Public Sector Wage Policy and Labour Market Equilibrium: a Structural Model

Jake Bradley
University of Bristol

Fabien Postel-Vinay
University of Bristol,
Paris School of Economics,
CEPR and IZA

Hélène Turon
University of Bristol
and IZA

April 2011

VERY PRELIMINARY

Abstract

The goal of this paper is to develop and estimate a job search model that incorporates a sizable public sector in a labour market with search frictions. We extend the Burdett Mortensen (1998) model to include a non-infinitesimally small firm (the public sector). Moments of the wage distribution and the employment rate in the public sector are taken as exogenous policy parameters. Overall wage distribution and employment rate are determined within the model, taking into account firm and worker heterogeneity. Job offer arrival rates are sector specific and transition rates depend on the worker's decision to accept alternative employment in the same or different sector by comparing the value of employment in the current and prospective jobs. Job destruction rates are also sector specific. The parameters of the model are estimated using the method of simulated moments (MSM) with British data. These parameter estimates allow us to make counterfactual predictions of the effects of public sector wage and employment policy on overall wage distribution and employment levels.

1 Introduction

This paper formulates a search-theoretic model that incorporates interaction between the public and private sectors. The wage offer distribution of the public sector is treated as an exogenous policy parameter and conditional on this, and exogenous transitional parameters. The offer and observed private sector wage distributions are then derived endogenously. Exploiting data from the British Household Panel Survey (BHPS) the model is then estimated by indirect inference. These estimates allow us to make counterfactual policy analysis of different public sector wage policies.

There has been very little done in modelling the public sector in a search or matching framework and nothing to our knowledge that estimates such a model. This is a considerably large oversight when one thinks that in our data 17% of employed individuals were employed by the public sector. It is of course naive to believe that with an employment share this large, the public sector will not influence wage determination and by extension overall employment. Instead, of modelling the behaviour of private sector firms explicitly, the literature thus far has been dominated by reduced form comparisons of the two sectors.[1] The general consensus of stylised facts emerging from the empirical literature is that the public sector wage distribution is more compressed than the private sector and workers receive a small public sector wage premium, which is more prevalent in low skilled workers. Also using the BHPS, Postel-Vinay and Turon (2007) largely confirm these findings of the literature. Modelling agents as forward-looking, they find a public sector wage premium of the 'value of a job' of between 2 and 3 percent. The 'value of a job' is defined as the sum of the discounted future income stream associated from employment in that job. This premium shrinks to effectively zero if they examine just the 'high-employability' individuals.

With these stylised facts being known for some time, it is fairly suprising that so little has been done in explicitly modelling the interaction between the two sectors. The existing literature that does this has largely been answering a different research question. Rather than explicitly modelling

[1] For a survey of the literature, see Bender (1998).

the private sector labour market's response to public wage policy, instead, they examine the effect of the public sector on the volatility of wages and employment over the business cycle. Since our model is derived at an economy's steady state it is not possible to shed further light over this. Two papers that have attempted to address this are by Quadrini and Trigari (2008) and Hörner et al. (2007). Both model search as directed to a particular sector and wages are determined by bargaining over the surplus from a match. Quadrini and Trigari examine a public sector wage policy that is acyclical (a single wage) and a pro-cyclical (government wage is an increasing function of private sector wages). Calibrating the model for the United Sates economy, they predict that the volatility of employment increased by two and four times, with the existence of the public sector over the periods 1970-2003 and 1945-1970, respectively. They postulate that this reduction over time is because of a more pro-cyclical policy adopted by the state. Horner et al model two economies, one where a benevolent social planner aims to maximise individual's welfare with public sector wages and employment (amongst other matters); the other in the absence of a public sector. The equilibrium of the model allows the authors to draw two conclusions. Firstly, that the public sector has an ambiguous effect of overall employment and secondly, that in more turbulent times there will be higher unemployment in the economy with the public sector. The latter result comes from the individuals being risk averse and therefore crowding into the safer sector (the public sector) in uncertain times.

To our knowledge, other than this paper, the extent of the literature that explicitly models private sector firm behaviour for a given public wage setting policy is Albrecht et al (working...) and Burdett (forthcoming). Albrecht et al extend the canonical Diamond-Mortensen-Pissarides model (Pissarides, 2000) to incorporate the public sector posting an exogenous number of job vacancies. They also introduce match-specific productivity and heterogeneity in worker's human capital. Burdett (forthcoming) is similar to ours in the sense that firms post wages rather than bargain over the surplus. However, in Burdett's model the public sector set a single wage and a number of vacancies to fill. In our model the public sector's policy is to post wages from a distribution of wages. This allows us to have wage differences in the public sector and a continuous private sector wage distribution. Unlike the two models discussed we allow for differences in cross-sector job destruction and job offer arrival rates. Crucially, this paper is unique in the literature insofar as the parameters of the model are estimated, unlike any of the structural models discussed.

Methodologically, a similar paper to ours is Meghir et al. (2010) who develop an equilibrium wage posting model to determine the interaction between a formal and informal sector in a developing country. Here the two sectors vary in the degree of regulatory tightness, the formal sector firms incurring additional costs to wages in the form of corporation tax, income tax, social security contributions, severance pay and unemployment insurance. While firms in the informal sector are not exposed to these labour market regulations they do face the chance (with given probability) of incurring a non-compliance cost. Unlike this model, private sector firms endogenously select into either the formal or informal sector and the equilibrium wage offer distributions of both are determined endogenously. Similarly to this paper, Meghir et al. simulate their model and estimate it using indirect inference.

The paper is organised as follows. In the next section we derive the equilibrium structural model. Section 3 gives an overview of the properties of the data used for estimation. Sections 4 and 5 outline the estimation protocol and present the estimation results. In Section 6 we use the results obtained, to run counterfactual policy analysis and in Section 7 we conclude.

