Wage and Price Dispersion: On-the-job-search meets Monopolistic Competition

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Abstract

I extend a simple model with on-the-job-search in the labor market by monopolistic competition in the goods market. It is well known that the simple on-the-job-search model predicts a wage distribution which is convex where empirical evidence suggests that it is hump-shaped. My model is able to predict a hump-shaped wage distribution. The intuition for the decline of the density function at the top wages is straightforward. In equilibrium high-price-firms will be the ones to pay high wages, however these firms cannot be large in number since the high-price output will not be sold in large quantities in equilibrium. Finally, the fact that both wages and prices are endogenous to the model allows interesting policy experiments. I study the quantitative effect on employment, output, price and wage dispersion after a change in the reservation wage and after an increase in the job offer arrival rate λ.

Contents

1 Introduction

There is a growing empirical literature on the phenomenon that similar workers receive different wages [Mor03]. Even after controlling for firm and worker heterogeneity in observables and unobservables ([PR02]) and when looking at specific occupations in small geographical areas ([AHV07]) wage dispersion persists.

This is the reason why a class of models which incorporate equilibrium wage dispersion has become increasingly popular. The common feature of these models is that firms and workers meet randomly and wages are posted by firms. Workers search for jobs while they are unemployed and continue to search for better jobs once they are employed. Even in a world with identical workers and firms this will result in an equilibrium with wage dispersion [1]. The rough intuition for that is the following. Employers face a tradeoff between the number of workers they can hire and the profit per worker they can make. The more people they want to hire in equilibrium, the higher the wage they have to pay. This leads to an equilibrium where firms are indifferent between posting high wages, attracting many workers and making a low profit on each worker on the one hand or posting low wages, attracting few workers but making a big profit on each worker on the other.

It is well known that the simplest on-the-job-search model predicts a wage distribution which is convex where empirical evidence suggests that it is hump-shaped (see Figure 1). There exist several extensions to the Burdett-Mortensen framework which generate unimodal earnings densities with a decreasing right tail. I briefly review the most prominent ones. As Burdett and Mortensen already show in their seminal paper allowing for differences in productivity can generate unimodal earnings densities. Bontemps and others [BRVDB00] do a non-parametric structural estimation of productivity dispersion. They emphasize that they need to rely on non-parametric estimation techniques to obtain a good match of the actual wage distribution. Christensen at al. [CW05] allow for endogenous search effort. They show that with convex search costs, search effort is a decreasing function of the wage rate. This feature generates the decreasing right tail of the wage

[1] I will refer to this model type without any heterogeneities as the standard model from now on

distribution. Hornstein, Krusell and Violante [AHV07] assess the success of various search models in explaining frictional wage dispersion and conclude that on-the-job search models which feature endogenous search effort or the ability of employers to make counteroffers or to offer wage-tenure-contracts seem most promising.

I propose an alternative model. Simply incorporating a goods market with monopolistic competition a la Dixit-Stiglitz yields wage densities which resemble the empirical distributions. I show how differences in the level of product differentiation in the goods market translate to differences in the resulting wage distribution. The major advantages of my model are:

- the resulting wage distribution only depends on two parameters which are related to the goods market
- it links price dispersion to wage dispersion and thereby generates testable hypotheses
- it captures a link between demand for products and employment which is a good starting point for interesting policy experiments

