Under-Employment and the Trickle-Down of Unemployment*

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Abstract

The unemployment rate misses an important characteristic of the labor market: under-employment. In the US, a substantial fraction of workers are under-employed, i.e., employed in jobs for which they are over-qualified, and that fraction is strongly counter-cyclical. Under-employment is a puzzle for standard labor market models, be it the Mortensen-Pissarides model or the competitive search model, and we propose a search model in which firms can rank applicants and hire their preferred candidate. With ranking, job competition is biased and favors the most skilled applicants, so that high-skill workers can find a job more easily by moving down the job ladder, i.e., becoming under-employed. In this framework, unemployment trickles down from the high-skill groups to the low-skill groups, so that high-skill workers enjoy not only higher expected income but also lower income volatility. A quantitative version of the model generates plausible fluctuations in under-employment and shows that the trickle-down of unemployment can be a powerful redistributive mechanism.

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The recession left millions of college-educated working in coffee shops and retail stores.[1]

To get by, the unemployed are accepting positions [in] retail jobs that don't match their skills. [...] There are more jobs, but there are also a lot of very well-qualified people applying for them. [...] There is still great competition. When you apply for a job, there are so many people applying.[2]

While the unemployment rate is the traditional gauge of the labor market, it misses an important dimension of the state of the labor market: under-employment. In the US, a substantial fraction of workers are under-employed, i.e., employed in jobs for which they are over-qualified, and that fraction –the under-employment rate– is strongly counter-cyclical, increasing markedly in slack labor markets. As shown in figure 1, the fraction of US college graduates working in lower skill-requirement occupations increased from 34 percent in 2008 to 38 percent in 2012. This change represents an increase of about 12 million under-employed workers in fours years, almost twice as large as the increase in unemployment of about 7 million workers.

While under-employment finds a large echo in the media, as suggested by our opening quote, there is surprisingly little work on the properties of under-employment and its implications for business cycle fluctuations. In this paper, we study the phenomenon of under-employment and show that the under-employment rate is an important, yet overlooked, characteristic of the labor market, and that the unemployment numbers alone can substantially underestimate the toll of a recession on the labor market.

First, we study empirically the phenomenon of under-employment in the US and show three stylized facts: (i) under-employment is strongly counter-cyclical, (ii) under-employment is very costly, an under-employed worker earning between 30 and 40 percent less than his non-under-employed counterpart, and (iii) under-employment is a highly persistent state, with under-employed workers spending an average of about five years in under-employed jobs.

Second, we aim to understand the economic mechanisms behind the existence and cyclicality of under-employment. We argue that under-employment constitutes a puzzle for canonical search models of the labor market, be it a search and matching model a la Mortensen and Pissarides (1994) or a competitive search model a la Moen (1997). In those models, given the substantial wage loss associated with under-employment, under-employment can only arise in equilibrium if low-qualification jobs are relatively more abundant than high-qualification jobs. We show that this

[1]The Wall Street Journal, 26 March 2013

[2]Bloomberg.com, May 22, 2014

is not the case, so that the existence (and cyclicality) of under-employment cannot be rationalized by standard search models.[3]

We argue that standard search models cannot explain the existence (and cyclicality) of under-employment, because they rely on the assumption that matching is random. With random matching, high-skill workers find low-qualification jobs at the same rate as low-skill workers, giving them little incentive to move down the job ladder given the associated wage loss. Instead, we propose a search model with an alternative matching process in which firms can rank applicants and hire their preferred candidate. In addition of being intuitively appealing, the notion of ranking is supported by evidence that firms prefer more qualified workers when facing multiple applicants. First, using experimental data compiled by Kroft et al. (2013), we show that when workers apply for a low-qualification job, workers with stronger resumes are substantially more likely to be called back by the firm. Second, high-skill workers command a wage premium of about 15 percent over their lower-skill peers working in the same occupation (defined at the three-digit level), consistent with the idea that high-skill workers generate a higher surplus.

With ranking, when multiple workers compete for the same job opening, the firm chooses the candidate that will generate the highest surplus. In this sense, job competition is biased and favors the high-skills, because a high-skill worker applying to a low-skill job is systematically hired over competing low-skill applicants. With skill-biased job competition, a high-skill worker moves down the occupational ladder, in order to find a job more easily and escape competition from his high-skill peers. In other words, under-employment exists, not because low-qualification jobs are more abundant, but because the competition to get a low-qualification job is, from the viewpoint of high-skill workers, less intense.

The model has some interesting implications that differ from those of traditional search models with random matching. In particular, under-employment has distributional implications. When high-skill workers move down the job ladder, they take the jobs of less-skill individuals, who are in turn driven out their market and further down the occupational ladder. Through this process, unemployment trickles down from the upper-occupation groups to the lower-occupation groups. As a result, different skill groups may be affected asymmetrically by business cycle fluctuations. A higher skill level not only guarantees a higher expected income, but it also provides

[3]In addition, we provide some evidence that the labor market for low-qualification jobs is more cyclical than the labor market for high-qualification jobs. In standard search models, high-skill workers would be more tempted to move down the job ladder in booms where low-qualification jobs are relatively more abundant, thereby leading to a counterfactually counter-cyclical under-employment rate.

a lower volatility of income, because high-skill workers can partially smooth out adverse labor demand shocks by moving down the occupational ladder. In contrast, under-employment exacerbates the income volatility faced by lower-skill workers.

The key ingredients of our model are heterogeneity across workers and jobs, coordination frictions, and wage competition between workers. Workers differ in their skill level, and islands differ in their productivity level. Workers can direct their search to a given island, that can be more or less productive and more or less congested. In each island, there are coordination frictions: some vacancies will receive multiple applications while other vacancies will have no applicants, and not every worker will get a job. Importantly, hiring is *non-random*: when a vacancy receives multiple applications, the firm ranks applicants who compete for the job, and the firm hires the most profitable applicant.

At the root of under-employment in our model lies a trade-off between the output of a job in that island and the level of competition for that job. In productive but congested islands, output is high but workers' bargaining position is low, because workers compete against each other and leave most of the surplus to the firm. In a less productive island, production is lower, but *even when congestion is high*, a high-skill worker is ranked above the other applicants and can easily get a job, because he is an exceptional candidate, being "better" than most other workers in that island.

We show that under-employment is generally inefficient, because when deciding to search for a job in a low-productivity island, a high-skill worker does not take into account how he can disproportionately hurt the labor market opportunities of low-skill workers. This (negative) externality, which we label a *ranking externality*, stems from the (little studied) fact that heterogeneous workers can face *different* labor market prospects in the *same* labor market. Because firms can rank applicants, a high-skill worker in the low-productivity island always faces better labor market prospects than low-skill workers.[4] Although the optimal allocation (subject to the same coordination frictions) calls for some level of under-employment in order to maximize the matching probability of the most-skilled workers, the low-productivity island is too attractive to high-skill workers and, and there is too much under-employment in the decentralized allocation. As a result, compared to the constrained optimal allocation (subject to the same coordination frictions), too many high-skill workers are "wasted" working for low productivity firms, too many low-skill workers end up unemployed, and there are too many low productivity firms and too few high productivity firms.

[4]This property of the model is in sharp contrast to models with random hiring (Mortensen and Pissarides (1994)): in those models, high-skill and low-skill workers face the *same* labor market prospects in the *same* island.

To get some intuition about the ranking externality and how the presence of a high-skill worker can disproportionately hurt the labor market of low-skill workers, it is helpful to use an illustrative example with Albert –a high-skill worker– and Bob –a low-skill worker–. Imagine that the high-productivity island is little congested with many jobs, but that the low-productivity island is very congested with few jobs and many low-skill workers (including Bob). Imagine that there are yet no high-skill workers in the low-productivity island. In that case, the low-productivity island looks completely different to Bob and Albert: the market is very congested for Bob, but it is very attractive for Albert, because he faces no competition. As a result, Albert may decide to take a job in the low-productivity island for a very small gain (Albert's market was little congested to begin with) while imposing a substantial cost to Bob and his peers (Bob's labor market was very congested to begin with). In other words, the ranking externality is strongest when the marginal high-skill applicant is most different from the average applicant, i.e., most "unique", as is the case in our example where Albert is the only one of his kind.

In a final section, we take our stylized model to the data and find that it can generate plausible fluctuations in under-employment, as well as plausible fluctuations in the wage premium and wage loss associated with under-employment. We then quantify the skill-group asymmetry generated by trickle down of unemployment, and we find that under-employment can be a major redistributive mechanism in which shocks get passed from the high skills to the low skills. Our illustrative simulation shows that the trickle-down of unemployment lowers the volatility of high-skill workers' expected income by about 20 percent but raises the volatility of low-skill workers' expected income by 15 percent.

The phenomenon of under-employment relates closely to the phenomenon of over-education,[5] which goes back to the 1970s when the supply of educated workers seemed to outpace its demand in the labor market, apparently resulting in a substantial reduction in the returns to schooling.[6] That higher educated workers earn a wage premium over lower-educated workers employed in the same occupation has been studied extensively by that literature. In fact, McGuinness (2006) concludes that the existence of a wage premium is a robust finding and even conjectures that under-employment exists because low-tech jobs are easier to find than high-tech jobs, consistent with the hypothesis proposed in this paper.

[5]Since we take the education level as given and study the resulting worker allocation problem, we prefer to use the term under-employment to differentiate our focus from the over-education literature which focuses on the returns from schooling.

[6]See Freeman (1976), Thurow (1975), Sicherman (1991), Sicherman and Galor (1990) , and McGuinness (2006) and Leuven and Oosterbeek (2011) for reviews.

Our approach to modeling under-employment builds on the search and matching literature with multiple islands and heterogeneous agents, as in Albrecht and Vroman (2002) and Gautier (2002), and on the competitive search literature with heterogeneous agents (Shi (2001, 2002), Shimer (2005) and Eeckhout and Kircher (2010)), in which firms post wage offers and workers can direct their search to the markets with the most attractive alternatives. In contrast to these papers, we relax the assumption of random matching. Our modeling of non-random hiring, in which firms can choose among different applicants, builds on Blanchard and Diamond (1994)'s idea of ranking, in which firms rank applicants according to some observed criteria. In our model, the productivity of each applicant is observed, and firms can rank applicants according to their productivity level.

One technical contribution of the paper is to propose a tractable and intuitive bargaining setup that can capture wage negotiations with (i) multiple, and (ii) heterogeneous, applicants in a non-random hiring setting. Our modeling of wage negotiation is related to the competing-auction theories of Shimer (1999) and Julien et al. (2000), in which job candidates auction their labor services to employers. In our setup, wage bargaining departs from the standard Nash-bargaining outcome (e.g., Pissarides (2000)), because firms can collect applications and make applicants compete for the job. As a result, the outside option of the firm, and thus the surplus extracted by the firm depends on the number and on the type (i.e., quality) of the other applicants.

Finally, the idea that unemployment may trickle-down to lower layers received some empirical support in Gautier et al. (2002) and more recently Beaudry et al. (2013). In particular, Beaudry et al. (2013) argue that, around the year 2000, the demand for skill underwent a reversal, which led high-skill workers to move down the occupational ladder and push low-skilled workers even further down the occupational ladder.

The remainder of this paper is structured as follows. In section 1, we study the properties of under-employment. In Section 2, we argue that under-employment is a puzzle for standard search models. Section 4 presents a model of under-employment, first in partial equilibrium and then in general equilibrium. Section 5 discusses the optimality of the decentralized allocation. Section 6 brings the model to the data, and the final section briefly concludes.

1 The anatomy of under-employment

This section argues that under-employment is an important, yet overlooked, characteristic of the labor market, and we present four stylized facts about under-employment in the US.

First, while about 35 percent of US workers are underemployed, a roughly constant fraction over the past 30 years, under-employment is highly counter-cyclical. In fact, fluctuations in under-employment are as large, if not larger, than fluctuations in the unemployment rate. Thus, business cycle fluctuations affect the labor market not only through the unemployment rate but also through the under-employment rate. Second, under-employment is a highly persistent state for workers who decide to move down the job ladder. Third, moving down the job ladder implies a substantial wage loss, although high-educated workers earn a premium over low-educated workers employed in the same occupation. Fourth, while moving down the job ladder entails a wage loss, it also likely comes with a higher job finding probability for high-skill job seekers.

1.1 The cyclicality of under-employment

To measure under-employment, we classify as under-employed an individual with a bachelor (or more) who is employed in an occupation that only requires a high-school degree (or less).[7] To measure an individual's education level, we use micro data from the CPS, and to measure the degree requirement of an occupation, we use data from the BLS on the education requirements by occupation at the three-digit level. As described in the Occupational Outlook Handbook (OOH), the education requirements are determined from federal and state regulations or from the typical path of entry into a job. Importantly, the definition is kept fixed over time.

Figure 1 plots the under-employment rate, defined as the fraction of workers employed in occupations for which they are over-qualified. In dashed, we also report the unemployment rate. Two important facts stand out. First, under-employment is strongly counter-cyclical, increasing in recessions and periods of slack labor market.[8] This is particularly apparent in the last recession when the under-employment rate increased by about 5 percentage points between 2009 and 2013, which amounts to about 12 more million workers being under-employed. This is a large number, even larger than the increase in the number of unemployed of about 7 million people over the same time period.[9] In other words, the increase in unemployment during a recession is only one aspect of the effect of a recession on the labor market. Another

[7] Changing the education threshold to separate the high educated from the low educated, for instance by requiring some years of college to be considered high educated, gives very similar conclusions. Only the average level of under-employment is changed.

[8] Another interesting aspect of underemployment is that it seems to lag the unemployment rate.

[9] Between 2008 and 2010, the unemployment rate increased by 5 percentage point, i.e., by about 7 million people.

substantial, but so far overlooked, aspect is the large increase in the fraction of workers who are under-employed, in line with our opening quote from the Wall Street Journal.

A second fact apparent in figure 1 is that the level of under-employment is high, with about 35 percent of employed workers being under-employed, and that that level has remained roughly stable over the past 30 years. However, one can notice a slight downward trend starting in the early 90s that appears to lag the downward trend in the unemployment rate.

1.2 The permanence of under-employment

We now show that under-employment is not a transitory stage for high-educated workers, as they stay under-employed for an average of 5 years.

Using matched CPS micro data, we can follow workers' labor force status over successive months and compute their transition probabilities, in particular their transition rate to unemployment or their transition rate to another occupation with different degree requirements.[10] As shown in table 1, the monthly job separation rate of high-educated individuals employed in low-tech jobs (i.e., requiring only a high-school degree) is about .6 percent, implying that it takes about 14 years before that individual becomes unemployed.[11] Looking at the job-to-job transition rate reveals a similar picture. The transition rate of high-educated workers employed in a low-tech occupation and transitioning to a high-tech occupation (requiring a bachelor or more) is about 1 percent. This means that it takes about 8 years for a high-educated individual in a low-tech occupation to transit directly to a high-requirement job. We conclude that under-employment is hardly a transitory state, as an individual in an occupation requiring a high-school degree or less remains employed in that job for more than 5 years (from a total exit rate of 1.6 percent).[12]

[10]The CPS is a rotating panel where individuals are surveyed for four consecutive months, left out for eight months, and then surveyed again for four consecutive months. It is thus possible to match 6/8 (in practice less because of attrition) of the survey records in the microdata files across months, allowing us to measure monthly transition probabilities between labor market status. See Shimer (2012).

[11]The average time to wait before transiting to unemployment is given by 1/.006, which gives about 14 years.

[12]These findings are in line with suggestive evidence from Verhaest and Schatteman (2010) who follow school leavers and investigate their first seven years on the labor market. They find that under-employment is highly persistent: about 40 percent of the individuals who became under-employed upon graduation were still under-employed seven years later. There is, however, some evidence that over-qualified workers are less satisfied with their current job, and willing to quit (Hersch 1991). This willingness does not really translate into a significantly higher job separation rate than their less skilled peers in the same occupations.

1.3 The wage cost of under-employment

Third, we show that high-educated workers incur a substantial wage loss when moving down the job ladder, even though high-educated workers earn a premium over low-educated workers employed in the same occupation.

Specifically, we use MORG CPS micro data between 1980 and 2012 and measure for each employed individual the educational attainment, the occupation code at the three-digit level, and the declared wage. Denote $\omega_{1,1}$ the wage of type-1 workers employed in low-tech occupations, $\omega_{2,1}$ the wage of type-2 workers employed in low-tech occupations, and $\omega_{2,2}$ the wage of type-2 workers employed in high-tech occupations.

Table 2 presents summary statistics about wages. Two facts stand out.

First, high educated workers who move down the job ladder to take a low-requirement job suffer a wage loss of about 30 percent. We find that those results are robust to the addition of controls such as age, sex, unemployment duration, state and previous occupation fixed effects. Interestingly, after controlling for observable characteristics, the wage cost of under-employment is counter-cyclical, being higher in recessions.

Second, high-educated and low-educated workers employed in the same occupation (defined at the three-digit level) are treated differently by firms: high-educated workers receive a premium of about 15 percent over low-educated workers.[13] Similarly, this premium is obtained controlling for age, sex, unemployment duration, state and *current* occupation fixed effects.[14] Finally, and related to the counter-cyclical cost of under-employment, the wage premium enjoyed by high-educated workers is pro-cyclical, being lower in recessions.

1.4 Under-employment and higher job finding probability

Finally, we argue that under-employment allows high-skill job seekers to find a job more easily, because high-skill workers who move down the job ladder are preferentially hired over lower skill applicants.

