Spousal Bargaining Power:
Decoupling Gender Norms and Earning Status

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Abstract

The collective model provides a spousal bargaining framework of household labor supply, but most empirical studies assume that husbands bargain with wives, leaving unclear the extent to which gender norms surrounding labor supply interact with empirical estimates. I estimate collective labor supply models for different-sex and same-sex married couples in order to quantify the role of gender norms in spousal bargaining as distinct from that of earning status within the couple. My findings suggest that gender norms do significantly decrease wives' bargaining power within the couple, with a much larger negative effect on working wives with non-working husbands. My results imply that policies aimed at reducing institutional inequalities, such as the gender wage gap, may be particularly effective among different-sex couples in which the husband works but may not be as effective for other couples.

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1 Introduction

The collective labor supply model introduced by Chiappori (1988, 1992) provides a spousal bargaining framework of household labor supply that requires the researcher to divide couples along one dimension so that one spouse can bargain with the other. Most empirical studies divide different-sex couples by sex (e.g., Chiappori, Fortin, and Lacroix 2002; Moreau and Donni 2002; Vermeulen 2006; Blundell et al. 2007; Donni and Moreau 2007; Cherchye, De Rock, and Vermeulen 2012; Gayle and Shephard 2019). However, institutional factors in the labor market, such as the gender wage gap and traditional gender norms surrounding labor supply, suggest that husbands are more likely to be primary earners in their households, meaning that husbands' estimated bargaining power may reflect both gender norms and earning status in the household. It is, therefore, unclear to what extent gender norms influence bargaining power within couples separately from each spouse's earning status in the household. For example, Bartels and Shupe (2018) conclude that earning status in the household, rather than sex, is a more influential driver of responses to work incentives, and Baldwin, Allgrunn, and Ring (2011) suggest that the traditional male-female division in household labor supply has become less useful over time.

Disentangling the role of gender norms in spousal bargaining power and labor supply can not only illuminate the extent to which gender inequality drives intra-household inequality and economic outcomes, but can also help inform policies aimed at decreasing inequality and inform expectations about labor supply elasticities. For example, if observed differences in men's and women's labor supply are entirely attributable to gender norms, then policies aimed at addressing institutional inequalities, such as reducing marginal tax rates for secondary earners, may have little effect on women's labor supply leading to small elasticity estimates. On the other hand, if observed labor supply differences are not affected by gender norms, then policies aimed at addressing institutional inequalities may be particularly effective.

In this paper, I estimate collective labor supply models for different-sex and same-sex married couples to quantify the role of gender norms in spousal bargaining power over labor supply. Although this is my main goal and contribution in this paper, there are two other contributions to the literature. First, I provide updated collective labor supply estimates for same-sex married couples relative to the pathbreaking work by Oreffice (2011), who uses data on same-sex cohabiting couples from the 2000 U.S. decennial census. The institutional context Oreffice (2011) studies pre-dates any legal access to same-sex marriage in the U.S., meaning the comparison of same-sex cohabiting partners' collective labor supply parameters to different-sex married spouses' parameters does not as cleanly identify the role of gender norms.[1] Second, I provide updated collective labor supply estimates from the model outlined by Donni (2003), which allows for both non-participation and non-linear budget constraints due to taxation. Moreau and Donni (2002) and Bloemen (2010) are the only others to estimate this model, to the best of my knowledge, and did so using French data from 1994 and Dutch data from 1990-2001, respectively. This model is useful in my context because there were substantial tax changes for same-sex married couples during my sample period, for which the model can account, and which I use to identify the unrestricted labor supply parameters.[2]

I use the 2012–2019 American Community Surveys to construct a sample of different- and same-sex married couples in which both spouses are between 25 and 60 years old. The 2012 American Community Survey is the first of the U.S. Census Bureau surveys to explicitly identify same-sex married couples in the data, whereas prior Census Bureau surveys suffered from substantial measurement error that made it difficult to reliably identify same-sex married couples (Black et al. 2007; Gates and Steinberger 2010). I divide different-sex couples by sex, as is common in this literature, and use a machine learning LASSO approach to divide same-sex couples by predicted earning status in the household. Identification of the sharing rule rests upon a distribution factor, defined as "variables that affect the household members' bargaining position but not preferences or the joint budget set" (Chiappori, Fortin, and Lacroix 2002). I use the age difference between the two spouses as a distribution factor in this paper.[3] Identification of the effect of gender norms on spousal bargaining power comes from the fact that bargaining over labor supply between different-

1. Massachusetts was the first state to legalize same-sex marriage, and did so in 2004.

2. Friedberg and Isaac (Forthcoming) and Isaac (2020) study these tax changes in more detail and estimate their effects on marriage and labor supply among same-sex couples, respectively.

3. Browning, Chiappori, and Weiss (2014) list the age difference between spouses is listed as a distribution factor that has been used elsewhere (page 204). Oreffice (2011) also uses the age difference between partners as a distribution factor when estimating a collective model of labor supply among same-sex cohabiting couples.

sex spouses necessarily includes differences in the spouses' sexes, whereas these differences are not present during bargaining between same-sex spouses. My empirical strategy allows me to recover the structural Marshallian labor supply parameters as well as the relative Pareto weights on the utility functions of wives in different-sex couples and predicted lower earners in same-sex couples.

I estimate significant Marshallian hours elasticities for almost all spouses.[4] My estimates imply negative and significant Marshallian elasticities for husbands in different-sex couples ranging from -0.02 to -0.17, indicating backward bending labor supplies. In contrast, I estimate positive and significant Marshallian elasticities for wives in different-sex couples, predicted primary earners in male couples, and most other working spouses in same-sex couples, indicating the traditional upward sloping labor supply. The estimates imply a Marshallian elasticity of 0.14 for wives, 0.06 for predicted higher earners in male couples, and 0.04 for predicted higher earners in female couples and predicted lower earners in male couples.

In couples in which the husband works, my estimates imply that wives in different-sex couples have a statistically significant 4% smaller Pareto weight on their utility in the collective household maximization problem, indicating lower bargaining power over labor supply for wives, relative to husbands, in different-sex couples. If the husband does not work then my estimates imply that wives' bargaining power over labor supply is instead a dramatic 31% lower, relative to husbands. I estimate no significant difference between the Pareto weights of same-sex spouses.

My Pareto weight estimates for different-sex and same-sex couples are statistically different from each other in most situations, suggesting that gender norms do significantly influence spousal bargaining power over labor supply within the couple. The substantial difference in relative bargaining power between wives with working vs. non-working husbands suggests that gender norms also meaningfully interact with earning status in the couple. Traditional gender norms likely influence men to work, and my estimates imply that when married men satisfy this traditional role then gender norms exert a much smaller negative effect on their wives' bargaining power. How-

4. The exception is for predicted lower earners in female couples and other non-working spouses in same-sex couples.

ever, when this traditional role is reversed, so that the wife works and the husband does not, then gender norms exert a much larger negative effect on wives' bargaining power. These results suggest that wives with non-working husbands may sacrifice bargaining power in the couple due to the interaction of gender norms and earning status. My results also suggest that past studies that have assumed that bargaining power is divided by sex in different-sex couples may exhibit biased estimates of bargaining power or sharing rule parameters because they are confounded by the interaction between gender norms and earning status.

My results suggest that policies aimed at addressing institutional inequalities may be particularly effective among different-sex couples in which the husband works because gender norms have a much smaller impact on wives' bargaining power over labor supply. For example, reducing the gender wage gap, which makes husbands more likely to be primary earners in their households, may be influential in reducing observable differences between male and female labor supply in these couples. However, my results suggest that this may not be the case for other couples because gender norms exert a much larger effect on wives' bargaining power.

It is important to note that spouses bargain over whether and how much to work in the context of this paper. The role of gender norms, therefore, is currently limited to this type of spousal bargaining, but future work includes expanding the analysis to consider interactions with child care.

2 The Collective Labor Supply Model

In this paper, I estimate the collective model of labor supply presented by Donni (2003), which extends of the models from Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) to allow for non-participation and non-linear budget constraints due to taxation. The collective labor supply model is empirically useful because, by first specifying a functional form for the spouses' unrestricted labor supply functions, it is possible identify the Marshallian labor supply functions, the indirect utility functions, and the Pareto weight for each spouse's utility, as demonstrated below. In this section I outline the collective model, along with its assumptions and restrictions, and reproduce the main propositions from Chiappori (1988, 1992), Chiappori, Fortin, and Lacroix (2002), and Donni (2003) below where necessary.

There are two individuals in the household indexed by i ($i = 1,2$), with vectors of preference factors $\mathbf{z}$, labor supplies L^i, gross hourly wages of w_i, household non-labor income of y, and aggregate Hicksian consumption of C^i. Assume that the price of consumption is normalized to one and the total time available to each individual is normalized one, so that $1-L^i$, with $0 \leq L^i \leq 1$, denotes individual i's leisure.

Donni (2003) makes the following two assumptions:

Assumption 1. *Each household member is characterized by specific utility functions of the form* $u^i(1-L^i, C^i)$. *These functions are both strongly concave, infinitely differentiable, and strictly increase in all their arguments on* $\mathbb{R}^3_{++}$, *with* $lim_{C^i \to 0} u^i(1-L^i, C^i) = \lim_{L^i \to 1} u^i(1-L^i, C^i) = -\infty$.

Assumption 2. *The outcome of the decision process is Pareto efficient*

Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) also assume Pareto efficiency, and it forms the foundation of the collective approach.

Under assumptions 1 and 2, there exists a Pareto weight, μ, such that household behavior is a solution to the problem:

$$\begin{aligned} \max_{L^1,L^2,C^1,C^2} \quad & u^1(1-L^1, C^1, \mathbf{z}) + \mu u^2(1-L^2, C^2, \mathbf{z}) \\ \text{subject to} \quad & \delta : h(L^1, L^2; w_1, w_2, y) \geq C^1 + C^2 \\ & 0 \leq L^1 \leq 1, \quad 0 \leq L^2 \leq 1, \quad C^1 \geq 0, \quad C^2 \geq 0, \end{aligned} \tag{$\bar{P}$}$$

where $h(\cdot)$ is infinitely differentiable, increasing in all its arguments, and concave in L^1 and L^2.

At this point, it should be noted that Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) provide canonical results of the collective model under linear budget constraints and assumptions 1 and 2 when both spouses work (i.e., ignoring non-participation).[5] A key result from Chiappori (1988) is that the household problem $\bar{P}$ is equivalent to individual maximization prob-

5. Using the notation of problem $\bar{P}$, the linear budget constraint would be $h(L^1, L^2; w_1, w_2, y) = L^1 w_1 + L^2 w_2 + y$.

lems in which the spouses split household non-labor income according to a sharing rule and then, conditional on the sharing rule, maximize their individual utilities subject to their relevant budget constraints. Chiappori (1992) builds upon this framework and provides testable restrictions on labor supply functions that allow for identification of the sharing rule. Chiappori, Fortin, and Lacroix (2002) introduce the concept of distribution factors, defined as "variables that affect the household members' bargaining position but not preferences or the joint budget set," which generate a new set of testable restrictions on labor supplies and allow for more straightforward identification of the sharing rule. It is here where Donni (2003) extends upon the framework from Chiappori, Fortin, and Lacroix (2002) to non-participation and non-linear budget constraints.