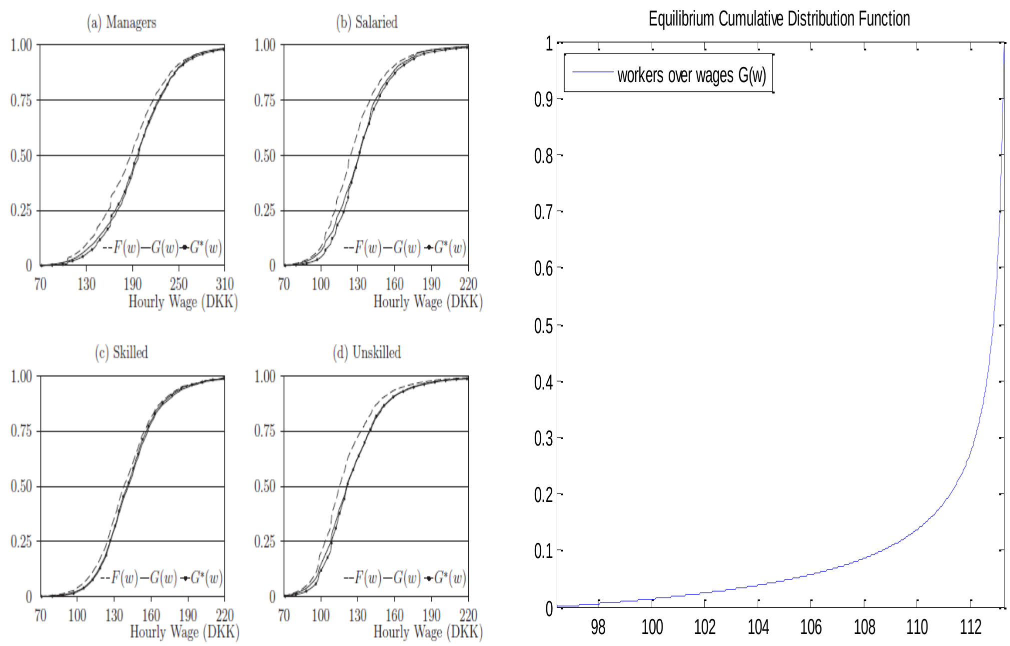

Figure 1: LHS: CDFs by Occupation, taken from [CW05].
RHS: CDF prediction of the standard on-the-job-search model.

2 The Model

2.1 The labor market - the standard Burdett-Mortensen framework

2.1.1 Workers

I briefly review the main features of the Burdett-Mortensen [BM98] model [2]. Employed and unemployed workers randomly receive job offers at a constant frequency λ. Offers are drawn from a wage distribution $F(w)$ which will turn out to be an equilibrium object. Workers accept an offer, whenever it exceeds their current earnings and is not beaten by a higher offer. The probability of receiving x offers during a time interval Δ is given by a Poisson distribution with mean λ:

$$Pr\{X = x\} = \frac{e^{-\lambda\Delta}(\lambda\Delta)^x}{x!} \tag{1}$$

Therefore the probability $P(F(w), \lambda\Delta)$ that an unemployed accepts a wage offer w is equal to the probability that w is the highest offer received:

$$P(F(w), \lambda\Delta) = \sum_{x=0}^{\infty} F(w)^x \frac{e^{-\lambda\Delta}(\lambda\Delta)^x}{x!} = e^{-\lambda\Delta[1-F(w)]} \tag{2}$$

The probability that a randomly selected worker accepts a wage offer w is then given by:

$$h(w) = [u + (1-u)G(w)]P(F(w), \lambda\Delta), \tag{3}$$

where $G(w)$ denotes the cumulative distribution function of workers over wages and u denotes the unemployment rate. Matches get separated at an exogenous probability δ plus an endogenous probability $Q(F(w), \lambda\Delta)$ that the worker leaves for a better wage offer:

$$Q(F(w), \lambda\Delta) = 1 - P(F(w), \lambda\Delta). \tag{4}$$

Following Mortensen [Mor03] I let the the time unit Δ tend to zero, i.e. I analyze the model in continuous time. Note that as Δ tends to zero, unemployed accept a wage offer whenever it exceeds the economy wide reservation wage b, which will be true for all equilibrium wage offers made, since there

[2] A detailed model description is provided by Mortensen [Mor03] , chapters 1 to 3

is no point for employers to make offers which will never be accepted. There is no option value of waiting for better offers here, since wage offers continue to arrive at any employment level with the same frequency λ and are drawn from the same distribution $F(w)$. The steady state unemployment rate u is pinned down by equating inflows to and outflows from unemployment:

$$\begin{aligned} \delta(1-u) &= \lambda u \\ u &= \frac{\delta}{\delta+\lambda}. \end{aligned} \tag{5}$$

The steady state distribution of workers over wages must satisfy:

$$\lambda F(w)u = (\delta + \lambda[1-F(w)])(1-u)G(w).$$

This yields the following equilibrium relationship between the wage offer distribution and the distribution of workers over wages:

$$G(w) = \frac{\delta F(w)}{\delta + \lambda[1-F(w)]}. \tag{6}$$

2.1.2 Employers optimal wage choice

Employers must commit to one wage policy. Note that the value of hiring a worker is given by:

$$J(p,w)(1+r\Delta) = (p-w)\Delta + [1-\delta\Delta - Q(F(w),\lambda\Delta))]J(p,w),$$

where p is the price and $(p-w)$ is hence the profit when employing one worker, see equation (14). For simplicity I set the interest rate r to zero. Let again the time unit Δ tend to zero and use (2) and (4) to obtain the value of hiring a worker at wage w:

$$J(p,w) = \frac{p-w}{\delta + \lambda(1-F(w))} \tag{7}$$

Firms optimal wage choice must maximize expected profit per worker contacted:

$$\pi(p,w) = h(w)J(p,w).$$

Recall equation (3) which becomes $h(w) = u + (1-u)G(w)$ as $\Delta \to 0$ and use the steady relationships (6) and (5) to obtain:

$$\pi(p,w) = \frac{\delta}{\delta + \lambda[1-F(w)]^2}(p-w) \tag{8}$$

The first order condition is:

$$\delta + \lambda[1 - F(w(p)) = 2(p - w(p))\lambda F'(w(p))] \tag{9}$$

The optimal wage policy $w^*(p)$ must solve this equation.