Specifically, we present evidence supporting the idea that firms prefer "better", i.e., more productive, workers. We use a dataset recently constructed by Kroft et al. (2013) on the call back rate of firms. Kroft et al. (2013) examine the effect of

[13] This finding is consistent with previous findings on the effect of over-education on wages, in which over-educated workers were found to earn more than comparably matched workers in the same occupations (Duncan and Hoffman (1981)).

[14] These numbers are in line with findings from the over-education literature, which, in addition, concluded that the wage premium cannot be explained by unobserved heterogeneity (McGuinness (2006)).

unemployment duration on callback rates using fictitious job applications in which duration is manipulated exogenously. However, because the fictitious resumes were also varied in their quality (in particular, education level of the candidate) and because four resumes (two of the high types, and two of the low types) were sent to the same job posting, it is possible to test whether better resumes of higher quality face higher call back rates for the very same job position. Table 3 presents the results and shows that higher quality resumes have a callback rate that is 35 percent higher than lower quality resumes. While the mapping from callback rates to job finding propensities is not straightforward, this evidence strongly suggests that firms do rank candidates by resume's quality/productivity, so that high-skill job seekers may have an easier time finding a job in a lower requirement occupation than their low-skill peers.

2 Under-employment: a puzzle for standard search models

The previous section highlighted four new stylized facts about the labor market: (i) a substantial fraction of workers are under-employed and the under-employment rate is counter-cyclical, (ii) under-employment is a persistent state for workers, (iii) under-employment involves a substantial wage loss but high-educated workers earn a premium over their low-educated colleagues, and (iv) high-skill job seekers likely have a higher job finding rate in lower requirement occupations.

In this section, we first argue that under-employment constitutes a puzzle for canonical search models of the labor market, whether it is the search and matching model of Mortensen and Pissarides (1994) (MP) or a competitive search model a la Moen (1997)), because in those models, matching is random and independent of the worker's type. We then argue that departing from random matching can help address this puzzle.

2.1 Under-employment and random matching

Consider a labor market divided in two islands, a high-tech island, island 2, and a low-tech island, island 1. In each island, and this is the key assumption in standard search models, matching is random and the matching probability is independent of the worker type.

Denote f_1 the job finding rate of a high-skill worker in island 1 and U_1 (resp. W_1) the value of being unemployed (resp. employed) in island 1, and f_2 and U_2 (W_2) the corresponding values in the high-tech island 2.

High-skill workers must choose in which labor market to look for a job, so that if under-employment exists in equilibrium, high skill workers must be indifferent between all islands, which implies $U_1 = U_2$ or:[15]

$$\frac{f_1}{r+\lambda+f_1}(w_1-b) = \frac{f_2}{r+\lambda+f_2}(w_2-b) \tag{1}$$

in which b denotes the value of home production including the unemployment benefits, λ denotes the job separation rate and r denotes the rate at which the future is discounted.

If, as we saw previously, under-employment entails a wage loss for high-skill worker, i.e., if the wage they receive in island 1 is lower than the wage they receive island 2 ($w_1 < w_2$), they much have a higher job finding rate in island 1 than in island 2 ($f_1 > f_2$). In canonical search frameworks, since matching is random, the job finding rate is identical across workers, and $f_1 > f_2$ implies that finding a job should be faster for low-requirement occupations, both for high- and low-skill workers. However, as illustrated in Figure 2, this implication of the model is not supported by the data: finding a job in a low-requirement occupation takes, on average, just as long as finding a job in a high-requirement occupation. In other words, the existence of under-employment cannot be explained by the fact that the low-tech island 1 has a higher job finding rate than the high tech island.

Thus, given the wage loss associated with under-employment, standard search models cannot explain the existence of under-employment. [16] Stated differently, in canonical search models with random matching, the arbitrage equation (1) is violated and the existence of under-employment is a puzzle.[17]

2.2 Matching with ranking

To explain the phenomenon of under-employment, this paper proposes to depart from the hypothesis that matching is strictly random. Instead, we allow firms to gather multiple applications, rank them and then hire the most productive workers,

[15]Assuming that unemployment benefits are identical across islands and that separation rates are similar across islands. Recall that in an island with random search and constant job separation rate λ_i, the value of working in island i is given by $rW_i = w_i + \lambda_i (U_i - W_i)$ and $rU_i = b + f_i (W_i - U_i)$, which gives $rU_i = \frac{f_i}{r+\lambda_i+f_i}(w_i - b)$.

[16]This would be true in a model a la Mortensen-Pissarides where Nash bargained wage or in a competitive search model with wage posting.

[17]Quantitatively, with an income loss of under-employment of about 35 percent, a monthly separation rate of .1 percent and an annual interest rate of 4 percent, for the arbitrage equation (1) to be satisfied, the difference in job finding probabilities between the high-tech and low-tech islands must be such, that if the monthly job finding rate in the low tech island is 0.3 (the observed average rate), the job finding rate in the high-tech island should be about 0.04, implying an average unemployment duration of about two years, much higher that the average duration of 17 weeks observed over the past 40 years (figure 2).

consistent with our fourth stylized fact that firms appear to prefer “better”, i.e., more productive, workers.[18]

In addition to being intuitively appealing, this matching process is consistent with qualitative evidence on firms’ recruitment behavior, which conclude that firms usually interview many applicants at once (Barron et al. 1985, Barron and Bishop 1985) and do preferentially hire more qualified candidates (Behrenz 2001, Raza and Carpenter 1987).

While the rest of the paper will develop this idea of non-random hiring more rigorously in a theoretical framework, it is easy to see why ranking can help rationalize the existence of equilibrium unemployment.

With ranking, even if the islands are equally congested, high-educated workers have a higher job finding rate in island 1 than in island 2, because they are systematically hired over competing lower-skill applicants. As a result, the arbitrage equation (1) *can* be verified (and under-employment can exist in equilibrium), because the wage loss associated with under-employment is compensated by a shorter unemployment spell.

3 A model of under-employment

This section presents the structure of the model and describes its three key ingredients: (i) heterogeneity across workers and jobs, (ii) search frictions, and (iii) wage competition between workers.

3.1 Preferences, technology and market structure

For clarity of exposition, the model is static and consists of one period. However, in the appendix, we show how the model can be generalized to a dynamic setting in the steady-state case.

There are two types of risk neutral agents in the economy, workers and firms, and the economy is divided into N islands indexed by $n = 1 \dots N$.

Firms operating in island n are characterized by a technology T_n. Islands are indexed such that island N has the highest technology level, and island 1 the lowest. A firm consists of one vacancy, and a firm can enter an island n by posting a vacancy at a cost $c_n > 0$. The number of firms/vacancies in each island will be determined endogenously by firm entry.

Workers are divided into N different types characterized by different productivity

[18]The idea of ranking draws on Blanchard and Diamond (1994), which allowed firms to rank applicants according to the length of their unemployment spell.

levels. Workers of type N are the most productive. There is a mass L_n of agents of type n. A worker with a job provides inelastically one unit of labor to the firm and receives a salary ω. A worker without a job receives 0.[19]

A firm with technology T_i paired with a worker of type j produces $\varphi_{i,j}$ for any $i, j \in \{1, \dots, N\}^2$. We further impose some restrictions on the degree of complementarity between workers' skill and firms' technology such that high-skilled workers are pushed towards more efficient firms.

3.2 Coordination frictions

It takes time to match workers with jobs. To capture this search friction, we assume that each worker must settle to one island and can apply to at most one job. In a large anonymous market, workers cannot coordinate on which firm to apply to, leading to coordination frictions in each island. Some firms will get multiple applications, while others receive none. Some firms will receive applications from workers of different types, while others will receive applications from workers of the same type. Unmatched jobs and workers produce nothing and get 0 payoff.

With workers applying at random in a market with many workers and firms, the matching process is described by an urn-ball matching function (Butters 1977), in which each application (ball) is randomly allocated to a vacancy (urn). With a large number V of vacancies and a large number L of homogeneous applicants, the probability for a firm to be matched with exactly a applicants follows a multinomial distribution which can be approximated with a Poisson distribution

$$P(a) = \frac{q^a}{a!} e^{-q}$$

with $q = L/V$ the queue length, i.e. the job candidate to vacancy ratio.

The probability that a firm has exactly one applicant is then $P(1) = qe^{-q}$ and the probability that a worker finds a firm with no other candidate is $\frac{V}{L} P(1) = e^{-q}$.

This urn-ball matching function can be easily generalized to heterogeneous applicants. For instance, when there are two types of applicants, say, L_1 of type 1 and L_2 of type 2, we can proceed in a similar way and define the probability $P(a_1, a_2)$ that a firm faces a_1 applicants of types 1 and a_2 of type 2. Defining $q_1 = L_1/V$ and $q_2 = L_2/V$, we have

$$P(a_1, a_2) = \frac{q_1^{a_1}}{a_1!} e^{-q_1} \frac{q_2^{a_2}}{a_2!} e^{-q_2}.$$

In this setup, we can derive a number of probabilities that will be useful later. The

[19] As shown in the appendix for the dynamic model, the assumption of no unemployment benefits/home production can be easily relaxed and is only used here for analytical simplicity.

probability that a worker of type 1 is the only applicant to a job is $\frac{V}{L_1}P(a_1 = 1, a_2 = 0) = \frac{V}{L_1}q_1 e^{-q_1-q_2} = e^{-q_1-q_2}$. Similarly, the probability that a worker type 1 is the only applicant of his type but with applicants of the other type is $e^{-q_1}(1 - e^{-q_2})$. Finally, the probability that a worker type 1 faces other applicants of his type is $1 - e^{-q_1}$.

The general formula with N types follows easily from a similar reasoning. The probability $P(a_1, \ldots, a_N)$ that a firm faces a vector $(a_1, \ldots, a_N)$ of applicants is

$$P(a_1, \ldots, a_N) = \frac{q_1^{a_1}}{a_1!}e^{-q_1} \ldots \frac{q_1^{a_N}}{a_N!}e^{-q_N}.$$

3.3 Wage negotiation and hiring decision

In this section, we describe the wage bargaining process taking place between a firm and its (possibly multiple) job candidate(s). Specifically, we present a tractable and intuitive bargaining setup that can capture wage negotiations with (i) multiple and (ii) heterogeneous applicants in a non-random hiring setting.

A key dimension captured by the wage negotiation process is that, because workers compete against each other to get a job, the bargaining position of an applicant is endogenous and depends on (i) the tightness of the market (capturing the degree of competition a worker faces) and (ii) the quality of the unemployment pool (capturing the type of competitors an applicant faces).

There is perfect information, and all agents observe the pool of applicants and their types. We posit that, with probability $1 - \beta$, the firm makes a take-it-or-leave-it offer to its preferred candidate, and, with probability β, each candidate makes a take-it-or-leave-it offer to the firm. When the firm faces only one candidate, the expected payoffs are similar to a standard Nash bargaining.[20] With more than one applicant, however, the candidates compete and bid down their wages.

What are the outcomes of our game? With probability $1 - \beta$, the firm sends an offer and captures all the surplus: the best candidate is hired with a wage equal to her outside option. If there are more than one applicant of the best type, the firm picks one at random. With probability β, applicants compete for the job by sending each an offer to the firm. The results of such game is that the best applicant sends an offer to the firm where she captures all the surplus above what could be produced by the second-best candidate.

[20] In this respect, we differ from Rubinstein (1982) in which the firm and the applicant engage in an infinite horizon game of alternating offers. In this framework, β is the relative impatience of the worker and the firm. β can also be interpreted as the relative degrees of risk aversion (Binmore et al. 1986).

The advantage of our bargaining game is that, in the cases of one-to-one matching, it coincides with Rubinstein (1982): the worker receives a share β of the surplus. With more than one applicant, it is as if the firm could negotiate with the best applicant under the threat of hiring the second-best candidate.[21]

There are then four cases that determine the negotiated wage:

1. The firm has only one applicant. The worker expects to receive a share β of the surplus:
$$\omega = \beta\varphi.$$

2. The firm has more than one applicants of *identical types*. Since the best applicant and the second-best applicant are identical, the firm gets all the surplus from the match, and the best applicant gets its reservation wage:[22]
$$\omega = 0.$$

3. If the firm has more than one applicants of *different types*. There are two subcases:

   (a) The firm has more than one applicants of the higher-type (for any number of lower-type applicants). The game is identical to case 2, and the firm gets all the surplus: $\omega = 0$.

   (b) The firm has one applicant of the higher-type and (one or many) lower-type applicants. Denote φ_2 the output generated by the high-type and $\varphi_1 < \varphi_2$ the output generated by the second-best applicant.

   The high-type applicant sends an offer that would make the firm indifferent between her and the second-best candidate: The outside option of the firm is to hire the second-best applicant and get all the surplus of such match, i.e., φ_1. As a result, with probability β, the worker gets the

[21]Our assumptions provide a very tractable wage bargaining rule. Allowing for a more general negotiation wage rule would complicate the analysis but would not affect the crucial property that the outside option of the firm depends on the quality and number of the other applicants. For instance, one could think of a more general framework in which the firm starts a game of alternating offers with the best applicant, and that, if these negotiations break-down, the firm starts a game of alternating offers with the second-best applicant, then with the third-best if these negotiations break-down, etc.. While more cumbersome, the outcome of the negotiation would be qualitatively similar: the best-applicant would get the job and his wage would depend negatively on the productivity and number of other applicants.

[22]The outcome of this bargaining game can be also seen as workers bidding themselves down to their reservation wage. Note that because the firm gets all of the surplus, in this case, the worker is indifferent between employment and unemployment

surplus generated over hiring the second-best applicant, i.e.,

$$\omega = \beta(\varphi_2 - \varphi_1)$$

3.4 Timing

The timing of events is as follows. (1) Each worker chooses which island to send an application to. In parallel, each potential firm entrant decides whether to post a vacancy in any given island; (2) In each island, applications are randomly allocated to vacancies; (3) A wage negotiation ensues between the firm and its (possibly multiple) applicants; (4) Firm-worker matches are formed and production starts. Finally, firms pay workers and realize profits.

To summarize, the firm's share of the surplus depends on the number *and* quality of applicants. If the firm can make applicants compete for the job, it is going to extract most of the surplus. However, it will not systematically capture all of the surplus because of worker heterogeneity. If one (and only one) applicant is strictly better (i.e., more productive) than the others, that applicant can extract some of the surplus, because the firm has a strict preference for that candidate and *ranks* him higher.

4 Equilibrium with with $N = 2$: two worker types and two firm types

In this section, we describe the equilibrium allocation characterizing our economy. For ease of presentation, throughout the main body of the paper, we will only focus on the case with $N = 2$ –two islands (i.e., two firm types) and two worker types–, since the $N = 2$ case already contains the most interesting lessons from our model. Characterizing the equilibrium in a higher N model is relatively straightforward and is discussed in a separate online Appendix.

4.1 Partial equilibrium with exogenous labor demand

In order to first clarify how workers decide on which island to search, we start with the partial equilibrium (PE) with exogenous labor demand, i.e., taking the number of firms/vacancies in each island as given.

Definition 1. *Partial Equilibrium Allocation.*

Each worker of each type $i \in \{1, \ldots, N\}$ decides on which island $n \in \{1, \ldots, N\}$ to search for a job, and the equilibrium is given by a sequence of worker choices $C_i : [0, L_i] \mapsto \{1, \ldots, N\}$, a matching function between applicants and firms and a sequence of allocations and wages.

Since we consider equilibria with under-employment, the Appendix describes the conditions that ensure the existence of a strictly positive rate of under-employment, and the absence of over-employment.[23]

4.2 Some useful notations

We first introduce some useful notations. Let $h_n = \frac{L_n}{L_{n-1}}$ denote the size of the pool of type n workers relative to that of type $n-1$. For instance, a low h_n captures a situation in which the distribution of worker types is pyramidal: there are few high types and many low types. This means that high types will not crowd each other out (i.e., compete against each other), when they move down the occupation ladder. In other words, h_n captures how type n workers can "dilute" themselves in the islands below.

Let $q_n = \frac{L_n}{V_n}$ denote the ratio of type n individuals to job openings in island n. q_n can be seen as an "initial" queue length in island n, i.e., an hypothetical queue length corresponding to the case where workers of type n were assigned to island n and not allowed to move.

Denote x_n the fraction of workers in island n who move to island $n-1$ to look for a job. Starting from an "initial" number L_n of job candidates in island n, the number of workers type n who end up searching in island n is $(1-x_n)L_n$, and the number of workers type n who search in an island below is x_nL_n. In island $n-1$, the ratio of searchers from island n to vacancies is thus given by $h_nx_nq_{n-1}$. In island n, the ratio of searchers from island n to vacancies is given by $q_n(1-x_n)$.

Finally, denote $E\omega_{n,m}$ the expected income of an individual of type n searching in island m.

4.3 Equilibrium with $N = 2$: two worker types and two firm types

This section characterizes the equilibrium allocation and presents some comparative statics exercises to illustrate the mechanisms underlying the equilibrium.

Equilibrium In equilibrium, workers allocate themselves across islands until the point where the expected income in each island are equalized. Specifically, the equilibrium with two types of workers and firms is characterized by the following Proposition:

[23] Alternatively, in a model with a strictly positive rate of over-employment and no under-employment, our skill-biased job competition would occur in the high-tech island. The main difference implied by this different configuration is that unemployment would then trickle up.

