources

This section describes the estimation of permanent and transitory covariances between two income sources, using male head's earnings and spousal earnings as an example. We index head's earnings as income source m, spouse's earnings as income source f.

$$y_{it}^{m} = \alpha_{t}^{m} \mu_{i}^{m} + \beta_{t} \nu_{it}^{m}$$

$$y_{it}^{f} = \alpha_{t}^{f} \mu_{i}^{f} + \beta_{t} \nu_{it}^{f}$$

$$cov(\mu_{i}^{m}, \mu_{i}^{f}) = \rho_{\mu}^{mf}$$

$$cov(\nu_{it}^{m}, \nu_{it}^{f}) = \rho_{\nu}^{mf}$$

$$(11)$$

Our method for estimating the covariances between the permanent and transitory components of these two sources is a direct extension of the method described above for estimating the parameters of a single source.

The variance-covariance matrix of the two income sources is a 2T by 2T matrix with the upper and lower T by T upper diagonal matrices being the variance-covariance matrices for each income source, and the off diagonal T by T elements

being the covariance matrix between two income sources. The following is an illustrative example for a two period 4 by 4 matrix:

The parameters can again be estimated using OLS to minimize the squared deviations between the empirical and theoretical covariances. Since this structure introduces an additional  $T^2$  covariances but only two additional parameters, it is clearly identified.

#### 2.1.4 Receipt of Positive Income

Thus far, we have followed the literature by estimating the covariance structure of income for persons with positive income. This, however, ignores instability that results from losing a source of income either permanently or transitorily. In this section we take a small and preliminary step toward filling this gap by modeling permanent and transitory components of receipt of positive income from source k. For example, non-receipt of source k may have become more permanent or more transitory. This change at the extensive margin would not be captured directly by the preceding models that focus on the intensive margin among those who receive source k..

At this stage our aim is only to describe this additional source of transitory fluctuations. We fully realize that changes at the extensive margin may also have selection effects at the intensive margin. For example an increase in the transitory fluctuations between zero and positive wive's earnings may increase the transitory variance of earnings among working wives. Addressing this selection effect requires additional identifying assumptions. At this point our aim is only to describe this additional process that has largely been ignored in the literature on changes in the covariance structure of positive income. The next step is clearly to estimate the permanent and transitory components of income net of these selection effects

Let  $D_{it}^k$  be a dichotomous variable indicating positive income from source k This indicator variable is also determined by a permanent and a transitory component

$$D_{it}^{k} = I(\tilde{\alpha}_{t}^{k}\tilde{\mu}_{i}^{k} + \tilde{\beta}_{t}^{k}\tilde{\nu}_{it}^{k}) \tag{12}$$

$$= I(\tilde{\nu}_{it}^k > \gamma_t^k \tilde{\mu}_i^k) \tag{13}$$

where  $\gamma_t = \frac{-\tilde{\alpha}_t}{\tilde{\beta}_t}$ . The permanent and transitory components of this propensity are allowed to differ from the components of positive income. We, therefore, allow receipt of public assistance to become more permanent (i.e. rise  $\tilde{\alpha}_t^k$  relative to  $\tilde{\beta}_t^k$  so  $\tilde{\gamma}_t^k$  increases) while the transitory component of public assistance among those who receive public assistance may becomes more transitory (i.e.  $\alpha_t^k$  falls relative to  $\beta_t^k$  so  $\gamma_t^k$  declines).

Given this structure, the probability of receiving income from source k is given by

$$\Pr(D_{it}^k = 1) = pr(\tilde{\nu}_{it}^k > \tilde{\gamma}_t^k \tilde{\mu}_i^k) \tag{14}$$

where  $\tilde{\gamma}_t^k$  and  $\sigma_{\tilde{\mu}}$  are identified <sup>4</sup>

<sup>&</sup>lt;sup>4</sup>As in other discrete choice models, one of the variances is unidentified and set to 1.

$$Pr(D_{it} = 1) = pr(\tilde{\nu}_{it} > \tilde{\gamma}_t \tilde{\mu}_i)$$
(15)

$$= \frac{\nu - E(\nu)}{\sigma_{\nu}} > \gamma_t \left(\frac{\mu_i - E(\mu)}{\sigma_{\mu}}\right) \left(\frac{E(\mu) - E(\nu)}{\sigma_{\mu}}\right) \left(\frac{\sigma_{\mu}}{\sigma_{\nu}}\right)$$
(16)

so multiplying  $\sigma_{\nu}$  and  $\sigma_{\mu}$  by k or adding k to  $E(\mu)$  and  $E(\nu)$  leads to the same predicted probability. The log likelihood for his random effects model with factor loadings is given by

$$L(\tilde{\gamma}_t^k, \sigma_{\tilde{\mu}}^k) = \sum_i \int \sum_t \left[ (1 - F(\tilde{\gamma}_t^k \tilde{\mu}_i^k))^{D_{it}^k} \left[ F(\tilde{\gamma}_t^k \tilde{\mu}_i^k)^{(1 - D_{it}^k)} d\tilde{\mu}^k \right] \right]$$
(17)

where the unobservable  $\tilde{\mu}_i^k$  is integrated out using Gaussian quadrature.