2.2 The goods market

The novelty of this paper is that I combine the classical on-the-job-search model with monopolistic competition on the goods market which is modeled here, as standard (see Benassy [Ben91]), via a Dixit-Stiglitz utility [DS77] function.

2.2.1 Demand Side

I assume that the demand side of the goods market is characterized by imperfect elasticity of substitution which give rise to the possibility of price dispersion in equilibrium. Households maximize their utility:

$$U = (\int_0^1 q_j^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{10}$$

subject to their budget constraint

$$\int_0^1 (p_j q_j) dj = I, \tag{11}$$

where $j \in [0,1]$ denotes the product variety and I is some fixed level of income. Households first order conditions are:

$$q_i = U\lambda^{-\epsilon}(p_i)^{-\epsilon}, \ \bigvee i$$

where λ is the lagrange multiplier on the budget constraint and U is the aggregate utility level. Household optimality requires that for every pair (i,j):

$$\frac{p_j}{p_i} = (\frac{q_i}{q_j})^{1/\epsilon}. \tag{12}$$

Combining the household optimality conditions (12) with the household budget constraint (11) yields the equilibrium demand for product type i [3]:

$$q_j = \frac{I(p_j)^{-\epsilon}}{\int_0^1 p_i^{(1-\epsilon)} di}. \tag{13}$$

[3] Since $q_i = (\frac{p_j}{p_i})^\epsilon q_j \Rightarrow p_i q_i = (p_j)^\epsilon (p_i)^{(1-\epsilon)} q_j \Rightarrow \sum_{i=1}^n p_i q_i = \sum_{i=1}^n (p_i)^{1-\epsilon}(p_j)^\epsilon q_j \Rightarrow I = (p_j)^\epsilon q_j \sum_{i=1}^n (p_i)^{1-\epsilon}$

2.2.2 Supply Side

Each variety is offered by a large number of firms. The output of any firm is simply given by its labor input:

$$q = N \tag{14}$$

3 Model Solution

3.1 Definition of equilibrium

An equilibrium must satisfy the following three conditions:

- free entry conditions are fulfilled
- supply equals demand in the goods market
- firms choose wages optimally

3.2 Free entry conditions

From the discussion so far it is clear, that each hire yields positive profits for the hiring firm. If there were no costs of posting vacancies, firms would post infinitely many vacancies. To guarantee that at some point it does not pay off for firms to post more vacancies in a given sector we assume convex recruiting costs $c(v)$. Firms will only continue to post vacancies until the marginal benefit equals the marginal cost:

$$c'(v) = \pi^*(p), \tag{15}$$

where $\pi^*(p)$ denotes the profits per worker implied by the optimal wage choice $w^*(p)$. The solution to this equation yields the number of vacancies posted in each price sector $v(p)$. Obviously, any potential entrant firm must post vacancies in order to be able to produce output. This condition therefore ensures that entry will no longer be attractive once the number of vacancies in the sector equals $v(p)$. Equation (15) ensures that entry to a given price-sector is free.

Given a price sector, entry is thus limit by condition (15). I assume that prices are continuously distributed on the interval $[p_0, p_1]$. Clearly, p_0 and p_1 must be such that it will never be attractive for a firm to establish a new price sector outside of this price interval. Let ϕ be a parameter which captures fixed costs of production. The fixed costs of the firm which is paying the reservation wage b is assumed to equal $(1+\phi)b$. Setting $p_0 = b(1+\phi)$ ensures that no firm will want to charge a lower price, since a lower price would yield negative profits. The maximum price p_1 is obtained by the fact that a firm can not operate if the demand for its output at the given price falls below one unit. That is, p_1 solves:

$$1 = \frac{I p_1^{-\epsilon}}{\int_0^1 p_i^{(1-\epsilon)} di}. \tag{16}$$

3.3 Supply equals demand

Supply must equal demand at each sector. Recall equation (14) to note that total supply of a price-sector is determined by total employment in this sector. Equilibrium thus requires that the employment-shares correspond to the output-shares across sectors. Calculating the cumulative output shares using equation (13) and using the link between the cdf of wage offers and the cdf of workers over wages established in (6) one can show that:

