Self-Employment, Wage Employment and Informality in a Developing Economy

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Abstract

We construct a simple model incorporating a range of labor market phenomena that obtain in developing economies. The model is an adaptation of the Lazear (2005) 'jack-of-all-trades' formulation of entrepreneurship. Each individual chooses between self-employment, wage employment, and entrepreneurship (employing others). Each is endowed with two characteristics: the first affects the individual's productivity in self-employment, while both affect their productivity in entrepreneurship. Small firms are informal, paying a market-clearing wage; large firms are informal, paying a higher wage. We specify conditions under which voluntary and involuntary self-employment, and voluntary and involuntary informal wage employment coexist. Involuntary informal entrepreneurship may also obtain.

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1 Introduction

Recent empirical research indicates the diversity of employment relationships within urban labor markets in developing economies, which typically include significant segments of both voluntary and involuntary self-employment, and formal and informal wage employment. For example, World Bank (2007) finds that, on average, in Latin America and the Caribbean 24% of urban employment is informal self-employment, with substantial voluntary and involuntary elements, the former being the larger; and 30% is informal wage employment, also with substantial voluntary and involuntary elements, the latter being the larger. Similarly, in Côte d'Ivoire, Günther and Launov (2011), find that, defining informal work as informal wage employment plus self-employment, the voluntary:involuntary split is 55:45. However, the theoretical literature generally focuses on specific forms of employment relationship, e.g., modeling either self-employment or informal wage employment, but not both. In the present paper we attempt to develop this literature by constructing a simple model that covers each of the relationships referred to above.

The model is an adaptation of the Lazear (2005) 'jack-of-all-trades' formulation of entrepreneurship. We assume that each individual can allocate his or her labor to one of three activities: self-employment, wage employment, or entrepreneurship (running a firm and providing wage employment to others). An individual is characterized in terms of two skills, Y and Z, where, loosely speaking, Y is the ability to produce and sell an output, while Z is managerial ability. An individual's success as a self-employed worker

would depend on the amount y of skill Y he or she possesses; but success as an entrepreneur would depend on applying both skills together, specifically, on the value of $min(y, z)$, where z is the amount of Z he or she possesses.[1] An individual may alternatively take wage employment, in which everyone is assumed to be equally able.

We begin by formulating the supply function of an individual to the three activities. Because of the role of $min(y, z)$, the response to increases in y, with z held constant, can be non-monotonic: as y rises, first wage employment, then self-employment, then entrepreneurship, but then self-employment again is preferred. The model also generates a demand for wage labor: for those individuals who choose entrepreneurship, the demand for labor is an increasing function of $min(y, z)$. Given the joint distribution of y and z across individuals, and a flexible wage rate, we characterize the labor market equilibrium and examine its comparative statics.

We then assume that the wage rate is fixed above the market-clearing level. For simplicity, we first examine the case in which there is no informal wage employment, and then go on to the more complex case in which, as in Rauch (1991), the minimum wage rate only applies for firms above a certain size, with firms above this size being regarded as formal, and smaller firms as informal. We show that informal and formal wage employment can coexist with voluntary and involuntary self-employment. If self-employment

[1] Since we are concerned with relatively small firms, the productive and sales skills of the entrepreneur will generally matter for a firm's success. De Mel, McKenzie and Woodruff (2008) suggest that the case for a jack-of-all-trades characterization is stronger if the market for business services is thin, as typically obtains in developing economies. See Nichter and Goldmark (2009) on the characteristics of successful entrepreneurs in developing economies.

is chosen it is involuntary in the sense that it is not the individual's first choice (though it is voluntary in the sense that it is chosen freely from the remaining options).[2] We also find that involuntary entrepreneurship may obtain, i.e., individuals who are rationed out of formal wage employment may choose, as the second-best, to run a firm and employ others. We do not specify the rationing scheme that allocates to formal jobs a subset of the individuals who would like them. However, a potential inefficiency exists (in addition to the distortion caused by the minimum wage rate) in that individuals who would be relatively productive in self-employment may gain wage jobs in which their skills have no impact.

Following Rauch (1991), many papers in the informality literature develop models based on Lucas (1978), where individuals differ with respect to a single ability parameter. These include Fortin, Marceau and Savard (1997), Amaral and Quintin (2006), Galiani and Weinschelbaum (2006) and De Paula and Scheinkman (2008). However, these contributions do not allow for self-employment, treating informal wage labor as the only type of informal work. A recent paper by Gollin (2008) is an exception. He allows for only one ability parameter, but incorporates self-employment into the Lucas framework by assuming that an individual may split his or her time between running a firm (with no employees) and working as a wage employee in another firm. However, his focus is different to ours, solving numerically a dynamic equilibrium model of capital accumulation to examine the relationship between aggregate productivity, firm size and the extent of

[2] In this formulation, informal activity may be classified as encompassing involuntary self-employment, and possibly voluntary self-employment, as well as informal wage employment.

self-employment.

Informal self-employment is modeled by Loayza and Rigolini (2006) and Fiess, Fugazza and Maloney (2010) in order primarily to examine whether it behaves cyclically or counter-cyclically. Each assume that a Lucas-type ability parameter affects an individual's output in self-employment. Loayza and Rigolini treat individuals and firms interchangeably, so that each firm constitutes self-employment, but they assume that an individual can choose between formality, which has a fixed cost of compliance, and informality, with the affect of greater ability on output being greater if formality is chosen. Fiess et al. formulate a two-sector dynamic macro-model, with an informal nontradables sector in which there is self-employment, with output depending on ability, and a formal tradables sector in which there is wage employment.

These models of informality are generally based on the assumption of homogeneity, in the sense that either, because of segmentation, all informal work is involuntary (e.g., Rauch, 1991; Fortin et al.,1997) or, because the labor market is unsegmented, all informal work is involuntary (e.g., Amaral and Quintin, 2006; Loayza and Rigolini, 2006).[3] However, Fields (1990) classifies informality into upper and lower tiers, one segmented and one not, and the evidence cited by World Bank (2007) and others is consistent with this view. Although our model stresses heterogeneity of employment status, it does not correspond neatly to this two-tier distinction, as it includes

[3]A separate branch of the literature develops search-and-matching models of informality. In the formulation by Albrecht, Navarro and Vroman (2009), e.g., informality is equated with unregulated self-employment, and ability is assumed only to affect an individual's productivity if they are in a formal sector job. Informal sector employment is assumed to preclude search for a formal jobs. It is examined how workers respond to informal or formal job offers according to their ability and, using simulations, the effects of different tax policies are analyzed.

informal wage employment, involuntary self-employment (which is usually regarded as informal) and voluntary self-employment (which is sometimes regarded as informal).

