Structural Change in Production Networks and Economic Growth

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Abstract: This paper studies patterns of structural change in production networks for intermediate goods (input-output network) and new capital goods (investment network) and how these changes impact the aggregate growth rate of the economy. In each network, the fraction of production coming from goods sectors (e.g. manufacturing) has been declining over time, with a corresponding rise in the fraction of production coming from services sectors (especially in information services and professional/technical services). We study these changes within a multi-sector neoclassical growth model that admits a balanced growth representation with constant aggregate growth and allows for structural change in production networks. Our framework allows us to study the growth contribution of each production network. We find that, relative to intermediate inputs, "effective" productivity gains in the production of investment goods were gaining in importance over the period 1980-2010, while this pattern reversed in 2010. Our framework highlights that these patterns are in part driven by an exogenous component—direct relative productivity gains in specific sectors—and an endogenous component—structural change in the two endogenous production networks.

JEL: E23, O14, O40, O41
Keywords: structural change, input-output network, investment network, economic growth, technical change, balanced growth

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1. Introduction

Production networks—the distribution across sectors of the production and purchases of goods used in production—play a central role in shaping economic fluctuations and growth.[1] However, with changes in technology and the organization of production (innovation, automation, outsourcing, etc.) these production networks also change over time. In this paper, we study structural change in production networks and its implications for economic growth.

We focus on the impact of structural change occurring in two production networks, the input-output network of intermediate inputs and the investment network for new capital. We start by documenting how these production networks have changed over time in the U.S. We find that from 1947 to 2019 there has been a significant reduction in the amount of both intermediates and investment produced by goods sectors (i.e. manufacturing), offset by a significant increase in production of intermediates and investment by service sectors. These changes are large and have occurred within many sectors. In particular, the production of both intermediates and investment has become more concentrated in professional/technical services and information services. We also document that these changes are observed in other high income countries.

Given that the shift in production networks towards services coincides with the well-known pattern of structural transformation in consumption expenditures, we study these changes using an extension to the multi-sector neoclassical growth model, which has been commonly applied to study structural transformation.[2] In our model, each sector produces output using a combination of capital, labor, and intermediate goods. Output in each sector can be used for consumption, investment, or intermediates. Intermediates and investment in each sector are constructed from a bundle of intermediates or investment goods purchased from different sectors in the economy. Critically, the combination of intermediate and investment goods is modeled as a constant elasticity of substitution function, meaning that changes in relative prices (induced by changes in technology) can generate changes in the composition of intermediates and investment production (i.e. changes in production networks).

We characterize the assumptions that are needed for our model to admit an aggregate balanced growth

[1] Among many possible examples, see Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Acemoglu, Ozdaglar and Tahbaz-Salehi (2017), Baqaee and Farhi (2020), Baqaee and Farhi (2019), Bigio and La'o (2020), Foerster, Hornstein, Sarte and Watson (2019), and vom Lehn and Winberry (2022).

[2] For a survey of that literature, see Herrendorf, Rogerson and Valentinyi (2014).

path which coexists with structural change in production networks. Along the balanced growth path, the aggregate growth rate of the economy depends on a weighted average of the growth rates in sectors engaged in the production of investment and intermediate goods. Since we use CES aggregators, a critical assumption necessary for the aggregate balanced growth path is that technical change in at least one sector is non-constant as in Herrendorf, Rogerson and Valentinyi (2021) (henceforth HRV).[3] We provide empirical evidence to validate this and several of the necessary assumptions for the balanced growth path. Further, the balanced growth path allows for the aggregate capital stock and aggregate quantity of intermediates to grow at different rates. These differences reflect potentially different growth rates in intermediates-specific and investment-specific technical change. We show that the relative price of intermediates to investment can provide a simple heuristic to assess the relative importance of each type of technical change in the aggregate rate of growth.

With this balanced growth representation, we calibrate the model to obtain the best fit to structural change observed in the data. This exercise sheds new light on several features of growth and structural change. We find that recent growth has been increasingly driven by TFP growth in sectors producing intermediate goods. This suggests that production networks play an increasingly important role in shaping economic growth, which, for example, may be relevant for economies emerging from supply-chain issues during the COVID-19 pandemic.

Finally, we observe that although a balanced growth path representation is tractable and convenient for studying these long run patterns, it also necessarily abstracts from significant heterogeneity across sectors, such as the intensity with which they use different intermediate and investment inputs in production. As a result, we plan to also solve a perfect foresight transition path exercise for the U.S economy which allows for greater heterogeneity across sectors and a more empirical realism for a larger number of sectors. We expect to find that our main conclusions continue to hold and are likely even stronger in this exercise, with growth becoming increasingly intermediates-oriented over time.

Related Literature. Our paper is at the intersection of two large literatures—the study of production networks as propagation mechanisms for economic fluctuations and growth and the study of long-run structural

[3]This assumption is a departure from the framework considered in Ngai and Pissarides (2007), who show that with constant TFP growth rates in each sector, the multi-sector neoclassical growth model cannot have structural change in the production of intermediate goods along the balanced growth path.

transformation from manufacturing to services. Many papers in the production networks literature have emphasized the role of static production networks for shaping fluctuations. The large literature on structural change has focused on multi-sector models that either abstract from production networks (or treat them implicitly in a "value added" specification) or do not allow for these networks to change over time. Our contribution lies in studying the intersection of these two phenomena.

While a significant literature has focused on how production networks play important roles in propagating short-run fluctuations over the business cycle, there has also been significant attention given to the role of production networks, particularly the input-output network, in shaping long-run growth.[4] Our paper is closely related to Ngai and Samaniego (2009), who focus on how technical change in sectors producing intermediate goods can impact the composition of long run growth, particularly the role played by investment-specific technical change, as described in Greenwood, Hercowitz and Krusell (1997).[5] Similarly, the balanced growth path of our model highlights how composition of aggregate economic growth depends on the growth rate of technical change in all sectors producing investment and intermediate goods. Our work is also closely related to recent work by Foerster et al. (2019), who study a multi-sector model with fixed production networks, and show the important role played by production networks, particularly the investment network, in determining which sectors' TFP growth has the largest effect on aggregate growth. They also consider growth accounting exercises, analyzing which sectors have contributed to economic growth in the postwar period, with a particular focus on the recent slowdown in per capita GDP growth. While they study the impact of a static production network for changing patterns of TFP growth across sectors, we focus instead on the impact of *changes* to the intermediates and investment production networks on the patterns of TFP growth over time.

While comparatively little attention has been paid to structural change in production networks over time, we are not the first to observe these patterns. Berlingieri (2013) documents the rising importance of the professional/business services sector in the production of intermediate goods and notes it may be tied to a rise in outsourcing. Galesi and Rachedi (2019) observe a similar pattern and study how changes in the composition of intermediate goods may matter for the effectiveness of monetary policy in recent decades.

[4]See, for example, Ngai and Samaniego (2009), Moro (2012), Moro (2015), Foerster et al. (2019), and Duarte and Restuccia (2017).

[5]This channel can also be generally observed in Ngai and Pissarides (2007).

vom Lehn and Winberry (2022) document changes in the U.S. investment network over time, though focus on the propagation impacts of a fixed network for changing business cycle patterns over time.

HRV is closest to our paper in that it embeds structural change in investment at the two-sector level (effectively, a simple investment network) into a standard model of structural change and studies the joint implications of structural change in consumption and investment. In their framework, consumption and investment are measured in "value-added" terms, where the input-output networks are implicitly embedded in measurement. In contrast, we explicitly model the investment production network. We also study a larger number of sectors and focus on the commonality of sectors where both intermediates and investment production is growing and consider non-balanced growth implications of the model.

2. Data & Empirical Evidence

Our study of structural change focuses on two production networks—the production and purchases of intermediate goods (the input-output network) and the production and purchases of new capital goods (the investment network). These networks are measured as matrices, where element (i, j) of the matrix reports the total expenditures by sector j on goods or services that were produced by sector i.

2.1. Data

Our primary data source for measuring these production networks in the U.S. are the Make and Use Tables from the BEA Input Output Database, which contains data for each sector on the value of gross output, value added, intermediate input purchases from each other sector, and final uses (consumption, investment, etc.) for each sector's production.[6] The database begins in 1947 and we study patterns running through 2020. For measuring the investment network, we also use data from vom Lehn and Winberry (2022), who use BEA Input Output and BEA Fixed Assets data to construct a time series of the investment network that covers a similar time frame and level of sector detail; we extend their data to run through 2020.[7] Additional details regarding data sources and measurement of these production networks is available

[6]This data includes imported intermediates, consumption and investment. We do not attempt to remove these because our primary interest is in the sectors producing inputs to intermediates and investment and not in the country of origin for production.

[7]As we note in Appendix A, we modify the investment network as constructed in vom Lehn and Winberry (2022) by utilizing the Make Table to map commodities produced into the sectors producing them. The investment network in vom Lehn and Winberry

Table 1: Average Share of Intermediates and Investment Production: Avg. 1947-2019

Goods Producing Sectors (NAICS Codes)	% of Prod. Int.	% of Prod. Inv.	Service Producing Sectors (NAICS Codes)	% of Prod. Int.	% of Prod. Inv.
Agriculture, forestry, fishing and hunting (11)	5.2	0.0	Wholesale trade (42)	5.4	3.7
Oil and gas extraction (211)	3.0	0.3	Retail trade (44-45)	1.8	1.3
Mining, except oil and gas (212)	1.2	0.3	Transport and warehousing (48-49, minus 491)	5.5	0.9
Support activities for mining (213)	0.1	2.0	Information (51)	4.2	5.8
Utilities (22)	2.3	0.5	Finance and insurance (52)	7.1	0.1
Construction (23)	1.7	36.2	Real estate (531)	4.4	1.7
Wood products (321)	1.3	0.5	Rental and leasing services (532-533)	1.2	0.0
Non-metallic mineral products (327)	1.5	0.1	Professional and technical services (54)	4.9	9.7
Primary metals (331)	4.4	0.1	Management of companies and enterprises (55)	2.8	0.1
Fabricated metal products (332)	3.8	1.2	Administrative support and waste services (56)	3.1	0.1
Machinery (333)	1.6	8.7	Educational services (61)	0.2	0.2
Computer and electronic products (334)	2.2	6.5	Health services (62)	0.2	0.1
Electrical equipment manufacturing (335)	1.1	1.3	Arts, entertainment and recreation services (71)	0.4	0.1
Motor vehicles manufacturing (3361-3363)	3.0	8.7	Accommodation services (721)	0.5	0.0
Other transportation equipment (3364-3369)	1.4	4.4	Food services (722)	0.9	0.0
Furniture and related manufacturing (337)	0.3	1.1	Other private services (81)	1.5	0.1
Misc. manufacturing (339)	0.8	1.2	Federal government (n/a, but incl. 491)	1.0	0.5
Food and beverage manufacturing (311-312)	4.4	0.1	State and local government (n/a)	1.3	0.8
Textile manufacturing (313-314)	1.8	0.3			
Apparel manufacturing (315-316)	0.6	0.0			
Paper manufacturing (322)	2.3	0.0			
Printing products manufacturing (323)	1.1	0.5			
Petroleum and coal manufacturing (324)	2.6	0.1			
Chemical manufacturing (325)	4.2	0.4			
Plastics and rubber products (326)	1.6	0.1			

Notes: The table reports the average share of total intermediate and investment production by 43 consistent sectors. Individual components may not exactly sum to totals due to rounding. Sectors are classified according to the NAICS-based BEA codes, with 2007 NAICS codes listed in parentheses. Government sectors as defined by the BEA do not have naturally corresponding NAICS codes. See Appendix A for details of the data construction.

in Appendix A.

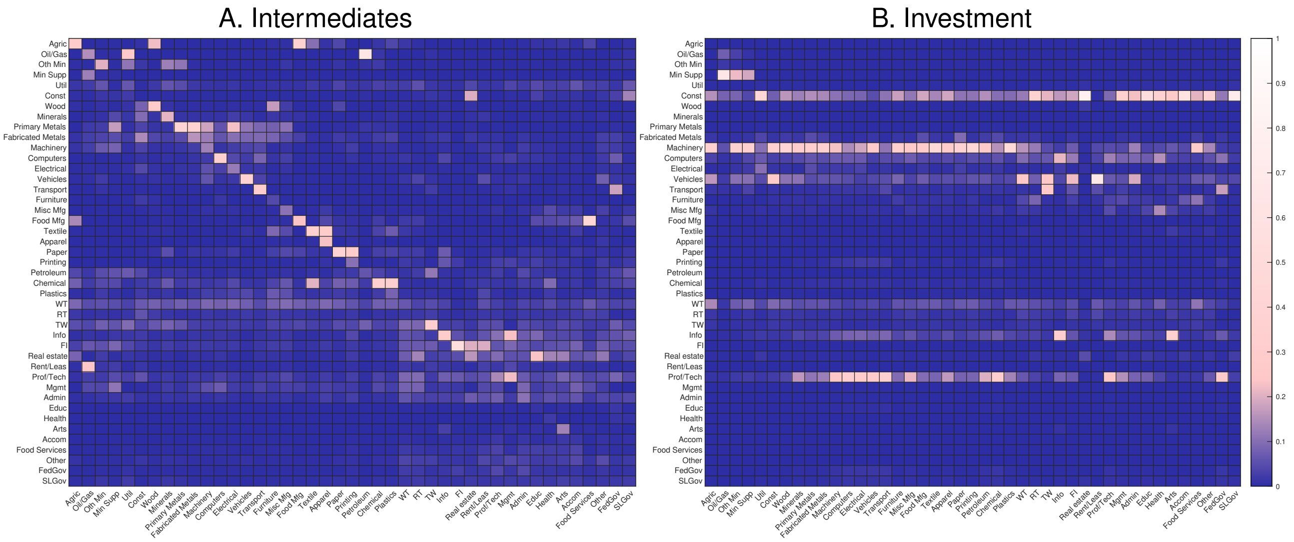
Our data provides consistent coverage of 43 NAICS-defined sectors of the economy, including agriculture and government; Table 1 lists each of the 43 sectors and their corresponding NAICS codes.[8] We consider evidence both at this level of disaggregation, as well as at a more aggregated two-sector level. When we aggregate to two sectors, we define "goods" sectors as all agriculture, mining, utilities, construction and manufacturing sectors (25 in total) and define "services" as all remaining sectors (18 in total).

To visualize the average distribution of producers of intermediate and investment goods, Figure 1 plots

(2022) does not distinguish between commodities produced and industries producing them. While this adjustment is necessary to more accurately represent the network structure of production, its quantitative impacts on the measured investment network are minor because the Make Table is largely diagonal.

[8]More recent vintages of the BEA Input Output database allow for greater sectoral detail, but given our interest in structural change over the long-run, we focus on these 43 sectors, which can be observed for all years of the data.

Figure 1: Heatmaps of Average Scaled Production Networks, 1947-2020



Notes: Panel A shows the input-output network for intermediate goods while panel B shows the investment network for new capital goods. Each (i, j) element in the matrix shows the fraction of total expenditure by sector j (columns) coming from producing sector i (rows).

a heatmap of the average input-output network (panel A) and investment network (panel B) from 1947-2020. To focus on the sectors which are particularly important producers, the heatmap follows a common convention in the networks literature and scales each column so that the rows sum to 1; thus each entry (i, j) in Figure 1 represents the share of total expenditures in sector j produced by sector i. This scaling abstracts from information regarding the distribution of expenditures across sectors; however, this information is implicit in the average share of total intermediates and investment produced by each sector from 1947-2020, reported in Table 1, and we incorporate this information when assessing structural change in these networks.

Both the input-output and investment network are fairly sparse; for any given sector, the majority of investment and intermediates are purchased from a small set of sectors.[9] For the investment network, the distribution of investment producers is fairly similar across sectors—most sectors purchase investment goods from a collection of prominent investment hubs. However, for the input-output network, there is much more sector-specificity as to which sectors are important intermediates suppliers. In particular, we observe that goods sectors play a larger role as intermediates producers for other goods sectors and services sectors

[9] vom Lehn and Winberry (2022) provide additional discussion of these investment hubs and explicitly contrast the sparseness of these two networks using eigenvalue centrality measures and the skewness of weighted outdegrees.

similarly play a larger role as intermediates producers for other services sectors. This includes a non-trivial diagonal component in the network, suggesting that within-sector production of intermediate inputs is important for many sectors.

2.2. Structural Change in Production Networks

Our primary interest with these production networks is to understand how the producers of intermediates and investment goods have changed over time. To measure structural change in these production networks, we compute the share of total production value (measured in current dollars) in intermediates or investment that was produced by each sector.[10] We then measure changes in this share of production from 1947 to 2019.[11] Each sector's share of total production reflects the role that sector plays in the production of intermediates or investment goods for each purchasing sector (as visualized in Figure 1), as well as the distribution of intermediates or investment spending across those purchasing sectors.

