Do I have what it takes?
Equilibrium Search With Uncertainty About the Self

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Abstract

This paper develops a search model of the labor market in which uncertainty about job finding probabilities leads unemployed to become discouraged and exit the labor market. Negative updating after unsuccessful search implies that the beliefs of the unemployed about themselves, and consequently their subjective job finding probabilities, decline with unemployment duration. The assumptions about uncertainty and updating are motivated by experimental behavioral evidence. The resulting negative duration dependence in subjective job finding probabilities also affects starting wages through workers' threat points. Discouragement generates structural flows from unemployment to nonparticipation as a consequence of rational choices rather than stochastic shocks. Unemployment, labor market tightness, job offer arrival rates, and wages are determined endogenously in equilibrium. The implications provide a novel interpretation of recent empirical evidence.

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1 Introduction

It is a commonplace that unemployment is a pervasive phenomenon in Europe. High unemployment levels are associated to sclerotic and frictional labor markets, with low flows between unemployment and employment. Much research has been devoted to investigating the structure and determinants of transitions between employment and unemployment, the reasons for the large proportions of long-term unemployed workers, and the role of high and long entitlements to unemployment benefits and pervasive employment protection in generating these outcomes (see *e.g.* Bean, 1994; Ljungqvist and Sargent, 1998; and Machin and Manning, 1999). Beyond its economic and fiscal costs, unemployment is socially undesirable because it is a stressful experience. Many unemployed exhibit substantial declines in well-being, mental health and self-esteem (see, *e.g.*, Gerdtham and Johannesson, 2003; Bjorklund and Eriksson, 1998; Mathers and Schofield, 1998; and Winkelmann and Winkelmann, 1998). Substantial fractions of the unemployed, around 30 percent in Germany according Atkinson and Micklewright (1991), eventually give up to search for new employment and leave the labor market to nonparticipation instead of employment.

Yet, surprisingly little work has been devoted to investigate the relationship between search decisions and flows between unemployment and nonparticipation in frictional labor markets. In the empirical literature, the usual approach is to allow for a transition from unemployment to nonparticipation only at stochastically determined moments, such as child birth, disability, the passing of an early retirement age threshold and the like. However, this approach cannot generate transitions that occur after some time in unemployment and without any exogenous stochastic change in the individual environment. To our knowledge the only study estimating a structural search model in which nonparticipation is a permanent option and involves a true choice rather than an invocation of a stochastic shock is the paper by Frijters and van der Klaauw (2006). They model the individual search environment as non-stationary in order to account for non-stochastic transitions from unemployment to nonparticipation. In their approach an individual's value of being unemployed declines as the duration of unemployment lengthens, either because the arrival rate of job offers to that individual declines, or because the distribution of wage offers that the individual receives shifts to the left. Eventually, if the value of unemployment is too low an individual may stop searching for new employment and leave the labor market. The estimation results obtained with data for Germany show negative duration dependence in the transition from unemployment to employment as consequence mainly in wage offers, while there is only weak evidence for negative duration dependence in job offers. They also find some differences between men and women, with women exhibiting fewer job offers and lower wage offers than men, and stronger negative effects of unemployment duration on the wage offers received.While highlighting the empirical importance of duration dependence and unobserved heterogeneity for search and participation decisions, the Frijters and van der Klaauw (2006) model has little to offer in terms of explaining where the non-stationarity and heterogeneity in individuals' unemployment valuation comes from. Candidates for explaining the results are discouragement as consequence of declining psychological well-being, human capital depreciation or stigma.

There is not much theoretical literature that could clarify these issues. Only very few models take nonparticipation explicitly into account in an environment with search frictions and unemployment. Existing equilibrium search models mostly look at individuals who are heterogeneous concerning the value of leisure or home production, and who decide about their labor market participation. Flows between unemployment and nonparticipation are driven by exogenous changes in the labor market environment, such as cyclical variations.[1] Only two recent general equilibrium search models by Garibaldi and Wasmer (2005) and Pries and Rogerson (2004) are able to generate permanent structural flows between activity and inactivity even when aggregate labor market conditions are unchanged. These structural flows are caused by exogenous shocks to the individual productivity in home production, or exogenous shocks to the individual costs of labor market participation, respectively.[2] However, to our knowledge, there is no theoretical equilibrium search model that resembles the approach of Frijters and van der Klaauw (2006) and models transitions from unemployment to inactivity as true choices between two alternatives without resorting to exogenous stochastic variation. Nor is there a theoretical framework that can rationalize the empirical findings of Frijters and van der Klaauw (2006) with

[1] Examples for models in this vein are Bowden (1980), McKenna (1987), Pissarides (2000, chapter 7) and Sattinger (1995).

[2] In another recent paper, Rosholm and Toomet (2005) investigate the role of unemployment benefits for transitions from unemployment to non-participation. In their partial equilibrium model, discouragement arises as consequence of shocks to the job offer arrival rate. In a different context, Calmfors and Lang (1995) consider exogenous shocks to psychological well-being to generate discouragement and negative duration dependence to study the role of active labor market programs in a union wage-setting framework.

respect to negative duration dependence in job contacts and wages.

