

# Labor market regulation, innovation through experimentation, and growth

– Preliminary and incomplete, comments welcome –

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## Abstract

This paper analyzes the effect of firing costs on productivity growth, a topic that is much less researched than the impact on the level of productivity. Two features stand out: a) growth occurs endogenously through experimentation and selection, and b) exempting exiting firms from firing costs can raise growth, but the net present value of current and future consumption is still lower than in the economy without firing costs.

The model of growth through experimentation is based on recent evidence on the importance of job turnover, on firm heterogeneity, and on the contribution of entry to aggregate productivity growth. Firms receive idiosyncratic productivity shocks and therefore differ in productivity and employment. Growth occurs endogenously through a spillover from incumbents to entrants; mean productivity of entrants is a constant fraction of that of incumbents.

In this context, besides inducing a misallocation of labor, firing costs discourage exit of low-productivity firms and thereby reduce the spillover to entrants, leading to slower growth. However, exempting exiting firms from firing cost speeds up the exit of inefficient firms and thereby growth.

For a quantitative evaluation, the model is calibrated to the US economy, as a low firing cost benchmark. Then the effects of introducing firing costs are explored. Preliminary results suggest that always applying firing costs has static and dynamic costs, while exempting exiting firms enhances growth, but not by enough to compensate static losses.

## 1 Introduction

This paper analyzes the effect of labor market regulation on productivity growth, a topic that is much less researched than the impact on the level of productivity or on employment. For this scope, a heterogeneous-firm model with endogenous growth is developed. Here, labor market regulation will not only have an effect on the efficiency of the allocation of labor across plants,

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or on the incentive to work or to search, but will affect the endogenous growth of aggregate productivity.

Recent empirical research on firm dynamics has highlighted the importance of entry and exit and the heterogeneity of firms and plants. For example, Dwyer (1998) finds that productivity differs by a factor 3 between establishments in the 9th and the 2nd decile of the productivity distribution in the US textile sector. Foster, Haltiwanger and Krizan (2001) (FHK) find that in the Census of Manufactures, more than a quarter of the increase in aggregate productivity between 1977 and 1987 was due to entry and exit. This is even more pronounced in the retail sector, as they find in their (2002) paper.<sup>1</sup> This paper takes this evidence as a point of departure and develops a model of endogenous growth through selection and experimentation.

This work is closely related to the seminal paper of Hopenhayn and Rogerson (1993), and the more recent ones by Alvarez and Veracierto (2001), Veracierto (2001), and Samaniego (2005). However, these papers employ a setting of exogenous growth and concentrate on the effect of employment protection legislation on the static efficiency of the allocation of labor. The mechanism of growth through experimentation and selection used here suggests itself for a heterogeneous-firm model, particularly in the light of the evidence on entry and growth reported by Foster et al. (2001), Haltiwanger, Jarmin and Schank (2003), and Bartelsman et al. (2004).<sup>2</sup>

In the model, firms receive idiosyncratic productivity shocks and therefore differ in their productivity and employment. Growth occurs endogenously because mean productivity of entrants is a constant fraction of that of incumbents, while exiting firms are less productive, an ordering established by e.g. Foster et al. (2001). This relationship implies that productivity spills over from incumbents to entering firms, and the severity of the selection process for incumbents will affect the distribution of incumbents, that of entrants, and growth. The setup can be interpreted as growth through experimentation and selection. Both entry and exit matter. The entry process injects fresh blood into the economy every period, and only relatively productive (innovative) entrants will survive. The exit process eliminates the least efficient firms, thereby improving the pool of firms from which productivity spills over to entrants. Clearly, having more productive entrants on average will raise growth, but this is not all. An increase in the variance of the distribution of entrants also benefits aggregate productivity in this framework by making

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<sup>1</sup>Some extensive surveys of methods and results on firm-level dynamics for developed and developing countries are Baldwin (1995), Roberts and Tybout (1996), Sutton (1997), Haltiwanger (1997), Caves (1998), Foster et al. (2001, 2002), Bartelsman and Doms (2000), Tybout (2000), Bartelsman, Scarpetta and Schivardi (2003), Bartelsman, Haltiwanger and Scarpetta (2004), and references therein.

<sup>2</sup>Gabler (2005) develops a very similar framework and uses it for measuring the extent of embodied versus disembodied technical change.

very high draws more likely, while very low draws will be cut off by exit anyway. Since entry is costly, there is a static loss besides this dynamic gain, as from some point on, there may be a waste of resources in “excess entry.”

In this context, labor market regulation affects the exit incentives of firms. It is well-known from the literature on firm dynamics and employment protection cited above that firing costs induce a static distortion by misallocating labor. Basically, the most productive firms do not employ enough, and declining firms do not lay off quickly enough, relative to the situation without firing costs. Samaniego (2005) remarks that the exact implementation of labor market regulation makes a difference and gives an overview over the implementation of firing costs in various countries. Exempting exiting firms from firing costs can provide an incentive to exit instead of just shrinking in the face of a negative productivity shock, and thereby amplifies the effect of the externality on the economy. While this just reduces the distortion in a model without growth, it can actually enhance growth in a model of growth through experimentation and selection. The net effect on welfare will depend on the relative size of the static and the dynamic effects.

To quantitatively evaluate the impact of labor market regulation on observed differences in productivity growth and in the behavior of entrants, the model is calibrated to the US economy. Then the effects of introducing firing costs of one year’s wages, close to the level observed in Germany and in many other continental European countries, is evaluated. Preliminary results show that when firing costs are always charged, they lead to a large welfare loss equivalent to around 8% of consumption. Exempting exiting firms leads to a static loss but dynamic gains, with the first dominating the second, so that the net effect is still a welfare loss.

Relating labor market regulation to growth crucially relies on the presence of idiosyncratic shocks to firms. Plain heterogeneity or even exogenous exit shocks would not be sufficient since firing costs work on the endogenous exit margin. In that sense, the development of the framework of growth through experimentation and selection is a necessary step in this analysis.

Moreover, besides the usual effect on labor force participation, results for the economy where firing costs are always charged fit qualitative EU-US patterns in relative firm size, relative productivity of entrants. It has to be said that patterns on turnover and post-entry growth do not fit the empirical patterns, though this could be due to unmodeled differences in entry barriers (affecting turnover) and financing constraints (affecting post-entry growth).

The paper is organized as follows. After a brief review of the facts on firm dynamics (Section 2), a simple heterogeneous firm model with growth by experimentation and selection is set up

(Section 3). In Section 4, the model is solved for optimal behavior of all agents, equilibrium is defined, and an algorithm for calculating it is given. In the following section, the model is calibrated, and in Section 6, the effects of firing costs are explored. Section 7 concludes.

## 2 Some facts

To motivate the choice of framework, it is useful to repass some facts on firm dynamics. Particularly since the book of Davis, Haltiwanger and Schuh (1996), the role of job turnover for the study of labor markets is firmly established. Yet this leaves open in which framework to study it. One option are matching models in the Mortensen-Pissarides tradition, which allow a detailed modelling of the worker side and labor market instrument he or she faces. Another one are heterogeneous firm models, first applied to employment protection legislation by Hopenhayn and Rogerson (1993).

The second type of model allows linking firing costs to the selection mechanism and its effect on aggregate productivity, so that way is taken here. In addition, the empirical literature by now has reported a wealth of facts that can be used for building firm dynamics models consistent with empirical facts. Recently, this data has been made comparable across countries in the OECD firm level project, attempting to adjust for the idiosyncrasies of national statistical agencies.

Very briefly, I will review some crucial facts on firm dynamics.<sup>3</sup> As a starting point, the literature on firm-level empirics agrees that idiosyncratic shocks are the main driver of firms' fate. Evidence for this comes from three different pieces of evidence. First of all, contemporaneous entry and exit rates are positively correlated for most industries. This is not consistent with a model of industry-level of aggregate shocks, since then positive shocks would encourage entry, negative ones exit, and the contemporaneous correlation would be negative. Secondly, productivity heterogeneity among firms within industries exceeds that between industries. For instance, Dwyer (1998) finds that the ratio between the 9th and the 2nd productivity decile in the U.S. textile industry is around 3. Moreover, productivity is very heterogeneous even for firms within narrowly defined industries and markets, and with similar technologies. Finally, market leadership within industries changes frequently, suggesting that firms receive firm-specific innovations to their productivity. All the while, productivity is very persistent. This observation will be crucial for motivating assumption 1 that firm-specific productivity follows a random walk.