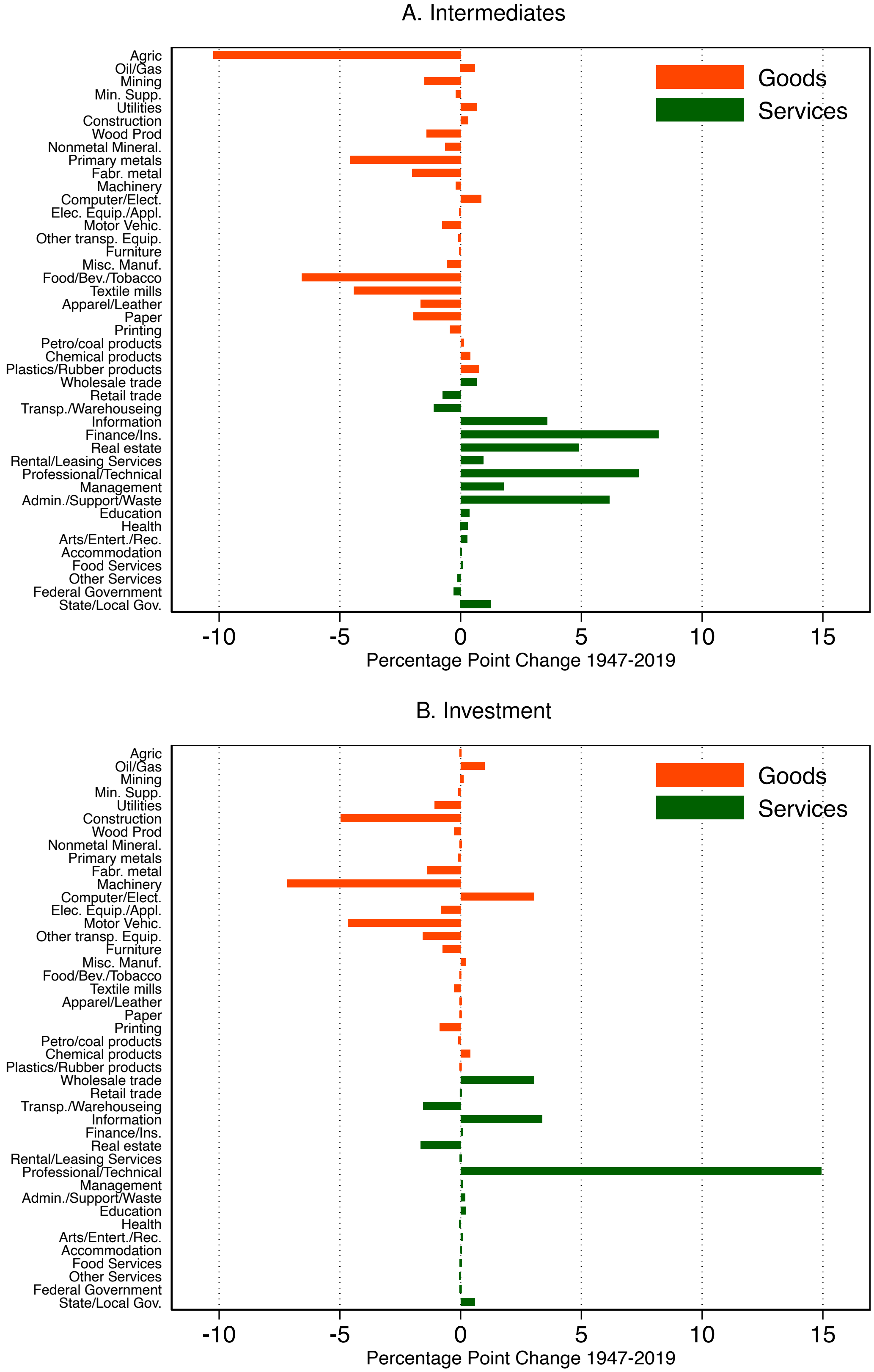
Figure 2 plots the difference between in 1947 and 2019 in the share of total production value coming from each sector in the input-output network (upper panel) and the investment network (bottom panel). Each network has seen significant structural change over this time period, with a significant increase in the share of production coming from services sectors (green bars) and an offsetting decrease in the share of production from goods sectors (red bars). For intermediate goods, the largest increases in production share are in information services, finance/insurance, real estate, professional/technical services, and administrative support services; the largest decreases occurred in agriculture, primary metals, food and beverage manufacturing, and textile manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest decreases are in machinery, construction, and motor vehicle manufacturing.

Given that changes in these production networks are primarily characterized by a shift in production from goods to services sectors, we aggregate the data to two sectors, goods and services. This allows us to

[10] That is, in each year, we take the unscaled version of the matrices plotted in Figure 1, sum along all columns to compute total production of intermediates or investment by each sector, and then compute each producing sector's fraction of total production value.

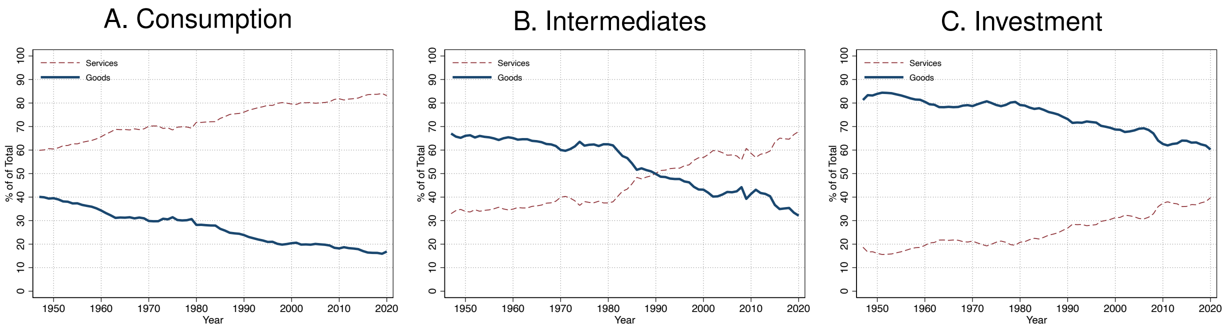
[11] Although our data runs through 2020, we compute these long differences through the year 2019 to avoid any unusual end point with the onset of the COVID-19 pandemic in 2020. More generally, using changes in a moving average of the share of production done by each sector generates similar results. We report the entire time series of changes at a more aggregated level (goods and services) in Figure 3, and for selected sectors at a more detailed level in Appendix A.

Figure 2: Changes in Production Share of Intermediates and Investment: 1947-2019



Notes: Each bar represents the change in the share of intermediates (upper panel) or investment (lower panel) produced by each sector between 1947 and 2019. Red bars: goods sectors; green bars: services sectors.

Figure 3: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, 1947-2020



Notes: Each plots the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue, solid line) and the services sector (red, dotted line). A full listing of which sectors are included in the goods and services sectors can be seen in Table 1.

document how the share of production of intermediate and investment produced by each broad sector has changed over time. Figure 3 plots the time series of the fraction of total production value of consumption, intermediates, and investment produced by goods and services.[12]

While there is heterogeneity in the level of how much production comes from the services sector across consumption, intermediates, and investment, the trends in these shares over time are similar. Services has long produced the majority of consumption expenditures, whereas it has only become the majority of intermediates production since 1990 and still produces a minority of all investment goods. But for each final use of output, the fraction produced by the services sector is rising over time. These changes are substantial in magnitude, with the services share of investment and consumption rising by around 20 percentage points and the services share in intermediates increasing by nearly 35 percentage points.

This rising services share in the production of intermediates and investment reflects two changes. First, changes within sectors generate a rising services share, as each sector reallocates its spending on intermediates and investment away from goods sectors to services sectors. Second, changes in the distribution of total spending on intermediates and investment across sectors, increases the share of production by services, given that total spending by services sectors has risen over time and the intensity of services usage is higher

[12] Both our measures of investment and consumption production by each sector are more consistent with the "final expenditure" approach in the structural change literature, as we focus on final producers of consumption and investment and do not incorporate input-output relationships in the measurement of consumption or investment production (as in the "value added" approach of Herrendorf, Rogerson and Valentinyi (2013)). However, to measure these final expenditures, we use income side data measured in producer's prices, not purchaser's prices from final expenditures. Later in this section, we discuss structural change in consumption and investment value added and how structural change in intermediates production affects these measures.

Table 2: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment

				Decomposition	
	1947	2019	Δ	within	between
Intermediates	0.33	0.67	0.34	0.19	0.15
				(55%)	(45%)
Investment	0.19	0.38	0.19	0.19	-0.00
				(100%)	(0%)

Notes: The table reports the shift-share decomposition described in equation (1) for the production share of services. Individual components may not exactly sum to totals due to rounding.

among services sectors (primarily for intermediates).[13]

We quantify the importance of these two channels for structural changes in production networks in two ways. First, we do a shift-share decomposition of the services production share to isolate changes occurring within and between sectors. Define I_{st} to be the total expenditure on intermediates (or investment) produced by services (s) in year t and I_t to be total spending on intermediates in year t. Changes in the fraction of total intermediates spending being produced by services, $Serv_t = I_{st}/I_t$ can be decomposed into changes in the share of intermediates spending on services within each purchasing industry j, $Serv_{jt} = I_{sjt}/I_{jt}$, and the importance of industry j's spending on intermediates as a fraction of all intermediates expenditures, $\omega_{jt} = I_{jt}/I_t$. This decomposition can be expressed using the standard formula for a shift-share decomposition:

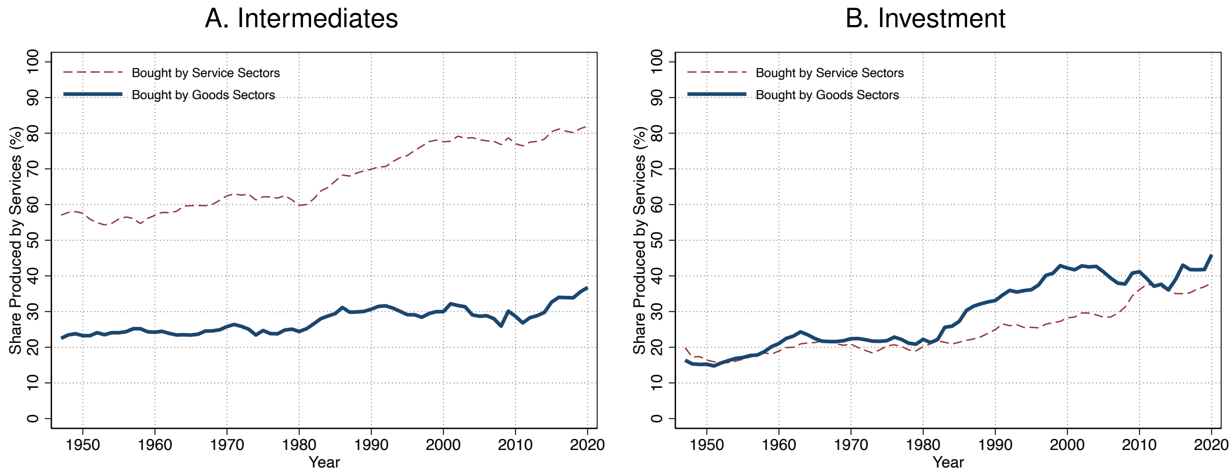
$$\Delta Serv_t = \underbrace{\sum_j \left(\overline{\omega}_j \Delta Serv_{jt}\right)}_{\text{within}} + \underbrace{\sum_j \left(\overline{Serv}_j \Delta \omega_{jt}\right)}_{\text{between}} \tag{1}$$

where $\Delta x = x_{2019} - x_{1947}$ is the change in x and $\overline{x} = (x_{2019} + x_{1947})/2$ the average of x in the two periods 1947 and 2019. We perform the decomposition using spending patterns across all goods and services; results using all 43 sectors are similar, and are available in Appendix A.

The results of this decomposition are presented in Table 2 for changes in the services share of production

[13] For this discussion, we focus solely on intermediates and investment, because for consumption, there does not exist a "between sector" component to structural change, since final users of consumption are households, who are not naturally identified as connected to a particular production sector.

Figure 4: Within Sector Changes in the Services/Goods Share of Production, Intermediates and Investment, 1947-2020



Notes: Each figure plots the fraction of services production of either investment or intermediates for expenditures by the goods sector (blue, solid line) and the services sector (red, dotted line). A full listing of which sectors are included in the goods and services sectors can be seen in Table 1.

in intermediates and investment between 1947 and 2019. For investment, all changes over time in the services production share are due to changes within sectors; for intermediates, the within sector component contributes roughly half of the change over time.[14] This evidence accords with the production network patterns shown in Figure 1, which showed stronger heterogeneity across goods and services sectors for how intermediates are produced, and thus a larger role for between sector changes in the services share of production.

The second way we illustrate the relative importance of within and between sector changes is to plot the service share of intermediates and investment expenditures within both goods and services sectors in Figure 4. The increase in the service share of production is observed within goods and services sectors for both intermediates and investment. However, the within-sector changes in the production of intermediates have been much smaller in goods than in services, whereas the magnitude of within sector changes across goods and services sectors is similar for the services share of investment production.[15] We address both within and

[14]As seen in Appendix A, when this decomposition is done using heterogeneity across all 43 sectors, still roughly 75% of the changes in investment are attributed to within-sector changes. The within-sector change component for intermediates is only slightly lower, still accounting for about 50% of the change.

[15]In Appendix A, we provide added detail about which sectors have seen the largest increase in the share of their intermediates and investment spending on services production.

between sector channels for structural change, including sectoral heterogeneity in the intensity of services usage as intermediate inputs and investment in quantitative applications of our model in Section 6.

Finally, we analyze the importance of changes in the input-output network for "value-added" measures of structural change in consumption and investment. As explained in Herrendorf et al. (2013), a value-added approach to measuring sectoral production of consumption and investment focuses not only on the sectors who produce the final good or service, but the network of sectors contributing intermediate goods needed to produce that final good. Value-added vectors of sectoral production of consumption and investment (denoted in current dollars), $\mathbf{c^{VA}}$ and $\mathbf{x^{VA}}$, are constructed using input-output data using the following equations:

$$\mathbf{c^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}\mathbf{c} \tag{2}$$

$$\mathbf{x^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}\mathbf{x} \tag{3}$$

where $\mathbf{c}$ and $\mathbf{x}$ are vectors of final production of consumption and investment, respectively, by each sector, $\mathbf{I}$ is the identity matrix, $\mathbf{v}$ is a diagonal matrix of the share of value added in gross output in each sector, and $\mathbf{\Gamma}$ is a matrix of input-output relationships, where the $(i, j)th$ element of $\mathbf{\Gamma}$ is the ratio of intermediates purchased by sector j from sector i to the total gross output in sector j.[16] The BEA Input Output database has information sufficient to compute all of these objects.[17] The matrix $(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}$ is commonly referred to as the total requirements matrix, or Leontief inverse, and captures the full chain of direct and indirect contributions of each sector to final production of consumption and investment goods.[18]

Given this construction of value-added consumption and investment, structural change in consumption value added or investment value added can occur because of changes in $\mathbf{v}$, reflecting changes in the ratio of value added to gross output in each sector, changes in $(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}$, reflecting changes in input-output networks, or changes in $\mathbf{c}$, reflecting changes in the final producers of consumption or investment goods. To decompose the importance of each of these changes, we consider a counterfactual exercise where we allow

[16] That is, the $(i, j)th$ element of $\mathbf{\Gamma}$ is the $(i, j)th$ entry of the scaled input-output network presented in Figure 1 multiplied by the ratio of total spending on intermediates to total gross output in sector j.

[17] We first compute all of these objects at the 43 sector level, and then analyze structural change at the 2 sector level, aggregated up from consumption value added and investment value added constructed at the 43 sector level.

[18] Our definition of the total requirements matrix takes as given that we have already properly adjusted both the intermediate input payments and final uses (consumption, investment) in the Use Table by the Make table to account for which sectors produce which final commodities; details of how we construct these objects are available in Appendix A.

Table 3: Decomposing Structural Change in Services Share of Value Added Measures of Consumption and Investment

Services Share of:	1947	2019	Δ	% of Total
Consumption Value Added	0.60	0.83	0.23	
Final Prod. only	0.60	0.76	0.16	70%
Input-Output only	0.60	0.70	0.11	46%
VA share only	0.60	0.58	-0.02	-6%
Investment Value Added	0.35	0.53	0.18	
Final Prod. only	0.35	0.48	0.13	71%
Input-Output only	0.35	0.45	0.10	54%
VA share only	0.35	0.32	-0.03	-20%

Notes: This table reports the share of value added based consumption and investment produced by the services sector and how this changes over time due to changes in each component of the value added measure (as seen in Equations (2) and (3)). Changes generated by each of the three components—final producers ("Final Prod. only"), the Total Requirements Matrix ("Input-Output only"), and value added shares of gross output ("VA shares only")—are computed by holding fixed all other components at their values in 1947 and varying one component at a time. The "% of total" column refers to the change in each component divided by the change in total consumption or investment value added. Because of the non-linear nature of the decomposition, the total of each individual component will not sum to the actual total.

only one of each of these three components to vary over time and hold fixed the two other components at their values in the initial year of our data, 1947. Since the construction of consumption and investment value added is non-additive in nature, the contributions of each of these three terms will not necessarily sum to 1.