2 The Model

We consider a model of wage-posting akin to Burdett and Mortensen (1998). However, a key aspect of our approach is that we allow for the possibility that firms are not all infinitesimally small. Instead there exists a mass point in the distribution of firm sizes, the public sector having a finite share of total employment. Thus, if employed, a worker can be in one of two states, $s \in p, g$, the private or public sectors. Although the offer distribution of wages of private sector firms is determined endogenously, the offer distribution of wages in the public sector is a policy parameter of the model and considered exogenous.

Transition parameters are also treated as exogenous and are allowed to vary across states. There are three classes of transitional parameters; the first two are very familiar to the literature, job offer arrival rates and job destruction shocks. For a given individual, the job offer arrival rate can take one of four values. Offers arrive at different rates depending on what your current state is and where the offer comes from, whereas, job destruction shocks vary only according to your current state (as all end in a spell of unemployment). The third class is a 'reallocation shock' (Jolivet et al., 2006). A worker loses his job and is instantaneously offered a new one. As the worker's outside option is now unemployment he will accept this offer so long as it is above the wage making him indifferent between working in this sector and unemployment. Similar to the job offer arrival rates a given individual will have four different reallocation shocks, varying according to his current state and where the offer comes from. We include this class of parameter as it allows us to account for job-to-job transitions that result in a lower wage, something which is observed in the data. Individuals only move job if they experience a reallocation shock or a job offer, while the arrival rate of these is exogenous, the choice of acceptance is determined endogenously.

We will introduce both cross-firm and cross-worker heterogeneity. Firms vary in their productivity, which is drawn from an exogenous distribution and is constant across the job match. Workers vary in their vector of transitional parameters. A finite number of classes of worker exist, k, which will be determined optimally in the estimation procedure. Within each class a worker faces identical transition parameters, but these will vary across an individual's class. A worker's class is fixed indefinitely. Consistent with the majority of wage posting models, time is continuous, the workers are risk-neutral and get flow utility depending on their wage level. Private sector firms are profit maximisers.

The notation used is largely consistent with the previous literature, δ being the job destruction shock and λ the job offer arrival rate. The reallocation shock is denoted by τ. These parameters vary across the class of individual k and across the particular state-to-state transition. To explain the two states between which the particular worker transits a two letter index is used. The first letter designates the sector of origin and the second the sector of destination. So for example, λ_{pg}^k is the arrival rate of offers from the public sector to private sector employees of type-k, and R_{ug}^k is the reservation wage of type-k unemployed workers facing an offer from the public sector.

2.1 Workers

There exists a [0,N] continuum of infinitely lived, risk neutral workers. There are K different types of workers whose transitional parameters differ across types, and N^k of each type-k, $N = \sum^K N^k$. There is heterogeneity in these workers in terms of their propensity to transit between states of employment and unemployment as well as in the flow utility they derive from being unemployed. At any one point in time a worker can be in one of three states, either unemployed or employed in the public or private sector. We do not extend the model to allow workers to opt out of the labour force. The model is derived when the economy is in steady state, the number steady state number of workers in each sector are denoted as, N_u, N_g and N_p, respectively.

Workers receive job offers at a Poisson rate that depends on the worker's type and current state. A job is fully characterised by a wage w. The job faces three potential shocks: A job destruction shock δ_s^k, after which the worker becomes unemployed and gets a flow utility b^k. A reallocation shock, τ_{ss}^k where he ends his current job in sector s and instantaneously receives a new job from sector s with a wage drawn from the appropriate job offer distribution. The public sector offers wages from an exogenously determined distribution, $F_g(w)$, while the distribution of wage offers from the private sector $F_p(w)$ is determined endogenously. A worker will accept a job offer if that job gives him a greater future value then he receives in his unemployment. Finally, the worker is also exposed to a job offer arrival rate, $\lambda_{ss\prime}^k$ which is analogous to the reallocation shock. However, he accepts this offer if the job gives him greater future value than his current job, rather than unemployment. The transiton rates described are all exogenous and search is modelled as random.

2.1.1 Reservation Wages

An individual's utility is entirely characterised by the present discounted future stream of wages, since for a given worker, the transitional parameters will be constant if he moves job within a sector. The acceptance decision for an offer within the worker's current sector is therefore entirely determined by the comparison between the worker's current wage and the new wage being offered. If the new offer, x, is higher than the worker's current wage, w, he will accept and otherwise reject. However, since a change in sector is not only associated with a different wage but also with a change in transitional parameters, the acceptance decision is not so trivial when the job offer is from another sector. Thus depending on the two sets of transition parameters, an individual may accept a job offer from a different sector with a wage cut, or conversely, require a higher wage in order to accept the job offer. These acceptance decisions can be characterised by a set of reservation wages, defining the thresholds above which (alternative) job offers are accepted.

When employed, the reservation wage will be a function of the worker's current wage. Denote $W_p^k(w)$ ($W_g^k(w)$) as a type-k worker's valuation of a private (public) sector job paying a wage w and U^k the value of unemployment for a worker of type-k. The minimum public sector wage offer that makes a worker of type-k currently earning a wage w in the private sector indifferent between staying in his current job and taking up the offer is denoted as $R_{pg}(w)$. Formally $W_g^k(w) = W_p^k\left(R_{pg}^k(w)\right)$. We will confirm shortly that $W_p^k(\cdot)$ and $W_p^k(\cdot)$ are increasing functions of w, which ensures the reservation property of $R_{pg}^k(w)$. Similarly $R_{gp}^k(w)$ denotes the minimum offer from a private sector employer to attract a worker of type-k currently employed in the public sector at wage w. Using these two expressions we can derive an interesting property, that the two reservation wages described are inverses of each other, $W_p^k\left(R_{pg}^k\left(R_{gp}^k(w)\right)\right) = W_g^k\left(R_{gp}^k(w)\right) = W_p^k(w)$, so that

$$R_{pg}^k\left(R_{gp}^k(w)\right) = w. \tag{1}$$

The reservation wage of an unemployed type-k worker receiving an offer from the public (private) sector is the wage at which they are indifferent between unemployment and the public (private) sector. Formally, the two reservation wages solve the equality, $U^k = W_p^k\left(R_{up}^k\right) = W_g^k\left(R_{ug}^k\right)$. Hence applying (1) to this equality one can derive a second property of the reservation wages:

$$R_{pg}^k\left(R_{up}^k\right) = R_{ug}^k. \tag{2}$$

Note, that the analogous property for $R_{gp}^k(.)$ also holds. The expressions derived here will be used to derive the reservation wages in the following section.