Proposition 1. *With $N = 2$, there is a unique equilibrium allocation of workers satisfying*

- *Type 2 workers are indifferent between islands 1 and 2, and x_2, the share of type 2 workers searching in island 1, is given by the arbitrage condition*

$$A(x_2) = -E\omega_{2,2} + E\omega_{2,1} = 0$$

with

$$\begin{cases} E\omega_{2,2} = \beta e^{-q_2(1-x_2)}\varphi_{2,2} \\ E\omega_{2,1} = \beta e^{-q_1 x_2 h_2} e^{-q_1}\varphi_{2,1} + \beta e^{-q_1 x_2 h_2}(\varphi_{2,1} - \varphi_{1,1})\left[1 - e^{-q_1}\right] \end{cases}$$

- *Type 1 workers only look for jobs in island 1 and their expected income is*

$$E\omega_{1,1} = \beta e^{-q_1(1+x_2 h_2)}\varphi_{1,1}$$

Proof. Appendix. □

Each worker searches for a job in the island that provides him with the highest expected wage: in equilibrium, a type 2 worker is indifferent between looking for a job in island 2 and looking for a job in island 1, while a type 1 worker strictly prefers looking for a job in island 1. The arbitrage condition, $A(x_2) = 0$, determines, x_2, the equilibrium allocation of type 2 workers across the two islands.

Figure 3 depicts the equilibrium allocation of type 2 workers as the intersection of the $E\omega_{2,1}$ curve, the expected wage earned in island 1, and the $E\omega_{2,2}$ curve, the expected wage earned in island 2. The $E\omega_{2,2}$ curve is increasing in x_2: an increase in the fraction of type 2 workers searching in island 1 lowers congestion in island 2, which lessens the competition type 2 workers face in island 2 and increases the expected wage. In contrast, an increase in the fraction of type 2 workers searching in island 1 makes island 1 more congested, which increases the competition workers face in island 1 and lowers the expected wage.

Comparative Statics We now present some comparative static exercises to illustrate the mechanisms underlying the equilibrium allocation. First, we study how an island-specific shock has different implications for high-skills and low-skills, and how unemployment can trickle down from the high skill groups to the low skill groups. High-skill workers can use under-employment to smooth shocks, but they hurt low-skill workers in the process. Second, we illustrate how the level of under-employment also depends on (i) the variance across workers' productivity, and (ii) the distribution of worker types.

Consider first an adverse labor demand shock affecting the high productivity island, i.e., an increase in the queue length q_2. As shown in figure 4, it shifts the expected wage earned in island 2, the $E\omega_{2,2}$ curve, down and generates a higher equilibrium x_2 as high-skill workers use under-employment to smooth the shock and the associated decline in expected income. As a result, low-skill (type 1) workers see a decrease in their expected earnings (as shown by the $E\omega_{1,1}$ curve), because they face more competition from type 2 workers searching in island 1. In other words, a shock affecting the high type group *trickles down* to the lower type group.

Consider now an adverse labor demand shock affecting the low productivity island, i.e., an increase in the queue length q_1. As shown in figure 5, it affects the expected income of type 1 workers by shifting down the the $E\omega_{1,1}$ curve. But is also shifts the expected wage of type 2 workers in island 1, the $E\omega_{2,1}$ curve, down and thus generates a lower equilibrium x_2, as type 2 workers smooth the effect of the shock in island 1 by adjusting their under-employment rate. In turn, this lower under-employment rate dampens the effect of the shock on low skill workers.

Interestingly, if one considers an adverse aggregate shock (or an experiment in which island-specific shocks are positively correlated), the effect of the shock on under-employment is a priori ambiguous and depends on the shocks' properties. However, when an aggregate shock increases congestion in both islands in a similar way, the burden it imposes on high-skill workers is generally lower in island 1 thereby increasing the under-employment rate. The rationale is that, from a high-skill viewpoint, congestion is less detrimental in island 1 because competition for vacancies is less intense than in island 2. We will come back to this point in our quantitative evaluation of the model.

The model has other interesting lessons about the determinants of the equilibrium level of under-employment. In particular, the higher the degree of heterogeneity among worker skill-groups, the higher the under-employment rate. For instance, consider a decrease in the productivity of low skill workers, $\varphi_{1,1}$, would increase the productivity difference between high-skill and low-skill workers and increase type 2 workers' expected wage in island 1. Indeed, the higher the productivity difference between type-1 and type-2, the more valuable to the firm is a type-2 worker relative to a type-1, so that type 2 can extract more surplus from the firm when they compete with type 1 workers for a job. With an inward shift of the $E\omega_{2,1}$ curve, more type 2 workers move down to island 1, and their expected wage is higher. Thus, the higher the degree of heterogeneity among workers, the higher the under-employment rate.

The distribution of worker types, the "shape of the pyramid", also affects the allocation and the extent of under-employment: a lower h_2, i.e., a pyramid with a larger base, allows type 2 workers to dilute themselves more easily in island 1. As a result, it leads to an outward shift of the $E\omega_{2,1}$ curve and thus to more under-employment.

4.4 General equilibrium with Endogenous Labor Demand

We now characterize the general equilibrium (GE) with endogenous labor demand. Embedding the model in a GE context is interesting for two reasons. First, it allows us to show that the basic intuition drawn from the PE case is not over-turned by GE forces. Second, it allows us to contrast our model (with skill-biased job competition) with the canonical search and matching model (Mortensen and Pissarides (1994), thereafter MP). As we will see, a key lesson is that our model reduces to the (random search) standard MP model as worker heterogeneity vanishes. However, with worker heterogeneity, the models' predictions differ. With non-random search and skill-biased job competition, the under-employment of high-skill workers can hurt the labor market perspectives of low-skill workers, whereas the opposite occurs in random search models.

We assume that there is an arbitrarily large mass of potential entrants who can settle in island n by adopting technology T_n. A firm still consists of one vacancy, and a firm can enter an island n by posting a vacancy at a cost $c_n > 0$. With free entry, firms will enter in each island n until the point where expected profits, denoted π_n, equal the fixed cost c_n. The number of firms/vacancies in each island will thus be determined endogenously by firm entry.

Definition 2. *General Equilibrium Allocation with Endogenous Firm Entry.*

Each worker of each type $i \in \{1, \dots, N\}$ decides on which island $n \in \{1, \dots, N\}$ to search for a job, and each potential firm entrant in island $j \in \{1, \dots, N\}$ decides whether or not to post a vacancy in island n. The equilibrium is given by a sequence of choices $C_i : [0, L_i] \mapsto \{1, \dots, N\}$ for each worker i, a sequence of choices $F_j : [0, \infty) \mapsto \{0, 1\}$ for each potential entrants in island j, a matching function between applicants and firms and a sequence of allocations and wages.

Equilibrium The equilibrium is characterized by the following Proposition:

Proposition 2. *With $N = 2$, there is a unique equilibrium allocation satisfying*

- *The arbitrage conditions characterizing the allocation of workers*

- *Type 2 workers are indifferent between islands 1 and 2, and x_2, the share of type 2 workers searching in island 1, is given by the arbitrage condition*

$$A(x_2, q_1) = -E\omega_{2,2} + E\omega_{2,1}(x_2, q_1) = 0 \qquad (L^S)$$

- *Type 1 workers only look for jobs in island 1: $x_1 = 0$.*

- *Firms' free entry conditions (market clearing) in islands 1 and 2*

$$\begin{cases} \pi_1(x_2, q_1) = c_1 & (L_1^D) \\ \pi_2(x_2, q_2) = c_2 & (L_2^D) \end{cases}$$

Proof. Appendix. □

The expected profit for a firm with technology 1 is given by

$$\pi_1(x_2, q_1) = \varphi_{2,1} - e^{-x_2 h_2 q_1} \left[\left(\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}^{-q_1} \right) (1 + (2-\beta) x_2 h_2 q_1) + (2-\beta) \varphi_{1,1} q_1 e^{-q_1} \right]$$

and the expected profit for a firm with technology 2 is

$$\pi_2(x_2, q_2) = \left(1 - e^{-(1-x_2)q_2} - (1-x_2) q_2 e^{-(1-x_2)q_2} \right) \varphi_{2,2} + (1-x_2) q_2 e^{-(1-x_2)q_2} (1-\beta) \varphi_{2,2}.$$

The GE allocation is thus the triple (x_2, q_1, q_2) determined by firms' free entry conditions in islands 1 and 2, and the arbitrage equation between islands 1 and 2 for type 2 workers. In the appendix, we show that one can represent the determination of this equilibrium allocation in a graphical manner, in a similar fashion to the Mortensen-Pissarides model.

Expected wage curves in general equilibrium To gain some intuition about the mechanics of the GE allocation, we depict the equilibrium worker allocation (x_2) as the intersection of the expected wage curves, as in the PE case, and we discuss how GE effects affect our PE conclusions.

Similarly to figure 3 in the PE case, figure 6 depicts the equilibrium underemployment rate as the intersection the $E\omega_{2,1}$ curve, the expected wage earned in island 1, and the $E\omega_{2,2}$ curve, the expected wage earned in island 2. The following corollary captures formally how the expected wage in island 1 or 2 depends on the share of type 2 workers searching in island 1.

Corollary 1. *Expected income of type 2 workers*

The expected income of type 2 workers searching in island 2, $E\omega_{2,2}(x_2, q_2(x_2))$, is independent of x_2. The expected income of type 2 workers searching in island 1, $E\omega_{2,1}(x_2, q_1(x_2))$, is strictly decreasing in x_2 with $\left|\frac{dE\omega_{2,1}}{dx_2}\right| < \left|\frac{\partial E\omega_{2,1}}{\partial x_2}\right|$.

Proof. Appendix. □

The $E\omega_{2,2}$ curve, the expected wage earned in island 2, is now flat, i.e., the expected income in island 2 is independent of the number of high-skill workers searching in island 1.

This result comes from the fact that the equilibrium queue length is independent of the number of job seekers, exactly as in a standard Mortensen-Pissarides model with homogeneous workers. Recall that island 2 is an homogeneous island only populated by type 2 workers. With free entry, it is easy to see from the firm's no profit condition that the equilibrium queue length $q_2(1 - x_2)$ is independent of the supply of type-2 workers.[24] Specifically, free entry, or (L_2^D), pins down the equilibrium queue length in island 2 –$q_2(1 - x_2)$–, regardless of the number of type 2 workers (i.e., regardless of $1 - x_2$). This is similar to what happens in standard search and matching models (Mortensen and Pissarides 1994) where the supply of (homogeneous) labor has no effect on the equilibrium queue length. Even though a higher number of type 2 workers improves the matching probability of a firm, free entry ensures that more firms enter the market in order to keep profits constant.[25]

This result points to a more general property of our model in GE: our model reduces to the canonical random search model when workers are homogeneous.[26]

Turning to the $E\omega_{2,1}$ curve, and contrasting with the PE case, an important property of the model remains in GE: the wage schedule of high-skill workers looking in island 1 is decreasing in x_2. This is in stark contrast with standard search models with heterogeneous workers, in which the $E\omega_{2,1}$ curve would be upward slopping. Indeed, worker heterogeneity gives rise to a "quality effect", in that firms respond to changes in the average productivity of the unemployment pool. Specifically, as more high-skill workers search in island 1, this raises firms' probability to meet high-skill

[24]Recall that the queue length is number of job seekers over the number of job openings and that the number of job seekers in island 2 is given by $L_2(1 - x_2)$.

[25]In a search and matching model, at a given vacancy level, an increase in the number of job seekers (coming from say out of the labor force, as in Pissarides (2000), Chapter 5) raises firms matching probability, i.e., reduces hiring costs, and leads more firms to enter the market, keeping profit and thus the queue length unchanged.

[26]A technical difference between our framework and the Mortensen-Pissarides model is that, in our set-up, an increase in the supply of workers also improves the bargaining position of the firm (as workers compete against each other when negotiated the wage). This difference has no consequence on the equilibrium queue length, because the bargaining position is also solely a function of the queue length $q_2(1 - x_2)$. As a result, no matter the level of $1 - x_2$, free entry ensures that the queue length adjusts to keep profits (including the fix cost) nil.

applicants (who generate a higher surplus than low skill applicants), which raises firms' profits, and leads to more job creation.[27] Thus, in random search model, the quality effect would lead to an upward slopping wage schedule. This does not happen in our framework, because hiring is not random and the wage schedule $E\omega_{2,1}$ is still decreasing in the number of high-skill workers searching in island 1.[28] However, the quality effect is still present in our model, and the wage schedule is flatter with endogenous firm entry than in PE. As under-employment increases, the average productivity of the unemployment pool increases, which fosters firm entry and limits the increase in congestion generated by the inflow of workers.
Turning to type 1 workers, figure 7 plots the expected wage curve of type 1 workers, both in PE and in GE, and the following corollary captures formally how the expected wage in island 1 depends on the share of type 2 workers searching in island 1.

Corollary 2. *Expected income of type 1 workers*

The expected income of type 1 workers searching in island 1, $E\omega_{1,1}(x_2, q_1(x_2))$, is a non-monotonic function of x_2; decreasing over $[0, x_2^*]$ and increasing over $[x_2^*, 1]$ with $x_2^* \in [0, 1]$.

Proof. Appendix. □

As in the PE case, the expected income of low-skill workers declines with the share of high-skill workers looking in island 1, at least for x_2 low enough. This property of the model is again in stark contrast with a random search model, in which an increase in the quality of the unemployment pool leads to more job creation, which raises the job finding rate of *all* job seekers. This quality effect is present in this model, but it is dominated, at least for low values of x_2, by the effect of skill-biased job competition. Because high-skill workers are systematically hired over competing low-skill applicants, an increase in x_2 implies a lower expected income for low-skill workers. However, as x_2 increases and the pool becomes more homogeneous (i.e., becomes dominated by high-skill workers), the degree of heterogeneity in the unemployment pool diminishes and the skill-biased job competition effect becomes weaker. This explains why, for large values of x_2, an increase in the number of

[27] Such a "quality effect" is well known in random search models with heterogeneous workers. See e.g., Albrecht and Vroman (2002).

[28] With non random hiring and skill-biased job competition, the wage schedule of high-skill workers is downward slopping, because the expected income of high skill workers is driven by their uniqueness, as it determines both their ability to find a job easily (by being preferably hired over low skill workers) and to obtain a wage premium over low skill workers. As the number of high skill workers increase, they become less unique, leading to a lower job finding rate (as they face more competition from their peers) and a lower wage premium.

type 2 workers can raise low skill workers' labor market prospects, as predicted by a standard search model with heterogeneous workers.

Comparative statics in general equilibrium Using our graphical representations, we can discuss the re-distributional effects of under-employment in general equilibrium. First, we show that the asymmetry across skill-groups generated by under-employment is even more acute in GE than in PE. Specifically, in GE, high-skill workers are *never* affected by shocks hitting the low productivity island, while low-skill workers are *always* affected by shocks hitting the high productivity island. Second, through the "quality effect", under-employment may dampen or exacerbate the effect of an island specific shock on low-skill workers.

Consider first an adverse labor demand shock affecting the high productivity island, i.e., an increase in the vacancy posting cost c_2. As shown in Figure 8, an increase in c_2 shifts down the $E\omega_{2,2}$ curve. As under-employment increases, this increases congestion for low-skill workers who move down their wage curve. However, thanks to the quality (GE) effect, job creation is higher in island 1, and the expected wage of low-skill workers goes down less or may even go up. This time, GE effects dampened the crowding-out coming from the larger number of high-skill workers searching in island 1.

Consider now an adverse labor demand shock in island 1, i.e., an increase in c_1 that shifts down the expected wage curves $E\omega_{1,1}$ and $E\omega_{2,1}$ (Figure 9). The income of type 2 workers is not affected as $E\omega_{2,2}$ remains constant thanks to GE effects, and high-skill workers are completely insulated from a shock affecting the low-tech island. However, the reduction in job openings in island 1 reduces the number of type 2 workers present in island 1, which further reduces the number of job openings, and further reduces the number of type 2 workers in island 1, etc.. In other words, the negative labor demand shock is magnified by the deterioration in the average quality of the pool of applicant, and type 1 workers suffer a larger drop in income (larger downward shift in the $E\omega_{1,1}$ curve). This time, GE effects exacerbate the effect of the shock on type 1 workers. Nonetheless, the lower under-employment rate dampens the effect of the shock on low skill workers by lowering congestion in island 1 (movement along the $E\omega_{1,1}$ curve).

5 Constrained optimal allocation

We now consider the efficiency property of the decentralized allocation. We study the problem of a planner who can only allocate workers across islands in order to maximize total output (net of the cost of posting vacancies), while taking into account firms' free entry condition. Importantly, the planner is constrained by the existence of coordination frictions in each island and by the impossibility for firms to commit to a posted wage.

The following proposition states that the decentralized allocation is, in general, not efficient, and that there is too much under-employment.

Proposition 3. *When $N = 2$, the constrained optimal allocation (x_2^*, q_1^*) is characterized by the firms' free entry conditions in islands 1 and 2, and*

$$\begin{aligned} A(x_2^*, q_1^*) &= -E\omega_{2,2} + E\omega_{2,1} \\ &= (1-\beta)\, h_2 q_1^* \varphi_{1,1} e^{-q_1^*(2x_2^* h_2 + 1)} \left(\varphi_{2,1} - \varphi_{1,1}\right) \frac{1}{\frac{\partial \pi_1(x_2^*, q_1^*)}{\partial q_1}} \\ &\geq 0 \end{aligned} \tag{2}$$

with the expression for $\frac{\partial \pi_1(x_2,q_1)}{\partial q_1} > 0$ given in the appendix.