$$F(w(p)) = F(p) = \frac{G(p)(\delta+\lambda)}{\delta+\lambda G(p)}. \tag{17}$$

Given p_0, p_1 and the production function (14) we get a condition for the distribution of workers over prices in equilibrium:

$$G(p) = \frac{(\frac{p_1}{p})^{\epsilon}(p p_0^{\epsilon} - p^{\epsilon} p_0)}{p_1 p_0^{\epsilon} - p_1^{\epsilon} p_0} \tag{18}$$

3.4 Optimal wage choice of firms

Given the goods market equilibrium condition (17) firms' first order condition with respect to wages (9) can be rewritten as:

$$\delta + \lambda[1 - F(p)] = 2(p - w(p))\lambda \frac{F'(p)}{w'(p)}. \tag{19}$$

The optimal wage policy is the function $w^*(p)$ which solves this first order differential equation. The unique solution is pinned down by the boundary condition: $w^*(p_0) = b$. It implies expected profits per worker contacted of:

$$\pi^*(p) = \frac{\delta}{\delta + \lambda[1 - F(p)]^2}(p - w^*(p)) \tag{20}$$

3.5 The number of operating firms in equilibrium

The share of operating firms in each price-sector in equilibrium is pinned down by $v(p)$ (which solves (15)) and the goods market equilibrium given by (17). In particular it must hold that

$$F(p) = \frac{\int_{p_0}^{p} v(z)\gamma(z)dz}{\int_{p_0}^{p_1} v(z)\gamma(z)dz}.$$

Differentiation with respect to p yields and rearranging yields the equilibrium distribution of firms over prices:

$$\gamma(p) = \alpha \frac{F'(p)}{v(p)} \tag{21}$$

where the constant α is pinned down by the condition that $\int_{p_0}^{p_1} \gamma(z)dz$ must equal one.

3.6 Benchmark Calibration

The job offer arrival rate λ is set equal to 0.04. This is the value reported by Van den Berg and Ridder [RVDB97] and also in the range of the estimates by [BRVDB00] [4]. The job destruction rate is set to $\delta = 0.004$, again in line with [BRVDB00] and yielding a steady state unemployment rate of $u = 9\%$. Income I and the reservation wage b can be seen as mere scaling parameters. The two parameters which are key in determining the shape of the distribution of workers over wages in equilibrium are the fixed cost parameter ϕ and the elasticity of substitution ϵ. Recall that the markup parameter relates the

[4] [BRVDB00] estimate a wide variety of values for λ and δ for different industries. They also find that the job arrival rate is different for unemployed and employed with λ being often ten times higher for the unemployed than for the employed. However in the interest of keeping the theoretical model as simple as possible I stick to one common job offer arrival rate here, as also suggested by the empirical estimates of [RVDB97]

reservation wage to the lowest price via: $p_0 = b * (1 + \phi)$. I set ϕ equal to 0.5 in the benchmark case. The elasticity of substitution is set equal to 3 in the benchmark case which reflects the median value estimated by Broda and Weinstein [BW06] for substitution elasticities in different product categories. I show how my results change for different values of ϕ and ϵ.

Let me emphasize here that I have a minimal set of parameters - the frictional parameters λ and δ, the fixed cost parameter ϕ and the elasticity of substitution ϵ - which pin down the entire model, in particular the distribution of workers over wages in equilibrium.

4 Results

Two parameters are key in determining the shape of the wage distribution: the substitution elasticity ϵ and the markup parameter ϕ.

4.1 The role of the elasticity of substitution ϵ for the distribution of wages

Recall that ϵ is the parameter in the Dixit Stiglitz utility function which captures the substitution elasticity between differently priced products. A market with a low ϵ is a market with a high degree of product differentiation. One intuitively expects prices to be more dispersed in such a market. Condition (16) turns out to precisely reflect this intuition. Figure 2 depicts the resulting negative relationship between the substitution elasticity and the maximum price p_1 [5]. Figure 3 nicely shows how introducing monopolistic competition affects the wage distribution compared to the one-price model. Obviously, the more monopolistic the market gets ($\epsilon \downarrow$), the wider will be the spread of the wage distribution as a result of the wider price spread. Note, that the wage spread will always increase by less than the price spread. The intuition is straightforward. The weaker the price competition on the product market ($\epsilon \downarrow$), the weaker will be the wage competition on the labor market. A look at the optimal wage choice in figure 5 illustrates this: at any given

[5] Obviously, equation (16) is non-linear and the solution is not unique. However, for all parametrizations under study only one of the solutions turned out to be a real positive number, which was my natural choice for the maximum price.