2.1.2 Bellman Equations

The value function for a type-k unemployed worker is defined by the following Bellman equation:

$$rU^k = b^k + \lambda_{up}^k \int_{R_{up}^k}^{+\infty} \left[W_p^k(x) - U^k\right] dF_p(x) + \lambda_{ug}^k \int_{R_{ug}^k}^{+\infty} \left[W_g^k(x) - U^k\right] dF_g(x), \tag{3}$$

where r is the rate of time preference, constant across workers. The first term, b^k is the flow utility an individual gets from being in unemployment, this varies across the type-k of the individual. Offers arrive from the public (private) sector at a rate of λ_{up}^k (λ_{up}^k). Wage offers, x are drawn from the private sector from an endogenous distribution, $F_p(w)$ which will be derived from the firm side later. Wage offers in the public sector are drawn from the distribution $F_g(\cdot)$ which we consider to be an exogenous policy decision. An unemployed worker will accept a job offer if the wage is higher than the workers reservation wage for that sector, the lower bound of the integral. The second and third terms represent the expected gain for a worker of typr-k of receiving a job offer from the private and public sectors, respectively. Below is the example for a private sector employee:

$$\begin{aligned} rW_p^k(w) = {} & w + \delta_p^k\left\{U^k - W_p^k(w)\right\} + \lambda_{pp}^k \int_{w}^{+\infty} \left[W_p^k(x) - W_p^k(w)\right] dF_p(x) \\ & + \lambda_{pg}^k \int_{R_{pg}^k(w)}^{+\infty} \left[W_g^k(x) - W_p^k(w)\right] dF_g(x) + \tau_{pp}^k \int_{R_{up}^k}^{+\infty} \left[W_p^k(x) - W_p^k(w)\right] dF_p(x) \\ & + \tau_{pg}^k \int_{R_{ug}^k}^{+\infty} \left[W_g^k(x) - W_p^k(w)\right] dF_g(x) \end{aligned} \tag{4}$$

So, for the case of a type-k worker employed in the private sector earning a wage w has discounted value from employment given by the right hand side of (4). The first term w is the instantaneous wage paid in the current private sector firm. The next term, is the loss of value an individual would get if he were to transit into unemployment $\left\{U^k - W_p^k(w)\right\}$ multiplied by the probability of such an event occurring, the private sector job destruction rate, δ_p^k. At rate λ_{pp}^k the worker receives an offer from another private sector firm, where the offer is drawn from the distribution $F_p(x)$. If this offer is greater than his current wage w he will accept it. Given the offer is received and it meets his acceptance criteria given, the individual will make an unambiguous gain in value given by $\left[W_p^k(x) - W_p^k(w)\right]$. The next term represents the equivalent, except for offers from the public sector. Thus the wage is drawn from a different distribution and the acceptance criteria, the lower bound of the integral is instead $R_{pg}^k(w)$. The final two terms are the expected changes in value associated with a reallocation shock. A reallocation shock happens at a Poisson rate of $\tau_{pp}^k + \tau_{pg}^k$. If a worker rejects this offer his outside option is unemployment, therefore in this instance the workers optimal acceptance strategy will be to accept wages higher than R_{up} and R_{ug} depending on whether the new job offer is in the private or public sector. An analogous Bellman equation defines the value function for a type-k worker employed in the public sector.

The value functions given by (3) and (4) and the analogous Bellman for the public sector allow us to obtain the reservation wage required to leave the private for the public sector and vice-versa as a function of the transition parameters. This is done using the identity $W_p^k\left(R_{pg}^k(w)\right) = W_g^k(w)$ and $W_g^k\left(R_{gp}^k(w)\right) = W_p^k(w)$ and assuming differentiability of the value functions. This manipulation is performed in the appendices and the solution for a type-k private sector worker's reservation wage from the public sector solves the following non-linear ordinary differential equation (ODE).

$$R_{pg}^{k\,\prime}(w) = \frac{r + \Delta_g^k + \lambda_{gp}^k \overline{F}_p(w) + \lambda_{gg}^k \overline{F}_g\left(R_{pg}^k(w)\right)}{r + \Delta_p^k + \lambda_{pp}^k \overline{F}_p(w) + \lambda_{pg}^k \overline{F}_g\left(R_{pg}^k(w)\right)}, \tag{5}$$

with initial condition $R_{pg}^k\left(R_{up}^k\right) = R_{ug}^k$, and $\Delta_p^k = \delta_p^k + \tau_{pp}^k + \tau_{pg}^k$, and $\Delta_g^k = \delta_g^k + \tau_{gp}^k + \tau_{gg}^k$ by analogy. Similarly, $R_{gp}^k(w)$ is defined by an analogous ODE. It should be noted that R_{up}^k and R_{ug}^k themselves depend on the functions $R_{pg}^k(\cdot)$ and $R_{gp}^k(\cdot)$ as they are obtained by solving $W_s^k\left(R_{us}^k\right) = U^k$ for

$s = p$ or g. However, they also depend on the b^ks, which are free parameters, so those reservation wages can themselves be estimated as free parameters.