Heterogeneity translates into a large rate of firm turnover (entry + exit divided by the number of firms), ranging from 13% in Germany to 22% in the U.S.. However, many entrants

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<sup>3</sup>Sources are cited in footnote 1.

do not survive long – the four-year survival rate is 63% in the U.S.. Yet those that do survive grow quickly. For example, in the U.S., they attain 140% of their original size 7 years after entry. Most importantly, reallocation is productivity-enhancing. This is a robust observation for the U.S.. For example, as quoted earlier, entry and exit contributed 25% of productivity growth in the U.S. manufacturing sector between 1977 and 1987, and more in the retail sector. These last observations together provide the motivation for a process of entry through experimentation and selection, where the interaction of entry and exit, and particularly the elimination of low-productivity firms, is crucial for aggregate productivity.

Finally, most of investment occurs in lumps and spikes and not smoothly as in representative-firm models; much of it is sunk. Moreover, the resale value of assets is very low (Ramey and Shapiro 1998). This motivates a model that follows this evidence through to the extreme assumption that all investment is made at entry and is sunk.

### 3 The Model Economy

In the following, a very simple heterogenous firm model is set up, optimal behavior of all agents is described, and the growth mechanism is explained. After that, equilibrium is defined and an algorithm for calculating it is given.

Time is discrete. The economy is populated by a continuum of infinitely-lived consumers of measure one, a continuum of active firms of endogenous measure, a large pool of potential entrants, and a sector of perfectly competitive portfolio firms.

Consumers value consumption and dislike working; this is summarized in the period utility function  $u(c_t, n_t) = \ln c_t - \theta n_t$ . They discount the future using a discount factor  $\beta < 1$ . They can consume or invest in shares  $a_t$  of the portfolio firms that pay a net return  $r_t$ ; income comes from wages and the return to the portfolio.

The portfolio firms finance investment in the firms in the economy. Since the sector is competitive, they don't make any profits and return net profits of the production sector to consumers as return on assets. Given perfect competition and symmetry, they all hold the same portfolio (the market portfolio) and pay the same return  $r_t$  on assets. Hence, they can be summarized into one representative portfolio firm. That firm will hold the market portfolio, i.e. invest in all firms.

**Firms:** Firms produce a numéraire good using labor as their only, variable input, with a positive and diminishing marginal product. To remain active, firms also incur a fixed operating

cost  $c_t^f$  each period; this grows over time at the growth rate of output,  $g$ . Moreover, there is an exogenous probability  $\delta$  that a firm's production facilities break down after a period's production, forcing the firm to exit; this affects all firms in the same way.

Firms differ in productivity. This arises because each firm receives idiosyncratic productivity shocks; more precisely, its productivity follows a random walk. This is a very simple way of capturing the role of idiosyncratic shocks established by the empirical literature. It also renders the persistence of firm level productivity found in the data.<sup>4</sup> This production technology can be summarized in the production function

$$\hat{y}_{it} = \exp(\hat{s}_{it}) \hat{n}_{it}^\alpha, \quad 0 < \alpha < 1, \quad (1)$$

where  $\hat{y}_{it}$  denotes output of firm  $i$  in period  $t$ ,  $\exp(\hat{s}_{it})$  is its productivity level, and  $\hat{n}_{it}$  employment.

**Assumption 1** *Productivity evolves according to*

$$\hat{s}_{it} = \hat{s}_{i,t-1} + \epsilon_{it}, \quad (2)$$

where the innovation  $\epsilon$  is distributed normally with mean zero and variance  $\sigma^2$ .

**Firing costs:** Adjusting employment is costless in the benchmark case. This will be compared to the case with employment protection legislation (EPL) in the form of firing costs of  $c^n$  times a period's wages for each worker fired. In the following, two cases will be considered, one where firing costs always have to be paid upon firing a worker, including upon exit, and a second one where firing costs only have to be paid if the firm also remains active in the subsequent period; i.e. exiting firms are exempted from firing costs. For ease of reference, introduce the indicator function  $\mathbb{1}_x$  that equals one if firing costs also have to be paid when the firm exits, and zero otherwise.<sup>5</sup> An active firm's profit function  $\pi(s_{it}, n_{i,t-1}, w_t)$  can then be written in a general way as

$$\hat{\pi}_{it} = \max_{\hat{n}} \{ \exp(\hat{s}_{it}) \hat{n}_{it}^\alpha - \hat{w}_t \hat{n}_{it} - c_t^f - g(\hat{n}_{it}, \hat{n}_{i,t-1}) \}, \quad (3)$$

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<sup>4</sup>There is no very clear agreement what "persistence" quantitatively means in this context. Dwyer (1996) illustrates some of the difficulties encountered in estimation.

<sup>5</sup>In general, I will use the indicator function  $\mathbb{1}_z$  under the convention that it equals one if the subscript  $z$  is true, and zero otherwise.

where  $w_t$  denotes the period- $t$  wage and the function  $g(\hat{n}_{it}, \hat{n}_{i,t-1})$  gives firing costs. It is given by

$$g(\hat{n}_{it}, \hat{n}_{i,t-1}) = c^n \hat{w}_t \cdot \begin{cases} \max(0, \hat{n}_{i,t-1} - \hat{n}_{it}) & \text{if } \mathbb{1}_x = 1, \\ \max(0, \hat{n}_{i,t-1} - \hat{n}_{it}) & \text{if } \mathbb{1}_x = 0 \wedge \hat{n}_{it} > 0, \\ 0 & \text{if } \mathbb{1}_x = 0 \wedge \hat{n}_{it} = 0. \end{cases} \quad (4)$$

The dependence of  $g(\cdot)$  on previous period's employment makes the employment choice a dynamic decision when there are firing costs, and implies that a firm's individual state variables are  $(s_{it}, n_{i,t-1})$ .

At the end of any period, firms can decide to exit. This is costless in the benchmark case and when exiting firms are exempt from firing costs ( $\mathbb{1}_x = 0$ ); otherwise ( $\mathbb{1}_x = 1$ ), the exiting firm has to cover the firing cost for reducing its workforce from  $\hat{n}_{i,t-1}$  to 0. As shown below, it will be optimal for firms to exit if their productivity is below a certain threshold, with  $\mathbb{1}_x = 0$ , this threshold will depend on past employment.

Entering firms have to pay a sunk entry cost  $c_t^e$  that grows at the same rate as output. This can be interpreted as an irreversible investment into setting up production facilities.<sup>6</sup> Then, they draw their initial productivity  $\hat{s}_{it}^0$  from a normal distribution with mean  $\hat{s}_t^0$  and variance  $\sigma_e^2$ . Call its *pdf*  $f_{s^0}$ . The following assumption is crucial.

**Assumption 2** *The mean  $\hat{s}_t^0$  of the distribution of entrants' productivities always is a constant  $\kappa$  below the mean productivity of all active firms,  $\hat{s}_t$ :*

$$E\hat{s}_t^0 = E(\hat{s} | \hat{s} \in \mu_t) - \kappa, \quad (5)$$

where  $\mu_t$  denotes the period- $t$  productivity distribution of all firms, incumbents and entrants, after realization of the productivity shocks.

Both distributions will shift upwards over time, but at a constant distance  $\kappa$ . (This relationship is illustrated in figure 6.) The stylized fact that the productivity of entrants lies slightly below that of incumbents (see e.g. Foster et al. 2001) suggests a slightly negative  $\kappa$ , so this will be assumed in the following discussion.

This construction can be interpreted in several ways. For instance, entrants' productivity will be related to the technological and institutional conditions in an economy; these are already

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<sup>6</sup>Empirical evidence shows that in practice, a large part of investment is irreversible in the sense that the resale value of assets is very low. This is more pronounced the more specific and the less tangible the asset, and the thinner the resale market, for evidence see e.g. Ramey and Shapiro (1998). The growth rate  $g$  can be rationalized in the following way. On the balanced growth path considered here, the expected productivity of entrants grows at a rate  $g$ . As a consequence, firm value also rises at a rate  $g$ , and to obtain a stationary distribution and constant firm turnover, the entry cost has to rise at the same rate.

captured in the productivity distribution of incumbents. Moreover, it can be thought that entrants try to imitate or outdo the best incumbents. Even if this is not always successful, it would cause their distribution of productivities to be correlated with that of incumbents. In this sense, entrants are engaged in a process of experimentation, but anchored to incumbents at the mean. Fundamentally, the dependence of entrants' productivity on that of incumbents constitutes an externality; incumbents' productivity spills over to entrants. This externality generates endogenous growth in this model; as the most productive entrants survive and the others exit again, as widely observed, aggregate productivity rises through the combination of entry, selection, and exit.