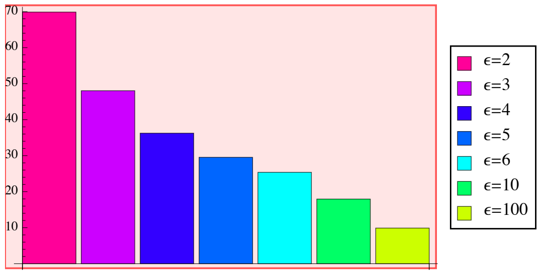

Figure 2: Maximum prices which result from the condition that there must be at least one unit demand for the most expensive product

price (here denoted as p relative to the corresponding p_1 to make things comparable) the worker's wage share $w(\overline{p}) = \frac{w}{p}$ is increasing in ϵ. All substitution elasticity scenarios feature a wage policy which prescribes decreasing relative wages from some price level onwards (absolute wages, however, will by optimality always increase in the price, see Mortensen [Mor03] p. 21 for a nice proof). This is interesting, since it implies that it is the high-price/high-wage firms which make a profit at the expense of worker's wage shares - not the firms which operate at the low end of the wage distribution. Finally, note that whereas for high values of ϵ the distribution corresponds to the standard model, for reasonably low values of ϵ we get the hump-shaped density function which is suggested by empirical evidence (compare again figure 1 from the introduction). The standard model can thus be seen as the limiting case ($\epsilon \to \infty$) of my model.

4.2 Markup ϕ

Increases in the markup parameter ϕ change the resulting distribution in two aspects. First, the price spread increases in ϕ which translates to an increased wage spread. Second, the peak of the wage density function moves to higher values. This feature provides a nice empirical check. It predicts that if fixed costs increase in an industry we should not only see prices rising but also mean wages and at the same time observe an increase in the spread of the wage distribution. An interesting special case is given for zero fixed costs, i.e. a parameter $\phi = 0$. In this case the hump shape of the density function vanishes and is replaced by a monotone decreasing density function. Figure

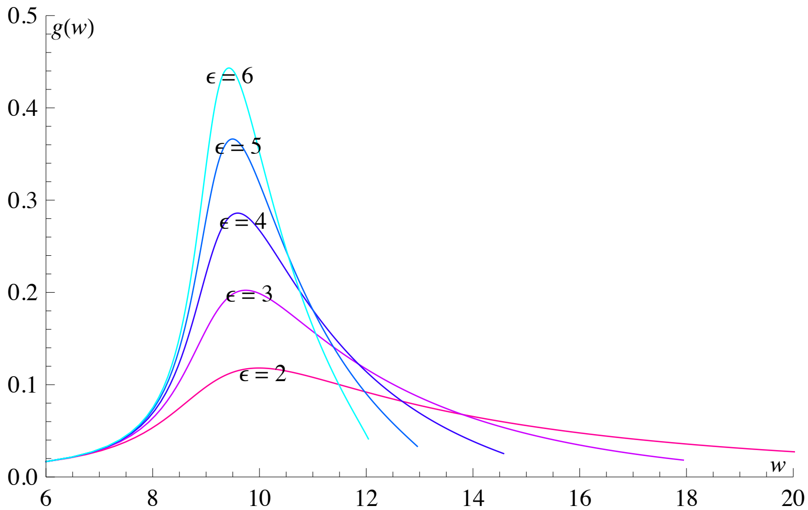

Figure 3: Pdf of workers over wages in equilibrium for varying degrees of substitution elasticity ϵ.

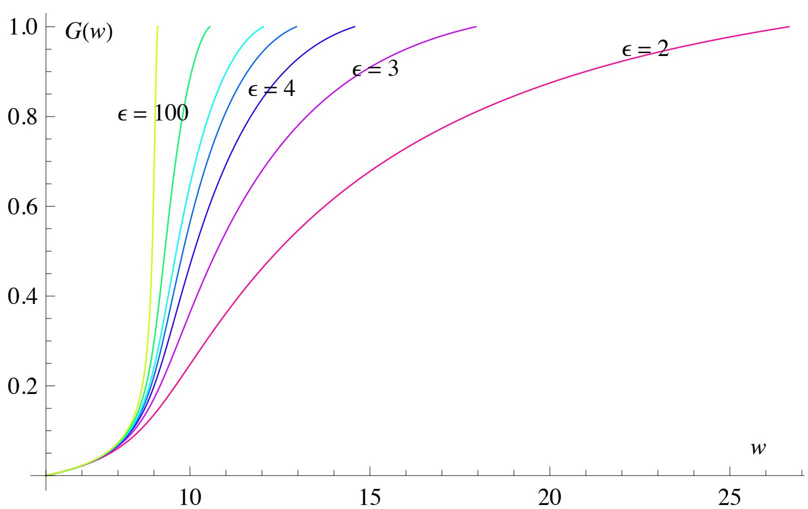

Figure 4: Cdf of workers over wages in equilibrium for varying degrees of substitution elasticity ϵ