Table 3 presents the contribution of each of these three forces to the change in the share of consumption value added and investment value added produced by the services sector between 1947 and 2019. We find that the contribution of changes in the input-output network account for roughly 45-55% of the rising share of services production of consumption and investment value added. This total contribution is potentially slightly inflated because the contribution of each component sums to more than the total change in the services share of consumption and investment value added. However, if we compute the contribution of changes in the input-output network as a fraction of the sum of changes in each component, we still find that input-output changes account for 40-50% of structural change in consumption and investment value added.

Appendix A contains additional empirical results regarding structural change in production networks, include a time series of expenditure shares for sectors whose production share of intermediates or investment has risen or fallen the most and detailed results on where within-sector changes in purchases from services sectors have been the largest. We also use data from the World Input Output Database (Timmer, Dietzenbacher, Los, Stehrer and De Vries (2015)) and show that these changes in production networks are not unique to the United States, but are observed more broadly throughout other high income nations in Europe and Asia.

3. Model

This section describes a multi-sector extension of the neo-classical growth model with intermediate inputs in production and network production of investment. The model is a discrete-time generalization of HRV with N production sectors and extensions to account for production networks in intermediate inputs. We first consider a general version of the model, but discuss additional assumptions and restrictions necessary for a balanced growth path in Section 4.

3.1. Technology

For each sector j, gross output, Q_{jt}, is produced using capital, K_{jt}, labor L_{jt}, and a bundle of intermediate goods M_{jt} according to the following Cobb-Douglas production function:

$$Q_{jt} = A_{jt} \left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j} \right)^{\alpha_j} M_{jt}^{1-\alpha_j}, \tag{4}$$

where A_{jt} is exogenous TFP in sector j.

The intermediates bundle for each sector, M_{jt}, is produced by an "intermediates bundling" sector for sector j, which aggregates intermediate goods from all sectors using a CES technology:

$$M_{jt} = A_{jt}^{M} \left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}} \right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}, \tag{5}$$

where ϵ_{Mj} is the elasticity of substitution between sectoral inputs in the production of intermediate goods for sector j, $\omega_{ij} \in (0,1)$ (with $\sum_i \omega_{Mij} = 1$) determines the relative importance of inputs from each sector in producing intermediates, M_{ijt} represents intermediate inputs used in sector j from sector i at time t, and A_{jt}^{M} represents exogenous intermediates-specific technical change for sector j. Although a Cobb-Douglas specification for bundling intermediate goods is common in many multi-sector growth models, we allow for a CES specification to allow for the study of structural change in the input-output network.[19] We specify the bundling of intermediate goods as a separate sector with its own technical change to allow for relative price movements in intermediate goods compared to the implied price index using the price of final goods in each sector.

Capital is sector-specific and evolves according to a standard law of motion, with depreciation rate δ_j in sector j:

$$K_{jt+1} = (1 - \delta_j)K_{jt} + X_{jt}. \tag{6}$$

Investment, X_{jt}, is produced in an "investment bundling" sector for sector j's capital with the following aggregation technology:

$$X_{jt} = A_{jt}^X \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}} \right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}, \tag{7}$$

where ϵ_{Xj} is the elasticity of substitution between sectors in the production of investment in sector j and $\omega_{Xj} \in (0, 1)$ (with $\sum_i \omega_{Xij} = 1$) determines the relative importance of inputs from each sector in producing investment. A_{jt}^X represents exogenous investment-specific technical change for sector j's capital good. Again, the specification of investment production as CES allows for structural change in the production of investment and the presentation of a separate "bundling" sector allows for movements in the relative price of investment.

3.2. Preferences

There is an infinitely lived representative household with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t U(\{C_{jt}\}_{j=1}^N), \tag{8}$$

[19] Johnson (2014), Boehm, Flaaen and Pandalai-Nayar (2019), Atalay (2017), Carvalho, Nirei, Saito and Tahbaz-Salehi (2021), Miranda-Pinto (2021) and Miranda-Pinto and Young (2022) also allow for a CES structure over intermediates in the context of studying short-run fluctuations.

where $0 < \beta < 1$ is the discount rate and C_{jt} is the consumption good produced by sector j. We assume that the period utility function, $U(\{C_{jt}\}_{j=1}^{N})$ follows a log CES structure, with

$$U(\{C_{jt}\}_{j=1}^{N}) = \ln\left(\left[\sum_{j}\omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C - 1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C - 1}}\right), \tag{9}$$

where ϵ_C represents the elasticity of substitution between consumption goods and $\omega_{Cj} \in (0,1)$ determines the relative importance of consumption goods from each sector in aggregate consumption. Our framework abstract from preferences over leisure; we assume the household inelastically supplies one unit of labor each period.

Alternatively, household preferences could be specified with a PIGL indirect utility function, as introduced to the structural change literature in Boppart (2014). However, as can be seen in the next section, along the balanced growth path, the form of household preferences have no impact on either the growth rate of GDP or structural change in production networks.[20] Given this result and the focus of our paper, we keep the structure of household preferences is simple as possible and adopt the more tractable log CES case as our baseline.[21]

3.3. Equilibrium

We study the competitive equilibrium of this economy with representative profit-maximizing firms in all markets. The price of final output in each production sector j is denoted by P_{jt}; the price of the intermediates bundle is given by P_{jt}^{M}. The household owns the capital stock and accumulates capital in sector j by purchasing new investment goods from the investment bundling firm for sector j at price P_{jt}^{X}. The household rents sector-specific capital to each sector j at a rental price R_{jt}. Since labor is common to each sector and freely mobile, there is a single wage paid to the household, denoted by W_t. We provide a full listing of equilibrium conditions in Appendix B.

[20]However, the aggregate balanced growth path derived in the subsequent section is identical under PIGL preferences where we restrict the number of sectors in preferences to two, because the PIGL specification only allows for two distinct income elasticities. Results available upon request.

[21]More recently, Comin, Lashkari and Mestieri (2021) consider structural change under a non-homothetic CES preference structure. However, as shown in their paper, this specification is only consistent with a balanced growth path (or "constant growth path") as $t \to \infty$.

In equilibrium, each labor, capital, intermediate bundling and investment bundling market clears. Additionally, market clearing for final production in each sector, j, implies the following relationship:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{10}$$

This resource constraint for final output implies that each sector's final good can be used as consumption for the household, or an input to intermediate and investment bundles for all other sectors.

Given constant returns and competitive markets, it is straightforward to show that the price indices for the bundle of intermediate goods, P_{jt}^M, and the bundle of investment goods, P_{jt}^I, will be given by:

$$P_{jt}^M = \frac{1}{A_{jt}^M} \left(\sum_i \omega_{Mij} P_{it}^{1-\epsilon_{Mj}} \right)^{\frac{1}{1-\epsilon_{Mj}}} \tag{11}$$

$$P_{jt}^X = \frac{1}{A_{jt}^X} \left(\sum_i \omega_{Xij} P_{it}^{1-\epsilon_{Xj}} \right)^{\frac{1}{1-\epsilon_{Xj}}}. \tag{12}$$

Furthermore, straightforward manipulation of the first order conditions for each sector's production can generate the following expression for the price of the final good produced by each sector j:

$$P_{jt} = \frac{1}{A_{jt}} \left(\frac{R_{jt}}{\theta_j \alpha_j} \right)^{\theta_j \alpha_j} \left(\frac{W_t}{(1-\theta_j)\alpha_j} \right)^{(1-\theta_j)\alpha_j} \left(\frac{P_{jt}^M}{1-\alpha_j} \right)^{1-\alpha_j}. \tag{13}$$

Finally, we describe the equilibrium conditions that dictate structural change in production networks. Manipulating first order conditions for the intermediates and investment bundling sectors, the share of expenditures by the bundling sectors for sector j on inputs purchased from sectors i can be written as:

$$s_{ijt}^M \equiv \frac{P_{it} M_{ijt}}{P_{jt}^M M_{jt}} = \omega_{Mij} \left(\frac{P_{it}}{A_{jt}^M P_{jt}^M} \right)^{1-\epsilon_{Mj}}, \tag{14}$$

$$s_{ijt}^X \equiv \frac{P_{it} X_{ijt}}{P_{jt}^X X_{jt}} = \omega_{Xij} \left(\frac{P_{it}}{A_{jt}^X P_{jt}^X} \right)^{1-\epsilon_{Xj}}. \tag{15}$$

The share of intermediates (investment) expenditures by sector j on sector i's products is defined as s_{ijt}^M (s_{ijt}^X), and depends on the relative prices of each sector's final output and the scale and elasticity parameters in the bundling sectors. Empirically, s_{ijt}^{M} (s_{ijt}^{X}) corresponds to a column j in the scaled input-output (investment) network plotted in Figure 1. Thus, given movements in relative prices across sectors, there can be structural change in production networks, depending on the interaction of these price movements with production parameters.

The overall composition of intermediate good and investment good production across sectors will also depend on the distribution of intermediates and investment spending across sectors. That is, the aggregate shares of intermediate good and investment good purchases that are produced by sector i (defined as s_{it}^{M} and s_{it}^{X}, respectively) are thus given by:

$$s_{it}^{M} = \sum_{j} \frac{P_{jt}^{M} M_{jt}}{\sum_k P_{kt}^{M} M_{kt}} s_{ijt}^{M}, \tag{16}$$

$$s_{it}^{X} = \sum_{j} \frac{P_{jt}^{X} X_{jt}}{\sum_k P_{kt}^{X} X_{kt}} s_{ijt}^{X}. \tag{17}$$

Changes in s_{it}^{M} and s_{it}^{X} over time (like those plotted in Figure 2) thus reflect both changes in production processes within sectors (changes in s_{ijt}^{M} and s_{ijt}^{X}) and changes in the composition of spending across all sectors. As our shift-share decomposition in Section 2.2 reveals, both margins of adjustment appear empirically relevant in practice, at least for the production of intermediates.

4. Balanced Growth Path

We now consider the joint evolution of economic growth and structural change along an aggregate balanced growth path (ABGP). We do this in three steps. First, we describe necessary assumptions for the existence of such a path and the implications of these assumptions for prices, technology and aggregate quantities. We later relax some of these assumptions for some of our quantitative exercises in Section 6.2. Second, we state a proposition establishing the existence and nature of the ABGP. Finally, we describe the economic implications of the ABGP, including the joint dynamics of economic growth and structural change in production networks.

4.1. Assumptions and Model Implications

The general setup presented in the previous section allows for heterogeneity in sectoral production functions and in the bundling of investment and intermediate goods. While considering this heterogeneity is interesting, it also generates different capital and labor intensities across sectors that are only consistent with a balanced growth path in the limit of the economy (see for example, Acemoglu and Guerrieri (2008), Alvarez-Cuadrado, Van Long and Poschke (2017), or Alvarez-Cuadrado, Van Long and Poschke (2018)).[22] Thus, the key assumptions needed for establishing the existence of a balanced growth path entail imposing homogeneity in production functions across sectors and in the bundling of new investment.

Assumption 1. *The parameters of the sectoral production functions are the same across all sectors, i.e.* $\alpha_j = \alpha$ *and* $\theta_j = \theta$ *for all* j.

Assumption 2. *The parameters governing the evolution of capital—both parameters of the investment bundling sectors and the depreciation rate—are the same for all sectors* j*, i.e.* $\delta_j = \delta$, $\omega_{Xij} = \omega_{Xi}$ *and* $\epsilon_{Xj} = \epsilon_X$ *for all* j*. Furthermore, technical change in each sector's investment bundling is the same, i.e.* $A^X_{jt} = A^X_t$.

With these two sets of assumptions, we see from equation (12) that the price of the price of new investment will be equated across sectors, i.e. $P^X_{jt} = P^X_t$. Furthermore, since the parameters governing the evolution of capital are equated across sectors, this implies capital is no longer sector-specific, but is common across sectors with a single rental rate, i.e. $R_{jt} = R_t$. The collective implication is that, as seen from equation (13), the relative price of output in sectors, j and k, depends only on sectoral TFP differences and differences in the price of the intermediates bundles in those two sectors:

$$\frac{P_{jt}}{P_{it}} = \frac{A_{it}/\left(P^M_{it}\right)^{1-\alpha}}{A_{jt}/\left(P^M_{jt}\right)^{1-\alpha}} \tag{18}$$

Given these manipulations, the Lemma 1 relates relative prices of final output in each sector to the price of intermediates and investment bundles:

[22] However, Herrendorf, Herrington and Valentinyi (2015) argues that many of these features only have second order impacts on quantitative patterns of structural change.

Lemma 1. *Given assumptions 1 and 2, the price of sector j's product relative to the price of the bundle of intermediates in each sector, P_{it}^{M}, and to the price of the bundle of investment, P_{t}^{X}, can be written as:*

$$\frac{P_{jt}}{P_{t}^{X}} = \frac{\tilde{B}_{t}^{X}}{\tilde{A}_{jt}} \tag{19}$$

$$\frac{P_{jt}}{P_{it}^{M}} = \frac{\tilde{B}_{it}^{M}}{\tilde{A}_{jt}} \tag{20}$$

where $\tilde{A}_{jt} \equiv \frac{A_{jt}}{(P_{jt}^{M})^{1-\alpha}}$, $\tilde{B}_{it}^{M} \equiv A_{it}^{M}\left(\sum_{k}\omega_{ki}^{M}\tilde{A}_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ *and* $\tilde{B}_{t}^{X} \equiv A_{t}^{X}\left(\sum_{k}\omega_{k}^{X}\tilde{A}_{kt}^{\epsilon_{X}-1}\right)^{\frac{1}{\epsilon_{X}-1}}$.

Proof. See Appendix B. □

Given the above relationship between relative prices and technology, $\tilde{B}_{t}^{X}$ and $\tilde{B}_{it}^{M}$ represent investment-specific and intermediate-specific TFP. In each case the expression contains an exogenous component, A_{t}^{X} or A_{it}^{M}, and an endogenous component constructed from the sum of adjusted sectoral productivities, $\tilde{A}_{it}$, weighted in proportion to that sector's role as a producer of investment or intermediates. These adjusted sectoral productivities capture both direct advances in technical change at sectors producing final investment or intermediates, as well as indirect advances in technical change, captured in the prices of intermediate goods used to produce final investment or intermediates. This second component is endogenous as movements in these adjusted sectoral productivities indicate changing relative prices of the inputs to the bundle of investment and intermediates, and will evolve as these relative prices change over time. This component is, of course, further endogenous because it yet contains the endogenous prices of each sector's intermediates bundle. In general, without further assumptions, we cannot solve for the price of intermediates in closed form; however, later in this section, we consider a special case where a closed form representation exists.[23]

Finally, given the above assumptions and relationships, we derive an expression for aggregate GDP in this economy. Nominal sectoral value added in sector j is defined as nominal sectoral gross output minus

[23] More specifically, if we take as the numeraire the aggregate investment good (which we do later on), the price of the intermediate bundle in sector i is given by:

$$P_{it}^{M} = \frac{A_{t}^{X}}{A_{it}^{M}}\left(\sum_{k}\omega_{k}^{X}\left(\frac{A_{kt}}{(P_{kt}^{M})^{1-\alpha}}\right)^{\epsilon_{X}-1}\right)^{\frac{1}{\epsilon_{X}-1}}\left(\sum_{k}\omega_{ki}^{M}\left(\frac{A_{kt}}{(P_{kt}^{M})^{1-\alpha}}\right)^{\epsilon_{Mi}-1}\right)^{\frac{1}{1-\epsilon_{Mi}}}$$

This produces a system of N non-linear equations which can be solved to obtain price of each intermediates bundle. However, without further assumptions, this system of nonlinear equations does not have a closed form solution.

expenditures on intermediates:

$$P_{jt}^{V} V_{jt} = P_{jt} Q_{jt} - P_t^M M_{jt} \tag{21}$$

where V_{jt} represents real value added in sector j and P_{jt}^V. Thus, aggregate GDP, Y_t, denoted in units of the numeraire, is given by $Y_t = \sum_i P_{it}^V V_{it}$. We take the aggregate investment good as the numeraire in the economy.

With these definitions, the following lemma presents a closed form expression for aggregate GDP in the economy:

Lemma 2. *Given Assumptions 1 and 2, aggregate GDP, denoted in units of the numeraire (the aggregate investment good), is given by:*

$$Y_t = \sum_i P_{it}^V V_{it} = \mathcal{A}_t K_t^{\theta} \tag{22}$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}}$ *and* $K_t = \sum_j K_{jt}$. *Furthermore, the following aggregate equilibrium conditions hold:*

$$R_t = \theta \mathcal{A}_t K_t^{\theta - 1} \tag{23}$$

$$W_t = (1-\theta) \mathcal{A}_t K_t^{\theta} \tag{24}$$

Proof. See Appendix B. □

Lemma 2 provides a closed-form representation of aggregate GDP that follows the same production structure as sectoral value added. Importantly, the implicit aggregate TFP term, $\mathcal{A}_t$ only depends on aggregate investment-specific TFP, $\tilde{B}_t^X$. Thus, aggregate growth only depends on technical change in the production of investment, either directly at final producers of investment, or at their intermediate suppliers (reducing the price of the intermediates bundle for investment producers).[24]

[24] This is a consequence of making aggregate investment the numeraire. If instead, aggregate consumption is the numeraire, as in Greenwood et al. (1997), then technical change in consumption production can matter for aggregate growth as well (although at the cost of being inconsistent with the Kaldor facts—see Duernecker, Herrendorf and Valentinyi (2021)). More generally, defining aggregate GDP as in (22), implies that the traditional Hulten's Theorem (Hulten (1978)) will not hold for this economy, because movements in output prices have first-order implications for aggregates. However, in Section 7, when we consider off-balanced growth path exercises, we consider a measure of aggregate GDP which is not sensitive to the choice of numeraire and for which Hulten's Theorem holds for aggregate productivity.