Under a non-linear budget constraint, Donni (2003) defines the shadow wages and shadow income as:

$$\omega_1(w_1, w_2, y) = \frac{\partial h(\bar{L}^1, \bar{L}^2; w_1, w_2, y)}{\partial L^1} \tag{1}$$

$$\omega_2(w_1, w_2, y) = \frac{\partial h(\bar{L}^1, \bar{L}^2; w_1, w_2, y)}{\partial L^2} \tag{2}$$

$$\eta(w_1, w_2, y) = h(\bar{L}^1, \bar{L}^2; w_1, w_2, y) - \sum_i \bar{L}^i \omega_i, \tag{3}$$

and puts forth the following lemma:

Lemma 1. *Let $(\bar{L}^1, \bar{L}^2)$ be a pair of labor supplies consistent with collective rationality conditionally on the budget constraint in problem $\bar{P}$. Then, there exist a pair of functions $(\bar{C}^1, \bar{C}^2)$ and a pair of functions (ρ^1, ρ^2), with $\sum_i \rho^i = \eta$, such that $(\bar{L}^i, \bar{C}^i)$ is a solution to:*

$$\begin{aligned} \max_{L^i, C^i} \quad & u^i(1 - L^i, C^i, \mathbf{z}) \\ \text{subject to} \quad & \gamma : L^i \omega_i + \rho^i = C^i \\ & 0 \leq L^i \leq 1 \\ \text{for any} \quad & (w_1, w_2, y) \in \mathbb{R}^3_{++} \end{aligned} \tag{P^i}$$

At an interior solution, the unrestricted labor supplies ($\bar{L}^1$ and $\bar{L}^2$) can be re-written as Marshallian labor supplies:

$$\bar{L}^1 = \lambda^1(\omega_1(w_1,w_2,y),\rho(w_1,w_2,y)) \tag{4}$$

$$\bar{L}^2 = \lambda^2(\omega_2(w_1,w_2,y),\eta(w_1,w_2,y)-\rho(w_1,w_2,y)), \tag{5}$$

where $\rho = \rho^1$ and $\eta - \rho = \rho^2$. Donni (2003) shows that we can further write λ^i as a function of only the shadow variables (ω_1,ω_2,η) using the Implicit Function Theorem by making the following assumption:

Assumption 3. *Labor supplies $\bar{L}^1(w_1,w_2,y)$ and $\bar{L}^2(w_1,w_2,y)$ and the budget constraint $h(L^1,L^2;w_1,w_2,y)$ are such that* $\begin{vmatrix} \frac{\partial\omega_1}{\partial w_1} & \frac{\partial\omega_2}{\partial w_1} & \frac{\partial\eta}{\partial w_1} \\ \frac{\partial\omega_1}{\partial w_2} & \frac{\partial\omega_2}{\partial w_2} & \frac{\partial\eta}{\partial w_2} \\ \frac{\partial\omega_1}{\partial y} & \frac{\partial\omega_2}{\partial y} & \frac{\partial\eta}{\partial y} \end{vmatrix} \neq 0$ *for any (w_1,w_2,y).*

If assumption 3 is satisfied, then we can write the labor supplies in (4) and (5) as:

$$\hat{L}^1(\omega_1,\omega_2,\eta) = \lambda^1(\omega_1,\varphi(\omega_1,\omega_2,\eta)) \tag{6}$$

$$\hat{L}^2(\omega_1,\omega_2,\eta) = \lambda^2(\omega_2,\eta-\varphi(\omega_1,\omega_2,\eta)), \tag{7}$$

where $\varphi(\omega_1(w_1,w_2,y),\omega_2(w_1,w_2,y),\eta(w_1,w_2,y)) = \rho(w_1,w_2,y)$.

Having defined the shadow wages, shadow income, and the unrestricted labor supplies, Marshallian labor supplies, and sharing rule (as functions of the shadow variables), the canonical results from Chiappori (1992) and Chiappori, Fortin, and Lacroix (2002) follow; namely, that the partial derivatives of the sharing rule are identifiable (with or without distribution factors) as functions of the shadow variables. In this paper, I use a distribution factor, s, for identification. Before deriving the structural parameters, define:

$$A = \frac{\frac{\partial\hat{L}^1}{\partial\omega_2}}{\frac{\partial\hat{L}^1}{\partial\eta}}, \qquad B = \frac{\frac{\partial\hat{L}^2}{\partial\omega_1}}{\frac{\partial\hat{L}^2}{\partial\eta}}, \qquad C = \frac{\frac{\partial\hat{L}^1}{\partial s}}{\frac{\partial\hat{L}^1}{\partial\eta}}, \qquad D = \frac{\frac{\partial\hat{L}^2}{\partial s}}{\frac{\partial\hat{L}^2}{\partial\eta}},$$

Chiappori, Fortin, and Lacroix (2002) present the proposition below, which I reproduce using the shadow variable notation from above:

Proposition 1. *Take any point such that* $\frac{\partial \hat{L}^1}{\partial \eta} \cdot \frac{\partial \hat{L}^2}{\partial \eta} \neq 0$. *Then the following results hold: (i) If there exists exactly one distribution factor such that* $C \neq D$, *the following conditions are necessary for any pair* $(\hat{L}^1, \hat{L}^2)$ *to be solutions of* (P^i) *for some sharing rule* φ*:*

$$\frac{\partial}{\partial s}\left(\frac{D}{D-C}\right) = \frac{\partial}{\partial y}\left(\frac{CD}{D-C}\right) \tag{8a}$$

$$\frac{\partial}{\partial \omega_1}\left(\frac{D}{D-C}\right) = \frac{\partial}{\partial y}\left(\frac{BC}{D-C}\right) \tag{8b}$$

$$\frac{\partial}{\partial \omega_2}\left(\frac{D}{D-C}\right) = \frac{\partial}{\partial y}\left(\frac{AD}{D-C}\right) \tag{8c}$$

$$\frac{\partial}{\partial \omega_1}\left(\frac{CD}{D-C}\right) = \frac{\partial}{\partial s}\left(\frac{BC}{D-C}\right) \tag{8d}$$

$$\frac{\partial}{\partial \omega_2}\left(\frac{CD}{D-C}\right) = \frac{\partial}{\partial s}\left(\frac{AD}{D-C}\right) \tag{8e}$$

$$\frac{\partial}{\partial \omega_2}\left(\frac{BC}{D-C}\right) = \frac{\partial}{\partial \omega_1}\left(\frac{AD}{D-C}\right) \tag{8f}$$

$$\frac{\partial \hat{L}^1}{\partial \omega_1} - \frac{\partial \hat{L}^1}{\partial \eta}\left(\hat{L}^1 + \frac{BC}{D-C}\right)\left(\frac{D-C}{D}\right) \geq 0 \tag{8g}$$

$$\frac{\partial \hat{L}^2}{\partial \omega_2} - \frac{\partial \hat{L}^2}{\partial \eta}\left(\hat{L}^2 + \frac{AD}{D-C}\right)\left(-\frac{D-C}{D}\right) \geq 0 \tag{8h}$$

(ii) Under the assumption that conditions 8a–8h hold and for a given $\mathbf{z}$, *the sharing rule is defined up to an additive function* $\kappa(\mathbf{z})$ *depending only on the preference factors* $\mathbf{z}$. *The partial derivatives of the sharing rule with respect to wages, non-labor income, and the distribution factor are given by:*

$$\begin{aligned}\frac{\partial \varphi}{\partial \eta} &= \frac{D}{D-C}\\ \frac{\partial \varphi}{\partial s} &= \frac{CD}{D-C}\\ \frac{\partial \varphi}{\partial \omega_1} &= \frac{BC}{D-C}\\ \frac{\partial \varphi}{\partial \omega_2} &= \frac{AD}{D-C}\end{aligned} \tag{9}$$

Donni's (2003) extension of the collective labor supply model to non-linear budget constraints, combined with proposition 1 above from Chiappori, Fortin, and Lacroix (2002), constitute the theoretical results needed for identification in this paper when both household members work. In order to extend the model to non-participation, Donni (2003) first implicitly defines the reservation wage for member i as the marginal rate of substitution along the axis $L^i = 0$ (conditional on φ^i):

$$\varpi^i(\omega_1, \omega_2, \eta) = -\frac{u_L^i(1, \varphi^i(\omega_1, \omega_2, \eta)}{u_C^i(1, \varphi^i(\omega_1, \omega_2, \eta)},$$

and makes the following assumption:

Assumption 4. *Preferences and the sharing rule are such that*

$$\max_{i=1,2}(|\varpi^i(\omega_1^*, \omega_2^*, \eta) - \varpi^i(\omega_1^\circ, \omega_2^\circ, \eta)|) \leq \max_{i=1,2}(|\omega_i^* - \omega_i^\circ|)$$

for any $(\omega_1^*, \omega_2^*, \eta)$ *and* $(\omega_1^\circ, \omega_2^\circ, \eta) \in \mathbb{R}^3_{++}$.

Assumption 4 ensures that, for any η, there exists a single pair of wages, denoted $\hat{\omega}_1$ and $\hat{\omega}_2$, such that both household members are indifferent between working and not working, and ensures that, for each member i, there exists a function $\gamma^i(\omega_j, \eta)$ such that member i participates in the labor market if and only if $\omega_i > \gamma^i(\omega_j, \eta)$. These elements partition $\mathbb{R}^3_{++}$ into four sets: the set in which both spouses work (denoted P or the spouses' participation set), the set in which only spouse 1 works (denoted N_2 or spouse 2's non-participation set), the set in which only spouse 2 works (denoted N_1 or spouse 1's non-participation set), and the set in which neither spouse works (denoted N or the spouses' non-participation set).

Given assumption 4, Donni (2003) puts forth the following lemma:

Lemma 2. *Let us assume that* $\lim_{\omega_i \uparrow \gamma^i} \bar{L}^j_\eta \neq 0$ *and* $b_i \cdot \gamma^i_\eta \neq -1$ *for any* $(\omega_1, \omega_2, \eta) \in I_i$ *and* $\bar{L}^j_\eta \neq 0$ *for any* $(\omega_1, \omega_2, \eta) \in int(N_i)$. *Then the sharing rule is identified up to a constant on* N_i.[6]

In the next section, I assume a parametric specification for the unrestricted labor supply functions and derive the sharing rule implied by these functions.

6. Donni (2003) defines $b_1(\omega_1, \eta) = A(\omega_1, \gamma^2(\omega_2, \eta), \eta)$ and $b_2(\omega_2, \eta) = B(\gamma^1(\omega_1, \eta), \omega_2, \eta)$, where A and B are defined above.

3 Parametric Specification and Estimation

For convenience, I follow Oreffice (2011) and assume the parametric specification of the unrestricted labor supplies below. I first consider the situation in which both spouses work and then consider, in turn, the situations in which only one spouse works.

3.1 When Both Spouses Work

$$L^1 = a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \mathbf{a}_5 \mathbf{z} \tag{10}$$

$$L^2 = b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + \mathbf{b}_5 \mathbf{z} \tag{11}$$

ω_i is individual i's net-of-tax hourly wage rate, η is the couple's virtual income, s is the age difference between the spouses (the distribution factor), and $\mathbf{z}$ includes controls for the presence and number of children, the state unemployment rate, and indicator variables for the individual's level of education, year, and state of residence.[7] The set-up of the regressions means that the constant term will effectively control for the individual's sex in different-sex couples.

Equations 10 and 11 lead to the following functions for A, B, C, and D under the definitions in Section 2:

$$A = \frac{a_2}{a_3 \omega_2}, \qquad B = \frac{b_1}{b_3 \omega_1}, \qquad C = \frac{a_4}{a_3}, \qquad D = \frac{b_4}{b_3}$$

These definitions imply that conditions 8a–8f are automatically satisfied because the derivatives are zero, but it does imply other testable restrictions. Namely, the condition that $\frac{\partial \hat{L}^1}{\partial \eta} \cdot \frac{\partial \hat{L}^2}{\partial \eta} \neq 0$ requires that $a_3 b_3 \neq 0$, and the condition that $C \neq D$ requires that $\frac{a_4}{a_3} \neq \frac{b_4}{b_3}$.