2.1.3 Flow-Balance Equations

We assume that the economy is in steady-state. That is, the flows in and out of any given sector, for each class of workers, are equal. The steady-state population in each sector for type-k workers are denoted as N_u^k, N_p^k and N_g^k. Thus the balanced flow in and out of unemployment for type-k workers is given by:

$$\begin{aligned}\left[\lambda_{up}^k \overline{F}_p\left(R_{up}^k\right) + \lambda_{ug}^k \overline{F}_g\left(R_{ug}^k\right)\right] N_u^k &= \left[\delta_p^k + \tau_{pp}^k F_p\left(R_{up}^k\right) + \tau_{pg}^k F_g\left(R_{ug}^k\right)\right] N_p^k \\ &+ \left[\delta_g^k + \tau_{gp}^k F_p\left(R_{up}^k\right) + \tau_{gg}^k F_g\left(R_{ug}^k\right)\right] N_g^k\end{aligned} \tag{6}$$

The left hand side of (6) is the rate at which workers leave unemployment toward the two sectors of employment. This occurs when a worker receives a job offer from a given employment sector and the associated wage offer is higher than his appropriate reservation wage. The first term of the right hand side is the flow of workers transitting from the private sector to unemployment; either because of a job destruction shock, δ_p^k or because of a reallocation shock with the corresponding wage offer below their reservation wage. Similarly, the second term is from the public sector to unemployment, and analogous to the first. A worker of type-k can only be in one of three states, u, p or g so: $N_u^k + N_p^k + N_g^k = N^k$, where N^k is the total population of type-k workers.

The flow-balance equations for employed workers are more cumbersome. The equation for type-k private sector employees earning less than w is given in the Appendix, equation (16). Below is the expression differentiated with respect to the wage rate, w, given in the appendices.

$$\begin{aligned}&\frac{d}{dw}\left\{\left[\Delta_p^k + \lambda_{pp}^k \overline{F}_p(w)\right] N_p^k G_p(w \mid k)\right\} + N_p^k g_p(w \mid k)\, \lambda_{pg}^k \overline{F}_g\left(R_{pg}^k(w)\right) \\ &\qquad - N_g^k \lambda_{gp}^k G_g\left(R_{pg}^k(w) \mid k\right) f_p(w) = \left\{N_u^k \lambda_{up}^k + N_p^k \tau_{pp}^k + N_g^k \tau_{gp}^k\right\} f_p(w).\end{aligned} \tag{7}$$

This would be a fairly straightforward ODE in $g_p(w \mid k)$ (the probability distribution of observed wages in the private sector), if it was not for the term featuring $G_g\left(R_{pg}^k(w) \mid k\right)$.

This term can be derived by manipulation of the flow balance equation for type-k public sector workers earning less than $R_{pg}^k(w)$ (instead of w). This manipulation is performed in the Appendix, (19). Plugging this solution into (7), we obtain an ODE that defines $G_p(w \mid k)$. Note that by considering $w \to +\infty$ in the latter equation, one obtains (6).

An additional hurdle at this point is the determination of N_p^k and N_g^k (with $N_u^k = N^k - N_p^k - N_g^k$). Those numbers are needed to solve for $G_p(\cdot \mid k)$ in the ODE resulting from the combination of (7) and the isolation of $N_g^k G_p\left(R_{pg}^k(w) \mid k\right)$, given in the appendix. Now N_p^k and N_g^k are jointly defined by the balance of flows in and out of employment (6), and the flow balance in and out of, say, the private sector, which is given by evaluating the flow-balance equation of type-k private sector workers (given in the appendix) at $w \to +\infty$:

$$\begin{aligned} N_p^k \Delta_p^k + N_p^k \lambda_{pg}^k \int_{R_{up}^k}^{+\infty} \overline{F}_g\left(R_{pg}^k(x)\right) dG_p(x \mid k) \\ - N_g^k \lambda_{gp}^k \int_{R_{ug}^k}^{+\infty} \overline{F}_p\left(R_{gp}^k(x)\right) dG_g(x \mid k) = \left\{N_u^k \lambda_{up}^k + N_p^k \tau_{pp}^k + N_g^k \tau_{gp}^k\right\} \overline{F}_p\left(R_{up}^k\right). \quad (8) \end{aligned}$$

The distribution, $G_g(x \mid k)$, can be derived by using the identity $R_{pg}^k\left(R_{gp}^k(w)\right) = w$ applied to the derivation of $G_p\left(R_{pg}^k(w) \mid k\right)$ in the appendix. The latter equation involves $G_p(\cdot \mid k)$, which in turns depends on N_p^k and N_g^k, so that those three objects have to be solved for simultaneously. This is done using an iterative procedure.

2.2 Private Sector Firms

There exists a [0,1] continuum of private sector firms who are profit maximisers and heterogeneous in their level of productivity, y, where $y \sim \Gamma(\cdot)$ in the population of firms. From our analysis of workers we can deduce the mean size of a private sector firm offering a wage w as the ratio of the observed wage distribution and offer distribution, summed across all worker types.

$$\ell_p(w) = \sum_k N_p^k \frac{g_p(w \mid k)}{f_p(w)}$$

By setting a wage w, a firm will be of size $\ell_p(w)$ and thus have total profit given by $\pi(w; y) := (y - w)\,\ell_p(w)$ Although the profit per worker is decreasing in wages, the size of firm increases in wages. To solve the firm's problem we use the first order condition:

$$w + \frac{\ell_p(w)}{\ell_p'(w)} = y \tag{9}$$

Thus if wages are increasing in productivity, then we can infer that $F_p(w) = \Gamma(y(w))$, where the relationship $y(w)$ is given by (9). Thus the distribution of wage offers in the private sector, $F_p(w)$ can be retrieved if we know the distribution of productivity across firms.

3 Data

We split this section into two distinct parts. First, we give a brief description of the sample we use for estimation. Then, we describe the composition and dynamics of the labour market, distinguishing between the two sectors; in order to provide the motivation for this paper and to justify the model described in the previous section.

3.1 The Sample

The data used in the analysis are taken from the BHPS, a longitudinal data set of British households. Data were first collected in 1991 and the households selected were determined by an equal probability sampling mechanism. Since then, there have been 18 further waves, collected annually. The model outlined is derived under a steady state assumption. Therefore it is necessary that the time period used is short and to has roughly constant shares in each of the three states across time. We also restrict our analysis to male workers because women are more likely to select out of the labour force as well as into part-time employment, and neither states are modelled explicitly in the paper. Below is a graph of the sector sizes across the BHPS, for males between the ages of 20 and 60. It should be noted, as will be seen later, that there is a much larger share of female workers in the public sector, so the share of public sector workers may appear deceptively small.