4.2. An Aggregate Balanced Growth Path

We adpot the same aggregate balanced growth path (ABGP) definition as in Ngai and Pissarides (2007) and HRV, where all aggregates (denoted in units of the numeraire, the price of aggregate investment) must grow at a constant rate. This means that K_t, Y_t, W_t, R_t, and X_t will grow at a constant (though not necessarily equal) rate. Total consumption expenditures are defined in units of the numeraire, $E_t = \sum_i P_{it}C_{it}$, and thus E_t also grows at a constant rate along the ABGP.

For any variable X_t, the gross growth rate between time periods t and $t+1$ is defined as $\gamma_{t+1}^X \equiv \frac{X_{t+1}}{X_t}$. Along the ABGP, we drop the time subscripts for variables growing at a constant rate. With these definitions, we state the following proposition.

Proposition 1. *Assume that Assumptions 1 and 2 hold and that $\gamma_t^{\mathcal{A}} > \frac{1-\delta}{\beta}\ \forall t$. An aggregate balanced growth path exists where*

$$\gamma^K = \gamma^X = \gamma^Y = \gamma^E = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{25}$$

and $\gamma^R = 0$ if and only if $\gamma^{\mathcal{A}}$ is constant.

Proof. See Appendix B. □

Furthermore, the balanced growth path exhibits capital deepening with respect to intermediates, $\gamma^K > \gamma_t^M$, if the growth rate of investment-specific TFP is greater than the growth rate of intermediates specific TFP, $\gamma_t^{B^X} > \gamma_t^{B^M}$.

Given the aggregate production function derived in Lemma 2, the result for the aggregate growth rate of the economy is unsurprising. As in the one-sector growth model, the economy's aggregate growth rate purely depends on the growth rate of aggregate TFP. The requirement that $\gamma_t^{\mathcal{A}} > \frac{1-\delta}{\beta}$ is standard, and holds for most reasonable parameter values.[25]

Explicitly modeling the production networks of intermediate and investment goods provides a number of insights. First, aggregate TFP, $\mathcal{A}_t$, only depends on technical change that increases the production frontier for investment, either through directly expanding production at final investment producers or through reducing costs of intermediate inputs for final investment producers. Technical change in sectors that solely

[25]For example, with annual depreciation of about 10% and annual discounting at a 3% real rate, this condition would require that $\gamma_t^{\mathcal{A}} - 1 > 0.9/(1/(1+0.03)) - 1 = -0.073$ or that aggregate TFP growth exceeds -7%.

produce consumption goods has no bearing on the growth rate of aggregates along the ABGP. This result is similar to the one in Ngai and Pissarides (2007), when the multi-sector growth model is extended to allow for intermediate goods.

However, our aggregate balanced growth path differs from that of Ngai and Pissarides (2007), who argue that endogenous structural change in production networks is not possible along the aggregate balanced growth path. The key distinction between our two frameworks that allows for structural change in production networks along the balanced growth path is that we do not require that all exogenous technical change terms grow at a constant rate. In fact, as discussed in more detail in HRV, at least one of the exogenous technical change processes *must* grow at a particular non-constant rate in order to satisfy the necessary and sufficient condition that $\gamma^{\mathcal{A}}$ remains constant. Imposing constant growth in all exogenous technical change terms would restore the result of Ngai and Pissarides (2007). We discuss structural change and its relationship with the aggregate growth path in more detail in the following section.

We emphasize that Proposition 1 requires no restrictions on the intermediates bundling processes across sectors; the balanced growth path admits arbitrary heterogeneity in the input-output network. However, with heterogeneity in the bundling process for intermediates across sectors, it is not possible to separate out in closed form the contribution of the direct and indirect portions of investment-specific TFP growth. The following assumption and subsequent lemma consider a special case in which such separation is possible, allowing for a separate assessment of the contributions of technical change at final investment producers and technical change at intermediate suppliers of investment producers to aggregate growth.

Assumption 3. *The parameters of the intermediates bundling sectors are the same for all sectors j, i.e. $\omega_{Mij} = \omega_{Xi}$ and $\epsilon_{Mj} = \epsilon_M$ for all j and that technical change in each sector's intermediates bundling is the same, i.e. $A^M_{jt} = A^M_t$*

Lemma 3. *Assume that assumptions 1-3 hold. Then the implied aggregate technical change term, $\mathcal{A}_t$ can be written as $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B^X_t \left(B^M_t\right)^{\frac{1-\alpha}{\alpha}}$, where $B^M_{it} \equiv A^M_{it}\left(\sum_k \omega^M_{ki} A^{\epsilon_{Mi}-1}_{kt}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ and $B^X_t \equiv A^X_t\left(\sum_k \omega^X_k A^{\epsilon_X-1}_{kt}\right)^{\frac{1}{\epsilon_X-1}}$. Further, with a single investment good and a single intermediate good, the relative productivities in investment and intermediates are equal to the inverse ratio of relative prices, $\frac{B^X_t}{B^M_t} = \frac{P^M_t}{P^X_t} = P^M_t$ (given the choice of aggregate investment as the numeraire).*

Proof. See Appendix B. □

Abstracting from heterogeneity in intermediates bundling allows us to write aggregate technical change purely in terms of exogenous technical change processes and to separate out direct technical change in investment production, B_t^X, from technical change in intermediates production, B_t^M. In the empirically reasonable case where $\alpha \approx 0.5$, and thus $\mathcal{A}_t \propto B_t^X B_t^M$, the result of Lemma 3 implies that movements in the relative price of intermediates to investment can provide information about which source of technical change—intermediates or investment—is playing a larger role in the aggregate rate of growth.[26] We provide evidence on how these relative prices have evolved over time and what this implies for sources of technical change in Section XX6.

4.3. Implications of Balanced Growth for Structural Change and Growth Rates

A key feature of the ABGP is that it allows for structural change in production networks while the aggregate growth rate of the economy remains stable. Because of assumptions 1 and 2, the share of each sector's intermediates or investment production in the aggregate (equations (16) and (17)) can be rewritten as:

$$s_{it}^M = \sum_j \frac{P_{jt}^M M_{jt}}{\sum_k P_{kt}^M M_{kt}} s_{ijt}^M = \sum_j \frac{P_{jt}^M M_{jt}}{\sum_k P_{kt}^M M_{kt}} \omega_{Mij} \left(\frac{\tilde{B}_{jt}^M}{A_{jt}^M \tilde{A}_{it}} \right)^{1-\epsilon_{Mj}}, \tag{26}$$

$$s_{it}^X = \omega_{Xi} \left(\frac{\tilde{B}_t^X}{A_t^X \tilde{A}_{it}} \right)^{1-\epsilon_X}. \tag{27}$$

Because the ABGP admits arbitrary heterogeneity in the input-output network, the structural change expression for intermediates remains fairly involved, depending both on within sector patterns of structural change (which may be heterogeneous across sectors) and the overall composition of the buyers of intermediates. However, in the case of investment, because assumptions 1 and 2 imply that each sector's production technology is identical and that there is a single investment good used for all sectors, structural change is identical across all sectors (i.e. $s_{it}^X = s_{ijt}^X \; \forall j$). Therefore, the ABGP rules out the composition channel for structural change in the investment network. However, as observed in Table 2, this channel played a minimal

[26] We also note that in this special case, it is straightforward to show that along the balanced growth path, the quantities of capital and intermediates need not grow at the same rate and that neither the aggregate price or quantity of intermediates need grow at a constant rate. Along the balanced growth path, total *spending* on intermediates grows at the same constant rate as the aggregate capital stock, i.e. $\gamma^K = \gamma_t^{P_M} \gamma_t^M$, but the price and quantity of intermediates may grow at non-constant rates over time.

role for structural change in the investment network.

More generally, structural change in production networks will play a role in the overall structural change observed in GDP and employment, the empirical patterns of which have been discussed extensively in the literature. As structural change in these quantities are not the primary focus of our paper, we relegate a discussion of the role that structural change in production networks play for these to Appendix C.

Importantly, structural change in production networks will play a role in shaping the composition of the aggregate growth rate of the economy. For illustrative purposes, we discuss the special case where assumption 3 holds, implying no heterogeneity in intermediates bundling technologies. This special case helps build intuition for the role that structural change plays in shaping the aggregate growth rate, however, in our quantitative exercises, we do not impose these simplifying assumptions.

In this special case, along the ABGP, the growth rate of aggregates is given by $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} = \left(\gamma_t^{B^X}\left(\gamma_t^{B^M}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\theta}}$. The dependence of the aggregate growth rate on both $\gamma_t^{B^X}$ and $\gamma_t^{B^M}$ establishes a direct connection between the aggregate growth rate and structural change in production networks. With some straightforward algebra, these two aggregate growth rates can be expanded as follows:

$$\gamma_t^{B^M} = \gamma_t^{A^M}\left(\sum_i s_{it-1}^M (\gamma_{it}^A)^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}} \tag{28}$$

$$\gamma_t^{B^X} = \gamma_t^{A^X}\left(\sum_i s_{it-1}^X (\gamma_{it}^A)^{\epsilon_X - 1}\right)^{\frac{1}{\epsilon_X - 1}} \tag{29}$$

These expressions establish that, in addition to the production share parameters on capital (θ) and value added (α), the growth rate of aggregates depends on four components: growth in exogenous technical progress in intermediates ($\gamma_t^{A^M}$), growth in exogenous technical progress in investment ($\gamma_t^{A^X}$), and two weighted sums of technical progress in each individual sector i, where the weights are determined by the composition of intermediates production (s_{it}^M) and the composition of investment production (s_{it}^X). Of course, the very definition of the ABGP implies that γ^Y is a constant; thus structural change in production networks will not impact the growth rate in the long run.[27] But the composition of the growth rate may

[27] However, in exercises in Section 6 where we depart from the assumptions necessary for the balanced growth path, structural change in production networks could lead to a change in the aggregate growth rate of the economy. We revisit that possibility with

depend on potentially differential rates of technical change in intermediates and investment production.

Before moving to measurement of these components of economic growth, we make two points about how the CES structure for intermediates and investment will matter for the composition of economic growth. First, the distribution of intermediates and investment expenditures across different producers of these goods, s_{it}^M and s_{it}^X, will obviously matter for the aggregate growth rate expression above. Particularly, the correlation between this distribution and the distribution of TFP growth rates in each sector, γ_{it}^A will matter for economic growth. For example, if the share of intermediates expenditures is high on sectors experiencing rapid technological change, then this will lead to faster economic growth coming from the endogenous portion of intermediates technical change. The underlying share parameters ω_{Mi} and ω_{Xi} in the CES aggregators for intermediates and investment will thus be important for shaping the composition of economic growth.

Second, as relative prices change with different rates of technical change across sectors, the distribution of intermediates and investment expenditures will change over time. The rate of this change will depend on the elasticities of substitution in the CES aggregators for intermediates and investment, ϵ_M and ϵ_X. We establish this formally with the following lemma.

Lemma 4. *Assume that assumptions 1-3 hold. Further, assume that TFP growth in each sector is constant over time and that the initial values of TFP are all normalized to unity. Holding all other parameters fixed, the growth rate of the endogenous component of intermediates,* $\left(\sum_i s_{it-1}^M (\gamma_i^A)^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}}$*, is weakly increasing in* ϵ_M*. The same result holds for the growth rate of the endogenous component of investment; all other parameters fixed, it is increasing in* ϵ_X*.*

Proof. See Appendix B. □

The higher is the elasticity of substitution in either intermediates or investment, the faster the CES bundle of each sector's TFP growth will grow. The intuition for this result can be seen from considering the limiting cases for these CES functions of growth rates when sector TFP growth rates are constant over time. For example, if $\epsilon_M < 1$, implying the gross complements case, then as $t \to \infty$, the growth rate of the endogenous component of intermediates technical change will converge to the *slowest* TFP growth rate

those exercises.

among producers of intermediates.[28] This is because movements in relative prices induced by differential rates of technical change will ultimately cause s_{it}^M to converge to one for the slowest growing sector. In contrast, in the gross substitutes case (i.e. $\epsilon_M > 1$), this growth rate will converge to the *fastest* TFP growth rate, as all expenditures ultimately become concentrated on this sector. Because the growth rate in the limit is increasing in the elasticity of substitution and the rate of convergence to that limit is increasing the further away the elasticity of substitution is from one, the result of the lemma obtains.[29]

A key implication of Lemma 4 is that if $\epsilon_M > \epsilon_X$ and the initial distribution of expenditures is identical across intermediates and investment (i.e. $s_{i0}^M = s_{i0}^X \; \forall i$), then the growth rate of the endogenous component of technical change in intermediates will be larger than the growth rate in the endogenous component of investment, implying potentially significant changes in composition in the aggregate growth rate along the balanced growth path. Given the empirical evidence already presented, the assumption of $s_{i0}^M = s_{i0}^X \; \forall i$ is unlikely to hold, and heterogeneity in the input-output network will imply additional richness than what is characterized in Lemma 4. Nonetheless, differential elasticities of substitution between intermediates and investment imply rates of structural change and endogenous changes in the components of aggregate growth. Further exploration of these parametric differences and their implications for long run growth requires quantitative analysis, which is the focus of Section 5.

5. Measurement, Calibration and Structural Change

We now measure the exogenous technical change processes in the data, estimate the parameters of the CES bundling technologies and evaluate the model's fit to observed patterns of technical change in production networks.

[28] Note that this need not be the slowest TFP growth rate among all sectors, as not all sectors need contribute to the production of intermediate goods.

[29] We note that the restrictions that TFP growth be constant in each sector are sufficient conditions to prove this result, but not necessary. The necessary and sufficient condition is that the covariance across sectors in the log-level of sectoral TFP and the growth rate of sectoral TFP raised to the power $\sigma - 1$ have the sign of $\sigma - 1$ in all time periods. For sufficiently small variations in growth rates in each sector over time, this is likely to hold. Further discussion is available in the proof in Appendix B.

5.1. Measuring Technical Change

There are three types of exogenous technical change to be measured—exogenous technical change in each sector's production (sectoral TFP), exogenous technical change in the production of intermediate goods, and exogenous technical change in investment goods. We describe how we measure each of these in turn.

First, we follow the procedures of vom Lehn and Winberry (2022) and measure sector-level productivity as the Solow residual of real gross output net of the primary inputs, constructed in log differences from equation (4):

$$\Delta \ln(A_{jt}) = \Delta \ln(Q_{jt}) - \alpha_j \theta_j \Delta \ln(K_{jt}) - \alpha_j(1-\theta_{jt}) \ln L_{jt} - (1-\alpha_j) \ln M_{jt}. \tag{30}$$

Our measures of real gross output, real intermediates and employment are taken from the BEA GDP by Industry database and the real capital stock is constructed using the perpetual inventory method using sectoral real investment, implied depreciation rates and initial values of the capital stock from the BEA Fixed Assets Database. Values of the production parameters are assigned using evidence on cost shares observed in BEA Input Output data — α_j using the ratio of nominal value added to nominal gross output in each sector and $1-\theta_j$ using the sectoral share of compensation in value added.[30] We normalize the level of A_{jt} to be 1 in 1947. When we only consider two sectors, we aggregate sectoral TFP using a Tornqvist index with weights of each sector's share of gross output.

Second, to measure exogenous technical change in the production of intermediate and investment goods, we follow HRV and use a Tornqvist index approximation to measure exogenous technical change independent of the parameters of the CES function.[31] Specifically, this implies that exogenous technical change in

[30] As in vom Lehn and Winberry (2022), we allow α_j, θ_j, and δ_j to vary over time in measurement, to more precisely isolate changes in technical change from changes in the production technology or depreciation rates. For the growth rate of TFP in any two years, we use the average of cost shares or implied depreciation rates for those two years. This implies that our measurement is inconsistent with our model, where these parameters are fixed over time. However, if we hold these parameters fixed, the impact on our measures of sector-specific TFP is negligible.