World Bank (2007) also notes the attachment of significant value to the non-pecuniary benefits of independent work, including the desire for flexibility, and so we include a non-pecuniary benefit for both the self-employed and entrepreneurs in the model. An implication is that in equilibrium, depending on ability, some individuals could earn more in informal wage employment than they could had they been self-employed, while for others the reverse is true. This is consistent with the mixed empirical evidence on which of these types of earnings is the higher (see, e.g., Agénor, 2005).

Section 2 sets out the model of individual labor supply. Section 3 examines labor market equilibrium on the assumption that the wage is market-clearing. Section 4 introduce a binding minimum wage for formal employment and Section 5 concludes. Proofs are given in the Appendix.

2 The Labor Supply Model

Consider a large population of individuals, in which each person is characterized in terms of two skills, Y and Z. Y may be thought of as an individual's ability to produce and sell, and Z as his or her managerial skill. An individual's levels of Y and Z are written $y \in (0, \bar{y}]$ and $z \in (0, \bar{z}]$, respectively. The density function of skills across the population is $f(y, z)$. Throughout, for simplicity, we assume that $f(.)$ is continuous and positive for all y and z.

Any individual may have one of three possible occupations: wage employment, self-employment or entrepreneurship. We assume a self-employed

person does not employ any others - rather, any employment of others qualifies the person to be categorized as an entrepreneur.[4] Regardless of an individual's (y, z)-characteristics, he or she has the same ability to do wage work as any other person. However, for self-employment and entrepreneurship, ability matters. If a person with characteristics (y, z) is self-employed, he or she produces the quantity y; that is, for self-employment 'the ability to produce and sell' matters, but 'managerial skills' do not. If, alternatively, he or she is an entrepreneur, the relevant measure of skill is $\min(y, z) \equiv A$; that is, a balance of both types of skill matters. Such a person runs a firm for which the production function is

$$x = Al^{\alpha}, \quad \alpha \in (0, 1), \tag{1}$$

where x is output and l is the number of people the firm employs.

Let q and p be the prices for the output of the self-employed and entrepreneurial firms, respectively, and let w be the money wage rate. An entrepreneur's profit is therefore $px - wL$, which, given (1), is maximized at $l = \hat{l}(A)$, where

$$\hat{l}(A) = \left(\frac{Ap\alpha}{w}\right)^{\frac{1}{1-\alpha}}. \tag{2}$$

We assume that both self-employment and entrepreneurship give an individual a non-pecuniary benefit, v, which may be thought of as a benefit from independence. Thus, letting UW, US and UE denote the utility from

[4] Our assumption is consistent with the definition that Lazear (2005) gives of an entrepreneur as being conceptually distinct from a self-employed person. The World Bank (2007) finds that a large proportion of microenterprises have no paid employees, the figures, e.g., for Mexico and Brazil, being 80% and 87%, respectively

working, self-employment and entrepreneurship, respectively, we have

$$\begin{aligned} UW &= w; \\ US &= qy + v; \\ UE &= pA\hat{l}^{\alpha} - w\hat{l} + v. \end{aligned} \tag{3}$$

We shall only consider cases in which $w > v$, which is necessary for wage employment to exist in equilibrium.

If the two production sectors produce the same good or service, we may expect self-employment, much of which may be classified as informal, to produce a good of lower quality, or at least not of higher quality, than the entrepreneurial sector (see Banerji and Jain, 2007). If we regard prices q and p as quality-adjusted, we may therefore expect that $q \leq p$ (though our analysis still applies if this inequality does not hold).[5]

Consider an individual's preferences across the three occupations. He or she prefers wage work if $UW > \max(UE, US)$.[6] Using (2) and (3), we have

$$UW > UE \text{ if } A < \frac{1}{p}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{w-v}{1-\alpha}\right)^{1-\alpha} \equiv B(w); \tag{4}$$

$$UW > US \text{ if } y < \frac{1}{q}(w-v) \equiv C(w). \tag{5}$$

Note that $B(w) - C(w) \gtreqless 0$ as $q/p \gtreqless Q(w)$, where

$$\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{w-v}{w}\right)^{\alpha} \equiv Q(w). \tag{6}$$

[5] In practice self-employment covers a wide range of activity. Thus we may interpret self-employed production with a low y as relating to, e.g., construction work or street vending, while that with a high y might relate, e.g., to professional work. A similar comment applies to entrepreneurial output.

[6] Throughout, we simplify the exposition by only considering strong preference.

Since $\alpha^{\alpha}(1-\alpha)^{1-\alpha} \in (1/2, 1)$, $Q(w) \in (0, 1)$. Thus, if $q < p$, either $q/p > Q(w)$ or $q/p < Q(w)$, and this is reflected in the two cases distinguished below; but if $q \geq p$, $q/p > Q(w)$.

Self-employment is preferred if $US > \max(UW, UE)$. The converse of (5) is $y > C \Rightarrow US > UW$. Also, from (2) and (3),

$$\begin{aligned} UE &\gtrless US \text{ as } A \gtrless [D(w)]^{\alpha} y^{1-\alpha} \equiv \tilde{z}(y), \text{ where} \qquad (7) \\ D(w) &\equiv \frac{w}{p\alpha}\left(\frac{q}{p(1-\alpha)}\right)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

Using (4)-(7), we obtain our first proposition.

