Labor supply distortions of pension fund recovery policies*

Roel Mehlkopf†

April 21, 2009

Abstract

This paper studies the distortions in labor supply choices that are induced by the recovery policies of pension funds. Pension funds recover from funding shortfalls and surpluses by making adjustments in the contribution rate charged to participants and/or adjusting the value of pension accruals received by participants in return. Deviations between contribution and accrual rates imply that the pension fund levies a net tax or provides a net subsidy on the labor supply of its participants. The resulting distortions in labor supply choices cause labor earnings to become more positively correlated to stock returns, thereby reducing the risk-bearing capacity of participants. Recovery policies prevent pension funds from taking optimal advantage of the risk premium in financial markets. Labor supply distortions are eliminated if financial gains and losses are levied upon participants through lump-sum transfers, rather than through transfers that are proportional to labor supply. My analysis suggests that this policy improvement results in an ex ante welfare gain of almost a full percentage point in terms of certainty equivalent consumption.

Keywords: Saving, investment, labor supply, life cycle, pension funds.
JEL classification: D91, G11, G23, H23

---

*The author thanks Frank de Jong and Lans Bovenberg for their help and encouragement.
†Tilburg University, CentER and Netspar. Email: R.J.Mehlkopf@TilburgUniversity.nl.

1 Introduction

Bodie, Merton, and Samuelson (1992) have shown that the labor-leisure choice plays an important role in intertemporal saving and investment decision making over the life cycle. They describe that *income effects* in labor supply provide a buffer against wealth shocks, causing labor supply flexibility to be a pure blessing in their analysis. A negative wealth shock causes the marginal utility from working to increase, whereas a positive wealth shock has an opposite effect. The resulting pro-cyclical labor supply behavior causes labor earnings to become more negatively correlated to stock returns, increasing the individual's appetite for risk taking. In the analysis of Bodie, Merton, and Samuelson (1992) an increase in labor supply flexibility therefore enables individuals to benefit more from the risk premium in financial markets, resulting in an increase in ex ante welfare levels.

In this paper I study the shadow side of labor supply flexibility for individuals that are saving for retirement in a collective pension fund. Pension funds apply recovery policies by which financial gains and losses are levied upon the future earnings of participants through adjustments in contribution and accrual rates. Given that the effective wage rate of a pension fund participant is equal to the wage rate minus the contribution rate plus the accrual rate, deviations between contribution and accrual rates induce a wage differential. Pension funds thus recover from financial shocks by levying a net tax or providing a net subsidy upon the labor earnings of participants, thereby introducing *substitution effects* in labor supply decisions. These substitution effects in labor supply have exactly the opposite implications as the income effects in labor supply analyzed by Bodie, Merton, and Samuelson (1992). Recovery policies induce pro-cyclical labor supply behavior because labor supply is taxed in bad economic times (during which the funding status needs to recover from financial losses) and subsidized in good times. Pro-cyclical labor supply behavior causes the labor earnings of participants to become more positively correlated to stock returns, reducing the risk bearing capacity of pension fund participants. The recovery policies of pension funds thus prevent participants from taking full advantage of the risk premium in financial markets, resulting in a decline in ex ante welfare levels.

My analysis recognizes that recovery policies also have a welfare improving effect for a pension fund participant when compared to an individual investor: recovery policies enable pension fund participants to alleviate the borrowing constraint[1]. By recouping current financial shocks upon the future labor earnings of participants, the human capital of a pension fund participant can be exposed to stock market risk. An individual investor, who I assume to borrowing constrained, is unable to do so because of the inability to use human capital as collateral[2]. The exposure to risky assets of an individual investor is therefore limited by amount of accumulated financial wealth, preventing the individual investor from taking optimal advantage of the risk premium in financial markets.

In my analysis, the individual investor as well as the pension fund participant find themselves unable to realize the first best solution: the individual investor is borrowing constrained whereas the participant in the pension fund suffers from distortions in the marginal wage rate against which labor is supplied. I find that the ex ante welfare loss resulting from imposing a borrowing constraint on the first-best solution is equal to 2.8% of certainty equivalent consumption whereas the welfare loss from labor supply distortions for a pension fund participant amounts to about a third ($0.8\%/2.8\% \approx 1/3$) of that. The recovery policies of pension funds thus remain to be welfare improving when taking into account distortionary effects.

Even though the recovery policies of pension funds thus improve welfare levels compared to an individual investor, my analysis is not without important critique on recovery policies. Recovery policies can be substantially improved if financial shocks are recouped on participants by *lump-sum transfers*, rather through transfers that are proportional to labor supply. Lump-sum transfers do not induce substitution effects in labor supply such that labor supply distortions are fully eliminated. My analysis suggests that moving to recovery policies that are fully based upon lump-sum transfers results in an ex ante welfare gain of 0.8%.

Under perfectly competitive labor markets, employers with an unfair link between con-

[1]Life cycle models typically find that at young ages the optimal amount to be invested in high yielding assets such as stocks is higher than the financial wealth of individuals. Typically being unable to borrow money for stock investments (where there is no collateral, unlike is the case for money borrowed for investments in real estate), the inability to borrow against future earnings is a binding constraint for savers in individual retirement accounts.

[2]I abstract here from sophisticated investment strategies that make use of call options to attain a high exposure to equity with a relatively small amount of capital invested.

tribution and accrual rates find themselves forced to offer compensating wage differentials to plan participants. After all, any difference to the market wage level induces an influx or outflow of employees if competition is perfect. Under these conditions it is thus the employer who fully bears the funding risk of the pension plan[3]. However, the employer is unlikely to (fully) bear the funding risk of the pension plan for two reasons. First, the workings of actual labor markets are a far cry from perfectly competitive labor markets, due to the costs of switching employers, the accumulation of company-specific human capital and the fact that pay schemes are often based upon seniority instead of productivity. Additionally, alternative employers may have the same pension fund (pension funds can cover a complete sector or industry) or a pension fund with a similar funding status. For all these reasons, workers are likely to bear a part of the wage differential themselves. For simplicity, I abstract fully from compensating wage differentials provided by the employer.

The complexity of my analysis is dramatically reduced by the fact that income effects in labor supply decisions are ignored. The motivation for this model simplification is that I am not so much interested in absolute labor supply levels but merely in the *difference* between in the labor supply choices of the pension fund participant and those in the first-best strategy. The income effects in labor supply of the pension fund participant are very similar to those in the first-best strategy, such that income effects in labor supply are of second order importance. My analysis therefore solely focuses on substitution effects in labor supply decisions[4].

For wage differentials to affect labor supply decisions, the wage-elasticity of labor supply should be positive. There is overwhelming evidence in the empirical labor supply literature[5]

[3]Rauh (2006) shows that such risk sharing by the employer leads to a reduction in the profitability of the plan sponsor through inefficient decision making on capital expenditures.

[4]Given that income effects in labor supply have a stabilizing effect on wealth, the absence of income effects causes the model to overestimate the volatility in labor earnings, resulting in welfare costs of labor supply distortions that are too high. This effect may be regarded as being offset by the fact that I assume the wage rate of the participant offered by the employer to be constant over time during the working period, thereby clearly underestimating the volatility in labor earnings.

[5]Findings on labor supply elasticities are often obtained from empirical studies on the impact of a change in marginal rate of payroll taxes on the labor supply decisions of workers. Such estimates for labor supply elasticities can be used as parameters for the labor supply responses with respect to changes in contribution rates, since they directly affect disposable income levels. However, one could question whether these parameters are also useful when studying the effects of changes in accrual rates. The literature on behavioral economics has found convincing evidence that individuals suffer from *hyperbolic discounting*, suggesting that

that this indeed is the case. Recent overviews of Blundell and MaCurdy (1999) and Alesina, Gleaser, and Sacerdote (2005) find a consensus in the empirical labor supply literature that the compensated labor supply elasticity of male workers is low and that 0.2 is a reasonable estimate, whereas the median estimate found for the labor supply of women is about 1.0. Due to the large variation in the estimates found for females, a conservative parameter model parameter of about 0.5 is often applied. In this paper I will work with a labor supply elasticity of 1.0 as the default parameter. This appears to be a high estimate at first sight, but I have two reasons to work with such a high estimate. First, labor supply distortions induced by pension fund are likely to have a higher impact on the labor supply behavior of individuals than distortions induced by taxes, since it may be easy for workers to evade recovery policies by moving to self-employment or moving to an employer with an actuarially fair retirement savings scheme. Second, my model takes the perspective of a pension fund participant who participates in the labor force continuously until retirement, thereby abstracting from a participation decision of workers. This ignores micro-econometric evidence that the labor market participation of individuals responds to financial rewards (Heckman (2003), Saez (2002)). Although less often estimated, the elasticities of labor market participation are substantially higher than those of the number of hours worked. Eissa and Liebman (1996) find a participation elasticity of 0.6, while Meyer and Rosenbaum (2001) find an elasticity of 0.7.