If $\beta < 1$ and $\varphi_{2,1} - \varphi_{1,1} > 0$, the decentralized allocation (x_2, q_1) is inefficient and has too much under-employment: $x_2 > x_2^*$.

Proof. See appendix. □

Contrasting the decentralized and centralized allocations To better understand the externalities present in this economy, it is useful to contrast the worker's problem and the planner's problem. Type 2 workers allocate themselves between islands 1 and 2 up until

$$E\omega_{2,2} = E\omega_{2,1}. \tag{3}$$

In contrast, the planner wishes to allocate workers in order to maximize total output, while satisfying firms' zero profit condition. With free entry, we have $\pi = y - \omega = c$ so that maximizing total output is identical to maximizing the total wage bill Ω. The planner's problem is thus to maximize the wage bill while satisfying firms' free entry conditions

$$\begin{cases} \max_{x_2} \Omega \\ \Omega = (1-x_2) h_2 E\omega_{2,2} + h_2 x_2 E\omega_{2,1}(x_2, q_1(x_2)) + E\omega_{1,1}(x_2, q_1(x_2)) \end{cases} \tag{4}$$

with $q_1(x_2)$ is given by fims' free entry condition in island 1: $\pi_1(x_2, q_1(x_2)) = c_1$.

Finally, note that we omitted one externality in our discussion: since type 2 workers do not internalize how their descent affects $E\omega_{2,2}$, the wage of high-skill workers who remained in the high-skill island, the marginal high-skill applicant should also exert an externality on $E\omega_{2,2}$. This externality has no consequence however, because $E\omega_{2,2}$ is constant thanks to general equilibrium effects. Indeed, and implicit in the planner's problem (4), we used the fact that the free entry condition in island 2 ensures that $E\omega_{2,2}$ is constant and independent of x_2.

Contrasting (3) with (4), we can see that type 2 individuals are solving the planner's problem treating $E\omega_{2,1}(x_2, q_1(x_2))$ and $E\omega_{1,1}(x_2, q_1(x_2))$ as independent of x_2. In other words, type 2 workers do not internalize how their descent affect the wages of other workers , i.e., how an increase in x_2, the fraction of type 2 workers in island 1, affects (i) competition between types, (ii) competition across types, and (iii) job creation in island 1. An increase in x_2:

1. *directly* affects the wage of type 2 workers $E\omega_{2,1}(x_2, .)$: a higher share of type 2 workers raises competition between type 2 workers, which lowers the expected wage $E\omega_{2,1}$. This is a (negative) within-type externality.

2. *directly* affects the wage of type 1 workers $E\omega_{1,1}(x_2, .)$: a higher share of type 2 workers raises the quality of the competition faced by type 1 workers, which lowers $E\omega_{1,1}$, the expected wage of type 1 workers. This is a (negative) across-type externality.

3. *indirectly* affects wages through firms' job creation condition: $E\omega_{2,1}(., q_1(x_2))$ and $E\omega_{1,1}(., q_1(x_2))$: a higher share of type 2 workers raises firm's profit, which leads through the no-profit condition to more job openings, and thus to higher wages for both type 1 and type 2. This is a (positive) job creation externality.

Intuition: the ranking externality

To get some intuition about the (in)efficiency property of the decentralized allocation, the efficiency discussion can be summarized in terms of one single externality, that we label a *ranking externality.*

The key element behind the ranking externality is the fact that high-skill workers and low-skill workers face *different* labor markets in the *same* island: with non-random hiring, a high-skill worker always gets the job when in competition with a low-skill worker, because he is ranked first. As a result, a high-skill worker's decision to move down the occupation ladder are motivated by different factors than those determining the labor market prospect of low-skill workers. Consider for instance a labor market with many low-skill workers but no high-skill worker. In that case, high-skill and low-skill face completely opposite labor market prospects: the market is very congested (and unattractive) for low-skill workers, but it is very attractive for high-skill workers, because they face no competition.

Since the gain from moving down the ladder for a high-skill is disconnected from the loss inflicted on the other workers, the decision to move down the ladder is disconnected from the loss inflicted on the other workers, and the ranking externality can be large. The magnitude of the ranking externality depends on how a marginal high-skill applicant differs from the average applicant in the low-productivity island. The externality is strongest (as in our example) when the marginal high-skill applicant is most different from the average applicant, i.e., most "unique".

To see this more formally, we can contrast (3) with (4). The decentralized allocation is efficient if and only if

$$\frac{d\Omega_1}{dx_2} \equiv \frac{dE\omega_{1,1}(x_2, q_1(x_2))}{dx_2} + x_2 h_2 \frac{dE\omega_{2,1}(x_2, q_1(x_2))}{dx_2} = 0. \tag{5}$$

Expression $\frac{d\Omega_1}{dx_2}$ captures the ranking externality, i.e., the effect of a marginal high-skill worker searching in the low-tech island on the labor markets of (i) low-skill workers ($\frac{dE\omega_{1,1}}{dx_2}$) and (ii) high-skill workers ($x_2 h_2 \frac{dE\omega_{2,1}}{dx_2}$) weighted by their relative population shares. In essence, it captures the congestion externalities net of the compensating effect of the job creation externality.

Using the expression for $\frac{d\Omega_1}{dx_2}$, one can visualize in figure 10 how the externality evolves with the presence of high-skill workers in the low-tech island. Starting from a world with no high-skill workers in the low-tech island ($x_2 = 0$), the presence of one high-skill worker imposes a large cost to low-skill workers: since a high-skill is systematically ranked above competing low-skill applicants, the high-skill applicant systematically gets the job when in competition with a lower-skill applicant, and the labor market of the low skills deteriorates sharply ($\frac{dE\omega_{1,1}}{dx_2} < 0$).

As the number of high-skill workers increases, high-skill workers become less unique and the high-skills start competing against each other for their premium over the low-skills. As each high-skill worker becomes less unique, his wage premium over an average applicant deteriorates, and the ranking externality, which stems from the difference between a marginal high-skill and the average worker in island 1, becomes less strong.

As x_2 increases further, the job creation externality becomes stronger (because firms are more likely to face a high-skill applicant) and compensates the increased congestion in the labor market. In fact, the income of low-skill workers starts increasing for x_2 large enough. However, this cannot compensate the increased congestion between the high-skill workers, and the ranking externality remains negative.[29]

[29] Recall that the ranking externality is nil in an homogeneous world, because as in a standard MP model, the equilibrium queue lengths is independent of the number of job seekers, i.e., firms exactly compensate the arrival of more skill workers by posting more vacancies to keep the queue length

In fact, it is only when the number of high-skills becomes arbitrarily large compared to the number of low-skills, that job creation exactly compensates the increased congestion. Specifically, as x_2h_2 increases further and becomes arbitrary large (if h_2 is arbitrary large), the labor market in the low-tech island resembles that of an homogeneous labor market with only high skill workers, and as in search models with homogeneous labor, the marginal high-skill applicant has no effect on the equilibrium queue length (as we describe in more detail below) and $\frac{d\Omega_1}{dx_2} \to 0$, i.e., the ranking externality converges to zero. Intuitively, with x_2h_2 large, the marginal high-skill applicant is identical to the average unemployed in the low-productivity island, and the ranking externality is nil.

This intuition for why $\frac{d\Omega_1}{dx_2} \to 0$ when island 1 is mainly populated by (homogeneous) high-skill workers also explains why type 2 workers exert no externality on workers in island 2. In that (homogeneous) island, the marginal high-skill applicant is always identical to the average unemployed, and the ranking externality is always nil.

6 Quantitative exercise

In this section, we take our model to the data. After discussing the calibration of a dynamic version of the model, we first show that the model can generate plausible fluctuations in under-employment, as well as plausible fluctuations in the wage premium and wage loss associated with under-employment. Then, we quantify the magnitude of a novel implication of the model: the asymmetry generated by under-employment. While high-skill workers can smooth labor demand shocks by moving down the job ladder, low-skill workers suffer from this trickle-down of unemployment by experiencing higher income volatility.

6.1 Calibration of the dynamic model

In this quantitative section, we rely on our benchmark general equilibrium model with 2 islands –high tech and low-tech–.

In order to bring our static model to the data, we first generalize the model a

constant. Thus, in an homogenous world, the congestion and job creation externalities exactly compensate each other. With worker heterogeneity, job creation can never fully compensate the congestion externality, because, while firms' profits increase with the number of high-skill workers, the presence of low-skill workers limits the increase in profits, because firms always face a non-zero probability of receiving just one application from a low-skill applicant and then ending up with a low-skill worker and low profit. It is only in the limit case where the share of high-skill workers goes to one and the unemployment pool becomes homogeneous that the job creation externality exactly compensates the congestion externality.

dynamic case, focusing on the steady-state as in standard search models (Rogerson et al. 2005). The only difference with the static model is that, in this dynamic version, matched workers separate from their job at an exogenous rate λ that can depend on the firm-worker type. We leave the details of the dynamic generalization in the appendix, since the qualitative properties of the model are unchanged in a dynamic setting.[30]

We then need to calibrate the model parameters, in particular the match productivity of each firm-worker pair $\varphi_{1,1}$, $\varphi_{2,1}$, $\varphi_{2,2}$.

To do so, we use our model's predictions on wages for each firm-worker match. Specifically, from the CPS Merged Outgoing Rotation Groups, we observe the wages paid in each island to each type of worker $w_{1,1}$, $w_{2,1}$, $w_{2,2}$ over 1980-2013. As detailed in the Appendix, the model allows us to express $w_{1,1}$, $w_{2,1}$, $w_{2,2}$ as functions of the model parameters φ_{11}, $\varphi_{2,1}$, $\varphi_{2,2}$, as well as $q_{1,t}$ and $q_{2,t}$, the queue lengths in each island as defined in Section 4, the fraction of high-skill workers searching in the low-tech island, $x_{2,t}$, and the ratio to high-skill to low-skill job seekers, $h_{2,t}$.

We back out values for $\varphi_{1,1}$, $\varphi_{2,1}$ and $\varphi_{2,2}$ by matching the average wages observed over 1980-2013 using as input data on $q_{1,t}$, $q_{2,t}$, $x_{2,t}$ and $h_{2,t}$.

To measure $q_{1,t}$ and $q_{2,t}$ in each island, we use job openings data from the Help Wanted OnLine (HWOL) dataset provided by the Conference Board and available since 2006. The HWOL provides information on the number of job openings by occupation group, so that combined with CPS data on the number of job seekers by occupation,[31] we can measure the job seekers to job openings ratio by occupation over 2006-2012. While occupation groups are different from degree requirements, we use the fact that for the four main occupation groups (professional, services, construction and sales), there is a relatively straightforward mapping between occupation and degree requirements. Specifically, services and professional occupations require a bachelor's degree in most cases (it is true for more than 90 percent of the 3-digit occupations which compose those two categories), and construction and sales typically require less than a high school degree (it is true for more than 75 percent of the 3-digit occupations which compose those two categories). We then obtain the queue lengths $q_{1,t}$ and $q_{2,t}$ over 2006-2012.

We do not observe $x_{2,t}$ the fraction of high-skill workers *searching* in the low-tech island. Instead, we observe (again from CPS micro data) $\hat{x}_{2,t}$, the fraction of

[30]The only difference with the static version is that workers' reservation wage –the wage making workers indifferent between working at that wage or remaining unemployed– is no longer 0 (or the exogenous value of unemployment benefits) but is endogenous and depends on the labor market allocation.

[31]We measure the number of job seekers by occupation with the number of unemployed who report that occupation as a previous occupation.

high-skill workers *employed* in the low-tech island. In a dynamic model, that latter quantity is different from x_2, the fraction of type-2 workers *searching* in island 1 at time t. However, as described in the appendix, one can use a stock-flow model to express $\hat{x}_{2,t}$ as a function of $x_{2,t}$ and the model's variables and parameters.

Finally, $h_{2,t}$ is the ratio of high-skill job seekers to low skill job seekers and is observable from CPS micro data.

6.2 Accounting for under-employment over the cycle

In this exercise, our goal is to evaluate whether our stylized model of under-employment can generate plausible fluctuations in under-employment over the cycle.

Our input variables are vacancy posting in each island, i.e., the "initial" queue lengths $q_{1,t}$ and $q_{2,t}$. Since we only observe the queue lengths over a relatively short interval, and to make our exercise a little more illustrative, we use the fact that we can observe the aggregate queue length $q = \frac{U}{V}$ over 1976-2012 using the composite Help-Wanted index built from Conference Board data as measure of total job openings (Barnichon 2010). Over 2006-2012, the aggregate queue length can explain fluctuations in $q_{1,t}$ and $q_{2,t}$ reasonably well, so that we use aggregate q to backcast $q_{1,t}$ and $q_{2,t}$ over 1976-2006.[32]

Although clearly imperfect, this illustrative exercise allows us to evaluate the variations in under-employment predicted by our model over the past 40 years, and to contrast it with the actual movements in under-employment.

The upper-panel of figure 11 plots the back-casted time series q_1 (low-skill) and q_2 (high-skill) over 1976-2012, along with the actual series (dashed lines) over 2006-2012. An interesting observation is that low-skill occupations appear to be more cyclically sensitive than high-skill occupation. This further supports our conclusion in section 2 that a model with random search and multiple islands will have a hard time explaining the increase in under-employment observed during the Great Recession, because the low-tech island appears to have suffered a larger drop in labor demand than the high-tech island.

The lower-panel of figure 11 plots x_2 over 1976-2012, the model simulated fraction of high-skill workers looking for a job in the low-tech sector, as well as $\hat{x}_{2,t}$ the corresponding fraction of high-skill workers employed in the low-tech sector. We can see that the model does a good job at explaining the counter-cyclical behavior of underemployment over the past 40 years, with the fluctuations in under-employment comparable to those observed in Figure 1.

[32]Specifically, we use the simple regression $\ln q_{i,t} = \alpha \ln q_t + \beta + \varepsilon_{i,t}$, $i = 1, 2$, estimated over 2006-2012, and we use the prediction of the model to back up $q_{i,t}$ between 1976 and 2012.

In addition, table 4 shows that the wage premium gained by high skills over low skills is pro-cyclical is consistent with the data (see table 2), with the wage premium being 13% smaller during recessions. Indeed, as more high-skill workers move down the job ladder, they become less "unique" and compete more and more against each other for the same job, which drives down their average realized wage. For the same reason, the average wage loss incurred by high-skills when moving down the ladder is counter-cyclical, being 13 percent higher in recessions.

6.3 Asymmetric effects of shocks across skill groups

We now turn to a counter-factual experiment, in which we aim to quantify the cost imposed by under-employment on low-skill workers. Specifically, we consider a counter-factual world in which the under-employment rate is constant at its average level, i.e., a world in which x_2 remains constant in the face of labor demand shocks (the fluctuations in $q_{1,t}$ and $q_{2,t}$), so that high-skill workers cannot smooth labor demand shocks by varying their under-employment rate.

We start from our baseline world with counter-cyclical under-employment, as simulated in the previous section. We can see that type-2 workers enjoy higher expected income that type-1 workers, a combination of their higher job finding probability (thanks to under-employment) and of their wage premium over type-1 workers (Table 5).

Moreover, the volatilities of the expected income of high skill and low-skill workers (as measured by the coefficient of variation) are comparable. However, this result masks the contribution of under-employment in redistributing volatility from the high-skills to the low skills.

To highlight the role of under-employment in the re-distribution of shocks from high-skill workers to low-skill workers, we now consider the counter-factual experiment in which $x_{2,t}$ is kept constant at its average level over 1976-2012. Looking at the last row of table 5, we can see that under-employment can be a major redistributive mechanism in which shocks get passed from the high skills to the low skills. Absent counter-cyclical movements in under-employment (i.e., absent any possibility for high-skill workers to smooth shocks by redistributing it to the low skills), the expected income of high skill workers would have been 23 percent more volatile (as measured by the coefficient of variation). Conversely, the expected income of low skill workers would have been 15 percent less volatile.

7 Conclusion

While the unemployment rate is the traditional gauge of the labor market, this paper shows that under-employment is an important, yet overlooked, characteristic of the labor market.

We study empirically the phenomenon of under-employment in the US and show that (i) under-employment is strongly counter-cyclical, (ii) under-employment is very costly, an under-employed worker earning between 30 and 40 percent less than his non-under-employed counterpart, and (iii) under-employment is a highly persistent state, with under-employed workers spending an average of about five years in under-employed jobs.

We then argue that under-employment constitutes a puzzle for canonical search models of the labor market, be it a search and matching model a la Mortensen and Pissarides (1994) or a competitive search model a la Moen (1997)), and instead propose a search model with non-random hiring in which firms can rank applicants and hire their preferred candidate. In such a model, job competition is biased and favors the high-skills, who are systematically hired over competing low-skill applicants. With skill-biased job competition, under-employment exists in equilibrium, because some high-skill workers move down the occupational ladder in order to find a job more easily. Through this process, unemployment trickles down from the upper-occupation groups to the lower-occupation groups, so that high-skill workers enjoy not only higher expected income but also lower income volatility.