Let $\Delta = a_3 b_4 - a_4 b_3$. If the above restrictions are satisfied, then the partial derivatives of the sharing rule, φ, are given by:[8]

$$\frac{\partial \varphi}{\partial \eta} = \frac{a_3 b_4}{\Delta}, \qquad \frac{\partial \varphi}{\partial s} = \frac{a_4 b_4}{\Delta}, \qquad \frac{\partial \varphi}{\partial \omega_1} = \frac{a_4 b_1}{\omega_1 \Delta}, \qquad \frac{\partial \varphi}{\partial \omega_2} = \frac{a_2 b_4}{\omega_2 \Delta} \tag{12}$$

7. The education level groups are "exactly high school education," "some college education," and "college education or more," with the omitted category being "less than high school education."

8. Appendix A presents the derivation of these parameters.

Solving this system of differential equations yields the sharing rule:

$$\varphi = \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z}) \tag{13}$$

It is also possible to derive the Marshallian labor supplies that are consistent with the unrestricted labor supplies in equations 10 and 11. These functions should take the following form:

$$\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z} \tag{14}$$

$$\lambda^2 = \beta_1 \log \omega_2 + \beta_2(\eta - \varphi) + \beta_3 \mathbf{z} \tag{15}$$

Given the form of φ in equation 13, the parameters above can be recovered as: $\alpha_1 = \frac{a_1 b_4 - a_4 b_1}{b_4}$, $\alpha_2 = \frac{\Delta}{b_4}$, $\beta_1 = \frac{a_4 b_2 - a_2 b_4}{a_4}$, and $\beta_2 = -\frac{\Delta}{a_4}$.[9]

Finally, we can also recover the indirect utility functions and the Pareto weight, μ. Stern (1986) shows that the the Marshallian labor supplies in equations 14 and 15 correspond to the following indirect utility functions:

$$V^1(\omega_1, \varphi, \mathbf{z}) = \left(\frac{e^{\alpha_2 \omega_1}}{\alpha_2}\right)(\alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z}) \tag{16}$$

$$V^2(\omega_2, \eta - \varphi, \mathbf{z}) = \left(\frac{e^{\beta_2 \omega_2}}{\beta_2}\right)(\beta_1 \log \omega_2 + \beta_2(\eta - \varphi) + \beta_3 \mathbf{z}) \tag{17}$$

As noted by Chiappori (1988), and appealing to the Envelope Theorem, $\frac{\partial V^1}{\partial \varphi} = \delta$, where δ is the Legrange multiplier from the household problem $\bar{P}$. Similarly, because the utility of individual 2 is multiplied by the Pareto weight (μ), $\frac{\partial V^2}{\partial(\eta - \varphi)} = \frac{\delta}{\mu}$. It is, therefore, possible to identify the Pareto weight as:

$$\mu = \frac{\frac{\partial V^1}{\partial \varphi}}{\frac{\partial V^2}{\partial(\eta - \varphi)}} = \frac{e^{\alpha_2 \omega_1}}{e^{\beta_2 \omega_2}} \tag{18}$$

9. Appendix B presents the derivations of these parameters.

3.2 When Only Spouse 1 Works

Next, consider the situation in which only spouse 1 works. Following Donni (2003), I assume that if spouse 2 does not work then spouse 1's unrestricted labor supply function switches to:

$$L^{1,s} = A_0 + A_1 \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \mathbf{A}_5 \mathbf{z}, \tag{19}$$

and the sharing rule switches to:

$$\varphi^{1,s} = K_1 \log \omega_1 + K_2 \log \omega_2 + K_3 \eta + K_4 s + \mathbf{K}(\mathbf{z}) \tag{20}$$

In order for $L^{1,s}$ and $\varphi^{1,s}$ to be continuous along spouse 2's participation frontier, it must be the case that:

$$L^{1,s} = L^1 + g \cdot L^2 \tag{21a}$$

$$\varphi^{1,s} = \varphi + h \cdot L^2, \tag{21b}$$

where g and h are free parameters. Donni's (2003) proof of identification shows that the partial differential equation $\frac{\partial \varphi^{1,s}}{\partial \omega_2} - A \frac{\partial \varphi^{1,s}}{\partial \eta} = 0$ holds within spouse 2's non-participation set, and I show in Appendix C that this implies that $h = g \frac{b_4}{\Delta}$. Therefore, the sharing rule is identified within spouse 2's non-participation set.

The parameters of L^2 (spouse 2's unrestricted labor supply function) remain the same as in equation 11, and Appendix C shows that the parameters of $L^{1,s}$ (spouse 1's new unrestricted labor supply function) can be recovered as:

$$\begin{aligned} A_0 &= a_0 + g b_0, \qquad & A_1 &= a_1 + g b_1, \qquad & A_2 &= a_2 + g b_2 \\ A_3 &= a_3 + g b_3, \qquad & A_4 &= a_4 + g b_4, \qquad & A_5 &= a_5 + g b_5 \end{aligned} \tag{22}$$

The sharing rule in this situation becomes:

$$\varphi^{1,s} = \frac{1}{\Delta}(A_4 b_1 \log \omega_1 + A_2 b_4 \log \omega_2 + A_3 b_4 \eta + A_4 b_4 s) + \kappa(\mathbf{z}) \tag{23}$$

Where $\Delta = A_3b_4 - A_4b_3 = a_3b_4 - a_4b_3$. The Marshallian labor supplies again take the following form:

$$\lambda^1 = \alpha_1^{1,s} \log \omega_1 + \alpha_2^{1,s} \varphi + \alpha_3^{1,s} \mathbf{z} \tag{24}$$

$$\lambda^2 = \beta_1^{1,s} \log \omega_2 + \beta_2^{1,s} (\eta - \varphi) + \beta_3^{1,s} \mathbf{z} \tag{25}$$

In this situation, given the form of $\varphi^{1,s}$ in equation 23, the parameters above can be recovered as: $\alpha_1^{1,s} = \frac{A_1b_4 - A_4b_1}{b_4}$, $\alpha_2^{1,s} = \frac{\Delta}{b_4}$, $\beta_1^{1,s} = \frac{A_4b_2 - A_2b_4}{A_4}$, and $\beta_2^{1,s} = -\frac{\Delta}{A_4}$.

Finally, the indirect utility functions and the Pareto weight in this situation take the same forms as equations 16, 17, and 18 with the relevant parameter definitions of $\alpha_1^{1,s}$, $\alpha_2^{1,s}$, $\beta_1^{1,s}$, and $\beta_2^{1,s}$.

3.3 When Only Spouse 2 Works

Finally, consider the situation in which only spouse 2 works. Following the same process as above, I assume that if spouse 1 does not work then spouse 2's unrestricted labor supply function switches to:

$$L^{2,s} = B_0 + B_1 \log \omega_1 + B_2 \log \omega_2 + B_3 \eta + B_4 s + \mathbf{B}_5 \mathbf{z}, \tag{26}$$

and the sharing rule switches to:

$$\varphi^{2,s} = P_1 \log \omega_1 + P_2 \log \omega_2 + P_3 \eta + P_4 s + \pi(\mathbf{z}) \tag{27}$$

In order for $L^{2,s}$ and $\varphi^{2,s}$ to be continuous along spouse 1's participation frontier, it must be the case that:

$$L^{2,s} = L^2 + j \cdot L^1 \tag{28a}$$

$$\varphi^{2,s} = \varphi + k \cdot L^1, \tag{28b}$$

where j and k are free parameters. Donni's (2003) proof of identification shows that the partial differential equation $\frac{\partial \varphi^{2,s}}{\partial \omega_1} - B \frac{\partial \varphi^{2,s}}{\partial \eta} = -B$ holds within spouse 1's non-participation set, and I show in Appendix C that this implies that $k = j\frac{a_4}{\Delta}$. Therefore, the sharing rule is identified within spouse

1's non-participation set.

The parameters of L^1 (spouse 1's unrestricted labor supply function) remain the same as in equation 10, and Appendix C shows that the parameters of $L^{2,s}$ (spouse 2's new unrestricted labor supply function) can be recovered as:

$$\begin{aligned} &B_0 = b_0 + ja_0, \qquad B_1 = b_1 + ja_1, \qquad B_2 = b_2 + ja_2 \\ &B_3 = b_3 + ja_3, \qquad B_4 = b_4 + ja_4, \qquad B_5 = b_5 + ja_5 \end{aligned} \tag{29}$$

The sharing rule in this situation becomes:

$$\varphi^{2,s} = \frac{1}{\Delta}(a_4 B_1 \log \omega_1 + a_2 B_4 \log \omega_2 + a_3 B_4 \eta + a_4 B_4 s) + \kappa(\mathbf{z}) \tag{30}$$

Where $\Delta = a_3 B_4 - a_4 B_3 = a_3 b_4 - a_4 b_3$. The Marshallian labor supplies again take the following form:

$$\lambda^1 = \alpha_1^{2,s} \log \omega_1 + \alpha_2^{2,s} \varphi + \alpha_3^{2,s} \mathbf{z} \tag{31}$$

$$\lambda^2 = \beta_1^{2,s} \log \omega_2 + \beta_2^{2,s} (\eta - \varphi) + \beta_3^{2,s} \mathbf{z} \tag{32}$$

In this situation, given the form of $\varphi^{2,s}$ in equation 30, the parameters above can be recovered as: $\alpha_1^{2,s} = \frac{a_1 B_4 - a_4 B_1}{B_4}$, $\alpha_2^{2,s} = \frac{\Delta}{B_4}$, $\beta_1^{2,s} = \frac{a_4 B_2 - a_2 B_4}{a_4}$, and $\beta_2^{2,s} = -\frac{\Delta}{a_4}$.

Finally, the indirect utility functions and the Pareto weight in this situation take the same forms as equations 16, 17, and 18 with the relevant parameter definitions of $\alpha_1^{2,s}$, $\alpha_2^{2,s}$, $\beta_1^{2,s}$, and $\beta_2^{2,s}$.

3.4 Estimation

I estimate the parameters of the model in three steps. I first estimate the reduced form labor supply parameters among dual-earner couples for each spouse separately using OLS (equations 10 and 11). I do this separately for spouses in different-sex couples, male couples, and female couples, which allows these unrestricted labor supply parameters to differ. I then use these parameters to predict hours worked for spouses in single-earner couples, which produces a predicted L^1 and L^2 for each spouse. I use these predicted values to estimate the switching parameters, g and j, using

OLS (equations 21a and 28a). Finally I use the estimates from the dual-earner couples and the estimates of the switching parameters to back out the reduced form labor supply parameters for single-earner couples (equations 19 and 26).

In order to quantify the role of gender norms, it is possible to compare the Pareto weight of wives in different-sex married couples to the Pareto weight of lower earners in same-sex couples. This comparison assumes that, all else equal, the only remaining unobserved influence on spousal bargaining power are gender norms between different-sex spouses, which are present in the Pareto weight for wives in different-sex couples, but not present in the Pareto weight for predicted lower earners in same-sex couples.

4 Data

I use data from the 2012–2019 American Community Surveys to construct a sample of different- and same-sex married couples in which both spouses are between 25 and 60 years old, so as to limit attention to labor supply of prime-age workers.[10] My main sample includes 5,019,276 individuals across both dual- and single-earner couples.