[31] Specifically, for two time periods t and $t-1$, we first take a log first order approximation of the bundling equation about the values in time period t, and use that approximate equation to construct log differences between periods t and $t-1$ (which causes terms involving the approximation point to drop out). Then, we construct the approximation again, however, about the values in time period $t-1$, and again construct log differences. Averaging these two approximations together yields the Tornqvist approximation.

intermediates and investment is measured as:

$$\Delta \ln(A_{jt}^{M}) = \Delta \ln(M_{jt}) - \sum_{i} \overline{\frac{P_{it}M_{ijt}}{P_{jt}^{M}M_{jt}}} \Delta \ln(M_{ijt}) \tag{31}$$

$$\Delta \ln(A_{jt}^{X}) = \Delta \ln(X_{jt}) - \sum_{i} \overline{\frac{P_{it}X_{ijt}}{P_{jt}^{X}X_{jt}}} \Delta \ln(X_{ijt}) \tag{32}$$

where $\Delta \ln(X_t)$ indicates the log difference in X between time periods t and $t-1$ and $\overline{\frac{P_{it}M_{ijt}}{P_{jt}^{M}M_{jt}}}$ and $\overline{\frac{P_{it}X_{ijt}}{P_{jt}^{X}X_{jt}}}$ are the averages of those ratios in periods $t-1$ and t. We measure real intermediates and investment spending in each sector, M_{jt} and X_{jt}, using BEA GDP by Industry and BEA Fixed Assets data. Measures of the real inputs to intermediates and investment purchased from each sector, M_{ijt} and X_{ijt}, are constructed using the $(i, j)th$ entries of the input-output and investment networks, deflated using the implicit price index for gross output in sector i. Aggregated series of technical change, either for the whole economy in balanced growth exercises or at the two sector level, are constructed using a Tornqvist index with either shares of total intermediates spending or investment spending used as weights.

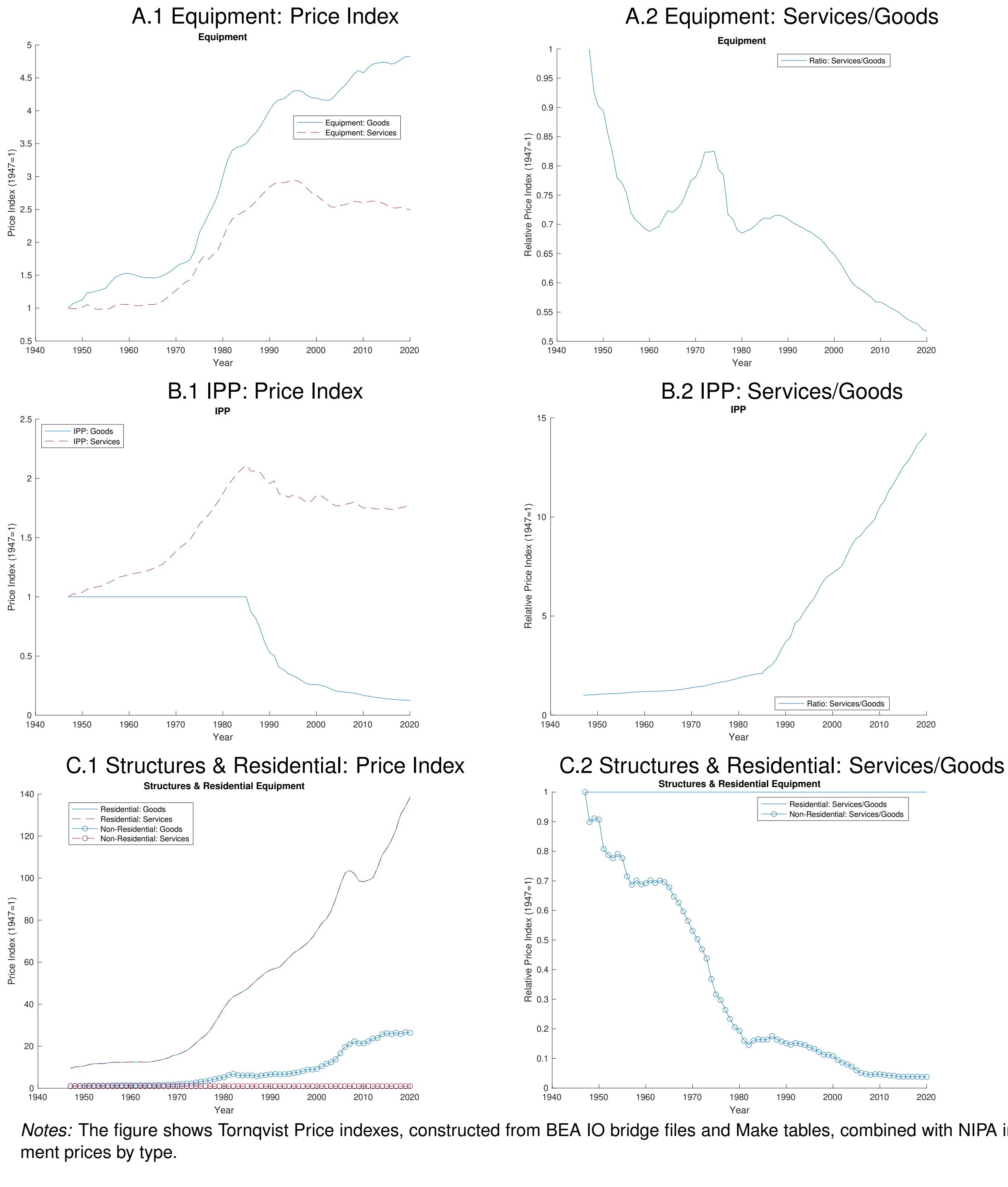
5.2. Calibration

- Describe calibration of non-CES parameters
- Describe estimation of CES structure parameters using first order conditions and basic OLS
- Present results for just 2 sectors vs. all 43 sectors
- Emphasize differences in estimation when using full variation and within-sector variation
- Estimate separate elasticity of substitution parameters for each sector and describe heterogeneity in those elasticities (particularly in goods vs. services for intermediate goods)

Equipment Investment: Goods/Services Prices. The equipment investment price index for goods and services is constructed using a Törnqvist index where the prices come form NIPA investment prices (NIPA Table 5.5.4) and the weights are constructed from the BEA's PEQ bridge scaled by the BEA's make table (more detail at some point).

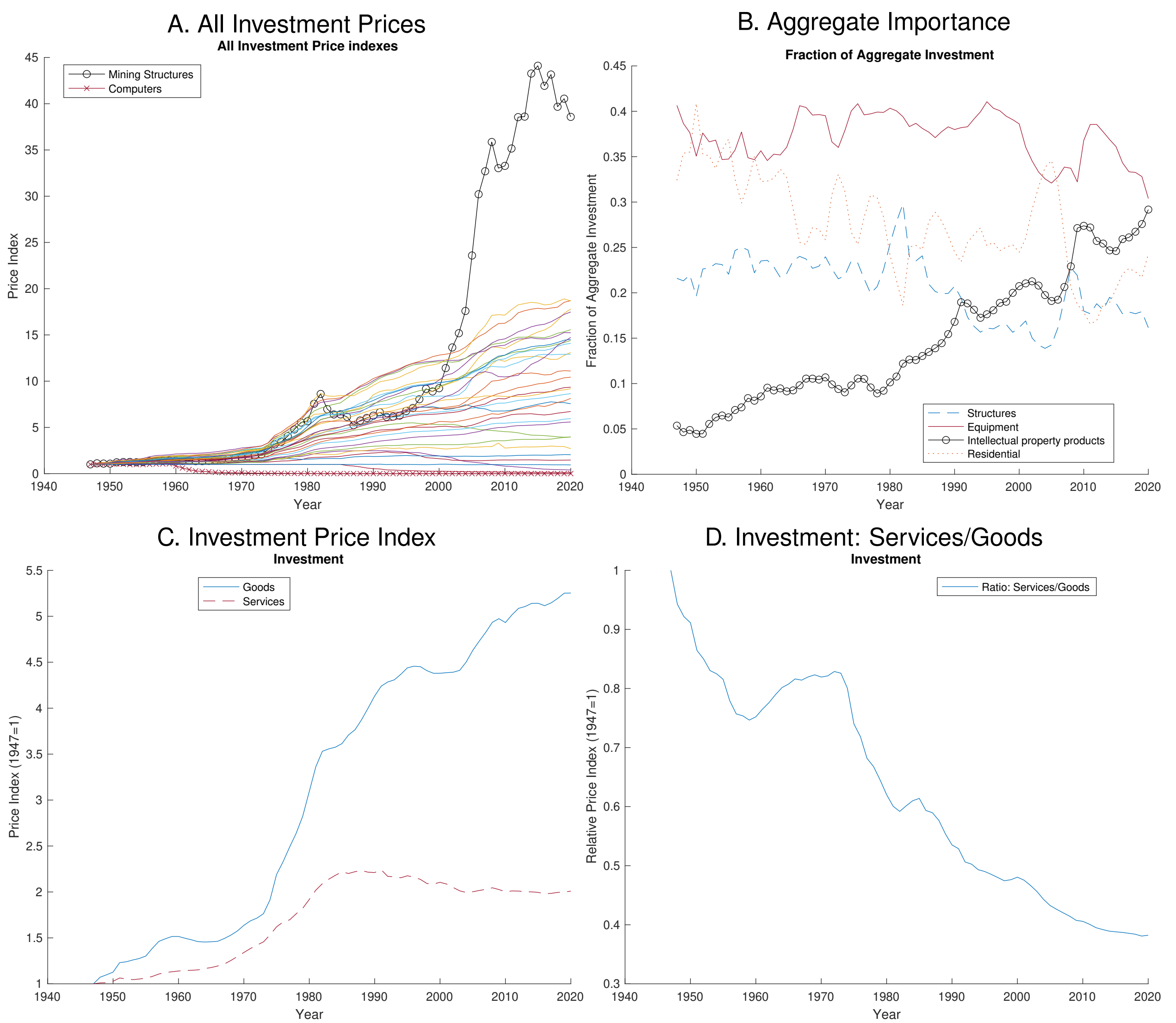
Panel C of Figure 7 shows the price indexes for the goods and services sector. Panels A and B show how we impute missing price data and fix a level jump due to re-classification in 2009.

Figure 5: Sectoral Price of Investment: 1947-2020



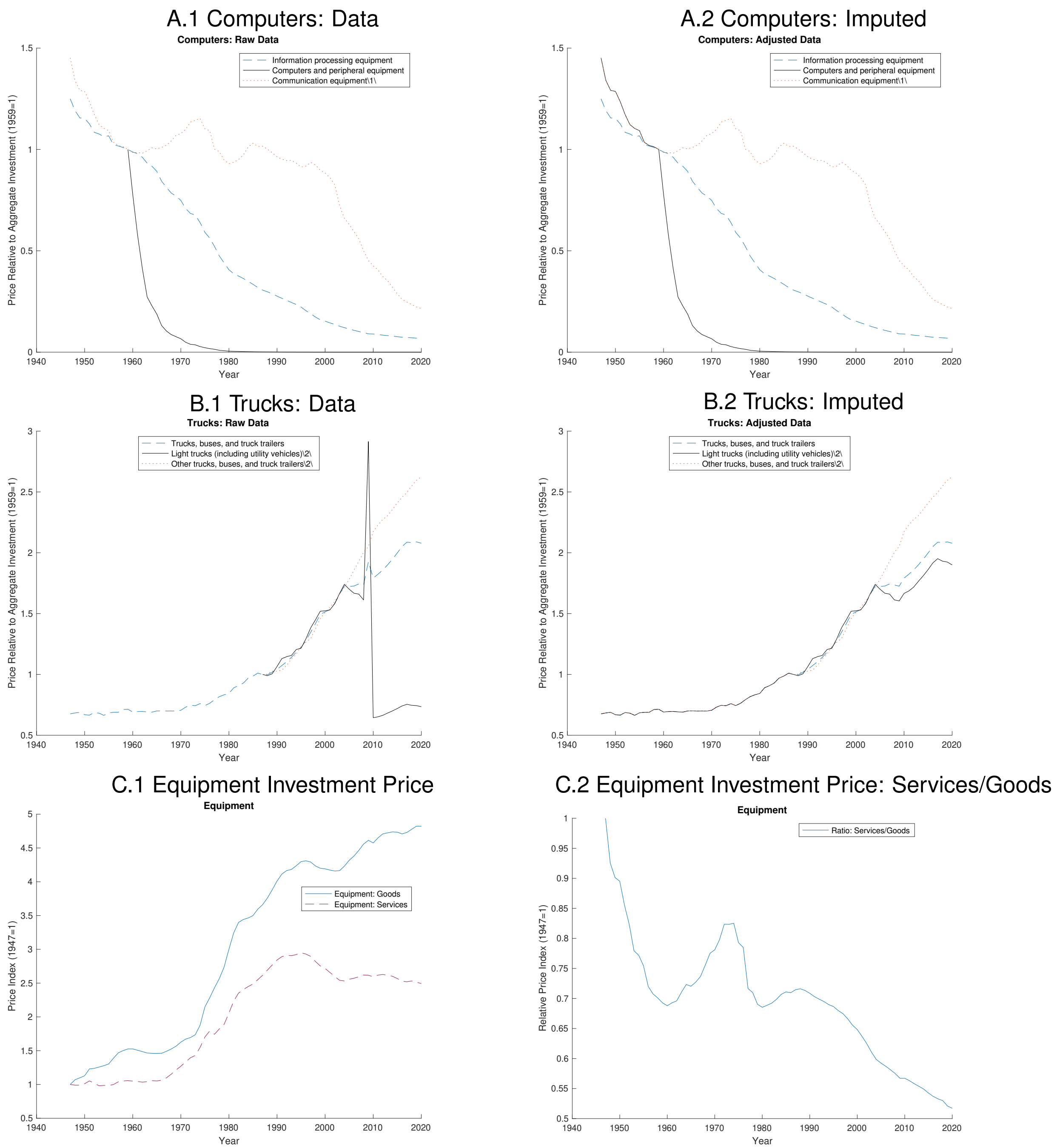
Notes: The figure shows Tornqvist Price indexes, constructed from BEA IO bridge files and Make tables, combined with NIPA investment prices by type.

Figure 6: Sectoral Price of Investment: 1947-2020



Notes: The figure shows Tornqvist Price indexes, constructed from BEA IO bridge files and Make tables, combined with NIPA investment prices by type.

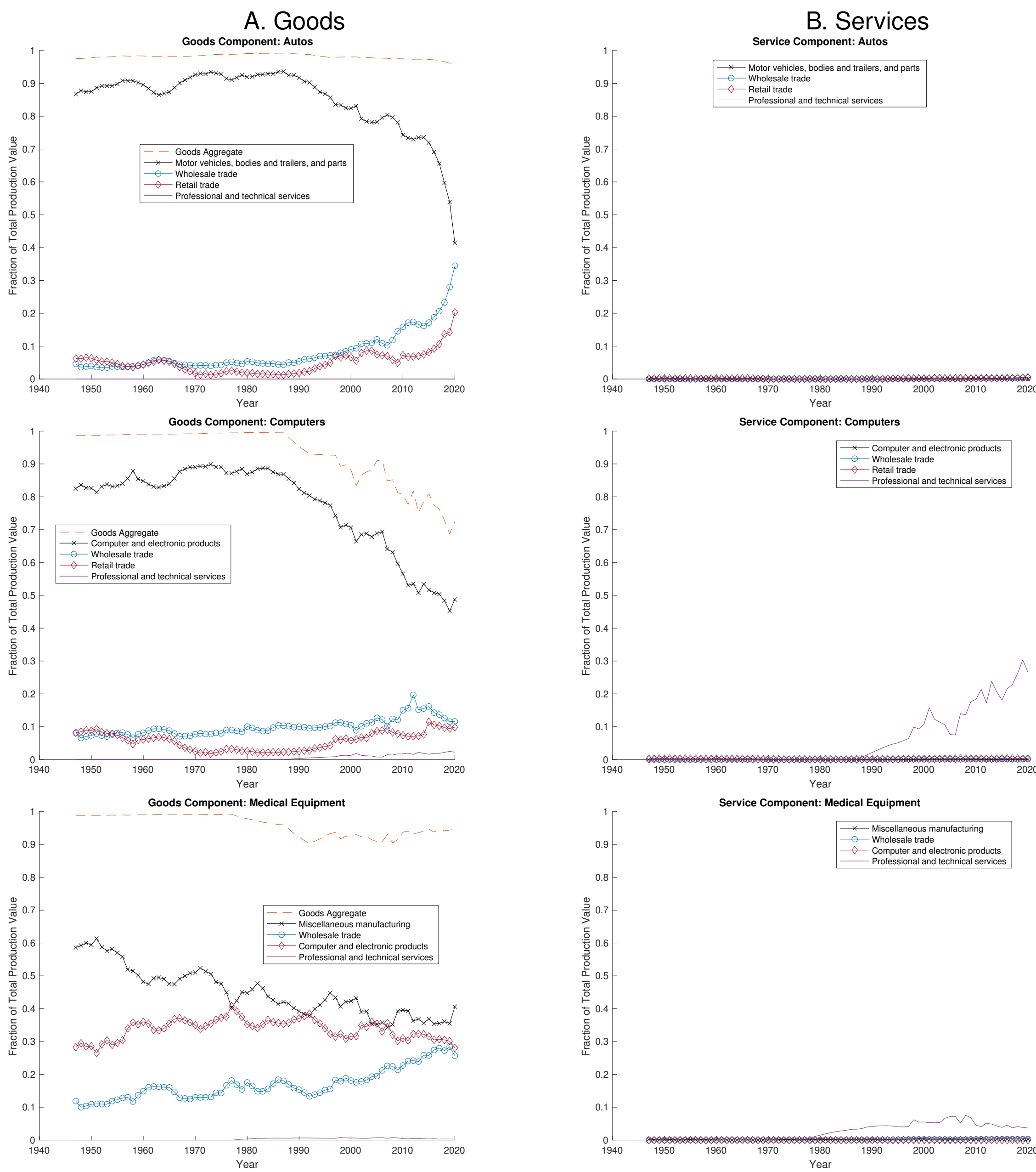
Figure 7: Sectoral Price of Equipment Investment: 1947-2020



Notes: The figure shows price indexes. Panels A and B plot prices relative to the aggregate price of investment. Panel A shows how we impute data for computers prior to 1959. Panel B shows how we impute data for the two detailed truck categories prior to 1987 and how we fix a jump due to reclassification in 2009.