This paper contributes to the theory on saving and investing over the life cycle. Most prior work in this field builds on the seminal contributions of Merton (1969) and Samuelson (1969) and treat labor earnings as exogenous (Viceira (2001), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2007)). The analysis of Bodie, Merton, and Samuelson (1992) provides the insight that income effects in labor supply increase the risk bearing capacity of individuals and recent contributions have built on their analysis. Farhi and Panageas (2007) and Choi, Shim, and Shin (2008) treat the case where labor supply flexibility is restricted to choice of an irreversible and indivisible retirement date. Cvitanic, Goukasian, and Zapatero

---

the utility value attributed to pension accruals may be substantially lower compared to the corresponding market value. This suggests that the pension fund policy induces distortions in labor supply *even* in the situation where contributions are equal to the market value of pension accruals. My analysis abstracts from such complications.

(2007) allow for more general preferences for the investor and the model of Gomes, Kotlikoff, and Viceira (2008) features realistically calibrated non-traded labor income. My paper is the first to show that, for a pension fund participant, labor supply flexibility also induces effects in labor supply that reduce the investor's appetite for risk taking and decrease ex ante welfare levels. Labor supply flexibility is thus only full of blessings as the analysis of Bodie, Merton, and Samuelson (1992) suggests.

This paper also contributes to the literature discussing the advantages and disadvantages of collective pension funds in comparison to individual retirement saving schemes (Teulings and de Vries (2006), Gollier (2007), Cui, de Jong, and Ponds (2007), Bovenberg, Nijman, Teulings, and Koijen (2007)). The advantages of collective pension funds include lower transaction costs, possibilities to share (non-traded) risks between overlapping and non-overlapping generations, the alleviation of the borrowing constraint and overcoming problems related to adverse selection and the bounded rationality of individuals. On the other hand, collective pension funds typically fail to provide tailor-made pension arrangements, apply suboptimal policy rules with respect to consumption smoothing over the life cycle and are often incomplete and not transparent. This paper contributes to our knowledge on the welfare effects of pension fund policies by pointing out that the recovery policies of pension funds can be welfare improving, also when taking into account the related labor supply distortions. At the same time this paper stresses the importance of improving recovery policies. Substantial welfare gains are realized by levying financial gains and losses upon participants through lump-sum transfers, rather than through transfers that are proportional to labor supply.

The structure of the paper is as follows. Section 2 introduces the financial market, the life-cycle and the preferences of the individual. Section 3 introduces the first best decision making rules. Section 4 derives the optimal strategy for an individual investor who faces a borrowing constraint. Section 5 derives the optimal policy for the pension fund and shows that the pension fund model corresponds to the first-best solution if labor supply is inelastic ($\epsilon = 0$) and that it corresponds to the solution of the individual investor if labor supply is infinitely elastic ($\epsilon = \infty$). Finally, section 6 concludes.

2 Financial market, the life-cycle and individual preferences

2.1 Financial market

The individual and the pension fund have the same investment opportunities. The financial market consists of two assets: a riskless asset which yields an instantaneous real net rate of return r and a risky asset whose real price $P(t)$ follows a Brownian Motion with instantaneous volatility σ and risk premium λ:

$$\frac{dP(t)}{P(t)} = (r + \sigma\lambda)dt + \sigma dZ(t) \tag{2.1}$$

for all t and where $Z(t)$ is a standard Wiener process for which it holds that

$$Z(s) - Z(t) \sim N(0, s - t) \tag{2.2}$$

for all $s > t$. The risk premium λ represents the expected excess return (over the riskfree rate r) per standard deviation of the excess return and is also referred to as the Sharpe ratio. Investments in the risky asset introduce uncertainty while at the same time increasing expected return if the risk premium is positive.

The market value at time t of a payoff $X(s)$ with $s \geq t$ is given by $\mathbf{E}_t\left[\frac{M(s)}{M(t)}X(s)\right]$, where $M(t)$ represents the stochastic discount factor at time t and is defined by the following stochastic differential equation:

$$\frac{dM(t)}{M(t)} = -rdt - \lambda dZ(t) \tag{2.3}$$

Throughout this paper I assume that the real riskfree rate equals 2% ($r = 0.02$), the volatility of stock returns equals 20% ($\sigma = 0.2$) and the risk premium on the stock is 20% ($\lambda = 0.2$). As a result, the expected return of risky investments in excess of the riskfree rate equals 4%.

2.2 The life cycle

Throughout this paper I consider an individual who starts working at age $t_0 = 25$, works for a 40-year period until the age of $R = 65$ and enjoys a 20-year retirement period before dying at the deterministic age of $T = 85$. The labor supply level $h(t)$ of the individual at time t during the working period is a decision variable, with $h(t) \geq 0$ for all $t_0 \leq t < R$. No labor is supplied in the retirement period, i.e. $h(t) = 0$ for all $R \leq t \leq T$. The real wage level per unit of labor supply at age t is deterministic and is denoted by $w(t)$. The wage level is inelastic with respect to labor supply levels, implying that the production function of the economy features infinite elasticity of substitution between capital and labor.

2.3 Individual preferences

Preferences of the individual are given by time-separable expected utility from consumption $C(t)$ and labor supply $h(t)$:

$$U(t) = \mathbf{E}_t \left[\int_t^T e^{-\beta(s-t)} u(C(s), h(s)) ds \right] \tag{2.4}$$

for $t_0 \leq t \leq T$, where β represents the individual's rate of time preference and where the instantaneous utility function at age t is given by:

$$u(C(t), h(t)) = \frac{1}{1-\gamma} \left(C(t) - \frac{\epsilon}{\epsilon+1} h(t)^{\frac{\epsilon+1}{\epsilon}} + \eta \right)^{1-\gamma} \tag{2.5}$$

where η represents a constant that will be defined at a later point in this section, and where ϵ represents the intratemporal labor supply elasticity with respect to the wage level. Thus, an decrease in the wage level at time t by one percent results in a decline in the labor supply level at time t of ϵ percent. Originating from Greenwood, Hercowitz, and Huffman (1988), the specification in (2.5) features the property that income effects in labor supply decision making are eliminated from labor supply supply decision. This is rather convenient given that, as explained in the introduction, income effects in labor supply are only of secondary importance for the purposes of my analysis.

Due to the absence of income effects, labor supply decisions are solely driven by the intra-temporal tradeoff between consumption and leisure[6]. In absence of a wage differential labor supply decisions and fully determined by the marginal wage rate $w(t)$:

$$h(t) \equiv h^*(t) = w(t)^\epsilon \tag{2.6}$$

[6]Leisure is used here in a broad sense of the term, that is any non market (or non traded) activity such as home production, work in the black economy or, indeed, having fun.

for all $t_0 \leq t < R$. Equation (2.6) implies that labor supply levels are constant over the life cycle if the wage rate is constant. Negative labor supply levels are ruled out by the model as long as marginal wage levels remain positive. Whereas the labor supply of the individual investor are given by equation (2.6), those of the pension fund participant will be different because an ex post wage differential is induced by the recovery policy of the pension fund.

In section 5 the welfare losses from the labor supply distortions of pension fund policies will be derived for various choices for the elasticity of labor supply parameter ϵ. In order to prevent that a change in the labor supply elasticity parameter ϵ affects the interpretation of the risk aversion parameter γ, I define parameter η in the utility function in equation (2.5) as:

$$\eta = \frac{\epsilon}{\epsilon+1} \left(h^*(t)\right)^{\frac{\epsilon+1}{\epsilon}} = \frac{\epsilon}{\epsilon+1} w(t)^{\epsilon+1} \tag{2.7}$$

where $h^*(t)$ is defined by equation (2.6). By substitution of equation (2.7) into equation (2.5) it follows that preferences simplify into standard time-separable utility with constant relative risk aversion over consumption only if labor supply levels are given by equation (2.6). Thus, in absence of labor supply distortions, the level of relative risk aversion and the elasticity of intertemporal substitution with respect to marginal consumption are given by γ and $1/\gamma$ respectively, regardless of the choice for the parameter of labor supply elasticity ϵ. Although this desirable feature of the utility function does not hold anymore in the presence of labor supply distortions, the effects of a change in the parameter labor supply elasticity ϵ will be small if wage differential induced by the pension fund policy is not too large. Indeed, in section 5 I will show that distortions to the marginal wage rate induced by model for the pension fund policy lie in the interval $[-9\%, +16\%]$ with 95% certainty if the labor supply elasticity ϵ is equal to one.