Table 1 presents demographic summary statistics for different- and same-sex couples in my sample. Different-sex couples are, on average, slightly younger than male couples but slightly older than female couples, slightly less educated, and have a larger age difference between spouses. The most notable difference is the presence of children. While over 60% of different-sex couples have any children, only 38% of female couples and 19% of male couples have children. Conditional on having children, however, the averages are more similar across all couples.

Estimating the model with labor force participation decisions requires a method of assigning wages to non-workers. In addition, earnings are endogenous to labor supply, meaning that higher wage and tax rates will be correlated with higher labor supply. Therefore, I predict earnings for each spouse in order to address both of these problems. The accuracy of these predictions is important to accurately address endogeneity concerns and control for the influence of the non-

10. If a same-sex couple reports themselves to be married even though they reside in a state that does not recognize same-sex marriages, then I assume the couple married in a state that did recognize same-sex marriages.

working spouse's latent wage, which Mullainathan and Spiess (2017) note is a useful context for a machine learning LASSO approach to predict earnings since it is "effectively a prediction exercise." The LASSO is a model selection method that uses a penalized regression to select the covariates that best predict earned income using OLS (Tibshirani 2011).[11] This approach enables me to include a large number of covariates and interactions while allowing the LASSO to select the subset of variables that best fit the data. Variables that I included, but which the LASSO may have ultimately ignored, include five year age groups, four education level groups, dummies for race, sex, occupation, college major, and state, as well as pairwise interactions between all of these variables.[12]

I use the LASSO to predict earnings in levels for each spouse. I limit the prediction sample to individuals observed in 2012 with positive earnings so that predicted earnings do not reflect labor supply changes influenced by policy variation during the sample period. I use these predicted earnings for two purposes. First, I use predicted earnings to divide spouses in same-sex couples by predicted earning status, so that predicted higher earners bargain with predicted lower earners. In contrast, I divide spouses in different-sex couples by sex, as is common in this literature, so that husbands bargain with wives.

Second, I use predicted earnings to compute a predicted shadow wage rate and income for each spouse, including non-workers. I first divide predicted earnings by 2,080 (52 weeks multiplied by 40 hours per week) to obtain a predicted measure of each individual's full-time gross wage rate: $w_{it} = \frac{\text{Predicted earnings}}{2080}$.[13] To account for non-linear taxation, Donni's (2003) model requires shadow wages and shadow income, ω_1, ω_2, and η, respectively, defined in equations 1–3. I follow Moreau and Donni (2002), and define, for a household with taxable income in the k^{th} bracket, $\omega_i = w_i(1-t_k)$ and $\eta = y - T(B_k) + t_k B_k$, where t_k is the federal marginal tax rate in bracket k, B_k is the lower income limit of bracket k, and $T(B_k)$ is the federal tax revenue corresponding to B_k. I

11. The LASSO is similar to a ridge regression, but uses an L1 norm constraint rather than the L2 norm constraint of the ridge regression. Friedberg and Isaac (Forthcoming) and Isaac (2020) also use a LASSO approach in similar contexts, and more detail about the methodology can be found there.

12. Although the prediction process differs, the goal of this process is similar to that used by Delhommer and Hamermesh (2021), who predict earning potentials for same-sex spouses in order to calculate the marital surplus.

13. Dividing predicted earnings by 2,080 also avoids using a noisy and endogenous measure of self-reported annual hours of work when quantifying predicted gross wage rates.

obtain the t_k, B_k, and $T(B_k)$ parameters for each tax year from the NBER TAXSIM program based on simulated married households with varying levels of earned income.[14] This process, therefore, takes into account numerous tax credits and deductions based on earned income when generating the tax brackets for each tax year, rather than using the statutory income tax brackets, which would result in much coarser measurement of the tax parameters. I also account for federal recognition of same-sex marriages when applying these definitions of ω_i and η to same-sex couples.[15]

Table 1 also displays the observed and predicted labor supply summary statistics for different- and same-sex couples in my sample. Different-sex couples tend to have lower observed earnings, work fewer hours, have less observed non-labor income, and exhibit a larger observed hourly wage gap between spouses, on average, relative to same-sex couples. Table 1 also makes it clear that the prediction process tends to understate earnings and, therefore, gross and after-tax hourly wage rates and virtual income. I also obtain more compressed variation in predicted earnings, wage rates, and virtual income relative to the observed values.

5 Results

I estimate collective models for different-sex and same-sex married couples following the empirical specification in equations 10 and 11 for the unrestricted labor supply equations. Section 3 details the derivations of the Marshallian labor supplies, the derivatives of the sharing rule, and the Pareto weight on the second household member. In what follows, I will use "husband/wife" to refer to different-sex spouses and "predicted higher/lower earner" to refer to same-sex spouses.

5.1 Unrestricted Labor Supply Parameters

Table 2 presents coefficient estimates for the unrestricted labor supply equations. Of primary importance are the coefficients on the distribution factor: the age difference between spouses. The coefficients on the distribution factor are significant and opposite-signed between spouses, as

14. These simulated households vary only in their total earned income; I do not consider other sources of income when obtaining these tax parameters. Figures of the tax brackets generated by this process are available upon request.

15. Same-sex married couples were still required to file federal taxes as two single individuals in tax years 2011 and 2012. Same-sex married couples were required to file joint federal taxes beginning in tax year 2013 following the *United States v. Windsor* Supreme Court ruling.

is expected, among different-sex couples and dual-earner male couples, and are opposite-signed but only sometimes significant among single-earner male couples (statistically different at the 1% level) and all female couples (not statistically different).[16] This indicates that the distribution factor does differentially affect spousal labor supply in different-sex couples and male couples, but may not do so among female couples. The differential effects of the distribution factor on each spouse's labor supply provides identification of the sharing rule, as outlined in section 2. Therefore, identification of the sharing rule and spousal bargaining power is likely to be strongest when comparing different-sex couples to male couples.

The coefficients on the individual's predicted own net wage are mostly positive and significant. The exceptions are the own wage coefficients for predicted lower earners who are the only earner in their household, which is positive but insignificant, and for wives who are the only earner in their household, which is negative and significant. The cross-net wage effect is mixed across spouses, but is significant among different-sex couples. Among different-sex couples, the cross-net wage effect is negative for dual-earner wives and single-earner husbands but positive and significant for dual-earner husbands and single-earner wives. This pattern suggests that dual-earner wives and single-earner husbands view their spouse's labor supply as substitutable for their own, but that dual-earner husbands and single-earner wives instead view their spouse's labor supply as complementary. The cross-net wage coefficients are mostly insignificant among same-sex couples, suggesting little evidence that same-sex spouses adjust their labor supply in response to their spouse's net wage.

5.2 Sharing Rule and Marshallian Labor Supply Parameters

Table 3 presents coefficient estimates of the sharing rule derivatives. Among different-sex couples in which the husband works, a $1 increase in the husband's net wage (an increase of $2,198 annually at the mean) translates into the transfer of $2,364–$2,625 more income to his spouse, which

16. Note that the coefficients on the distribution factor exhibit opposite patterns between different-sex and same-sex spouses. In different-sex spouses, the coefficient on the age difference is negative for husbands and positive for wives, whereas the coefficient is positive for predicted primary earners and negative for predicted secondary earners in same-sex couples. However, these patterns are consistent with Oreffice (2011), who finds that the age difference between spouses is opposite-signed for same-sex cohabiting couples relative to different-sex married couples.

outweighs the annual increase in earnings. However, this transfer decreases to \$1,052 among couples in which the husband does not work, suggesting a more even split among these couples. Among different-sex couples in which the wife works, a \$1 increase in the wife's net wage (an increase of \$1,779 annually at the mean) translates into the transfer of \$27–278 to herself, which changes to a \$863 loss among couples in which the wife does not work. This suggests that wives receive a larger premium (in the form a larger fraction of additional income transfered to them) when their husband's net wage increases relative to their own. In addition, across all different-sex couples, the sharing rule indicates that a \$1 increase in the couple's virtual income translates into the transfer of \$0.05–0.33 to the husband, with the remainder going to the wife. Finally, a greater age difference between an older husband and a younger wife results in a transfer to the husband. The age difference transfer is largest when he works, ranging from \$1,483–\$1,646, and drops to \$241 when he does not work.

Among same-sex couples, the derivatives of the sharing rule are not statistically significant at conventional levels. The standard errors of these coefficients are much larger relative to those from different-sex couples, and the lack of precision may be due to the demands of the theoretical model combined with smaller sample sizes of same-sex couples.

Table 4 presents coefficient estimates of the structural parameters in the Marshallian labor supply equations. The coefficient on log own net wage (α_1 and β_1 in equations 14 and 15) are negative and significant for husbands, but positive and significant for wives in different-sex couples (regardless of whether they work), predicted higher earners in male couples (regardless of whether they work), and for most other working spouses in same-sex couples. These coefficients imply negative and significant Marshallian elasticities for husbands ranging from -0.02 to -0.17, indicating backward bending labor supplies. In contrast, the Marshallian elasticities for wives, predicted primary earners in male couples, and most other working spouses in same-sex couples are positive and significant, indicating the traditional upward sloping labor supply. The estimates imply a Marshallian elasticity of 0.14 for wives, 0.06 for predicted higher earners in male couples, and 0.04 for predicted higher earners in female couples and predicted lower earners in male couples, which are all significant at conventional levels.

The share of unearned income also only significantly affects different-sex spouses, although these coefficients are negative and of a similar magnitude among other couples despite the larger standard errors. The effect of virtual income is negative, as theory predicts, indicating that a larger share of virtual income decreases hours worked.

5.3 Pareto Weights and the Role of Gender Norms

Given the structural parameter estimates above, it is also possible to estimate the Pareto weight on the wife's utility ($\tilde{\mu}$) and on the predicted lower earner's utility (μ).[17] The estimated difference between these Pareto weights, therefore, is my estimate of the effect of gender norms on bargaining power. Note, however, that spouses bargain over whether and how much to work in the context of this paper. The role of gender norms, therefore, is currently limited to this type of spousal bargaining, but future work includes expanding the analysis to consider interactions with child care.

As discussed above, the coefficient on the distribution factor is only significantly different among different-sex couples and male couples, suggesting that identification of bargaining power is strongest when comparing these groups. I therefore limit the comparisons and estimates of bargaining power below to different-sex couples and male couples.

I estimate $\hat{\tilde{\mu}} = 0.96$ (s.e. = 0.003) for wives in different-sex couples in which the husband works, compared to $\hat{\mu} = 1.01$ (s.e. = 0.01) in dual-earning male couples and $\hat{\mu} = 1.06$ (s.e. = 0.15) in male couples where only the predicted higher earner works. The Pareto weight for wives is statistically different than 1, indicating that wives with working husbands have a significant 4% smaller weight put on their utilities, relative to their husbands. In contrast, the Pareto weight on predicted lower earners is not significantly different from 1 in male couples where the predicted higher earner works, meaning that I cannot reject that the Pareto weights are equal for these spouses. The Pareto weight estimates are also significantly different from each other, implying that gender norms do

17. Recall that $\mu = \frac{e^{\alpha_2 \omega_1}}{e^{\beta_2 \omega_2}}$.

significantly decrease wives' bargaining power in these couples.

In male couples where only the predicted lower earner works, the Pareto weight is $\hat{\mu} = 1.00$ (s.e. = 0.01), which is also not statistically different than 1. In contrast, the Pareto weight for wives in different-sex couples in which only the wife works falls to $\hat{\hat{\mu}} = 0.69$ (s.e. = 0.09), indicating that wives' bargaining power is 31% lower than their non-working husbands'.

