Structural Change within the Services Sector and the Future of Cost Disease*

Georg Duernecker (Goethe University Frankfurt and CEPR)
Berthold Herrendorf (Arizona State University, CEPR, and CESifo)
Ákos Valentinyi (University of Manchester, CEPR, and KRTK KTI)

June 21, 2022

Abstract

Baumol (1967) observed that developed economies suffer from cost disease, i.e., aggregate productivity growth falls because structural change reallocates production to services with low productivity growth. We document that cost disease importantly contributed to the productivity growth slowdown in the postwar U.S. To assess how severe cost disease may become, we build a model of structural change among the goods sector and broad services sectors. Calibrating the model to the postwar U.S. implies that broad categories of services are substitutes and the services with low productivity growth do not take over production. Simulating the calibrated model forward implies that future cost disease will be less severe than past one.

Keywords: Cost Disease; Productivity Growth Slowdown; Services; Structural Change; Substitutability.

JEL classification: O41; O47; O51.

*This paper includes material from our earlier manuscript "Unbalanced Growth Slowdown". We have received useful feedback from Zsófia Bárány, Aspen Gorry, Hannes Malmberg, Marti Mestieri, Rachel Ngai, Richard Rogerson, V. Kerry Smith, Michael Sposi, and Gustavo Ventura, as well as the audiences of presentations at ASU, the BEA, CEPR's ESSIM 2017, CERGE–EI Prague, the Christmas Meeting of the German Expat Economists 2016, the Cowles Conference on Macro 2019, the Economic Growth and Fluctuations Group of the Barcelona Summer Forum 2017, Erasmus University Rotterdam, the European Monetary Forum 2016 at the Bank of England, the Federal Reserve Banks of Atlanta and St. Louis, the Growth and Macro Development Conference in Madrid 2018, the RIDGE Workshop on Growth and Development 2016, the SED Meeting 2017, Southern Methodist University, Sabanci University, the Stockholm School of Economics, the Universities of Barcelona, Frankfurt, Konstanz, Manchester, Munich, North Carolina at Charlotte, and Western Ontario, the Vienna Macroeconomic Workshop 2017, Washington University, and the World Bank. Valentinyi thanks the Hungarian National Research, Development and Innovation Office (Project KJS K 124808). All errors are our own.

1 Introduction

Cost disease occurs in developed economies when aggregate productivity growth slows down because structural change reallocates production to services sectors with relatively low productivity growth. In the initial statement of cost disease, Baumol (1967) drew particular attention to the fact that production is even reallocated to services that have productivity growth close to zero. Subsequent evidence confirmed that indeed several such sectors expanded in terms of employment or value added shares; see for example Nordhaus (2008). If the services sectors that have productivity growth close to zero were to slowly take over the economy, then that would reduce long-run aggregate productivity growth all the way down to close to zero. Whether that is going to happen depends on the strengths of the forces that determine the sectoral composition. Although authors like Smith (1978) recognized this point in the 1970s, it is fair to say that we still do not have a goods sense of whether, and under what conditions, the "apocalyptic" scenario may occur. In this paper, we therefore ask how important the effect of cost disease on U.S. productivity growth will be in the future.

We start our analysis by documenting that the past effect of structural change on aggregate productivity growth was quantitatively important. We find for the postwar U.S. that structural change reduced the average annual growth rates of aggregate TFP and of labor productivity by 0.24 and 0.16 percentage points, respectively. We note that the size of this past effect is in the ballpark of the existing evidence; see for example Nordhaus (2008). We also find that the effect of structural change on aggregate productivity growth varied largely across subperiods. By and large, it followed an inverted U-shape that peaked during the productivity growth slowdown of the 1980s.

Our main result is that the future effect of structural change on aggregate productivity growth will be less severe than the past one. Establishing our main result requires an equilibrium model that captures how uneven sectoral productivity growth leads to changes in the sectoral composition, and how these changes affect aggregate productivity growth. Most existing models study structural change among the three broad sectors agriculture, manufacturing, and services; see for example the benchmark model presented in Herrendorf et al. (2014). In contrast, Jorgenson and Timmer (2011) argued convincingly that structural change within the services sector is more important for aggregate outcomes in developed countries. In the U.S., for example, the services sector comprises around 4/5 of aggregate value added and the industries of the services sector are not at all homogeneous in terms of growth of productivity and employment. It is therefore of first-order importance to model what happens *within* the services sector. We disaggregate economic activity into three different sectors: the goods sector produces tangible output whereas two services sectors produce intangible output; the two services sectors differ in their productivity growth: "progressive" services comprise the services industries with above average productivity growth in services whereas "stagnant" services comprise the services industries with below average productivity growth in services.

The literature on structural change has shown that changes in the sectoral composition of the economy result from how uneven productivity growth, which is determined by technology, interacts with income and substitution elasticities, which are determined by preferences. Our formulation of preferences has two crucial features: the elasticity of substitution between the two services may differ from the elasticity of substitution between aggregate services and goods ("nested preference structure"); the income elasticities of goods and the two services may differ from one even in the long run ("persistent non-homotheticities"). The second feature takes seriously the evidence of Boppart (2014), who established that persistent non-homotheticities are of first-order importance in the context of structural change.

Connecting our model to aggregate data from the postwar U.S. economy, we find the standard features that goods are necessities, aggregate services are luxuries, and goods and aggregate services are complements. We also find the new features that progressive services are necessities, stagnant services are luxuries, and the two services are substitutes. Since the new features are critical for our results, we go to great lengths to provide empirical support for them. We first offer micro evidence from the Consumer Expenditure Survey (CEX henceforth) that the new features are present at the household level. We then show that a careful calibration to macro data implies the new features.

We find that the future effect of structural change on productivity growth remains limited and are smaller than the past effect. Specifically, our theoretical analysis shows the novel result that if the two services subsectors are substitutes then the stagnant services can take over our model economy only if their productivity growth is sufficiently *high*. In contrast, if their productivity growth is below a threshold value, then households substitute away from the stagnant services so strongly that their share is driven down to zero. For plausible parameter values, our model therefore rules out that future aggregate productivity growth is driven all the way down to zero. The surprising part of this analytical result is that it holds although stagnant services are luxuries and irrespective of the strength of the persistent income effects that are implied by non-homothetic preferences. Turning to our quantitative results, when we simulate our model forward, we find that the productivity-growth slowdown caused by structural change will be smaller in the future than it was in the past. We establish that this result is robust to the details of the specification of our model and the parameter choices.

The intuition for why the future effect of structural change on productivity growth remains limited is as follows. In the past, the main effect of structural change on productivity growth came from the reallocation of employment and value added rom the goods sector to the services sector. This effect was quantitatively sizeable because the reallocation was strong and average productivity was considerably higher in goods than in services production. In the future, similar reallocation will play only a limited role because by now the goods sectors has shrunk to merely a fifth of the economy so that there is not much left to reallocate to services. Instead, it will matter more what happens within the services sector. We find that, consistent with our theoretical result, substitutability among progressive and stagnant services implies that stagnant services will not take over the economy. Therefore, the future effect of structural change on productivity growth remains limited.

One may wonder whether there is an advantage of working with our disaggregation into stagnant and progressive services instead of working with existing disaggregations of services. Examples include the distinction between traditional and non-traditional services as suggested by Duarte and Restuccia (2020); market and non–market services as used by the guidelines of the System of National Accounts; high-skill-intensive and low-skill-intensive services as suggested by Buera and Kaboski (2012) and Buera et al. (2021). We establish that none of these alternatives is as informative about productivity growth as our two-sector split. As an additional argument in support of our two-sector split, we also establish that the implications of the calibrated model for future productivity growth are robust to relaxing our nesting structure and to disaggregating services further. These results suggest that our two-split into progressive and stagnant services is suitable for answering our question.

Our work contributes to the debate about whether the past slowdown in productivity growth is temporary or permanent. To begin with, Antolin-Diaz et al. (2017) established statistically that aggregate productivity growth is declining. Moreover, Gordon (2016) argued that we picked the "low–hanging fruit" (e.g., railroads, cars, and airplanes) during the "special century 1870–1970" and that more recent innovations pale in comparison. Bloom et al. (2020) provided evidence that indeed ideas seem to become harder and harder to find. In addition, Fernald and Jones (2014) and Fernald (2016) pointed out that the engines of economic growth like education or research and development require the input of time which cannot be increased ad infinitum. The tendency in the debate is to conclude that low productivity growth is likely to be the future norm. Our contribution to the debate is to assess an additional reason for productivity growth slowdown: changes in the sectoral composition of the economy resulting from structural change.

The rest of the paper is organized as follows. We start our analysis by presenting evidence on cost disease and by establishing the stylized facts of structural change within services (Section 2). We then develop our model and theoretically characterize its equilibrium dynamics (Section 3). We supplement our theoretical results with a calibration of our model and a quantitative analysis of the calibrated model (Section 5). We also offer micro evidence in favor of the new features of our utility function (Section 4), conduct robustness analysis (Section 6), and review the related literature (Section 7). We conclude in Section 8. An Appendix contains background information about the construction of real value added in the data, the proofs of our theoretical results, and details of our quantitative analysis.

2 The Past Effect of Structural Change on U.S. Productivity

While we are focused on assessing the possible future effects of structural change on productivity growth, a natural starting point for our analysis is to confirm that the effects of structural change importantly contributed to the past productivity growth slowdown. Nordhaus (2008) provided initial evidence to that effect based on BEA data for 1948–2001. Unfortunately, BEA data are not ideal in the current context because they do not offer a complete series of quality adjusted labor services by industry. Using hours data that are not quality adjusted leads to a conceptual problem when conducting counterfactuals that reallocate workers among sectors with different average human capital. In effect, it requires the assumption that a reallocated worker immediately adjusts her human capital to the new sector's average level, which is unpalatable. We therefore use WORLD KLEMS data, which offer quality-adjusted labor services by industry for the U.S. during 1947–2017. Quality adjusted labor services are the sum of raw hours of different labor categories weighted with their relative rental prices. Since WORLD KLEMS ends 2017, we extend the relevant statistics until 2019 using the BEA–BLS Industry Level Production Accounts. In doing so, we follow the methodology behind the data construction of WORLD KLEMS that Jorgenson et al. (2013) describe.

2.1 Productivity Accounting Framework

In this subsection, we derive the relationship between productivity growth at the aggregate and the sectoral level and use it to assess the quantitative importance of cost disease in the postwar U.S. by calculating the counterfactual aggregate productivity growth without structural change. We employ the two most common measures of productivity: total factor productivity (TFP) and labor productivity. We derive the key relationships in continuous time because that is more convenient. We then approximate them in discrete time because that is required for the data work. Our derivation draws on the productivity accounting framework of Nordhaus (2001).

We start by deriving some basic accounting relationships akin to what is familiar from aggregate growth accounting. This will serve to obtain relationships between the growth of aggregate and sectoral TFP and of aggregate and sectoral labor.

There are N sectors that produce sectoral value added, Y_{nt}, from sectoral capital, K_{nt}, and sectoral labor, H_{nt}, according to a constant-returns-to-scale sectoral technology F_n:

$$Y_{nt} = A_{nt} F_n(K_{nt}, H_{nt}), \tag{1}$$

where A_{nt} is sectoral TFP and $n \in \{1, \ldots, N\}$. Aggregate value added (GDP) is a composite of sectoral value added according to a constant-returns-to-scale aggregator F:

$$Y_t = F(Y_{1t}, \dots, Y_{Nt}). \tag{2}$$

Assuming perfect competition, the first-order conditions of the representative sectoral firms are:

$$P_{nt} A_{nt} \frac{\partial F_n(t)}{\partial K_{nt}} = R_{nt}, \tag{3a}$$

$$P_{nt} A_{nt} \frac{\partial F_n(t)}{\partial H_{nt}} = W_{nt}, \tag{3b}$$

$$P_t \frac{\partial F(t)}{\partial Y_{nt}} = P_{nt}, \tag{3c}$$

where P_{nt} is the price of value added from sector n, R_{nt} and W_{nt} are the rental prices of capital and labor, and P_t is the price level of GDP.

We derive the relationships between TFP growth and labor growth in three steps. First, differentiating the log of (1) with respect to time and using (1) and (3a)–(3b), we obtain:

$$\frac{\dot{Y}_{nt}}{Y_{nt}} = \frac{\dot{A}_{nt}}{A_{nt}} + \frac{R_{nt} K_{nt}}{P_{nt} Y_{nt}} \frac{\dot{K}_{nt}}{K_{nt}} + \frac{W_{nt} H_{nt}}{P_{nt} Y_{nt}} \frac{\dot{H}_{nt}}{H_{nt}}. \tag{4}$$

Second, differentiating the log of (2) with respect to time and using (2) and (3c), we obtain:

$$\frac{\dot{Y}_t}{Y_t} = \sum_{n=1}^{N} \frac{P_{nt} Y_{nt}}{P_t Y_t} \frac{\dot{Y}_{nt}}{Y_{nt}}. \tag{5}$$

Third, combining (4) and (5) yields the familiar growth-accounting expression at the aggregate level:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} + \frac{R_t K_t}{Y_t} \frac{\dot{K}_t}{K_t} + \frac{W_t H_t}{Y_t} \frac{\dot{H}_t}{H_t}, \tag{6}$$

where

$$\frac{\dot{A}_t}{A_t} \equiv \sum_{n=1}^{N} \frac{P_{nt} Y_{nt}}{P_t Y_t} \frac{\dot{A}_{nt}}{A_{nt}}, \tag{7a}$$

$$\frac{\dot{K}_t}{K_t} \equiv \sum_{n=1}^{N} \frac{R_{nt} K_{nt}}{R_t K_t} \frac{\dot{K}_{nt}}{K_{nt}}, \qquad \frac{\dot{H}_t}{H_t} \equiv \sum_{n=1}^{N} \frac{W_{nt} H_{nt}}{W_t H_t} \frac{\dot{H}_{nt}}{H_{nt}}, \tag{7b}$$

$$R_t \equiv \frac{\sum_{n=1}^{N} R_{nt} K_{nt}}{K_t}, \qquad W_t \equiv \frac{\sum_{n=1}^{N} W_{nt} H_{nt}}{H_t}. \tag{7c}$$

In words, aggregate TFP growth is a weighted average of sectoral TFP growth where the weights are the shares of sectoral value added in aggregate value added. Aggregate labor growth is a weighted average of sectoral labor growth where the weights are the shares of the sectoral payments to labor in the aggregate payments to labor.

Next, we derive the relationship between aggregate and sectoral labor productivity growth, which are defined as:

$$\frac{\dot{LP}_t}{LP_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{H}_t}{H_t}, \tag{8}$$

$$\frac{\dot{LP}_{nt}}{LP_{nt}} = \frac{\dot{Y}_{nt}}{Y_{nt}} - \frac{\dot{H}_{nt}}{H_{nt}}. \tag{9}$$

Using (5), (7b), and (8)–(9), we obtain:

$$\begin{aligned}\frac{\dot{LP}_t}{LP_t} &= \sum_{n=1}^{N} \frac{P_{nt}Y_{nt}}{P_tY_t}\frac{\dot{Y}_{nt}}{Y_{nt}} - \sum_{n=1}^{N} \frac{W_{nt}H_{nt}}{W_tH_t}\frac{\dot{H}_{nt}}{H_{nt}} \\ &= \sum_{n=1}^{N} \frac{P_{nt}Y_{nt}}{P_tY_t}\frac{\dot{LP}_{nt}}{LP_{nt}} - \sum_{n=1}^{N} \left(\frac{W_{nt}H_{nt}}{W_tH_t} - \frac{P_{nt}Y_{nt}}{P_tY_t}\right)\frac{\dot{H}_{nt}}{H_{nt}}. \end{aligned} \tag{10}$$

Aggregate labor productivity growth has two components. The first component is analogous to TFP growth, that is, a weighted average of sectoral labor productivity growth where the weights are the shares of sectoral value added. It captures how aggregate productivity growth is affected by the allocation of labor to sectors with different *growth rates* of labor productivity. For example, average productivity growth is larger if more labor is allocated to a sector with a higher than average rate of productivity growth. The second component is a weighted average of the growth rates of sectoral labor where the weights are the differences between the sectoral shares of value added and of labor compensation. It captures how aggregate productivity growth is affected by the reallocation of labor among sectors with different *levels* of labor productivity, as measured by $P_{nt}Y_{nt}/(P_tY_t) - W_{nt}H_{nt}/(W_tH_t)$. For example, average productivity growth is larger in a period in which labor is reallocated to a sector with a higher than average level of productivity. This term is absent for aggregate TFP.