Throughout this paper I assume a rate of time preference of 2% ($\beta = 0.02$) and a relative risk aversion of five ($\gamma = 5$). The wage-elasticity of labor supply is assumed equal to one ($\epsilon = 1.0$). The wage rate is assumed constant during the working period and is normalized to unity ($w(t) = 1$ for all $t_0 \leq t < R$).

3 The first best solution

3.1 The optimization problem

Let the financial wealth that has been accumulated at age t ($t_0 \leq t \leq T$) be denoted by $F(t)$. The budget constraint on financial wealth is given by the following stochastic differential equation:

$$dF(t) = \begin{cases} (rF(t) + \sigma\lambda X(t))dt + \sigma X(t)dZ(t) + (h(t)w(t) - C(t))dt & \text{if } t_0 \leq t < R \\ (rF(t) + \sigma\lambda X(t))dt + \sigma X(t)dZ(t) - C(t)dt & \text{if } R \leq t \leq T \end{cases} \tag{3.1}$$

with $F(t_0) = 0$. The first and the second term on the right hand size represent respectively the expected and the unexpected return on financial wealth. The final term represents net savings by the individual at age t. Abstracting from a bequest motive, the condition for final wealth is given by:

$$F(T) = 0 \tag{3.2}$$

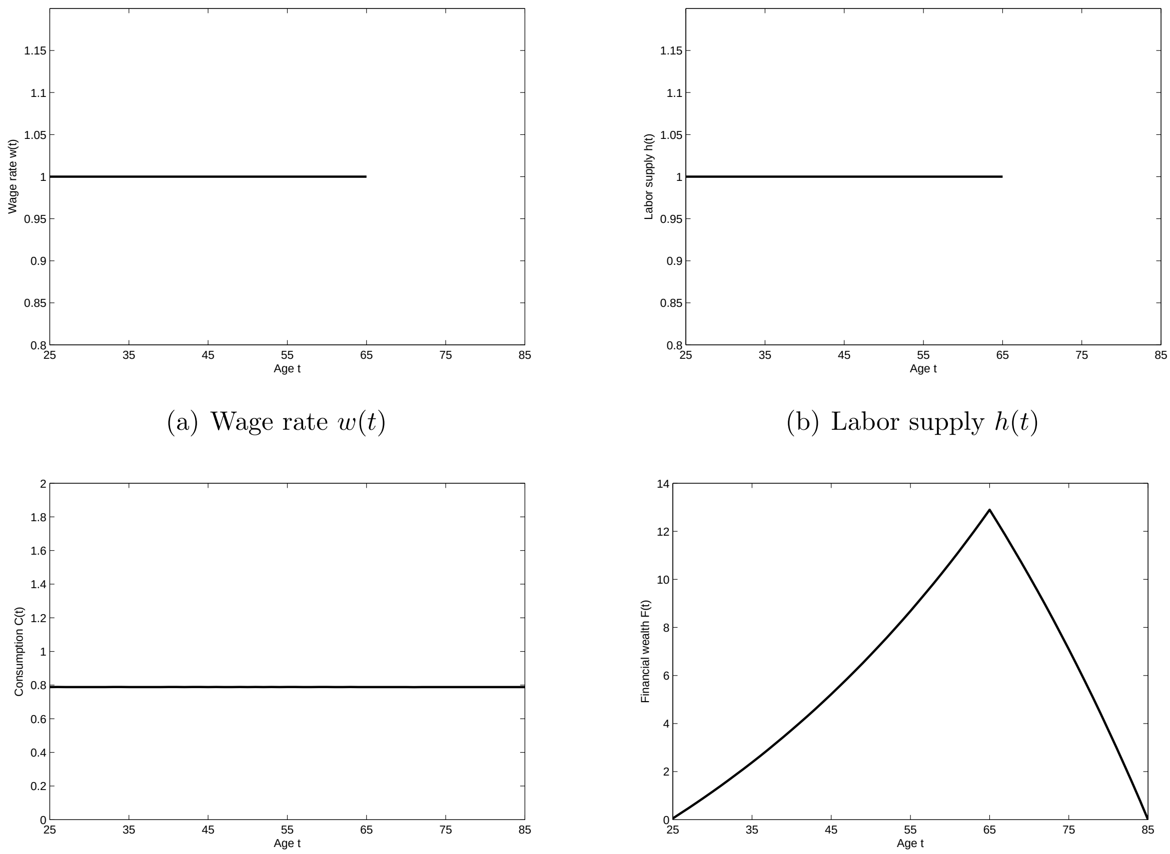
The optimization problem is to maximize utility as given by equation (2.4) with respect to the decision variables $\{ C(t), X(t) \}$ under the constraints in equations (3.1) and (3.2).

3.2 The first-best decision rules

Bodie, Merton, and Samuelson (1992) provide a general solution for the simultaneous optimization of labor supply, consumption and investment choices over the life cycle. Due to the absence of income effects in labor supply, the optimal solution features deterministic labor supply choices as given by equation (2.6). The decision making problem therefore reduces into a consumption-investment decision making problem with deterministic labor income, which is solved by Samuelson (1969) in a discrete time setting and by Merton (1969) in a continuous time setting. For completeness, this section provides a short overview of the Samuelson-Merton solution.

In absence of risk taking, i.e. if $X(t) = 0$ for all $t_0 \leq t \leq T$, the optimal consumption path is smooth over the life cycle is characterized by its growth rate:

$$\frac{dC(t)}{C(t)} = \frac{r - \beta}{\gamma} dt \tag{3.3}$$



(a) Wage rate $w(t)$

(b) Labor supply $h(t)$

(c) Consumption $C(t)$

(d) Financial wealth $F(t)$

Figure 1: *The wage rate $w(t)$, labor supply level $h(t)$, the consumption level $C(t)$ and financial wealth $F(t)$ over the life cycle in absence of risk taking for the default model parameters.*

for all $t_0 \leq t \leq T$. It follows from equation (3.3) that consumption levels are increasing over the life-cycle if the interest rate r is larger than the individual's rate of time preference β and decreasing in the opposite case. The extent to which this difference affects the slope of the consumption profile is determined by the willingness of the individual to substitute consumption over time intertemporally, i.e. the elasticity of intertemporal substitution of consumption $1/\gamma$.

Figure 1 illustrates optimal decision making in absence of risk taking for the default parameters. The default interest rate is equal to the default rate of time preference, resulting in a flat consumption profile. During the working period, the labor income of the individual exceeds the consumption level, such that the individual has positive savings. These savings result in the accumulation of financial wealth, required to finance consumption in the retirement period during which there are no labor earnings. Labor earnings are flat over the working life due to the absence of distortions in labor supply decisions in the individual retirement savings scheme.

In the presence of risk taking, the optimal investment rule is characterized by the property that the optimal amount $X(t)$ invested in stocks at age t equals a fixed fraction $\lambda/(\gamma\sigma)$ of the sum of financial wealth $F(t)$ and human wealth $H(t)$:

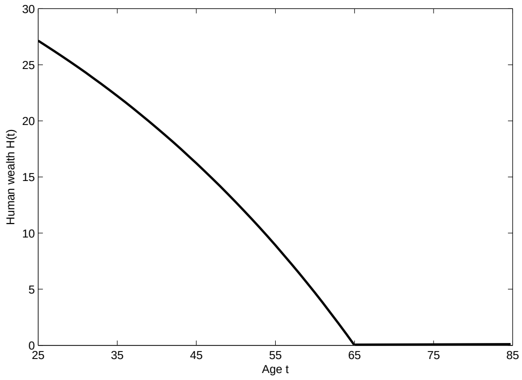
$$X(t) = \frac{\lambda}{\gamma\sigma}(F(t) + H(t)) \tag{3.4}$$

for all $t_0 \leq t \leq T$, where the human wealth $H(t)$ of the individual at age t represents the market value of future labor earnings:

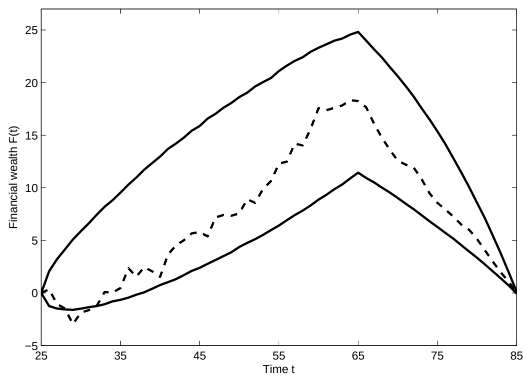
$$H(t) = \begin{cases} \mathbf{E}_t \left[\int_t^R \frac{M(s)}{M(t)} w(t)h(t)ds \right] & \text{for } t_0 \leq t < R \\ 0 & \text{for } R \leq t \leq T \end{cases} \tag{3.5}$$

where labor supply levels $h(t)$ are given by equation (2.6).