Through this construction, this model provides an attempt to explain theoretically findings about the entry and post-entry process in the U.S. compared to other countries: It is well-known (see e.g. Geroski 1995) that in the U.S., entrants start small and then expand significantly, if they survive. This goes along with a more spread-out productivity distribution, particularly of entrants. In continental Europe, in contrast, entrants are larger relative to incumbent firms, and do not grow so much subsequently (see e.g. Bartelsman et al. 2004). In addition, their productivity distribution has lower variance.<sup>7</sup>

In this context, more intense experimentation, parametrized by a higher  $\sigma_e^2$ , can increase growth because the distribution of entrants will be more spread out, and the probability of drawing extreme, including very high, productivities rises. On the other hand, the larger probability of bad draws means that the entry process will consume more resources.

Finally, the model does not allow for existing firms to invest into improving their productivity. It is plausible, however, that the model still captures some of these activities. By now, there is significant evidence that productivity increases within firms usually go hand in hand with reorganization that involves investment, firing, and hiring (for evidence, see e.g. Brynjolfsson and Hitt 2000), and thereby comes close to a plant closure and reopening. To the extent that exiting firms improve productivity by closing existing facilities and opening new ones with expected higher productivity, the model hence captures ongoing innovative activities.

**Timing:** The structure of the economy implies the following timing. At the end of any period, firms decide if they stay or exit, and potential entrants decide whether to enter. In the beginning of the next period, incumbent firms receive their productivity innovations and entrants draw their initial productivity. All firms pay the fixed operating cost  $c_t^f$ , entrants in addition pay the

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<sup>7</sup>The last fact is documented for Germany by Haltiwanger et al. (2003) who compare Germany and the U.S.; no data is available on other countries.



entry cost  $c_t^e$ . Firms demand labor, workers supply it, and the wage adjusts to clear the labor market. Production occurs, agents consume, and profits are realized. Firms that reduced labor or exited at the end of the previous period pay the firing cost. After this, the whole process resumes. Hence, the dynamic choices of entry, exit, and employment are all made based on firms' expectations of future productivity.

**Stationarize the economy:** The analysis will focus on the balanced growth path of this economy, where output, consumption, wages, and average productivity grow at a constant rate  $g$ , the firm productivity distribution shifts up the productivity scale at the same rate, the shape of the firm productivity distribution is invariant, and employment, the firm employment distribution, the interest rate, the share of entrepreneurs, turnover and other dynamic characteristics of the firm distribution are constant. The growth rate  $g$  is endogenous and will be obtained later. Then, all growing variables can be made stationary by adjusting them by their (cumulative) growth rates, i.e.

$$z_t = \hat{z}_t e^{-gt} = z \quad (6)$$

for any variable  $z$  growing at rate  $g$ ,  $x_t = \hat{x}_t = x$  for any variable  $x$  that does not grow, and

$$s_{it} = \hat{s}_{it} - gt \quad (7)$$

for productivity. This implies that in the stationarized economy, firm productivity evolves according to

$$s_{it} = s_{i,t-1} - g + \epsilon_{it}. \quad (8)$$

Intuitively, firm productivity now follows a random walk with downward drift (for positive growth rates) because the whole firm productivity distribution shifts up at rate  $g$ , so in expectation, firms fall back by  $g$  every period relative to the distribution. In short, growing variables wear a hat, variables of the stationarized economy don't. The sunk costs entry cost  $c^e$  and the fixed operating cost  $c^f$  are constants now. To save on time subscripts, drop the subscript  $t$  and denote next period's values by a superscript  $+$  and last period's values by a superscript  $-$ . In the remainder, the analysis will be in terms of this stationary equilibrium.

The assumption of a continuum of firms that are all independently affected by the same stochastic process, together with the absence of aggregate uncertainty, allows invoking the Glivenko Cantelli Theorem (see e.g. Billingsley 1986). This greatly simplifies the analysis because it implies that the underlying probability distribution can be used to describe the evolution

of the cross-sectional distribution. As a consequence, although the identity of firms with any  $(s, n^-)$  is not determined, their measure is deterministic.

## 4 Equilibrium

In this section, optimal behavior of all agents is derived. Based on this, equilibrium is defined and an algorithm for calculating it is given, and the growth rate is derived.

### 4.1 Optimal Behavior

In this economy, consumers maximize utility by choosing asset holdings and labor supply, and firms maximize the expected discounted flow of profits by choosing employment, entry, and exit.

**Consumers:** The consumer problem is completely standard. Utility maximization yields the Euler equation

$$1 + g = \beta(1 + r_t), \quad (9)$$

implying that the prevailing gross interest rate in the economy is  $1 + r = (1 + g)/\beta$ . Moreover, consumers supply labor in accordance with the first order condition  $c = w/\theta$ .

**Employment:** Active firms face a standard dynamic optimization problem. This is particularly simple in the case with no firing costs, since then labor demand is completely static, and a firm's productivity  $s \in S$  is its only state variable. Call labor demand for this case  $n_0(s, w)$ . With firing costs, last period's employment  $n^- \in N$  also becomes a state variable for the firm. Aggregate state variables are the wage  $w$  and the growth rate  $g$ , since the latter affects the productivity evolution of all firms. The Bellman equation for the more general problem is

$$V(s, n^-, w, g) = \max_{n, X} \left\{ \pi(s, n^-, w) - c^f + \frac{1}{1+r} \max(E[V(s^+, n, w, g)|s, n^-, g], -\mathbb{1}_X c^n wn) \right\}, \quad (10)$$

where the profit function includes adjustment cost of labor, and the inner max operator indicates the option to exit. The choice variable  $X$  captures a firm's exit decision, say  $X = 1$  for exiting firms and  $X = 0$  for continuing ones.

This is a standard problem, existence and uniqueness of the value function follow from standard arguments. Two properties carry over from the profit function: The value function is increasing and convex in  $s$  given an  $n^-$ , and weakly decreasing in  $n^-$  given  $s$  if there are firing costs. Whereas the employment policy  $n(s, n^-, w, g)$  increases monotonically in  $s$  in the frictionless economy, it features a constant part around  $n^-$  when  $c^n > 0$ . Intuitively, when a firm's

productivity changes a little, it will not immediately adjust employment because productivity will change again, and reducing employment then would command an additional firing cost. When firms are exempted from paying the firing cost upon exit, firms that suffer such a large, negative productivity shock that they are forced to exit, will not adjust employment downward immediately, but instead do so costlessly upon exit. So given an  $n^-$ , the employment policy is constant for  $s$  very low or around  $n^-$ , and strictly increasing elsewhere. The employment policy function and the law of motion for  $s$  jointly define a transition function  $Q : S \times N \rightarrow S \times N$  that moves firms over productivity and employment states, and there is an associated function  $q : (S \times N) \times (S \times N) \rightarrow [0, 1]$  that gives the probability of going from state  $(s, n^-)$  to state  $(s^+, n)$ .<sup>8</sup> Clearly, the value of incumbent firms is decreasing in the wage  $w$  and in the aggregate growth rate  $g$ . The same holds for the employment policy, except that it is independent of  $g$  in the frictionless case.

Figures 1 to 4 illustrate the employment policies. Figures 1 and 3 have productivity  $s$  on the  $x$ -axis, past employment  $n^-$  on the  $y$ -axis, and employment, i.e.  $n(s, n^-, w, g)$ , on the  $z$ -axis. Figures 2 and 4 are slices of figures 1 and 3. They have productivity on the  $x$ -axis and employment on the  $y$ -axis, and each line stands for a level of  $n^-$ , with higher lines (in the  $y$ -direction) belonging to higher levels of  $n^-$ . The line with crosses is the employment policy in the economy without firing costs. It is clear that employment of continuing firms rises in  $s$  given  $n^-$ . Moreover, for each  $n^-$ , there is a neighborhood of  $n^-$  where employment is not changed, and is different from the firing-cost-free economy, for a neighborhood of  $n_0^{-1}(s, w)$ . Finally, when exiting firms are exempt from firing costs, they continue to employ  $n^-$ , since it is more profitable to lay off workers for free upon exit. This explains the high values of  $n$  for low  $s$  in figures 3 and 4; as  $s$  rises above the exit threshold  $s_x$ , firms expect to continue, and do lay off workers.