Just a silly graph below:

Figure 8: Where is the Value of Key Investment Goods Made? 1947-2020



Notes: The figure highlights expenditures on three example commodities (autos, computers, and medical equipment) within highest spending industries. Panel A reports the portion of value "made" in goods industries, while panel B reports the portion "made" in service industries. Note that margin sectors (wholesale/retail trade and transportation) are classified as goods producing sectors.

5.3. Structural Change

- Present fit estimates and time series for two sector trends
- Present bar chart fit estimates akin to Figure 2 for the full 43 sector estimates
- Be clear that estimation is of course designed to fit these, but degree of fit in the time series is encouraging overall for a relative prices story for structural change in production networks
- Discuss misses in the 43 sector estimates (computer equipment manufacturing, for example) – comes from single elasticity of substitution setup.

6. Growth Accounting with Structural Change

6.1. Balanced Growth Path Accounting

- Present technical change in intermediates and investment and describe how these have evolved over time (including how much of each is endogenous vs. exogenous)
- Present estimates of the relative importance of intermediates and investment technical change and how those evolve over time.
- Compare estimates under different elasticity estimates.
- Analyze what forces drive things – mix in sectors producing intermediates and investment and their TFP growth, magnitudes of initial shares, differential elasticities (Relate back to Lemma 4 on differential elasticities), etc.
- Compare results to a simple measure of relative price of investment to intermediates and highlight similarities and rising importance of intermediates specific technical change since 2010.
- Discuss validity of assumption regarding constancy of growth in composite technical change

6.2. Unbalanced Growth Accounting

- Previous exercises had some restrictions: (1) may not actually be on BGP. Structural change only affected composition of growth on BGP; could matter more generally in an unbalanced growth path.

(2) Assumption of homogeneous production networks and complementarities in those networks is strong. Significant heterogeneity could matter – for example, which sectors are the intermediate producers to investment is the type of connection that matters most (see for example, Foerster, Sarte and Watson (2011))

- Solve an unbalanced growth path exercise with heterogeneity in production networks and complementarities where agents have perfect foresight over future TFP growth (use algorithm from Maliar, Maliar, Taylor and Tsener (2020))

- Key insight for accounting is that contribution of intermediate goods to growth depends on parameter α; set $\alpha = 0$ for counterfactuals to isolate relative components of technical change.

7. Robustness

- Discuss choice of numeraire – we use investment to preserve constant K/Y ratio along BGP. But that choice is not innocuous (see Duernecker et al. (2021)).

- Present alternative balanced growth representation with price of aggregate consumption as numeraire. This will allow non-investment/intermediates producer sectors' TFP to matter for aggregate growth... but shouldn't change things too much (Ngai and Samaniego (2009), for example, find that once intermediate goods are accounted for, more than 90% of growth comes from investment-specific technical change.

- Discuss how you can use these two representations as a way to approximately think about growth slowdown and what matters for that (compare to findings of Foerster et al. (2011)).

8. Conclusion

References

Acemoglu D, Carvalho VM, Ozdaglar A, Tahbaz-Salehi A. 2012. The network origins of aggregate fluctuations. *Econometrica* **80**: 1977–2016.

Acemoglu D, Guerrieri V. 2008. Capital deepening and nonbalanced economic growth. *Journal of political Economy* **116**: 467–498.

Acemoglu D, Ozdaglar A, Tahbaz-Salehi A. 2017. Microeconomic origins of macroeconomic tail risks. *American Economic Review* **107**: 54–108.

Alvarez-Cuadrado F, Van Long N, Poschke M. 2017. Capital–labor substitution, structural change, and growth. *Theoretical Economics* **12**: 1229–1266.

Alvarez-Cuadrado F, Van Long N, Poschke M. 2018. Capital-labor substitution, structural change and the labor income share. *Journal of Economic Dynamics and Control* **87**: 206–231.

Atalay E. 2017. How important are sectoral shocks? *American Economic Journal: Macroeconomics* **9**: 254–80.

Baqaee DR, Farhi E. 2019. The macroeconomic impact of microeconomic shocks: beyond hulten's theorem. *Econometrica* **87**: 1155–1203.

Baqaee DR, Farhi E. 2020. Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics* **135**: 105–163.

Berlingieri G. 2013. Outsourcing and the Rise in Services. CEP Discussion Papers dp1199, Centre for Economic Performance, LSE. URL https://ideas.repec.org/p/cep/cepdps/dp1199.html

Bigio S, La'o J. 2020. Distortions in production networks. *The Quarterly Journal of Economics* **135**: 2187–2253.

Boehm CE, Flaaen A, Pandalai-Nayar N. 2019. Input linkages and the transmission of shocks: Firm-level evidence from the 2011 tōhoku earthquake. *Review of Economics and Statistics* **101**: 60–75.

Boppart T. 2014. Structural change and the kaldor facts in a growth model with relative price effects and non-gorman preferences. *Econometrica* **82**: 2167–2196.

Carvalho VM, Nirei M, Saito YU, Tahbaz-Salehi A. 2021. Supply chain disruptions: Evidence from the great east japan earthquake. *The Quarterly Journal of Economics* **136**: 1255–1321.

Comin D, Lashkari D, Mestieri M. 2021. Structural change with long-run income and price effects. *Econometrica* **89**: 311–374.

Duarte M, Restuccia D. 2017. Relative prices and sectoral productivity. *Journal of the European Economic Association* .

Duernecker G, Herrendorf B, Valentinyi A. 2021. The productivity growth slowdown and kaldor's growth facts. *Journal of Economic Dynamics and Control* **130**: 104200.

Foerster A, Hornstein A, Sarte PD, Watson MW. 2019. Aggregate implications of changing sectoral trends. Technical report, National Bureau of Economic Research.

Foerster AT, Sarte PDG, Watson MW. 2011. Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy* **119**: 1–38.

Galesi A, Rachedi O. 2019. Services deepening and the transmission of monetary policy. *Journal of the European Economic Association* **17**: 1261–1293.

Greenwood J, Hercowitz Z, Krusell P. 1997. Long-run implications of investment-specific technological change. *The American economic review* : 342–362.

Herrendorf B, Herrington C, Valentinyi A. 2015. Sectoral technology and structural transformation. *American Economic Journal: Macroeconomics* **7**: 104–33.

Herrendorf B, Rogerson R, Valentinyi A. 2013. Two perspectives on preferences and structural transformation. *American Economic Review* **103**: 2752–89.

Herrendorf B, Rogerson R, Valentinyi A. 2014. Growth and structural transformation. In *Handbook of economic growth*, volume 2. Elsevier, 855–941.

Herrendorf B, Rogerson R, Valentinyi Á. 2021. Structural Change in Investment and Consumption—A Unified Analysis. *The Review of Economic Studies* **88**: 1311–1346. ISSN 0034-6527.
URL https://doi.org/10.1093/restud/rdaa013

Hulten CR. 1978. Growth accounting with intermediate inputs. *The Review of Economic Studies* **45**: 511–518.

Johnson RC. 2014. Trade in intermediate inputs and business cycle comovement. *American Economic Journal: Macroeconomics* **6**: 39–83.

Maliar L, Maliar S, Taylor JB, Tsener I. 2020. A tractable framework for analyzing a class of nonstationary markov models. *Quantitative Economics* **11**: 1289–1323.

Miranda-Pinto J. 2021. Production network structure, service share, and aggregate volatility. *Review of Economic Dynamics* **39**: 146–173.

Miranda-Pinto J, Young ER. 2022. Flexibility and frictions in multisector models. forthcoming in *American Economic Journal: Macroeconomics* .

Moro A. 2012. Biased technical change, intermediate goods, and total factor productivity. *Macroeconomic Dynamics* **16**: 184–203.

Moro A. 2015. Structural change, growth, and volatility. *American Economic Journal: Macroeconomics* **7**: 259–94.

Ngai LR, Pissarides CA. 2007. Structural change in a multisector model of growth. *American Economic Review* **97**: 429–443.

Ngai LR, Samaniego RM. 2009. Mapping prices into productivity in multisector growth models. *Journal of Economic Growth* **14**: 183–204.

Timmer MP, Dietzenbacher E, Los B, Stehrer R, De Vries GJ. 2015. An illustrated user guide to the world input–output database: the case of global automotive production. *Review of International Economics* **23**: 575–605.

vom Lehn C, Winberry T. 2022. The investment network, sectoral comovement, and the changing us business cycle. *The Quarterly Journal of Economics* **137**: 387–433.

Appendix A. Measurement Details and Additional Empirical Results

Appendix A.1. Measurement Details

Our primary data sources for measuring production networks and related data are the BEA Input-Output database, specifically, the time series of Make and Use tables from 1947-2020, and the time series of the investment network generated in vom Lehn and Winberry (2022). The Make and Use tables from the BEA Input Output database can be downloaded at the BEA's website here: https://www.bea.gov/industry/input-output-accounts-data; data from vom Lehn and Winberry (2022) containing the time series of the investment network can be found here: https://doi.org/10.7910/DVN/CALDHX. These data provide details for 43 NAICS-defined sectors of the economy, including agriculture and government; Table 1 lists each of the 43 sectors and their corresponding NAICS codes. More recent vintages of the BEA Input Output database allow for greater sectoral detail (and it is possible to construct more detailed investment networks for recent years), but given our interest in structural change over the long-run, we focus on these 43 sectors, which can be observed for all years of the data.

We use the Make and Use tables from the BEA to measure input-output relationships in the following way. The core of the Use table is a square matrix that reports intermediate input expenditures by different sectors (organized along columns) on specific commodities (organized along rows). These commodities are named and assigned NAICS codes based on which sectors are major producers of the given commodity, but more than one sector may be involved in the production of a given commodity. The mix of sectors which produce a given amount of each commodity are observed in the Make table, which is a square matrix reporting commodities along columns and the amounts of each commodity produced by each sector along rows. Our final input-output network combines the data from these two tables, using the product of the Make matrix and the Use matrix detail on intermediate input payments.[32]

The investment network data from vom Lehn and Winberry (2022) reports the matrix of sectoral spending and production of new investment; see that paper for construction details. We follow those construction details provided to extend the investment network through the year 2020. However, the raw investment matrix from vom Lehn and Winberry (2022) is still organized with commodities along each row, not sectors, and needs to also be adjusted using the Make matrix. Thus, the final investment network data we use is the

[32]Formally, the final input-output matrix in any given year is the matrix product of a scaled Make table, where each column is scaled by its sum (thus summing to 1) and the unscaled Use table.

product of the Make matrix and the investment network data from vom Lehn and Winberry (2022).[33]

The Use tables from the BEA also contain information on the final uses of each commodity produced, including consumption. To measure structural change in consumption, we construct final consumption produced by each sector as the product of the Make matrix and the private final consumption vector in the Use table.[34] The Use table also has information on the final use of each commodity as new investment, however, we use sums of the data from the investment network (which is closely tied to this figure) to compute total production of investment by each sector.

Finally, we make note of how we use data from the BEA Make and Use tables to construct value added measures of consumption and investment (as described in equations (2) and (3). As noted above, the final vectors of consumption and investment are available from the Use tables and the investment network data. To compute the fraction of value added in gross output for each sector, we use data in the Use table on nominal value added and nominal gross output for each sector. To compute the total requirements matrix, or Leontief inverse, we simply scale our final input-output data (adjusted by the Make matrix) by the total gross output of each sector (as opposed to the total spending on intermediates), which gives us the matrix $\mathbf{\Gamma}$. Because we initially adjust both final consumption and the input-output data by the Make matrix, the formulas in equations (2) and (3) look slightly different from those reported in Herrendorf et al. (2013), however the methods are identical.[35]

Appendix A.2. Additional Detail on Section 2 Results

In this subsection, we report three additional empirical results: time series detail for sectors whose production share of investment or intermediates has increased or decreased the most, detail on how the services share of intermediates and investment is changing within all 43 sectors in our data, and the shift-share decomposition for the share of services in the production of investment or intermediates using data from all 43 sectors.

[33] Formally, the final investment matrix in any given year is the matrix product of a scaled Make table, where each column is scaled by its sum (thus summing to 1) and the unscaled investment network data.

[34] Again, formally, we use a scaled Make matrix for this multiplication. We do not include government consumption or make adjustments for imports/exports in computing final consumption, though this would have minor impact on our results.

[35] We note that, in principle, one of the reasons why the total requirements matrix might change over time are because the Make matrix has changed over time. However, we have explored counterfactuals holding the Make matrix fixed over time and found that changes in this matrix are minor and have virtually no impact on value added measures of structural change.

First, the four panels of Figure A.1 report the time series patterns sectors whose share of production of intermediates or investment has increased the most (right panels) or decreased the most (left panels). As reported in Section 2, for intermediate goods, the largest increases in production share are in information services, finance/insurance, real estate, professional/technical services, and administrative support services; the largest decreases occurred in agriculture, primary metals, food and beverage manufacturing, textile manufacturing, and paper manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest decreases are in machinery, construction, and motor vehicle manufacturing. While there is certainly heterogeneity in the changes over time in each of these production shares, we note that each sector's changes in production shares appears to be part of a gradual long run trend and not some permanent spike occurring in a particular year.

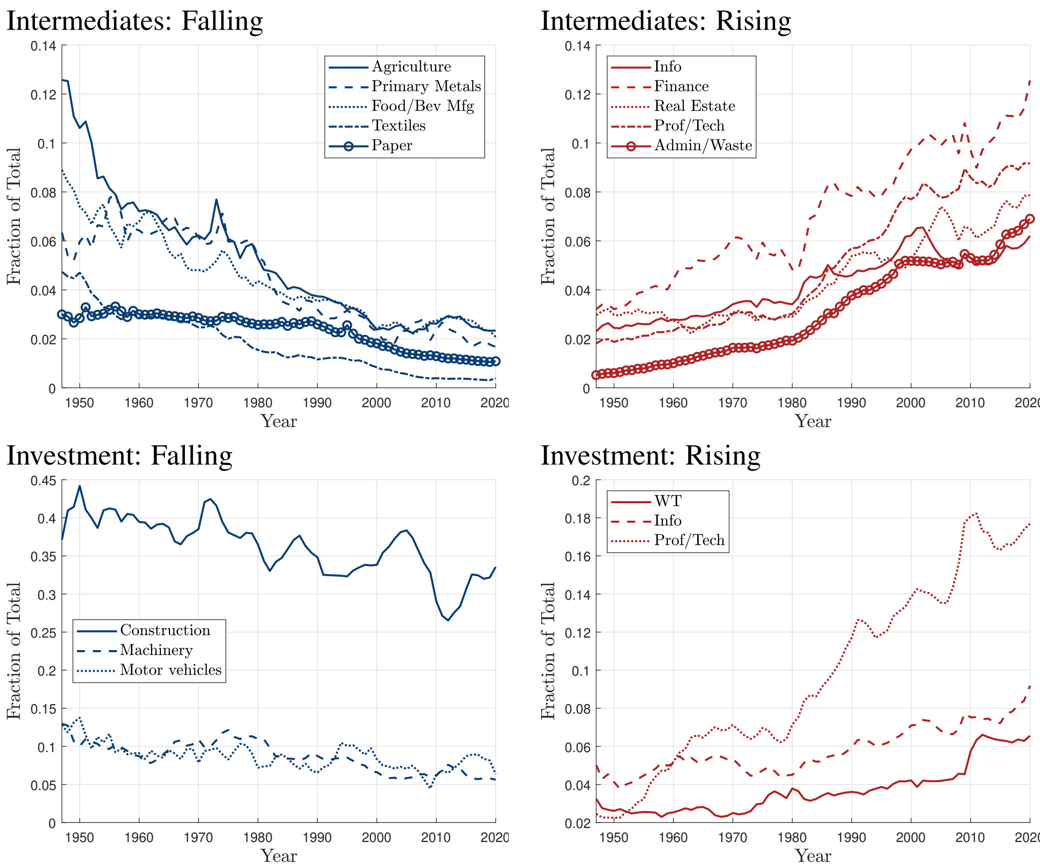
Second, in Figure A.2, we present bar charts showing the change in the services share of production of intermediates (top panel) and investment (bottom panel) within all 43 sectors in our data between 1947 and 2019. Although there is significant heterogeneity in how much the services share of production of intermediates or investment has changed in each sector, it is increasing in the vast majority of sectors.