2.2 Measuring the Effect of Structural Change on the Growth of U.S. Productivity

To prepare for the data work to follow, we now approximate the continuous-time expressions of aggregate productivity growth in discrete time. Growth rates become log differences so that the growth rate of a generic variable X is given by:

$$\Delta \log X_t \equiv \log X_t - \log X_{t-1}. \tag{11}$$

Shares become averages of shares over two adjacent periods:

$$S(P_{nt}Y_{nt}) \equiv \frac{1}{2}\left(\frac{P_{nt-1}Y_{nt-1}}{P_{t-1}Y_{t-1}} + \frac{P_{nt}Y_{nt}}{P_tY_t}\right),$$
$$S(W_{nt}H_{nt}) \equiv \frac{1}{2}\left(\frac{W_{nt-1}H_{nt-1}}{W_{t-1}H_{t-1}} + \frac{W_{nt}H_{nt}}{W_tH_t}\right).$$

The discrete-time equivalents of expressions (7a) and (10) then are:

$$\Delta \log A_t \equiv \sum_{n=1}^{N} S(P_{nt}Y_{nt})\Delta \log A_{nt}, \tag{12}$$

$$\Delta \log LP_t = \sum_{n=1}^{N} \left[S(P_{nt}Y_{nt})\Delta \log LP_{nt} + \left(S(P_{nt}Y_{nt}) - S(W_{nt}H_{nt})\right)\Delta \log H_{nt}\right]. \tag{13}$$

We are now ready to measure the effect of structural change on productivity growth. We start with TFP growth. The counterfactual TFP growth without structural change follows by assuming that the shares of sectoral value added would not have changed. Replacing $S(P_{nt}Y_{nt})$ in equation (12) by $S(P_{n0}Y_{n0})$ from the initial period, we obtain

$$\Delta \log A_t(\text{counterfactual}) \equiv \sum_{n=1}^{N} S(P_{n0}Y_{n0})\Delta \log A_{nt}. \tag{14}$$

We continue with labor-productivity growth. Without structural change, the shares of sectoral labor would not have changed either and sectoral labor would have grown like aggregate labor. Replacing $S(P_{nt}Y_{nt})$ with $S(P_{n0}Y_{n0})$ and $\Delta \log H_{nt}$ with $\Delta \log H_t$ in equation (13), we obtain:

$$\begin{aligned}\Delta \log LP_t(\text{counterfactual}) &= \sum_{n=1}^{N} \left[S(P_{n0}Y_{n0})\Delta \log LP_{nt} + \left(S(P_{nt}Y_{nt}) - S(W_{nt}H_{nt})\right)\Delta \log H_t\right] \\ &= \sum_{n=1}^{N} S(P_{n0}Y_{n0})\Delta \log LP_{nt}. \end{aligned} \tag{15}$$

The second equality follows because the shares add up to one:

$$\sum_{n=1}^{N} S(P_{nt}Y_{nt}) = \sum_{n=1}^{N} S(W_{nt}H_{nt}) = 1.$$

An obstacle to connecting (14)–(15) to the data is that average human capital generally differs across sectors. It is then unclear what human capital counterfactual one ought to assign to workers when one constructs a counterfactual that reallocates them across sectors. We address this issue by expressing labor productivity in terms of efficiency units of labor that control for the effect of differences in sectoral human capital on labor productivity. We can then reallocate efficiency units without the need to specify what happens to the average human capital of reallocated workers. Note that expressing labor productivity in terms of efficiency units has the added advantage that the human capital produced in the education sector affects total efficiency units in the economy but not labor productivity in other sectors.

Calculating counterfactual labor productivity growth requires us to specify the value-added shares of the initial period. To smooth out business-cycle fluctuations of the sectoral shares, we take averages over the first five postwar shares:

$$S(P_{n0}Y_{n0}) \equiv \sum_{t=1948}^{1952} \frac{S(P_{nt}Y_{nt})}{5}.$$

We then evaluate the effect of structural change on productivity during the seven decades falling into the time period 1949–2019. The effect of structural change on the *level* of labor productivity at the end of the sample is also informative, as it reflects the accumulated effects on the labor productivity growth rates. To calculate the final productivity level, we normalize the productivity level in 1949 to one and accumulate the corresponding growth rates over the following seventy years.

We implement the previous results on U.S. data during 1949–2019. The raw data for 1947–2014 are from WORLD KLEMS and they contain information about real value added and efficiency units. We extend the data until 2019 using the BEA-BLS Industry Level Production Accounts. WORLD KLEMS has 65 industries while the BEA–BLS data has only 63 industries. To make the industry structure of the two data sets consistent we aggregate the relevant four industries into two industries in WORLD KLEMS. As a result, we have information for 63 industries implying that $N = 63$ in the previous formulas.

Table 1 shows the results for TFP growth. The upper part reports growth rates in percent and the lower part reports levels. The third column report the actual TFPs with structural change. The fourth column reports the counterfactual TFP without structural change. The last column reports the difference between the two.

The main finding of the table is that the overall effect of structural change on aggregate productivity growth has been sizeable. In particular, the last column of the row 1949–2019 shows that without structural change, average annual TFP growth would have been 0.24 percentage points larger than it actually was. After seven decades, this leads to a 0.27 percentage points, or 18%, higher TFP level in 2019. To put these numbers into perspective, note that Jones and Vollrath (2013, page 236) estimate the drag of natural resources on the annual growth rate of postwar U.S. GDP per capita to have been 0.31 percentage points, which is not that much bigger than the drag of structural change on the growth rate of TFP.

An additional finding of the table is that the effect of structural change differs widely across subperiods. The largest drag on TFP growth is 0.48 pp during 1979–1989. Moreover, by and

Table 1: The Effects of Structural Change on TFP in the Postwar U.S.

		With Struct.	Without Change	Without – With
	1949–2019	0.57	0.81	0.24
	1949–1959	1.00	1.08	0.09
	1959–1969	1.10	1.30	0.21
Growth Rates	1969–1979	0.35	0.53	0.17
(in %)	1979–1989	0.28	0.76	0.48
	1989–1999	0.50	0.84	0.34
	1999–2009	0.25	0.51	0.26
	2009–2019	0.48	0.61	0.13
Levels	1949	1.00	1.00	0.00
	2019	1.49	1.76	0.27

large, the drag on TFP growth follows an inverted U-shape. These findings are interesting for two reasons. First, the 1970s and 1980s were the main decades of the productivity growth slowdown that has received so much attention in the literature. We find that structural change was a major contributor to the productivity growth slowdown. While actual TFP growth slowed from 1.10% in the 1960s to 0.35% in the 1970s and 0.28% in the 1980s, counterfactual TFP growth slowed from 1.30% in the 1960 to 0.53% but then sped up again to 0.76%. In other words, without structural change, the productivity growth slowdown would have been considerably milder and shorter. Second, the inverted U-shape implies that the growth drag of structural change has been subsiding in recent decades. This suggests that the productivity drag is not getting progressively worse, implying that Baumol's "apocalyptic" scenario of ultimate near-zero productivity growth has not been born out of the data so far. Our analysis in the model part will reach a similar conclusion for the future.

We now turn to the effect of structural change on labor-productivity growth. Labor productivity growth is of interest because calculating it requires less information than calculating TFP growth, implying that often labor productivity growth is all that is available. Moreover, structural change is driven by changes in aggregate income and in relative sectoral prices, both of which are determined by sectoral capital accumulation in addition to sectoral TFP growth. Since sectoral labor-productivity growth reflects both, it summarizes the driving forces of income and relative prices.

Table 2 shows that actual labor-productivity growth is larger than TFP growth; over the whole period, labor productivity grew by 1.51% whereas TFP grew by only 0.57%. This is expected because labor productivity growth reflects capital accumulation in addition to TFP growth. The table also shows that the effect of structural change is larger on TFP growth than on labor-productivity growth. Moreover, two subperiods are outliers with a positive effect of structural change on productivity growth. Particularly striking are the 1970s during which structural change increased productivity growth by 0.2 percentage points. This unexpected finding is likely to be caused by the large increase in the prices of primary inputs after the oil price shocks in the 1970s, which seems to have reduced most strongly the value added of industries that shrink with structural change. Be that as it may, over the whole period, the growth drag of structural change is 0.16 for labor-productivity growth compared to 0.24 for TFP growth. While this may seem somewhat surprising, closer inspection reveals two reasons for it. First, it turns out that many industries with lower than average labor-productivity growth have higher than average labor-productivity levels, implying that the direct effect of uneven sectoral labor-productivity growth on aggregate productivity growth is dampened by the indirect effect of uneven labor-productivity levels; see the discussion after expression (10). Note that over time the importance of such dampening declines, as sufficiently many years of relatively low sectoral labor-productivity growth erode relatively high initial sectoral labor-productivity levels. Second, it also turns out that many industries with lower than average labor-productivity growth have higher than average capital-to-labor growth. This means that labor-productivity growth exceeds TFP growth for industries with relatively low TFP growth, which reduces the effect of structural change on labor productivity growth further.

Table 2: The Effects of Structural Change on Labor Productivity in the Postwar U.S.

		With Struct.	Without Change	Without – With
Growth Rates (in %)	1949–2019	1.51	1.67	0.16
	1949–1959	2.15	2.07	-0.08
	1959–1969	2.40	2.55	0.15
	1969–1979	1.44	1.24	-0.20
	1979–1989	1.27	1.63	0.36
	1989–1999	1.46	1.94	0.48
	1999–2009	1.39	1.62	0.23
	2009–2019	0.45	0.62	0.16
Levels	1949	1.00	1.00	0.00
	2019	2.88	3.21	0.33

2.3 Discussion

The effects of structural change on the growth of TFP and labor productivity that we find are somewhat lower than those of Nordhaus (2008), whose estimates over the period 1948–2001 came out between 0.27 and 0.89 depending on the method. There are several reason for the difference. First, we consider the longer time period 1949–2019, and the strength of the effect is relative weak during the last couple of decades. Second, we consider real labor productivity per *efficiency unit* whereas he considered real labor productivity per *hour worked*. Our labor productivity measure thus purges the effects of differences in sectoral human-capital per hour worked which his productivity measure contains. Lastly, we used a somewhat different measure for the effect of structural change from him: whereas we compare the counterfactual productivity growth rates under the initial industry composition with the actual observed productivity growth, he compared the counterfactual productivity growth rates under the initial and under the final industry compositions with each other. This tend to increase the size of the effect because the productivity growth rates with the final industry composition tend to be smaller than the productivity growth rates with the actual industry composition.

The fact that structural change importantly reduced productivity growth in the postwar U.S. raises the question why the recent literature on structural change has paid relatively little attention to the phenomenon. The likely reason is the strong focus of the literature on aggregate balanced growth path (BGPs). In many models of structural change, an aggregate BGP exists if GDP growth is measured in terms of a current numeraire. Since by construction productivity growth is constant along an aggregate BGP, it is tempting to conclude that the productivity growth slowdown is not an issue. Duernecker et al. (2021) show that this conclusion is misleading; if one measures GDP growth as it is done in the data, then the productivity growth slowdown resulting from structural change plays an important role even along standard aggregate BGPs.[1] In our quantitative analysis, we therefore make sure to measure GDP and labor productivity in the same way in the model as it is done in the WORLD KLEMS data we use.

2.4 Disaggregating Services

Assessing the future strength of the effect of structural change on productivity growth requires a model of structural change that balances two considerations. On the one hand, a "convincing" model ought to capture the main effects, in particular within the already sizeable services sector. On the other hand, a "realistic" model with dozens of sectors, subsectors, and industries would be impenetrable. As a compromise between the considerations, we propose a three-sector split that first disaggregates the economy into the broad sectors of goods and services and then disaggregate services further into the sub-sectors that have fast and slow productivity growth. We use the standard definition of the goods sector as comprising all industries that produce tangible value added, namely, agriculture, construction, manufacturing, mining, and utilities. The services sector comprises the remaining industries, which produce intangible value added. The services sector with fast (slow) productivity growth contains all services industries that have av-

[1] In independent work, Leon-Ledesma and Moro (2020) also observed that a productivity growth slowdown results from structural change if value added is measured as it is in the data. We will discuss their work in more detail in Section 7 below.

erage productivity growth above (below) the average productivity growth of the service sector over the postwar period.[2] Following Baumol et al. (1985), we call the two services subsectors sectors "progressive" and "stagnant" services, although many stagnant services industries have low but positive productivity growth. Table 3 lists the services industries in declining order of their average productivity growth rates; progressive services industries are above the line and stagnant services industries are below the line.

Several industries have negative productivity growth. It is important to realize that negative productivity growth may naturally occur for at least three reasons. First, we measure labor productivity as value added per efficiency unit, instead of the more familiar measure of value added per worker. It is then possible to have negative labor productivity growth according to our measure and positive productivity growth according to the other measure. For that to happen, sectoral human capital must grow sufficiently strongly, for example because the industry becomes more intensive in high-skilled labor and less intensive in capital. Second, tighter regulation or increased labor hoarding can generate negative productivity growth. For example, it is often argued that it is getting increasingly expensive to construct highways because of increasingly tighter regulation regarding noise generation and pollution externalities. Third, there is widespread mismeasurement of quality improvements in the service industries. To see the effect of mismeasurement of quality, note that nominal value added is observed. Underestimating quality improvements then implies that the price is too high and the real value added is too low. This can lead to negative measured productivity growth.[3] One example with negative productivity growth in our data is "Food services & drinking places". To see how negative productivity growth can happen in this industry as a result of mismeasurement, suppose that over time high-end restaurants increasingly replace low-end restaurants. High-end restaurants are more labor intensive and offer a higher quality product than low-end restaurants. Suppose further that the BEA measures the increase in labor input but does not measure the increase in the quality of the product. In that case, measured labor productivity in the industry may fall although actual labor productivity does not.

We continue by providing two arguments in support of using our three-sector split: that it is reasonably robust over time and that it speaks directly to differences in productivity growth. Starting with the first argument, we split our period into the two subperiods 1947–1983 and 1983–2019 and calculate the average industry productivity growth rates in the first and the second half of the sample. Figure 1 plots the result. The south-west (north-east) quadrant depicts the industries that are stagnant (progressive) in both subperiods. 22 industries stayed in their classification in both subperiods. In contrast, 15 industries changed classification between the two subperiods: 4 industries moved from stagnant in the first subperiod to progressive in the

[2]Note that using the median instead of the average would not affect the classification at all.

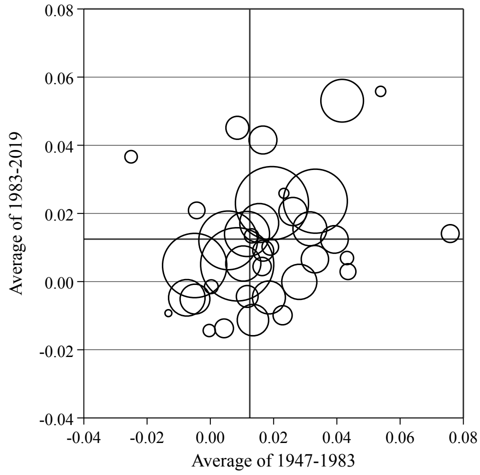
[3]Note that we show below that mismeasured quality improvements do not overturn our main result, but reinforce them.

Table 3: Two–Sector Splits of Services

Services Industries	**Progr. (0) vs. Stagnant (1)**	**Market (0) vs. Non-market (1)**	**Low-skill (0) vs. High-skill (1)**	**Prod. Growth (in %)**
Pipeline transportation	0	0	0	5.48
Broadcasting & telecommunications	0	0	0	4.74
Air transportation	0	0	0	4.50
Publishing industries, except internet (incl. software)	0	0	1	2.91
Wholesale trade	0	0	0	2.84
Securities, commodity contracts, & investments	0	0	1	2.68
Rental & leasing services & lessors of intangible assets	0	0	0	2.58
Waste management & remediation services	0	0	0	2.51
Water transportation	0	0	0	2.46
Administrative & support services	0	0	0	2.35
Rail transportation	0	0	0	2.33
Social assistance	0	1	1	2.32
Retail trade	0	0	0	2.12
Truck transportation	0	0	0	1.98
Insurance carriers & related activities	0	0	1	1.63
Performing arts, spectator sports, museums, & rel. act.	0	0	1	1.46
Management of companies & enterprises	0	0	1	1.40
Warehousing & storage	0	0	0	1.32
Motion picture & sound recording industries	0	0	1	1.29
Miscellaneous professional, scientific, & technical serv.	0	0	1	1.28
Accommodation	1	0	0	1.04
Federal government	1	1	0	0.89
Computer systems design & related services	1	0	1	0.83
Federal Reserve banks, credit intermediation, & rel. act.	1	0	0	0.79
Ambulatory health care services	1	1	1	0.69
Real estate	1	1	0	0.68
Educational services	1	1	1	0.65
Data processing, internet publishing, & other info. serv.	1	0	1	0.58
Legal services	1	0	1	0.37
Hospitals & nursing & residential care facilities	1	1	1	0.11
State & local government	1	1	1	-0.01
Amusements, gambling, & recreation industries	1	0	0	-0.06
Other transportation & support activities	1	0	0	-0.47
Food services & drinking places	1	0	0	-0.50
Other services, except government	1	0	0	-0.61
Transit & ground passenger transportation	1	0	0	-0.74
Funds, trusts, & other financial vehicles	1	0	1	-1.13

Note: Based on average annual labor productivity growth in the postwar U.S.;
37 services industries from WORLD KLEMS.

Figure 1: Industry Productivity Growth in the First and Second Half of the Sample



second subperiod; 11 industries moved from progressive in the first subperiods to stagnant in the second subperiod. The figure suggests that the industries that changed classification did not have high value added shares. That is crucial because the Törnqvist indexes of the productivity growth rates of the progressive and stagnant services subsectors are the value-added-weighted averages of the relevant industry growth rates. The suggestion of the figure is confirmed by numbers: the average value added share in services value added over the entire period of the 4 industries is 5.9% and of the 11 industries is 14.9%. Thus, taken together, the 15 industries that changed classification make up about 21% of total services value added and the 22 industries that did not change classification make up about 79% of total services value added. In other words, our classification is reasonably robust over time.

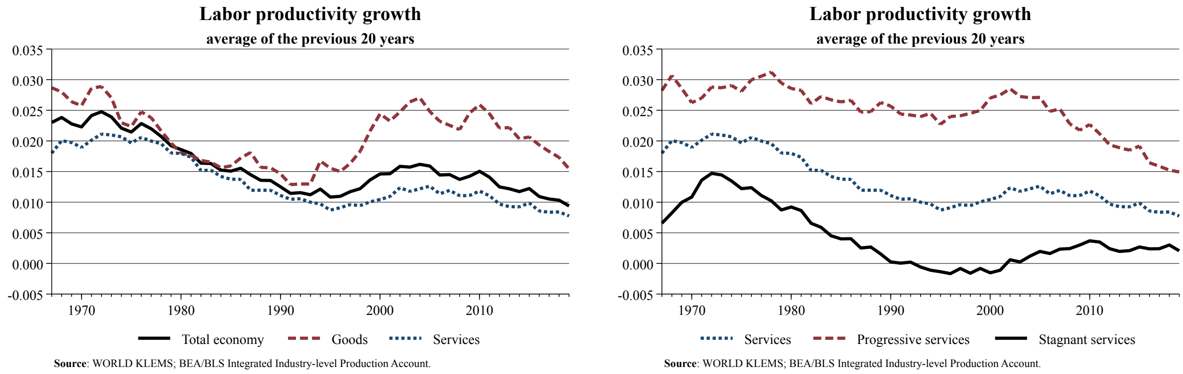
Turning now to the second argument, it turns out that alternative disaggregations of services that exist in the literature are not as informative about productivity growth as our three-sector split. To begin with, the split into traditional versus non-traditional services as suggested by Duarte and Restuccia (2020) is based on final expenditure categories and therefore does not directly speak to value added. Two other popular splits are market versus non–market services, as used by the guidelines of the System of National Accounts, and high-skill- versus low-skill-intensive services, as suggested by Buera et al. (2021).[4] Although they are both based on value added, Table 3 shows that they capture differences in productivity growth only imperfectly. In particular, the six services industries with the fastest productivity growth and with the slowest productivity growth are market services. Moreover, nine high-skilled services have above average productivity growth whereas ten high-skilled services have below average productivity growth.