**Exit:** Firms will exit if the expected value of continuing conditional on current states is less than or equal to that of exiting. The latter is constant in  $s$ . Since the value function is strictly increasing in  $s$  for any  $n^-$ , there is a unique threshold

$$s_x(n^-, w, g) = \{s | E[V(s^+, n, w, g) | s, n^-, g] = -\mathbb{1}_x c^n w n\} \quad (11)$$

below which firms will exit. Taking into account the exit decision leads to a modification of the transition function to become  $Q_x : \bar{S} \times N \rightarrow (\bar{S} \cap \underline{S}) \times N$ , where now the support of the productivity state  $s$  is partitioned into  $\bar{S} = \{s | s \geq s_x(n^-, w, g)\}$  (continue) and  $\underline{S} =$

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<sup>8</sup>Although both  $Q$  and  $q$  also depend on  $w$  and  $g$ , these arguments are omitted here and in the following.

$\{s | s < s_x(n^-, w, g)\}$  (exit). The latter is an absorbing state. Note that the partition may differ across different elements of  $N$ . The probability of going from  $(s, n^-) \in \bar{S} \times N$  to  $(s^+, n) \in (\bar{S}(N) \cap \underline{S}(N)) \times N$  then is given by a function  $q_x(\cdot)$ .

As the value function is weakly decreasing in  $n^-$ , this also holds for  $s_x(n^-, w, g)$ . It is clear that with no firing costs upon exit, the value of exit is higher, and the exit threshold will therefore also be higher. This means that  $s_x(\cdot)$  will strictly decrease in  $n^-$  wherever  $V(\cdot)$  does so, and be constant in  $n^-$  where  $V(\cdot)$  is constant. With  $\mathbb{1}_x$  the picture is less clear-cut because the exit value also decreases in  $n^-$ ; in the benchmark economy it turns out that  $s_x(\cdot)$  is constant in  $n^-$  for that case. In this sense, exempting exiting firms from firing costs provides incentives to exit; and these will particularly benefit low-productivity firms. By improving the distribution of surviving firms, this can boost growth, as shown below.

**Entry:** Potential entrants will enter if the expected net value of doing so is non-negative. So in equilibrium, the free entry condition

$$E[V^e(s^0, w, g)] = c^e \quad (12)$$

holds. Since the distribution of  $s^0$  and  $c^e$  are exogenous features of technology, this equation will pin down the wage, given a growth rate. If the wage was below (above) its equilibrium value, there would be additional (less) entry, driving up (down) the wage.

All firms' decisions combined and the process for idiosyncratic shocks yield the law of motion for the firm productivity distribution. Defining  $\mu(s, n^-)$  as the measure of firms that has productivity  $s$  now and had employment  $n^-$  last period, its law of motion is given by

$$\tilde{\mu}(s^+, n) = \int_N \int_{\bar{S}(N)} (1 - \delta) \tilde{\mu}(s, n^-) q_x(s^+, n | s, n^-) ds dn^- + \eta(s^+) \mathbb{1}_{n=n(s^+, 0, w, g)}. \quad (13)$$

The integral describes the motion of incumbents. Exit is captured by the transition function  $q_x(\cdot)$  and by the restriction of the domain of the integral to  $N \times \bar{S}(N)$ . Entry is given by  $\eta(\cdot)$ . It only contributes to  $\tilde{\mu}(s^+, n)$  for  $n$  that are optimal for entrants, given  $s^+$ ,  $w$  and  $g$ , as indicated by the indicator function. For later use, also denote the integral by  $\tilde{\mu}'$ . For computational purposes, it will be practical to decompose the firm distribution  $\tilde{\mu}(\cdot)$  into the product of a probability distribution  $\mu(\cdot)$  (with total measure 1) and the number of firms  $M$ . Discretizing the state space and denoting the identity matrix by  $I$ , the ergodic firm distribution can then easily be obtained as  $\tilde{\mu} = (I - Q'_x)^{-1} \eta$  in the case without firing costs.

Next, I define the equilibrium of the economy, describe the growth rate, and outline an algorithm for calculating equilibrium.

## 4.2 Equilibrium Definition

Define a *stationary competitive equilibrium* of the stationarized economy as real numbers  $w$ ,  $g$  and  $M$ , functions  $n(s, n^-, w, g)$ ,  $V(s, n^-, w, g)$ , and  $s_x(n^-, w, g)$ , and probability distributions  $\mu(s, n^-)$  such that:

- (i) Consumers choose consumption, asset holdings, and labor supply optimally, so the interest rate is given by equation (9);
- (ii) all active firms choose employment optimally according to the employment policy  $n(\cdot)$ , yielding value  $V(\cdot)$  as described by equation (10) for all  $(s, n^-, w, g)$ ;
- (iii) exit is optimal:  $s_x(\cdot)$  is given by equation (11) and firms exit if they draw an  $s < s_x(\cdot)$ , given  $n^-$ ,  $w$  and  $g$ ;
- (iv) entry is optimal and free: given a distribution  $f_{s^0}$  over entrants' productivities  $s^0$ , an entry cost  $c^e$ ,  $w$  and  $g$ , firms enter until the net value of entry equals its cost (equation (12));
- (v) the labor market clears: given  $w$  and  $g$ , aggregate labor demand  $M \int_N \int_S \mu(s, n^-) n(s, n^-, w, g) ds dn^-$  equals supply as chosen by households;
- (vi) the firm distribution evolves according to the law of motion given by equation (13); and
- (vii) the firm distribution is stationary:  $\mu(s, n^-) = \mu^+(s, n^-)$  for all  $(s, n^-)$ .

Existence of equilibrium for similar economies is proven e.g. by Hopenhayn (1992); the proof here would proceed along very similar lines.

## 4.3 The growth rate

The final missing object is the growth rate  $g$ . Not surprisingly, this will fundamentally be driven by the distance  $\kappa$  between entrants' and incumbents' mean productivity  $s$ . (This is illustrated in figure 6.) Intuitively, what happens is the following: In the growing economy, the productivity of incumbents follows a random walk, so it is constant in expectation. Those firms that draw sufficiently negative shocks will have to exit. Entrants, on average, are less productive than incumbents (for negative  $\kappa$ ). However, for  $\kappa$  small enough in absolute value, there will always be some lucky entrants that draw a high initial productivity  $s^0$ , while some unlucky ones draw a low  $s^0$ . The latter will exit quickly, while the former will raise average productivity of the economy. This could be interpreted as some entrants having discovered a better way of producing

their product, or a better product in a heterogeneous-product setting. With higher average productivity of the economy, next period's entrants will draw from a better distribution, and the economy's productivity distribution will keep shifting upwards, as illustrated in Figure ???. The presence of some more productive firms will raise the wage and thereby the exit threshold  $s_x$  so that the distribution, while shifting, will conserve its shape.

The structure of the model does not allow solving for  $g$  as a function of the other parameters explicitly, but some more formal discussion is still possible. To simplify the discussion, it is conducted for the frictionless case. The reasoning carries over to the case with firing costs, at the cost of significantly more complicated notation. Consider the growing economy on its balanced growth path. In a slight abuse of notation, denote the expectation of a random variable by the expectation of its distribution, i.e. write  $E\mu$  instead of  $E(s), s \in \mu$ . The growth rate is then defined as

$$g \equiv E\hat{\mu}^+ - E\hat{\mu}. \quad (14)$$

With some tedious but simple algebra (see Appendix), this can be transformed into

$$g = (1 - \delta)E\hat{\mu}' - E\hat{\mu} - \frac{e}{1 - e}\kappa, \quad (15)$$

where  $e$  is the entry (= exit) rate. It is clear, as suggested above, that the growth rate is decreasing in  $\kappa$  and in the breakdown probability  $\delta$ . The first two terms give the difference between mean productivity of surviving firms and incumbents present before, and are thus a selection effect. The tougher market selection is, and the more firms at the low end of the productivity distribution exit, the more aggregate productivity grows. What is the role of entry here? Note that  $E\mu'$  is the expectation of a truncated distribution. Given a truncation point, this rises in the variance of the distribution. Hence, a more dispersed distribution of entrants in the previous period will increase  $E\mu'$ . In the entry period itself, entry has a negative effect through  $\kappa$ . So the positive effect of entry arises not immediately upon entry, but only in combination with market selection in subsequent periods. For related reasons, the entry rate enters in two ways. By increasing current entry, it lowers the growth rate. But on the balanced growth path considered here, more entry also implies more exit, driving up the exit threshold and thereby average productivity of surviving firms, so its net effect is ambiguous. Its contribution is more positive the smaller  $\kappa$  in absolute value. <sup>9</sup>

After having described the growth rate, it is appropriate to reconsider the role of the assumptions made. Assumption 1 is clearly necessary for obtaining endogenous growth by selection.