Finally, in Table A.1, we report the shift-share decomposition (equation (1) and Table 2) where instead of looking at within and between sector changes for just 2 sectors, goods and services, we decompose the within and between sector changes in the services production share across all 43 sectors in our data. With this additional detail, we see that changes in sectoral composition, the between component, explains a larger fraction of the rise in the services share of production in both intermediates and investment. However, we still see that the vast majority of the rising services share in investment production is occurring within sectors (75%, as opposed to 100% in Table 2) and the within-sector component of the increase in the services share of intermediates production is about half.

Appendix A.3. International Evidence

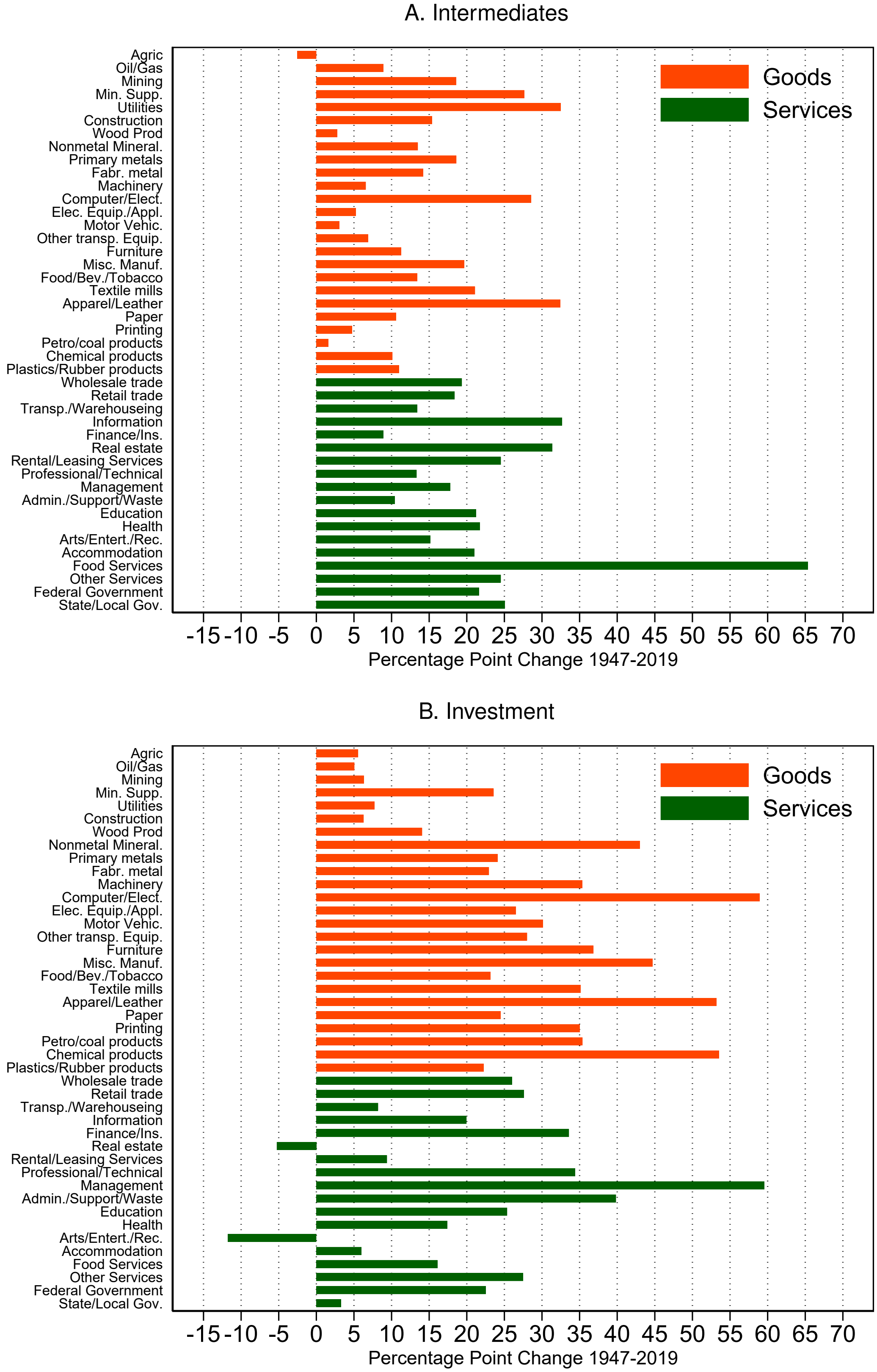
Appendix B. Equilibrium Conditions, Derivations and Proofs

Figure A.1: Time Series Changes in Production Share of Intermediates and Investment, Additional Sector Detail



Notes: Each line represents the a given sector's share of total production of intermediates (top rows) or investment (bottom rows). Right panels with red lines show sectors whose production share has increased the most; left panels with blue lines show sectors whose production share has decreased the most.

Figure A.2: Changes in Services Production Share of Intermediates and Investment Within Sectors: 1947-2019



Notes: Each bar represents the change in the share of intermediates (upper panel) or investment (lower panel) produced by the services sectors between 1947 and 2019 within each of the 43 sectors. Red bars: goods sectors; green bars: services sectors.

Table A.1: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment, 43 Sector Detail

				Decomposition	
	1947	2019	Δ	within	between
Intermediates	0.33	0.67	0.34	0.17 (51%)	0.16 (49%)
Investment	0.19	0.38	0.19	0.15 (75%)	0.05 (25%)

Notes: The table reports the shift-share decomposition described in equation (1) for the production share of services, but with within-sector and between-sector changes across all 43 sectors in our data. Individual components may not exactly sum to totals due to rounding.

Appendix B.1. Full Listing of Equilibrium Conditions

Appendix B.1.1. Household Problem

The household's problem is

$$\max_{C_{jt},K_{jt+1}} \sum_{t=0}^{\infty} \beta^t log \left(\left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C-1}} \right),$$

$$\text{s.t. } \sum_j P_{jt}C_{jt} + \sum_{j=1}^{N} P_{jt}^X \left(K_{jt+1} - (1-\delta_j)K_{jt} \right) \leq W_t + \sum_j R_{jt}K_{jt}.$$

The first order conditions for this problem, $\forall j$, are

$$\frac{P_{jt}^X}{E_t} = \frac{\beta}{E_{t+1}} \left(R_{jt+1} + P_{jt+1}^X (1-\delta_j) \right) \tag{B.1}$$

$$\frac{P_{jt}}{E_t} = \left(\omega_{Cj} \frac{C_t}{C_{jt}} \right)^{\frac{1}{\epsilon_C}} \frac{1}{C_t} \tag{B.2}$$

where total consumption, C_t, and total expenditures (denoted in units of the numeraire), E_t, are given by:

$$C_t \equiv \left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C - 1}{\epsilon_C}} \right]^{\frac{\epsilon_C}{\epsilon_C - 1}} \tag{B.3}$$

$$E_t = \sum_j P_{jt} C_{jt}. \tag{B.4}$$

Appendix B.1.2. Production Firm Problem

The profit maximization problem for the representative production firm in sector j is given by

$$\max_{L_{jt}, K_{jt}, M_{jt}} P_{jt} Q_{jt} - W_t L_{jt} - R_{jt} K_{jt} - P_{jt}^M M_{jt}.$$

where $Q_{jt} = A_{jt} \left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j} \right)^{\alpha_j} M_{jt}^{1-\alpha_j}$.

The first order conditions for this problem are

$$W_t = \alpha_j (1 - \theta_j) \frac{P_{jt} Q_{jt}}{L_{jt}} \tag{B.5}$$

$$R_{jt} = \alpha_j \theta_j \frac{P_{jt} Q_{jt}}{K_{jt}} \tag{B.6}$$

$$P_{jt}^M = (1 - \alpha_j) \frac{P_{jt} Q_{jt}}{M_{jt}}. \tag{B.7}$$

Appendix B.1.3. Bundling Firm Problems

For each sector j, there are two bundling firms: one that produces the intermediate good used by sector j, M_{jt}, and one that produces the investment (purchased by the household) for capital specific to sector j, X_{jt}.

The profit maximization problem of the intermediates bundling firm for sector j is given by:

$$\max_{M_{ijt}} P_{jt}^M M_{jt} - \sum_i P_{it} M_{ijt},$$

where the bundle of intermediates used by sector j, M_{jt} is given by:

$$M_{jt} = A^M_{jt} \left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}} \right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}} . \tag{B.8}$$

The first order conditions for this problem are, for each sector i:

$$P_{it} = P^M_{jt} \left(A^M_{jt} \right)^{1-\frac{1}{\epsilon_{Mj}}} \left(\omega_{Mij} \frac{M_{jt}}{M_{ijt}} \right)^{\frac{1}{\epsilon_{Mj}}} . \tag{B.9}$$

Obtaining the expression for the price of the intermediates bundle sold to sector j, P^M_{jt}, as reported in equation (11), follows from solving the first order conditions for M_{ijt}, plugging into the expression for M_{jt}, and solving for P^M_{jt}.

The profit maximization problem and first order conditions for the investment bundling are symmetric and are given by:

$$\max_{X_{ijt}} P^X_{jt} X_{jt} - \sum_i P_{it} X_{ijt}$$

$$P_{it} = P^X_{jt} \left(A^X_{jt} \right)^{1-\frac{1}{\epsilon_{Xj}}} \left(\omega_{Xij} \frac{X_{jt}}{X_{ijt}} \right)^{\frac{1}{\epsilon_{Xj}}} \tag{B.10}$$

where the bundle of investment for capital specific to sector j, X_{jt} is given by:

$$X_{jt} = A^X_{jt} \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}} \right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}} . \tag{B.11}$$

Similarly, the expression for the price of the investment bundle for sector j's capital shown in equation (12) can be obtained by solving the first order conditions for X_{ijt}, plugging into the expression for X_{jt}, and solving for P^X_{jt}.

Appendix B.1.4. Market Clearing Conditions

In equilibrium, each labor, capital, intermediate bundling and investment bundling market clears. To conserve on notation, market clearing is built into how the capital, intermediates and investment problems have been written down. With the household inelastically providing unitary labor supply each period, labor market clearing is simply given by $\sum_j L_{jt} = 1$. That leaves market clearing for final production in each sector j, which is given by:

$$C_{jt} + \sum_j M_{jit} + \sum_j X_{jit} = Q_{jt}. \tag{B.12}$$

We also note that evolution of capital in each sector is given by the standard accumulation equation:

$$K_{jt+1} = (1 - \delta_j)K_{jt} + X_{jt}. \tag{B.13}$$

Appendix B.1.5. Sectoral Value Added and Prices

For each production sector j, constant returns to scale implies

$$W_t L_{jt} + R_{jt} K_{jt} + P_{jt}^M M_{jt} = P_{jt} Q_{jt}. \tag{B.14}$$

Therefore, the accounting definition of nominal value added is simply

$$p_{jt}^V V_{jt} = P_{jt} Q_{jt} - P_{jt}^M M_{jt} = W_t L_t + R_{jt} K_{jt}. \tag{B.15}$$

To obtain real value added, we use a discrete time application of the Divisia index definition, which differentiates the accounting definition of nominal value added holding prices fixed:

$$\begin{aligned} P_{jt}^V V_{jt} \Delta \ln V_{jt} &= P_{jt} Q_{jt} \Delta \ln Q_{jt} - P_{jt}^M M_{jt} \Delta \ln M_{jt} \\ \alpha_j \Delta \ln V_{jt} &= \Delta \ln Q_{jt} - (1 - \alpha_j) \Delta \ln M_{jt} \\ \Delta \ln V_{jt} &= \frac{1}{\alpha_j} \Delta \ln A_{jt} + \theta_j \Delta \ln K_{jt} + (1 - \theta_j) \Delta \ln L_{jt}. \end{aligned}$$

Cumulating this expression yields that real value added is given by $V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}$.[36]

[36] We have normalized the implicit time-invariant constant in cumulating this expression to 1.

Finally, we can write the price index for value added in sector j, P_{jt}^V, as follows:

$$
\begin{aligned}
P_{jt}^V &= \frac{P_{jt}Q_{jt} - P_{jt}^M M_{jt}}{V_{jt}} \\
&= \frac{P_{jt}V_{jt}^{\alpha_j}\left(\left(\frac{(1-\alpha_j)P_{jt}}{P_{jt}^M}\right)^{\frac{1}{\alpha_j}} V_{jt}\right)^{1-\alpha_j} - P_{jt}^M\left(\frac{(1-\alpha_j)P_{jt}}{P_{jt}^M}\right)^{\frac{1}{\alpha_j}} V_{jt}}{V_{jt}} \\
&= P_{jt}^{\frac{1}{\alpha_j}}\left(P_{jt}^M\right)^{1-\frac{1}{\alpha_j}}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{1}{1-\alpha_j} - 1\right) \\
&= \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P_{jt}^M}\right)^{\frac{1-\alpha_j}{\alpha_j}}
\end{aligned}
$$

where we use the fact that $Q_{jt} = V_{jt}^{\alpha_j} M_{jt}^{1-\alpha_j}$ and the fact that $M_{jt} = \left(\frac{(1-\alpha_j)P_{jt}}{P_{jt}^M}\right)^{\frac{1}{\alpha_j}} V_{jt}$ (from the first order conditions for intermediates, shown in equation (B.7)).

Appendix B.2. Proof of Lemma 1

With Assumptions 1 and 2, we have:

$$
\frac{P_{jt}}{P_{it}} = \frac{A_{it}/\left(P_{it}^M\right)^{1-\alpha}}{A_{jt}/\left(P_{jt}^M\right)^{1-\alpha}} = \frac{\tilde{A}_{it}}{\tilde{A}_{jt}}
$$

where $\tilde{A}_{jt} \equiv \frac{A_{jt}}{\left(P_{jt}^M\right)^{1-\alpha}}$.

With this relationship, the lemma is straightforward to prove by manipulation of the expression for the price of investment (equation (12), though now common to all sectors due to Assumptions 1 and 2):

$$
\begin{aligned}
P_t^X &= \frac{1}{A_t^X}\left(\sum_k \omega_{Xi} P_{kt}^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} \\
&= P_{jt}\frac{1}{A_t^X}\left(\sum_k \omega_{Xk}\left(\frac{P_{kt}}{P_{jt}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} \\
&= P_{jt}\frac{1}{A_t^X}\left(\sum_k \omega_{Xk}\left(\frac{\tilde{A}_{jt}}{\tilde{A}_{kt}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} \\
&= P_{jt}\tilde{A}_{jt}\frac{1}{A_t^X}\left(\sum_k \omega_{Xk}\left(\tilde{A}_{kt}\right)^{\epsilon_X-1}\right)^{\frac{1}{1-\epsilon_X}}.
\end{aligned}
$$

Hence,

$$
P_{jt}A_{jt} = P_t^X A_t^X \left(\sum_k \omega_{Xk}\left(\tilde{A}_{kt}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}.
$$

Defining $\tilde{B}_X(t) \equiv A_t^X \left(\sum_k \omega_{Xk}\tilde{A}_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$, the result of the lemma obtains.

The proof for the price and technical change in intermediate goods follows identical steps as the above, with $\tilde{B}_{it}^M \equiv A_{it}^M \left(\sum_k \omega_{Mki}\tilde{A}_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$.

Appendix B.3. Proof of Lemma 2

Aggregate GDP, Y_t, denoted in units of the numeraire, is given by:

$$
Y_t = \sum_i P_{it}^V V_{it}
$$

where V_{it} is real value added in sector i and P_{it}^V is the price of value added in sector i.

As shown in Appendix B.1, sectoral real value added and its price, can be written as:

$$V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}$$

$$P_{jt}^V = \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}} \left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P_{jt}^M} \right)^{\frac{1-\alpha_j}{\alpha_j}}$$

Given Assumptions 1 and 2 (implying that $\alpha_j = \alpha$ for all sectors), and these expressions for real value added and its price, we can write aggregate GDP as:

$$\begin{aligned} Y_t &= \sum_i P_{it}^V V_{it} \\ &= \sum_i \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{A_{it} P_{it}}{\left(P_{it}^M\right)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \left(\frac{K_{it}}{L_{it}} \right)^{\theta} L_{it} \end{aligned}$$

Because of the common rental rate and wage, the capital to labor ratios will be equated across sectors, and with an aggregate labor supply of 1, will simply be equal to the aggregate stock of capital, $K_t = \sum_i K_{it}$. Further, from Lemma 1 and our choice of the price of investment as our numeraire, we have that $\frac{A_{it}P_{it}}{\left(P_{it}^M\right)^{1-\alpha}} = P_{it}\tilde{A}_{it} = \tilde{B}_t^X$. With this, we can rewrite the above expression for GDP as:

$$\begin{aligned} Y_t &= \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}} (K_t)^{\theta} \sum_i L_{it} \\ &= \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}} K_t^{\theta} \\ &= \mathcal{A}_t K_t^{\theta} \end{aligned}$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}}$.