[4]We use the BEA–BLS Industry Level Production Account, 1987–2019, to construct the last two-sector split. Industries of the high-skill-intensive services sector pay a higher share of labor compensation to skilled workers than the services sector does on average. Skilled workers are those who have at least a college degree.

Looking ahead, our analysis below will establish two further arguments in favor of using our three-sector split: it leads to a demand system with sensible parameter values that are consistent with both micro and macro evidence; it has similar out-of-sample implications for future productivity growth as a more disaggregated sector split.

Figure 2: Postwar U.S. Productivity Growth

Goods versus Services



2.5 Stylized Facts

Figure 2 plots for our disaggregation the average productivity growth over the preceding twenty years in the postwar U.S.[5] Two features stand out. First, there are substantial differences in sectoral productivity growth. Over the whole postwar period 1947–2019, average productivity growth in goods exceeded that of services even though average productivity growth in progressive services exceeded that in goods. To be precise, average annual productivity growth rates were: 1.53% in the aggregate; 2.05% in the goods sector; 1.24% in the services sector; 2.35% in progressive services; 0.33% in stagnant services.[6] Second, there is substantial variation across subperiods. While goods and stagnant services experienced a slowdown in productivity growth in the 1970s, productivity growth of progressive services started to slow down at least ten years later and mostly after 2000. Lastly, right before the Great Recession, average productivity growth in the goods sector over the preceding twenty years had almost recovered, but

[5]Note that since our data start in 1947, the averages over the preceding 20 years start in 1967. Note too that taking averages over the preceding 10, instead of 20, years would not alter the qualitative properties of the figure overall.

[6]Note that the low productivity growth of stagnant services industries may in part reflect unmeasured quality improvements; see for example Byrne et al. (2016). We initially ignore this possibility and take the numbers from WORLD KLEMS at face value. In Subsection 6.2 below, we then show that this way of proceeding yields an upper bound of the future productivity-growth effect of structural change. Since our main conclusion is that the future effect of structural change on productivity growth remains limited, unmeasured quality improvements do not overturn it.

afterwards it started to slow down again.

At a more disaggregate level, there is even more heterogeneity: Table 3 from above shows that the seven top performing services industries all exceeded 2.5% average annual productivity growth whereas the bottom seven services industries all showed negative average annual labor productivity growth. The large heterogeneity among the productivity growth in the services sector confirms the observation of Baumol et al. (1985) that it "contains some of the economies most progressive activities as well as its most stagnant". In comparison, there is considerably less heterogeneity among the productivity growth of the goods sector where only Forestry, Fishing and Related Activities had negative average productivity growth. Since it is safe to assume that neither one of them is going to grow much in size in the future, we choose not to disaggregate the goods sector and instead to focus on what happens within the services sector.

We now turn to documenting the stylized facts of structural change for our disaggregation. Figure 3 plots the sector compositions along with the relative prices and productivities in the postwar U.S. economy. The reference year is 1947 for all graphs (that is, real value added and labor services are expressed in 1947 dollars) and the relative prices in 1947 are normalized to one. The upper panel is about goods versus services and shows the usual patterns: the shares of services in employment and in value added increased; the relative price of services increased while the relative productivity of services decreased. The lower panel is about stagnant and progressive services and shows several new patterns: the share of stagnant services in total services employment increased; the share of stagnant services in total services value added increased until the 1970s and then flattened out; the relative price of stagnant services increased over the whole period, with an acceleration after 1970s; the relative productivity of progressive services increased over the whole period, with an acceleration after 1970s. The new patterns will be crucial when we discipline the parameters of our model below.

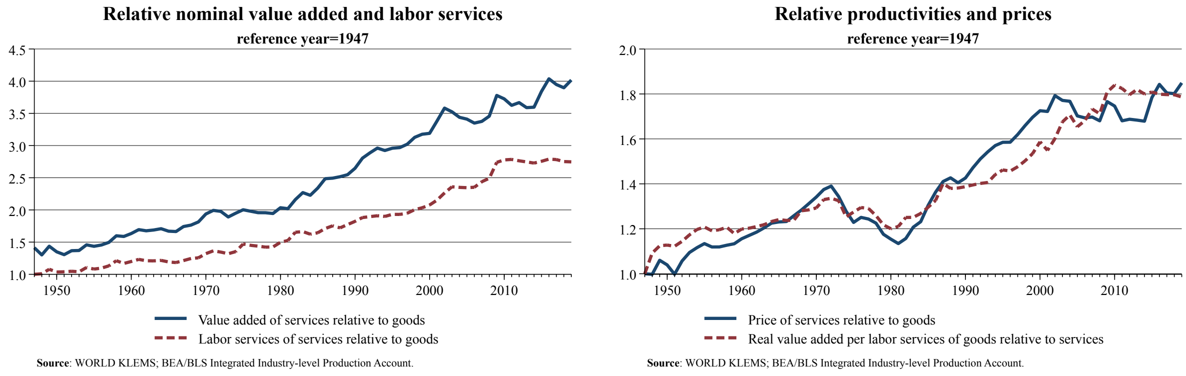
3 Model

We have established that the past growth effect of structural change is quantitatively important. We have also constructed a three-sector split that is suitable for analyzing the future growth effect of structural change and we have established the stylized facts of structural change for that three-sector split. We now turn to constructing a model that will help us analyze the macro implications of structural change among the three sectors.

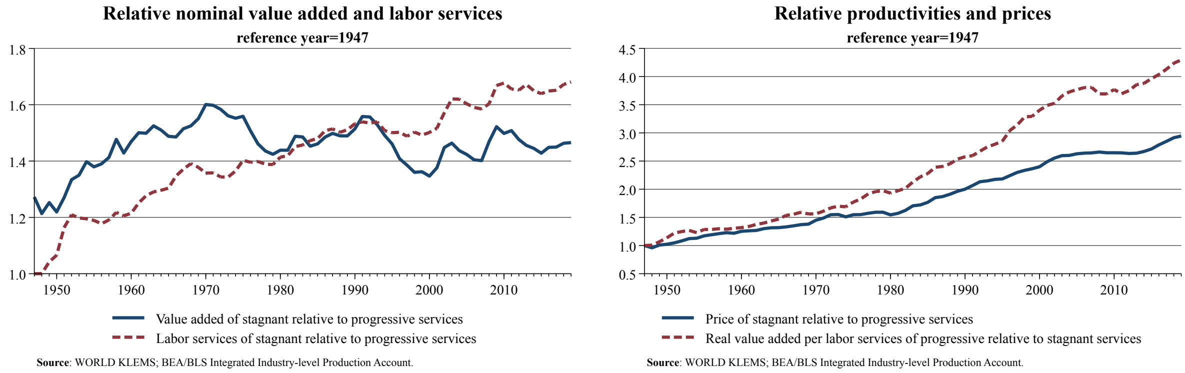
We note that the common term cost disease has the connotation of an inefficient outcome that occurs in a "sick" economy and should be "cured", if possible. A different interpretation is that cost disease is the consequence of structural change that occurs as an efficient outcome in a "healthy" economy. While the distinction is relevant if one is interested in welfare and policy implications, we do not speak to it here but focus on the question of how structural change

Figure 3: Postwar U.S. Structural Transformation

Goods versus Services



Progressive versus Stagnant Services



may affect future productivity growth. While somewhat narrower, our question is interesting in its own right in light of the lively debate about the productivity growth slowdown that we referenced in the introduction.

3.1 Environment

The sectors produce goods, progressive services, and stagnant services and are indexed by g, p, and u. Note that we use the index u for stagnant ("unprogressive") services because s is taken for aggregate services. In each sector, value added is produced with labor services:

$$Y_{it} = A_{it} H_{it}, \quad i = g, p, u, \tag{16}$$

where Y_i, A_i, and H_i denote value added, total factor productivity, and labor services in sector i, respectively. Note that the linear specification (16) implies that sectoral TFP equals labor productivity in our model, $A_{it} = Y_{it}/H_{it}$.

There is a measure one of identical households. Each household is endowed with a finite number of labor services that are inelastically supplied and can be used in all sectors.

We take the value-added perspective and formulate utility over the sectoral value added components of final expenditures, instead of over sectoral final expenditures. This is possible because every final-expenditure bundle may be decomposed into its value-added components via the use of input-output tables; see Herrendorf et al. (2013) for more details. Taking the value-added perspective implies that a maintained assumption for our analysis is that the input-output relationships that link final expenditures to value added are relatively stable over time.

The utility function consists of two nested, non–homothetic CES functions. The utility from the consumption of goods and aggregate services, C_{gt} and C_{st}, is given by:

$$C_t = \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}}. \tag{17a}$$

Aggregate services are given by a non–homothetic CES aggregator of the consumption from the two service sub–sectors, C_{pt} and C_{ut}:

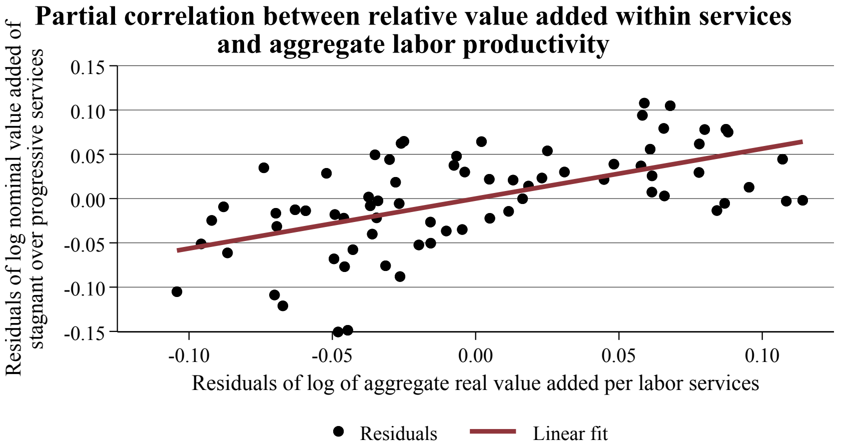
$$C_{st} = \left(\alpha_p^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_p - 1}{\sigma_s}} C_{pt}^{\frac{\sigma_s - 1}{\sigma_s}} + \alpha_u^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_u - 1}{\sigma_s}} C_{ut}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}. \tag{17b}$$

α_i are weights, $\sigma_i \geq 0$ are the elasticities of substitution, and $\varepsilon_i > 0$ capture income effects.

The non–homothetic CES utility functions we are using go back to the work of Hanoch (1975) and Sato (1975) on implicitly additive utility and production functions. They have recently been introduced to the literature on structural change by Comin et al. (2021). For $\varepsilon_i = 1$, the expressions in (17) reduce to the standard CES utility that implies homothetic demand functions for each consumption good. For $\varepsilon_i \neq 1$, the level of utility, C_t, affects the weight attached to the consumption goods. The nested structure of (17a)–(17b) is a novel feature of our work.

The most important feature of equations (17a)–(17b) for our purposes is that the implied income elasticities remain different from one even if consumption grows without bound. Boppart (2014) and Comin et al. (2021) established that the income elasticities of goods and services in rich countries like the U.S. remain different from one. Figure 4 establishes that the income elasticities of our two service subsectors also remain different from one. In particular, after taking out the effects of relative prices changes on the composition of services, the figure plots the ratio of the residual value added in the stagnant services and in the progressive services against the aggregate value added per labor services. Clearly, there is a positive long-run slope, which would be inconsistent with having a utility function like the Stone Geary that converges to a homothetic CES utility function as consumption expenditure grow without limit.

Figure 4: Persistent Income Effects within Services



Source: WORLD KLEMS; BEA/BLS Integrated Industry-level Production Account.
Note: Residuals on the y-axis are from regressing the log difference of nominal value added of stagnant services and progressive services on the corresponding log difference of prices. Residuals on the x-axis are from regressing the log of aggregate real value added per labor services on the same log difference of prices.

We complete the description of the environment with the resource constraints:

$$C_{it} \leq Y_{it}, \quad i = g, p, u, \tag{18a}$$
$$\sum_{i=g,p,u} H_{it} \leq H_t. \tag{18b}$$

3.2 Discussion

Our model abstracts from investment and international trade. One implication is that by construction GDP equals consumption and the features of preferences shape the reallocation among sectors. How suitable a model with this feature is for answering our question depends on whether it can match the sectoral reallocation within GDP. Below, we will confirm the result of previous work that it can deliver this match, at least for the U.S. In particular, the long-run trends in the changes of the sectoral shares within consumption and investment are similar, the trade share is small, and the economy is close to a BGP. As a result, one can find a preference specification which captures structural change in GDP without separately considering investment and international trade.

Liberalizations of international trade lead to productivity growth, because they imply access to the most advanced technologies. The exogenous sectoral labor productivity processes that we feed into our model will reflect the productivity effect of liberalizations of international trade. What our model does not capture is that international trade may lead to differences between the sectoral value added that the domestic economy produces and the sectoral value added that it absorbs. This is not likely to be of first-order importance when the trade share is as small as it is in the U.S.

3.3 Equilibrium Analysis

In the data, the nominal labor productivities per efficiency unit are not equalized across sectors, which leads to contemporaneous effects of structural change on aggregate productivity. To capture them, we introduce a sector-specific wedge τ_{it} that firms pay per unit of wage payments and that is rebated to households through a lump-sum transfer. In particular,

$$T_t = \sum_{i=g,p,u} \tau_{it} W_t H_{it},$$

where W_t denotes the economy-wide wage per unit of labor services. With the wedge, the problem of firm $i = g, p, u$ becomes:

$$\max_{H_{it}} P_{it} A_{it} H_{it} - (1 + \tau_{it}) W_t H_{it}.$$

The first–order conditions imply that

$$\frac{P_{it}}{P_{gt}} = \frac{(1 + \tau_{it}) A_{gt}}{(1 + \tau_{gt}) A_{it}}, \quad i = p, u. \tag{19}$$

Combining this with the specification of the production function in (16), we obtain:

$$\frac{P_{it} C_{it} / H_{it}}{P_{gt} C_{gt} / H_{gt}} = \frac{1 + \tau_{it}}{1 + \tau_{gt}}, \quad i = p, u. \tag{20}$$

As intended, the wedges imply gaps between the nominal sectoral labor productivities; a sector with a relatively large wedge has relatively large labor productivity. Note that, as usual, only relative wedges matter. We will therefore set $\tau_{gt} = 0$ in the quantitative part of our analysis.

To solve the household problem, we split it into two "layers". The outer layer of the problem is about allocating a given C_t between C_{gt} and C_{st}. Solving the outer layer amounts to:

$$\min_{C_{gt}, C_{st}} P_{gt} C_{gt} + P_{st} C_{st} \quad \text{s.t.} \quad \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}} \geq C_t, \quad C_t \text{ given,}$$

Appendix B.2 shows that the first–order conditions imply:

$$\frac{P_{st} C_{st}}{P_{gt} C_{gt}} = \frac{\alpha_s}{\alpha_g} \left(\frac{P_{st}}{P_{gt}} \right)^{1 - \sigma_c} C_t^{\varepsilon_s - \varepsilon_g}, \tag{21a}$$

$$P_t = \left(\alpha_g C_t^{\varepsilon_g - 1} P_{gt}^{1 - \sigma_c} + \alpha_s C_t^{\varepsilon_s - 1} P_{st}^{1 - \sigma_c} \right)^{\frac{1}{1 - \sigma_c}}, \tag{21b}$$

where P_t is the aggregate price index and $P_t C_t \equiv \sum_{i=g,p,u} P_{it} C_{it}$.

The inner layer of the household problem is about allocating a given C_{st} between C_{pt} and C_{ut}. Solving the inner layer amounts to:

$$\min_{C_{pt},C_{ut}} P_{pt}C_{pt} + P_{ut}C_{ut} \quad \text{s.t.} \quad \left(\alpha_p^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_p - 1}{\sigma_s}} C_{pt}^{\frac{\sigma_s - 1}{\sigma_s}} + \alpha_u^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_u - 1}{\sigma_s}} C_{ut}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \geq C_{st}, \quad C_t, C_{st} \text{ given.}$$

Note that in solving the inner problem, C_t is taken as given. Appendix B.1 shows that the first–order conditions imply that

$$\frac{P_{ut}C_{ut}}{P_{pt}C_{pt}} = \frac{\alpha_u}{\alpha_p} \left(\frac{P_{ut}}{P_{pt}} \right)^{1-\sigma_s} C_t^{\varepsilon_u - \varepsilon_p}, \tag{22a}$$

$$P_{st} = \left(\alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s} + \alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}. \tag{22b}$$

where P_{st} is the price index of services.

The solutions to the minimization problem make economic sense only if the consumption index $C_t = C(C_{gt}, C_{pt}, C_{ut})$ that follows by substituting (17b) into (17a) satisfies the basic regularity conditions such as monotonicity and quasi-concavity. To ensure that this is the case, we restrict the parameters as follows:

Assumption 1

- *If* $\sigma_c < 1$*, then* $\sigma_c < \min\{\varepsilon_g, \varepsilon_s\}$ *and* $\varepsilon_s > 1$*. If* $\sigma_c > 1$*,* $\sigma_c > \max\{\varepsilon_g, \varepsilon_s\}$ *and* $\varepsilon_s < 1$*.*
- *If* $\sigma_s < 1$*, then* $\sigma_s < \min\{\varepsilon_p, \varepsilon_u\}$*. If* $\sigma_s > 1$*, then* $\sigma_s > \max\{\varepsilon_p, \varepsilon_u\}$*.*

Proposition 1 *The expenditure function*

$$E_t(C_{gt}, C_{pt}, C_{ut}, C_t) \tag{23}$$

$$\equiv P_t C_t = \left(\alpha_g C_t^{\varepsilon_g - \sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - \sigma_c} \left(\alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s} + \alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} \right)^{\frac{1-\sigma_c}{1-\sigma_s}} \right)^{\frac{1}{1-\sigma_c}}.$$

is continuous, strictly increasing, concave, homogenous of degree one and differentiable in prices if prices are strictly positive. If Assumption 1 holds, then the expenditure function is also strictly increasing in C_t*.*

Proof in Appendix B.2.