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<sup>9</sup>Note that equation (15) cannot be used directly for calculating the equilibrium since  $Q_x$ , and thereby  $\mu'$ , depends on  $g$ . This is why finding  $g$  is a fixed-point problem.

Under the alternative, an autoregressive process (or draws from a constant distribution), an ergodic productivity distribution arises, and the growth rate is zero. Assumption 2 is equally necessary. The alternative would be a distribution of entrants' productivity that is independent of that of incumbents. Now suppose that the productivity distribution was shifting to the right at a constant rate. Then the exit threshold  $s_x$  would shift at the same rate. This means, however, that within finite time the exit threshold would lie above a large fraction of the mass of entrants' productivity distribution. As a result, almost all entrants would immediately re-exit, or not enter in the first place. Yet, exit would still occur, and the industry thin out and vanish in finite time with probability one. Hence, there is no such balanced growth path. So, the realistic assumption that entrants' productivity is related to that of incumbents ensures that the distribution of entrants keeps up with those incumbents that have had good realizations of the shock. Moreover, firms with low productivity are eliminated more quickly as the distribution shifts to the right. To summarize, both assumptions 1 and 2 are necessary for a balanced growth path with positive growth.

#### 4.4 Algorithm for finding equilibrium

As just seen, the equilibrium  $g$  is determined by  $\kappa$ , essentially a parameter of technology, other endogenous variables, and other parameters that shape the firm distributions. In the implementation, I face the problem that  $\kappa$  is unknown. Evidence on the relative productivity of entrants cannot be used directly as a proxy because it measures post-entry relative productivity, not the underlying parameter of technology relating potential but unrealized to active projects. Hence, in calculating equilibrium, a  $\kappa$  has to be found that is consistent with findings on the relative productivity of entrants, after selection through the entry process.

In the numerical implementation, the state space  $S \times N$  is discretized into a grid of  $200 \times 200$  points. Using more points does not significantly affect results. The  $N$  grid is chosen such that it is constituted by the optimal employment quantities chosen by a firm in the frictionless economy for the points in  $S$ . Then, firm value can be obtained for each  $(s, n^-)$  pair given  $g$  and  $w$ . This yields the exit thresholds  $s_x(n^-, w, g)$  as defined in equation (11), and the transition function  $Q_x$ , given  $g$  and  $w$ . For any fixed  $g$ , equation (12) yields the equilibrium wage  $w$ , and thereby the equilibrium exit threshold and transition function. Using these, the ergodic firm productivity distribution can be obtained; in the frictionless case directly as  $\tilde{\mu} = (I - Q'_x)^{-1}\eta$  and in the case with firing cost by iteration on the law of motion for  $\mu$  (equation (13)). When the relative productivity of entrants fits its empirical value, the right  $g$  was chosen, otherwise, the whole

process is repeated for different  $g$ . Again, the process converges in a few iterations.<sup>10</sup>

Knowing  $g$ , the underlying parameter  $\kappa$  can be calculated from its definition in equation (5). The result is interesting in itself, since it gives some insight into the entry technology, and particularly the strength of spillovers from active to new firms and the severity of the selection process. Interpretation of this number is not easy, but a back-of-the-envelope calculation can help: If entrants and incumbents are equally productive (which is close to empirical, post-entry estimates),  $\kappa$  should be close to zero; if entrants are slightly less productive, it should be slightly negative, e.g. on the order of  $-0.1$  for a relative productivity of  $0.9$ . Yet it will be seen below that in the benchmark economy,  $\kappa$  is larger in absolute value, indicating a severe selection process. So many inefficient projects are weeded out before entry or in a very early stage that despite a low  $\kappa$ , entrants are almost as productive as incumbents after 5 years of market selection.

## 5 Benchmark Economy

Before analyzing the effects of firing costs, the model has to be calibrated to infer  $\kappa$ . I calibrate to the U.S. non-agricultural business sector as a no-firing cost benchmark. According to the World Bank's Doing Business database, firing costs are zero in the U.S., and the "Difficulty of Firing" index is among the lowest worldwide. This is similar for other measures of employment protection. Calibrating models of firm dynamics has become easier in recent years due to increased availability of empirical evidence on firm dynamics.<sup>11</sup> As usual, I calibrate the model by matching some baseline parameters to values that are commonly used in the literature, and choosing the remaining ones such that the distance between a set of model moments and equivalent data moments is minimized, where distance is the mean squared relative deviation. The fact of dealing with distributions leads to some practical difficulties. First, to obtain model moments, the whole model has to be solved repeatedly for all parameter combinations under consideration. Second, the distance between between model and data moments is a highly nonlinear function of the parameters with many local minima. To find the global minimum, a genetic algorithm as laid out in Dorsey and Mayer (1995) is used. The calibration reported here is far from perfect due to time constraints. Qualitatively, results should still carry over, though.

The parameter values adopted from the literature are  $0.64$  for the labor share  $\alpha$  and  $0.95$  for the discount factor  $\beta$ . The disutility of labor  $\theta$  is set such that the employment rate fits

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<sup>10</sup>Empirical measures of entrants' relative productivity define as entrants firms that enter within the last 5 years; this measure is also applied here.

<sup>11</sup>In principle, structural estimation of the model using firm panel data would be an alternative.



the value of 66% reported by the BLS. The five parameters that remain to be assigned are the variance of the log productivity distribution of entrants ( $\sigma_e^2$ ), the variance of the the idiosyncratic productivity shock hitting incumbents ( $\sigma^2$ ); the fixed operating cost ( $c^f$ ), the entry cost ( $c^e$ ); and the breakdown probability  $\delta$ . In addition, there is the value of  $\kappa$  to be inferred.

The data equivalent that allows matching the latter is the relative productivity of entrants. Foster et al. (2001) report this to be about 90% of that of all active firms, counting as entrants firms that entered within the last five years and are still active. This statistic allows inferring  $\kappa$  in the way described above.

The remaining parameters are chosen to match three moments referring to the entire economy, and two moments related to entry and post-entry behavior: the rate of plant turnover, average plant size, dispersion of the productivity distribution, the four-year survival rate of entrants, and their seven-year growth rate. These moments are chosen because each captures a different aspect of the firm distribution and its dynamics and therefore allow a relatively full description. Average plant size and the dispersion of the productivity distribution are closely related to the mean and variance of the firm productivity distribution. The rate of plant turnover describes its dynamic behavior. The survival rate of entrants indicates the severity of the selection process, and their growth rate the contribution they can make to aggregate productivity.

The main data source in the following discussion will be Bartelsman et al. (2004) (BHS). That paper and others of a partly overlapping set of authors use national data on firm dynamics coordinated in the OECD Firm-Level Project as to make the reported measures comparable; an important issue since national statistical agencies use different data collection procedures and cutoffs, and calculate different statistics. Data reported by BHS are closest to allowing cross-country comparisons.

The rate of plant turnover is the sum of entering and exiting plants in a year over the total number of active plants in that year; it is a crucial dynamic feature of the plant distribution. BHS report it to be 22% yearly in the U.S.. Although cross-country differences in this measure are small according to BHS, the U.S. value is clearly at the high end of the cross-country distribution, consistent with the findings of earlier studies such as Cable and Schwalbach (1991). Average plant size (employment) provides a measure of the mean of the firm distribution. BHS report it to be 26.4 for the whole U.S. economy. The dispersion of the productivity distribution helps pin down the variance of the incumbents' productivity shock. For lack of more data, I use the measure from Dwyer (1998) of a ratio of 3 between the 9<sup>th</sup> and the 2<sup>nd</sup> deciles of the plant productivity distribution. His measure is for the U.S. textile industry, but other studies

Table 1: Calibration: Model statistics, Targets (U.S.), values for Germany, all data for 1990s

Statistic	model	U.S.	GER	source
Plant turnover rate	14.2%	22%	13%	BHS
Average plant size	25.6	26.4	17	BHS
Productivity dispersion	3.17	3		Dwyer (1996)
Four-year survival rate of entrants	70%	63%	68%	BHS
Seven-year growth rate of entrants	54%	40%	25.4%	BHS
Relative productivity of entrants	90%	90%		FHK
Labor force participation	66%	66%		BLS
Sources:				
not used in calibration:				
Output per capita growth	2.5%	1.7%	VMMTY	
TFP growth	1.6%	0.91%	1.22%	VMMTY
TFP growth (1995-2000)		1.21%	0.91%	VMMTY
Employment-weighted firm turnover	6.3%	6.97%	3.89%	BHS
Job turnover	28.5%	28%		BLS

BHS: Bartelsman et al. (2004), BLS: Bureau of Labor Statistics (<http://data.bls.gov>), FHK: Foster et al. (2001), VMMTY: van Ark, Melka, Mulder, Timmer and Ypma (2003), Tables 1, 20.

report results in the same ballpark for other countries and industries (see e.g. Roberts and Tybout 1996).