A quantitative version of the model generates plausible fluctuations in under-employment and shows that the trickle-down of unemployment can be a major redistributive mechanism as shocks get redistributed from the high skills to the low skills.

Davis and Wachter (2011) recently showed that the cumulative earnings losses associated with job displacement are (i) substantial and (ii) even larger if the displacement occurs during a recession. While Davis and Wachter (2011) show that standard search models cannot rationalize such large losses, under-employment may provide an explanation for this finding. In recessions, more high-skill workers move down the job ladder, and suffer substantial income losses given the persistence of the the under-employment status. While under-employment is an optimal choice for high-skill workers in our model, it is easy to imagine cases for which high-skill workers are forced down the occupation ladder to a greater extent, because of borrowing constraints or reputation considerations associated with long unemployment spells (Kroft et al. 2013). Then, if moving back up the ladder is difficult because of asymmetric information or skill deterioration, under-employment opens the door to large welfare costs of business cycle fluctuations.

Finally, the possibility of under-employment opens new, and so far unexplored, returns to education. First, if, ceteris paribus, firms always prefer more educated workers (as is the case in our model), more educated workers have more bargaining power, and they can extract a larger share of a match surplus than less educated workers, i.e., they receive a higher labor income share. Second, a higher education level may not only guarantee a higher expected income, but it may also provide a lower volatility of income, because highly educated workers can partially smooth out adverse labor demand shocks by moving down the occupational ladder. Exploring these additional returns to education is an important goal for future research.

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Table 1. Transitions out of under-employment.

TRANSITIONS	All sample (1980-2013)	Normal periods (1980-2013)	Recessions (1980-2013)
$P(E_{2,1} - E_{2,2})$	.0119	.0121	.0105
	[.0025]	[.0022]	[.0038]
$P(E_{2,1} - U)$	.0049	.0047	.0059
	[.0015]	[.0013]	[.0021]
$P(E_{1,1} - U)$	.0084	.0081	.0104
	[.0022]	[.0019]	[.0029]
$P(E_{2,2} - U)$	.0025	.0025	.0028
	[.0007]	[.0007]	[.0008]

Transitions are computed here as monthly probabilities. $E_{i=2,j=1}$ denotes employment for an individual with a higher degree than a high school degree ($i = 2$) in an occupation that only requires at most a high school degree ($j = 1$). Standard deviations are between brackets.

Table 2. Wage differences by education and occupation's degree requirements.

WAGE PREMIUMS	All sample (1980-2013)	Normal periods (1980-2013)	Recessions (1980-2013)
$(\omega_{2,2} - \omega_{2,1})/\omega_{2,1}$			
No controls	.365	.392	.214
Controls (age, sex, state, occup.)	.289	.278	.373
	(.021)	(.022)	(.094)
$(\omega_{2,1} - \omega_{1,1})/\omega_{1,1}$			
No controls	.292	.286	.323
Controls (age, sex, state, occup.)	.147	.148	.135
	(.013)	(.013)	(.053)

$\omega_{i=2,j=1}$ denotes the hiring wage of an individual with a higher degree than a high school degree ($i = 2$) in an occupation that only requires at most a high school degree ($j = 1$). The premiums without controls are simply computed from the average monthly hiring wages. The premiums with controls are computed using an OLS specification controlling for age, sex and year and state fixed effects. Standard errors between parentheses are clustered at the occupation level. For $\frac{\omega_{2,2}-\omega_{2,1}}{\omega_{2,1}}$, we control for the previous occupation while, for the premium $\frac{\omega_{2,1}-\omega_{1,1}}{\omega_{1,1}}$, we add current occupation fixed effects.

Table 3. Callback rate as a function of applicants' type.

CALLBACK RATE	(1)	(2)	(3)
High-type	.0153**	.0135**	.0176**
	(.0056)	(.0055)	(.0060)
	[.0439]	[.0439]	[.0439]
Fixed-effects		City	Ad
Controls (gender, age)	Yes	Yes	Yes
Observations	11,724	11,724	11,724

Significantly different than zero at † 90% confidence, * 95% confidence, ** 99% confidence. Standard errors between parentheses are clustered at the ad level. The average callback rates over the sample are shown between brackets.

Table 4. Model simulation: Wage differences across skill groups and island levels.

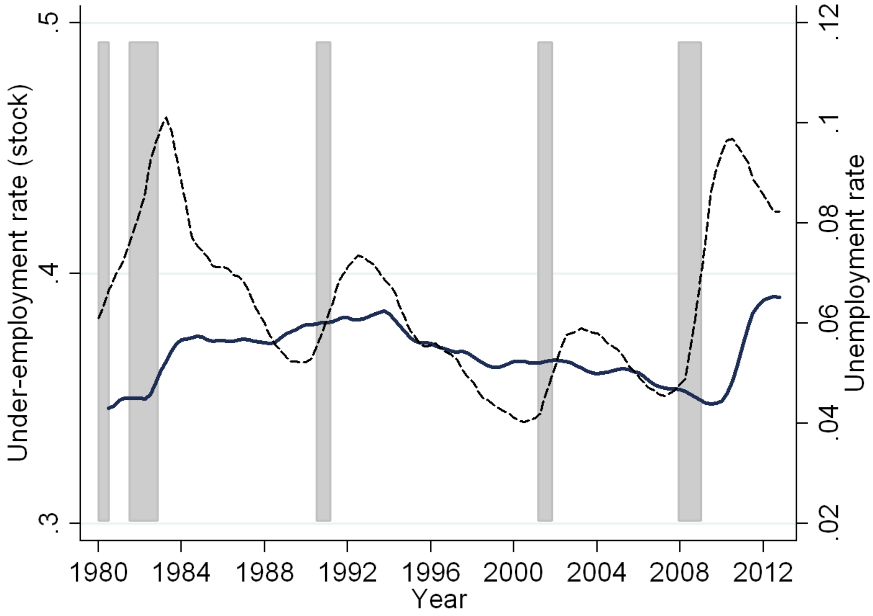
WAGE PREMIUMS	All sample (1976-2012)	Recessions (1976-2012)
$(\omega_{2,2} - \omega_{2,1})/\omega_{2,1}$	.38	.43
$(\omega_{2,1} - \omega_{1,1})/\omega_{1,1}$	.31	.27

These premiums are the average premiums implied by our model simulations over the period 1976-2012. These results can be compared to the empirical premiums as measured over the period (1980-2012) in table 2.

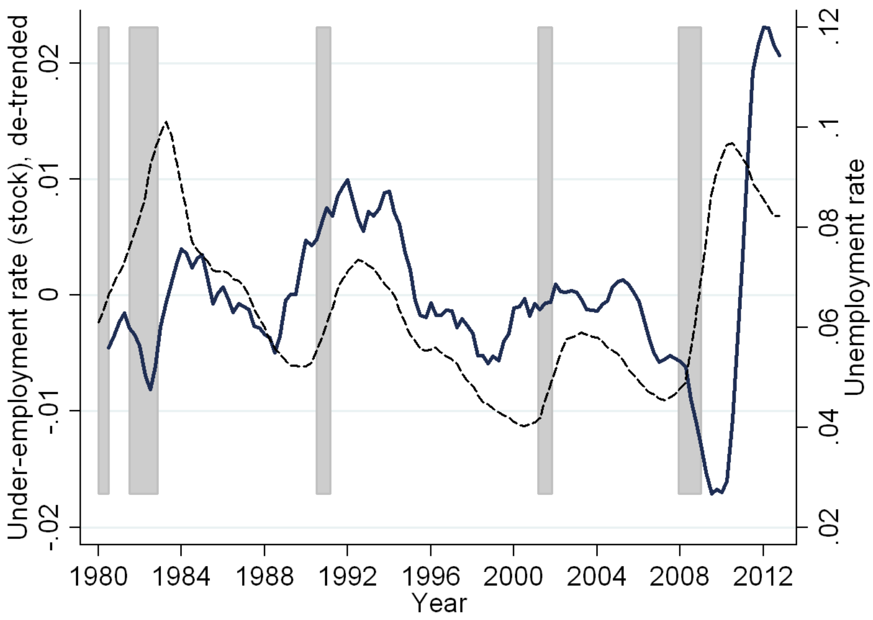
Table 5. Model simulation: Expected income across skill groups.

EXPECTED INCOME	High-skill workers $E\omega_{2,2}$	Low-skill workers $E\omega_{1,1}$
Mean	.82	.51
Constant Variation	.13	.11
Constant Variation, fixed x_2	.16	.09

These statistics over the evolution of expected wages are implied by our model simulations over the period 1976-2012.



(a) Under-employment rate (plain) and unemployment rate (dashed)



(b) Detrended under-employment (plain) and unemployment rate (dashed).

Figure 1. Under-employment in the US

Source: Current Population Survey, 1980-2012. Under-employment (plain dark line) is defined as the fraction of individuals with at least some college education working in occupations requiring at most a high school degree. Detrended under-employment is HP filtered under-employment. The dashed line is the unemployment rate.

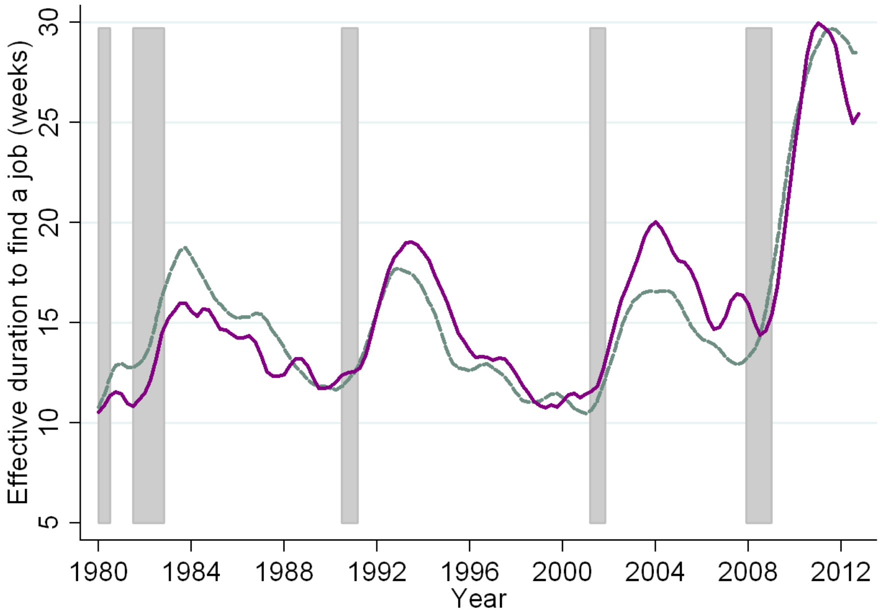


Figure 2. Effective duration to find a job (high versus low-requirement occupations).

Source: Current Population Survey, 1980-2012. The effective duration to find a job is computed as the average unemployment duration (in weeks) of just hired workers. The plain line (resp. dashed line) is the effective duration to find a job for high-requirement (resp. low-requirement) occupations.

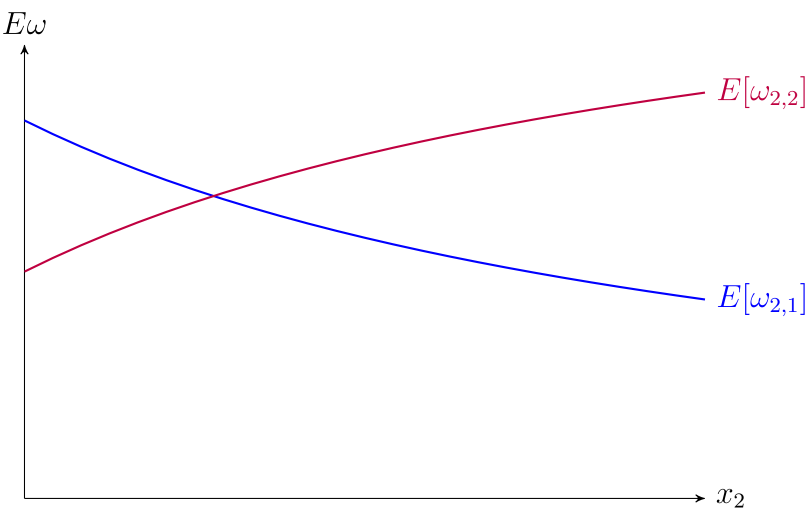


Figure 3. Partial Equilibrium.

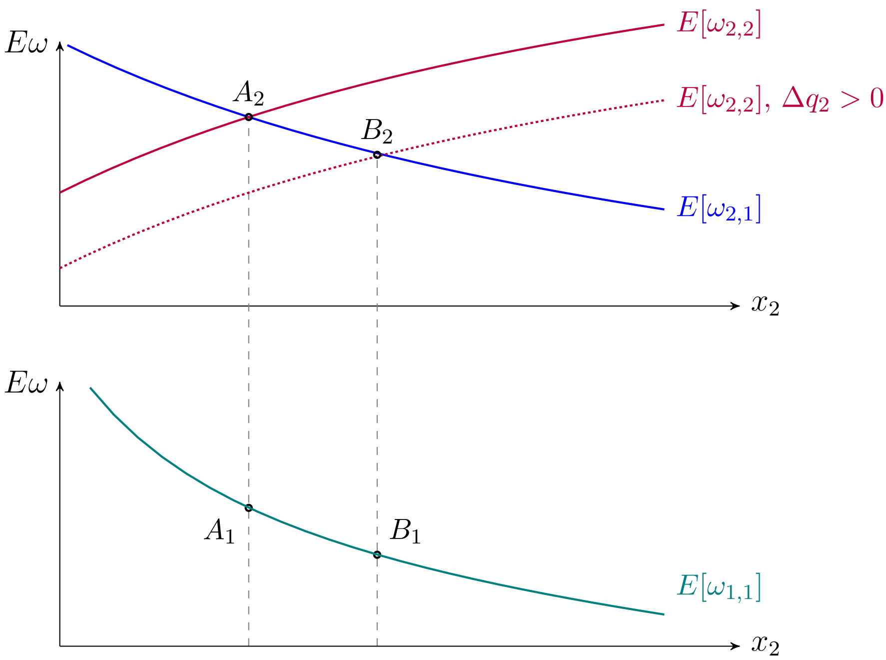


Figure 4. Partial Equilibrium – shock on $\Delta q_2 > 0$.

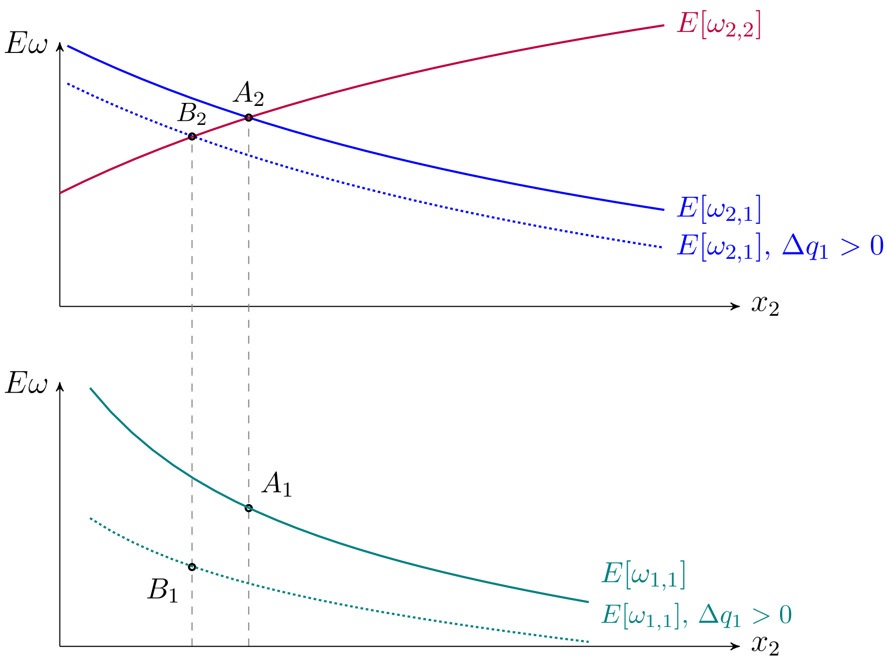


Figure 5. Partial Equilibrium – shock on $\Delta q_1 > 0$.

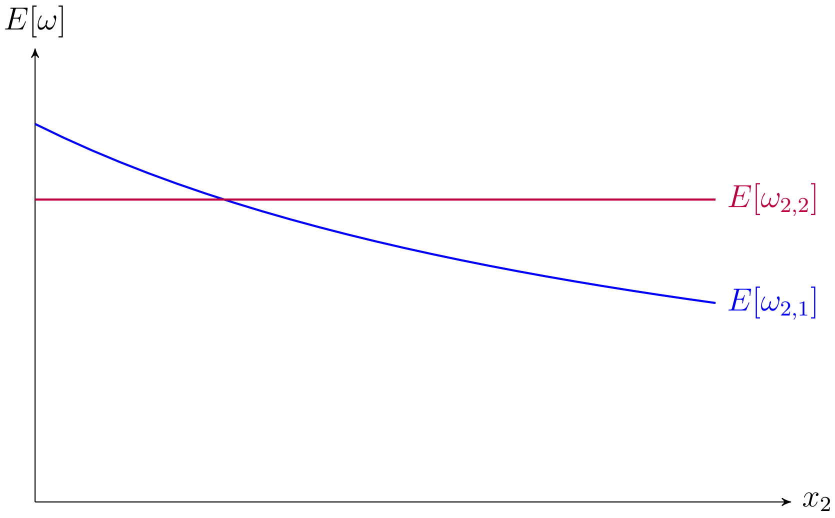


Figure 6. General Equilibrium – wages for type-2 workers.