[Figure 1 about here]

From 1991 to the early part of this century, there was a general decline in the public sector, probably as a result of the continuation of Thatcher's drive towards privatisation. Over a ten year period the public sector share declined from 20% to remain approximately constant at 17% from 2002 to 2006. After this, there was an expansion of unemployment, probably a result of the global recession. Thus, this paper will focus on the relatively stable period, between 2002 and 2006, where the size of three states modelled are approximately constant.

3.2 Composition and Dynamics

3.2.1 Composition

In order to give an idea of differences across sector, the table below contains information on the composition, wages and dynamics of each sector. Individuals are included here if across the time period considered (2002-2006) they did not opt out of the labour force at any stage and if they are between the ages of 20 and 60. The total number of observations at this point is 33,174.

Table 1: Descriptive Statistics

	Private Sector	Public Sector	Unemployment
size of each sector	65.51%	30.04%	4.44%
women in each sector	37.27%	57.51%	28.32%
part-time workers in each sector	15.29%	21.08%	n/a
average log hourly wages	2.19	2.36	n/a
standard deviation of log hourly wages	0.48	0.44	n/a
proportion who make any job transition in a given year	15.15%	5.90%	53.20%

The table above has two implications. Firstly, it is broadly consistent with the stylized facts presented in the introduction, particularly, the first and second moments of the wage distribution in the public and private sector. Although, presented in Table 1 are just descriptive statistics, no heterogeneity has been controlled for. The mean wage could suggests there may be a public sector premium and the standard deviation suggests a higher level of inequality in the private sector.

Also, the greater rate of job transition in the private sector is consistent with the Postel-Vinay and Turon (2007) assertion that although more equal, there is a greater persistence of wages in the public sector. Secondly, it shows a clear selection of workers into a specific sector. Women appear to be selecting into part-time and public sector jobs, while also being far less likely to be unemployed. One would conjecture that women select into safer, more flexible jobs and out of the labour force entirely. There is a large existing literature on the labour market participation of women and since this model neither differentiates between sex nor models non-participation, the sample will be homogenised to include only men.

3.2.2 Worker Turnover

The picture given in Table 1 about the transition rates between sectors is by no means complete. The table below shows a cross-sector annual transition matrix for men between the age of 20 and 60. Derived across the whole sample, this matrix shows the probability of being in the state given in the row at time t and the state given by the column in time t plus a year. Thus the diagonal element is the probability an individual will be in the same sector in one year's time. There is a large amount of employment to employment transitions that are associated with a change in sector, particularly from the public to private sector. There is a 9% chance a public sector worker will be a private sector worker, in a year's time. Another point to be drawn from the transition matrix is the likelihood of a public sector worker to be unemployed in a year's time (2%) compared with a private sector worker (3.4%). This suspected additional job security of the public sector was another motivation of the paper. That for instance, exogenous risk preferences will determine search and thus different classes of people will have different exogenous transition parameters. Although, again, it should be noted that these are only raw descriptive statistics and once individual heterogeneity is accounted for, the perceived higher security in the public sector may disappear.

Table 2: Annual Transition Matrix

	U_{t+1}	G_{t+1}	P_{t+1}
U_t	53.43%	8.26%	38.31%
G_t	2.01%	88.75%	9.24%
P_t	3.42%	1.92%	94.65%

3.2.3 Wage Dynamics

A further discrepancy with labour models and reality (as observed in the data) is the dynamics of wage rates. With a few exceptions[2] models of the labour market treat each employer-employee match as characterised by a constant wage rate through time. However, wages typically increase with job tenure. This paper does nothing to explain changes in wages within the firm. However, as described previously this paper can explain job-to-job transitions that are associated with wage reductions, either because of the 'reallocation shock' or because the change is also associated with a change in sector. To justify our assumption that change in wages are always associated with a a chane in job, see Figure (2).

Figure (2) represent two cumulative distribution functions for the change in log wages in a year for those who have moved jobs in that time (the movers) and those who have not (the stayers). If our model was to be believed, the stayers, the solid line, should be degenerate at zero. If one remains in their job there is no reason their wage rate should change. The movers, the dashed line should have both increasing and decreasing wages, as discussed. While the stayers distribution function is not degenerate at zero there is significantly less volatility in their function than the corresponding one for job movers. Finally, approximately a quarter of wage changes occur with an associated decrease in wage. This non-insignificant amount provides some justification for the inclusion of the two types of wage reductions modelled in this paper.

[2]There are two broad classes of models that explain the positive wage-tenure relationship. The sequential auction model (Postel-Vinay and Robin, 2002), where firms can make counteroffers to poachers of their workers. Thus staying in a single firm may lead to discrete jumps in wage. The second is where firms offer contacts contingent on tenure. Stevens (2004) shows that if employers offer workers contracts contingent on tenure they are able to eliminate inefficient separations and extract all rents from the employer-employee match. Burdett and Coles (2003) introduce risk aversion into Stevens model and restore the result of Burdett and Mortensen (1998) where in equilibrium homogeneous firms will offer different contracts.

[Figure 2 about here]

4 Estimation: Warning, very preliminary

The model is fully characterised by the parameter vector:

$$\theta = \left(\left(r, b^k, \delta_p^k, \delta_g^k, \lambda_{up}^k, \lambda_{ug}^k, \lambda_{pp}^k, \lambda_{pg}^k, \lambda_{gp}^k, \lambda_{gg}^k, \tau_{up}^k, \tau_{ug}^k, \tau_{pp}^k, \tau_{pg}^k, \tau_{gp}^k, \tau_{gg}^k, F_g\left(w\right), \Gamma\left(y\right) \right) k = 1, ..., K \right)$$

Conditional on θ, all other paramters of the model can be obtained:

$$\left(\left(N_u^k, N_p^k, N_g^k, R_{up}^k, R_{ug}^k, R_{pg}^k, R_{gp}^k, F_p\left(w\right), G_p\left(w\right), G_g\left(w\right) \right) k = 1, ..., K \right).$$

To make the estimation simpler, we will estimate $F_g\left(\cdot\right)$ and $\Gamma\left(y\right)$ parametrically assuming they are both mixes of two log-normal distributions. Thus there are six parameters associated with the two distribution, giving a total of $(21 \times K)$ parameters needed to estimate.