The substantial difference in relative bargaining power between wives with working vs. non-working husbands suggests that gender norms meaningfully interact with earning status in the couple. Traditional gender norms likely influence men to work, and my estimates imply that when married men satisfy this traditional role then gender norms exert a much smaller negative effect on their wives' bargaining power. However, when this traditional role is reversed, so that the wife works and the husband does not, then gender norms exert a much larger negative effect on wives' bargaining power. These results suggest that wives with non-working husbands may sacrifice bargaining power in the couple due to the interaction of gender norms and earning status.

6 Conclusion

In this paper, I estimate collective labor supply models for different-sex and same-sex married couples to quantify the role of gender norms in spousal bargaining power. In doing so, I also provide updated collective labor supply estimates for same-sex married couples relative to the pathbreaking work by Oreffice (2011), who used data on same-sex cohabiting couples from the 2000 U.S. decennial census. I corroborate Oreffice's (2011) conclusion that labor supply in same-sex couples is consistent with the collective labor supply model, although the model may be a better fit for male couples rather than female couples. Additionally, I provide updated collective labor supply estimates from the model outlined by Donni (2003), which allows for both non-participation and non-linear budget constraints due to taxation, and which is useful in my context because there were substantial tax changes for same-sex married couples during my sample period.

My estimates imply that gender norms decreases the relative bargaining power of wives in different-sex couples and that gender norms interact with earning status in the couple. Wives with working husbands experience a 4% decrease in bargaining power due to gender norms, but working wives with non-working husbands experience a 31% decrease in bargaining power. In contrast, I estimate no significant difference between the Pareto weights of same-sex spouses. Note, however, that spouses bargain over whether and how much to work in the context of this paper. The role of gender norms, therefore, is currently limited to this type of spousal bargaining, but future work includes expanding the analysis to consider interactions with child care.

My results suggest that policies aimed at addressing institutional inequalities may be particularly effective among different-sex couples in which the husband works because gender norms have a much smaller impact on wives' bargaining power over labor supply. For example, reducing the gender wage gap, which makes husbands more likely to be primary earners in their households, may be influential in reducing observable differences between male and female labor supply in these couples. However, this may not be the case for couples in which only the wife works because gender norms exert a much larger negative effect on wives' bargaining power. My estimates also suggest that past studies that have assumed that bargaining power is divided by sex in different-sex couples may exhibit biased estimates of bargaining power or sharing rule parameters because they are confounded by the interaction between gender norms and earning status.

References

Baldwin, Alex, Michael Allgrunn, and Raymond Ring. 2011. "Does the Male-Female Partition Still Apply to Household Labor Supply?" *International Journal of Applied Economics* 8 (1): 46–54.

Bartels, Charlotte, and Cortnie Shupe. 2018. "Drivers of Participation Elasticities across Europe: Gender or Earner Role within the Household?" *IZA Discussion Paper No. 11359.*

Black, Dan, Gary Gates, Seth Sanders, and Lowell Taylor. 2007. "The Measurement of Same-Sex Unmarried Partner Couples in the 2000 U.S. Census." *California Center for Population Research On-Line Working Paper Series.*

Bloemen, Hans G. 2010. "An Empirical Model of Collective Household Labour Supply with Non-Participation." *The Economic Journal* 120 (543): 183–214.

Blundell, Richard, Pierre-André Chiappori, Thierry Magnac, and Costas Meghir. 2007. “Collective labour supply: Heterogeneity and non-participation.” *The Review of Economic Studies* 74 (2): 417–445.

Browning, Martin, Pierre-André Chiappori, and Yoram Weiss. 2014. *Economics of the Family.* Cambridge University Press.

Cherchye, Laurens, Bram De Rock, and Frederic Vermeulen. 2012. “Married with children: A collective labor supply model with detailed time use and intrahousehold expenditure information.” *American Economic Review* 102 (7): 3377–3405.

Chiappori, Pierre-André. 1988. “Rational household labor supply.” *Econometrica* 56 (1): 63–90.

———. 1992. “Collective labor supply and welfare.” *Journal of Political Economy* 100 (3): 437–467.

Chiappori, Pierre-André, Bernard Fortin, and Guy Lacroix. 2002. “Marriage market, divorce legislation, and household labor supply.” *Journal of Political Economy* 110 (1): 37–72.

Delhommer, Scott M., and Daniel S. Hamermesh. 2021. “Same-Sex Couples and the Gains to Marriage: The Importance of the Legal Environment.” *Journal of Policy Analysis and Management.*

Donni, Olivier. 2003. “Collective household labor supply: nonparticipation and income taxation.” *Journal of Public Economics* 87 (5): 1179–1198.

Donni, Olivier, and Nicolas Moreau. 2007. “Collective Labor Supply: A Single-Equation Model and Some Evidence from French Data.” *Journal of Human Resources* 42 (1): 214–246.

Friedberg, Leora, and Elliott Isaac. Forthcoming. “Same-Sex Marriage Recognition and Taxes: New Evidence About the Impact of Household Taxation.” *Review of Economics and Statistics.*

Gates, Gary J., and Michael D. Steinberger. 2010. “Same-Sex Unmarried Partner Couples in the American Community Survey: The Role of Misreporting, Miscoding and Misallocation.” *Working Paper.*

Gayle, George-Levi, and Andrew Shephard. 2019. “Optimal Taxation, Marriage, Home Production, and Family Labor Supply.” *Econometrica* 87 (1): 291–326.

Isaac, Elliott. 2020. “Suddenly Married: Joint Taxation and the Labor Supply of Same-Sex Married Couples After *U.S. v. Windsor*.” *Working Paper.*

Moreau and Donni. 2002. “Estimation d’un modèle collectif d’offre de travail avec taxation.” *Annales d’Économie et de Statistique,* no. 65: 55–83.

Mullainathan, Sendhil, and Jann Spiess. 2017. “Machine Learning: An Applied Econometric Approach.” *Journal of Economic Perspectives* 31 (2): 87–106.

Oreffice, Sonia. 2011. “Sexual orientation and household decision making, Same-sex couples’ balance of power and labor supply choices.” *Labour Economics* 18 (2): 145–158.

Stern, Nicholas. 1986. “On the specification of labour supply functions.” In *Unemployment, Search and Labour Supply,* edited by Richard Blundell and Ian Walker, 143–189. Cambridge: Cambridge University Press.

Tibshirani, Robert. 2011. “Regression shrinkage and selection via the lasso: a retrospective.” *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 73 (3): 273–282.

Vermeulen, Frederic. 2006. “A collective model for female labour supply with non-participation and taxation.” *Journal of Population Economics* 19 (1): 99–118.

Table 1: Summary Statistics

	Different-sex couples		Male same-sex couples		Female same-sex couples	
	Husbands	Wives	Predicted primary earners	Predicted secondary earners	Predicted primary earners	Predicted secondary earners
Age	45.058	43.458	45.246	44.208	43.947	43.391
	(9.262)	(9.277)	(8.562)	(9.781)	(8.677)	(10.007)
Less than HS education	0.062	0.036	0.012	0.039	0.013	0.033
	(0.242)	(0.187)	(0.107)	(0.193)	(0.112)	(0.179)
Exactly HS education	0.315	0.254	0.107	0.263	0.115	0.264
	(0.465)	(0.436)	(0.309)	(0.440)	(0.319)	(0.441)
Some college education	0.221	0.245	0.159	0.265	0.179	0.261
	(0.415)	(0.430)	(0.365)	(0.441)	(0.383)	(0.439)
College degree or more	0.401	0.465	0.723	0.433	0.694	0.441
	(0.490)	(0.499)	(0.448)	(0.496)	(0.461)	(0.497)
Any children	0.636	0.609	0.196	0.192	0.385	0.381
	(0.481)	(0.488)	(0.397)	(0.394)	(0.487)	(0.486)
Conditional number of children	1.980	1.895	1.870	1.832	1.725	1.716
	(0.997)	(0.922)	(1.003)	(0.946)	(0.942)	(0.943)
Partners' age difference	1.834	1.848	0.967	0.879	0.429	0.485
	(4.244)	(4.209)	(7.227)	(7.230)	(6.019)	(5.972)
Annual hours worked	2198.176	1779.015	2125.957	2021.873	2033.056	1959.493
	(629.250)	(700.259)	(663.137)	(698.530)	(646.126)	(679.514)
Observed earnings	79970.563	47200.767	98368.618	77098.005	71672.883	59012.174
	(84468.623)	(51491.188)	(105156.086)	(90676.705)	(72638.318)	(60982.119)
Predicted earnings	71931.059	42437.272	87566.099	65008.176	68693.344	51588.915
	(32583.287)	(17201.210)	(31297.465)	(30811.588)	(26655.348)	(26155.712)
Observed gross hourly wage	37.598	27.552	47.534	38.547	35.964	31.691
	(98.664)	(93.806)	(80.482)	(61.166)	(53.016)	(111.812)
Predicted gross hourly wage	34.582	20.403	42.099	31.254	33.026	24.802
	(15.665)	(8.270)	(15.047)	(14.813)	(12.815)	(12.575)
Observed after-tax hourly wage	25.551	18.859	32.692	26.528	24.834	21.941
	(66.008)	(60.310)	(55.840)	(40.959)	(36.005)	(76.278)
Predicted after-tax hourly wage	23.788	14.081	29.670	22.163	23.215	17.541
	(10.579)	(5.628)	(10.218)	(10.297)	(8.663)	(8.734)
Reported non-labor income	5000.809	4933.527	7298.919	7322.517	5920.821	5697.289
	(23896.072)	(22379.701)	(30690.781)	(30962.322)	(24754.704)	(24166.987)
Observed virtual income ($10,000s)	1.410	1.373	1.340	1.315	1.227	1.201
	(2.498)	(2.329)	(3.148)	(3.105)	(2.433)	(2.413)
Predicted virtual income ($10,000s)	1.340	1.309	1.242	1.219	1.159	1.131
	(2.411)	(2.257)	(3.062)	(3.022)	(2.398)	(2.377)
State unemployment rate	5.446	5.424	5.045	5.019	4.958	4.952
	(1.782)	(1.776)	(1.555)	(1.538)	(1.565)	(1.565)
Observations	2,733,234	2,245,784	9,954	9,400	10,743	10,161

Notes: The data come from the 2012–2019 American Community Surveys and include different-sex and same-sex married, childless couples in which both spouses are working and between 25-–60 years old.