Proposition 1 *The first preference among the three options - wage employment, self-employment and entrepreneurship - of an individual with characteristics (z, y) is as follows: (i) for $q/p > Q(w)$, wage employment if $y < C(w)$; self-employment if either $y \in (C(w), D(w))$ or both $y > D(w)$ and $z < \tilde{z}(y)$; entrepreneurship otherwise; (ii) for $q/p < Q(w)$, wage employment if either $y < B$, or both $y \in (B(w), C(w))$ and $z < B(w)$; self-employment if $y > C(w)$ and $z < \tilde{z}(y)$; entrepreneurship otherwise.*

This is illustrated in Figures 1 and 2, where in Figure 1 $q/p > Q(w)$ and in Figure 2 $q/p < Q(w)$. For simplicity, it is assumed in these figures that $\bar{y} = \bar{z}$.[7] The values $C(w)$, $B(w)$ and $D(w)$ play significant roles here and, in discussing the figures, it will be useful to refer to the inequality derived as

[7]Without this symmetry it is simply necessary to trim the Figures 1(a) and 2(a) accordingly. For example if $\bar{y}$ were reduced by δ we would delete the area in each panel (a) between $\bar{y} - \delta$. When, below, we interpret these figures as representing labor market equilibrium for uniform distributions of the skills, such a trimming would affect the equilibrium value of w, so that the values of B, C and D change, but the appropriately redrawn figures would still have the same general characteristics as Figures 1(a) and 2(a).

an intermediate step in the proof:

$$q/p \lesseqgtr Q(w) \Leftrightarrow C(w) \gtreqless B(w) \gtreqless D(w). \tag{8}$$

Consider Figure 1(a), in which $q/p > Q(w)$. W, SE and E denote the respective sets of characteristics for which wage employment, self-employment and entrepreneurship are the first preference. For individuals with $y < C(w)$ self-employment and entrepreneurship both offer relatively low rewards and so wage employment is preferred. For $y > C(w)$, however, either self-employment or entrepreneurship is preferred. In this range of y, we might expect that, since Y and Z are perfect complements in production, the area on the diagram for which entrepreneurship is preferred would be a square at the north-east corner of the figure for $y > D(w)$, with self-employment occupying an L-shaped area around it to the south-west, with at least one of Y and Z taking lower values than in the entrepreneurship-square. This is not quite what obtains, however, for when $y > D(w)$, along the borderline on which an individual is indifferent between self-employment and entrepreneurship, which is defined by $z = \tilde{z}(y)$, we find that $dz/dy > 0$.[8]

[Figure 1 about here]

An implication of this result is that there is a range of z for which, as y is raised, first wage employment is preferred, then self-employment, and then

[8]When $y = D$, $UE = US$ on the 45°- line, and, from this point, $z = \tilde{z}(y)$ slopes up. Intuitively, this is because entrepreneurship exhibits increasing returns to A, whereas self-employment income exhibits constant returns to y. Starting at the south-west corner of set E, an equal positive increment to y and z has a greater effect on UE than it does on US, while the same increment to y alone has a positive effect on US, but no effect on UE. Therefore, points on the 45°-line belong to E while points due east belong to S. The border, $z = \tilde{z}(y)$, between the two sets must therefore slope up. The difference in returns is also the reason that $\tilde{z}''(y) < 0$.

entrepreneurship, but finally there is a switch back to self-employment. This is illustrated in panel (b), which plots the individual's utility for a given value of z, $z = z_1$. The switch from self-employment to entrepreneurship occurs at $y = z_1$, i.e., when y is high enough to create the relatively balanced skill-set that the jack-of-all-trades property of entrepreneurship requires. But if y is sufficiently great ($y > \tilde{y}(z_1)$, where $\tilde{y}(z_1)$ is the inverse of $\tilde{z}$) the payoff from using only this skill - in self-employment - dominates.

Figure 2 illustrates the corresponding results for $q/p < Q(w)$. The pattern of preferences shown in Figure 2(a) differs from those in 1(a) in two significant respects, both of which are the result of the relatively low value of q/p, i.e., of self-employment being relatively unattractive compared to entrepreneurship, in 2(a) for any given w. First, for $z > B(w)$, as y is reduced the individual's preference shifts from entrepreneurship straight to wage employment, without an intermediate preference for self-employment. Second, for $y \in (B(w), C(w))$ a similar direct shift of preferences is found as z is reduced. Panel (b) plots the corresponding levels of utility for $z = z_2$.

[Figure 2 about here]

3 Labor Market Equilibrium

Aggregating over all (y, z), we obtain the supplies of labor to the three activities expressed as proportions of the whole population. We denote the total supplies to wage employment, self-employment and entrepreneurship by L^s, SE^s and E^s, respectively. For each entrepreneur the demand for labor is given by $\hat{l}\,(A)$ in (2) and thus we obtain the total demand for labor, L^d (see

the Appendix for details).

Lemma 1 *The comparative statics of the supply and demand for wage labor are as follows:*

$$L_p^d > 0;\ L_q^d < 0;\ L_w^d < 0;\ L_q^s < 0;\ L_w^s > 0;\ L_v^s < 0;$$
$$L_v^d \begin{cases} = 0 & for\ q/p > Q(w) \\ > 0 & for\ q/p < Q(w) \end{cases};\ L_p^s \begin{cases} = 0 & for\ q/p > Q(w) \\ < 0 & for\ q/p < Q(w) \end{cases}.$$

The demand for wage labor is increasing in the price of the firms' output and decreasing in the money wage. It is decreasing in the price paid for the output of the self-employed because a higher price for this output makes entrepreneurship relatively less attractive. If the benefit from independence, v, is greater, then entrepreneurship (as well as self-employment) is more attractive relative to wage employment. But it is only if there are individuals on the margin of choice between entrepreneurship and wage employment (i.e., if $q/p < Q(w)$) that this is associated with more individuals choosing to be entrepreneurs.

The supply of wage labor is increasing in the money wage rate, and decreasing in the price of the self-employed output and the benefit from independence. If the output price p is higher then, again, provided there are individuals on the margin of choice between entrepreneurship and wage employment (i.e., if $q/p < Q(w)$), wage employment becomes less attractive relative to entrepreneurship for these individuals, and so the supply of wage labor is lower.

For equilibrium in the labor market, $L^s = L^d \equiv L$. Since $L_w^d < 0$ and $L_w^s > 0$, a unique equilibrium exists with positive employment if the L^s

and L^d curves cross. Note that, depending on whether the market-clearing wage rate w^* is such that $q/p > Q(w^*)$ or $q/p < Q(w^*)$, Figure 1(a) or 2(a), respectively, can be interpreted as representing this equilibrium. With $w = w^*$, a labor market equilibrium obtains with the set of entrepreneurs indicated by E in the figure generating an aggregate demand for wage labor that equals the supply of wage labor indicated by set W.

The next proposition gives the comparative statics of the labor market equilibrium, with w adjusting endogenously.