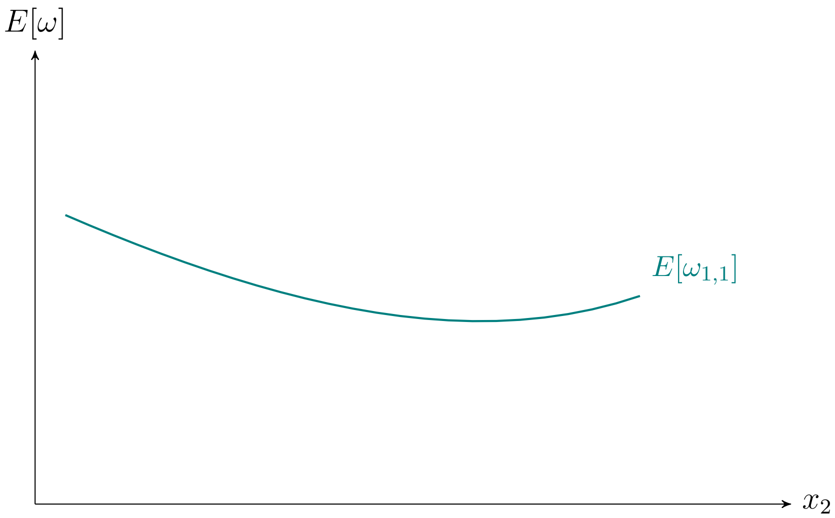


Figure 7. General Equilibrium – wages for type-1 workers.

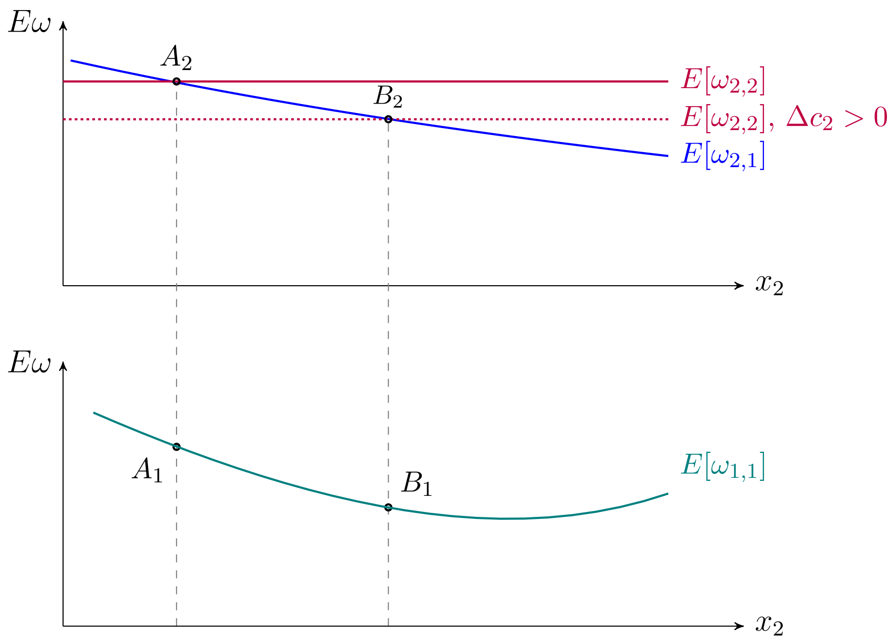


Figure 8. General Equilibrium – shock $\Delta c_2 > 0$.

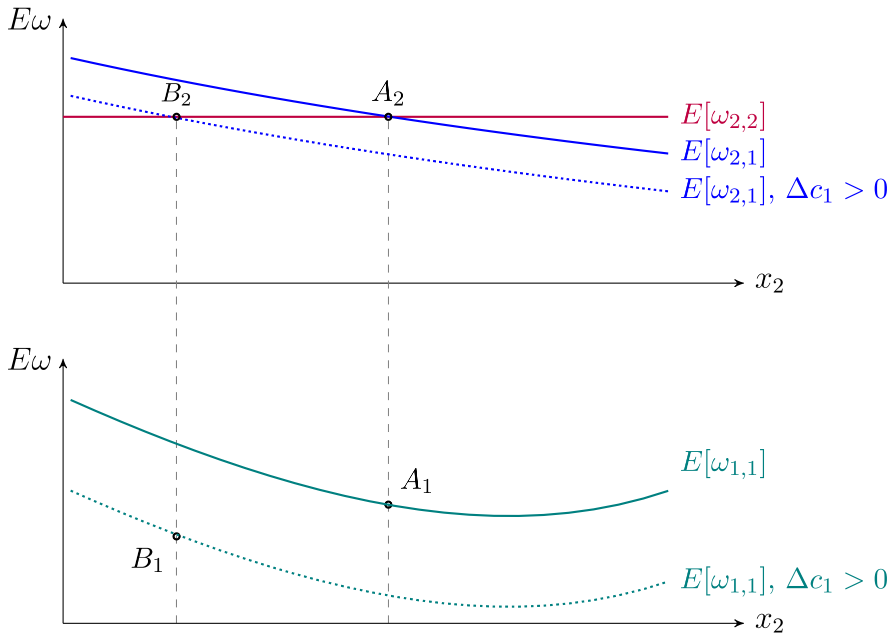


Figure 9. General Equilibrium – shock $\Delta c_1 > 0$.

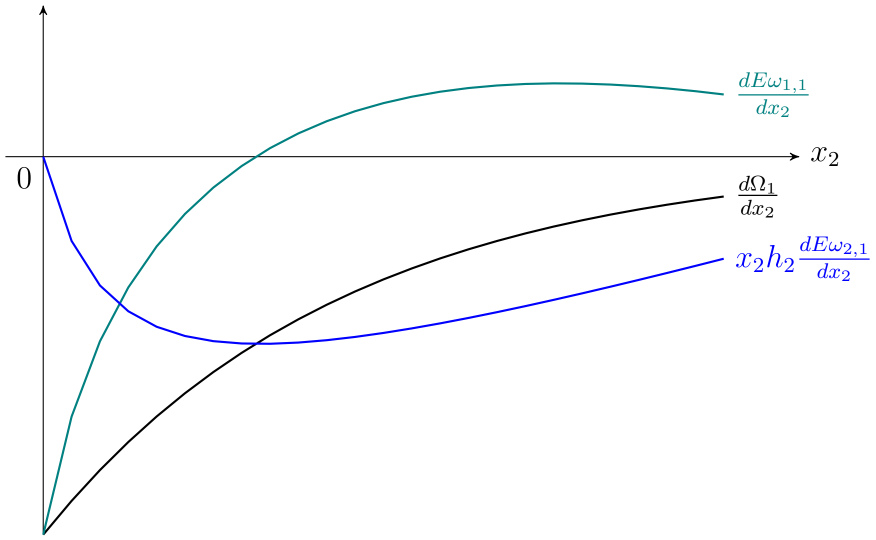


Figure 10. Inefficiency – ranking externality $\frac{d\Omega_1}{dx_2} = \frac{dE\omega_{1,1}}{dx_2} + x_2 h_2 \frac{dE\omega_{2,1}}{dx_2}$.

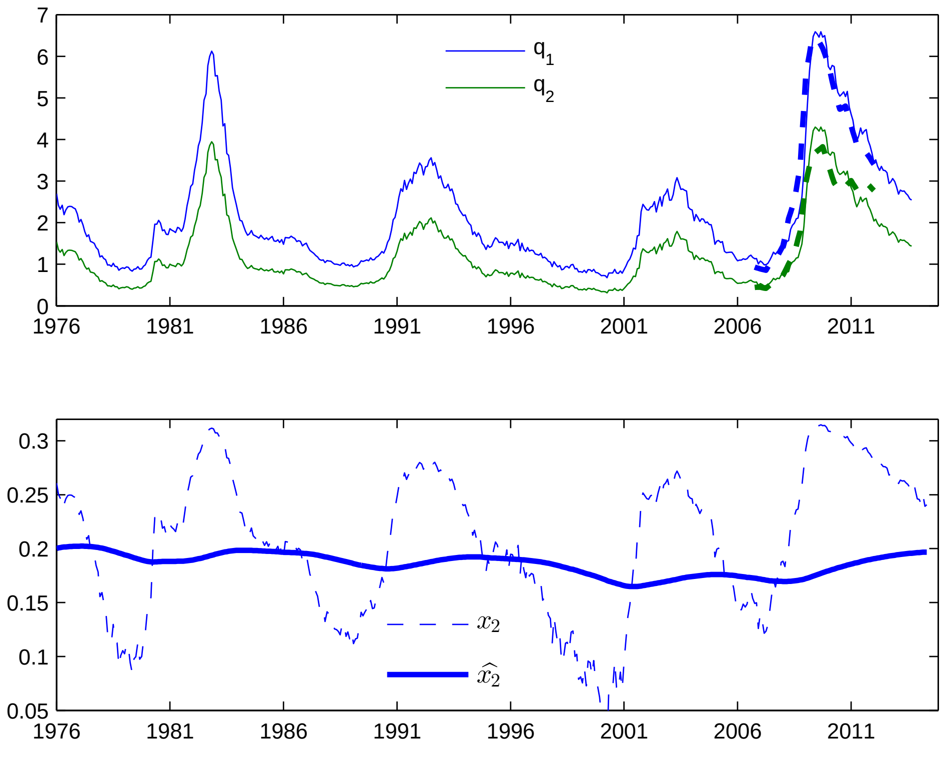


Figure 11. Queue lengths across islands (top panel – dashed lines are from HWOL data, solid lines are inferred from data on aggregate queue length) and simulated under-employment rate $\hat{x}_2$ (bottom panel – solid line is the fraction $\hat{x}_2$ of high-skill workers employed in island 1, dashed line is the fraction x_2 of high-skill workers searching in island 1).

A Appendix

A.1 Conditions to ensure under-employment in equilibrium

We derive here the conditions that ensure that the equilibrium we consider is an under-employment equilibrium. Our conditions boil down to ensuring that the equilibrium is not at a corner solution in which either everyone or no one is under-employed.

We ensure that some type 2 workers descend to island 1, and that no type 1 workers are tempted to search in island 2. In partial equilibrium, this is given by:

$$e^{-q_2}\varphi_{2,2} < (\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1}) \quad (C_2^p)$$
$$\frac{\varphi_{1,2}}{\varphi_{2,2}}\frac{\varphi_{2,1}-\varphi_{1,1}+\varphi_{1,1}e^{-q_1}}{\varphi_{1,1}e^{-q_1}} < 1$$

Intuitively, the first condition ensures that, when $x_2 = 0$ (no under-employment), the expected wage in island 1 is higher than the expected wage in island 2. The second condition guarantees that, as long as type 2 workers are indifferent between the two islands, then type 1 workers always prefer island 1. This condition is guaranteed when type 1 workers are particularly unproductive in high-tech jobs, i.e. $\varphi_{1,2}$ is small.

In general equilibrium, we impose a similar set of conditions. First, we set c_2 such that, in equilibrium, $E\omega_{2,2}(x_2)$ is lower than the expected wage $E\omega_{2,2}(x_2 = 0)$ when there is no under-employment. Formally, the condition can be written as follows:

$$E\omega_{2,2} < (\varphi_{2,1} - \varphi_{1,1}) + E\omega_{1,1}(x_2 = 0) \quad (C_2^g)$$

Second, we set c_1, such that the queue $\underline{q_1}$ associated with all type 1 workers in island 1 and no type 2 workers verify:

$$\frac{\varphi_{1,2}}{\varphi_{2,2}}\frac{\varphi_{2,1}-\varphi_{1,1}+\varphi_{1,1}e^{-\underline{q_1}}}{\varphi_{1,1}e^{-\underline{q_1}}} < 1$$

A.2 Proofs

Proof. Proposition 1

Consider first the problem of type 2 workers. A type 2 worker has two choices, he can (i) look for a job in island 2, his "home island", or (ii) look for a job in island 1, i.e., move down the occupation ladder. We now consider these two possibilities.

When a type 2 worker looks for a job in island 2, he faces two possible outcomes: (a) with probability $e^{-q_2(1-x_2)}$, he is the only applicant and receives $\beta\varphi_{2,2}$, or (b), with probability $1 - e^{-q_2(1-x_2)}$, he is in competition with other workers and receives 0 (regardless of whether he ends up employed or unemployed). The expected payoff of a worker type 2 who searches for a job in island 2, $\omega_{2,2}$, is thus

$$E\omega_{2,2} = \beta e^{-q_2(1-x_2)}\varphi_{2,2}$$

The expected wage is increasing in x_2. When a lot of type 2 workers descend to island 1, it becomes easier for the ones who stayed in 2 to be the only applicant to a job and receive a high wage. When a type 2 worker looks for a job in island 1, he faces three possible outcomes: (a) with probability $e^{-q_2x_2h_2}e^{-q_1(1-x_1)}$, he is the only applicant and receives $\beta\varphi_{2,1}$. Note that he produces less than in his "home" island and thus receives a lower wage than would have been the case if he had been the only applicant to a type 2 firm, (b) with probability $1-e^{-q_2x_2h_2}$, he is in competition with other type 2 workers and receives 0 (regardless of whether he ends up employed or unemployed), and (c) with probability $e^{-q_2x_2h_2}\left(1 - e^{-q_1}\right)$, he is in competition with type 1 workers only and receives $\beta(\varphi_{2,1} - \varphi_{1,1})$.[33] The expected payoff of a worker type 2 who searches for a job in island 1, $\omega_{2,1}$, is thus

$$E\omega_{2,1} = \beta e^{-q_2x_2h_2}e^{-q_1}\varphi_{2,1} + \beta e^{-q_2x_2h_2}(\varphi_{2,1} - \varphi_{1,1})\left[1 - e^{-q_1}\right]$$

The expected wage in island 1 is decreasing in x_2: when there are fewer type 2 workers in island 1, there is less competition in island 1, and type 2 workers can expect a higher wage. In equilibrium, a type 2 worker must be indifferent between looking for a job in island 2 or in island 1. This arbitrage condition, $A(x_2)$, determines, x_2, the equilibrium allocation of workers

$$A(x_2) = -e^{-q_2(1-x_2)}\varphi_{2,2} + e^{-q_1x_2h_2}e^{-q_1}\varphi_{2,1} + e^{-q_1x_2h_2}(\varphi_{2,1} - \varphi_{1,1})\left[1 - e^{-q_1}\right] = 0 \quad (6)$$

Consider now the problem of type 1 workers. Type 1 workers could choose to move up the occupation ladder and search for a job in island 2. This will not happen as long as there are type 2 workers in island 1. Given our initial assumption on the returns of type 1 to their skills in island 2, as long as type 2 workers are indifferent between their "home" island and the island below, type 1 workers will always prefer to remain in 1. Indeed, $E\omega_{1,2} = e^{-q_2(1-x_2)}\varphi_{1,2}$ and equation (6) implies:

$$E\omega_{1,2} = E\omega_{2,2}\frac{\varphi_{1,2}}{\varphi_{2,2}} = e^{-q_1x_2h_2}\frac{\varphi_{1,2}}{\varphi_{2,2}}(\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1})$$

[33] As noted earlier, despite the presence of competing applicants, a single type 2 applicant can extract some of the surplus thanks to due to his productivity advantage over the other applicants.

As a consequence,

$$E\omega_{1,2} = \frac{\varphi_{1,2}}{\varphi_{2,2}} \frac{\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1}}{\varphi_{1,1}e^{-q_1}} E\omega_{1,1}$$

and $\frac{\varphi_{1,2}}{\varphi_{2,2}} \frac{\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1}}{\varphi_{1,1}e^{-q_1}} < 1$ by assumption.

Unicity

Under condition (C_2^p), some workers of type 2 will always apply in island 1. We have already shown that, as long as type 2 workers are indifferent between the 2 islands, there cannot be workers of type 1 looking for jobs in island 2. As a consequence, the only variable that adjusts is the number of workers of type 2 applying in island 1.

The trade-off faced by type 2 workers is monotonic, i.e. as they apply more in island 1, their relative gain of doing so is strictly decreasing. As already discussed above, under condition (C_2^p), the relative gain of applying in island 1 is initially positive (for $x_2 = 0$). The relative gain is negative when $x_2 = 1$, because $\varphi_{2,2} > \varphi_{2,1}$. It only crosses once the x-axis, and the intersection defines the unique equilibrium.

□

Proof. Proposition 2

The workers no-arbitrage conditions are already derived in Proposition 1. The number of job openings is given by the free entry condition, and we only need to express the firm's expected profit as a function the the number of job openings V_1, or the "initial" queue length $q_1 = \frac{L_1}{V_1}$.

Consider first a firm that enters island 1. The firm's profit will depend on the number of applications it receives. There are 5 cases: (the outcome of the wage negotiation process in each case is described in detail in the Proof of Proposition 1)

1. The firm has no applicant. Profit is zero.

2. The firm has only one applicant. The firm gets a share $1 - \beta$ of the output, i.e. $(1 - \beta)\varphi_{1,1}$ if the applicant is of type 1 (which happens with probability $P(a_1 = 1, a_2 = 0) = q_1 e^{-q_1} e^{-q_2 x_2 h_2}$), and $(1 - \beta)\varphi_{2,1}$ if the applicant is of type 2 (which happens with probability $P(a_1 = 0, a_2 = 1) = q_2 x_2 h_2 e^{-q_2 x_2 h_2} e^{-q_1}$).