Table 2: Unrestricted Labor Supply Parameters

	Male couples				Female couples				Different-sex couples			
	Both partners work				Both partners work				Both partners work			
	Predicted higher earners	Predicted lower earners	Only predicted higher earner works	Only predicted lower earner works	Predicted higher earners	Predicted lower earners	Only predicted higher earner works	Only predicted lower earner works	Husbands	Wives	Only husband works	Only wife works
Log(predicted own net wage)	135.660***	72.456***	126.627***	44.417	74.779**	71.972***	74.180**	95.935*	102.905***	252.903***	120.051***	-43.391***
	(33.351)	(21.944)	(34.781)	(46.035)	(35.724)	(15.476)	(35.781)	(55.904)	(1.662)	(2.587)	(1.830)	(1.989)
Log(predicted spouse net wage)	2.485	-25.573	28.077	73.738***	15.874	-3.364	28.704	93.050***	10.846***	-97.516***	-33.620***	258.608***
	(16.492)	(24.997)	(23.666)	(23.453)	(13.806)	(19.952)	(23.259)	(24.368)	(1.219)	(1.195)	(2.313)	(2.669)
Couple's virtual income ($10,000s)	-1.350	-7.436**	-0.000	-0.001**	-10.400**	-25.573***	-0.001*	-0.004***	-9.164***	-11.585***	-0.001***	-0.002***
	(3.243)	(3.671)	(0.000)	(0.000)	(4.135)	(4.271)	(0.001)	(0.001)	(0.269)	(0.284)	(0.000)	(0.000)
Couple's age difference	3.157***	-6.294***	0.934	-4.665***	0.398	-1.006	0.218	-0.478	-4.093***	2.571***	-4.545***	0.419***
	(1.034)	(1.129)	(1.720)	(1.483)	(1.119)	(1.217)	(1.165)	(1.918)	(0.109)	(0.117)	(0.112)	(0.141)
Couple has children	-17.105	19.564	-10.195	10.739	-14.553	1.669	-14.255	-17.655	32.428***	-6.733***	33.611***	10.323***
	(36.447)	(37.371)	(38.830)	(41.902)	(24.883)	(25.449)	(25.206)	(41.781)	(1.452)	(1.639)	(1.481)	(1.857)
Number of children	-3.466	-23.597	-11.801	-25.385	-26.895**	-30.951**	-32.413**	-66.665***	3.755***	-56.516***	13.692***	-54.541***
	(18.289)	(18.475)	(19.965)	(20.684)	(12.778)	(13.007)	(15.202)	(22.759)	(0.630)	(0.762)	(0.773)	(0.832)
Exactly HS education	31.727	55.057	51.174	71.426	111.009	134.540***	134.993*	281.948***	72.244***	17.378***	69.188***	55.377***
	(76.985)	(45.067)	(79.154)	(60.387)	(69.620)	(49.063)	(78.043)	(109.397)	(2.443)	(3.164)	(2.509)	(3.540)
Some college education	-19.290	54.163	-0.159	44.211	139.951**	150.499***	166.780**	336.339***	80.188***	2.034	79.830***	44.211***
	(76.225)	(46.579)	(78.503)	(60.924)	(69.688)	(49.311)	(80.020)	(112.485)	(2.627)	(3.389)	(2.694)	(3.803)
College education or more	15.094	130.002***	61.012	137.790**	188.704**	171.089***	219.204**	421.667***	78.247***	11.462***	76.232***	52.618***
	(79.881)	(50.177)	(85.940)	(64.796)	(74.720)	(50.712)	(86.999)	(124.440)	(3.043)	(4.072)	(3.128)	(4.491)
Unemployment rate	28.471*	-29.493	18.054	-14.805	9.931	-34.212*	3.833	-21.024	-10.667***	-2.535***	-10.221***	-8.146***
	(17.264)	(19.182)	(19.481)	(22.292)	(17.248)	(18.034)	(19.580)	(29.196)	(0.847)	(0.926)	(0.863)	(1.037)
Switching parameter			0.353*	0.516**			0.178	1.328***			-0.176***	0.526***
			(0.210)	(0.256)			(0.258)	(0.298)			(0.008)	(0.013)

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. All specifications also include year and state fixed effects. The data come from the 2012–2019 American Community Surveys.

Table 3: Sharing Rule Derivatives

	Sharing rule: $\phi = \gamma_1 \log \omega_1 + \gamma_2 \log \omega_2 + \gamma_3 \eta + \gamma_4 s + \kappa(\mathbf{z})$								
	Both partners work			Only partner 1 works			Only partner 2 works		
	Male couples	Female couples	Different-sex couples	Male couples	Female couples	Different-sex couples	Male couples	Female couples	Different-sex couples
Derivative with respect to:									
Partner 1 net wage	-858.873	-28.756	-2,364.287***	-254.132	-15.787	-2,625.450***	1,491.768	820.201	-1,052.009***
	(1,064.959)	(176.951)	(67.545)	(546.225)	(116.049)	(68.923)	(1,875.130)	(1,323.523)	(54.952)
Partner 2 net wage	-225.162	-465.164	-278.317***	-2,544.351	-841.139	862.694***	-123.299	-163.137	-26.895***
	(1,479.486)	(758.383)	(33.549)	(2,771.572)	(1,387.945)	(68.609)	(807.914)	(788.569)	(9.549)
Virtual income	0.266	0.507	0.332***	0.783*	0.729	0.258***	0.197	0.241	0.054***
	(0.485)	(0.770)	(0.014)	(0.402)	(1.145)	(0.013)	(0.366)	(1.177)	(0.018)
Age difference	-6,214.927	-193.891	1,482.540***	-1,838.934	-106.444	1,646.304***	-4,606.450	-92.124	241.347***
	(4,664.908)	(301.656)	(54.668)	(3,393.970)	(418.892)	(66.557)	(3,569.820)	(237.186)	(77.395)

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. Section 3 details the derivations of the sharing rule derivatives above from the unrestricted labor supply equations. The derivatives with respect to ω_1 and ω_2 are calculated at the mean values of these variables.

Table 4: Structural Labor Supply Parameters

	Panel A: Both partners work					
	Male couples		Female couples		Different-sex couples	
	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners
Coefficient on:						
log(own net wage)	122.831***	77.409**	73.450**	112.132	-52.314***	259.718***
	(36.663)	(38.970)	(37.092)	(122.157)	(8.283)	(2.760)
log(Share of virtual income)	-0.001	-0.001	-0.002	-0.005	-0.003***	-0.002***
	(0.000)	(0.001)	(0.003)	(0.008)	(0.000)	(0.000)
Pareto weight[a]	1.007		1.041		0.960***	
	(0.008)		(0.212)		(0.003)	

	Panel B: Only partner 1 works					
	Male couples		Female couples		Different-sex couples	
	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners
Coefficient on:						
log(own net wage)	122.831***	261.616	73.450**	204.251	-52.314***	233.883***
	(36.663)	(442.088)	(37.092)	(766.486)	(8.283)	(2.693)
log(Share of virtual income)	-0.001	-0.003	-0.002	-0.009	-0.003***	-0.002***
	(0.000)	(0.006)	(0.003)	(0.040)	(0.000)	(0.000)
Pareto weight[a]	1.061		1.117		0.957***	
	(0.147)		(0.818)		(0.003)	

	Panel C: Only partner 2 works					
	Male couples		Female couples		Different-sex couples	
	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners
Coefficient on:						
log(own net wage)	165.721***	77.409**	154.588	112.132	-321.350**	259.718***
	(55.507)	(38.970)	(511.858)	(122.157)	(142.509)	(2.760)
log(Share of virtual income)	-0.001	-0.001	-0.004	-0.005	-0.017***	-0.002***
	(0.001)	(0.001)	(0.021)	(0.008)	(0.006)	(0.000)
Pareto weight[a]	1.002		0.989		0.685***	
	(0.011)		(0.602)		(0.092)	

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. Section 3 details the derivations of the structural parameters above from the unrestricted labor supply equations. The Marshallian hours elasticity is conditional on the share of unearned income.
a: Pareto weight test is $H_0 : \mu = 1$ so that significance stars indicate whether the pareto weight is significantly different than 1.

A Derivation of Sharing Rule Derivatives

Under the assumption that conditions 8a–8h hold for a given $\mathbf{z}$, the derivatives of the sharing rule are given by:

$$\frac{\partial\varphi}{\partial\eta} = \frac{D}{D-C}$$
$$\frac{\partial\varphi}{\partial s} = \frac{CD}{D-C}$$
$$\frac{\partial\varphi}{\partial\omega_1} = \frac{BC}{D-C}$$
$$\frac{\partial\varphi}{\partial\omega_2} = \frac{AD}{D-C}$$

Recall the definitions of A, B, C, and D are:

$$A = \frac{\frac{\partial\hat{L}^1}{\partial\omega_2}}{\frac{\partial\hat{L}^1}{\partial\eta}}, \qquad B = \frac{\frac{\partial\hat{L}^2}{\partial\omega_1}}{\frac{\partial\hat{L}^2}{\partial\eta}}, \qquad C = \frac{\frac{\partial\hat{L}^1}{\partial s}}{\frac{\partial\hat{L}^1}{\partial\eta}}, D = \frac{\frac{\partial\hat{L}^2}{\partial s}}{\frac{\partial\hat{L}^2}{\partial\eta}},$$

Under the function form in Equations 10 and 11, these values are:

$$A = \frac{a_2}{a_3\omega_2}, \qquad B = \frac{b_1}{b_3\omega_1}, \qquad C = \frac{a_4}{a_3}, \qquad D = \frac{b_4}{b_3}$$

Note that the denominator of the sharing rule derivates are the same ($D-C$), which can be written:

$$D-C = \frac{b_4}{b_3} - \frac{a_4}{a_3} = \frac{a_3b_4}{a_3b_3} - \frac{a_4b_3}{a_3b_3} = \frac{a_3b_4 - a_4b_3}{a_3b_3} = \frac{\Delta}{a_3b_3}$$

Where $\Delta \equiv a_3b_4 - a_4b_3$. Using this expression for $D-C$, the sharing rule derivatives can be written as:

$$\frac{\partial\varphi}{\partial\eta} = \frac{D}{D-C} = \frac{\frac{b_4}{b_3}}{\frac{\Delta}{a_3b_3}} = \frac{a_3b_4}{\Delta}$$
$$\frac{\partial\varphi}{\partial s} = \frac{CD}{D-C} = \frac{\frac{a_4b_4}{a_3b_3}}{\frac{\Delta}{a_3b_3}} = \frac{a_4b_4}{\Delta}$$
$$\frac{\partial\varphi}{\partial\omega_1} = \frac{BC}{D-C} = \frac{\frac{a_4b_1}{a_3b_3\omega_1}}{\frac{\Delta}{a_3b_3}} = \frac{a_4b_1}{\omega_1\Delta}$$
$$\frac{\partial\varphi}{\partial\omega_2} = \frac{AD}{D-C} = \frac{\frac{a_2b_4}{a_3b_3\omega_2}}{\frac{\Delta}{a_3b_3}} = \frac{a_2b_4}{\omega_2\Delta}$$

The above expressions are those in Equation 12, and solving this system of differential equations leads to the sharing rule in Equation 13.

B Derivation of the Marshallian Labor Supply Parameters

Recall that the sharing rule is:

$$\varphi = \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z})$$

The Marshallian labor supplies take the following form:

$$\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z}$$
$$\lambda^2 = \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z}$$

Beginning with λ^1, let $\alpha_2 = \frac{\Delta}{b_4}$. Expanding the expression for φ, we obtain:

$$\begin{aligned}
\lambda^1 &= \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z} \\
&= \alpha_1 \log \omega_1 + \frac{a_4 b_1}{b_4} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \frac{\Delta}{b_4} \kappa(\mathbf{z}) + \alpha_3 \mathbf{z} \\
&= \underbrace{\left(\alpha_1 + \frac{a_4 b_1}{b_4}\right)}_{=a_1} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \underbrace{\frac{\Delta}{b_4} \kappa(\mathbf{z}) + \alpha_3 \mathbf{z}}_{=\tilde{\alpha}_3(\mathbf{z})}
\end{aligned}$$

In order for λ^1 to be consistent with L^1, it must be the case that $\alpha_1 + \frac{a_4 b_1}{b_4} = a_1$, implying that $\alpha_1 = a_1 - \frac{a_4 b_1}{b_4} = \frac{a_1 b_4 - a_4 b_1}{b_4}$.