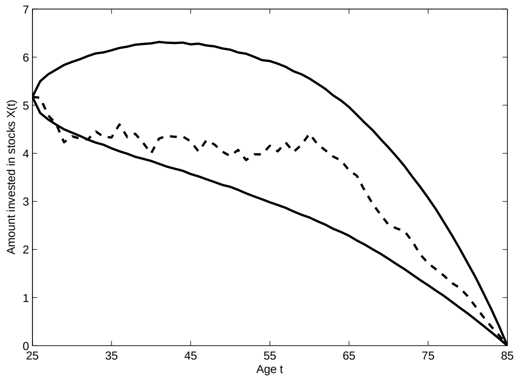
The consumption profile becomes stochastic in the presence of risk taking and is characterized by the way in which financial shocks are smoothed over consumption levels. In the optimal consumption rule, a financial shock at time t is levied proportionally on all consumption levels in the remaining lifetime, such that shocks to remaining consumption levels



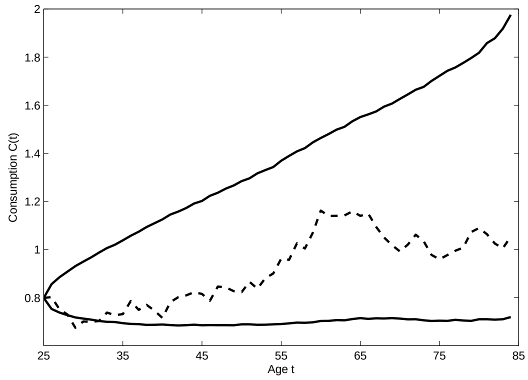
(a) Human wealth $H(t)$



(b) Financial wealth $F(t)$



(c) Investments in risky assets $X(t)$



(d) Consumption $C(t)$

Figure 2: *The 90% confidence interval (solid lines) and an example scenario path (dotted line) for human wealth $H(t)$, financial wealth $F(t)$, investments in risky assets $X(t)$ and consumption $C(t)$.*

are proportional to the wealth shock:

$$\frac{\partial C(s)/C(s)}{\partial Z(t)} = \frac{\partial W(t)/W(t)}{\partial Z(t)} = \frac{\lambda}{\gamma} \tag{3.6}$$

for all $t_0 \leq t < s \leq T$. The investment strategy in equation (3.4) causes relative changes in consumption levels with respect to economic shocks to be independent of age and to be increasing in the risk premium λ and decreasing in the parameter of risk aversion γ.

Figure 2 illustrates the optimal individual solution in the presence of risk taking. The figure shows 90% confidence intervals (solid lines) as well as an example scenario path (dotted line) for several variables. The figure illustrates that the optimal amount invested in stocks is well above financial wealth levels during the beginning of the life cycle, such that the individual has to borrow substantial amounts to get the optimal exposure to stock market risk. This is the result of the fact that young individuals posses little financial capital in comparison to the value of their human capital. The resulting short position in stocks results in the possibility of negative values for financial wealth. Future labor earnings are used to repay loans at a later stage in the working life. The amount of wealth that can be put at risk through investments in stocks is therefore limited by the market value of human wealth:

$$X(t) < H(t) \tag{3.7}$$

The optimal investment strategy thus requires the human capital of the individual to be used as a collateral to ensure that the loan is paid back. Since short positions in the investment portfolio are often difficult to implement for individual investors, the next subsection treats the optimization problem under a borrowing constraint.

4 The individual investor

4.1 The optimization problem

Adverse selection and moral hazard cause borrowing against future labor income often to be not possible for individual investors. Financial institutions are unable or unwilling to use human capital as a collateral to ensure that the loan is paid back. This subsection therefore

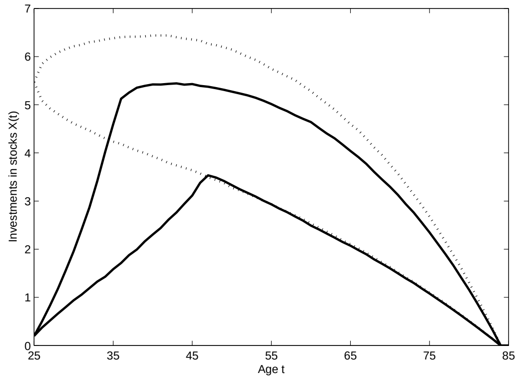


Figure 3: *90% confidence intervals for investments in risky assets $X(t)$ in presence of the borrowing constraint (solid lines) and in absence of the borrowing constraint (dotted lines).*

discusses the optimal solution under an exogenous borrowing constraint:

$$F(t) \geq 0 \tag{4.1}$$

for all $t_0 \leq t \leq T$. The borrowing constraint implies that the amount invested in stocks cannot exceed the financial wealth of the individual:

$$X(t) \leq F(t) \tag{4.2}$$

for all $t_0 \leq t \leq T$.

4.2 The optimal solution

Figure 3 illustrates that the borrowing constraint is binding during the beginning of the life cycle when little financial wealth has been accumulated. The borrowing constraint thereby causes the exposure to stock market risk for young individuals below the optimal exposure. This reduction in the risk-bearing capacity of the individual comes at substantial welfare costs because the individual investor is unable to optimally take advantage of the risk premium in the financial market. Welfare costs are expressed in terms of the percentage reduction in the certainty equivalent consumption level, i.e. the certain consumption level that yields an equivalent welfare level. The borrowing constraint results in a welfare loss of 2.8%, a result that stresses the importance of the optimal equity exposure during the beginning of the life cycle of the individual.

5 The pension fund policy

5.1 The optimization problem

The objective function of the pension fund is to maximize the utility of the participant. The investment opportunities of the pension fund are those introduced in subsection 2.1 and the life cycle characteristics and the preferences of the participant are those introduced in subsections 2.2 and 2.3 respectively. The model for the pension fund abstracts from wealth transfers between participants, such that the model features *inter*generational risk sharing nor *intra*generational risk sharing. This allows me to focus on a single participant, thereby greatly reducing the complexity of the model and allowing me to stick close to the analysis in sections 3 and 4.

I assume that participants are unable to save or borrow outside the pension fund, such that consumption levels during the working period are given by labor earnings after pension contributions and consumption levels in the retirement period are given by pension benefits:

$$C(t) = \begin{cases} (1 - \pi(t))h(t)w(t) & \text{if } t_0 \leq t < R \\ b(t) & \text{if } R \leq t \leq T \end{cases} \tag{5.1}$$

where $\pi(t)$ represents the contribution rate and is defined as the faction of labor earnings pledged to the pension fund during the working period and where $b(t)$ represents the rate at which pension benefits are received by the individual at age t in the retirement period. The contribution rate $\pi(t)$ and benefit rate $b(t)$ are decision variables of the pension fund.

Whereas there is only a single retirement savings account in the individual model of section 4, there are two accounts in the pension fund model treated in this section. First, there is the *individual* account $A(t)$ of the participant, which I will also refer to as the value of pension accumulations of the individual. Second, there is a *collective* account $S(t)$ which the participant does not consider to be his or her own wealth. The collective account is also referred to as the funding surplus (if $S(t)$ is positive) or the funding shortfall (if $S(t)$ is negative). The motivation for the introduction of two accounts is the observation that the value of the liabilities of a pension fund (i.e. the value of pension entitlements accumulated by participants) is in general unequal to the value of the assets. Deviations between the value of assets and liabilities result in a funding shortfall or a funding surplus. The ownership of the surplus of a pension fund is typically ambiguous. Moreover, the information fairs that are provided to pension fund participants typically contain information about the value of individual pension entitlements only. It is therefore assumed that the participant only takes into account mutations in the value of pension accumulations when determining labor supply decisions.