Next, two crucial statistics describing the post-entry process, and hence entrants' contribution to aggregate productivity, are entrants' survival and growth rates. The four-year survival rate, i.e. the proportion of entrants of a given year still active four years later, is 63% in the U.S. (BHS). This is lower than in most other industrialized countries, but higher than in many Latin American ones, though quantitatively, cross-country differences are not very large. They are more considerable for the post-entry growth rate. In the U.S., surviving entrants are 140% their original size (employment) 7 years after entry (BHS). This is more than in most other industrialized countries, but less than in many Latin American ones. Calibration targets, their values for Germany, and model values are given in Table 1. Adopted parameter values are given in Table 2.

The correct measurement of TFP growth raises a measurement issue. The growth rate  $g$  above refers to output, and to the shift of the productivity distribution. In measuring aggregate TFP, however, inputs also have to be taken into account. Inputs here are labor, which is constant, and the fixed factor. Just calculating aggregate TFP by aggregating over  $\mu(s)$  would neglect the role of the fixed factor. The fixed factor is modelled as being acquired at a sunk cost  $c_t^e$  that is paid in terms of the numéraire good and rises at a rate  $g$ . If the entry cost is thought of as an

Table 2: Calibration: Parameter values

Parameter	Value	Description
$\alpha$	64%	Labor share
$\beta$	0.95	Discount factor
$\theta$	1.17	Disutility of working
$\sigma_e^2$	0.51	Variance of log productivity distribution of entrants
$\sigma^2$	0.16	Variance of idiosyncratic productivity shock
$c^f$	2.2%	Fixed operating cost, % of avg firm output
$c^e$	152%	Cost of entry, % of avg firm output
$\delta$	3.5%	Probability of exogenous exit

entry investment, the sum of entry costs incurred by all firms active in any period  $t$  corresponds to capital. This also fits the national accounts measurement of capital. There, the productive capital stock is measured as the sum of the value of assets in the economy, using the “as new” value, i.e. the discounted value of income streams expected from the asset when it is acquired (for details, see OECD 2001). By the free entry condition (equation (12)), this corresponds exactly to the sum of entry investments,  $M c^e$ . Income that accrues to a fixed factor is measured residually in productivity measurement (Berndt and Fuss 1986), so the share attributable to the fixed factor is  $1 - \alpha$ . The growth rate of TFP then is  $g_{TFP} = g_Y - (1 - \alpha)g_{c^e} - \alpha g_n = \alpha g$ . The number implied by the calibration, 1.6% annually, is a bit on the high side compared to around 1% reported by van Ark, Inklaar and McGuckin (2002) for the U.S. in the 1980s and 90s, while the number of 1.39% reported for the years 1995-2000 comes closer to the model number. Clearly, the imperfect calibration is partly responsible for this bad fit. It might, however, also be that there are unmodeled obstacles to the selection mechanism holding back growth in practice. Finally, the model is pretty abstract, so no too exact fit can be expected.

For job flows, however, the calibration works pretty well; both the numbers for job turnover ((job creation + job destruction)/employment) and the job-weighted firm turnover rate are close to their empirical counterparts.

## 6 Firing costs and productivity growth

Because the growth rate is endogenous, frictions can affect not only the level, but also the growth rate of output. While some frictions, such as the entry cost, work at the entry margin, firing costs affect the exit margin. Firing costs introduce a cost of exiting (when  $\mathbb{1}_x = 1$ ). Given  $g$  and  $w$ , this implies a lower exit threshold  $s_x$ , meaning that more inefficient firms stay in the market.

This reduces  $E\mu'$  in equation (15), reducing the growth rate until in equilibrium, equation (15) holds again. A lower growth rate raises the value of entry, implying a higher equilibrium wage. The effect on the entry rate is ambiguous, since a higher value of entry might increase entry, but a lower growth rate also decreases exit, slowing down the flow of entry. Because of the cost of firing, firms take on less workers, and average firm employment and output decline. The production structure on the whole is less efficient since firms are smaller, but still have to cover the same fixed operating cost. The net effect on employment depends on the relative size of reductions in firm size (decreasing labor demand) versus the rise in the wage (increasing supply). For a welfare evaluation, both static and dynamic effects have to be taken into account. In the quantitative exercise, I calculate the equivalent variation of introducing firing costs. This expresses the welfare effect of introducing firing costs in terms of a reduction of consumption in the no-firing-cost economy that would have an equivalent welfare effect. It will turn out that although consumption actually rises, this is more than compensated by the fall in the growth rate, and consumers are worse off with firing costs.

When exiting firms are exempted from firing costs, firms can again employ much more freely. Should they face hard times, they could exit instead of firing employees, thereby avoiding the firing cost. Moreover, firing costs here don't affect the exit cost by construction. In effect, they have the opposite effect: charging firing costs to surviving firms but exempting exiting ones encourages low-productivity firms to exit, particularly if they have high past employment. This raises  $E\mu'$  in equation (15), and thereby the growth rate. Both the higher growth rate and the exit inducement raise the economy's firm turnover rate. The shorter expected lifetime of a firm decrease the wage, leading to lower employment, lower output, and lower consumption. Again, for welfare evaluation, both this and the higher growth rate have to be accounted for; it turns out that consumers are worse off even in this case, despite the higher growth rate, though less so than without the exit exemption.

With calibration results in hand, the effect of introducing firing costs can also be explored quantitatively. I conduct the exercise of introducing firing costs of  $c^n$  times the equilibrium wage for each worker fired.  $c^n$  is set to one, i.e. a year's wages. In practice, numbers differ across countries, workers, and episodes. However, the average over continental European countries is close to a year's wages according to the World Bank's Doing Business Database.

Results are reported in Table 3 and fit the qualitative patterns described above. The first column reports the results for the case where firing costs are always charged ( $\mathbb{1}_x = 1$ ), the second one for the case where exiting firms are exempt ( $\mathbb{1}_x = 0$ ). Numbers are relative to

Table 3: Results: Introducing firing costs (always:  $\mathbb{1}_x = 1$ , exit exemption:  $\mathbb{1}_x = 0$ , benchmark economy = 100)

	$\mathbb{1}_x = 1$	$\mathbb{1}_x = 0$
Growth rate of output	-0.7	0.8
Growth rate of TFP	-0.5	0.5
Wage	104.3	82.7
Output	118.2	84.6
Consumption	104.3	82.7
Employment rate	91.1	84.9
Number of firms	206.5	80.0
Welfare loss	8.2	4.9
Average firm employment	41.4	106.5
Average firm output	50.5	103.4
Relative productivity entrants	3.6	0.5
Productivity spread	0.0	-0.3
Productivity spread of entrants	0.0	0.0
Firm turnover rate	0.4	3.8
Exit rate of entrants	0.0	0.0
4-year survival rate of entrants	-4.3	-7.2
7-year growth rate of entrants	57.1	75.3

the benchmark economy, which is normalized to 100, except for growth rates and ratios, where differences are given. Numbers for wages, output, etc. are for the stationarized economy, so they cannot be compared directly. To properly address welfare evaluation, the equivalent variation is given. The number indicates what percentage of consumption would need to be taken away from consumers in the no-firing-cost case to make their welfare equivalent to that of consumers in each of the other two economies. The table lists first variables that refer to the aggregate economy, then to the firm distribution, and finally to dynamic behavior of firms and post-entry behavior.

The most salient result are the changes in growth rates. Introducing firing costs decreases the growth rate by 0.7 points when firing costs are always charged, but increases it by 0.8 points when exiting firms are exempt. The mechanism is as shown above: When firing costs are always charged, they increase the exit cost. That this keeps less efficient firms in the market can be deduced from the relative productivity of entrants: it is 3.6 points higher with firing costs. As the underlying technology is the same in both economies, this implies that in the firing-cost economy, the productivity distribution of entrants is worse, leading to a reduction in the growth rate. From a firm's point of view, this makes entry more profitable, since the firm expects to

Table 4: Results: Model comparison – reaction to introduction of firing costs

Model	growth rate	output	consumption	employment	welfare
$\mathbb{1}_x = 1$	-0.7				-8.2%
$\mathbb{1}_x = 0$	0.8				-4.9%
Hopenhayn and Rogerson (1993)	–	-4.6%	-4.6%	-2.5%	-2.4%
Veracierto (2001)	–	-7.9%	-6.0%	-7.9%	-2.9%
Samaniego (2005)	–	-20.3%	-10.0%	-11.2%	-2.3%

live longer; hence the wage is higher. It is also clear that firms are smaller on average, and there are more firms; firing costs discourage hiring.