The fact that E_t is strictly increasing in C_t implies that there a one-to-one mapping between C_t and E_t. Hence, standard duality theory implies that the regularity conditions of E_t from the previous proposition carry over to C_t:

Corollary 1 *The utility function* C_t *is strictly increasing in* C_{it} *and is quasi–concave.*

3.4 Equilibrium Dynamics

Although it is impossible to solve for the equilibrium dynamics in closed form, we are able to characterize the qualitative behavior of the model. We begin with structural change between goods and services. Since the model is formulated in discrete time, it is convenient to use growth factors, which we denote by "hats". For a generic variable X_t:

$$\widehat{X}_t \equiv \frac{X_{t+1}}{X_t} = 1 + \Delta \log X_t,$$

where $\Delta \log X_t$ is the growth rate defined in Subsection 2.2 above. Dividing (21a) for periods $t+1$ and t by each other, we obtain:

$$\widehat{\left(\frac{P_{st}C_{st}}{P_{gt}C_{gt}}\right)} = \widehat{\left(\frac{P_{st}}{P_{gt}}\right)}^{1-\sigma_c} \widehat{C}_t^{\varepsilon_s-\varepsilon_g}. \tag{24}$$

The first term on the right–hand side is the relative price effect and the second term is the income effect. Note that the latter depends only on the *difference* $\varepsilon_s - \varepsilon_g$, implying that the two ε_i will not be separately identified in our calibration and estimation exercises. Thus, we have to normalize one of them in such a way that we do not violate Assumption 1 given the choice of the other parameters.

We make the standard assumptions that goods and aggregate services are complements, goods are necessities, and services are luxuries:[7]

Assumption 2 $0 < \sigma_c < 1$ *and* $\varepsilon_s - \varepsilon_g > 0$.

Expression (24) shows that our model then generates the observed structural change from goods to services if P_{st}/P_{gt} and C_t both grow. Moreover, if P_{st}/P_{gt} and C_t both keep growing, then it we get the usual result from the structural change literature that the services sector takes over the economy in the limit.[8]

We continue with the structural change between the two services subsectors. Combining equations (20) and (22a), we obtain:

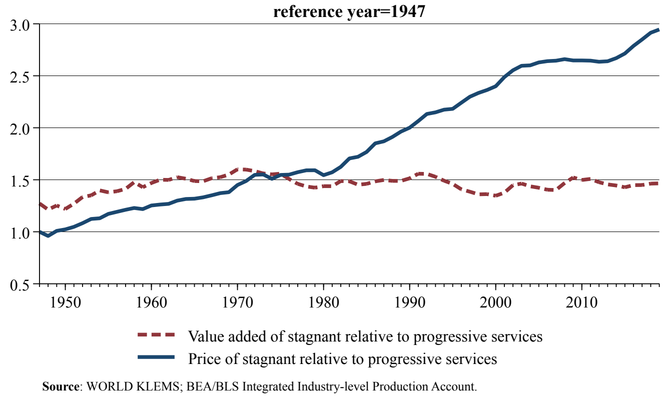
$$\widehat{\left(\frac{P_{ut}C_{ut}}{P_{pt}C_{pt}}\right)} = \widehat{\left(\frac{P_{ut}}{P_{pt}}\right)}^{1-\sigma_s} \widehat{C}_t^{\varepsilon_u-\varepsilon_p}. \tag{25}$$

Figure 5 shows the behavior of the relative expenditure and price of stagnant to progressive services. To generate this behavior, we assume that the two service subsectors are substitutes, progressive services are a necessity, and stagnant services are a luxury:

[7]See for example Echevarria (1997), Ngai and Pissarides (2007), and Herrendorf et al. (2013). Our calibration and estimation exercises below will generate parameter values that are consistent with this assumption.

[8]Our calibration below will generate parameter values that are consistent with P_{st}/P_{gt} and C_t both growing.

Figure 5: Relative Prices and Relative Expenditures in Services



Assumption 3 $1 < \sigma_s$ *and* $\varepsilon_u - \varepsilon_p > 0$.

Since P_{ut}/P_{pt} increases over time, Assumption 3 implies that the relative price effect, which is the first term on the right–hand side of equation (25), increases the expenditure of progressive relative to stagnant services. In contrast, the income effect, which is the second term on the right–hand side, increases the relative expenditure of stagnant services. The net effect is analytically ambiguous.

In the next sections, we will provide evidence from micro estimations and a macro calibration supporting the features in Assumptions 3. Here, we build some initial intuition for why they are required to replicate the patterns of structural change within the service sector. Figure 5 shows that until around 1970 the price of stagnant relative to progressive services increased along with the expenditure ratio of stagnant relative to progressive services. After 1970, the increase in the relative price accelerated while the expenditure ratio flattened out. The model is consistent with the patterns before and after 1970 if stagnant services are luxuries, progressive services are necessities, and the two services are substitutes. The different income elasticities then imply that the increasing C_t increases the ratio of stagnant over progressive services over the whole period. The substitutability implies that the increasing P_{ut}/P_{pt} decreases their ratio over the whole period. To replicate the observed pattern, the first effect must dominate before 1970 and the two effects must offset each other after 1970. That is consistent with the fact that after 1970 the increase in the relative price of stagnant to progressive services accelerates so that the substitution effect becomes stronger.

Alternative parameter constellations are not able to generate the observed patterns. In particular, if the two services are substitutes as above, then making stagnant services necessities and progressive services luxuries would imply that both effects decrease $P_{ut}C_{ut}/\ P_{pt}P_{pt}$, which would be counterfactual because the ratio first increases and then flattens out. That leaves the case in which the two services are complements. To see why it cannot work either, we first assume that stagnant services are luxuries and progressive services are necessities. The increases in C_t and in P_{ut}/P_{pt} would then both increase $P_{ut}C_{ut}/P_{pt}P_{pt}$ over the whole sample. Since the increase in P_{ut}/P_{pt} is stronger after 1970 while the increase in C_t is similar, $P_{ut}C_{ut}/P_{pt}P_{pt}$ would increases more after 1970 than before. That would be counterfactual because the ratio flattens out instead in the data. Next, we assume that stagnant services are necessities and progressive services are luxuries. The increase in P_{ut}/P_{pt} would again increase $P_{ut}C_{ut}/P_{pt}P_{pt}$ whereas the increase in C_t would now decrease $P_{ut}C_{ut}/P_{pt}P_{pt}$ over the whole sample. Since $P_{ut}C_{ut}/P_{pt}P_{pt}$ increased before 1970, the effect of P_{ut}/P_{pt} must have dominated the effect of C_t before 1970. Since the increase in P_{ut}/P_{pt} is stronger after 1970 while the increase in C_t is similar, the effect of P_{ut}/P_{pt} would dominate more after 1970 than before. As a result, $P_{ut}C_{ut}/P_{pt}P_{pt}$ would again increase more after 1970 than before, which would be counterfactual.

In the long run, we can go further and characterize analytically what happens within the services sector. In preparation of our main theoretical result, we introduce the following notation for expenditure shares:

$$\chi_{it} \equiv \frac{P_{it}C_{it}}{P_{gt}C_{gt} + P_{st}C_{st}}, \quad i \in \{g, s\},$$
$$\chi_{jt} \equiv \frac{P_{jt}C_{jt}}{P_{ut}C_{ut} + P_{pt}C_{pt}}, \quad j \in \{p, u\}.$$

To be able to derive a sharp analytical result, we impose that $\chi_{gt} = 0$ and $\chi_{st} = 1$. This is innocuous because $\lim_{t\to\infty} \chi_{gt} = 0$ in our model. To obtain the analytical result, we also impose that the wedges be zero. We will reintroduce them in the quantitative analysis conducted below and show that the analytical result survives. Appendix B.3 proves:

Proposition 2 *Let $\chi_{st} = 1$, $\tau_{ut} = \tau_{pt} = 0$, and $\widehat{A}_{ut} = \widehat{A}_u$ and $\widehat{A}_{pt} = \widehat{A}_p$ be constant. If the parameters satisfy Assumptions 2–3, then for all $\widehat{A}_p > 1$ there is a unique $\widehat{A}_u^* \in \left[1, \widehat{A}_p\right)$ such that:*

- *For all $\widehat{A}_u \in \left(0, \widehat{A}_u^*\right)$, $\lim_{t\to\infty} \chi_{pt} = 1$ and $\lim_{t\to\infty} \widehat{LP}_t = \widehat{A}_p$.*
- *For $\widehat{A}_u = \widehat{A}_u^*$, χ_{ut} and χ_{pt} are constant and $\lim_{t\to\infty} \widehat{LP}_t \in \left[\widehat{A}_u^*, \widehat{A}_p\right]$.*
- *For all $\widehat{A}_u \in \left(\widehat{A}_u^*, \widehat{A}_p\right)$, $\lim_{t\to\infty} \chi_{ut} = 1$ and $\lim_{t\to\infty} \widehat{LP}_t = \widehat{A}_u$.*

The key implication of the proposition is that the productivity growth factor is strictly above one in the limit: $\widehat{LP}_t \geq \widehat{A}_u^* > 1$. The reason is as follows. The progressive services sector takes over the model economy in the limit if the productivity growth of the stagnant services sector is sufficiently *weak*, that is, $\widehat{A}_u < \widehat{A}_u^*$. The stagnant services sector takes over the economy in the limit only if its productivity growth is sufficiently *strong*, that is, $\widehat{A}_u^* < \widehat{A}_u < \widehat{A}_p$. In other words, given Assumptions 2–3, Proposition 2 rules out Baumol's apocalyptic scenario that the stagnant services sector has almost zero productivity growth and takes over the economy.

To build intuition for the result of the proposition, consider what happens for a given $\widehat{A}_p$ when one lowers $\widehat{A}_u$. The income effect in favor of stagnant services becomes weaker because $\widehat{C}_t$ falls; the substitution effect against stagnant services becomes stronger because P_{ut}/P_{pt} rises. Therefore, both effects work in the same direction and against C_{ut} and in favor of C_{pt}. If $\widehat{A}_u$ is low enough, that is, $\widehat{A}_u < \widehat{A}_u^* < \widehat{A}_p$, then the combined effect is strong enough to drive χ_{ut} to zero and have C_{pt} take over the economy.

Importantly, the result of the proposition holds for any value of the productivity growth factor of progressive services larger than one, any positive value of the difference between the income elasticities of stagnant and progressive services, and any value of the elasticity of substitution between stagnant and progressive services larger than one. This is noteworthy because the effects that result from the non-homotheticity of preferences are persistent in our model, implying that the difference between the income elasticities of stagnant and progressive services does not converge to zero in the limit. Nonetheless, the proposition shows that even if the difference in the income elasticities is arbitrarily large, there is always a positive productivity growth rate threshold of stagnant services below which progressive services take over. Note that in standard models with Stone Geary preferences, such a result would be entirely expected because in the limit the difference between the income elasticities would converge to zero, the utility function would become homothetic, and only the substitution effect would operate. In that case, progressive services will trivially take over in the limit if both services are substitutes.

We note that our Proposition 2 differs from Proposition 1 of Comin et al. (2021). In particular, they have one non-homothetic CES aggregator of consumption categories with one elasticity of substitution. They assume that the elasticity of substitution between the consumption categories is smaller than one, that is, the consumption categories are *complements*.[9] In contrast, we have two nested non-homothetic CES aggregators and we allow the elasticities of substitution in the outer and inner nests to differ from each other. We find that the best match to the data is complementarity in the outer nest between goods and services and substitutability in the inner nest between stagnant and progressive services. In other words, while our outer nest is a special case of the specification with three consumption categories of Comin et al. (2021), our inner nest is distinct because the two services are substitutes. The purpose of our Proposition 2 is to characterize the limiting dynamics under substitutability in the inner nest. Our key result is that the substitutability in the inner nest puts a limit on how unproductive the services sector can be that takes over in the limit.

[9] Note that this is revealed in the proof of the proposition although it is not explicitly contained in the statement of the proposition.

4 Micro Evidence

Since the parameter constellation within services is key for the main result of our paper, and since we are not aware of existing micro evidence to support it, we now provide some micro evidence ourselves. We use quarterly data for the period 1999–2015 on household consumption expenditures from the CEX. Every household in the CEX is interviewed for up to four consecutive quarters. We apply standard selection criteria and consider urban households with heads between 25–65 years of age who have participated in all four interview rounds.[10] To account for top coding and outliers we drop households at the bottom and the top 5% of the income and the expenditure distributions. The total number of remaining observations is 87,017.

To be consistent with the formulation of our model, we adopt the value-added representation of expenditures and prices. Hence, we follow Buera et al. (2021) and use the input-output tables to translate observed consumption expenditure into value added. Since, total household expenditures and household's relative prices are likely to be endogenous, we follow Comin et al. (2021) and instrument with the household's income quintile, the household's after-tax income, and a "Hausman"-type relative-price instrument that uses price information from other regions than that of the household. According to Comin et al. (2021), *"These price instruments capture the common trend in U.S. prices while alleviating endogeneity concerns due to regional shocks (and measurement error of expenditure)"*.

4.1 Reduced-Form Estimation

The first natural step is to obtain reduced-form estimates of the slopes of the household-level Engel curves. These slopes are not identical to the ε_i in the model because the preferences we use do not aggregate in general and because consumption expenditure, which we use in the estimation, are not equal to GDP, which we use in the calibration. Nonetheless, the Engel curves are informative about the qualitative features of the underlying income effects. We follow Aguiar and Bils (2015) and estimate the following models:

$$\log(Y_{it}^n) = \alpha_i + \beta_i \log(E_t^n) + \gamma_i Z_t^n + \nu_{it}^n, \tag{26}$$

where n is the household superscript and the subscript $i \in \{g, s, p, u\}$ indicates goods, services, and progressive and stagnant services, respectively. E_t^n is total household income and Z_t^n is a vector of demographic variables including age, number of earners, and household size. The parameters of interest are the β_i, which measure the income elasticities of household expenditures of category i. We consider two different dependent variables: (a) the log of total household expenditure on good i: $\log(Y_{it}^n) = \log(P_{it}^n C_{it}^n)$; (b) the log deviation of total household expen-

[10] Aguiar and Bils (2015) and Comin et al. (2021) proceed in a similar way.

diture on good i from average expenditure on good i across all households in the same time period: $\log(Y_{it}^n) = \log(P_{it}^n C_{it}^n) - \log(\bar{P}_{it}\bar{C}_{it})$, which is the specification used by Aguiar and Bils. We estimate (26) by the Generalized Methods of Moments and by Instrumental Variables. The set of instruments includes the same variables as above.

The estimation results are in Table 4. Across all specifications, we obtain the robust result that $\beta_s > \beta_g$ and that $\beta_u > \beta_p$. In other words, the micro data confirm that services and stagnant services are luxuries and goods and progressive services are necessities. This is exactly what we concluded above from interpreting the macro evidence.

Table 4: Results of Reduced-form Estimation

	(1)		**(2)**	
Panel (a): Dependent variable is $\log(P_{it}^n C_{it}^n)$				
	IV	GMM	IV	GMM
β_g	0.76	0.77	0.77	0.78
	(0.003)	(0.005)	(0.003)	(0.005)
β_s	1.08	1.08	1.08	1.08
	(0.001)	(0.002)	(0.001)	(0.002)
β_p	1.03	1.04	1.07	1.07
	(0.003)	(0.005)	(0.003)	(0.004)
β_u	1.18	1.16	1.12	1.12
	(0.005)	(0.008)	(0.004)	(0.007)
Panel (b): Dependent variable is $\log(P_{it}^n C_{it}^n) - \log(\bar{P}_{it}\bar{C}_{it})$				
β_g	0.74	0.75	0.77	0.78
	(0.003)	(0.005)	(0.003)	(0.005)
β_s	1.02	1.05	1.08	1.08
	(0.002)	(0.004)	(0.001)	(0.002)
β_p	1.01	1.04	1.07	1.06
	(0.003)	(0.005)	(0.003)	(0.004)
β_u	1.06	1.06	1.12	1.12
	(0.005)	(0.007)	(0.004)	(0.006)
Household controls	Y		Y	
Region fixed effects	N		Y	
Year fixed effects	N		Y	
Quarter fixed effects	N		Y	

Note: SE clustered at household level; 87,017 observations.

4.2 Structural Estimation

Next, we use the model to derive structural estimation equations. We choose $\alpha_s = 1 - \alpha_g$, $\alpha_p = 1 - \alpha_u$, and $\varepsilon_g = \varepsilon_p = 1$. Note that the resulting parameter values will satisfy Assumption

1. Taking logs of (21a) and (22a) then gives:

$$\log(P^n_{st}C^n_{st}) = \log\left(\frac{1-\alpha_g}{\alpha_g}\right) + (1-\sigma_c)\log(P^n_{st}) - (1-\sigma_c)\log(P^n_{gt}) + (\varepsilon_s - 1)\log(C^n_t) + \log(P^n_{gt}C^n_{gt}) + \beta_s X^n_t + \delta_{sr} + \delta_{st} + \nu^n_{st}, \tag{27}$$

$$\log(P^n_{ut}C^n_{ut}) = \log\left(\frac{1-\alpha_p}{\alpha_p}\right) + (1-\sigma_s)\log(P^n_{ut}) - (1-\sigma_s)\log(P^n_{pt}) + (\varepsilon_u - 1)\log(C^n_t) + \log(P^n_{ut}C^n_{ut}) + \beta_u X^n_t + \delta_{ur} + \delta_{ut} + \nu^n_{ut}, \tag{28}$$

where n is the household superscript, X^n is the vector of household characteristics, δ_r and δ_t denote region and time fixed effects, and ν^n_i is the error term. We include in the vector X^n variables related to age, household size, and the number of earners. We allow for time fixed effects to absorb aggregate consumption shocks. The underlying assumption is that household heterogeneity in time-invariant demand can be fully explained by X^n and δ_r.