Similarly, moving to λ^2, let $\beta_2 = -\frac{\Delta}{a_4}$. Expanding the expression for φ, we obtain:

$$\begin{aligned}
\lambda^2 &= \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z} \\
&= \beta_1 \log \omega_2 - \frac{\Delta}{a_4} \eta + b_1 \log \omega_1 + \frac{a_2 b_4}{a_4} \log \omega_2 + \frac{a_3 b_4}{a_4} \eta + b_4 s + \frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3 \mathbf{z} \\
&= b_1 \log \omega_1 + \left(\beta_1 + \frac{a_2 b_4}{a_4}\right) \log \omega_2 + \left(\frac{a_3 b_4 - a_3 b_4 + a_4 b_3}{a_4}\right) \eta + b_4 s + \frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3 \mathbf{z} \\
&= b_1 \log \omega_1 + \underbrace{\left(\beta_1 + \frac{a_2 b_4}{a_4}\right)}_{=b_2} \log \omega_2 + b_3 \eta + b_4 s + \underbrace{\frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3 \mathbf{z}}_{=\tilde{\beta}_3(\mathbf{z})}
\end{aligned}$$

In order for λ^2 to be consistent with L^2, it must be the case that $\beta_1 + \frac{a_2 b_4}{a_4} = b_2$, implying that $\beta_1 = b_2 - \frac{a_2 b_4}{a_4} = \frac{a_4 b_2 - a_2 b_4}{a_4}$.

C Derivation of the Switching Parameters

Consider the situation in which only spouse 1 works. Following Donni (2003), I assume that if spouse 2 does not work then spouse 1's unrestricted labor supply function switches to:

$$L^{1,s} = A_0 + A_1 \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \mathbf{A}_5 \mathbf{z},$$

and the sharing rule switches to:

$$\varphi^{1,s} = K_1 \log \omega_1 + K_2 \log \omega_2 + K_3 \eta + K_4 s + \mathbf{K}(\mathbf{z})$$

In order for $L^{1,s}$ and $\varphi^{1,s}$ to be continuous along spouse 2's participation frontier, it must be the case that:

$$L^{1,s} = L^1 + g \cdot L^2$$

$$\varphi^{1,s} = \varphi + h \cdot L^2,$$

where g and h are free parameters. These conditions imply that:

$$\frac{\partial L^{1,s}}{\partial \omega_2} = \frac{\partial L^1}{\partial \omega_2} + g \frac{\partial L^2}{\partial \omega_2} \tag{33a}$$

$$\frac{\partial L^{1,s}}{\partial \eta} = \frac{\partial L^1}{\partial \eta} + g \frac{\partial L^2}{\partial \eta} \tag{33b}$$

$$\frac{\partial \varphi^{1,s}}{\partial \omega_2} = \frac{\partial \varphi}{\partial \omega_2} + h \frac{\partial L^2}{\partial \omega_2} \tag{33c}$$

$$\frac{\partial \varphi^{1,s}}{\partial \eta} = \frac{\partial \varphi}{\partial \eta} + h \frac{\partial L^2}{\partial \eta} \tag{33d}$$

Combined with the partial differential equation $\frac{\partial \varphi^{1,s}}{\partial \omega_2} - A \frac{\partial \varphi^{1,s}}{\partial \eta} = 0$, which holds within spouse 2's non-participation set, the functional forms of L^1 in equation 10 and L^2 in equation 11, and the sharing rule parameters in equation 12 we can see that:

$$
\begin{aligned}
&\frac{\partial \varphi}{\partial \omega_2}+h\frac{\partial L^2}{\partial \omega_2}=\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\left[\frac{\partial \varphi}{\partial \eta}+h\frac{\partial L^2}{\partial \eta}\right]\\
&\Rightarrow h=\frac{\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial \varphi}{\partial \eta}-\frac{\partial \varphi}{\partial \omega_2}}{\frac{\partial L^2}{\partial \omega_2}-\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}\\
&\Rightarrow h=\frac{\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{a_3 b_4}{\Delta}-\frac{a_2 b_4}{\omega_2 \Delta}}{\frac{\partial L^2}{\partial \omega_2}-\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}\\
&\Rightarrow h=\frac{\frac{b_4}{\Delta}\left[\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}a_3-\frac{a_2}{\omega_2}\right]}{\frac{\partial L^2}{\partial \omega_2}-\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}\\
&\Rightarrow h=\frac{\frac{b_4}{\Delta}\left[\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta}-\frac{\partial L^1}{\partial \omega_2}\right]}{\frac{\partial L^2}{\partial \omega_2}-\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}\\
&\Rightarrow h=\frac{b_4}{\Delta}\frac{\frac{\partial L^{1,s}}{\partial \omega_2}\frac{\partial L^1}{\partial \eta}-\frac{\partial L^1}{\partial \omega_2}\frac{\partial L^{1,s}}{\partial \eta}}{\frac{\partial L^2}{\partial \omega_2}\frac{\partial L^{1,s}}{\partial \eta}-\frac{\partial L^{1,s}}{\partial \omega_2}\frac{\partial L^2}{\partial \eta}}
\end{aligned}
$$

Plugging in equations 33a and 33b, we obtain:

$$
\begin{aligned}
&h=\frac{b_4}{\Delta}\frac{\left[\frac{\partial L^1}{\partial \omega_2}+g\frac{\partial L^2}{\partial \omega_2}\right]\frac{\partial L^1}{\partial \eta}-\frac{\partial L^1}{\partial \omega_2}\left[\frac{\partial L^1}{\partial \eta}+g\frac{\partial L^2}{\partial \eta}\right]}{\frac{\partial L^2}{\partial \omega_2}\left[\frac{\partial L^1}{\partial \eta}+g\frac{\partial L^2}{\partial \eta}\right]-\left[\frac{\partial L^1}{\partial \omega_2}+g\frac{\partial L^2}{\partial \omega_2}\right]\frac{\partial L^2}{\partial \eta}}\\
&\Rightarrow h=\frac{b_4}{\Delta}\frac{g\frac{\partial L^2}{\partial \omega_2}\frac{\partial L^1}{\partial \eta}-g\frac{\partial L^1}{\partial \omega_2}\frac{\partial L^2}{\partial \eta}}{\frac{\partial L^2}{\partial \omega_2}\frac{\partial L^1}{\partial \eta}-\frac{\partial L^1}{\partial \omega_2}\frac{\partial L^2}{\partial \eta}}\\
&\Rightarrow h=g\frac{b_4}{\Delta}
\end{aligned}
$$

The restriction that $L^{1,s} = L^1 + g \cdot L^2$ implies that:

$$
\begin{aligned}
L^{1,s} =& L^1 + gL^2 \\
=& a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \mathbf{a}_5 \mathbf{z} \\
& + gb_0 + gb_1 \log \omega_1 + gb_2 \log \omega_2 + gb_3 \eta + gb_4 s + g\mathbf{b}_5 \mathbf{z} \\
=& \underbrace{(a_0 + gb_0)}_{A_0} + \underbrace{(a_1 + gb_1)}_{A_1} \log \omega_1 + \underbrace{(a_2 + gb_2)}_{A_2} \log \omega_2 + \underbrace{(a_3 + gb_3)}_{A_3} \eta + \underbrace{(a_4 + gb_4)}_{A_4} s + \underbrace{(a_5 + gb_5)}_{A_5} \mathbf{z}
\end{aligned}
$$

The restriction that $\varphi^{1,s} = \varphi + h \cdot L^2$ and using $h = g\frac{b_4}{\Delta}$ implies that:

$$
\begin{aligned}
\varphi^{1,s} =& \varphi + g\frac{b_4}{\Delta} L^2 \\
=& \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z}) \\
& + g\frac{b_4}{\Delta}(b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + \mathbf{b}_5 \mathbf{z}) \\
=& \frac{1}{\Delta}[\underbrace{(a_4 b_1 + gb_1 b_4)}_{A_4 b_1} \log \omega_1 + \underbrace{(a_2 b_4 + gb_2 b_4)}_{A_2 b_4} \log \omega_2 + \underbrace{(a_3 b_4 + gb_3 b_4)}_{A_3 b_4} \eta + \underbrace{(a_4 b_4 + gb_4^2)}_{A_4 b_4} s] + \underbrace{\kappa(\mathbf{z}) + g\frac{b_4}{\Delta}\mathbf{b}_5 \mathbf{z}}_{\tilde{\kappa}^{1,s}(\mathbf{z})}
\end{aligned}
$$

Recall that the Marshallian labor supplies take the following form:

$$
\lambda^1 = \alpha_1^{1,s} \log \omega_1 + \alpha_2^{1,s} \varphi^{1,s} + \alpha_3^{1,s} \mathbf{z}
$$

$$
\lambda^2 = \beta_1^{1,s} \log \omega_2 + \beta_2^{1,s} (\eta - \varphi^{1,s}) + \beta_3^{1,s} \mathbf{z}
$$

Beginning with λ^1, let $\alpha_2^{1,s} = \frac{\Delta}{b_4}$. Expanding the expression for $\varphi^{1,s}$, we obtain:

$$
\begin{aligned}
\lambda^1 &= \alpha_1^{1,s} \log \omega_1 + \alpha_2^{1,s} \varphi^{1,s} + \alpha_3^{1,s} \mathbf{z} \\
&= \alpha_1^{1,s} \log \omega_1 + \frac{A_4 b_1}{b_4} \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \frac{\Delta}{b_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{1,s} \mathbf{z} \\
&= \underbrace{\left(\alpha_1^{1,s} + \frac{A_4 b_1}{b_4} \right)}_{=A_1} \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \underbrace{\frac{\Delta}{b_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{1,s} \mathbf{z}}_{=\tilde{\alpha}_3^{\mathbf{1,s}}(\mathbf{z})}
\end{aligned}
$$

In order for λ^1 to be consistent with $L^{1,s}$, it must be the case that $\alpha_1^{1,s} + \frac{A_4 b_1}{b_4} = A_1$, implying that $\alpha_1^{1,s} = A_1 - \frac{A_4 b_1}{b_4} = \frac{A_1 b_4 - A_4 b_1}{b_4}$.

Similarly, moving to λ^2, let $\beta_2^{1,s} = -\frac{\Delta}{A_4}$. Expanding the expression for $\varphi^{1,s}$, we obtain:

$$\begin{aligned}
\lambda^2 &= \beta_1^{1,s} \log \omega_2 + \beta_2^{1,s}(\eta - \varphi^{1,s}) + \beta_3^{1,s}\mathbf{z} \\
&= \beta_1^{1,s} \log \omega_2 - \frac{\Delta}{A_4}\eta + b_1 \log \omega_1 + \frac{A_2 b_4}{A_4} \log \omega_2 + \frac{A_3 b_4}{A_4}\eta + b_4 s + \frac{\Delta}{A_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{1,s}\mathbf{z} \\
&= b_1 \log \omega_1 + \left(\beta_1^{1,s} + \frac{A_2 b_4}{A_4}\right) \log \omega_2 + \left(\frac{A_3 b_4 - a_3 b_4 + a_4 b_3}{A_4}\right)\eta + b_4 s + \frac{\Delta}{A_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{1,s}\mathbf{z} \\
&= b_1 \log \omega_1 + \left(\beta_1^{1,s} + \frac{A_2 b_4}{A_4}\right) \log \omega_2 + \left(\frac{A_3 b_4 - A_3 b_4 + A_4 b_3}{A_4}\right)\eta + b_4 s + \frac{\Delta}{A_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{1,s}\mathbf{z} \\
&= b_1 \log \omega_1 + \underbrace{\left(\beta_1^{1,s} + \frac{A_2 b_4}{A_4}\right)}_{=b_2} \log \omega_2 + b_3\eta + b_4 s + \underbrace{\frac{\Delta}{A_4}\kappa(\mathbf{z}) + \beta_3^{1,s}\mathbf{z}}_{=\tilde{\boldsymbol{\beta}}_{\mathbf{3}}^{\mathbf{1,s}}(\mathbf{z})}
\end{aligned}$$

In order for λ^2 to be consistent with L^2, it must be the case that $\beta_1^{1,s} + \frac{A_2 b_4}{A_4} = b_2$, implying that $\beta_1^{1,s} = b_2 - \frac{A_2 b_4}{A_4} = \frac{A_4 b_2 - A_2 b_4}{A_4}$.