Proposition 2 *In equilibrium ($w = w^*$), $dw/dp > 0$, $dw/dq \gtrless 0$ and $dw/dv > 0$; and total wage employment L satisfies*

$$\begin{aligned} \frac{dL}{dp} &> 0 \begin{cases} \text{if } q/p > Q(w), \\ \text{if } q/p < Q(w) \text{ and } L_p^d L_w^s - L_w^d L_p^s > 0; \end{cases} \\ dL/dq &< 0; \\ \frac{dL}{dv} &= \begin{cases} < 0 & \text{if } q/p > Q(w), \\ \gtrless 0 & \text{if } q/p < Q(w). \end{cases} \end{aligned}$$

If the price p of the entrepreneurial output is higher then the demand for wage labor is greater, as is the equilibrium wage. Wage employment is therefore greater, subject, in the case of $q/p < Q(w)$, to a stability condition. If the price q of the output of the self-employed is higher, the relatively greater attractiveness of self-employment is associated with both the supply of and the demand for wage labor being smaller, the latter effect arising because the supply of entrepreneurship is smaller. Thus, wage employment is smaller, but the net effect on w^* may be of either sign. A greater desire for independence v implies a greater attractiveness of both self-employment (with a negative effect on the supply of wage labor) and entrepreneurship (with a positive

effect on the demand for wage labor). The latter effect implies a greater demand for wage labor, but as the supply of wage labor is smaller we cannot sign the effect on L, except for when $q/p > Q(w)$.

4 A Minimum Wage

We now examine the equilibrium that obtains when the wage rate w is fixed, e.g. by law, at $\bar{w}$, above the market-clearing level w^*. We first suppose that all entrepreneurial firms must pay $w = \bar{w}$. Then we amend the analysis, assuming instead that only firms above a threshold employment level must pay $\bar{w}$, which, following Rauch (1991), allows us to distinguish between formal and informal wage employment. We leave consideration of comparative statics until we examine the latter case.

4.1 All Firms Pay the Minimum Wage

With all entrepreneurial firms paying $w = \bar{w}$, the effect can be seen intuitively from Figures 1(a) and 2(a). From (6), $Q'(w) > 0$, and so $Q(\bar{w}) > Q(w^*)$. Assume first that $q/p > Q(\bar{w}) > Q(w^*)$, and interpret Figure 1(a) as showing the labor market equilibrium, with $w = w^*$. Consider the effect if, instead, $w = \bar{w}$. From (5), the higher wage rate would be associated with point $C(w)$ being further to the right, the supply of wage labor being greater. But, from (7), point $D(w)$ would also be further to the right, with some individuals on the margin of choice between entrepreneurship and self-employment now choosing self-employment (whereas with $w = w^*$ they would choose entrepreneurship). Thus, the demand for wage labor is lower than when $w = w^*$, some individuals being rationed out of wage employment. For

them, self-employment is the only alternative; i.e., we now have involuntarily self-employed as well as voluntarily self-employed individuals in the model.

If, instead $q/p < Q(w^*) < Q(\bar{w})$, as in Figure 2(a), a similar explanation applies, except in one significant respect, which is the result of q/p, the self-employed price relative to the entrepreneurial price, being relatively low here.[9] Whereas in Figure 1(a) the interior boundary of set E is shared entirely with set S, in Figure 2(a) part of this boundary is shared with set W, so that some (y, z)-bundles are near to or at the margin of choice between entrepreneurship and self-employment. Individuals endowed with these bundles choose wage employment when $w = w^*$; but if, with $w = \bar{w}$, they are rationed out of wage employment, they choose to be entrepreneurs, employing others, rather than being self-employed. This outcome obtains in Figure 2(a), but not Figure 1(a), because here q/p is relatively low. Thus, as well as involuntary self-employment, there is involuntary entrepreneurship in the model, the former (latter) being the outcome for those with a relatively high (low) endowment of y relative to z. These conclusions are summarized in the next proposition.

Proposition 3 *If there is a binding minimum wage rate $w = \bar{w} > w^*$ on all wage employment then involuntary self-employment may exist simultaneously with voluntary self-employment. If also $q/p < Q(\bar{w})$, involuntary entrepreneurship may exist simultaneously with voluntary entrepreneurship.*

In Figure 1(a), with $w = w^*$, the set of individuals indicated by W is in wage employment; but when $w = \bar{w}$ a subset of W fails to obtain wage em-

[9] One other case is feasible: it may be that $Q(\bar{w}) > q/p > Q(w^*)$, Figure 1(a) applying for $w = w^*$ but Figure 2(a) for $w = \bar{w}$.

ployment. The composition of this subset depends on the 'rationing scheme,' which is not determined in the model. Pareto efficiency, subject to the constraint $w = \bar{w}$, would require that the individuals from W with relatively low ability y obtain wage employment, while those with relatively high y be self-employed. Insofar as this rationing scheme does not obtain, the total output of the self-employed sector is lower than it would otherwise be. A similar argument applies in Figure 2(a), and it also applies to the identity of those who are involuntary entrepreneurs. Since $\hat{l}'(A) > 0$, an inefficient rationing scheme in this case also holds back the aggregate amounts of employment and output in entrepreneurial firms.

4.2 Informal Wage Employment

Now suppose that only firms above a certain threshold employment level, $l = l_0$, pay the minimum wage $\bar{w}$, whereas firms with $l \leq l_0$ pay the market-clearing wage $w = w_i$. Denote the former firms 'formal' and the latter 'informal'. In Rauch's model (in which skill is one-dimensional) there is a critical entrepreneurial skill level above which formality is chosen, with informality being chosen otherwise. In our model there is a critical level of A, $A = \tilde{A}$, that plays a similar role. This is the level of A at which the entrepreneur achieves the same utility from operating informally at the maximum employment level l_0 as from operating formally at the higher, profit-maximizing employment level $\hat{l}(A)$; i.e.,

$$UE(\tilde{A}, w_i, l_0) = UE(\tilde{A}, \bar{w}, \hat{l}(\tilde{A})). \tag{9}$$

Of individuals choosing entrepreneurship, those with $A > \tilde{A}$ choose formality. As in Rauch's model there is a gap in the size-distribution of firms at $A = \tilde{A}$.[10]

With this amendment to the model, individuals who allocate their labor according to their first choice, i.e., their allocation is voluntary, can be divided into four sets. These are[11]

1. formal entrepreneurs (employing $l \geq l_0$ at wage $\bar{w}$), denoted $E(f)$;
2. voluntary informal entrepreneurs (employing $l < l_0$ at wage w_i), denoted $E_V(i)$;
3. voluntary self-employed (those among the self-employed for whom $US > UW(\bar{w})$), denoted S_V;
4. formal employees (employed by formal firms at wage $\bar{w}$), denoted $W(f)$.