It is often legally or politically not possible for pension funds to provide their participants with pension entitlements that have a negative value and it is therefore imposed that the value of pension entitlements is not allowed to become negative:

$$A(t) \geq 0 \qquad \text{for all } S \leq t \leq T \tag{5.2}$$

The budget constraints of the two accounts of the pension fund participant are given by

$$dA(t) = \begin{cases} (rA(t) + \sigma\lambda X^A(t))dt + \sigma X^A(t)dZ(t) + \alpha(t)w(t)h(t)dt & \text{if } t_0 \leq t < R \\ (rA(t) + \sigma\lambda X^A(t))dt + \sigma X^A(t)dZ(t) - b(t)dt & \text{if } R \leq t \leq T \end{cases} \tag{5.3}$$

and

$$dS(t) = \begin{cases} (rS(t) + \sigma\lambda X^S(t))dt + \sigma X^S(t)dZ(t) + (\pi(t) - \alpha(t))w(t)h(t)dt & \text{if } t_0 \leq t < R \\ 0 & \text{if } R \leq t \leq T \end{cases} \tag{5.4}$$

where $A(t_0) = S(t_0) = A(T) = S(T) = 0$. X^A and X^S represent the amounts invested in stocks on respectively the individual account $A(t)$ and the collective account $S(t)$ and are both decision variables of the pension fund. Furthermore $\alpha(t)$ denotes the accrual rate at time t and is defined as the partial derivative of the value of pension accumulations $A(t)$ at time t with respect to labor supply level $h(t)$ at time t. Notice that $\alpha(t)$ represents not only the *absolute amount* accrued per unit of labor supply on the individual account $A(t)$ but also the market value of these pension accruals[7] because the return on pension accumulations $A(t)$ is fair in market terms for all $t_0 \leq t \leq T$. Future investment decisions thus do not affect the market value of pension rights accrued at present since the pension fund cannot create or destroy market value by investing in the financial market.

[7] In practice there may not be a uniquely defined market price of pension accruals. Pension funds typically offer benefit payments that are linked to inflation or wage levels and therefore cannot be replicated in financial markets. Additionally, pension funds may offer incomplete contracts to their participants in which the pension fund board can exercise discretion in choosing how policy rules are applied.

Deviations between contribution rates and accrual rates induce a *wage differential*: the effective wage rate against which labor is supplied by the participant is affected by the pension fund policy. Equation (2.6) suggests that the labor supply level of a pension fund participant is approximately[8] given by

$$h(t) \approx (w(t)(1 - \pi(t) + \alpha(t)))^{\epsilon} \tag{5.5}$$

for all $t_0 \leq t < R$, where $w(t)(1 - \pi(t) + \alpha(t))$ represents the effective wage rate of a pension fund participant per unit of labor supply: labor earnings minus pension contributions plus the value of pension accruals.

The essential difference between risk taking on the individual account $A(t)$ and risk taking on the collective account $S(t)$ is the way in which the resulting gains and losses are levied upon the participant. The gains and losses from risk taking on the individual account $A(t)$ are levied directly upon the value of pension entitlements and do therefore not affect the marginal effective wage rate against which labor is supplied. On the other hand, gains and losses from risk taking on the collective account have to be recouped upon the participant later in the working life by adjustments in the contribution and the accrual rate. If the contribution rate exceeds the accrual rate, the pension fund levies a net tax on the labor earnings of the participant such that a funding shortfall is recouped upon the participant. In the opposite case a net subsidy is provided on labor earnings such that a funding surplus is gradually reduced. The mechanism by which funding surpluses and shortfalls are recouped on participants is referred to as the *recovery policy* of the pension fund.

From equation (5.4) it follows that the market value of the financial surplus equals the market value of future net taxes or net subsidies on labor earnings at all times:

$$S(t) = \begin{cases} \int_t^R \mathbf{E}_t \left[\frac{M(s)}{M(t)} w(t) h(t) \left(\alpha(s) - \pi(s) \right) \right] ds & \text{if } t_0 \leq t < R \\ 0 & \text{if } R \leq t \leq T \end{cases} \tag{5.6}$$

All financial gains and losses that are incurred on the collective account must be recouped upon the participant during the working period, i.e. $S(t) = 0$ for all $t_0 \leq t < R$, since no labor is supplied by the participant during the retirement period. Furthermore, the absence

[8]The expression in equation (5.5) is only an approximation for labor supply decisions because the utility value of pension accruals only coincides with the corresponding market value in the first-best solution.

of intragenerational and intergenerational transfers implies that the ex ante market value of pension contributions is equal to the ex ante market value of pension accruals:

$$\mathbf{E}_0\left[\int_0^R \frac{M(t)}{M(0)} w(t)\pi(t)dt\right] = \mathbf{E}_0\left[\int_0^R \frac{M(t)}{M(0)} w(t)\alpha(t)dt\right] \tag{5.7}$$

The model feature that the recovery policy induces a wage differential is the result of the fact that it is imposed that the *marginal* net tax or net subsidy levied by the pension fund upon labor supply equals the *average* net tax or net subsidy. If financial gains and losses incurred on the collective account were to be levied upon the participant through *lump-sum transfers*, no labor supply distortions would be induced. Labor supply distortions exist in the pension fund model due to the fact that gains and losses from the collective account are levied on the participant through taxes and subsidies that are *proportional to labor supply.*

The optimization problem of the pension fund is to maximize the utility of the participant as given by equation (2.4) with respect to the variables $\{\ \pi(t),\ \alpha(t),\ X^A(t),\ X^S(t)\ \}$ during the working period and $\{\ b(t),\ X^A(t)\ \}$ during the retirement period under the constraints in equations (5.2), (5.3) and (5.4).

If the labor supply of the participant is elastic (i.e. $\epsilon > 0$) there exists no analytical solution for the optimal decision rules. The problem is therefore solved using numerical methods described in Appendix A.

5.2 The optimal policy

5.2.1 Special case: infinite elastic labor supply ($\epsilon = \infty$)

If the labor supply choice of the individual is infinitely elastic, then no more risk can be taken on the collective account $S(t)$ anymore, i.e. $X^S(t) = 0$ for all $t_0 \leq t \leq T$. After all, a positive exposure to stock market risk can result in a funding shortage which needs to be recouped on the participant trough a net tax on labor supply. This becomes impossible as the labor supply elasticity goes to infinity.

In absence of risk taking on the collective account, the model for the pension fund corresponds to the model for the individual investor under the borrowing constraint as treated in section 4. The risk-bearing capacity of the pension fund equals that of the borrowing

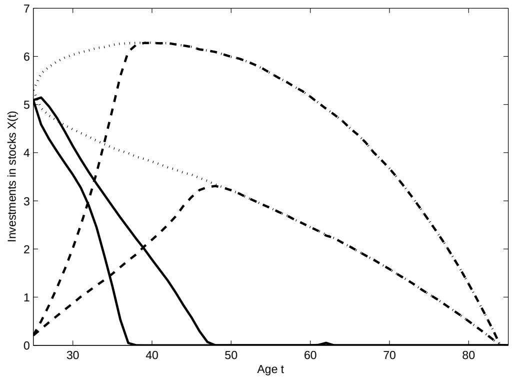


Figure 4: *90% confidence intervals for financial wealth $X^A(t)$ (dashed lines) and $X^S(t)$ (solid lines). Together, the amount invested in stocks by the pension fund sums up to the first-best allocation of the unconstrained individual (dotted lines).*

constrained individual investor and the welfare loss relative to the first-best strategy is equal to 2.8%.