Now consider the situation in which only spouse 2 works. Following Donni (2003), I assume that if spouse 1 does not work then spouse 2's unrestricted labor supply function switches to:

$$L^{2,s} = B_0 + B_1 \log \omega_1 + B_2 \log \omega_2 + B_3\eta + B_4 s + \mathbf{B}_5\mathbf{z},$$

and the sharing rule switches to:

$$\varphi^{2,s} = P_1 \log \omega_1 + P_2 \log \omega_2 + P_3\eta + P_4 s + \mathbf{P}(\mathbf{z})$$

In order for $L^{2,s}$ and $\varphi^{2,s}$ to be continuous along spouse 1's participation frontier, it must be the case that:

$$\begin{aligned}
L^{2,s} &= L^2 + j \cdot L^1 \\
\varphi^{2,s} &= \varphi + k \cdot L^1,
\end{aligned}$$

where j and k are free parameters. These conditions imply that:

$$\frac{\partial L^{2,s}}{\partial \omega_1} = \frac{\partial L^2}{\partial \omega_1} + j\frac{\partial L^1}{\partial \omega_1} \tag{34a}$$
$$\frac{\partial L^{2,s}}{\partial \eta} = \frac{\partial L^2}{\partial \eta} + j\frac{\partial L^1}{\partial \eta} \tag{34b}$$
$$\frac{\partial \varphi^{2,s}}{\partial \omega_1} = \frac{\partial \varphi}{\partial \omega_1} + k\frac{\partial L^1}{\partial \omega_1} \tag{34c}$$
$$\frac{\partial \varphi^{2,s}}{\partial \eta} = \frac{\partial \varphi}{\partial \eta} + k\frac{\partial L^1}{\partial \eta} \tag{34d}$$

Combined with the partial differential equation $\frac{\partial \varphi^{2,s}}{\partial \omega_1} - B\frac{\partial \varphi^{2,s}}{\partial \eta} = -B$, which holds within spouse 1's non-participation set, the functional forms of L^1 in equation 10 and L^2 in equation 11, and the sharing rule parameters in equation 12 we can see that:

$$\frac{\partial \varphi}{\partial \omega_1} + k\frac{\partial L^1}{\partial \omega_1} = \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\left[\frac{\partial \varphi}{\partial \eta} + k\frac{\partial L^1}{\partial \eta}\right] - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}$$

$$\Rightarrow k = \frac{\frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial \varphi}{\partial \eta} - \frac{\partial \varphi}{\partial \omega_1} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}}{\frac{\partial L^1}{\partial \omega_1} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta}}$$

$$\Rightarrow k = \frac{\frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{a_3 b_4}{\Delta} - \frac{a_4 b_1}{\omega_1 \Delta} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}}{\frac{\partial L^1}{\partial \omega_1} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta}} = \frac{\frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\left[1 + \frac{a_4 b_3}{\Delta}\right] - \frac{a_4 b_1}{\omega_1 \Delta} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}}{\frac{\partial L^1}{\partial \omega_1} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta}}$$

$$\Rightarrow k = \frac{\frac{a_4}{\Delta}\left[\frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}} b_3 - \frac{b_1}{\omega_1}\right]}{\frac{\partial L^1}{\partial \omega_1} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta}}$$

$$\Rightarrow k = \frac{\frac{a_4}{\Delta}\left[\frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta} - \frac{\partial L^2}{\partial \omega_1}\right]}{\frac{\partial L^1}{\partial \omega_1} - \frac{\frac{\partial L^{2,s}}{\partial \omega_1}}{\frac{\partial L^{2,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta}}$$

$$\Rightarrow k = \frac{a_4}{\Delta}\frac{\frac{\partial L^{2,s}}{\partial \omega_1}\frac{\partial L^2}{\partial \eta} - \frac{\partial L^2}{\partial \omega_1}\frac{\partial L^{2,s}}{\partial \eta}}{\frac{\partial L^1}{\partial \omega_1}\frac{\partial L^{2,s}}{\partial \eta} - \frac{\partial L^{2,s}}{\partial \omega_1}\frac{\partial L^1}{\partial \eta}}$$

Plugging in equations 34a and 34b, we obtain:

$$
\begin{aligned}
k &= \frac{a_4}{\Delta}\frac{\left[\frac{\partial L^2}{\partial \omega_1}+j\frac{\partial L^1}{\partial \omega_1}\right]\frac{\partial L^2}{\partial \eta}-\frac{\partial L^2}{\partial \omega_1}\left[\frac{\partial L^2}{\partial \eta}+j\frac{\partial L^1}{\partial \eta}\right]}{\frac{\partial L^1}{\partial \omega_1}\left[\frac{\partial L^2}{\partial \eta}+j\frac{\partial L^1}{\partial \eta}\right]-\left[\frac{\partial L^2}{\partial \omega_1}+j\frac{\partial L^1}{\partial \omega_1}\right]\frac{\partial L^1}{\partial \eta}} \\
\Rightarrow k &= \frac{a_4}{\Delta}\frac{j\frac{\partial L^1}{\partial \omega_1}\frac{\partial L^2}{\partial \eta}-j\frac{\partial L^2}{\partial \omega_1}\frac{\partial L^1}{\partial \eta}}{\frac{\partial L^1}{\partial \omega_1}\frac{\partial L^2}{\partial \eta}-\frac{\partial L^2}{\partial \omega_1}\frac{\partial L^1}{\partial \eta}} \\
\Rightarrow k &= j\frac{a_4}{\Delta}
\end{aligned}
$$

The restriction that $L^{2,s} = L^2 + j \cdot L^1$ implies that:

$$
\begin{aligned}
L^{2,s} =& L^2 + jL^1 \\
=& b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + \mathbf{b}_5 \mathbf{z} \\
&+ ja_0 + ja_1 \log \omega_1 + ja_2 \log \omega_2 + ja_3 \eta + ja_4 s + j\mathbf{a}_5 \mathbf{z} \\
=& \underbrace{(b_0 + ja_0)}_{B_0} + \underbrace{(b_1 + ja_1)}_{B_1} \log \omega_1 + \underbrace{(b_2 + ja_2)}_{B_2} \log \omega_2 + \underbrace{(b_3 + ja_3)}_{B_3} \eta + \underbrace{(b_4 + ja_4)}_{B_4} s + \underbrace{(b_5 + ja_5)}_{B_5} \mathbf{z}
\end{aligned}
$$

The restriction that $\varphi^{2,s} = \varphi + k \cdot L^1$ and using $k = j\frac{a_4}{\Delta}$ implies that:

$$
\begin{aligned}
\varphi^{2,s} =& \varphi + j\frac{a_4}{\Delta}L^1 \\
=& \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z}) \\
&+ j\frac{a_4}{\Delta}(a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \mathbf{a}_5 \mathbf{z}) \\
=& \frac{1}{\Delta}[\underbrace{(a_4 b_1 + ja_1 a_4)}_{a_4 B_1} \log \omega_1 + \underbrace{(a_2 b_4 + ja_2 a_4)}_{a_2 B_4} \log \omega_2 + \underbrace{(a_3 b_4 + ja_3 a_4)}_{a_3 B_4} \eta + \underbrace{(a_4 b_4 + ja_4^2)}_{a_4 B_4} s] + \underbrace{\kappa(\mathbf{z}) + j\frac{a_4}{\Delta}\mathbf{a}_5 \mathbf{z}}_{\tilde{\kappa}^{2,s}(\mathbf{z})}
\end{aligned}
$$

Recall that the Marshallian labor supplies take the following form:

$$
\begin{aligned}
\lambda^1 &= \alpha_1^{2,s} \log \omega_1 + \alpha_2^{2,s} \varphi^{2,s} + \alpha_3^{2,s} \mathbf{z} \\
\lambda^2 &= \beta_1^{2,s} \log \omega_2 + \beta_2^{2,s} (\eta - \varphi^{2,s}) + \beta_3^{2,s} \mathbf{z}
\end{aligned}
$$

Beginning with λ^1, let $\alpha_2^{2,s} = \frac{\Delta}{B_4}$. Expanding the expression for $\varphi^{2,s}$, we obtain:

$$\begin{aligned}
\lambda^1 &= \alpha_1^{2,s} \log \omega_1 + \alpha_2^{2,s} \varphi^{2,s} + \alpha_3^{2,s} \mathbf{z} \\
&= \alpha_1^{2,s} \log \omega_1 + \frac{a_4 B_1}{B_4} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \frac{\Delta}{B_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{2,s} \mathbf{z} \\
&= \underbrace{\left(\alpha_1^{2,s} + \frac{a_4 B_1}{B_4} \right)}_{=a_1} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \underbrace{\frac{\Delta}{B_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{2,s} \mathbf{z}}_{=\tilde{\alpha}_3^{2,s}(\mathbf{z})}
\end{aligned}$$

In order for λ^1 to be consistent with $L^{1,s}$, it must be the case that $\alpha_1^{2,s} + \frac{a_4 B_1}{B_4} = a_1$, implying that $\alpha_1^{2,s} = a_1 - \frac{a_4 B_1}{B_4} = \frac{a_1 B_4 - a_4 B_1}{B_4}$.

Similarly, moving to λ^2, let $\beta_2^{2,s} = -\frac{\Delta}{a_4}$. Expanding the expression for $\varphi^{2,s}$, we obtain:

$$\begin{aligned}
\lambda^2 &= \beta_1^{2,s} \log \omega_2 + \beta_2^{2,s} (\eta - \varphi^{2,s}) + \beta_3^{2,s} \mathbf{z} \\
&= \beta_1^{2,s} \log \omega_2 - \frac{\Delta}{a_4} \eta + B_1 \log \omega_1 + \frac{a_2 B_4}{a_4} \log \omega_2 + \frac{a_3 B_4}{a_4} \eta + B_4 s + \frac{\Delta}{a_4} \tilde{\kappa}(\mathbf{z}) + \beta_3^{2,s} \mathbf{z} \\
&= B_1 \log \omega_1 + \left(\beta_1^{2,s} + \frac{a_2 B_4}{a_4} \right) \log \omega_2 + \left(\frac{a_3 B_4 - a_3 b_4 + a_4 b_3}{a_4} \right) \eta + B_4 s + \frac{\Delta}{a_4} \tilde{\kappa}(\mathbf{z}) + \beta_3^{2,s} \mathbf{z} \\
&= B_1 \log \omega_1 + \left(\beta_1^{2,s} + \frac{a_2 B_4}{a_4} \right) \log \omega_2 + \left(\frac{a_3 B_4 - a_3 B_4 + a_4 B_3}{a_4} \right) \eta + B_4 s + \frac{\Delta}{a_4} \tilde{\kappa}(\mathbf{z}) + \beta_3^{2,s} \mathbf{z} \\
&= B_1 \log \omega_1 + \underbrace{\left(\beta_1^{2,s} + \frac{a_2 B_4}{a_4} \right)}_{=B_2} \log \omega_2 + B_3 \eta + B_4 s + \underbrace{\frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3^{2,s} \mathbf{z}}_{=\tilde{\beta}_3^{2,s}(\mathbf{z})}
\end{aligned}$$

In order for λ^2 to be consistent with L^2, it must be the case that $\beta_1^{2,s} + \frac{a_2 B_4}{a_4} = B_2$, implying that $\beta_1^{2,s} = B_2 - \frac{a_2 B_4}{a_4} = \frac{a_4 B_2 - a_2 B_4}{a_4}$.