All variables of the right-hand side of (27)–(28) except for P_{st} and C_t are observable. (22b) implies that P_{st} is a function of observables and of C_t. This leaves C_t as the only unobservable variable. It is important to realize that C_t is the consumption index implied by the model and is not in general equal to real consumption expenditures from the data.[11] A natural strategy to deal with this issues is to add the CES aggregator from the model, (17a), as an estimation equation:

$$C_t = \left(\alpha_g^{\frac{1}{\sigma_c}}(P_{gt}C_{gt})^{\frac{\sigma_c-1}{\sigma_c}} P_{gt}^{\frac{1-\sigma_c}{\sigma_c}} + (1-\alpha_g)^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s-1}{\sigma_c}} (P_{st}C_{st})^{\frac{\sigma_c-1}{\sigma_c}} P_{st}^{\frac{1-\sigma_c}{\sigma_c}}\right)^{\frac{\sigma_c}{\sigma_c-1}}. \tag{29}$$

We estimate (22b) together with (27)–(29) together. Importantly, our estimation strategy explicitly treats the consumption index C_t and expenditures E_t as different objects.

Table 10 in Appendix C reports the estimation results. Across all specifications, we obtain the robust result that $\varepsilon_s - \varepsilon_g > 0$, $\varepsilon_u - \varepsilon_p > 0$, and that they are statistically different from zero.[12] Thus, services and stagnant services are again luxuries and goods and progressive services are again necessities. We also obtain the robust result that $\sigma_c < 1 < \sigma_s$ and that they are statistically different from one, that is, goods and services are complements and progressive and stagnant services are substitutes. This is exactly what we concluded above from interpreting the macro evidence. We now show that the same patterns also result from a rigorous macro calibration.

[11] The working-paper version of this paper shows formally that the two indexes are different objects when the utility is non–homothetic, implying that their growth factors cannot directly be compared to each other.

[12] We have also tried other combinations of the fixed effects than those reported in the table but found that the results are broadly unchanged.

5 Quantitative Analysis

5.1 Calibration

We make the following normalizations: $A_{g,1947} = P_{i,1947} = 1$ for $i = g, p, u$ and $\tau_{gt} = 0$ for $t = 1947, ..., 2019$. We choose the other two wedges to match the observed relative nominal labor productivities according to equation (20):

$$\frac{\widetilde{VA_{it}/H_{it}}}{\widetilde{VA_{gt}/H_{gt}}} = 1 + \tau_{it}, \quad i = p, u, \quad t = 1947, ..., 2019, \tag{30}$$

where $VA_{jt} \equiv P_{jt}Y_{jt}$ is nominal value added in sector j and a tilde denotes an observation from the data. The upper left panel of Figure 6 shows the resulting wedges.

The normalizations $A_{g,1947} = P_{g,1947} = 1$ imply that nominal and real labor productivity of the goods sector equal one in 1947:

$$\frac{VA_{g,1947}}{H_{g,1947}} = \frac{Y_{g,1947}}{H_{g,1947}} = 1.$$

We choose $\{A_{gt}\}_{t=1948,...,2019}$ to match the observed growth of the real labor productivity of the goods sector after 1947 according to equation (16):

$$\frac{\widetilde{Y_{gt+1}/H_{gt+1}}}{\widetilde{Y_{gt}/H_{gt}}} = \frac{A_{gt+1}}{A_{gt}}, \quad t = 1947, ..., 2019. \tag{31}$$

We choose the other two sectoral TFPs, $\{A_{it}\}_{t=1947,...,2019}$ for $i = p, u$, to match the observed relative prices. Using equation (19), the wedges, and the normalizations $A_{g,1947} = P_{i,1947} = 1$, this gives:

$$\frac{\widetilde{P}_{it}}{\widetilde{P}_{gt}} = (1 + \tau_{it})\frac{A_{gt}}{A_{it}}, \quad i = p, u, \quad t = 1947, ..., 2019. \tag{32a}$$

The upper right panel of Figure 6 plots the implied sectoral TFPs.

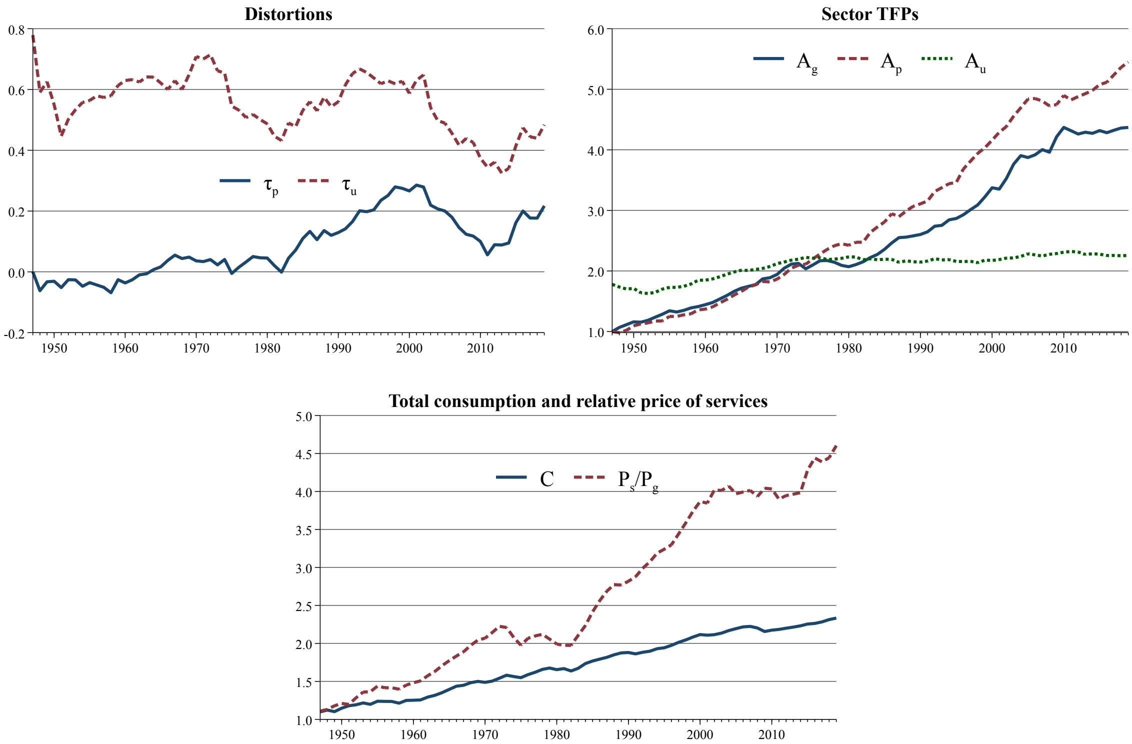
Combining (30) with (32) shows that the previous choices imply we have matched relative real productivities:

$$\frac{\widetilde{Y_{it}/H_{it}}}{\widetilde{Y_{gt}/H_{gt}}} = \frac{\widetilde{VA_{it}/H_{it}}}{\widetilde{VA_{gt}/H_{gt}}} \frac{\widetilde{P}_{gt}}{\widetilde{P}_{it}}, \quad i = p, u, \quad t = 1947, ..., 2019. \tag{33}$$

Given the normalizations, (31) implies we have matched real labor productivity in the goods sector in all years. Therefore, we have matched real labor productivity in all sectors and years.

We are left with ten parameters to calibrate: the four relative weights $\{\alpha_g, \alpha_s, \alpha_p, \alpha_u\}$, the

Figure 6: Implications of the Calibration



two elasticities $\{\sigma_s, \sigma_c\}$, and the four parameters governing the income effects $\{\varepsilon_g, \varepsilon_s, \varepsilon_p, \varepsilon_u\}$. As in the structural estimation, we normalize ε_g and ε_p, making sure that the choices satisfy Assumption 1. We also impose that $\alpha_s = 1 - \alpha_g$ and $\alpha_u = 1 - \alpha_p$.

This leaves six parameters, $\{\alpha_g, \alpha_p, \sigma_s, \sigma_c, \varepsilon_s, \varepsilon_u\}$ to calibrate. We calibrate them by jointly targeting the two nominal-value-added ratios given by (21a) and (22a) in all years:

$$\frac{VA_{st}}{VA_{gt}} = \frac{\alpha_s}{\alpha_g}\left(\frac{(1+\tau_{st})A_{gt}}{A_{it}}\right)^{1-\sigma_c} C_t^{\varepsilon_s-\varepsilon_g}, \tag{34}$$

$$\frac{VA_{ut}}{VA_{pt}} = \frac{\alpha_u}{\alpha_p}\left(\frac{(1+\tau_{ut})A_{pt}}{(1+\tau_{pt})A_{ut}}\right)^{1-\sigma_s} C_t^{\varepsilon_u-\varepsilon_p}, \qquad t = 1947, ..., 2019. \tag{35}$$

To be precise, we choose the six parameters to minimize the squared deviations of VA_{it}/VA_{jt} from $\widetilde{VA_{it}/VA_{jt}}$. When solving for the right-hand-side ratios, we take into account that the consumption index is given by (17a) and we impose that sectoral labor services satisfy the feasibility constraint (18b):

$$\sum_{i\in\{g,p,u\}} H_{it} = H_t = \sum_{i\in\{g,p,u\}} \widetilde{H}_{it}, \tag{36}$$

where the right-hand side is given by the data. The upper panels of Figure 7 show that we match well the trends of relative-nominal-sectoral value added. We also match well the non-targeted employment shares.

To compare aggregate productivity implied by the model with the data, we need a measure of GDP (aggregate value added) per labor service in the model. We emphasize that the utility index in the model differs from the Törnqvist index of GDP in WORLD KLEMS. Consistency requires that we apply the same measure of GDP in the model and in the data. We therefore use the utility index to solve the model, but the Törnqvist index to calculate the model-implied GDP that we compare with the GDP in the WORLD KLEMS data.[13]

Real and nominal GDP in the reference period 1947 follow from the normalizations $A_{g,1947} = P_{g,1947} = 1$ and the first-order conditions (19):

$$Y_{1947} = VA_{1947} = H_{1947} + \tau_{p,1947} H_{p,1947} + \tau_{u,1947} H_{u,1947}.$$

Real GDP in the other years follows from the accumulated, annual growth rates:

$$Y_T = Y_{1947} \frac{Y_{1948}}{Y_{1947}} \dots \frac{Y_T}{Y_{T-1}} = \exp\left(\log Y_{1947} + \sum_{t=1947}^{T-1} \Delta \log Y_t \right), \quad T = 1948, ..., 2019, \tag{37}$$

where the growth rates $\Delta \log Y_t$ are given by:

$$\Delta \log Y_t = \sum_{i=g,p,u} \frac{1}{2} \left(\frac{1}{\sum_{j=g,p,u} \frac{VA_{jt}}{VA_{it}}} + \frac{1}{\sum_{j=g,p,u} \frac{VA_{j,t+1}}{VA_{i,t+1}}} \right) \Big(\log Y_{i,t+1} - \log Y_{it} \Big). \tag{38}$$

Aggregate productivity follows by dividing the GDP measure (37) by total labor services from the data, $LP_t \equiv Y_t / \widetilde{H}_t$. We divide by $\widetilde{H}_t$ instead of by H_t because the quality–adjusted sectoral labor inputs from WORLD KLEMS are non-additive indexes:

$$H_t = \sum_{i \in \{g,p,u\}} \widetilde{H}_{it} \neq \widetilde{H}_t.$$

Since the difference between $\sum \widetilde{H}_{it}$ and $\widetilde{H}_t$ is quantitatively non negligible, we use $\sum \widetilde{H}_{it}$ when we solve the model but $\widetilde{H}_t$ when we compute the measure of model productivity that we compare with the data. Note that we must take $\widetilde{H}_t$ from the data because it does not have a counterpart that we could generate within the model.

The upper panel of Figure 7 shows that the calibrated series for (aggregate) productivity from the model and the data lie right on top of each other, implying that the model passes the "smell test" for being suitable for our purposes. The reason for why the model does well is that it matches closely both components of the GDP measure (37): by construction, it matches

[13]In Duernecker et al. (2021), we demonstrate that doing the same things in the model and the data is essential for capturing the productivity growth slowdown in a standard two-sector growth model.

perfectly the real sectoral productivity growth; it also matches closely the relative-nominal-sectoral value added and thus the Thörnqvist shares. And, of course, it matches the observed aggregate labor services by construction.

Table 5: Calibrated Preference Parameters

α_g	α_p	σ_c	σ_s	$\varepsilon_s - \varepsilon_g$	$\varepsilon_u - \varepsilon_p$
0.477	0.417	0.269	1.029	0.193	0.101

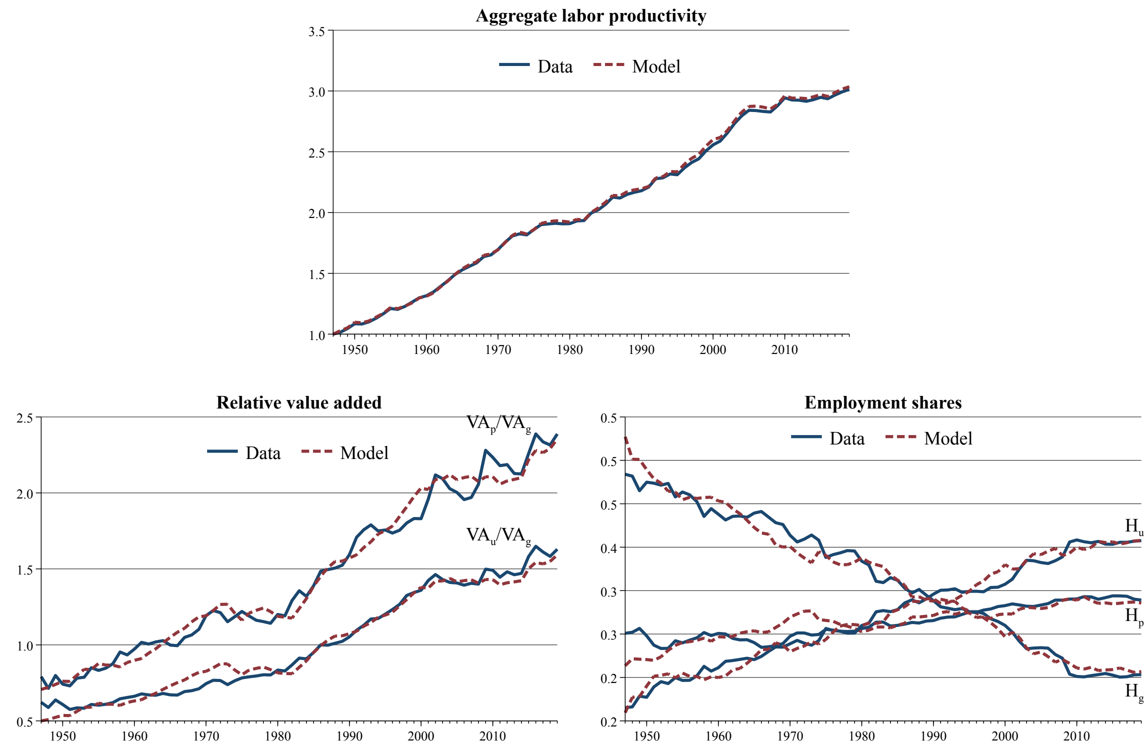
The calibrated parameters are in Table 5. We find that goods and services are complements ($\sigma_c < 1$); goods are necessities and services are luxuries ($\varepsilon_s - \varepsilon_g > 0$); services with high and low productivity growth are substitutes ($\sigma_s > 1$); progressive services are necessities and stagnant services are luxuries ($\varepsilon_u - \varepsilon_p > 0$). Three remarks about the calibration results are in order. First, for appropriate normalizations of ε_g and ε_p, Assumptions 1–2 are satisfied. For example, $\varepsilon_g = \varepsilon_p = 0.91$ works. Second, it is noteworthy that the parameters of the macro calibration have the same qualitative features as the structural micro estimates; see Tables 5 and Column (2) of Table 10 in Appendix C. As a note of caution, we should add that there is no sense in which the macro and micro values should exactly equal to each other, because the non-linear CES utility function we are using does not in general aggregate across different households. Third, the lower panel of Figure 6 shows that the calibrated parameters imply model sequences $\{C_t, P_{st}\}_{t=1947,...,2019}$ that have upward trends. This justifies our assumption in the theory part that both C_t and P_{st}/P_{gt} are increasing.

Note that our calibration implies that $\sigma_s - 1$ and $\varepsilon_u - \varepsilon_p$ are close to zero. If they were zero, then the inner nest would be a Cobb-Douglas function. We have therefore also calibrated the model while restricting the inner nest to a Cobb-Douglas function. It turns out that we can match the macro targets almost as well as with a non-homothetic CES as the inner nest. We nonetheless prefer to use the calibrated non-homothetic CES because the micro estimates of the previous section strongly suggest that $\sigma_s - 1$ and $\varepsilon_u - \varepsilon_p$ are statistically significantly different from zero.

5.2 Simulating the Model Forward

We now simulate our model forward to assess how severe cost disease may be in the future. To have roughly similar horizons for the past and the future, we compare the productivity growth slowdown during the seven past decades in 1947–2019, which we analyzed in Subsection 2.2, with the productivity growth slowdown during the seven future decades in 2019–2089, which our model implies. We assume that during 2019–2089, the variables $\{A_{gt}, A_{pt}, A_{ut}, \sum_{i\in\{g,p,u\}} \widetilde{H}_{it}, \widetilde{H}_t\}$ grow at the same constant, average rates as they did "in the past"

Figure 7: Value Added, Employment, and Aggregate Productivity – Model versus Data



and the wedges $\{\tau_{pt}, \tau_{ut}\}$ equal the average of their "past values". We consider four possibilities for what the past means: 1979–2019; 1989–2019; 1999–2019; 2007–2019. To speak to whether Baumol's "apocalyptic" scenario might happen, we add a counterfactual that imposes $\Delta \log A_u = 0$ while taking the past growth from 1999–2019 for all other variables as before. Since past wedges fluctuate, but do not show a clear trend, we will conduct robustness analysis in Subsection 6.1 below and establish that our results are not sensitive to the specification of the process of future wedges.