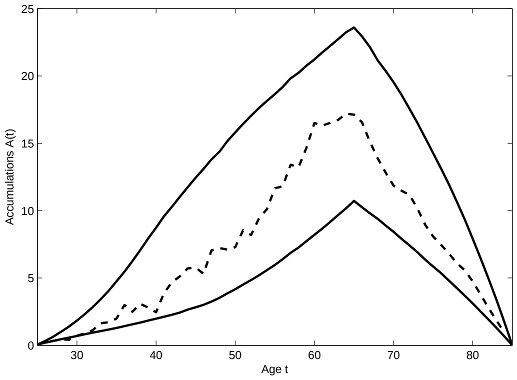
5.2.2 Special case: inelastic labor supply ($\epsilon = 0$)

In the case where the labor supply of the participant is inelastic with respect to the wage differential, the surplus can be freely used by the pension fund without any distortions being induced. Labor supply levels are at all times equal the first-best labor supply level in the unconstrained individual solution $h(t) = h^*(t)$, where $h^*(t)$ is defined by equation (2.6). Absence of distortions implies that the full human wealth of the individual can be used as collateral by the pension fund through investments on the collective account:

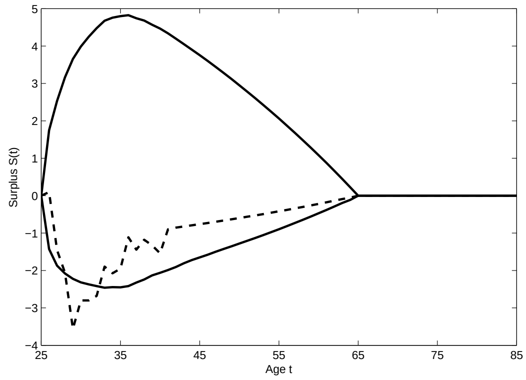
$$-S(t) > H(t) \tag{5.8}$$

Gains and losses incurred on the collective account are levied upon the participant through taxes and subsidies on labor earnings that are *non-distortionary*. The pension fund has the same risk-bearing capacity as the unconstrained individual investor and the first-best strategy can be perfectly replicated by the pension fund.

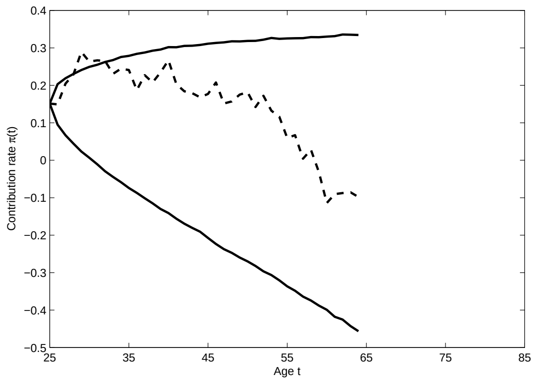
Figures 4 and 5 illustrate how the first-best consumption and investment rules are perfectly replicated by the pension fund policy. In the beginning of the working life, risk is



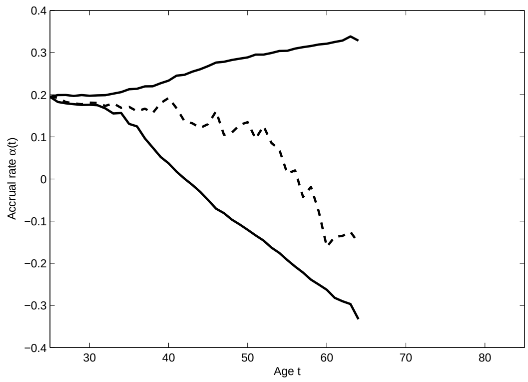
(a) Accumulations $A(t)$



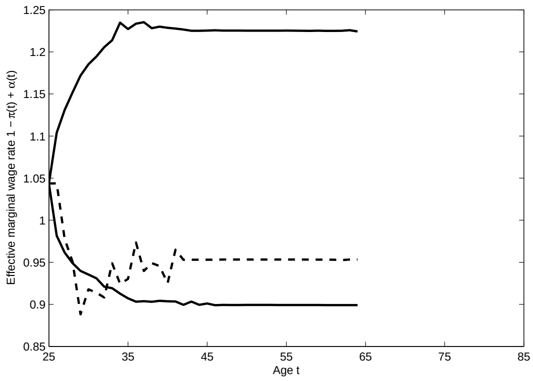
(b) Financial surplus $S(t)$



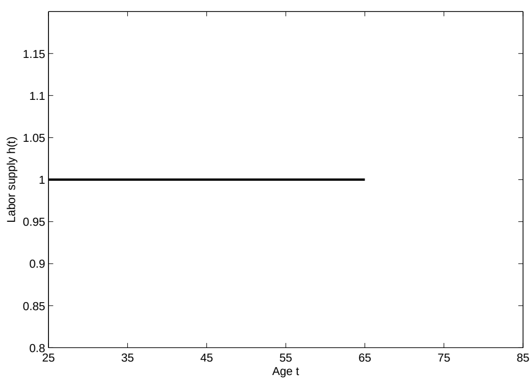
(c) Contribution rate $\pi(t)$



(d) Accrual rate $\alpha(t)$



(e) Effective wage rate $1 - \pi(t) + \alpha(t)$



(f) Labor supply $h(t)$

Figure 5: *90% confidence intervals (solid lines) and the example scenario path (dashed line) for pension accumulations $A(t)$, the financial surplus $S(t)$, the contribution rate $\pi(t)$, the accrual rate $\alpha(t)$, the effective marginal wage rate $1 - \pi(t) + \alpha(t)$ and the labor supply level $h(t)$.*

taken on the collective account, resulting in a positive or negative financial surplus. The financial surplus is levied upon the participant in the remaining working life by deviations between the contribution rate and the accrual rate[9] but the resulting wage differential leaves labor supply unaffected. By perfectly replicating the first-best consumption, investment and labor supply choices, the pension fund is able to realize the welfare level of the unconstrained individual investor.

5.2.3 General case: elastic labor supply $(0 < \epsilon < \infty)$

In this section I discuss the non-trivial case where the labor supply elasticity of the participant is positive but finite. This implies that risk taking on the collective account $S(t)$ results in distortions in labor supply decisions, such that it is less attractive to employ the surplus compared to the case where labor supply is inelastic.

The exposure to stocks that can be realized through risky investments on the collective account is reduced under elastic labor supply compared to the case of inelastic labor supply. Whereas the full human wealth can be put at risk under inelastic labor supply, changes in future earnings induced by the recovery policy have to be taken into account under elastic labor supply. The maximum revenue η per unit of time (corrected for the wage rate $w(t)$) that can be pledged from the participant by levying a net taxes on labor earnings is given by:

$$\eta w(t) \equiv \sup_{-\pi(t),\ \alpha(t)} \{(-\pi(t) + \alpha(t))w(t)h(t)\} \approx \frac{1}{\epsilon+1}\left(\frac{\epsilon}{\epsilon+1}\right)^{\epsilon} \tag{5.9}$$

where the approximation for the labor supply choice of a pension fund participant in equation (5.5) is substituted. From equation (5.9) it follows that the financial surplus is constrained from below

$$-S(t) > \mathbf{E}_t\left[\int_t^R \frac{M(s)}{M(t)}\eta w(s)ds\right] = \eta H(t) \tag{5.10}$$

where $H(t)$ is the non-stochastic human wealth at time t of the individual investor as defined by equation (3.5). The constraint in equation (5.10) implies that the exposure to stock

[9]Introducing intergenerational risk sharing to the model would cause confidence intervals for wage differentials to become more uniformly distributed over the working life because young workers share in the economic shocks that occurred before their time of entrance.