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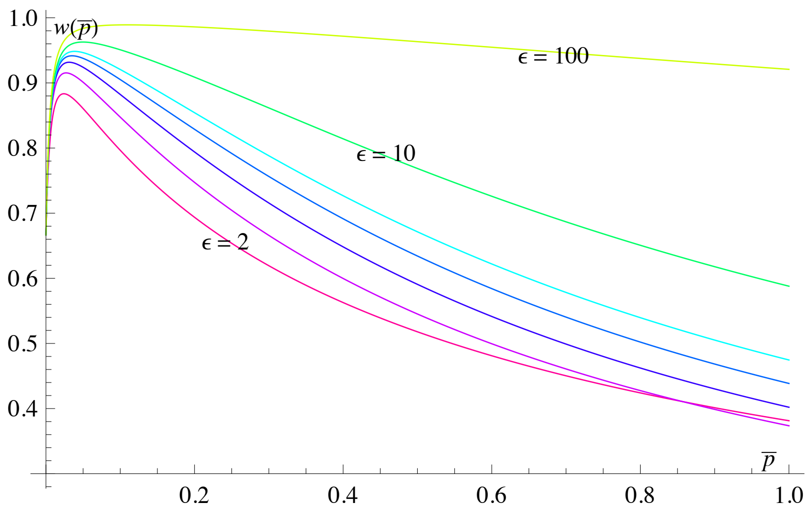


Figure 5: The optimal wage policy for varying degrees of substitution elasticity ϵ. Note that prices are normalized to the $[0, 1]$ interval and wages are expressed as shares of the revenue per labor unit, that is as shares of the price.

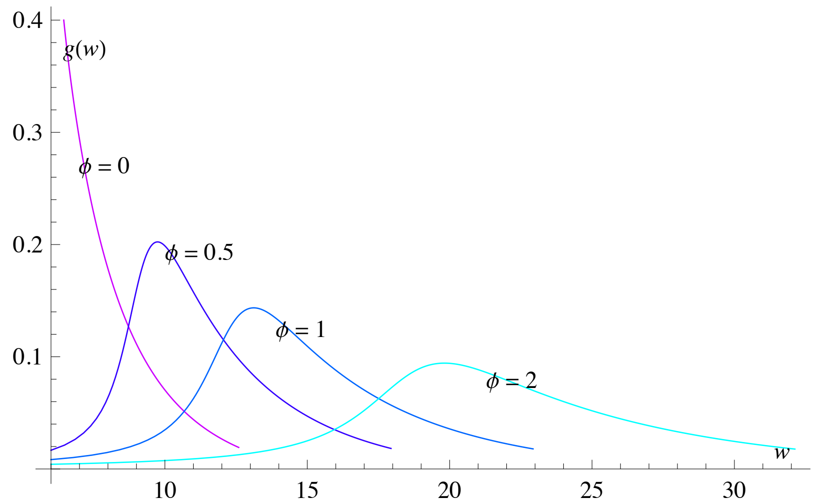


Figure 6: PDF of workers over wages for varying low price markup ϕ

8 illustrates why this is the case: $\phi = 0$ implies that firms in the lowest price-sector exercise zero monopsony power. Therefore the optimal wage policy, where wage is expressed as shares in the price, is strictly declining, whereas for all other scenarios, it is first decreasing and then declining. Hence, the concave part of the optimal wage policy is missing in the case $\phi = 0$ which corresponds to the missing increasing part of the pdf for $\phi = 0$.