#### 3 Data

We use data from the Panel Study of Income Dynamics (PSID), a longitudinal study of a representative set of family units in 1968 and their descendants. PSID began in 1968 and families have been interviewed ever since then, giving researchers a long period of observation of the earnings and income of these families. From 1968 to 1996, the PSID interviewed and re-interviewed families in the core sample every year. Since 1997 data were collected biennially. The PSID has experienced a significant amount of attrition, although there are no indications to date that this causes noticeable bias. In addition, because the sample is only representative of the U.S. population in 1968, it cannot capture the portion of the current population that has immigrated since then.

We examine the covariance structure of total family income and three family in-

come components: male head earnings, spouses earnings if applicable, and all other income, over income year 1970-2004. The later is a residual category that includes capital income, public and private transfers, and other family members' earnings. Our sample includes all families with male head aged between 20 and 59. To reduce measurement error, we trim two percent at the top and bottom of the relevant distribution. The trim at the top of the distribution ensures that all top-coded values are excluded. The trim at the bottom the distribution drops extremely low values that have a disproportional impact on the variance of log income. Therefore, when interpreting results we focus on income and earnings instability of the inner 96 percent of the distribution only.

#### 4 Results

We start by restricting the analysis to the permanent and transitory components of positive income from each source. In the following section we turn our attention to the permanent and transitory components of the propensity to receive each income source.

# 4.1 Permanent and Transitory Components of Positive Income Sources

We estimate permanent and transitory variance in each year for log family income, log head earnings, log spouse earnings, and log residual income (i.e., family income minus heads earnings and spouse's earnings). Figure 1 displays permanent and transitory variance in log family income. Over the past three decades, cross-sectional variance in log family income has increased substantially. This increase is a result of roughly

equal increases in the transitory variance and permanent variance. We also note that family income instability, as measured by transitory variance, moves together with business cycles. Family income instability displays significant spike one year after recession period in 1992 and 2001.

Figure 2 displays permanent and transitory variance in male head's earnings. Male head's earnings instability rises substantially in the late 1970s and early 1980s. This is followed by a flattening in the late 1980s. There is a subsequent slow decline in the transitory variance through the late 1990s, followed by an increase in the 2000s. This flattening of the trend in head's earnings instability while family income instability continued to increase is what motivates our study.

Figure 3 shows sharp declines in both the permanent and transitory variances in spousal earnings. <sup>5</sup> Spousal earnings instability showed a net decline between 1970 and 2000 but most of the decline occurred in the early 1970s. However, instability of spousal earnings rose in the more recent period, narrowing the rise in head's earnings. instability.

Finally Figure 4 displays the permanent and transitory variance in the residual components of family income. This residual category includes head and wife's capital income, public and private transfers, other family members' earnings, capital income and transfer income. This series shows much more modest trends and greater year to year fluctuations.

Taken together, changes in these marginal distributions show substantial differences across time, with no single source miring the changes in family income. Clearly, one must go beyond changes in these marginal distributions to account for the steady rise in family income instability. This includes an explicit consideration of the covari-

 $<sup>^5</sup>$ According to PSID definition, spousal earnings of male heads refers to married women's earnings.

ances between these income sources and the covariance structure of the propensity to have positive income from each source.

#### 4.2 Covariances across Income Sources

Figure 5 displays the covariance between the permanent and transitory components of log heads earnings and log spousal earnings. Since we do not observe spousal earnings for single males, results in figure 5 include only the sample of married couples. The covariance between couple's earnings is close to zero and fairly flat in the 1970s. There is a modest increase over the early 1980s and then a flattening since the mid 1990s. These changes reflect clear offsetting trends in the covariances of permanent and transitory income. The covariance of the permanent components of husbands and wives are always positive, which may reflect positive marital sorting. On the other hand, transitory covariance are consistently negative before the late 1990s. Such negative correlations between couple's transitory income suggests some household smoothing. Wives may work more to compensate for head's income loss due to transitory shocks. Such negative covariance start to rise in the early 1980's and become positive in the late 1990s.<sup>6</sup>

Figure 6 shows permanent and transitory covariance between head's earnings and residual family income. The covariance between head's earnings and other family income has increased over the past three decades. Increases in the overall correlation reflect increases in the correlation of both permanent and transitory components until the mid 1980s. However, it is only the correlation in the transitory component that continues to increase, as the correlation in the permanent components of head's earnings and residual income continues to decline since the mid 1980s.