Table 6 shows the future growth rates of aggregate labor productivity implied by our forward simulation.[14] The first important observation is that average productivity growth during 2019–2089 is predicted to stay far away from zero. Moreover, average productivity growth during 2069–2089 is not far away from that during 2019–2089, suggesting that the model generates only a limited future productivity-growth slowdown. A different way of seeing that is that, during 1949–2019, the average annual productivity growth slowdown was 0.16 percentage points; see the last column of Table 2 above. The forward simulation implies that between 2019–2039 and 2069–2089, the average annual productivity growth slowdown is at most 0.1 percentage points. These findings imply that the productivity growth slowdown in the near future is smaller than during the postwar period.

[14] Background information on the inputs into obtaining the table is in Table 11 in Appendix D.

Table 6: Past and Future Productivity Growth

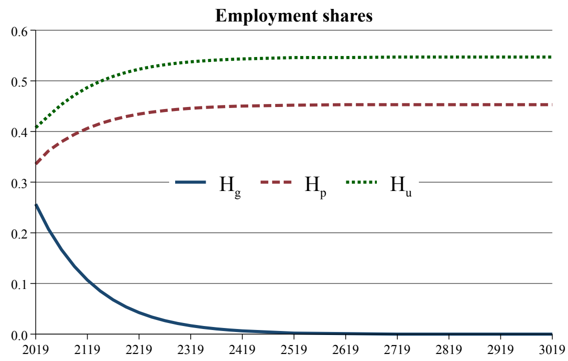
Periods	Postwar Productivity Growth				
	Data		Model		
1947–2019	1.53		1.54		
1979–2019	1.14		1.13		
1989–2019	1.01		1.09		
1999–2019	0.92		0.88		
2007–2019	0.52		0.47		
	Forward Simulation with Exogenous Variables Averaged over				
	1979–2019	1989–2019	1999–2019	2007–2019	1999–2019 $\Delta \log A_{ut} = 0$
2019–2089	0.94	0.95	0.78	0.46	0.69
2019–2039	0.99	0.99	0.81	0.47	0.73
2069–2089	0.89	0.91	0.74	0.45	0.65
2019–2029	0.99	0.99	0.82	0.47	0.74
2029–2039	0.99	0.99	0.81	0.46	0.72
2039–2049	0.97	0.97	0.79	0.46	0.71
2049–2059	0.94	0.95	0.78	0.46	0.69
2059–2069	0.92	0.93	0.76	0.45	0.67
2069–2079	0.90	0.93	0.75	0.45	0.66
2079–2089	0.89	0.90	0.74	0.45	0.65

Note: Productivity is real value added per labor services; all numbers are annual growth rates in %.

Productivity growth slowed down considerably during the Great Recession and has not fully recovered. A more pessimistic scenario for the future would be to use the average growth of the exogenous variables over the period 2007–2019. The results are in column 5. While future productivity growth is lower than before, that is expected because productivity growth during 2007–2019 was lower too. Importantly, the slowdown between the data during 2019–2039 and the model during 2069–2089 again remains modest: average productivity growth falls by just 0.02 percentage points from 0.47 to 0.45.

The future productivity growth slowdown remains limited even in the extreme case in which the stagnant services have zero productivity growth while all other exogenous variables growth like their averages over 1999–2019. Column 6 of Table 6 reports for this case that between 2019–2039 and 2069–2089 productivity growth slows down by only 0.2 percentage points. This is far smaller than the productivity growth slowdowns of 0.16 percentage points that resulted from structural change during the 1949–2019; see the last column of Table 2 above.

Figure 8: Future Sectoral Composition



5.3 Intuition

There are two reasons why our forward simulations imply that the future effects of structural change on aggregate productivity growth are smaller than the past effects. First, as we saw above, goods have higher productivity growth than services and structural change reallocates economic activity from goods to services. Over time, the importance of the implied productivity-growth slowdown declines. In particular, while between 1947 and 2019 the value-added share of the goods sector decreased by 25 percentage points from 45% to 20%, the simulation implies that until 2089 it will decrease further only by an additional 9 percentage points to 11%. Second, for the calibrated parameter values there is little reallocation within the services sector. In fact, our analysis implies that the value-added share of stagnant services in total services is not much different in 1947 and 2089 (59% versus 60%).

Our analysis suggests that for the near future there is no sign of Baumol's "apocalyptic" scenario that the stagnant services taking over the economy and driving productivity growth anywhere towards zero. To understand in more detail what happens, Figure 8 plots the employment shares when we simulate the model forward into the far future. While the employment share of the goods sector declines to zero in the very long run, the employment shares of both services sectors steadily increase without much change in the composition of services. Even after several hundred years, the employment shares of both services sectors remain interior. It is interesting to put the last statement into the context of Proposition 2, which established that for a given $\widehat{A}_p \in (1, \infty)$ there is a unique threshold value of productivity growth for stagnant services, $\widehat{A}_u^* \in (1, \widehat{A}_p)$, at which the composition of the services sector does not change in the limit. A natural interpretation of our quantitative results is that our calibration must generate parameter values very close to the threshold value from Proposition 2.[15] The numerical analysis brings out the additional key feature that the convergence to a corner with only stagnant services is too slow to be relevant in the foreseeable future.

[15]We cannot formally establish this because the Proposition is formulated for zero wedges whereas the calibrated model includes non-zero wedges.

6 Robustness Analysis

This section establishes that the previous findings are robust. The particular robustness exercises we conduct are: we change the future evolution of wedges; we take into account the possibility of underestimated quality improvements in services; we disaggregate further than into three sectors.

6.1 Different Wedges

We first establish that our predictions are robust to different specifications of future wedges. As the upper left panel of Figure 6 showed, the calibrated series of the wedges fluctuate without showing a clear trend. Above, we therefore assumed that the future values of the wedges equal the average values over past periods. To establish robustness, Table 7 explores all combinations of the minimum and maximum values of τ_p and τ_u over the past period 1999–2019. It is remarkable that the implied productivity growth rates hardly change. We conclude from these results that the values of wedges are not of first-order importance for our implied future productivity growth rates.

Table 7: Wedges and Future Productivity Growth

τ_p	τ_u	Productivity Growth 2019–2089	2019–2039	2069–2089
1999–2019		0.78	0.81	0.74
max	max	0.78	0.81	0.75
max	min	0.77	0.81	0.73
min	max	0.77	0.82	0.74
min	min	0.77	0.81	0.74

Note: productivity is real value added per labor services;
all numbers are average annual growth rates in %; max, min taken over 1999–2019;
line "1999–2019" corresponds to the baseline case in column "1999–2019" of Table 6.

6.2 Mismeasured Quality

We have already touched on the possibility that the lower productivity growth of some service industries may in part come from the fact that quality improvements in services are hard to measure. So far, we have taken the numbers from the data at face value and have ignored that possibility. In the current subsection, we look more seriously at the implications of underestimated quality improvements and substantiate the claim made in Subsection 2.5 above that our forward simulations provide an upper bound for how much structural change reduces productivity growth.

To entertain different degrees to which quality improvements in stagnant services are underestimated, and the related price increases of stagnant services are overestimated, we consider the following counterfactual price increases:

$$\Delta \log \overline{P}_u = \omega \Delta \log \widetilde{P}_u + (1-\omega) \Delta \log \widetilde{P}_p, \tag{39}$$

where $\omega \in [0, 1]$. If $\omega = 1$, then $\Delta \log \overline{P}_u = \Delta \log \widetilde{P}_u$ and quality improvements are properly estimated. If $\omega = 0$, then $\Delta \log \overline{P}_u = \Delta \log \widetilde{P}_p$ and quality improvements are underestimated so severely that the actual price increases in both services subsectors are the same and their relative price does not change at all. We vary ω between these extremes, recalibrate our model after replacing $\widetilde{P}_u$ in the data by the counterfactual $\overline{P}_u$, take the period 1999–2019 as the past from which we obtain the estimates of future exogenous processes, and redo the forward simulation.

Table 8 reports the results. Recall that $\omega = 1$ is the previous benchmark case and a lower value of ω corresponds to a more severe underestimation of quality of the value added produced in the service sector with low productivity growth. As ω decreases, the future productivity-growth slowdown becomes smaller and smaller. Therefore, our forward simulations do provide an upper bound of the actual effect of structural change on productivity growth when quality improvements are mismeasured.

Table 8: Quality Mismeasurement and Productivity Growth

ω	Productivity Growth		
	2019–2089	2019–2039	2069–2089
1.00	0.78	0.81	0.74
0.75	0.92	0.95	0.89
0.50	1.05	1.08	1.03
0.25	1.19	1.21	1.18

Note: productivity is real value added per labor services; all numbers are annual growth rates in %; ω is defined in (39); line "1.00" corresponds to the baseline case in column "1999–2019" of Table 6.

6.3 Finer Disaggregations

Our nested utility specification remains tractable and allows us to derive analytical results and build intuition for the main forces behind the productivity effect of structural change. However, its simplicity raises two questions: How restrictive is it that we first combine the services subsectors and then combine the resulting aggregate services with goods? How restrictive is it that we consider the three categories goods, progressive services, and stagnant services? In this subsection, we relax the nesting structure and increase the number of services sectors to establish that the predictions of our nested utility specification are fairly robust.

Hanoch (1975) and Sato (1975) formulated more general utility functions that allow for many sectors $i = 1, ..., I$, each of which with its own income effect and elasticity parameter. In our context, the relevant class of utility functions satisfies the following equation:[16]

$$1 = \sum_{i=1}^{I} \alpha_i^{\frac{1}{\sigma_i}} \left(\frac{C_{it}}{C_t^{\phi_i}} \right)^{1-\frac{1}{\sigma_i}}, \tag{40}$$

where C_t is a utility index that depends on the consumed quantities $C_{1t}, ..., C_{It}$; α_i are weights; ϕ_i govern the income effects; σ_i and σ_j govern the Allen-Uzawa elasticities of substitution σ_{ij} between i and j (see Hanoch for the explicit elasticity formula). To apply standard consumer theory, the utility function $C_t = U(C_{1t}, ..., C_{It})$ that is implicitly defined by (40) must be globally monotone and quasi–concave. Hanoch (1971) proved that this is the case if $\alpha_i, \phi_i, \sigma_i > 0$ for all $i = 1, \dots, I$; either $\sigma_i > 1 \ \forall i$ or $\sigma_i \in [0, 1] \ \forall i$.

To see that Hanoch's class encompasses our utility specifications, set $\phi_i = (\sigma_i - \varepsilon_i)/(\sigma_i - 1)$ in (40):

$$1 = \sum_{i=1}^{I} \alpha_i^{\frac{1}{\sigma_i}} \left(\frac{C_{it}^{\sigma_i - 1}}{C_t^{\sigma_i - \varepsilon_i}} \right)^{\frac{1}{\sigma_i}} \tag{41}$$

The specification of our outer layer, (17a), results if, in addition, we set $\sigma_i = \sigma_c$ and $I = 2$ and rearrange:

$$C_t = \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}}.$$

To obtain the specification of our inner layer, (17b), we set $I = 2$, $\sigma_s = \sigma_i$, and $\varepsilon_i = 1$. Rearranging (41) then gives the CES special case. Our precise specification results if one modifies the weights to $\alpha_p C_t^{\varepsilon_p - 1}$ and $\alpha_u C_t^{\varepsilon_u - 1}$:

$$C_{st} = \left(\left(\alpha_p C_t^{\varepsilon_p - 1} \right)^{\frac{1}{\sigma_s}} C_{pt}^{\frac{\sigma_s - 1}{\sigma_s}} + \left(\alpha_u C_t^{\varepsilon_u - 1} \right)^{\frac{1}{\sigma_s}} C_{ut}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}},$$

which is a version of (17b).

We can use (41) to generalize our analysis to category-specific elasticity parameters, $\sigma_i \neq \sigma_j$, and more than two services sectors, $I - 1$. To do this, we consider the goods sector along with an off-the-shelf split into the services sector classification used in the Economic Transformation Database from the GGDC:

1. Trade Services (Wholesale and Retail Trade; Accommodation and Food Service Activi-

[16]Except for the notation, this is equation (2.16) of Hanoch (1975, page 403). He calls the implied utility functions the constant-ratios-of-elasticities utility class, which is a subclass of his implicitly-additive utility functions.

ties);

2. Transport Services (Transportation and Warehousing);
3. Business Services (Information and Communication; Professional, Scientific and Technical Activities; Administrative and Support Service Activities);
4. Financial Services and Real Estate (Financial and Insurance Activities; Real Estate Activities);[17]
5. Government Services (Education; Human Health and Social Work Activities; General Government);
6. Other Services.

The rest of the analysis is exactly the same as before, so we do not repeat here. Appendix B.4 contains the solution to the household problem for the resulting split with goods and six services sectors. The calibrated parameter values satisfy Hanoch's assumptions because $\alpha_i, \phi_i > 0$ and $\sigma_i \in [0, 1]$ $\forall i = 1, \ldots, I$. Hence, the implied utility function is monotonically increasing in all arguments and quasi concave and the household problem is well defined.

Table 9 reports the simulation results and shows that by and large the previous results are robust. For comparison, the row that starts with "g, p, u" repeats the baseline case in column "1999–2019" of Table 6. The next row reports the new results if we disaggregate into six services sectors with their own elasticities. We can see that disaggregating reduces the final growth rate during 2069–2089 by just 0.01 percentage point (from 0.74 to 0.73) and increases the average growth rate during 2019–2089 by just 0.02 percentage points (from 0.78 to 0.80). The reason why the average growth rate increases compared to the benchmark case is that, for some reason, productivity growth is initially higher with six services sectors (0.87 versus 0.81). In other words, productivity growth does not fall as fast from its value during 1999–2019 as in the benchmark case, but it ends up at almost the same value in the far future. Moreover, it is still true that the 0.14 percentage point productivity growth slowdown from 0.87 to 0.73 is smaller than the 0.16 productivity growth slowdown that we found for 1949–2019 in Table 2 above. Lastly, the result that future productivity growth is well above zero remains clearly valid for goods and six disaggregated services sectors.

[17] This category is often referred to as FIRE. The GGDC splits it into Financial Services and Real Estate because the latter involves imputations that are not always reliable in less developed countries than the U.S. We keep it as one category to be consistent with the usual practice for the U.S. to view FIRE as one sector at this level of aggregation.

Table 9: Finer Dis-aggregation of the Services Sector and Productivity Growth

Sector Split	Substitution Elasticities	Productivity Growth 2019–2089	2019–2039	2069–2089
g, p, u	$\sigma_c \neq \sigma_s$	0.78	0.81	0.74
$g, i = 1, ..., 6$	$\sigma_g \neq \sigma_1 \neq ... \neq \sigma_6$	0.80	0.87	0.73

Note: Productivity is real value added per labor services; all numbers are annual averages in %; values of exogenous variables are averages over 1999–2019; first line corresponds to the baseline case in column "1999–2019" of Table 6.

7 Related Literature

Several papers from the recent literature on structural change are directly related to cost disease. The 2004-CEPR-working-paper version of Ngai and Pissarides (2007) mentioned that cost disease can lead to a GDP growth slowdown when GDP growth is calculated with constant relative prices. However, they did not pursue the growth slowdown further but framed their entire analysis in terms of a balanced growth path and constant GDP growth measured in a current numeraire. Moro (2015) provided an interesting model in which cost disease reduces GDP measured with the Fisher index. His analysis differs from our analysis because he focused on the role of differences in the sectoral intermediate-input shares in a cross section of middle- and high-income countries. In independent work, Leon-Ledesma and Moro (2020) asked to what extent structural change may lead to violations of the Kaldor growth facts. In their simulation results, based on the model of Boppart (2014), structural change leads to a growth slowdown of GDP measured with the Fisher index. Although there are obvious similarities with what we do, the following novel features set our work apart: we provide micro and macro evidence on structural change within services; we characterize analytically the limit behavior of a new model with structural change within services; we use our model to predict the future productivity-growth effect of structural change in the U.S.

Our work is also related to a literature on cross–country gaps in sectoral TFP or labor productivity; see for example Duarte and Restuccia (2010), Herrendorf and Valentinyi (2012), and Duarte and Restuccia (2020). The paper of Duarte and Restuccia (2020) is the most closely related one to our paper. The main difference between the papers is that Duarte and Restuccia (2020) study expenditure data from the International Comparisons Program of the Penn World Table for a cross section of countries in 2005 whereas we study value-added data for the time series of the postwar U.S. The main similarity between the papers is that both disaggregate the services sector into two subsectors and find that as GDP per capita changes – either in the cross section or the time series – one services subsector shows stronger productivity growth than the other and there is substitutability between the two services subsectors. In addition, we find that the services subsector with weaker productivity growth is a luxury whereas Duarte and Restuccia (2010) assume homotheticity.

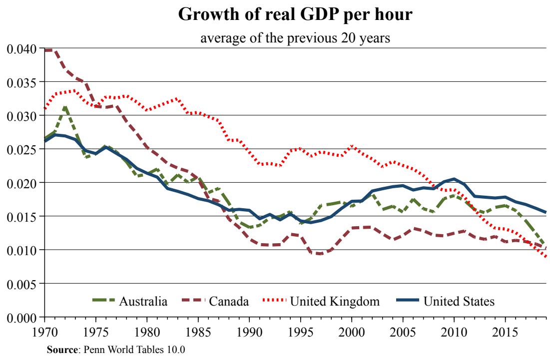
It is challenging to further compare our results with those of Duarte and Restuccia (2020) because our production-side data contain different information from their expenditure-side data. A first implication is that the sector classifications are not directly comparable between the two papers. In particular, our production-side data report real value added and efficiency units of labor by *industry* and sectors are defined as collections of similar *industries*. In contrast, their expenditure-side data report relative prices and final expenditure for different *expenditure categories* and sectors are defined as collections of similar *expenditure categories*. As a result, sectoral output in their paper is the gross output of similar expenditure categories which tends to reflect value added from more than one of our sectors. A second implication of using different datasets is that sectoral productivity is calculated differently because the expenditure-side data miss information on sectoral inputs. Duarte and Restuccia (2020) impute the missing inputs by adding input-output linkages to the method developed by Herrendorf and Valentinyi (2012). In contrast, our production side data has all the information required to calculate sectoral productivity without imputation and the utility function captures the compositional effects of input-output linkages.