For the remaining individuals, formal employment, with wage $\bar{w}$, is the first preference; but formal employment is not available. They therefore allocate their labor to other activities involuntarily. Each belongs to one of the following sets:[12]

[10] We assume that $\tilde{A} \leq \min(\bar{y}, \bar{z})$, i.e., that $(\bar{w}, l_0)$ are small enough for some individuals to possess the ability set required to choose formal entrepreneurship. The proof of the properties mentioned in the text is similar to that given by Rauch.

[11] Our notation for these sets is to write in parentheses f for formal and i for informal, and then to add a subscript V for voluntary and I for involuntary if a further distinction is necessary.

[12] There are no (y, z)-combinations for which both (i) formal employment is the first preference and (ii) formal entrepreneurship the second preference; i.e., involuntary formal entrepreneurship is not feasible. Individuals who choose entrepreneurship are in the highest A-range in the population, and, amongst these, individuals with $A > (\leq)\ \tilde{A}$ choose formality (informality). Thus, someone may be on the margin of choice between formal employment and informal entrepreneurship, slightly preferring the former, but because of formal employment rationing, engaging in the latter; but they cannot be on the margin of choice between formal employment and formal entrepreneurship.

1. involuntary informal entrepreneurs (employing $l < l_0$ at wage w_i), denoted $E_I(i)$;

2. involuntary self-employed (those among the self-employed for whom $UW(\bar{w}) > US$), denoted S_I;

3. informal employees (employed by informal firms at wage w_i), denoted $W(i)$.

With this more complex model, parallel to Proposition 3, we have the following.[13]

Proposition 4 *Suppose firms may be formal, with $l \geq l_0$ and paying wage $\bar{w}$, where $\bar{w} > w^*$, or informal, with $l < l_0$ and paying the market clearing wage w_i. Then informal wage employment, formal wage employment, voluntary self-employment and involuntary self-employment may all coexist in equilibrium. If $Q(\bar{w}) > \max[Q(w_i), q/p]$ involuntary informal entrepreneurship may also exist (along with voluntary formal and informal entrepreneurship).*

This is illustrated in Figure 3, which is a development of Figures 1(a) and 2(a), and can be interpreted as representing the equilibrium with endogenous adjustment of w_i. Three cases are distinguished: with $q/p > Q(\bar{w}) > Q(w_i)$ we have panel (i), which corresponds to the case shown in Figure 1(a); with $Q(\bar{w}) > q/p > Q(w_i)$ we have panel (ii), which is essentially a hybrid of the Figure 1(a)- and Figure 2(a)-cases; and with $Q(\bar{w}) > Q(w_i) > q/p$ we have panel (iii), which corresponds to Figure 2(a). Each of the panels can be explained in three steps.[14]

[13] The proof of this proposition is essentially a repeated application, first for $E(f)$, $E_V(i)$ and S_V, and then for the remaining sets, of the reasoning applied for Proposition 1.

[Figure 3 about here]

First, using equations (1)-(7) with $w = \bar{w}$ and (9), we determine the (y, z)-characteristics of the members of sets 1-3 from the list of voluntary allocations, i.e., of $E(f)$, $E_V(i)$ and S_V. These are shown unshaded in each panel.

Second, because the rationing scheme has not been specified, we note that membership of the $W(f)$ may come from anywhere in the shaded area in each panel. (The shaded area is the complement of $E(f) \cup E_V(i) \cup S_V$).

Third, disregarding temporarily the allocation of individuals to set $W(f)$, we treat the shaded area in the same way as we did the whole of (y, z)-space in Figures 1(a) and 2(a). Thus, for the individuals concerned, we show the choices made between the three options of entrepreneurship, self-employment and wage employment, given that all three options are involuntary in the sense that these individuals would have preferred to have formal wage employment, and that the wage rate available is w_i. Thus we determine the sets $E_I(i)$, S_I and $W(i)$, with the proviso that a selection of individuals with (y, z)-characteristics consonant with these sets, belong instead to set $W(f)$.

For a given (y, z)-distribution, we assume that w_i adjusts endogenously such that the supply of informal wage labour (from the set $W(i)$) equals the demand for informal wage labour (from the set $E_V(i) \cup E_I(i)$). The other allocations are determined simultaneously. The diagrams can then be interpreted as representing the equilibrium for three different cases.

It can be seen that relatively highly-skilled individuals with a balanced

[14] If $\bar{w}$ is not significantly above w^* the horizontal boundary of the set $E(f)$ will meet the upward-sloping boundary of set $E_V(i)$ and terminate there.

skill set become formal entrepreneurs, while those not quite so highly skilled and/or with not quite so balanced skill sets become voluntary informal entrepreneurs. Individuals with a high y, but sufficiently low z, become voluntarily self-employed. However, just as $\tilde{z}'(y) > 0$ in the model of Section 3 (see the explanation in note 8), in all three panels the sets $E_V(i)$ and S_V share an upward-sloping border. Consequently, a voluntarily self-employed person may have more of both skills than a voluntary informal entrepreneur - even though in self-employment it is only skill y that matters.[15]

In panel (i), the return to self-employment is relatively high ($q/p > Q(\bar{w}) > Q(w_i)$). As a result, there is no involuntary informal entrepreneurship, involuntary self-employment being preferred instead. However, the return to self-employment is not so high in panels (ii) and (iii) and so some involuntary entrepreneurship obtains, with the individuals concerned having lower values of $A = \min(y, z)$ than voluntary entrepreneurs. Roughly speaking, involuntary informal entrepreneurs have high values of z, but intermediate values of y, although, paralleling the result already noted for the corresponding voluntary sets $E_V(i)$ and S_V, an involuntarily self-employed person may have more of both skills than an involuntary informal entrepreneur.

As noted in Section 4.1 for the model without informal wage employment, the rationing scheme for formal wage employment may create an (additional)

[15] An implication is that, as with the simpler model of Figure 1, we have some non-monotonicity of choice when y is raised with z held constant. In panel (i) there can be a change from voluntary self-employment to involuntary informal entrepreneurship and back again, while in panel (ii) a similar result holds, but with the self-employment being involuntary. Note also that if $\bar{w}$ is not significantly above w^*, as mentioned in note 14, then a member of set S_V could also have more of both skills than a member of set $E(f)$. This too would result in a non-monotonicity.

inefficiency. In the current formulation, unless the formal wage employees are those with the smallest y endowments in the shaded area, some output by the involuntarily self-employed is forgone. Also, in panels (ii) and (iii), insofar as some individuals from the shaded area associated with $E_I(i)$ gain formal employment, there is a negative effect on the demand for informal wage labour and the supply of informal output. There is a negative effect on the informal wage rate w_i, and also on formal wage employment and output.