3. The firm has more than one applicants of type 1 (and no applicants of type 2). The firm gets all the surplus: $\varphi_{1,1}$. This happens with probability $e^{-x_2 h_2 q_1} \left[1 - e^{-q_1} - q_1 e^{-q_1}\right]$.

4. The firm has more than one applicants of type 2 (and no applicants of type 1). The firm gets all the surplus: $\varphi_{2,1}$. This happens with probability $1 - e^{-x_2 h_2 q_1} - x_2 h_2 q_1 e^{-x_2 h_2 q_1}$.

5. The firm has more than one applicants *of different types*. The most productive worker is hired and gets a share β of the surplus generated over hiring the second-best applicant. The firm generates a profit $\varphi_{1,1} + (1-\beta)(\varphi_{2,1} - \varphi_{1,1})$. This happens with probability $x_2 h_2 q_1 e^{-x_2 h_2 q_1}(1 - e^{-q_1})$.

The expected profit for a firm with technology 1 is then as stated in Proposition 3. Reasoning similarly gives the expected profit for a firm with technology 2. Consequently, free entry imposes two no-profit conditions in addition to workers' arbitrage equations described in Section 3.

The unicity of the equilibrium comes is a direct consequence of Corollaries 1 and 2 that we prove next. □

Proof. Corollary 1

First, it is straightforward from the expression of $\pi_2(x_2, q_2)$ that the free entry condition $\pi_2 = c_2$ imposes that $q_2(1 - x_2)$ is constant, so that the expected wage in island 2, $E\omega_{2,2}$, is constant. We can thus restrict our analysis to the arbitrage condition coupled with the free entry condition in island 1.

$$\begin{cases} E\omega_{2,2} = \left[\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1}\right] e^{-q_1 x_2 h_2} & (L^S) \\ \varphi_{2,1} - c_1 = \left[\left(\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1}\right)\left(1 + (2-\beta) q_1 x_2 h_2\right) + (2-\beta)\varphi_{1,1} q_1 e^{-q_1}\right] e^{-q_1 x_2 h_2} & (L_1^D) \end{cases}$$

The (L_1^D) equation defines a job creation function $q_1(x_2)$. As before, we only consider interior solutions, i.e. we impose that:

$$\left[\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1(1)}\right] e^{-q_1(1) h_2} < E\omega_{2,1} < \left[\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}e^{-q_1(0)}\right]$$

Under this condition, the relative gain of searching for a job in lower-tech island is positive for $x_2 = 0$ ($E\omega_{2,2} < E\omega_{2,1}$) and negative for $x_2 = 1$ ($E\omega_{2,2} > E\omega_{2,1}$).

Combining the (L_1^D) and (L^S) equations, it can be shown with a little bit of algebra that:

$$\begin{aligned} E\omega_{2,1}'(x_2) &= \frac{\partial E\omega_{2,1}}{\partial x_2} + q_1'(x_2)\frac{\partial E\omega_{2,1}}{\partial q_1} \\ &= \frac{(2-\beta) q_1 (E\omega_{2,1} - E\omega_1)}{[(2-\beta) q_1 x_2 h_2 - (1-\beta)] E\omega_{2,1} + (2-\beta) q_1 E\omega_1} q_1'(x_2) E\omega_1 < 0 \end{aligned}$$

and that $q_1'(x_2)\frac{\partial E\omega_{2,1}}{\partial q_1} > 0$.

This proves Corollary 1. Moreover, using that $E\omega_{2,1}(x_2) < 0$ with the fact that $E\omega_{2,2}(x_2)$ is constant guarantees the uniqueness of the equilibrium. □

Proof. Corollary 2

Combining the (L_1^D) and (L^S) equations, it can be shown with a little bit of algebra that:

$$\begin{aligned} E\omega_{1,1}^{'}(x_2, q_1(x_2)) &= \frac{\partial E\omega_1}{\partial x_2} + q_1^{'}(x_2)\frac{\partial E\omega_1}{\partial q_1} \\ &= \frac{[(2-\beta)q_1x_2h_2-(1-\beta)](E\omega_{1,1}-E\omega_{2,1})}{[(2-\beta)q_1x_2h_2-(1-\beta)]E\omega_{2,1}+(2-\beta)q_1E\omega_{1,1}} q_1^{'}(x_2)E\omega_{1,1} \gtrless 0 \end{aligned}$$

We can see that $E\omega_{1,1}(x_2, q_1(x_2))$ is not monotonically decreasing, implying that a larger number of high-skilled workers does not necessarily imply lower expected income for low-skilled workers. For $\beta < 1$, $E\omega_{1,1}(x_2, q_1(x_2))$ is initially decreasing and then increases once $(2-\beta)q_1x_2h_2 > (1-\beta)$. This proves Corollary 2. □

Proof. Proposition 3

The maximization program of the central planner can be written as follows (denote Y the aggregate output of the economy):

$$\max_{x_2, q_1, q_2} \{Y\}$$

subject to

$$\begin{cases} \pi_2(x_2, q_2) = c_2 \\ \pi_1(x_2, q_1) = c_1 \end{cases}$$

We can already simplify the program through two channels. First, with free entry, the aggregate profit of firms (net of investment costs) is zero. Consequently, the central planner equivalently maximizes the wage bill of workers. Second, free entry in island 2 imposes that q_2 is set such as to make $(1-x_2)q_2$ constant.

$$(1-x_2)q_2 = f^{-1}\left(\frac{c_2}{\varphi_{2,2}}\right)$$

The program then sums up to:

$$\max_{x_2, q_1} \left\{(1-x_2)h_2E\omega_{2,2} + h_2x_2e^{-q_1h_2x_2}\left[\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}^{-q_1}\right] + e^{-q_1h_2x_2-q_1}\varphi_{1,1}\right\}$$

subject to

$$\varphi_{2,1} - e^{-x_2h_2q_1}\left[\left(\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1}^{-q_1}\right)(1 + (2-\beta)x_2h_2q_1) + (2-\beta)\varphi_{1,1}q_1e^{-q_1}\right] = c_1$$

The first-order condition in x_2 leads to:

$$A(x_2, q_1) - B_{x_2}(x_2, q_1) - \lambda C_{x_2}(x_2, q_1) = 0$$

where

$$\begin{cases} B_{x_2}(x_2,q_1) = q_1 h_2 e^{-q_1 h_2 x_2}\left[x_2 h_2\left(\varphi_{2,1}-\varphi_{1,1}+\varphi_{1,1}^{-q_1}\right)+\varphi_{1,1}e^{-q_1}\right] \\ C_{x_2}(x_2,q_1) = q_1 h_2 e^{-q_1 h_2 x_2}\left[\left((2-\beta)q_1 x_2 h_2-(1-\beta)\right)\left(\varphi_{2,1}-\varphi_{1,1}+\varphi_{1,1}^{-q_1}\right)+(2-\beta)\varphi_{1,1}e^{-q_1}\right] \end{cases}$$

We can already see that profits losses are not entirely internalized by workers: the marginal gain in wages for workers of type 2 cannot fully translate in marginal profits for firms in island 1. The first-order condition in q_1 gives:

$$-B_{q_1}(x_2,q_1)-\lambda C_{q_1}(x_2,q_1)=0$$

where

$$\begin{cases} B_{q_1}(x_2,q_1) = x_2/q_1 B_{x_2}(x_2,q_1)+(1+x_2h_2)\varphi_{1,1}e^{-q_1x_2h_2-q_1} \\ C_{q_1}(x_2,q_1) = x_2/q_1 C_{x_2}(x_2,q_1)+\left[q_1(2-\beta)(1+x_2h_2)-(1-\beta)\right]\varphi_{1,1}e^{-q_1x_2h_2-q_1} \end{cases}$$

We can observe the symmetry between the expressions in x_2 and q_1. The basic intuition is that, since profits can be written as a function of q_1x_2, the externality generated by a change in x_2 will be partly compensated by an inverse change in q_1. Indeed, combining these two equations, we have that:

$$A(x_2,q_1)=\frac{B_{x_2}(x_2,q_1)C_{q_1}(x_2,q_1)-B_{q_1}(x_2,q_1)C_{x_2}(x_2,q_1)}{C_{q_1}(x_2,q_1)}$$

Once accounted for the expression of $B_{q_1}(x_2,q_1)$ and $C_{q_1}(x_2,q_1)$,

$$A(x_2,q_1)=\frac{(1-\beta)\,h_2q_1\varphi_{1,1}e^{-2q_1x_2h_2-q_1}\left(\varphi_{2,1}-\varphi_{1,1}\right)}{C_{q_1}(x_2,q_1)}$$

As a consequence, $A(x_2,q_1)$ is striclty positive as long as the surplus is not entirely given to workers, i.e. $\beta<1$, and workers are not equally productive, i.e. $\varphi_{2,1}-\varphi_{1,1}>0$. Coupled with the two free entry conditions, this equation characterizes the constrained optimum which does not coincide with the decentralized allocation. $A(x_2,q_1)>0$ implies that wages are higher in 1 than in 2. In other words, the decentralized allocation induces a lower x_2, a higher q_1 and a lower q_2. □

A.3 Graphical representation of the model in general equilibrium

We now show how to graphically represent the equilibrium allocation in GE. Recall that the GE allocation is determined by the triple (x_2,q_1,q_2).

We start with the straightforward characterization of the equilibrium in island 2, which is an homogeneous island only populated by type 2 workers. With free entry, it is easy to see that the equilibrium queue length $q_2(1-x_2)$ is independent of the supply of type-2 workers, exactly as in the canonical search and matching model.[34] Specifically, free entry, or (L_2^D), pins down the equilibrium queue length in island 2 –$q_2(1-x_2)$–, regardless of the number of type 2 workers (i.e., regardless of $1-x_2$). This is similar to what happens in standard search and matching models (Mortensen and Pissarides 1994) where the supply of (homogeneous) labor has no effect on the equilibrium queue length. Even though a higher number of type 2 workers improves the matching probability of a firm, free entry ensures that more firms enter the market in order to keep profits constant.[35] In other words, our model behaves similarly to the canonical search model when workers are homogeneous.[36]

Since free entry in island 2 fixes $q_2(1-x_2)$, characterizing the equilibrium allocation reduces to finding the pair (x_2, q_1) that satisfies (i) firms' free entry condition in island 1 and (ii) type 2 worker's arbitrage condition. Although one could depict the equilibrium in the (x_2, q_1) space, we prefer to depict it in the (x_2, V_1) space (recall that $V_1 = L_1/q_1$ with L_1 fixed), since it corresponds to the (U, V) space representation used in standard search and matching models.

As shown in figure 12, the equilibrium is then determined by the intersection of two curves: a "labor demand curve", (L_1^D), given by firms' free entry condition (also called job creation condition) as in search and matching models, and a "labor supply curve", (L^S), characterizing the number of type 2 workers in island 1 and given by the arbitrage condition of type 2 workers between islands 1 and 2.

The labor demand curve is upward sloping and non-linear. To understand the shape of the labor demand curve (L_1^D), it is again useful to go back to the standard Mortensen-Pissarides (MP) model, in which workers are homogeneous. Recall that the total number of job seekers in island 1 is given by $L_1(1 + x_2h_2)$. We can thus represent the labor demand curve, or job creation curve, in a similar fashion to MP models by plotting the job creation curve in (U, V) space. Starting from a world with only type 1 workers and $x_2 = 0$ (i.e., being to the left of the y-axis in figure 12), all workers are homogeneous and, as in the MP model, increasing the number of type 1 (increasing L_1) does not affect the equilibrium queue length V_1/L_1. As

[34]Recall that the queue length is number of job seekers over the number of job openings and that the number of job seekers in island 2 is given by $L_2(1-x_2)$.

[35]In a search and matching model, at a given vacancy level, an increase in the number of job seekers (coming from say out of the labor force, as in Pissarides (2000), Chapter 5) raises firms matching probability, i.e., reduces hiring costs, and leads more firms to enter the market, keeping profit and thus the queue length unchanged.

[36]A technical difference between our framework and the Mortensen-Pissarides model is that, in our set-up, an increase in the supply of workers also improves the bargaining position of the firm (as workers compete against each other when negotiated the wage). This difference has no consequence on the equilibrium queue length, because the bargaining position is also solely a function of the queue length $q_2(1-x_2)$. As a result, no matter the level of $1-x_2$, free entry ensures that the queue length adjusts to keep profits (including the fix cost) nil.

a result, the labor demand curve (dashed blue line) crosses the origin at 0. Now, consider the case where one adds type 2 workers and $x_2 > 0$. Because firms generate a higher profit when hiring type 2 workers than when hiring type 1 workers, an increase in x_2 generates a disproportionate increase in the number of firms in island 1, and the equilibrium queue length $\frac{V_1}{L_1(1+x_2h_2)}$ increases. In other words, the slope of the labor demand curve is initially increasing with x_2. This portion of the labor demand curve can be seen as capturing a "quality effect":[37] as the share of type 2 workers in island 1 increases, the quality (i.e., skill level) of the average applicant improves, and this leads to a disproportionate increase in job creation. Then, as the number of type 2 workers becomes large relative to the number of type 1, the labor market in island 1 resembles more to more to that of an homogeneous market with only type 2 workers, in which the queue length is independent of the number of type 2 and the slope of (L_1^D) is again independent of x_2 (dashed red line).

The labor supply curve is capturing how x_2 depends on V_1 and is also upward slopping: the larger the number of job openings, the less competition type 2 workers will face when searching in island 1, and the higher their expected wage. As a result, an increase in V_1 raises the incentive of type 2 to move down to island 1 and increases x_2.

A.4 Under-employment in a dynamic setting

In this section, we present our model of under-employment in a dynamic setting. For simplicity, we focus on the steady-state and only consider the $N = 2$ case.[38]

The generalization of the static case to a dynamic setting is relatively trivial, the only difference with the static version is that workers' reservation wage –the wage making workers indifferent between working at that wage or remaining unemployed– is no longer 0 (or the exogenous value of unemployment benefits) but is endogenous and depends on the steady-state labor market allocation.

Denote b the flow payment of unemployment benefits (or home production) identical across islands. Denote λ_1 the (exogenous) job separation rate of type-1 workers working in island 1, $\lambda_{2,1}$ the job separation rate of type-2 workers working in island 1, and $\lambda_{2,2}$ the job separation rate of type-2 workers working in island 2. Let r denote the workers' discount rate.

Denote $U_{2,1}$ the value function of being unemployed in island 1 for a type-2 worker, and $U_{2,2}$ the value function of being unemployed in island 2 for a type-2 worker. Denote $U_{1,1}$ the value function of being unemployed in island 1 for a type-1

[37]Such a "quality effect" is well known in random search models with heterogenous workers.
[38]Generalization to the $N = 3$ case (or higher) would proceed in an identical fashion.

worker. Denote $J_{2,1}$ the value for the firm in island 1 of hiring a type-2 worker, $J_{1,1}$ the value for the firm in island 1 of hiring a type-1 worker, $J_{2,2}$ the value for the firm in island 2 of hiring a type-2 worker. Regarding type-2 workers, note that since, in equilibrium, high-skill workers are indifferent between island 1 and 2 (ie., $U_{2,1} = U_{2,2} = U_2$), type-2 reservation wages are equal across islands.

The value of being unemployed and employed for a type i, U_i and W_i, satisfy

$$\begin{cases} rU_i = b + \int \max\left(W_i(w) - U_i, 0\right) dF_i(w) \\ rW_i = w_i + \lambda_i \left(U_i - W_i(w)\right) \end{cases} \tag{7}$$

where F's denote the (endogenous) distributions of negotiated wages.

Non-degenerate distributions F arises because the wage of a worker depends on the number of applicants competing for the same job. Denote b_1 (resp. b_2) the reservation wage of a type-1 (resp. type-2) worker. To find b_2 and b_1, we use $b_2 = rU_2 = rW(b_2)$ and $b_1 = rU_1 = rW(b_1)$ with (7) to get

$$b_2 = b + \frac{1}{r + \lambda_{2,2}} \int_{b_2}^{\infty} (w - b_2)\, dF_{2,2}(w) \tag{8}$$

and similarly

$$b_1 = b + \frac{1}{r + \lambda_1} \int_{b_1}^{\infty} (w - b_1)\, dF_1(w). \tag{9}$$

To derive the distributions F, it is instructive to start with the simpler case of an homogenous island where workers are all of the same type. This will be the case of island 2, since similarly to the static case, in an equilibrium with postive under-employment, no low-skill workers chooses to look for a job in the high-tech island. For clarity of exposition, we thus directly denote this case with subscript 2 to characterize the problem of the high-tech island. In an homogenous island, there are two cases: (i) a worker is alone to negotiate with the firm. In that case, she gets the wage $w_{2,2} = \beta\varphi_2 + (1-\beta)b_2$ with b_2 the reservation wage of workers in island i. Note that b_i is an endogenous object. Case (i) occurs with probability $e^{-q_2(1-x_2)}$. (ii) a worker faces other applicants, in which case she receives her reservation wage $w_{2,2} = b_2$ (whether she ends up getting the job or not in the end). Case (ii) occurs with probability $1 - e^{-q_2(1-x_2)}$. These two cases define the distribution $F_{2,2}(w)$.