4.1 Indirect Inference

Unfortunately θ cannot be estimated adequately using a closed form solution. Therefore, in order to estimate the structural model we turn to using methods of indirect inference. The principle of indirect inference is to find values of the structural parameters that minimise a function of the difference between a chosen set of moments from the data and data simulated with these values of the structural parameters.

The vector of moments being matched is derived from estimating a set of auxiliary models, which we will call, $\hat{\beta}_0$. The auxiliary models will be spelt out in the following section. Once the auxiliary models are specified and estimated, we assign initial (reasonable) values to the vector θ, which we call θ_0. We then simulate the model using these initial values to produce simulated data, $y^S\left(\theta_0\right)$. Using the simulated data, the auxiliary model is estimated, these estimates are a function of the initial parameter values of θ, $\hat{\beta}\left(\theta_0\right)$. The values of θ will be updated in order to minimise the distance between $\beta\left(\theta\right)$ and β_0, where each element is weighted by the standard deviation of each component in β_0. This is done until the value of θ converges[3], thus if it take N iterations to solve,

[3]This will be a special version of the Robbins and Monro (1951) learning algorithm.

θ_N is given by equation (10) below. For a comprehensive overview of methods of indirect inference, see Gourieroux et al. (1993).

$$\theta_N = \arg\min_{\theta} \|\hat{\beta}(\theta) - \hat{\beta}_0\| \tag{10}$$

4.2 Auxiliary Models

The selection of auxiliary models is key to indirect inference and is usually the most contentious stage of the estimation procedure. Clearly since we need to identify every element in θ, we require $dim(\theta) = dim(\beta)$. Thus we require $(21 \times K)$ parameters to estimate in our auxiliary model. We combine the following auxiliary models to make up the components of β.

4.2.1 Transition Rates

We estimate the probability a type-k worker transits between sectors. Using the same two letter index notation as before, we denote π as the probability of a transition. In the case of within sector transition, we also distinguish between a promotion and a demotion, so π^k_{pp-} is the probability a private sector worker moves to another private sector firm at a lower wage.

Table 3: Transitional Matched Moments

Auxiliary Model	Structural Model
π^k_{up}	$\lambda^k_{up}\bar{F}_p\left(R^k_{up}\right)$
π^k_{ug}	$\lambda^k_{ug}\bar{F}_g\left(R^k_{ug}\right)$
π^k_{pu}	$\delta^k_p + \tau^k_{pp}F_p\left(R^k_{up}\right) + \tau^k_{pg}F_g\left(R^k_{ug}\right)$
π^k_{gu}	$\delta^k_g + \tau^k_{gp}F_p\left(R^k_{up}\right) + \tau^k_{pg}F_g\left(R^k_{ug}\right)$
π^k_{pp+}	$\left(\lambda^k_{pp} + \tau^k_{pp}\right)\bar{F}_p(w)$
π^k_{pp-}	$\tau^k_{pp}\bar{F}_p\left(R^k_{up}\right)F_p(w)$
π^k_{gg+}	$\left(\lambda^k_{gg} + \tau^k_{gg}\right)\bar{F}_g(w)$
π^k_{gg-}	$\tau^k_{gg}\bar{F}_g\left(R^k_{ug}\right)F_g(w)$
π^k_{pg}	$\left(\lambda^k_{pg} + \tau^k_{pg}\right)\bar{F}_g\left(R^k_{pg}\right)$
π^k_{gp}	$\left(\lambda^k_{gp} + \tau^k_{gp}\right)\bar{F}_p\left(R^k_{gp}\right)$

To recover the transition rates of the auxiliary model we use simple method of moments. Each p is obtained by recovering the total number of that class of transition made across the whole sample and dividing by the total length of collective time (in months) spent in the initial state. For example, for p^k_{up},

$$p_{up}^{k} = \frac{\sum_{i=1}^{K} UP_i}{\sum_{i=1}^{K} ut_i}, \tag{11}$$

where UP_i is the total number of unemployment to private sector moves made by individual i observed across the saample and ut_i is the total number of months individual i spent in unemployment.

4.2.2 Wage Distributions

The wage distributions of the public and private sector are both fully observed in the data and endogenous to the model. Although endogenous, as was shown earlier the functions $G_p(w)$ and $G_g(w)$ do not have an analytical solution. However, given parameter values for θ^k they can be appropriately simulated. We therefore take the mean and variance of the two distributions as four further moments to match. The auxiliary estimates of the mean and variances of the private and public sector are denoted as μ_p, σ_p^2, μ_g and σ_g^2, respectively.

5 Results

in progress...

6 Counterfactual Policy Analysis

Using the estimated parameters of the structural model we simulate the effects of a change in the public sector wage policy. Using two metrics, employment and total welfare, we assess public sector wage policy and determine how the distribution of wage offers should change.

to be continued...

7 Concluding Remarks

forthcoming

8 Appendices

8.1 Workers

In this section, the first order ODE for $R^k_{pg}(w)$ and the expression for $N^k_g G_g\left(R^k_{pg}(w) \mid k\right)$ are derived. Firstly, the differential equation that defines a type-k private sector worker's public sector reservation wage.