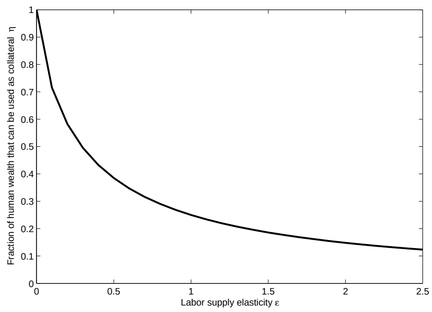


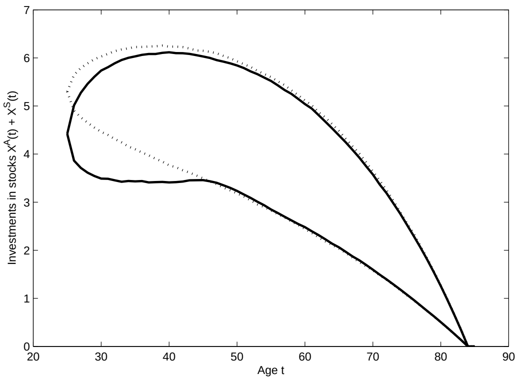
Figure 6: *The fraction η of human capital that can be put at risk for various parameter choices for labor supply elasticity ϵ.*

market risk that can be attained through investments in stocks on the collective account is limited by

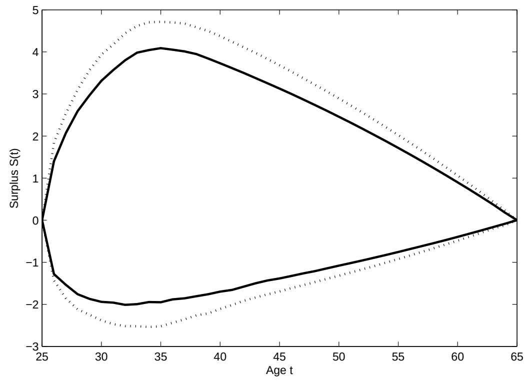
$$X^S(t) < \eta H(t) \tag{5.11}$$

It follows from equation (5.9) that positive finite levels of labor supply elasticity ($0 < \epsilon < \infty$ imply that parameter η is larger than zero but strictly smaller than unity, such it follows from comparing equations (3.7) and (5.11) that elastic labor supply implies a reduction in the maximum amount of risk that can be attained through the collective account. Figure 6 illustrates the percentage reduction $1 - \eta\%$ in this maximum amount for various parameter choices for labor supply elasticity ϵ. If the parameter of labor supply elasticity ϵ equals 1.0, the maximum amount that can be put at risk reduced by more than 70%. The case of infinite labor supply distortions ($\epsilon = \infty$) results in the situation where the human capital of the individual cannot be used as collateral at all ($\eta = 0$) and in the case where labor supply distortions are absent ($\epsilon = 0$) the constraints in equations (3.7) and (5.11) coincide.

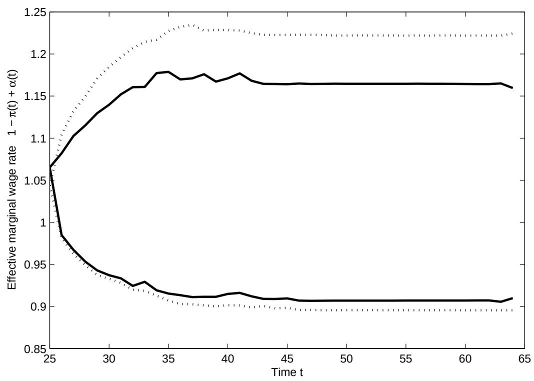
Figure 7 shows confidence intervals in the presence of labor supply distortions with $\epsilon = 1$ (solid lines) and in absence of labor supply distortions, i.e. $\epsilon = 0$ (dotted lines). Labor supply distortions cause the amount invested in stocks to decline, especially during the beginning of the working life where stock investments have to be taken on the collective account because little financial accumulations have been accumulated. In the presence of distortions it becomes less attractive to take risks on the collective account, resulting in smaller wage differentials and smaller funding surpluses and shortfalls compared to the case



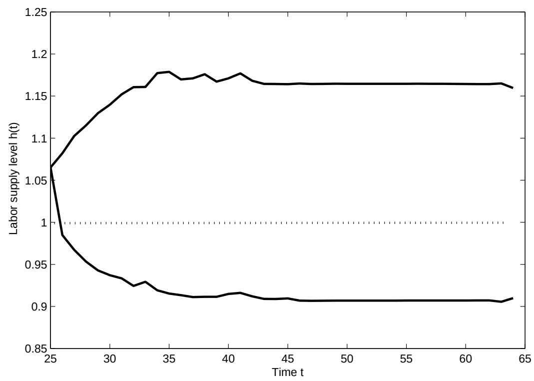
(a) Investments in stocks $X^A(t) + X^S(t)$



(b) Financial surplus $S(t)$



(c) Effective wage rate $1 - \pi(t) + \alpha(t)$



(d) Labor supply $h(t)$

Figure 7: *90% confidence intervals for stock investments $X^A(t)+X^S(t)$, the financial surplus $S(t)$, the effective wage rate $1 - \pi(t) + \alpha(t)$ and labor supply levels $h(t)$ in the presence of labor supply distortions with $\epsilon = 1$ (solid lines) and in absence of labor supply distortions, i.e. $\epsilon = 0$ (dotted lines).*

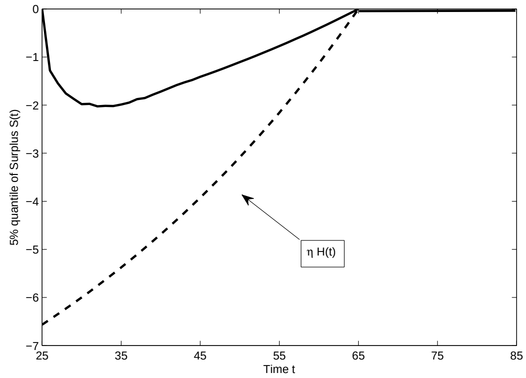


Figure 8: *5% quantile of the financial surplus $S(t)$ (solid line) and the constraint in equation* (5.10) *(dashed line) if* $\epsilon = 1$.

where distortions are absent. The constraint in equation (5.11) turns out not to hardly constraining for the default parameter $\epsilon = 1$, as is illustrated in Figure 8. That is: the unconditional probability that the constraint in equation (5.11) is binding is negligible. Only at levels of the parameter of labor supply elasticity ϵ above 1.5 the unconditional probability of hitting the constraint becomes substantial.

The intuition for the decline in stock investments reported in Figure 7 can be understood a comparison to the results of Bodie, Merton, and Samuelson (1992), who find that income effects in labor supply have a stabilizing effect on the wealth and increase the appetite for risk taking. The substitution effects in labor supply induced by the recovery policy work in exactly the opposite direction. Labor supply is decreased after a negative wealth shock, whereas it is increased after a positive shock. The substitution effects thus trigger destabilizing wealth effects by causing labor earnings to become *more positively correlated to stock returns*, thereby reducing the appetite for risk taking in the investment portfolio of the pension fund participant. Human capital thus becomes more stock-like rather than bond-like. This result is consistent with the findings of Cocco, Gomes, and Maenhout (2005), who show that the optimal exposure to stock returns is reduced as the human capital of an investor becomes more strongly correlated to stock returns. The reduction in investments in stocks is economically substantial: for a pension fund with a population that is uniformly distributed in age, the asset allocation decreases from 44% to 39% as a result of labor supply

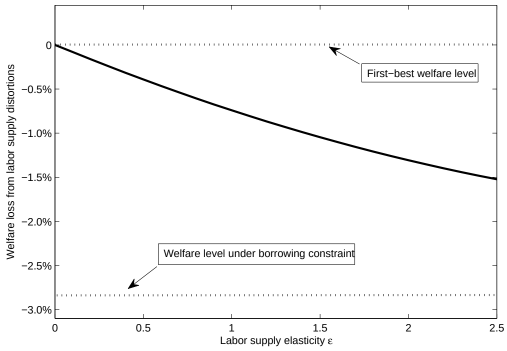


Figure 9: *The % change in the the certainty equivalent consumption level (ceteris paribus the leisure level, which is fixed at the levels of the first-best solution given by* (2.6) *for the calculation of the certainty equivalents) over the life cycle for various levels of the compensated wage-elasticity of labor supply* ϵ*.*

distortions.

Due to the reduction in the risk bearing capacity, the pension fund participant cannot take full advantage of the risk premium on financial markets, thereby reducing ex ante welfare levels. Figure 9 shows, for various levels of the elasticity of labor supply ϵ, the welfare of pension fund participation compared to the first best strategy. The model suggests that the welfare loss for the default parameter of the elasticity of labor supply ($\epsilon = 1.0$) is around 0.8% which is roughly a third ($0.8/2.8 \approx 1/3$) of the welfare gain that recovery policies generate by enabling individuals to borrow against human capital. Thus, although exposing human capital through recovery policies remains welfare improving for the participant, its attractiveness substantially reduces if one recognizes that such policies trigger distortions in labor supply decisions.

The welfare loss of pension fund participation can be reduced to zero if funding surpluses and shortfalls are levied upon the participant by lump-sum transfers, rather than by transfers that are proportional to labor supply. My analysis suggests that the welfare gains from moving to such 'lump-sum recouping' yields a welfare gain of 0.8%.