When exiting firms are exempt from firing, on the other hand, the growth rate increases. The relative productivity of entrants increases slightly, here, too. However, this goes along with a strong increase in turnover. As a result, the selection effect is much stronger, as also indicated by the increase in surviving entrants' seven-year growth rate. Moreover, firms actually are larger on average. In this sense, in this configuration firing costs work the way they were possibly politically intended: discouraging firing, but not hiring.

Nevertheless, welfare decreases in both cases; more so without the exemption. Moreover, because in the present context, firing costs have an impact on growth and thereby distort the economy both statically and dynamically, welfare effects are stronger than in other analyses of firing costs. Table 4 compares results obtained here to those obtained elsewhere. Hopenhayn and Rogerson (1993) is the seminal model of this literature and focussed attention on the static distortion induced by firing costs, due to the misallocation of labor. Veracierto (2001) extends this analysis by introducing flexible capital, and finds that it does not affect results significantly in the long run, but has some short-run effects. Samaniego (2005) is the first to distinguish between the application of firing costs to continuing and exiting establishments. In all cases, welfare losses are smaller since they are purely static; hence this paper contributes to the literature by introducing dynamic losses due to firing costs.

One additional, but nontrivial, step in the analysis would be desirable: the model developed here completely abstracts from benefits of firing costs. Alvarez and Veracierto (2001) find that severance payments, which affect firms in a way similar to firing costs, are welfare-improving in a world with search frictions because agents become unemployed less often. This more than compensates the static distortions. However, this result relies on an analysis where firing costs do not affect growth. Combining results obtained here with those by Alvarez and Veracierto

allows the conjecture that in a model with growth and search frictions, firing costs would very probably be harmful when always charged, since they would decrease growth on top of the static distortions, but might still be welfare-improving if not charged upon exit.

## **7 Conclusion and directions for further research**

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## Appendix

### A Derivation of growth rate

$$\begin{aligned}
g &= E\hat{\mu}^+ - E\hat{\mu} \\
&= \int_S \hat{\mu}^+ s \, ds - \int_S \hat{\mu} s \, ds \\
&= \int_S [(1 - \delta)\hat{\mu}' + e\eta] s \, ds - \int_S \hat{\mu} s \, ds \\
&= \int_S [(1 - \delta)\hat{\mu}' - \hat{\mu}] s \, ds - e \int_S \eta s \, ds \\
&= \int_S [(1 - \delta)\hat{\mu}' - \hat{\mu}] s \, ds - eE\eta \\
&= \int_S [(1 - \delta)\hat{\mu}' - \hat{\mu}] s \, ds - e \left( \int_S \hat{\mu}^+ s \, ds - \kappa \right) \\
&= \int_S [(1 - \delta)\hat{\mu}' - \hat{\mu}] s \, ds - e \left( \int_S \hat{\mu} s \, ds + g - \kappa \right) \\
(1 - e)g &= \int_S [(1 - \delta)\hat{\mu}' - (1 - e)\hat{\mu}] s \, ds - e\kappa \\
g &= \int_S \left[ \frac{1 - \delta}{1 - e} \hat{\mu}' - \hat{\mu} \right] s \, ds - \frac{e}{1 - e} \kappa \\
&= (1 - \delta)E\hat{\mu}' - E\hat{\mu} - \frac{e}{1 - e} \kappa
\end{aligned}$$

The first equality is the definition of  $g$ , the third follows from the law of motion for  $\mu$  (equation 13), the sixth from the definition of the distribution of entrants, and the seventh again from the definition of  $g$ . Note that  $\mu'$  is not a probability density because it has integral  $< 1$ , this is corrected for by dividing by  $1 - e$ . For computational purposes, it is useful to make the last step

$$g = E[(1 - \delta)(Q_x - I)\mu] - \frac{e}{1 - e} \kappa.$$

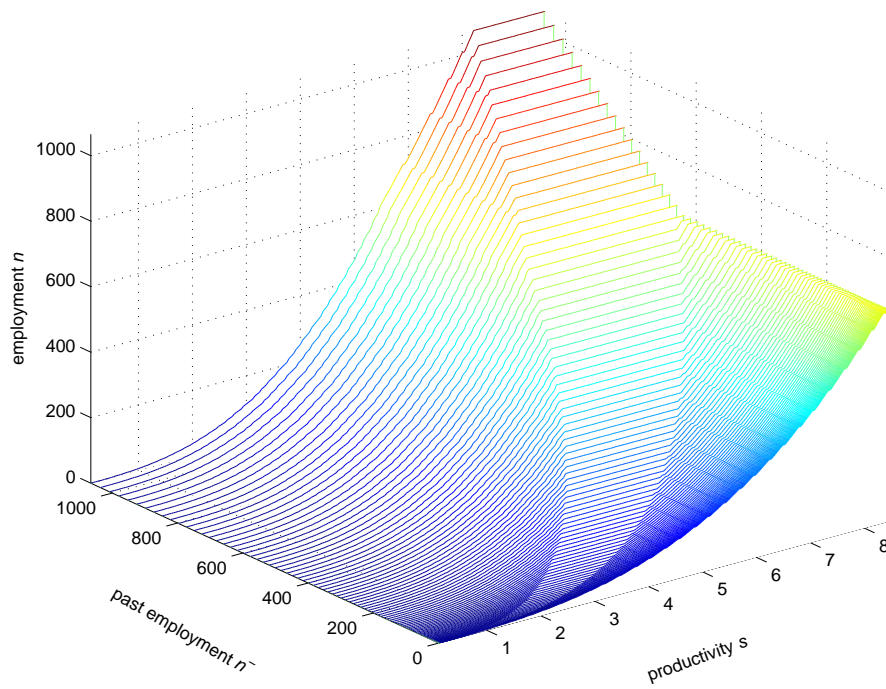


Figure 1: The employment policy function when firing costs are always charged ( $\mathbb{1}_x = 1$ )

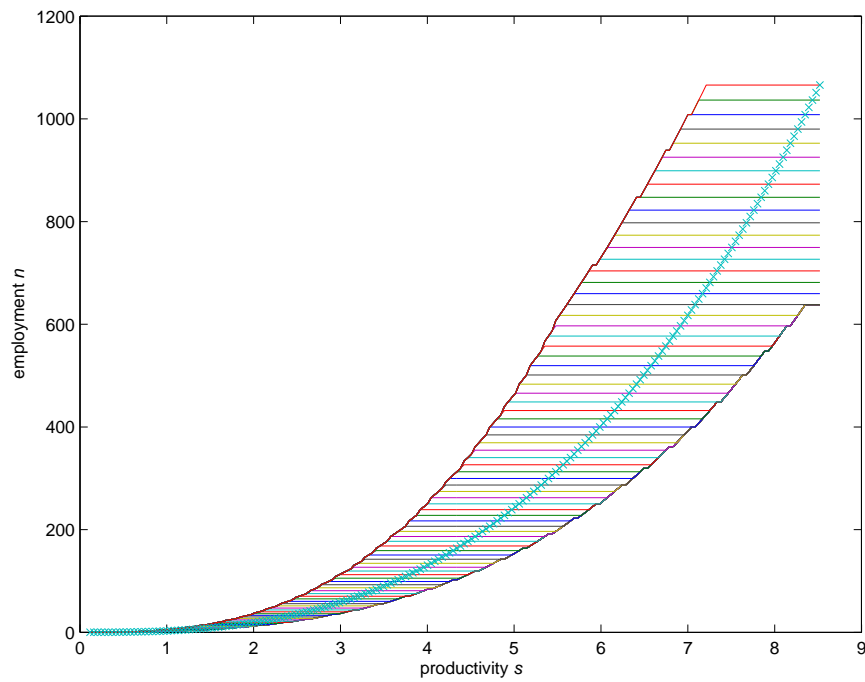


Figure 2: The employment policy function when firing costs are always charged (lines), and without firing costs (crosses)