5 Conclusion

We have developed a simple model of the urban labor market that encompasses many of the features observed in practice. In particular, we have specified conditions under which voluntary and involuntary self-employment, and voluntary and involuntary informal wage employment coexist. We have also found conditions under which there is involuntary informal entrepreneurship, where entrepreneurship is defined as providing wage employment to others. We intend to use this model to develop simulations of the evolution of these categories as the stock of skills improves and to relate the numerical results to the empirical evidence. This would also allow us to explore how the allocation of labor is related to the distribution of skills, including the correlation between the different skills.

Among the factors missing from the model are free labor provided by the family, and wealth and liquidity constraints that may hold back both self-employment and entrepreneurship. Also, it would be interesting to develop the model to include risk, in particular so that diversification of family labor in different activities could be modeled. Finally, we might separate what we

have called 'the ability to produce and sell' into two skills, with the ability to produce then affecting an individual's productivity in wage employment.

Appendix

Labor Market Equilibrium

$$L^s = \begin{cases} \int_0^C \int_0^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_0^B \int_0^{\bar{z}} f(y,z)dzdy + \int_B^C \int_0^B f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

$$SE^s = \begin{cases} \int_D^{\bar{y}} \int_0^{\tilde{z}(y)} f(y,z)dzdy + \int_C^D \int_0^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_C^{\bar{y}} \int_0^{\tilde{z}(y)} f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

$$E^s = \begin{cases} \int_D^{\bar{y}} \int_{\tilde{z}(y)}^{\bar{z}} f(y,z)dzdy & \text{for } q/p > Q(w); \\ \int_B^C \int_B^{\bar{z}} f(y,z)dzdy + \int_C^{\bar{y}} \int_{\tilde{z}(y)}^{\bar{z}} f(y,z)dzdy & \text{for } q/p < Q(w). \end{cases}$$

Inserting $\hat{l}(A)$ into each double integral in E^s we obtain labor demand, L^d. Using $A \equiv \min(y,z)$, this can be written

$$L^d = \begin{cases} \int_D^{\bar{y}} \int_{\tilde{z}(y)}^{y} \hat{l}(z) f(y,z)dzdy + \int_D^{\bar{y}} \int_y^{\bar{z}} \hat{l}(y) f(y,z)dzdy & q/p > Q(w) \\ \int_B^C \int_B^y \hat{l}(z) f(y,z)dzdy + \int_B^C \int_y^{\bar{z}} \hat{l}(y) f(y,z)dzdy & \\ + \int_C^{\bar{y}} \int_{\tilde{z}(y)}^{y} \hat{l}(z) f(y,z)dzdy + \int_C^{\bar{y}} \int_y^{\bar{z}} \hat{l}(y) f(y,z)dzdy & q/p < Q(w) \end{cases}$$

From Lemma 1, $L_w^d < 0$ and $L_w^s > 0$. Let $w = w^*$ be the equilibrium wage and consider the two cases, $q/p > Q(w^*)$ and $q/p < Q(w^*)$ separately. Given the continuity of $L^s(w)$ and $L^d(w)$, since $L_s(0) = 0$, labor market equilibrium with $w = w^*$ exists provided $L^d(0) > 0$, which we assume.

Proof of Proposition 1 Consider first the conditions under which wage employment is preferred. If $q/p > Q(w)$ then $C < B$. Since $A \leq y$, we have that $y < C(w) \Rightarrow A \leq y < C(w) < B(w)$; i.e., (5) is sufficient for (4) to be satisfied. If $q/p < Q(w)$ then $B(w) < C(w)$. To satisfy (4) and (5), we need either either $y < B(w)$ or $y \in (B(w), C(w))$ and $z < B(w)$ (since, $A \leq z$, so that $z < B(w)$ is sufficient for $A < B(w)$).

Now consider the conditions under which self-employment is preferred. We have seen that $y > C(w) \Rightarrow US > UW$, so now consider what is required for $US > UE$. First, suppose $A = z$; then, from (7), $UE > US$ if $A > [D(w)]^{\alpha} y^{1-\alpha} \equiv \tilde{z}(y)$. For this to be consistent with $A = z$ we require $y \geq \tilde{z}(y)$. Note that, for $y > 0$, $\tilde{z}(y)$ has a unique fixed point, $\tilde{z}(D(w)) = D(w)$, and that $\tilde{z}'(y) = (1-\alpha)[D(w)]^{\alpha} y^{-\alpha} > 0$, so that $\tilde{z}'(D(w)) = 1 - \alpha < 1$ Since also $\tilde{z}''(y) = -\alpha(1-\alpha)[D(w)]^{\alpha} y^{-\alpha-1} < 0$, this implies that $y \gtreqless D(w) \Leftrightarrow y \gtreqless \tilde{z}(y)$. Hence, if $y > D(w)$, we have $US > UE \Leftrightarrow z < \tilde{z}(y)$. Alternatively, suppose $A = y$. Then, from (7) $y < D(w) \Rightarrow US > UE$.

Hence $US > UE$ if either (i) $y < D(w)$ or (ii) $y > D(w)$ and $z < \tilde{z}(y)$. Therefore $US > \max(UW, UE)$ when either (i) $y \in (\min(C(w), D(w)), D(w))$ or (ii) $y > \max(C(w), D(w))$ and $z < \tilde{z}(y)$. But also, from (4), (6) and (7) we have that $B(w) - D(w) \gtrless 0 \Leftrightarrow q/p \lesseqgtr Q(w)$. Therefore, since $B(w) - C(w) \gtreqless 0$ as $q/p \gtreqless Q(w)$, we have $q/p \gtreqless Q(w) \Leftrightarrow C(w) \gtrless B(w) \gtrless D(w)$, and the conditions stated in the proposition under which self-employment is preferred follow.

The conditions under which entrepreneurship is preferred then follow.