The value function for private sector workers can be written as the below, a manipulation of (4).

$$\begin{aligned}\left\{r+\delta^k_p+\tau^k_{pp}F_p\left(R^k_{up}\right)+\tau^k_{pg}F_p\left(R^k_{ug}\right)\right\}W^k_p(w) &= w+\left\{\delta^k_p+\tau^k_{pp}F_p\left(R^k_{up}\right)+\tau^k_{pg}F_p\left(R^k_{ug}\right)\right\}U^k \\ &+\lambda^k_{pp}\int_w^{+\infty}\left[W^k_p(x)-W^k_p(w)\right]dF_p(x)+\lambda^k_{pg}\int_{R^k_{pg}(w)}^{+\infty}\left[W^k_g(x)-W^k_p(w)\right]dF_g(x) \\ &+\tau^k_{pp}\int_{R^k_{up}}^{+\infty}\left[W^k_p(x)-W^k_p(w)\right]dF_p(x)+\tau^k_{pg}\int_{R^k_{ug}}^{+\infty}\left[W^k_g(x)-W^k_p(w)\right]dF_g(x),\end{aligned} \tag{12}$$

Assuming differentiability:

$$W^{k\prime}_p(w)=\left[r+\delta^k_p+\tau^k_{pp}+\tau^k_{pg}+\lambda^k_{pp}\overline{F}_p(w)+\lambda^k_{pg}\overline{F}_g\left(R^k_{pg}(w)\right)\right]^{-1} \tag{13}$$

This also gives $W^{k\prime}_g(w)$ by analogy. Define $\Delta^k_p := \delta^k_p+\tau^k_{pp}+\tau^k_{pg}$, and Δ^k_g by analogy. Integrating by parts in (12) yields:

$$\begin{aligned}&\left(r+\Delta^k_p\right)W^k_p(w)=w+\Delta^k_pU^k+\lambda^k_{pp}\int_w^{+\infty}W^{k\prime}_p(x)\overline{F}_p(x)\,dx \\ &+\lambda^k_{pg}\int_{R^k_{pg}(w)}^{+\infty}W^{k\prime}_g(x)\overline{F}_g(x)\,dx+\tau^k_{pp}\int_{R^k_{up}}^{+\infty}W^{k\prime}_p(x)\overline{F}_p(x)\,dx+\tau^k_{pg}\int_{R^k_{ug}}^{+\infty}W^{k\prime}_g(x)\overline{F}_g(x)\,dx.\end{aligned} \tag{14}$$

Plugging the various value functions into the definition of $R^k_{pg}(w)$ given in the paper, one obtains the following, fairly complicated expression:

$$\begin{aligned}R^k_{pg}(w)&=\frac{r+\Delta^k_g}{r+\Delta^k_p}w+\left\{\frac{r+\Delta^k_g}{r+\Delta^k_p}\Delta^k_p-\Delta^k_g\right\}U^k \\ &+\left\{\frac{r+\Delta^k_g}{r+\Delta^k_p}\lambda^k_{pp}-\lambda^k_{gp}\right\}\int_w^{+\infty}W^{k\prime}_p(x)\overline{F}_p(x)\,dx+\left\{\frac{r+\Delta^k_g}{r+\Delta^k_p}\lambda^k_{pg}-\lambda^k_{gg}\right\}\int_{R^k_{pg}(w)}^{+\infty}W^{k\prime}_g(x)\overline{F}_g(x)\,dx \\ &+\left\{\frac{r+\Delta^k_g}{r+\Delta^k_p}\tau^k_{pp}-\tau^k_{gp}\right\}\int_{R^k_{up}}^{+\infty}W^{k\prime}_p(x)\overline{F}_p(x)\,dx+\left\{\frac{r+\Delta^k_g}{r+\Delta^k_p}\tau^k_{pg}-\tau^k_{gg}\right\}\int_{R^k_{ug}}^{+\infty}W^{k\prime}_g(x)\overline{F}_g(x)\,dx.\end{aligned} \tag{15}$$

Differentiating, we get an ODE in $R_{pg}^{k}(w)$:

$$R_{pg}^{k}{}'(w) = \frac{r + \Delta_g^k + \lambda_{gp}^k \overline{F}_p(w) + \lambda_{gg}^k \overline{F}_g\left(R_{pg}^k(w)\right)}{r + \Delta_p^k + \lambda_{pp}^k \overline{F}_p(w) + \lambda_{pg}^k \overline{F}_g\left(R_{pg}^k(w)\right)},$$

with initial condition $R_{pg}^k\left(R_{up}^k\right) = R_{ug}^k$. This is given as (5).

Next, the derivation of equation (7) and the isolation of the term $N_g^k G_g\left(R_{pg}^k(w) \mid k\right)$, which can be made derived. Below is the flow-balance equation for type-k private sector workers earning less than w.

$$\begin{aligned}
&N_p^k \Delta_p^k G_p(w \mid k) + N_p^k \lambda_{pg}^k \int_{R_{up}^k}^{w} \overline{F}_g\left(R_{pg}^k(x)\right) dG_p(x \mid k) \\
&\quad + N_p^k \lambda_{pp}^k \overline{F}_p(w) G_p(w \mid k) + N_p^k \tau_{pp}^k \overline{F}_p(w) G_p(w \mid k) + N_p^k \tau_{pg}^k \overline{F}_g\left(R_{ug}^k\right) G_p(w \mid k) \\
&\quad = N_u^k \lambda_{up}^k \left[F_p(w) - F_p\left(R_{up}^k\right)\right] + N_g^k \lambda_{gp}^k \int_{R_{ug}^k}^{R_{pg}^k(w)} \left[F_p(w) - F_p\left(R_{gp}^k(x)\right)\right] dG_g(x \mid k) \\
&\qquad + N_p^k \overline{G}_p(w \mid k) \tau_{pp}^k \left[F_p(w) - F_p\left(R_{up}^k\right)\right] + N_g^k \tau_{gp}^k \left[F_p(w) - F_p\left(R_{up}^k\right)\right],
\end{aligned}$$

which simplifies slightly as:

$$\begin{aligned}
&\left\{\Delta_p^k + \lambda_{pp}^k \overline{F}_p(w)\right\} N_p^k G_p(w \mid k) + N_p^k \lambda_{pg}^k \int_{R_{up}^k}^{w} \overline{F}_g\left(R_{pg}^k(x)\right) dG_p(x \mid k) \\
&\quad - N_g^k \lambda_{gp}^k \int_{R_{ug}^k}^{R_{pg}^k(w)} \left[F_p(w) - F_p\left(R_{gp}^k(x)\right)\right] dG_g(x \mid k) \\
&\quad = \left\{N_u^k \lambda_{up}^k + N_p^k \tau_{pp}^k + N_g^k \tau_{gp}^k\right\} \left[F_p(w) - F_p\left(R_{up}^k\right)\right]. \qquad (16)
\end{aligned}$$