<sup>&</sup>lt;sup>6</sup>This diminishing negative correlation between spousal transitory income shocks is consistent with recent findings that due to a greater positive co-movement of employment among couples, the overall value of marriage as a risk-sharing arrangement has diminished (Juhn and Potter 2007)

### 4.3 Permanent and Transitory Component of Receipt of Positive Income

To be completed.

#### 5 Conclusion

This paper seeks to understand the factors that account for the increase in family income instability over the past three decades. We examine the following factors that may contribute to the rising family income instability: changes in transitory variance in positive income of each income sources (male head's earnings, spouse's earnings, etc.), changes in the covariance between transitory shocks in positive income of each income pairs, and changes in transitory component of receiving positive income from each income source.

Our results point to a set of factors that are consistent with the rise in family income instability: increase in transitory variance of head's earnings in the late 1970's and early 1980's; increases in the covariance between head's transitory earnings and transitory residual income are consistent with the rise in family income instability. On the other hand, negative transitory covariances between head's and spouse's earnings, and decrease in married women's earnings instability, help smooth out income fluctuations hence reduce transitory variance in total family income. We also find that although male earnings instability is strongly correlated with business cycle, family income is less influenced by recessions due to household smoothing through spouse's earnings and unearned income.

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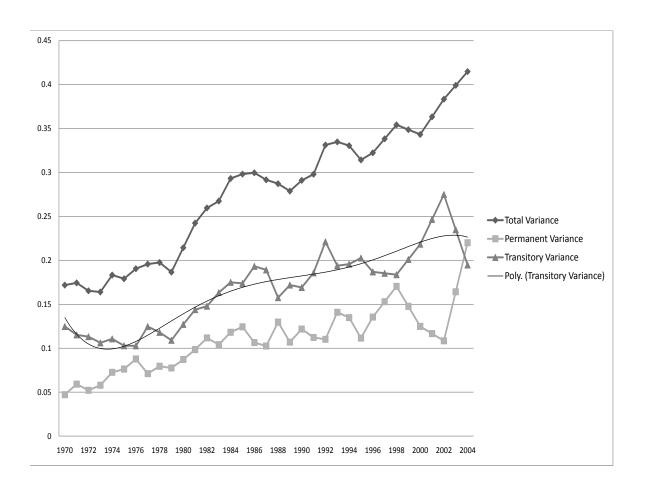