Proof of Lemma 1 First we find from (2), (4), (5) and (7) that $\hat{l}_p > 0$; $\hat{l}_q = 0$; $\hat{l}_w < 0$; $\hat{l}_v = 0$; $B_p(w) < 0$; $B_q(w) = 0$; $B_w(w) > 0$; $B_v(w) < 0$; $C_p(w) = 0$; $C_q(w) < 0$; $C_w(w) > 0$; $C_v(w) < 0$; $D_p(w) < 0$; $D_q(w) > 0$; $D_w(w) > 0$; $D_v(w) = 0$; $\tilde{z}_p < 0$; $\tilde{z}_q > 0$; $\tilde{z}_w > 0$; $\tilde{z}_v = 0$. Differentiating L^s and L^d by (p, q, w, v) and using these inequalities, the proposition is obtained.

Proof of Proposition 2 Writing labor supply and supply as $L^s(w, i)$ and $L^d(w, i)$, respectively, where $i = (p, q, v)$, when $w = w^*$, $dw/di = (L^d_i - L^s_i)/(L^s_w - L^d_w)$. Using Lemma 1 with this equation yields $dw/dp > 0$, $dw/dq \gtrless 0$ and $dw/dv > 0$. Thus, (i) $dL/dp = L^d_w(dw/dp) + L^d_p = (L^d_p L^s_w - L^d_w L^s_p)/(L^s_w - L^d_w)$; from Lemma 1, $L^d_w - L^s_w < 0$ and if $q/p > Q(w)$, $L^s_p = 0$ and the result for dL/dp follows; (ii) $dL/dq = (L^d_q L^s_w - L^d_w L^s_q)/(L^s_w - L^d_w) < 0$; (iii) $dL/dv = (L^d_v L^s_w - L^d_w L^s_v)/(L^s_w - L^d_w)$ and the result in the proposition follows.

Proof of Proposition 3 First suppose $q/p > Q(\bar{w})$. To show that voluntary and involuntary self-employment may coexist we first look for the set of involuntarily self-employed. As they are involuntary, $UW(\bar{w}) > \max(UE(\bar{w}), US)$, which, from Proposition 1, implies that $y < C(\bar{w})$. Those individuals with $y < C(\bar{w})$ who are rationed out of a job can choose only between (involuntary) self-employment and entrepreneurship. Again from Proposition 1, we have $y < D(\bar{w}) \Rightarrow US > UE(\bar{w})$. But then, since $C(\bar{w}) < D(\bar{w})$, we have that $y < C(\bar{w}) \Rightarrow y < D(\bar{w})$. Therefore, all individuals with $y < C(\bar{w})$ that cannot find wage work are involuntarily self-employed. From Proposition 1 there exists a further set of agents with either $y \in (C(\bar{w}), D(\bar{w}))$ or both $y > D(\bar{w})$ and $z < \tilde{z}(y)$, and they become voluntarily self-employed.

Now suppose $q/p < Q(\bar{w})$. To show that voluntary and involuntary self entrepreneurship may coexist we first look for the set of involuntary entrepreneurs. As they are involuntary, $UW(\bar{w}) > \max(UE(\bar{w}), US)$, which, from Proposition 1, implies that either $y < B(\bar{w})$ or both $y \in (B(\bar{w}), C(\bar{w}))$. Individuals satisfying these conditions who are rationed out of a job can choose only between (involuntary) self employment or entrepreneurship. Again from Proposition 1, we have that $US > UE(\bar{w})$ for $y > C(\bar{w})$ and $z < \tilde{z}(y)$. Then there exists a set of individuals with $\min(y, z) < B$, $y \in (D(\bar{w}), C(\bar{w}))$ and $z > \tilde{z}(y)$ for whom $UW(\bar{w}) > \max(UE(\bar{w}), US)$ and $UE(\bar{w}) > US$. From Proposition 1 there exists a further set of agents with $y > B(\bar{w})$ and $z > \tilde{z}(y)$ who are voluntarily entrepreneurs.