First order conditions for capital in each production sector give us $R_t = \theta\alpha\frac{P_{jt}Y_{jt}}{K_{jt}}$. Using our expressions for real sectoral value added and its price, as well as the result of Lemma 2, we can rewrite the this first order condition as:

$$
\begin{aligned}
R_t &= \theta\alpha\frac{P_{jt}Y_{jt}}{K_{jt}} \\
&= \theta\alpha\frac{\frac{1}{\alpha}P^V_{jt}V_{jt}}{K_{jt}} \\
&= \theta\frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}}\left(\frac{K_{jt}}{L_{jt}}\right)^{\theta-1}\left(\frac{P_{jt}A_{jt}}{(P^M_t)^{1-\alpha}}\right)^{\frac{1}{\alpha}} \\
&= \theta\frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}}(\tilde{B}^X_t)^{\frac{1}{\alpha}}\left(\frac{K_t}{L_t}\right)^{\theta-1} \\
&= \theta\mathcal{A}_t K_t^{\theta-1}
\end{aligned}
$$

Applying the same algebraic steps to the first order condition for labor demand (equation (B.5)) generates the other equation in the lemma, $W_t = (1-\theta)\mathcal{A}_t K_t^{\theta}$.

Appendix B.4. Proof of Proposition 1

Given the if and only if statement in the proposition, we must prove both the necessary and sufficient directions. We start with the necessary direction, showing if an ABGP exists, this requires that $\gamma^{\mathcal{A}}$ is constant and that $\gamma^K = \gamma^X = \gamma^Y = \gamma^E = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$

The requirement that $\gamma^{\mathcal{A}}$ be constant follows immediately from the aggregate production function expression from Lemma 2, $Y_t = \mathcal{A}_t K_t^{\theta}$. If Y_t and K_t grow at constant rates, that means that $\mathcal{A}_t$ must as well. Thus, the remainder of this direction of the proof entails showing that the growth rates of K_t, Y_t, W_t, X_t and E_t are all equal to $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and that the growth rate of R_t is zero.

Taking the Euler equation from the household's problem (see Appendix B.1), we have that:

$$\frac{E_{t+1}}{E_t} = \gamma^E_{t+1} = \beta\left(R_{t+1} + 1 - \delta\right) \tag{B.16}$$

This implies that a constant growth rate in household expenditures implies a constant rental rate of capital, R_t, along the ABGP.

Taking the ratio of first order conditions for capital in each sector, we have that:

$$\frac{K_{jt}}{L_{jt}} = \frac{\theta}{1-\theta}\frac{W_t}{R_t} \tag{B.17}$$

With our assumptions of common parameters across sectors, capital to labor ratios are equated, and since $\sum_j L_{jt} = 1$, we can write the aggregate capital stock, K_t, as $K_t = \frac{\theta}{1-\theta}\frac{W_t}{R_t}$. Since R_t is constant along the ABGP, this implies that $\gamma^K = \gamma^W$.

From Lemma 2, we have that $R_t = \theta \mathcal{A}_t K_t^{\theta-1}$. Taking the ratio of this simplified first order condition for capital across time periods yields:

$$\frac{K_{t+1}}{K_t} = \left(\frac{R_{t+1}}{R_t}\frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\right)^{\frac{1}{1-\theta}} \tag{B.18}$$

$$\gamma^K = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{B.19}$$

where the last step holds given the constant rental rate of capital along the ABGP.

Taking the ratio of the aggregate production function from Lemma 2 across time periods yields:

$$\frac{Y_{t+1}}{Y_t} = \frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\left(\frac{K_{t+1}}{K_t}\right)^{\theta} \tag{B.20}$$

$$\gamma^Y = \gamma^{\mathcal{A}}\left(\gamma^{\mathcal{A}}\right)^{\frac{\theta}{1-\theta}} = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{B.21}$$

which thus implies $\gamma^Y = \gamma^K$.

Now, turning to the capital accumulation equation, if we divide by K_t, we have:

$$\gamma^K = (1-\delta) + \frac{X_t}{K_t}$$

Since γ^K is a constant, this requires that the RHS be constant, or in other words, $\gamma^X = \gamma^K$.

The only remaining condition to verify here is that aggregate consumption expenditures, E_t, grow at the same rate as aggregate capital. We can write GDP using expenditure side accounting as $Y_t = E_t + X_t$, since all these aggregates are denoted in units of the numeraire. Since we know that $\gamma^Y = \gamma^K = \gamma^X$ and γ^E is constant, then $\gamma^E = \gamma^Y = \gamma^K$. This finishes the necessity direction of the proof.

We now consider the sufficiency direction required for the proof. We now show that if $\gamma^{\mathcal{A}}$ is constant, then an ABGP exists. We do this by construction. We set $\gamma^K = \gamma^X = \gamma^Y = \gamma^E = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and we set R_t to be a constant such that the Euler Equation holds:

$$R_{t+1} = \frac{1}{\beta}\gamma^E - (1-\delta) \tag{B.22}$$

Given our assumption that $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} > \frac{1-\delta}{\beta}$, this will produce a non-negative rental rate for capital.

Then, given an initial value of $\mathcal{A}_t$, this value of R implies a unique value for K_0 from (the rewritten first order conditions). It is then straightforward to construct X_0 to satisfy capital accumulation, given K_0 and γ^K. Finally, we can determine the initial condition for expenditures, using the expenditure side accounting relationship, with

$$\begin{aligned} E_0 &= Y_0 - X_0 \\ &= \mathcal{A}_0 K_0^{\theta} - K_0 \left(\gamma^K - (1-\delta)\right). \end{aligned}$$

Lastly, to show that transversality holds, we need that

$$lim_{t\to\infty}\beta^t \frac{K_t}{E_t} = 0 \tag{B.23}$$

Given that we have constructed the path such that $\gamma^K = \gamma^E, \frac{K_t}{E_t}$ will be a constant along this path and thus the limit will be satisfied. Thus completes the proof in the sufficiency direction.

Appendix B.5. Proof of Lemma 3

Given the assumption 3, that the parameters of the intermediates bundling sectors are the same for all sectors j, i.e. $\omega_{Mij} = \omega_{Xi}$ and $\epsilon_{Mj} = \epsilon_M$ for all j and that technical change in each sector's intermediates bundling is the same, i.e. $A_{jt}^M = A_t^M$, we start by revisiting the result of Lemma 1. Assumption 3 implies that there is now a single intermediate good in the economy, with a single price, P_t^M. As a result, given our definition of $\tilde{A}_{it} \equiv \frac{A_{jt}}{\left(P_{jt}^M\right)^{1-\alpha}}$, we have that

$$\frac{P_{jt}}{P_{it}} = \frac{\tilde{A}_{it}}{\tilde{A}_{jt}} = \frac{A_{it}}{A_{jt}}$$

Thus, we now have that $\tilde{B}_{it}^{M} = \tilde{B}_{t}^{M}$ and by the same logic as the above and the proof of Lemma 1, we have that:

$$\frac{B_t^X}{B_t^M} = \frac{P_t^M}{P_t^X}$$

where $B_{it}^M \equiv A_{it}^M \left(\sum_k \omega_{ki}^M A_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ and $B_t^X \equiv A_t^X \left(\sum_k \omega_k^X A_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$.

The final part left to show is that $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B_t^X \left(B_t^M\right)^{\frac{1-\alpha}{\alpha}}$. Given that, in the more general case, $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}}$, this amounts to showing that $\tilde{B}_t^X = \left(B_t^X\right)^{\alpha}\left(B_t^M\right)^{1-\alpha}$. Given the definition of $\tilde{B}_t^X$, we have that:

$$\begin{aligned}
\tilde{B}_X(t) &= A_t^X \left(\sum_k \omega_{Xk} \tilde{A}_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}} \\
&= A_t^X \left(\sum_k \omega_{Xk} \left(\frac{A_{kt}}{\left(P_t^M\right)^{1-\alpha}}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}} \\
&= \frac{B_t^X}{\left(P_t^M\right)^{1-\alpha}} \\
&= \frac{B_t^X}{\left(\frac{B_t^X}{B_t^M}\right)^{1-\alpha}} \\
&= \left(B_t^X\right)^{\alpha}\left(B_t^M\right)^{1-\alpha}
\end{aligned}$$

This completes the proof.

Appendix B.6. Proof of Lemma 4

We prove the result for intermediates and ϵ_M first; the result for investment with ϵ_X follows by the symmetry of the CES functions.

For ease of exposition of the proof, we define $g^M(\epsilon_M)$ as follows:

$$g^M(\epsilon_M) = \left(\sum_i s^M_{it-1}(\gamma^A_{it})^{\epsilon_M - 1}\right)^{\frac{1}{\epsilon_M - 1}}$$

Thus, the objective is to show that $g^M(\epsilon_M)$ is weakly increasing in ϵ_M. For ease of exposition, we also suppress the A superscript on γ^A_{it} and define $\gamma_i \equiv \gamma_{it}$.

We observe that $g^M(\epsilon_M)$ depends on ϵ_M in two ways—both directly, as an exponent on γ^A_{it} and in the exponent for the overall sum, but also indirectly, through its impact on s^M_{it-1}, which is itself a function of ϵ_M. Plugging in the definition for B^M_t from Lemma 1 to the definition of s^M_{it} in equation (26), s^M_{it-1} can be written as a function of exogenous values, including ϵ_M:

$$s^M_{it-1}(\epsilon_M) = \omega_{Mi} \frac{A^{\epsilon_M - 1}_{it-1}}{\sum_j^N \omega_{Mj} A^{\epsilon_M - 1}_{jt-1}}$$

Our goal is to show that for any $\epsilon_1 > \epsilon_2$, $g^M(\epsilon_1) \geq g^M(\epsilon_2)$. We show this in two steps. First, we define the function $\tilde{g}((\sigma, \epsilon_M) = \left(\sum_i s^M_{it-1}(\sigma)\gamma^{\epsilon_M - 1}_{it}\right)^{\frac{1}{\epsilon_M - 1}}$. We first show that for fixed σ and $\epsilon_1 > \epsilon_2$, $\tilde{g}((\sigma, \epsilon_1) \geq \tilde{g}((\sigma, \epsilon_2)$. The second step defines the function $\hat{g}(\epsilon_M, \sigma) = \left(\sum_i s^M_{it-1}(\epsilon_M)\gamma^{\sigma - 1}_{it}\right)^{\frac{1}{\sigma - 1}}$ and shows that $\hat{g}((\epsilon_1, \sigma) \geq \hat{g}((\epsilon_2, \sigma)$. Then, given these two substeps, the final result follows from the following sequence of inequalities:

$$g(\epsilon_1) = \tilde{g}(\epsilon_1, \epsilon_1) \geq \tilde{g}(\epsilon_1, \epsilon_2) = \hat{g}(\epsilon_1, \epsilon_2) \geq \hat{g}(\epsilon_2, \epsilon_2) = g(\epsilon_2) \tag{B.24}$$

We note that the lemma makes two additional assumptions—that TFP in each sector is normalized to 1 initially and that the growth rate of TFP is held constant over time. These assumptions are not needed until Step 2, and so we demonstrate Step 1 for the more general case without these additional assumptions.

Step 1: For $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma, \epsilon_1) \geq \tilde{g}(\sigma, \epsilon_2)$. This first step of the proof follows from an application of Jensen's inequality. Jensen's inequality tells us that for any convex function, $\phi(x)$, any real valued function $h(x)$, and any set of non-negative weights a_i with $\sum_i a_i = 1$, $\sum_i a_i \phi(h(x_i)) \geq \phi(\sum_i a_i h(x_i))$. The inequality is reversed in the case where $\phi(x)$ is concave.

First, begin with the case where $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $\frac{\epsilon_1 - 1}{\epsilon_2 - 1} > 1$. Define $\phi(x) = x^{\frac{\epsilon_1 - 1}{\epsilon_2 - 1}}$. This function is convex because $\frac{\epsilon_1 - 1}{\epsilon_2 - 1} > 1$. Define $h(x) = x^{\epsilon_2 - 1}$ and $a_i = s^M_{it-1}(\sigma)$.

Jensen's inequality thus implies the following result:

$$\left(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_2-1}\right)^{\frac{\epsilon_1-1}{\epsilon_2-1}} = \tilde{g}(\sigma,\epsilon_2)^{\epsilon_1-1} \leq \left(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_1-1}\right) = \tilde{g}(\sigma,\epsilon_1)^{\epsilon_1-1}$$

Exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1-1}$, which is a positive exponent, completes the result for this case.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 < \frac{\epsilon_1-1}{\epsilon_2-1} < 1$, then it must be that $\epsilon_1 < 1$. In this case, $\phi(x)$ is now concave, which reverses the above inequality. However, because $\epsilon_1 < 1$, the step of exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1-1}$ again reverses the inequality and ensures the result holds.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 > \frac{\epsilon_1-1}{\epsilon_2-1}$, then $\epsilon_2 < 1$ and $\epsilon_1 > 1$, and $\phi(x)$ is again convex and $\frac{1}{\epsilon_1-1}$ is a positive exponent, so the result still holds.

Finally, consider the case where either $\epsilon_1 = 1$ or $\epsilon_2 = 1$. Although $\tilde{g}(\epsilon_M)$ is undefined in this case, we consider instead the limiting result, defining $\tilde{g}(\sigma,0) = \prod_i \gamma_{it}^{s^M_{it-1}(\sigma)}$. Here we apply Jensen's inequality using $\phi(x) = \ln(x)$ and $h(x) = x^{\epsilon_1-1}$. If $\epsilon_2 = 1$ and $\epsilon_1 > 1$, then we have that:

$$(\epsilon_1-1)\ln(\tilde{g}(\sigma,\epsilon_1)) = \ln(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_1-1}) \geq (\epsilon_1-1)\sum_i s^M_{it-1}(\sigma)\ln(\gamma_{it}) = (\epsilon_1-1)\ln(g(\sigma,0))$$

Dividing both sides by $\epsilon_1 - 1$ and exponentiating both sides of the inequality yields the result.

In the case where $\epsilon_1 = 2$ and $\epsilon_2 < 1$, the same steps can be followed, replacing ϵ_1 with ϵ_2, but now since $\epsilon_2 < 1$, the final step of dividing both sides by $\epsilon_2 - 1$ will reverse the inequality, proving the result.

Thus completes step 1.

Step 2: For $\epsilon_1 > \epsilon_2$, $\hat{g}((\epsilon_1,\sigma) \geq \hat{g}((\epsilon_2,\sigma)$. To prove this inequality, we show that $\frac{\partial\hat{g}((\epsilon_M,\sigma)}{\partial\epsilon_M} \geq 0$.

Taking the partial derivative, we obtain the following result:

$$\begin{aligned}
\frac{\partial\hat{g}(\epsilon_M,\sigma)}{\partial\epsilon_M} &= \frac{1}{\sigma-1}\hat{g}(\epsilon_M,\sigma)^{2-\sigma}\sum_i \frac{\partial s^M_{it-1}(\epsilon_M)}{\partial\epsilon_M}\gamma_{it}^{\sigma-1} \\
&= \frac{1}{\sigma-1}\hat{g}(\epsilon_M,\sigma)^{2-\sigma}\left(\sum_i s^M_{it-1}\gamma_{it}^{\sigma-1}\ln(A_{it}) - \left(\sum_i s^M_{it-1}\gamma_{it}^{\sigma-1}\right)\left(\sum_i s^M_{it-1}\ln(A_{it})\right)\right) \\
&= \frac{1}{\sigma-1}\hat{g}(\epsilon_M,\sigma)^{2-\sigma} COV\left(\ln(A_{it}),\gamma_{it}^{\sigma-1}\right)
\end{aligned}$$

where $COV(\ln(A_{it}), \gamma_{it}^{\sigma-1})$ is the covariance between $\ln(A_{it})$ and $\gamma_{it}^{\sigma-1})$ where probability weights across sectors are defined by the shares s_{it-1}^{M}. To be able to consistently sign this covariance term requires assumptions of constant growth in each sector's TFP and initial conditions with each sector having the same level of TFP (i.e. $A_{i0} = 1$). Those assumptions are sufficient conditions (though not necessary ones) for knowing the sign of this covariance term will be the the sign of $\sigma - 1$.[37] More generally, provided that this covariance has the same sign as $\sigma - 1$, the result will go through. Since we know that $\hat{g}(\epsilon_M, \sigma)^{2-\sigma} > 0$ and the entire expression is multiplied by $\frac{1}{\sigma-1}$, this ensures that $\frac{\partial \hat{g}(\epsilon_M,\sigma)}{\partial \epsilon_M} \geq 0$. This completes step 2 of the proof.

Given the successful completion of steps 1 and 2, the proof for intermediates follows from the inequalities in equation (B.24) and the proof for investment follows by symmetry.

[37] Assuming that γ_i is constant and that $A_{i0} = 1$ ensures that $\ln(A_{it}) = t\ln(\gamma_i)$, which is an increasing function of γ_i. Thus, if $\gamma_i^{\sigma-1}$ is a increasing (decreasing) function of γ_i, then the covariance must be positive (negative). The increasing or decreasing nature of the function $\gamma_i^{\sigma-1}$ depends on the sign of $\sigma - 1$.