Differentiating we obtain equation (7):

$$\begin{aligned}
&\frac{d}{dw}\left\{\left[\Delta_p^k + \lambda_{pp}^k \overline{F}_p(w)\right] N_p^k G_p(w \mid k)\right\} + N_p^k g_p(w \mid k) \lambda_{pg}^k \overline{F}_g\left(R_{pg}^k(w)\right) \\
&\quad - N_g^k \lambda_{gp}^k G_g\left(R_{pg}^k(w) \mid k\right) f_p(w) = \left\{N_u^k \lambda_{up}^k + N_p^k \tau_{pp}^k + N_g^k \tau_{gp}^k\right\} f_p(w).
\end{aligned}$$

his would be a fairly straightforward ODE if it wasn't for the term featuring $G_g\left(R_{pg}^k(w) \mid k\right)$. Let us try and get rid of that term. Writing the flow-balance equation (16) for the public sector yields:

$$\left\{\Delta_g^k + \lambda_{gg}^k \overline{F}_g(w)\right\} N_g^k G_g(w \mid k) + N_g^k \lambda_{gp}^k \int_{R_{ug}^k}^{w} \overline{F}_p\left(R_{gp}^k(x)\right) dG_g(x \mid k)$$
$$- N_p^k \lambda_{pg}^k \int_{R_{up}^k}^{R_{gp}^k(w)} \left[F_g(w) - F_g\left(R_{pg}^k(x)\right)\right] dG_p(x \mid k)$$
$$= \left\{N_u^k \lambda_{ug}^k + N_g^k \tau_{gg}^k + N_p^k \tau_{pg}^k\right\} \left[F_g(w) - F_g\left(R_{ug}^k\right)\right]. \quad (17)$$

Now applying the latter equation at $R_{pg}^k(w)$ (instead of w), we get:

$$\left\{\Delta_g^k + \lambda_{gg}^k \overline{F}_g(w)\right\} N_g^k G_g\left(R_{pg}^k(w) \mid k\right) + N_g^k \lambda_{gp}^k \int_{R_{ug}^k}^{R_{pg}^k(w)} \overline{F}_p\left(R_{gp}^k(x)\right) dG_g(x \mid k)$$
$$- N_p^k \lambda_{pg}^k \int_{R_{up}^k}^{w} \left[F_g\left(R_{pg}^k(w)\right) - F_g\left(R_{pg}^k(x)\right)\right] dG_p(x \mid k)$$
$$= \left\{N_u^k \lambda_{ug}^k + N_g^k \tau_{gg}^k + N_p^k \tau_{pg}^k\right\} \left[F_g\left(R_{pg}^k(w)\right) - F_g\left(R_{ug}^k\right)\right]. \quad (18)$$

Adding (18) to (16):

$$N_p^k G_p(w \mid k) \left\{\Delta_p^k + \lambda_{pp}^k \overline{F}_p(w) + \lambda_{pg}^k \overline{F}_g\left(R_{pg}^k(w)\right)\right\}$$
$$+ N_g^k G_g\left(R_{pg}^k(w) \mid k\right) \left\{\Delta_g^k + \lambda_{gp}^k \overline{F}_p(w) + \lambda_{gg}^k \overline{F}_g\left(R_{pg}^k(w)\right)\right\}$$
$$= \left\{N_u^k \lambda_{ug}^k + N_g^k \tau_{gg}^k + N_p^k \tau_{pg}^k\right\} \left[F_g\left(R_{pg}^k(w)\right) - F_g\left(R_{ug}^k\right)\right]$$
$$+ \left\{N_u^k \lambda_{up}^k + N_p^k \tau_{pp}^k + N_g^k \tau_{gp}^k\right\} \left[F_p(w) - F_p\left(R_{up}^k\right)\right], \quad (19)$$

which can be solved for $N_g^k G_g\left(R_{pg}^k(w) \mid k\right)$. Plugging the solution into (7), we obtain an ODE defining $G_p(w \mid k)$. Note that by considering $w \to +\infty$ in the latter equation, one obtains (6).

References

Bender, Keith, "The Central Government-Private Sector Wage Differential," *Journal of Economic Surveys*, 1998, *12* (2), 177–220.

Burdett, Kenneth and Dale T Mortensen, "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, May 1998, *39* (2), 257–73.

_ **and Melvyn Coles**, "Equilibrium Wage-Tenure Contracts," *Econometrica*, September 2003, *71* (5), 1377–1404.

Gourieroux, C, A Monfort, and E Renault, "Indirect Inference," *Journal of Applied Econometrics*, Suppl. De 1993, *8* (S), S85–118.

Hörner, Johannes, L. Rachel Ngai, and Claudia Olivetti, "Public Enterprises And Labor Market Performance," *International Economic Review*, 05 2007, *48* (2), 363–384.

Jolivet, Gregory, Fabien Postel-Vinay, and Jean-Marc Robin, "The empirical content of the job search model: Labor mobility and wage distributions in Europe and the US," *European Economic Review*, May 2006, *50* (4), 877–907.

Meghir, Costas, Renata Narita, and Jean-Marc Robin, "Wages and Informality in Developing Countries," *mimeo UCL*, 2010.

Pissarides, Christopher, *Equilibrium Unemployment Theory (2nd edition)*, Cambridge, Massachusetts: MIT Press, 2000.

Postel-Vinay, Fabien and Hélène Turon, "The Public Pay Gap in Britain: Small Differences That (Don't?) Matter," *Economic Journal*, October 2007, *117* (523), 1460–1503.

_ **and Jean-Marc Robin**, "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, November 2002, *70* (6), 2295–2350.

Quadrini, Vincenzo and Antonella Trigari, "Public Employment and the Business Cycle," *Scandinavian Journal of Economics*, 03 2008, *109* (4), 723–742.

Robbins, Herbert and Sutton Monro, "A Stochastic Approximation Method," *Annals of-Mathematical Statistics*, 1951, *22* (3), 400–407.

Stevens, Margaret, "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *Review of Economic Studies*, 04 2004, *71* (2), 535–551.