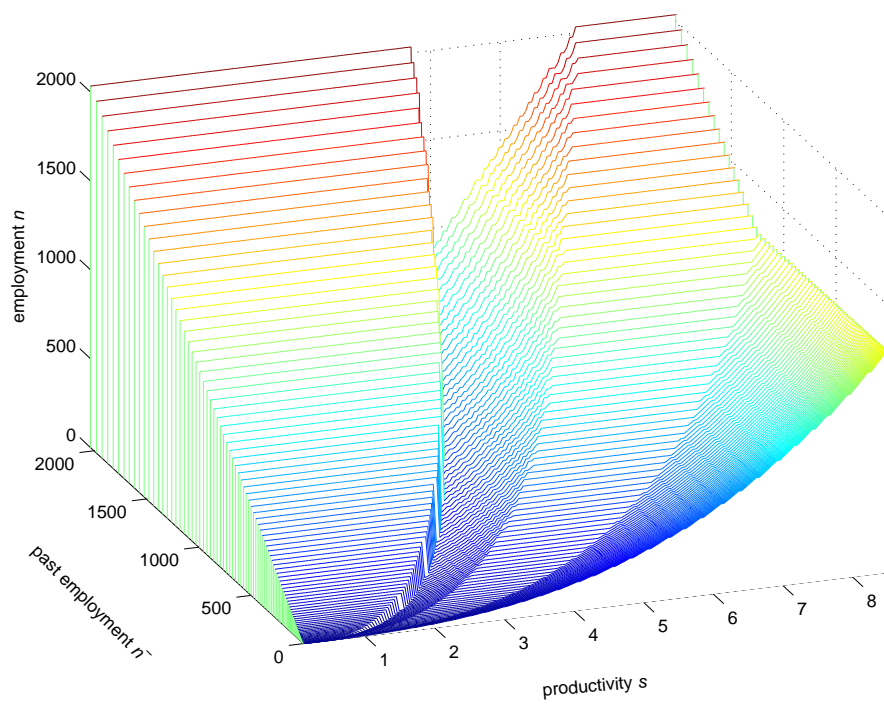


Figure 3: The employment policy function with firing cost exemption upon exit ( $\mathbb{1}_x = 0$ )

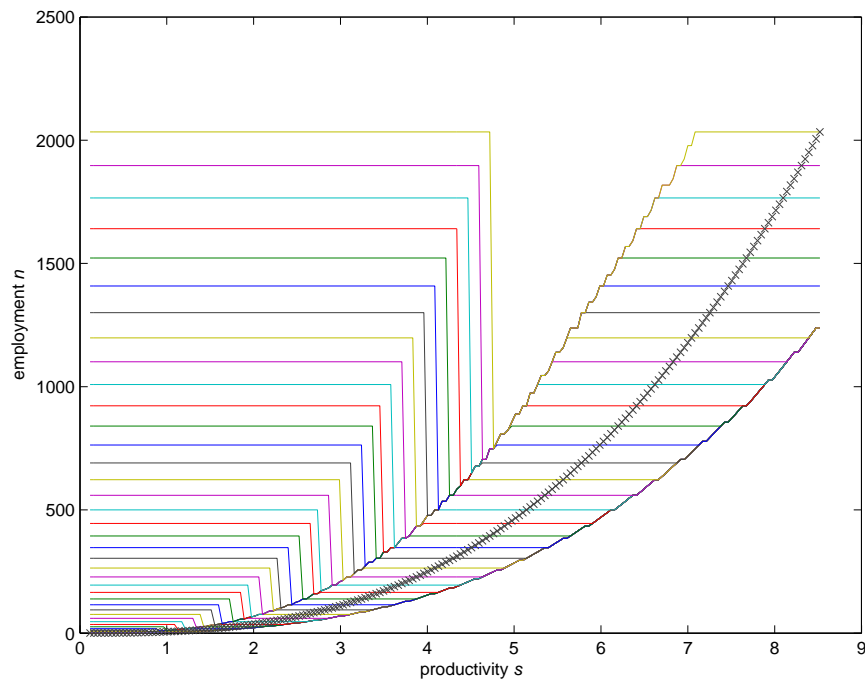


Figure 4: The employment policy function with firing cost exemption upon exit (lines), and without firing costs (crosses)

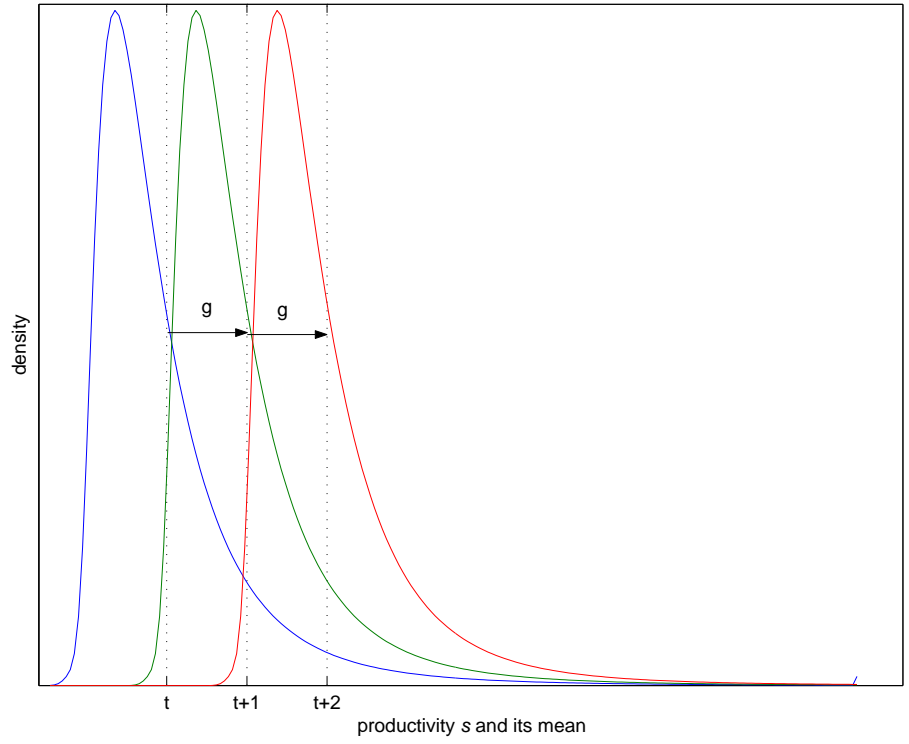


Figure 5: Growth through right-shifts of the firm productivity distribution

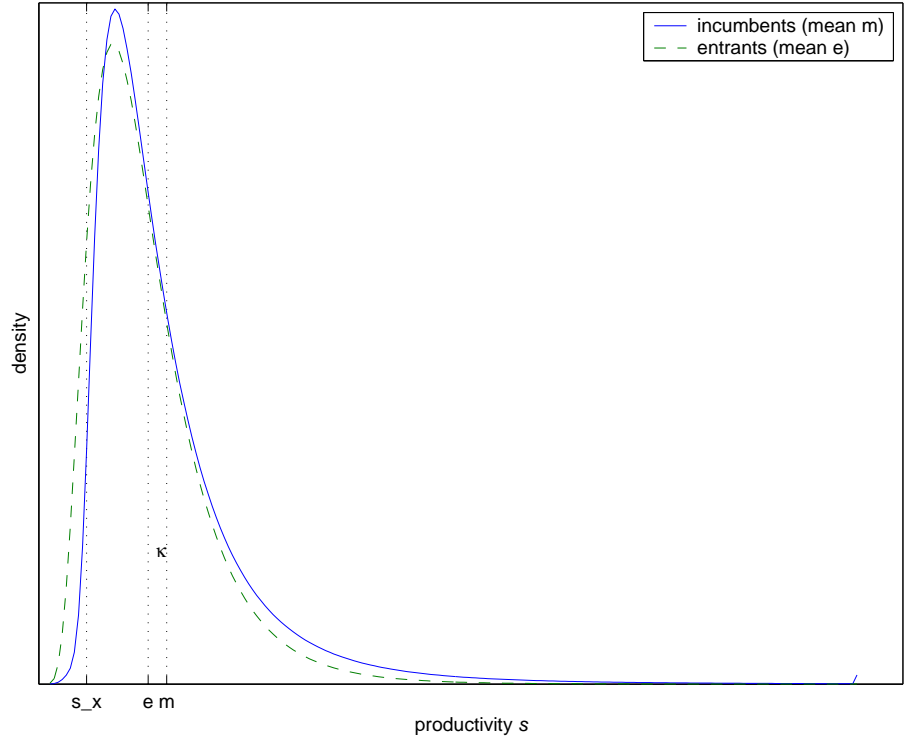


Figure 6: Productivity distribution of entrants and incumbents, difference:  $\kappa$