4.3 Policy Experiments

4.4 Changing the job offer arrival rate λ

As we have noted before, the firms which operate in the high-price sector make the highest profits, since they exercise most monopsony power. As often suggested (for example by Bontemps et al. [BRVDB00]) a potential policy to redistribute the rents of the matches at "high-price" firms is to increase λ, for example by subsidizing costs of moving and thereby facilitating job-to-job-transitions. However, as they point out, theories which do not model the origins of price or productivity dispersion face limitations in making statement about equilibrium effects of changing λ. Clearly, my model allows to study changes in λ which will by equation (18) also impact the steady state distribution of prices. I let λ vary from 0.04 to 0.3, where the highest

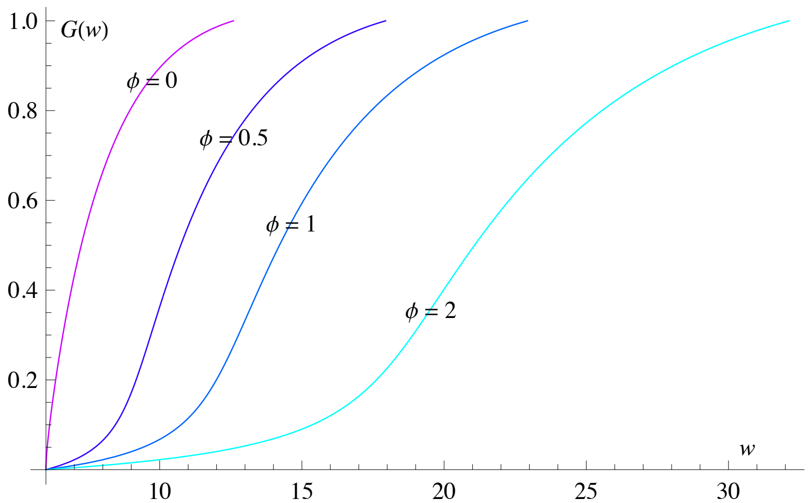


Figure 7: CDF of workers over wages for varying low price markup ϕ

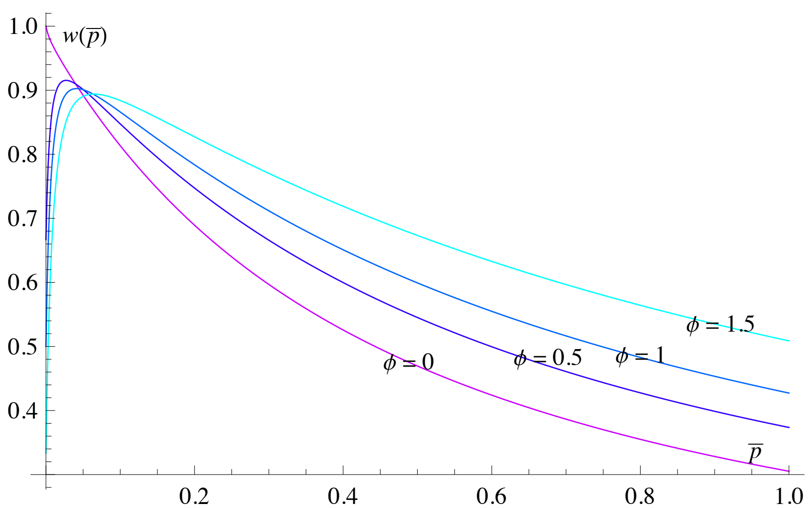


Figure 8: The optimal wage policy for varying markup levels ϕ. Note that prices are normalized to the $[0,1]$ interval and wages are expressed as shares of the revenue per labor unit, that is as shares of the price.