Figure 1: Total, Permanent and Transitory Variance in Log Family Income

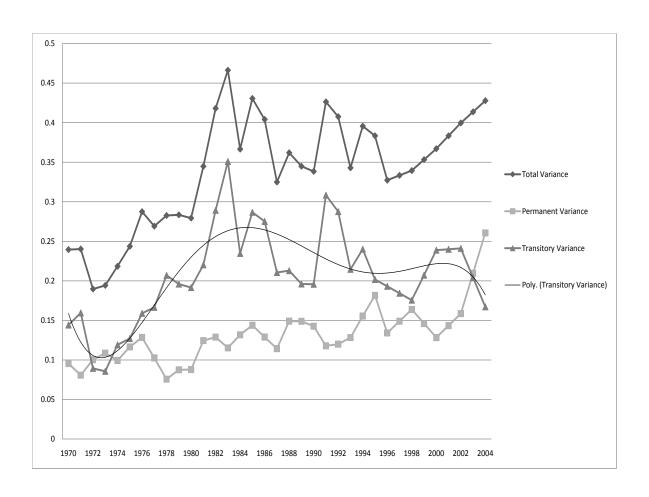


Figure 2: Total, Permanent and Transitory Variance in Log Male Head Earnings

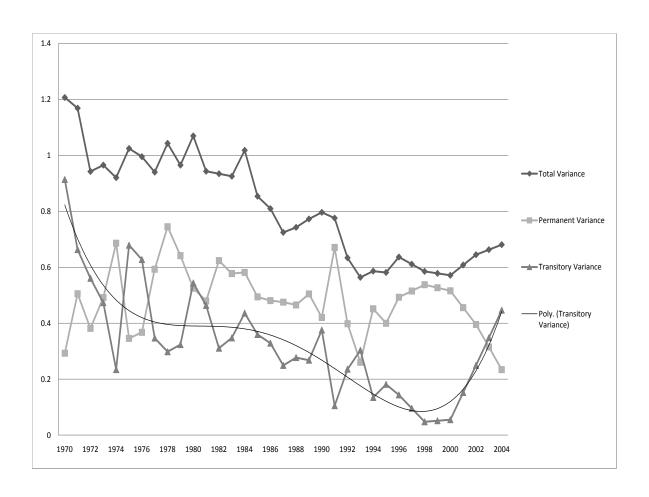


Figure 3: Total, Permanent and Transitory Variance in Log Spousal Earnings

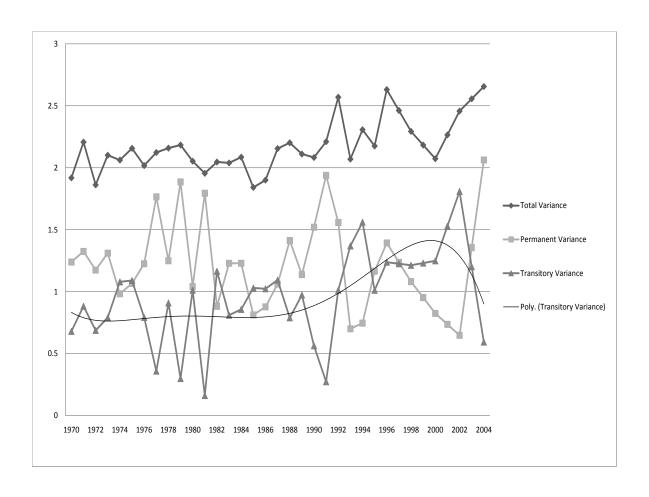


Figure 4: Total, Permanent and Transitory Variance in Log Other Income

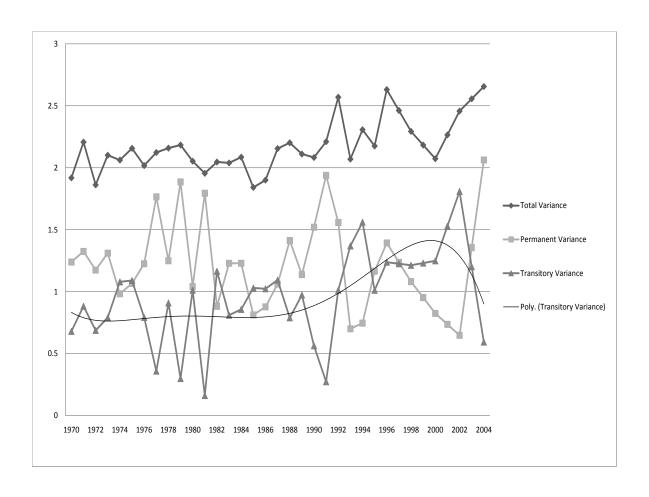


Figure 5: Total, Permanent and Transitory Variance in Log Other Family Income

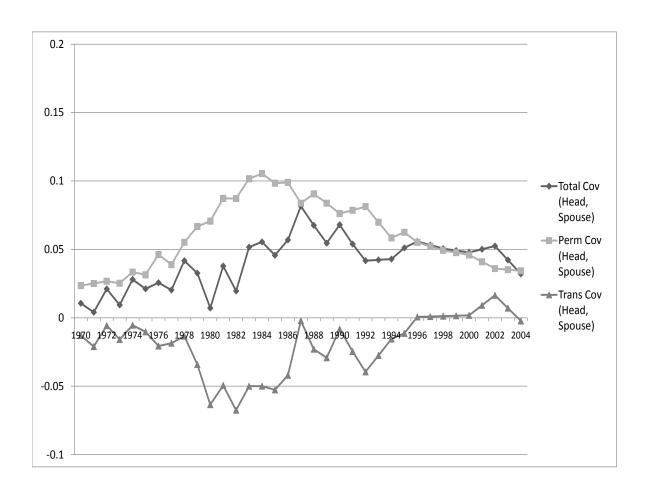


Figure 6: Total, Permanent and Transitory Covariance Between Log Male Head's Earnings and Log Spousal Earnings

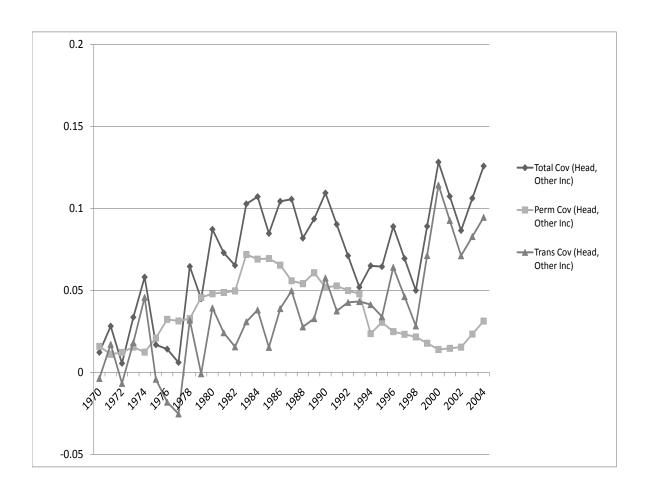


Figure 7: Total, Permanent and Transitory Covariance Between Log Male Head's Earnings and Log Other Income