In the current paper, we have abstracted from physical capital accumulation, which implies that the services sector takes over our economy in the limit and aggregate productivity growth falls to the services sector's productivity growth. In contrast, in many models of structural change with capital accumulation the services sector does not take over the economy in the limit because all investment is produced in manufacturing. Since investment does not disappear along a balanced growth path, the manufacturing sector remains at least as large as the investment sector. As long as productivity growth is larger in manufacturing than in services, aggregate productivity growth remains larger than productivity growth in the services sector. It is important to realize that this conclusion changes dramatically as soon as one takes into account that structural change also takes place within the investment sector. Acemoglu and Guerrieri (2008) and Herrendorf et al. (2021) show that the services sector then takes over the economy in the limit exactly as it does in the current model.

8 Conclusion

We have built and calibrated a multi-sector model and used it to analyze the effect of structural change on aggregate productivity growth in the postwar U.S. The key novel feature of our model is that we have disaggregated the services sector into progressive and stagnant services. We have documented micro and macro evidence that stagnant services are luxuries, progressive services are necessities, and stagnant and progressive services are substitutes. We have shown

Figure 9: The Productivity Growth Slowdown Outside the U.S.



that, as a result, stagnant services do not take over the economy in the limit if their productivity growth falls below a positive threshold level. We have found that in our model the future effect of structural change on productivity growth will be smaller than the past one.

As a natural first step, we have taken the sectoral growth rates as given and we have explored which consequences the implied changes in the sectoral composition have for future productivity growth. An interesting question for future work is why different sectors show different productivity growth. Young (2014) suggested that continuing selection of workers with different relative productivities may explain part of the differences in sectoral productivity growth. He estimated a Roy model to provide evidence for his thesis. A second interesting question for future work is to study whether the slow growing sectors will continue to grow slowly even when they comprise sizeable shares of the economy. We have made some initial progress on these questions in Herrendorf and Valentinyi (2015).

Our analysis raises the follow up question to what extent the results for the U.S. generalize to other countries. A natural starting point is to document that the productivity growth slowdown is a broader phenomenon that occurred also outside the U.S. To avoid mixing the productivity growth slowdown with declining GDP growth rates after catch-up dynamics that followed World War II, we focus on Australia, Canada, and the United Kingdom, which did not experience major war destruction. Figure 9 depicts the average annual growth rates of their labor productivities in the preceding 20 years. One can see clearly that the productivity growth slowdown was a broader than just a U.S. phenomenon. In fact, it looks more pronounced in the other three countries than in the U.S.[18] We think that an important task for future research is to study in detail the effects of structural change on productivity growth in other developed countries. Recent work by Buiatti et al. (2018), Sen (2019), and Duernecker and Sanchez-Martinez (2021) takes first steps in this direction.

[18]Note though that, because of data constraints, the productivity measure used in the figure is GDP per *hour worked*. If instead we used GDP per *labor services* for the U.S. as in the rest of the paper, then the strong rebound of GDP per-hour measure in the last two or so decades would be mitigated.

References

Acemoglu, Daron and Veronica Guerrieri, "Capital Deepening and Non-Balanced Economic Growth," *Journal of Political Economy*, 2008, *116*, 467–498.

Aguiar, Mark and Mark Bils, "Has Consumption Inequality Mirrored Income Inequality?," *American Economic Review*, 2015, *105*, 2725–2756.

Antolin-Diaz, Juan, Thomas Drechsel, and Ivan Petrella, "Tracking the Slowdown in Long–Run GDP Growth," *Review of Economics and Statistics*, 2017, *99*, 343–356.

Baumol, William J., "Macroeconomics of Unbalanced Growth: The Anatomy of the Urban Crisis," *American Economic Review*, 1967, *57*, 415–426.

—–, Sue Anne Batey Blackman, and Edward N. Wolff, "Unbalanced Growth Revisited: Asymptotic Stagnacy and New Evidence," *American Economic Review*, 1985, *75*, 806–817.

Bloom, Nicholas, Charles I. Jones, John Van Reenen, and Michael Webb, "Are Ideas Getting Harder to Find?," *American Economic Review*, 2020, *110*, 1104–1144.

Boppart, Timo, "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences," *Econometrica*, 2014, *82*, 2167–2196.

Buera, Francisco J. and Joseph P. Kaboski, "The Rise of the Service Economy," *American Economic Review*, 2012, *102*, 2540–2569.

—–, —–, Richard Rogerson, and Juan I. Vizcaino, "Skill Biased Structural Change," *forthcoming: Review of Economic Studies*, 2021.

Buiatti, Cesare, Joao B. Duarte, and Luis Felipe Sáenz, "Why is Europe Falling Behind? Structural Transformation and Services Productivity between Europe and the U.S.," mimeo, Univeristy of Illinois 2018.

Byrne, David M., John G. Fernald, and Marshall B. Reinsdorf, "Does the United States Have a Productivity Slowdown or a Measurement Problem," *Brookings Papers on Economic Activity*, 2016, pp. 109–157.

Comin, Diego, Martí Mestieri, and Danial Lashkari, "Structural Change with Long-Run Income and Price Effects," *Econometrica*, 2021, *89*, 311–374.

Duarte, Margarida and Diego Restuccia, "The Role of the Structural Transformation in Aggregate Productivity," *Quarterly Journal of Economics*, 2010, *125*, 129–173.

—— **and** ——, "Relative Prices and Sectoral Productivity," *Journal of the European Economic Association*, 2020, *18*, 1400–1443.

Duernecker, Georg and Miguel Sanchez-Martinez, "Structural Change and Productivity Growth in Europe – Past, Present and Future," Working Paper 9323, CESifo 2021.

——, Berthold Herrendorf, and Ákos Valentinyi, "The Productivity Growth Slowdown and Kaldor's Growth Facts," *forthcoming: Journal of Economic Dynamics and Control*, 2021.

Echevarria, Cristina, "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review*, 1997, *38*, 431–452.

Fernald, John G., "Reassessing Longer–Run U.S. Growth: How Low?," Federal Reserve Bank of San Francisco Working Paper 2016–18 2016.

—— **and Charles I. Jones**, "The Future of US Growth," *American Economic Review: Papers and Proceedings*, 2014, *104*, 44–49.

Gordon, Robert, *The Rise and Fall of American Growth: The US Standard of Living since the Civil War*, Princeton, New Jersey: Princeton University Press, 2016.

Hanoch, Giora, "CRESH Production Functions," *Econometrica*, 1971, *39*, 695–712.

——, "Production and Demand Models with Direct or Indirect Implicit Additivity," *Econometrica*, 1975, *43*, 395–419.

Herrendorf, Berthold and Ákos Valentinyi, "Which Sectors Make Poor Countries so Unproductive?," *Journal of the European Economic Association*, 2012, *10*, 323–341.

—— **and** ——, "Endogenous Sector–Biased Technological Change and Industrial Policy," Discussion Paper 10869, CEPR, London 2015.

——, Richard Rogerson, and Ákos Valentinyi, "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 2013, *103*, 2752–2789.

——, ——, and ——, "Growth and Structural Transformation," in Philippe Aghion and Steven N. Durlauf, eds., *Handbook of Economic Growth*, Vol. 2, Elsevier, 2014, pp. 855–941.

——, ——, and ——, "Structural Change in Investment and Consumption: A Unified Approach," *Review of Economic Studies*, 2021, *88*, 1311–1346.

Jones, Charles I. and Dietrich Vollrath, *Introduction to Economic Growth*, New York: W. W. Norton (3rd edition), 2013.

Jorgenson, Dale W. and Marcel P. Timmer, "Structural Change in Advanced Nations: A New Set of Stylised Facts," *Scandinavian Journal of Economics*, 2011, *113*, 1–29.

——, **Mun S. H, , and Jon D. Samuels**, "A Prototype Industry-Level Production Account for the United States, 1947–2010," mimeo, National Bureau of Economic Research, Cambridge, MA 2013.

Leon-Ledesma, Miguel and Alessio Moro, "The Rise of Services and Balanced Growth in Theory and Data," *American Economic Journal: Macroeconomics*, 2020, *12*, 109–146.

Moro, Alessio, "Structural Change, Growth, and Volatility," *American Economic Journal: Macroeconomics*, 2015, *7*, 259–294.

Ngai, L. Rachel and Christopher A. Pissarides, "Structural Change in a Multisector Model of Growth," *American Economic Review*, 2007, *97*, 429–443.

Nordhaus, William D., "Alternative Methods for Measuring Productivity Growth," Working Paper 8095, NBER 2001.

——, "Baumol's Disease: A Macroeconomic Perspective," *The B.E. Journal of Macroeconomics (Contributions)*, 2008, *8*.

Sato, Ryuzo, "The Most General Class of CES Functions," *Econometrica*, 1975, *43*, 999–1003.

Sen, Ali, "Structural Change within the Services Sector, Baumol's Cost Disease, and Cross-Country Productivity Differences," mimeo, Univeristy of Essex 2019.

Smith, V. Kerry, "Unbalanced Productivity Growth and the Growth of Public Services: A Comment," *Journal of Public Economics*, 1978, *10*, 133–135.

Young, Alwyn, "Structural Transformation, the Mismeasurement of Productivity Growth, and the Cost Disease of Services," *American Economic Review*, 2014, *122*, 3635–3667.

Appendix A Calculating Value Added with Thörnqvist Indexes

Before we explain how to calculate value added with Thörnqvist indexes, we note that using them to aggregate data quantities differs from using the non-homothetic aggregator specified by our model. This is a common issue when one connects multi-sector models with data. It implies that one cannot directly compare model generated quantities with data quantities. We deal with the issue in the usual way. First, we use the Thörnqvist index to aggregate data quantities as it is done in WORLD KLEMS. We describe the details in the following paragraphs. Second, we solve the model by using its non-homothetic aggregator. Third, we compare aggregate quantities from the model and the data by constructing the aggregate quantities from the model with the Thörnqvist index in the same way as in the data.

The WORLD KLEMS 2017 March Release contains nominal and real gross outputs, intermediate inputs, and capital and labor services for 65 industries. In a reference year, nominal and real variables are the same so that the usual relationships hold. Moreover, in the reference year, capital and labor inputs are normalized to equal nominal capital and labor compensation. For all other years, real variables are calculated by calculating their growth rates via Törnqvist indexes. This implies that real industry quantities are additive only in the reference year. In all other years, real value added no longer equals the difference between the real gross output and real intermediate inputs. Here, we describe how real value added at the industry level is constructed. Similar issues arise when one aggregates the 65 industries to the coarser three-sector split considered in the main analysis above.

The first step is to go to the reference year in which real and nominal value added are equal to each other. Real value added then results simply as the difference between gross output and intermediate inputs. The next step is to consider years other than the reference year. One may construct real quantities for these years by starting from the reference year and then applying annual growth rates of the real quantity. To see what is involved, define the growth rate of a generic variable X between periods t and $t+1$ as:

$$\Delta \log X_t \equiv \log X_{t+1} - \log X_t.$$

The growth rates of real gross output, real value added and real intermediate inputs in industry i are linked by the following identity:

$$\Delta \log GO_{it} = \left[1 - S(P_{it}^Z Z_{it})\right] \Delta \log Y_{it} + S(P_{it}^Z Z_{it}) \Delta \log Z_{it}, \tag{42}$$

where GO_{it}, Z_{it}, Y_{it}, P_{it}^{GO}, P_{it}^{Z} and P_{it} denote real gross output, real intermediate inputs, real value added, the price of real gross outputs, the price of real intermediate inputs and the price of real value added in industry i. Moreover, $S(P_{it}^Z Z_{it})$ denotes the averages over periods t and $t+1$ of the shares of industry i's nominal intermediate inputs in the industry's nominal gross output:

$$S(P_{it}^Z Z_{it}) = \frac{1}{2}\left(\frac{P_{it}^Z Z_{it}}{P_{it}^{GO} GO_{it}} + \frac{P_{it+1}^Z Z_{it+1}}{P_{it+1}^{GO} GO_{it+1}}\right).$$

Note that these shares are meaningful concepts because they are constructed in terms of nominal variables that are additive. We can calculate $\Delta \log(Y_{it})$ by solving equation (42) for $\Delta \log(Y_{it})$, and substituting in $GO_{it}, Z_{it}, P_{it}^{GO} GO_{it}, P_{it}^{Z} Z_{it}$ from WORLD KLEMS:

$$\Delta \log Y_{it} = \frac{\Delta \log GO_{it} - S(P_{it}^{Z} Z_{it}) \Delta \log Z_{it}}{1 - S(P_{it}^{Z} Z_{it})}. \tag{43}$$

Appendix B Derivations and Proofs

Appendix B.1 Equilibrium Conditions with Three Goods

The first–order condition to the outer and inner parts of the household's problem are:

$$P_{it} = \lambda_{ct} \alpha_i^{\frac{1}{\sigma_c}} C_{it}^{-\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_i - 1}{\sigma_c}} C_t^{\frac{1}{\sigma_c}}, \quad i = g, s, \tag{44a}$$

$$P_{jt} = \lambda_{st} \alpha_j^{\frac{1}{\sigma_s}} C_{jt}^{-\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_j - 1}{\sigma_s}} C_{st}^{\frac{1}{\sigma_s}}, \quad j = p, u. \tag{44b}$$

To derive (21a) and (22a), divide (44a) for s and g by each other and divide (44b) for h and l by each other, respectively. To derive (21b), multiply both sides of (44a) with C_{it} and add up the resulting equations:

$$P_{gt} C_{gt} + P_{st} C_{st} = \lambda_{ct} \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}} \right) C_t^{\frac{1}{\sigma_c}} = \lambda_{ct} C_t^{\frac{\sigma_c - 1}{\sigma_c}} C_t^{\frac{1}{\sigma_c}} = \lambda_{ct} C_t. \tag{45}$$

This equation implies that:

$$P_t = \frac{P_{gt} C_{gt} + P_{st} C_{st}}{C_t} = \lambda_{ct}. \tag{46}$$

Substituting the previous equation into (44a), we obtain:

$$P_{it}^{1-\sigma_c} = P_t^{1-\sigma_c} \alpha_i^{\frac{1-\sigma_c}{\sigma_c}} C_{it}^{\frac{\sigma_c - 1}{\sigma_c}} C_t^{(1-\sigma_c)\frac{\varepsilon_i - 1}{\sigma_c}} C_t^{\frac{1-\sigma_c}{\sigma_c}},$$

which implies that:

$$\alpha_i C_t^{\varepsilon_i - 1} P_{it}^{1-\sigma_c} = P_t^{1-\sigma_c} \alpha_i^{\frac{1}{\sigma_c}} C_{it}^{\frac{\sigma_c - 1}{\sigma_c}} C_t^{\frac{\varepsilon_i - 1}{\sigma_c}} C_t^{\frac{1-\sigma_c}{\sigma_c}}.$$

Adding over $i = g, s$ yields:

$$\begin{aligned} \alpha_g C_t^{\varepsilon_g - 1} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - 1} P_{st}^{1-\sigma_c} &= P_t^{1-\sigma_c} \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}} \right) C_t^{\frac{1-\sigma_c}{\sigma_c}} \\ &= P_t^{1-\sigma_c} C_t^{\frac{\sigma_c - 1}{\sigma_c}} C_t^{\frac{1-\sigma_c}{\sigma_c}} = P_t^{1-\sigma_c}, \end{aligned}$$

implying that the price index is given as

$$P_t = \left(\alpha_g C_t^{\varepsilon_g - 1} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - 1} P_{st}^{1-\sigma_c}\right)^{\frac{1}{1-\sigma_c}} .$$

This is (21b). Similar steps give (22b).

Appendix B.2 Proof of Proposition 1

We start by deriving the expenditure shares of the two services subsectors in total services expenditure. It is helpful to restate the first–order conditions for the inner and outer layer:

$$\frac{P_{ut}C_{ut}}{P_{pt}C_{pt}} = \frac{\alpha_u}{\alpha_p}\left(\frac{P_{ut}}{P_{pt}}\right)^{1-\sigma_s} C_t^{\varepsilon_u - \varepsilon_p}, \tag{47a}$$

$$P_{st} = \left(\alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} + \alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} . \tag{47b}$$

$$\frac{P_{st}C_{st}}{P_{gt}C_{gt}} = \frac{\alpha_s}{\alpha_g}\left(\frac{P_{st}}{P_{gt}}\right)^{1-\sigma_c} C_t^{\varepsilon_s - \varepsilon_g}, \tag{47c}$$

$$P_t = \left(\alpha_g C_t^{\varepsilon_g - 1} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - 1} P_{st}^{1-\sigma_c}\right)^{\frac{1}{1-\sigma_c}} , \tag{47d}$$

where P_{st} and P_t are the price indexes. Multiplying (21b) with C_t leads to

$$P_t C_t = \left(\alpha_g C_t^{\varepsilon_g - \sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - \sigma_c} P_{st}^{1-\sigma_c}\right)^{\frac{1}{1-\sigma_c}} . \tag{48}$$

Substituting out P_{st} with (47b) yields

$$E_t \equiv P_t C_t = \left(\alpha_g C_t^{\varepsilon_g - \sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - \sigma_c}\left(\alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} + \alpha_{pt} C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}\right)^{\frac{1-\sigma_c}{1-\sigma_s}}\right)^{\frac{1}{1-\sigma_c}} . \tag{49}$$

For a given C_t, the expenditure function is a nested CES function of prices. Hence, it satisfies the required properties with respect to prices. It is continuous, increasing, concave, homogenous of degree one and differentiable in prices. is clearly monotonically increasing in prices.