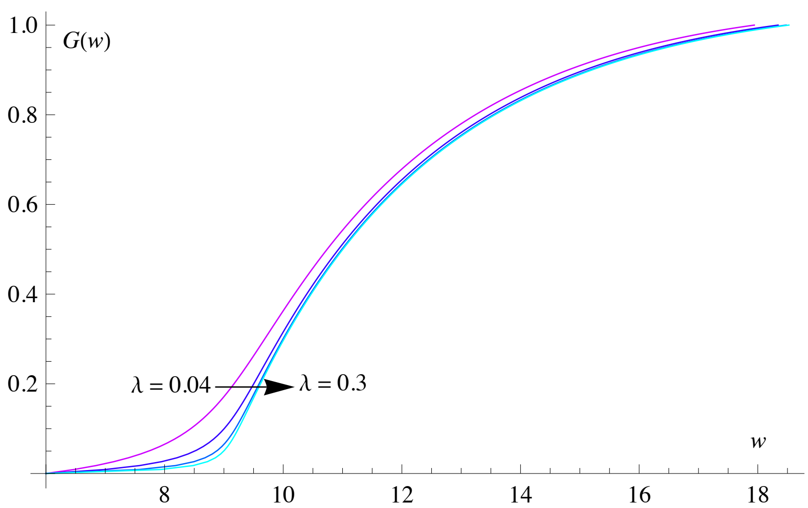
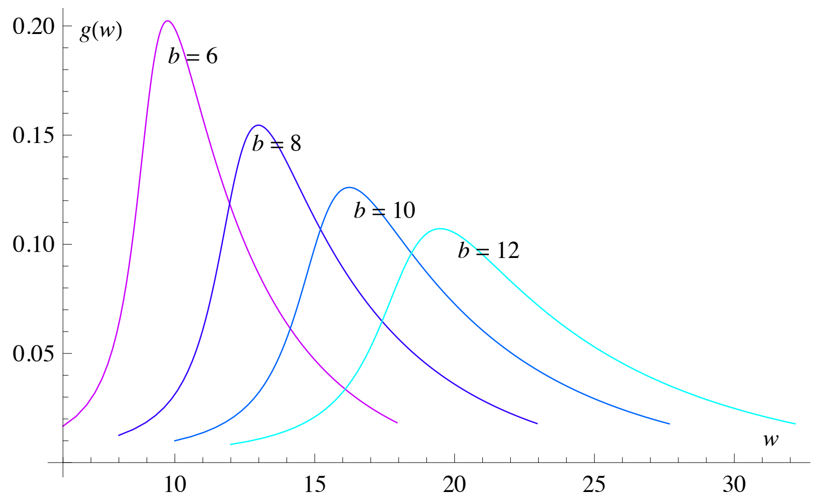


Figure 9: CDF of workers over wages for varying job offer arrival rates λ

value corresponds to an unemployment rate of only 1.3%, whereas the lowest λ corresponds to an unemployment rate of 9% as noted before. Effects are negligible compared to for example changes in fixed costs. I conclude that the effectiveness of policies which aim to affect λ seems very limited.

4.5 Introducing a minimum wage

As shown for example by Manning [Man03] minimum wages can potentially be employment increasing in a labor market environment where demand for workers is not perfectly competitive. The purpose of minimum wages in such an environment is to ensure that the monopsonistic power of firms, which arises due to some frictions in the labor market, is reduced and workers are guaranteed a certain share in firms' profits. Figure 10 shows that the minimum wage is successful in reducing firms' monopsonistic power, in the sense that at any given price level, workers receive a higher profits share with increasing rates of minimum wages. However, this comes at a cost of lower employment. As figure 12 shows, an increase the reservation wage by a factor x lowers employment by a factor almost as low as $\frac{1}{x}$. In this environment, where prices respond to increases in the reservation wage, a minimum wage will never feature positive employment effects despite the

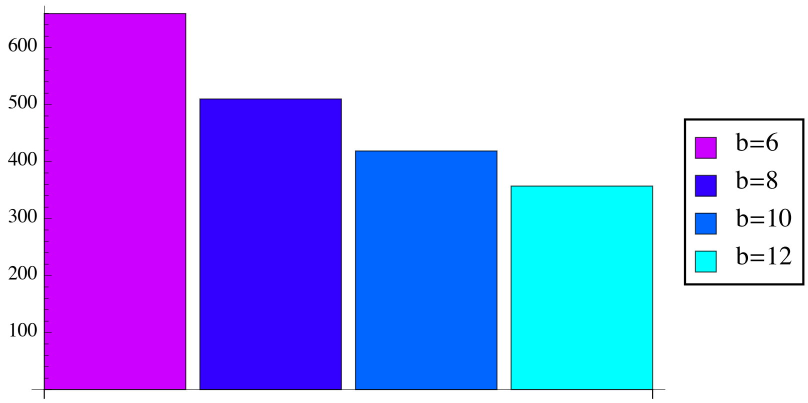


monopsonistic power which firms exercise.

5 Conclusion

I have shown that the introduction of a goods market with monopolistic competition for differentiated products in a standard model labor market model with on-the-job-search is able to replicate the main empirical features of wage distributions. Furthermore, incorporating the goods market in the model yields some nicely testable hypotheses. For example, we expect, that industries with higher fixed costs not only feature higher prices but also higher mean wages and wider spreads of the wage distribution.

The combination of the goods and labor market in one model with imperfect competition on both markets also gives scope for interesting policy analyses. For example, the model shows, that if prices can adjust in response to the introduction of a minimum wage, equilibrium employment will always fall, which is in contrast to the standard model with on-the-job-search where employment effects of a minimum wage might be positive. The effectiveness of policies which aim to facilitate job-to-job-transitions (as subsidizing moving costs for example) seems very limited.



Finally, this model is a good starting point for economic modelling of the link between the product demand, prices, wages and employment.

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