Consider now the case of an heterogenous island with two worker types, high-skill and low-skill. Since this will be, in equilibrium, the case of island 1, we denote this case with subscript 1. Denote $F_{2,1}(w)$ the distribution of wages of type-2 (high skill) workers searching in island 1 and $F_{1,1}(w)$ the distribution of wages of type-1 (low skills) workers searching in island 1.

To derive $F_{2,1}(w)$, there are three cases to consider. (i) a high-productivity worker is alone to negotiate with the firm. In that case, she gets the wage $w_{21} = \beta\varphi_{2,1} + (1-\beta)b_2$ with b_2 the reservation wage of workers in island 2. Case (i) occurs with probability $e^{-q_1}e^{-q_1 h_2 x_2}$. (ii) a high-productivity worker faces other applicants, but she is the only one of her kind (i.e., the only of type 2). In that case, she gets the wage $w_{2,1} = \beta\left(\varphi_{2,1} - (\varphi_1 - b_1)\frac{r+\lambda_{2,1}}{r+\lambda_1}\right) + (1-\beta)b_2$ with b_1 the reservation wage of worker type-1 in island 1.[39] Case (ii) occurs with probability $e^{-q_1 h_2 x_2}(1-e^{-q_1})$. (iii) a high-skill worker faces other applicants of the same type, in which case she receives her reservation wage $w_{2,1} = b_2$ (whether she ends up getting the job or not in the end). Case (ii) occurs with probability $1 - e^{-q_2 x_2 h_2}$. These three cases define the distribution $F_{2,1}(w)$.

We can derive a similar distribution $F_{1,1}(w)$ for type-1 workers.

Using (8) with the expression for $F_{2,2}(w)$ expressions for the reservations wages b_1 and b_2

$$\left\{\begin{array}{l} b_2 = \frac{b+\frac{\beta}{r+\lambda_{2,2}}e^{-q_2(1-x_2)}\varphi_{2,2}}{1+\frac{\beta}{r+\lambda_{2,2}}e^{-q_2(1-x_2)}} \\ b_1 = \frac{b+\frac{1}{r+\lambda_1}e^{-q_1}e^{-q_2 x_2 h_2}\beta\varphi_{1,1}}{1+\frac{\beta}{r+\lambda_1}e^{-q_1}e^{-q_2 x_2 h_2}} \end{array}\right. \tag{10}$$

which define the functions $b_1(q_1, x_2)$ and $b_2(q_2, x_2)$.

Using the fact that $W_{2,2}(w) - U_{2,2} = \frac{w_2 - b_2}{r+\lambda_{22}}$, the no-arbitrage conditions implies

$$rU_{2,1} = rU_{2,2}$$

with

$$\begin{aligned} rU_{2,2} &= \frac{\beta}{r+\lambda_{2,2}}\left[e^{-q_2(1-x_2)}\left(\varphi_{2,2} - b_2\right)\right] \\ rU_{2,1} &= \frac{\beta}{r+\lambda_{2,1}}\left[\begin{array}{c} e^{-q_1}e^{-q_1 h_2 x_2}\beta\left(\varphi_{2,1} - b_2\right) \\ +\left(\beta\left(\varphi_{2,1} - (\varphi_1 - b_1)\frac{r+\lambda_{2,1}}{r+\lambda_1}\right) + (1-\beta)b_2\right)e^{-q_1 h_2 x_2}(1-e^{-q_1}) \end{array}\right] \end{aligned}$$

which implicitly define a function $x_2(q_1, q_2)$ as in the static case.

The job creation condition can be obtained just as in the static case from the

[39] To see that, note that if the firm makes the offer (with probability β) the firm hires the only type-2 worker and pays her reservation wage b_2. When workers simulataneously make the offer (with probability $1-\beta$), the high-skill worker offers to work for a wage w satisfying $J_{2,1}(w) = J_1(w)$ so that the wage is as high as possible, making the firm indifferent between hiring the best worker and the second-best. Using that $J_{2,1}(w) = \frac{\varphi_{2,1} - w}{r+\lambda_{2,1}}$, we get $\frac{\varphi_{2,1} - w_{2,1}}{r+\lambda_{2,1}} = \frac{\varphi_1 - b_1}{r+\lambda_1}$ which gives $w_{2,1} = \varphi_{2,1} - (\varphi_1 - b_1)\frac{r+\lambda_{2,1}}{r+\lambda_1}$. Combining the two cases gives the negotiated wage.

free entry condition

$$V_i = 0, \quad i = 1, 2$$

with

$$\begin{cases} rV_1 = -c_1 + \int \max\left(J_1(\pi) - V_1, 0\right) dG_1(\pi) \\ rJ_{1,1}(\pi) = \pi + \lambda_1 \left(V_1 - J_{1,1}(w)\right) \\ rJ_{2,1}(\pi) = \pi + \lambda_{2,1} \left(V_1 - J_{2,1}(w)\right) \end{cases} \tag{11}$$

where the distributions of profits $G(\pi)$ can be defined by proceeding as previously and considering the different cases. For instance, in island 1, the distribution of profit is given by

$$\pi \underset{G}{\sim} \begin{cases} (1-\beta)\left(\varphi_{2,1} - b_2\right) \text{ with probability } q_1 x_2 h_2 e^{-q_2 x_2 h_2} e^{-q_1} \\ (1-\beta)\left(\varphi_{1,1} - b_1\right) \text{ with probability } q_1 e^{-q_2 x_2 h_2} e^{-q_1} \\ \varphi_{2,1} - b_2 \text{ with probability } 1 - e^{-x_2 h_2 q_2} - x_2 h_2 q_1 e^{-q_2 x_2 h_2} e^{-q_1} \\ \varphi_{1,1} - b_1 \text{ with probability } e^{-x_2 h_2 q_2}\left(1 - e^{-q_1} - q_1 e^{-q_1}\right) \\ (1-\beta)\left(\varphi_{2,1} - b_2\right) + \beta\left(\varphi_1 - b_1\right) \frac{r+\lambda_{2,1}}{r+\lambda_1} \text{ with probability } x_2 h_2 q_1 e^{-x_2 h_2 q_2}\left(1 - e^{-q_1}\right) \\ 0 \text{ otherwise.} \end{cases} \tag{12}$$

A.5 Calibration of the dynamic model

The model allows us to express $w_{1,1}$, $w_{2,1}$, $w_{2,2}$ as functions of the model parameters $\varphi_{1,1}$, $\varphi_{2,1}$, $\varphi_{2,2}$, as well as $q_{1,t}$ and $q_{2,t}$, $x_{2,t}$, and the ratio to high-skill to low-skill job seekers, $h_{2,t}$. Specifically, with a little bit of algebra, one can show that the wages satisfy

$$\begin{cases} w_{2,2,t} = b_{2,t} + \beta \frac{\varphi_{2,2} e^{-mq_{2,t}\left(1-x_{2,t}\right)}}{f\left(mq_{2,t}(1-x_{2,t})\right)} \\ w_{2,1,t} = b_{2,t} + \beta \frac{\left(\varphi_{2,1} - \varphi_{1,1} + \varphi_{1,1} e^{-mq_{1,t}}\right) e^{-mq_{1,t} h_{2,t} x_{2,t}}}{e^{-mq_{1,t} h_{2,t} x_{2,t}} f\left(mq_{1,t} h_{2,t} x_{2,t}\right)} \\ w_{1,1,t} = b_{1,t} + \beta \frac{\varphi_{1,1} e^{-mq_{1,t} h_{2,t} x_{2,t} - mq_{1,t}}}{e^{-mq_{1,t} h_{2,t} x_{2,t}} f\left(mq_{1,t}\right)} \end{cases}, \tag{13}$$

and that the job finding rates of type-2 and type-1 workers satisfy

$$\begin{cases} f_{2,t} = \left(1 - x_{2,t}\right) f_{2,2,t} + x_{2,t} f_{2,1,t} \\ f_{1,t} = f_{1,1,t} \end{cases} \tag{14}$$

with $f_{2,2,t}$, $f_{2,1,t}$, $f_{1,1,t}$ denoting respectively the job finding rate of type 2 workers searching in island 2, the job finding rate of type 2 workers in island 1,[40] and the

[40]Note that, while $f_{2,t}$ and $f_{1,t}$ are observed, we do not observe $f_{2,2,t}$ and $f_{2,1,t}$, since we do not know where high-skill workers are searching.

job finding rate of type 1 workers searching in island1, and given by

$$\begin{cases} f_{2,2,t} = mf(m(1-x_{2,t})q_{2,t}) \\ f_{2,1,t} = mf(mx_{2,t}h_{2,t}q_{1,t}) \\ f_{1,1,t} = me^{-mx_{2,t}h_{2,t}q_{1,t}}f(mq_{1,t}) \end{cases} \tag{15}$$

where the function $f(q)$ is[41]

$$f(q) = \frac{1-e^{-q}}{q}$$

and where m denotes the "matching efficiency" of the labor markets. Indeed, in order to bring an urn-ball matching function to the data (and match the observed average job finding rates to type-2 and type-1 workers), we proceed as Blanchard and Diamond (1994) and assume that workers send out an application with probability m, so that a job finding rate is given by $mf(mq)$.

Assuming that $q_{1,t}$, $q_{2,t}$, $x_{2,t}$ and $h_{2,t}$ are observables over 2006-2012, we can back out values for $\varphi_{1,1}$, $\varphi_{2,1}$ and $\varphi_{2,2}$ by taking expectations of (13) and using the average wages reported over the period.

We thus need data on $q_{1,t}$, $q_{2,t}$, $x_{2,t}$ and $h_{2,t}$. As described in the main text, we can measure $q_{1,t}$, $q_{2,t}$ and $h_{2,t}$, but not $x_{2,t}$. Instead, we measure the under-employment rate, $\hat{x}_{2,t}$, which is the fraction of high-skill workers employed in the low-tech island and is given by

$$\hat{x}_{2,t} = \frac{N_{2,1}(t)}{N_{2,1}(t) + N_{2,2}(t)} \tag{16}$$

where $N_{2,1}(t)$ is the number of type-2 workers employed in island 1 at time t, and $N_{2,2}(t)$ is the number of type-2 workers employed in island 2 at time t.

To link $x_{2,t}$ and $\hat{x}_{2,t}$, we can use a simple stock-flow model of under-employment. Specifically, we consider a continuous time environment in which data are available only at discrete dates. For $t \in \{0, 1, 2...\}$, we refer to the interval $[t, t+1[$ as 'period t'. We assume that during period t, the fraction of type-2 workers searching in island 1 is constant at $x_{2,t}$, the ratio of type-2 to type job sekers is constant at $h_{2,t}$ and that the job finding rates are constant at $f_{2,2,t}$, $f_{2,1,t}$ and $f_{1,1,t}$. The job separation rates of type-2 workers empleyed in island 2 and 1 are constant and respectively $\lambda_{2,2}$

[41] To obtain (15) and the expressions for the job finding rates, note that, in our setup, a worker always meets a firm, so that the job finding probability of a high-skill depends on the number of other applicants he is facing. Denoting q the ratio of high-type job seekers to vacancies, the number of hires of high-types is given by $V(1-e^{-q})$, and the probability that a high-type gets a job is $f(q) = \frac{1-e^{-q}}{q}$. We then obtain (15) by using that the number of high-type job seekers in island 2 is $(1-x_{2,t})L_{2,t}$ and that the number of high-type job seekers in island 1 is $x_{2,t}h_{2,t}L_{1,t}$. For low-skill workers, we get a similar expression, except that they must face no high-skill applicants, which happens with probability $e^{-mx_{2,t}h_{2,t}q_{1,t}}$.

and $\lambda_{2,1}$, and the job separation rate of type-1 workers is $\lambda_{1,1}$.

Over $\tau \in [0,1]$, the number of type 2 workers employed in island 2, $N_{2,2}(t+\tau)$, the number of type 2 workers employed in island 1, and $N_{2,1}(t+\tau)$, and the number of type 1 workers employed in island 1, $N_{1,1}(t+\tau)$, evolve according to

$$\begin{cases} \frac{dN_{2,2}(t+\tau)}{d\tau} = \left(1 - N_{2,2}(t+\tau) - N_{2,1}(t+\tau)\right) x_{2,t} f_{2,2,t} - \lambda_{2,2} N_{2,2}(t+\tau) \\ \frac{dN_{2,1}(t+\tau)}{d\tau} = \left(1 - N_{2,2}(t+\tau) - N_{2,1}(t+\tau)\right) (1 - x_{2,t}) f_{2,1,t} - \lambda_{2,1} N_{2,1}(t+\tau) \\ \frac{dN_{1,1}(t+\tau)}{d\tau} = \left(1 - N_{1,1}(t+\tau)\right) f_{1,1,t} - \lambda_{1,1} N_{1,1}(t+\tau) \end{cases} \tag{17}$$

with the job finding rates in each island given by (15).

With the observed under-employment rate $\hat{x}_{2,t}$ given by (16), we can combine (17) with expressions for the job finding rates (15), and solve the system of differential equation to express $\hat{x}_{2,t}$ as a function of $x_{2,t}$, $h_{2,t}$, $q_{2,t}$, $q_{1,t}$, $\hat{x}_{2,t-1}$ and the other constant parameters of the model. Inversing this relation, we can express $x_{2,t}$ as a function of $\hat{x}_{2,t}$ and the other observable variables of the model.

To calibrate the job separation rates $\lambda_{2,2}$, $\lambda_{2,1}$ and $\lambda_{1,1}$, we use the data reported in Table 1. In line with the literature, we calibrate unemployment benefits to equal 40 percent of the income earned in a given island by a worker of a given type, and where b_1 and b_2 are given by (9) and (8). Finally, we choose m to match the average level of $f_{1,t}$.

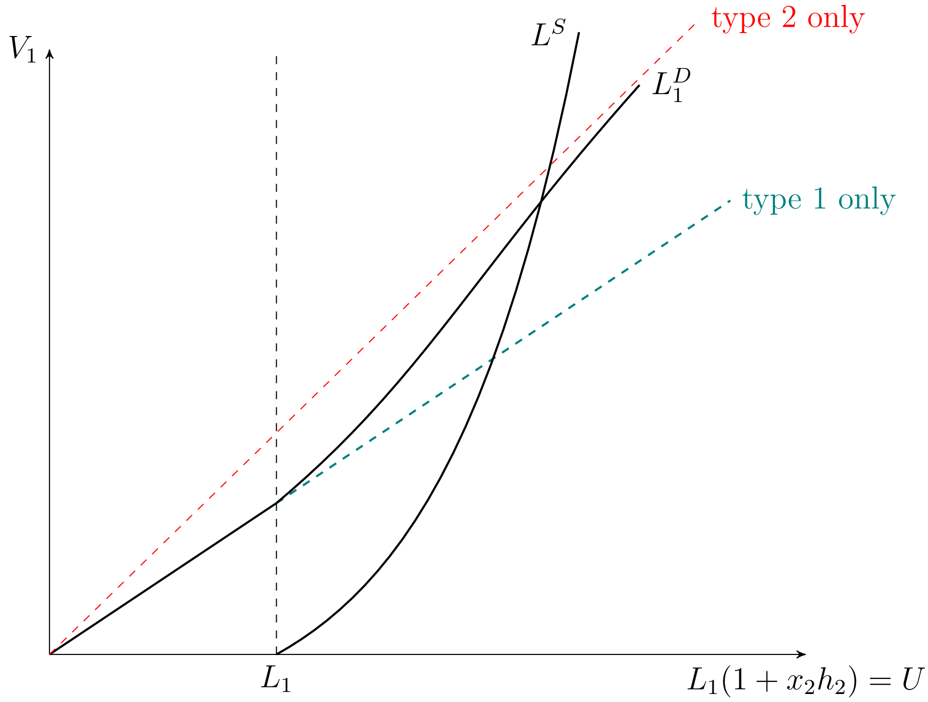


Figure 12. Labor market equilibrium – N=2.

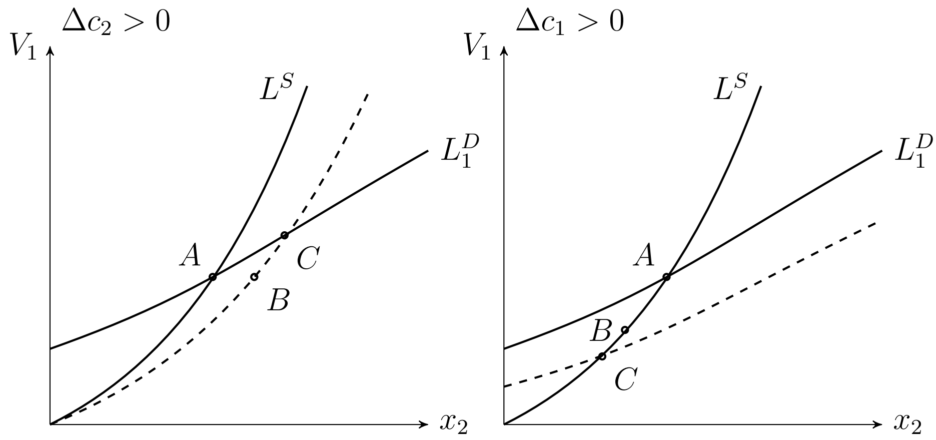


Figure 13. General Equilibrium – Comparative statics – N=2.