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Figures

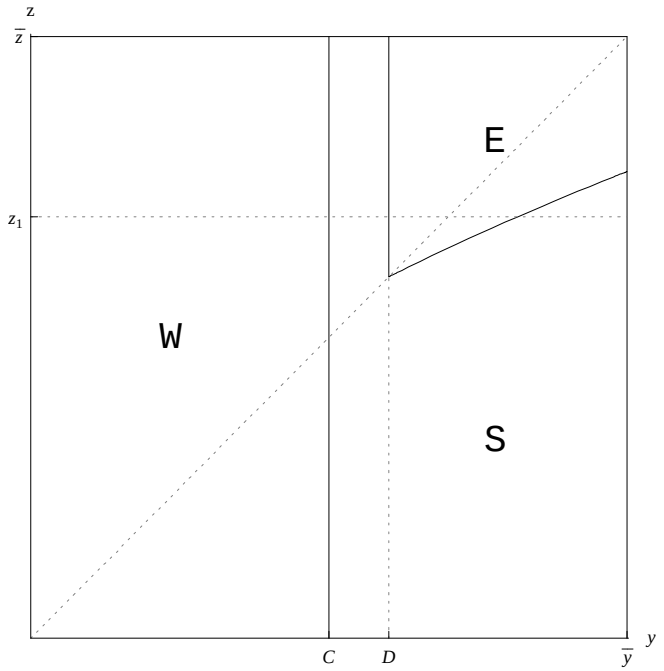

Figure 1(a): Labor allocation for $q/p > Q(w)$

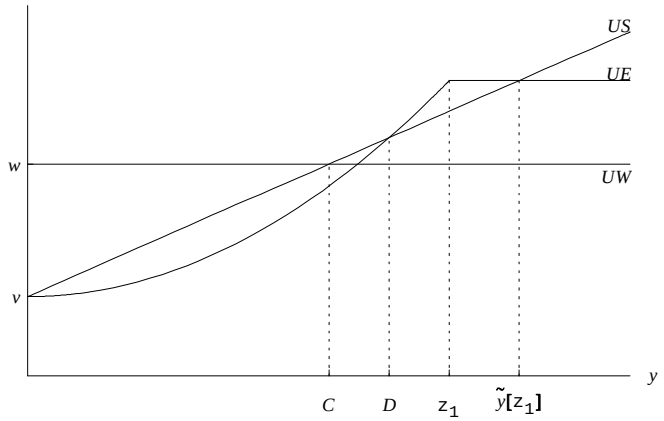

Figure 1(b): Utility for $q/p > Q(w)$

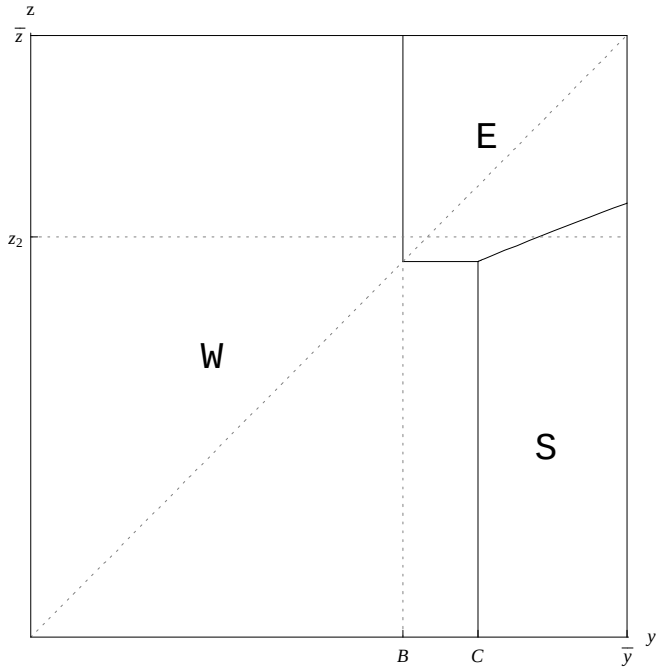

Figure 2(a): Labor allocation for $q/p < Q(w)$

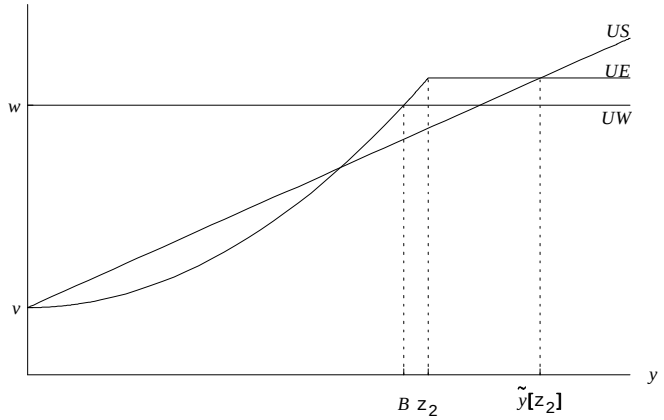

Figure 2(b): Utility for $q/p < Q(w)$

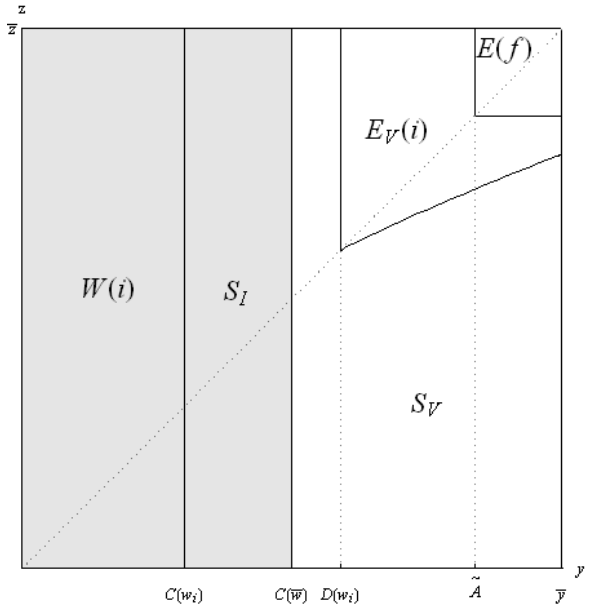

Figure 3(i): Labor allocation for $q/p > Q(\bar{w}) > Q(w_i)$

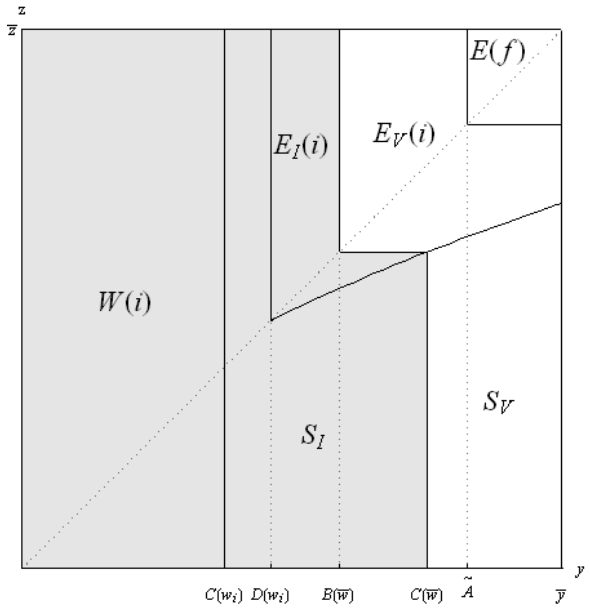

Figure 3(ii): Labor allocation for $Q(\bar{w}) > q/p > Q(w_i)$

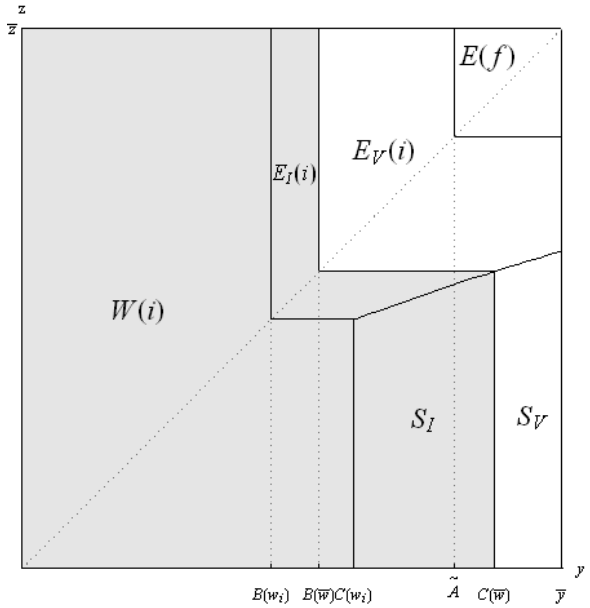

Figure 3 (iii): Labor allocation for $Q(\bar{w}) > Q(w_i) > q/p$