Next we show that the expenditure function is strictly increasing C_t. First we derive expressions for the expenditure shares. Note that (47a) implies that:

$$P_{ut}C_{ut} = \alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} \frac{P_{pt}C_{pt}}{\alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}}.$$

Adding $P_{pt}C_{pt}$ to both sides and rearranging yields:

$$P_{ut}C_{ut} + P_{pt}C_{pt} = \left(\alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} + \alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}\right) \frac{P_{pt}C_{pt}}{\alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}},$$

which can be solved for $P_{pt}C_{pt}/(P_{ut}C_{ut} + P_{pt}C_{pt})$ implying

$$\chi_{jt} \equiv \frac{P_{jt}C_{jt}}{P_{ut}C_{ut} + P_{pt}C_{pt}} = \frac{\alpha_j C_t^{\varepsilon_j - 1} P_{jt}^{1-\sigma_s}}{\alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} + \alpha_p C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}}, \qquad j \in \{p, u\}. \tag{50a}$$

A similar derivation shows that (47c) implies

$$\chi_{jt} \equiv \frac{P_{jt}C_{jt}}{P_{gt}C_{gt} + P_{st}C_{st}} = \frac{\alpha_j C_t^{\varepsilon_j - 1} P_{jt}^{1-\sigma_c}}{\alpha_g C_t^{\varepsilon_g - 1} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - 1} P_{st}^{1-\sigma_c}}, \qquad j \in \{g, s\}. \tag{50b}$$

Next, we take the derivative of E_t with respect to C_t:

$$\begin{aligned}\frac{\partial E_t}{\partial C_t} = \frac{1}{1-\sigma_c} \frac{E_t}{\alpha_g C_t^{\varepsilon_g - \sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s - \sigma_c} P_{st}^{1-\sigma_c}} &\Bigg[\alpha_g C_t^{\varepsilon_g - \sigma_c} P_{gt}^{1-\sigma_c} \frac{\varepsilon_g - \sigma_c}{C_t} + \alpha_s C_t^{\varepsilon_s - \sigma_c} P_{st}^{1-\sigma_c} \frac{\varepsilon_s - \sigma_c}{C_t} \\ &+ \frac{1-\sigma_c}{1-\sigma_s} \frac{\alpha_s C_t^{\varepsilon_s - \sigma_c} P_{st}^{1-\sigma_c}}{\alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} + \alpha_{pt} C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s}} \left(\alpha_u C_t^{\varepsilon_u - 1} P_{ut}^{1-\sigma_s} \frac{\varepsilon_u - 1}{C_t} \alpha_{pt} C_t^{\varepsilon_p - 1} P_{pt}^{1-\sigma_s} \frac{\varepsilon_p - 1}{C_t}\right)\Bigg].\end{aligned}$$

Using the expression for expenditure shares in (50a) and (50b), we can simplify this as:

$$\frac{\partial E_t}{\partial C_t} = \frac{E_t}{1-\sigma_c} \left[\chi_{gt} \frac{\varepsilon_g - \sigma_c}{C_t} + \chi_{st} \frac{\varepsilon_s - \sigma_c}{C_t} + \frac{1-\sigma_c}{1-\sigma_s} \chi_{st} \left(\chi_{ut} \frac{\varepsilon_u - 1}{C_t} + \chi_{pt} \frac{\varepsilon_p - 1}{C_t}\right)\right].$$

It follows that

$$\frac{\partial E_t}{\partial C_t} = \frac{E_t}{C_t} \left(\chi_{gt} \frac{\varepsilon_g - \sigma_c}{1-\sigma_c} + \chi_{st} \frac{\varepsilon_s - 1}{1-\sigma_c} + \chi_{st}\chi_{ut} \frac{\varepsilon_u - \sigma_s}{1-\sigma_s} + \chi_{st}\chi_{pt} \frac{\varepsilon_p - \sigma_s}{1-\sigma_s}\right). \tag{51}$$

It is easy to verify that Assumption 1 ensures that each term in the bracket is strictly positive. Hence the expenditure function is strictly increasing in C_t for all $\chi_{gt}, \chi_{st}, \chi_{pt}, \chi_{ut} \in [0, 1]$. **QED**

Appendix B.3 Proof of Proposition 2

Strategy of the Proof. We start by noting that in the limit, $\chi_{st} = 1$. We then proceed in four steps. In step 1, we establish the condition under which χ_{ut}/χ_{pt} is constant. In step 2, we show that there is a unique $\widehat{A}_u^* \in \left(1, \widehat{A}_p\right)$ such that χ_{ut}/χ_{pt} is constant in equilibrium. In step 3, we characterize the dynamics of χ_{ut}/χ_{pt} when $\widehat{A}_u \neq \widehat{A}_u^*$. In step 4, we show that the limit growth of GDP is as claimed.

Step 1. Equation (50a) implies that:

$$\frac{\chi_{ut}}{\chi_{pt}} = \frac{\alpha_u}{\alpha_p}\left(\frac{P_{ut}}{P_{pt}}\right)^{1-\sigma_s} C_t^{\varepsilon_u-\varepsilon_p}.$$

Rewriting the equation into growth factors gives:

$$\widehat{\left(\frac{\chi_{ut}}{\chi_{pt}}\right)} = \widehat{\left(\frac{P_{ut}}{P_{pt}}\right)}^{1-\sigma_s} \widehat{C}_t^{\varepsilon_u-\varepsilon_p}.$$

Using equation (19) and that the wedges are assumed to be zero, the previous equation implies that the expenditure shares are constant if and only if:

$$1 = \widehat{\left(\frac{\chi_{ut}}{\chi_{pt}}\right)} = \left(\frac{\widehat{A}_p}{\widehat{A}_u}\right)^{1-\sigma_s} \widehat{C}_t^{\varepsilon_u-\varepsilon_p}. \tag{52}$$

Step 2. Next, we establish a condition under which consumption growth implied by equation (52) is consistent with all equilibrium conditions given $\chi_{st} = 1$. To this end, we consolidate the two equilibrium conditions (47b) and (48) into one, using the expressions for expenditure shares relative to total, (50a) and (50b), the market clearing condition, and the firms' first-order conditions. We state all equations that follow in terms of growth factors ("hats") to be able to relate them to (52).

The equilibrium condition for the service price, (22b), implies that:

$$\left(\frac{P_{st+1}}{P_{st}}\right)^{1-\sigma_s} = \frac{\alpha_u C_{t+1}^{\varepsilon_u-1} P_{ut+1}^{1-\sigma_s} + \alpha_p C_{t+1}^{\varepsilon_p-1} P_{pt+1}^{1-\sigma_s}}{\alpha_u C_t^{\varepsilon_u-1} P_{ut}^{1-\sigma_s} + \alpha_p C_t^{\varepsilon_p-1} P_{pt}^{1-\sigma_s}}.$$

Dividing and multiplying each term in the nominator with the appropriate $C_t^{\varepsilon_j-1} P_{jt}^{1-\sigma_s}$ ($j \in \{p, u\}$), and using (50a), we obtain

$$\widehat{P}_{st}^{1-\sigma_s} = \chi_{ut} \widehat{C}_t^{\varepsilon_u-1} \widehat{P}_{ut}^{1-\sigma_s} + \chi_{pt} \widehat{C}_t^{\varepsilon_p-1} \widehat{P}_{pt}^{1-\sigma_s}. \tag{53}$$

Similarly, use (48) to obtain:

$$\widehat{E}_t^{1-\sigma_c} = \chi_{gt} \widehat{C}_t^{\varepsilon_g-\sigma_c} \widehat{P}_{gt}^{1-\sigma_c} + \chi_{st} \widehat{C}_t^{\varepsilon_s-\sigma_c} \widehat{P}_{st}^{1-\sigma_c}. \tag{54}$$

Now, setting $\chi_{gt} = 0$ and $\chi_{st} = 1$ as well as substituting (53) into (54) yields:

$$\widehat{E}_t^{1-\sigma_c} = \widehat{C}_t^{\varepsilon_s-\sigma_c}\left(\chi_{ut}\widehat{C}_t^{\varepsilon_u-1}\widehat{P}_{ut}^{1-\sigma_s} + \chi_{pt}\widehat{C}_t^{\varepsilon_p-1}\widehat{P}_{pt}^{1-\sigma_s}\right)^{\frac{1-\sigma_c}{1-\sigma_s}}$$
$$= \left(\chi_{ut}\left(\widehat{C}_t^{\frac{\varepsilon_u-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}\widehat{P}_{ut}\right)^{1-\sigma_s} + \chi_{pt}\left(\widehat{C}_t^{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}\widehat{P}_{pt}\right)^{1-\sigma_s}\right)^{\frac{1-\sigma_c}{1-\sigma_s}}. \tag{55}$$

To rewrite the left-hand side, we show that $\widehat{E} = \widehat{A}_g$ because for $\chi_{st} = 1$:

$$E_t = \sum_{j\in\{p,u\}} \frac{P_{jt}}{P_{gt}} Y_{jt} = \sum_{j\in\{p,u\}} \frac{A_{gt}}{A_{jt}} A_{jt} H_{jt} = A_{gt} \sum_{j\in\{p,u\}} H_{jt} = A_{gt}, \tag{56}$$

where we used that $Y_{jt} = A_{jt}L_{jt}$, that the firms' first-order conditions imply that $P_{jt}/P_{gt} = A_{gt}/A_{jt}$ given that we assumed $\tau_{jt} = 0$. Note that this step would not go through if we didn't restrict the wedges to equal zero. Even if they were positive constants, the wedges would introduce a proportionality factor which would change when the sectoral composition changes.

Turning now to the right-hand side of (55), we substitute out relative prices with relative productivities from (19). We then arrive at:

$$\widehat{A}_g^{1-\sigma_c} = \left(\chi_{ut}\left(\widehat{C}_t^{\frac{\varepsilon_u-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}\frac{\widehat{A}_g}{\widehat{A}_u}\right)^{1-\sigma_s} + \chi_{pt}\left(\widehat{C}_t^{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}\frac{\widehat{A}_g}{\widehat{A}_p}\right)^{1-\sigma_s}\right)^{\frac{1-\sigma_c}{1-\sigma_s}}, \tag{57}$$

which is equivalent to:

$$\widehat{A}_p = \widehat{C}_t^{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}\left[\chi_{ut}\left(\frac{\widehat{A}_p}{\widehat{A}_u}\right)^{1-\sigma_s}\widehat{C}_t^{\varepsilon_u-\varepsilon_p} + \chi_{pt}\right]. \tag{58}$$

Substituting the condition for constant χ_{ut}/χ_{pt}, (52), into (58) and solving, we obtain

$$\widehat{C} = \widehat{A}_p^{\frac{1}{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}}.$$

For an equilibrium with constant χ_{ut}/χ_{pt} to exist, $\widehat{C}$ has to satisfy this equation as well as equation (52):

$$\widehat{C}^* = \left(\frac{\widehat{A}_u^*}{\widehat{A}_p}\right)^{\frac{1-\sigma_s}{\varepsilon_u-\varepsilon_p}} = \left(\widehat{A}_p\right)^{\frac{1}{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}}. \tag{59}$$

Step 3. We now show that $1 < \widehat{A}_u^* < \widehat{A}_p$. Solving the second equation in (59) for $\widehat{A}_u^*$, we find:

$$\widehat{A}_u^* = \left(\widehat{A}_p\right)^{1+\frac{\frac{\varepsilon_u-\varepsilon_p}{1-\sigma_s}}{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}} = \left(\widehat{A}_p\right)^{\frac{\frac{\varepsilon_u-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}{\frac{\varepsilon_p-1}{1-\sigma_s}+\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}}}. \tag{60}$$

If the exponent of $\widehat{A}_p$ on the right-hand side is between 0 and 1, then the assumption that $\widehat{A}_p > 1$ implies that $1 < \widehat{A}_u^* < \widehat{A}_p$. To see that the exponent is indeed between 0 and 1, note that Assumptions 1–3 imply that:

$$\frac{\varepsilon_u - 1}{1-\sigma_s} + \frac{\varepsilon_s - \sigma_c}{1-\sigma_c} = \frac{\sigma_s - \varepsilon_u}{\sigma_s - 1} + \frac{\varepsilon_s - 1}{1-\sigma_c} > 0.$$

Moreover, note that Assumption 3 implies that:

$$\frac{\varepsilon_u - 1}{1-\sigma_s} + \frac{\varepsilon_s - \sigma_c}{1-\sigma_c} < \frac{\varepsilon_p - 1}{1-\sigma_s} + \frac{\varepsilon_s - \sigma_c}{1-\sigma_c}.$$

Step 4. It remains to characterize how χ_{ut} changes if $\widehat{A}_u$ does not satisfy condition (60). Two simple observations are useful. The first one is that the right-hand side of the condition (58) is increasing in $\widehat{C}_t$. The reason is that the assumed parameter values imply that:

$$\begin{aligned} &\varepsilon_u - \varepsilon_p > 0, \\ &\frac{\varepsilon_p - 1}{1-\sigma_s} + \frac{\varepsilon_s - \sigma_c}{1-\sigma_c} > 0. \end{aligned}$$

The second one is that, for any given $\widehat{A}_u$ and $\widehat{A}_p^*$, condition (52) implies that the consumption growth factor $\widehat{C}$ that would be consistent with constant χ_{ut}/χ_{pt} still satisfies:

$$1 = \left(\frac{\widehat{A}_p}{\widehat{A}_u}\right)^{\frac{1-\sigma_s}{\varepsilon_u-\varepsilon_p}} \widehat{C}.$$

Note that given the assumptions on the parameter values, $\widehat{C}$ is decreasing in $\widehat{A}_u$.

Now we are ready to characterise the behavior of χ_{ut} if (59) is not satisfied. Let $\widehat{A}_u \neq \widehat{A}_u^*$ while $\widehat{A}_p = \widehat{A}_p$. Consider first the case of $\widehat{A}_u < \widehat{A}_u^*$. Then, $\widehat{C} > \widehat{C}^*$ and the right-hand side of (58) is larger that the left-hand side. Since the right-hand side of (58) is monotonically increasing in $\widehat{C}_t$, there is a unique $\widehat{C}_t < \widehat{C}$ that satisfies (58). For that $\widehat{C}_t$, χ_{ut}/χ_{pt} is decreasing because the right-hand side of (52) is less than 1. In the other case, $\widehat{A}_u > \widehat{A}_u^*$, similar arguments imply that χ_{ut}/χ_{pt} is increasing.

Since $\chi_{ut}/\chi_{pt} \in [0, 1]$, the standard result applies that on a compact set every sequence has a limit. Since there is only one interior limit, it must be that $\lim_{t\to\infty} \chi_{ut} = 0$ or $\lim_{t\to\infty} \chi_{ut} = 1$ if

$\widehat{A}_u \neq \widehat{A}_u^*$. Since χ_{ut} is decreasing if $\widehat{A}_u < \widehat{A}_u^*$, it must be that $\lim_{t\to\infty} \chi_{ut} = 0$ if $\widehat{A}_u < \widehat{A}_u^*$. Since χ_{ut} is increasing if $\widehat{A}_u > \widehat{A}_u^*$, it must be that $\lim_{t\to\infty} \chi_{ut} = 1$ if $\widehat{A}_u > \widehat{A}_u^*$.

For $\chi_{jt} = 1$, (10) implies the following growth factors of C:

$$\Delta LP_j = \Delta A_j, \quad j = p, u. \tag{61}$$

QED

Appendix B.4 Equilibrium Conditions with Many Goods

Recall that the general utility function was characterized by condition (41):

$$1 = \sum_{i=1}^{I} \alpha_i^{\frac{1}{\sigma_i}} \left(\frac{C_{it}^{\sigma_i - 1}}{C_t^{\sigma_i - \varepsilon_i}} \right)^{\frac{1}{\sigma_i}} \tag{62}$$

Minimizing consumption expenditure subject to that constraint implies the first-order conditions:

$$P_{it} = \lambda_t \alpha_i^{\frac{1}{\sigma_i}} \frac{\sigma_i - 1}{\sigma_i} C_{it}^{-\frac{1}{\sigma_i}} C_t^{\frac{\varepsilon_i - \sigma_i}{\sigma_i}}.$$

Dividing them by the first-order condition for goods gives the relative demand for the six service industries $i = 1, ..., 6$:

$$\frac{P_{it}C_{it}}{P_{gt}C_{gt}} = \frac{\alpha_i^{\frac{1}{\sigma_i}} (\sigma_i - 1)/\sigma_i}{\alpha_g^{\frac{1}{\sigma_g}} (\sigma_g - 1)/\sigma_g} \frac{C_{it}^{\frac{\sigma_i - 1}{\sigma_i}}}{C_{gt}^{\frac{\sigma_g - 1}{\sigma_g}}} C_t^{\frac{\varepsilon_i - \sigma_i}{\sigma_i} - \frac{\varepsilon_g - \sigma_g}{\sigma_g}}. \tag{63}$$

We calibrate the model by using (62) together with (63) for $i = 1, ..., I$.

Appendix C Micro Evidence

Table 10: Results of Structural Estimation

	(1)	(2)
σ_c	0.61	0.46
	(0.03)	(0.06)
σ_s	1.23	1.51
	(0.04)	(0.06)
$\varepsilon_s - \varepsilon_g$	0.79	0.51
	(0.07)	(0.06)
$\varepsilon_u - \varepsilon_p$	0.49	0.59
	(0.04)	(0.02)
Household controls	Y	Y
Region fixed effects	N	Y
Year fixed effects	N	Y
Quarter fixed effects	N	Y

Note: SE clustered at household level; 87,017 observations.

Appendix D Inputs for the Simulations

Table 11: Inputs for Table 6

Exogenous Variables Based on	$\Delta \log A_{gt}$	$\Delta \log A_{pt}$	$\Delta \log A_{ut}$	$\Delta \log H_t$	$\frac{\sum_{i\in\{g,p,u\}} \widetilde{H}_{it}}{\widetilde{H}_t}$	τ_{pt}	τ_{ut}
2007–2019	0.73	1.05	0.04	0.24	-0.06	0.13	0.41
1999–2019	1.52	1.50	0.17	0.11	-0.10	0.17	0.46
1989–2019	1.76	1.91	0.16	0.20	-0.09	0.18	0.51
1979–2019	1.84	2.00	0.05	0.33	-0.08	0.15	0.51