6 Conclusions

The model developed in this paper suggests that labor supply flexibility plays a different role collective pension saving scheme than in an individual saving scheme. Our analysis produces a number of insights:

Labor supply distortions induced by recovery policies reduce the risk bearing capacity of participants in collective pension plans. If funding shortfalls and surpluses are recouped on participants through transfers that are proportional to labor supply, then recovery policies reduce the risk bearing capacity of fund participants. The analysis therefore suggests that sectors or industries facing labor market competition from employers with actuarially fair pension plans may need to reduce the risk taking in their pension funds, compared to the case where there is no such competition.

The welfare costs from recovery policies (labor supply distortions) can amount to a substantial fraction of the welfare benefits from recovery policies (alleviation of the borrowing constraint). For a stand alone pension fund with age-independent policy rules I find that the welfare costs of recovery policies (distortions in labor supply) are substantial, and equal roughly a third of the welfare benefits of recovery policies (alleviation of the borrowing constraint) for the default parameters of the model.

Substantial welfare gains can be realized if financial surpluses and shortfalls are recouped on participants by lump-sum transfers, rather than by transfers that are proportional to labor supply. If financial gains and losses incurred on the collective account are levied upon the participant through lump sum taxes and subsidies on labor earnings, then no labor supply distortions are induced by the pension fund anymore and a welfare gain of almost a full percentage point can be realized..

The main virtue of the present model, its simplicity, is also an important limitation. The way in which labor supply flexibility has been modeled in our paper is far from the workings of actual labor markets. A more realistic model would therefore allow limited flexibility in the possibilities of varying intertemporal labor and leisure, while on the other hand allowing for some flexibility in the participation decision and the retirement decision. Additionally, a more advanced model would recognize risk sharing by the employer and the possibility for participants to switch jobs to companies or industries with actuarially fair retirement saving schemes.

This paper underscores the importance of distortionary labor supply effects of recovery policies of pension funds. This simple observation, however, suggests others. First, it is still unclear from the current analysis whether labor supply flexibility is a blessing or a curse for a participant in a collective pension plan. In other words: it is unclear under which conditions the stabilizing income effects in labor supply are larger or smaller than the destabilizing substitution effects induced by the recovery policy. Secondly, our findings suggest that the attractiveness of risk sharing between non-overlapping generations reduces if one recognizes that the policies that implement such risk sharing facilities come to together with an increase in labor supply distortions. While the current work on risk sharing between non-overlapping generations (Gollier (2007)) finds that more risk sharing is always better, the optimal level of risk sharing will be bounded when labor supply effects are taken into account.

References

Alesina, A., E. Gleaser, and B. Sacerdote (2005): "Work and leisure in the U.S. and Europe, why so different?," *Harvard Institute of Economic Research, Discussion Paper Number 2068.*

Blundell, R., and T. MaCurdy (1999): "Labor Supply: A Review of Alternative Approaches," *in: O. Ashenfelter and D. Card (eds.), Handbook of Labor Economics Vol 3A, Amsterdam: Elsevier North Holland, pp. 1559-1695.*

Bodie, Z., R. Merton, and W. Samuelson (1992): "Labour supply flexibility and portfolio choice in a life cycle model," *Journal of Economics: Dynamics and Control*, 16, 427.

Bovenberg, A., T. Nijman, C. Teulings, and R. Koijen (2007): "Saving and investment over the life cycle: the role of individual and collective pensions," *De Economist, 2007, 155(4), pp. 347-415.*

Choi, K., G. Shim, and Y. Shin (2008): "Optimal portfolio choice, consumption-leisure and retirement choice problem with CES utility," *Mathematical Finance*, 18, 445.

Cocco, J., F. Gomes, and P. Maenhout (2005): "Consumption and Portfolio Choice over the Life Cycle," *Review of Financial Studies 18.*

Cui, J., F. de Jong, and E. Ponds (2007): "The value of intergenerational transfers within funded pension schemes," *under review of third submission for Journal of Public Economics.*

Cvitanic, J., L. Goukasian, and F. Zapatero (2007): "Optimal risk taking with flexible labor income," *Management Science, No. 10, pp. 1594-1603.*

Eissa, N., and J. Liebman (1996): "Labor Supply Response to the Earned Income Credit," *Quarterly Journal of Economics 61, pp. 605-637.*

Farhi, E., and S. Panageas (2007): "Saving and investing for early retirement: A theoretical analysis," *Journal of Financial Economics 83, pp 87-121.*

Gollier, C. (2007): "Intergenerational Risk-Sharing and Risk-Taking of a Pension Fund," *Journal of Public Economics, Volume 92, pp. 1463-1485.*

Gomes, F., J. Kotlikoff, and L. Viceira (2008): "Optimal Life-Cycle Investing with Flexible Labor Supply, A Welfare Analysis of Life-Cycle Funds," *American Economic Review, Papers and Proceedings, Vol. 98, pp. 297-303.*

Gomes, F., and A. Michaelides (2007): "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance, vol 60, pp. 869-904.*

Greenwood, J., Z. Hercowitz, and G. Huffman (1988): "Investment, capacity utilitzation, and the real business cycle," *American Economic Review 78, pp. 402-417.*

Heckman, J. (2003): "What has been learned about labor supply in the past twenty years?," *AEA Papers and Proceedings 83, 116-121.*

Merton, R. (1969): "Lifetime portfolio selection under uncertainty, the continuous time model," *Review of Economics and Statistics*, 51, 247.

Meyer, B., and D. Rosenbaum (2001): "Welfare, the Earned Income Tax Credit and the Labor Supply of Single Mothers," *Quarterly Journal of Economics 66, pp. 1063-1114,.*

Rauh, J. (2006): "Investment and Financing Constraints: Evidence from the Funding of Corporate Pension Plans," *The Journal of Finance.*

Saez, E. (2002): "Optimal Income Transfer Programs: Intensive Versus Extensive Labor Supply Responses," *Quarterly Journal of Economics, 117, 1039-1073.*

Samuelson, P. (1969): "Lifetime portfolio selection by dynamic stochastic programming," *Review of Economics and Statistics.*

Teulings, C., and C. de Vries (2006): "Generational Accounting, Solidarity and Pension Losses," *De economist, vol 154, pp 63-83.*

Viceira, L. (2001): "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labour Income," *Journal of Finance 56, pp 433-470.*

A Numerical solution method

If labor supply is elastic with respect to the wage level (i.e. if $\epsilon > 0$) then there exists no analytical solution for the decision rules of the pension fund. The problem is therefore solved on the basis of numerical methods. During the working period the problem contains two state variables ($A(t)$ and $S(t)$) while during the retirement period, in which the financial surplus is zero, $A(t)$ is the only state variable. I assume that the labor supply decision $h(t)$ of the pension fund participant is given by the approximation rule given in equation (5.5), such that there are four decision variables to be solved during the working period ($\pi(t)$, $\alpha(t)$, $X^A(t)$ and $X^S(t)$) and two decision variables during the retirement period ($b(t)$ and $X^A(t)$). I apply a discretization with respect to age as well discretization in both dimensions of the state space. The decision rules can now be represented on a numerical grid in the state space at all ages t on the age grid. I apply an exponentially spaced in $A(t)$-dimension, resulting in relatively many gridpoints at low values of $A(t)$. The state space is bounded from below in the $A(t)$-dimension as well as in the $S(t)$-dimension as a result of the constraints in equations (5.2) and (5.10) respectively.

The decision rules are solved via backward induction. For every age age t on the grid prior to age T, and for each point in the state space, I optimize utility at time t with respect to the decision variables using a grid search. Discretization of the utility function with respect to age implies that utility $U(t)$ at age t is given by the sum of utility gained at the present age t ($\Delta t u(C(t), h(t))$ and the discounted expected continuation value $e^{-\beta\Delta t}\mathbf{E}_t\left[U(t+\Delta t)\right]$. Utility gained from present consumption and leisure follows directly from the decision variables, whereas the expected continuation value can be computed once the problem at time $t + \Delta t$ is solved. If continuation values do not lie on the the state space grid, I evaluate the value function on the basis of cubic interpolation of the value function. In the last period the solution is trivial since the individual finds it optimal to consume all remaining wealth due to the absence of a bequest motive. This thus gives me the terminal condition for my backward induction procedure. I perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the risky asset return.

The 90% confidence intervals reported in the paper are generated by Monte-Carlo simulations on the basis of the numerically solved decision variables. If the simulated values for the state variables do not lie on the the state space grid, decision making is evaluated on the basis of cubic interpolation of decision rules represented on the state space grid.