In this paper, we provide a model that can generate structural flows from unemployment to nonparticipation in line with the empirical evidence, and that at the same time provides a behavioral explanation and interpretation of the empirical findings of Frijters and van der Klaauw (2006). The model builds on a single intuitive generalization of the standard search model in which it is typically assumed that unemployed workers know, with certainty, their objective chances of success in search. If uncertainty and learning are modelled at all in the standard model, it is usually in the form of uncertainty about market conditions, e.g., the shape of the wage offer distribution. In contrast, we hypothesize that individuals are uncertain about *themselves* when they enter the search process, lacking knowledge about their own abilities, and their abilities relative to those of competing applicants. We argue that this subjective aspect of job search is potentially very important for understanding search behavior, the nature of unemployment, and the structure of labor market flows. In our model workers are assumed to be uncertain of their own abilities, and to update beliefs about themselves based on previous search outcomes. The motivation for this assumption comes from a companion paper on search and learning (Falk, Huffman and Sunde, 2006), which provides experimental evidence that people are in fact uncertain about relative ability, even in a simple experimental setting. The experimental results show that people update their beliefs about themselves based on the new information provided by search outcomes. The updated beliefs in turn affect future search decisions. By incorporating individual uncertainty about the self into a standard equilibrium search framework, we derive a steady state equilibrium from individual decisions that are made in a – from the individuals' perspective – non-stationary environment.

In the model there are two types of individuals, high and low. High types have a sufficiently high probability of receiving a job offer such that search is always worthwhile if an individual is certain of being a high type. Low types have a sufficiently low probability of success such that it is never worthwhile to search. The key assumption of the model is that individuals do not know their type with certainty. As a result, search decisions are based on a subjective probability of success, which arises from the searcher's subjective beliefs about her type. Assuming rational updating, individuals update beliefs about their types based on successful or unsuccessful search outcomes. We show that for a given prior there is a unique cumulative unemployment duration after which it is rational for an individual to stop search, even if she is a high type, because subjective beliefs have converged sufficiently towards certainty of being a low type. This gives rise to rational inactivity, or discouragement. The transition from active search to discouragement and non-participation is an active decision rather than the outcome of a stochastic change in an individual's environment.[3]

On the demand side, firms post vacancies optimally given their expectations about wages, and wages are determined once a match occurs based on Nash bargaining. However, individuals' subjective beliefs about their type affect their threat point in the wage negotiations. This implies that in equilibrium the wage an individual can earn decreases endogenously with her unemployment duration. In equilibrium, wages, the level of unemployment, the number of vacancies, and the unemployment duration threshold for discouragement are all determined endogenously. Everything else equal, both the equilibrium level of unemployment and the share of inactive unemployed are increasing in unemployment benefits and decreasing in productivity. Moreover, in terms of responsiveness to variations in productivity, such as cyclical shocks, the model predicts a higher variability of unemployment than a standard model without uncertainty. This arises due to an additional feedback effect through the discouragement margin.

The model offers a new perspective on a collection of important empirical facts regarding search behavior. For example, the model predicts that workers are increasingly likely to stop search as duration of unemployment increases, because of declining self-confidence. The same mechanism generates negative duration dependence in subsequent wages. These predictions are consistent with the pattern of negative duration dependence frequently documented in field data (*e.g.*, Machin and Manning, 1999; Frijters and van der Klaauw, 2006). The typical explanations for these findings, based on human capital depreciation or stigma, have been difficult to corroborate with direct empirical evidence (*e.g.* the discussion in Machin and Manning, 1999). Based on experimental evidence, our model instead suggests that uncertainty about the self could be an important contributing factor, providing a behavioral foundation for the phenomenon of discouraged workers. Moreover, the paper provides a natural explanation for the gender differences in search behavior found by Frijters and van der Klaauw (2006) that are difficult to rationalize

[3] In a less extreme form, this result can be interpreted as individuals deciding to stop search for a particular kind of job, and redirecting efforts towards labor market segments that are less demanding in terms of ability but offer higher job finding chances.

with human capital depreciation or stigma. If women have lower beliefs about themselves, they are less attached to the labor market and have a weaker stance in wage bargaining, which implies stronger negative duration dependence in the wage offers than for men. This explanation is corroborated by the results of our search experiment, where we find that women exhibit lower confidence. This is also in line with previous experiments that have documented a gender gap in confidence in other settings (Gneezy, Niederle and Rustichini, 2003).[4]

Our model also implies a different interpretation of non-participation, or discouragement, than usually adopted in the literature. Because search outcomes are only a noisy signal of ability, some of the discouraged may in fact be types with high ability who have simply been unlucky. In that sense, leaving the labor force too early (or too late) is a mistake based on imperfect information. Discouraged individuals are thus more appropriately considered as part of the problem of involuntary unemployment, in contrast to definitions of unemployment based on the criteria of active search.[5] We also argue that the model provides an answer to a long-standing puzzle regarding the group of workers labelled as discouraged in US labor force surveys. Our model highlights the role of individual search histories for the participation decision, potentially explaining why some workers exhibit only a weak response to improvements in aggregate conditions (McElhattan, 1980; Jones and Riddell, 1999). Taken together, this subjective view of job search has consequences for policy interventions to reduce (long-term) unemployment. Rather than training measures for individuals with a long unemployment history designed to address a lack of objective ability, our model suggests early interventions that assist the unemployed in their search efforts and foster their subjective perception to be able to find new employment. In this respect, our approach reflects the evidence for the damaging effect of unemployment on subjective well-being referred to in the opening paragraph. Additional psychological costs of unsuccessful search in the form of a loss in "ego utility" (Köszegi, 2001 and 2006) caused by receiving negative information about the self would further aggravate the mechanism highlighted in our model. In fact, our experiment provides evidence for such a psychological cost, showing that people deny free information about their ability if this information

[4] These predictions are consistent with field evidence on gender differences in search behavior and the gender wage gap (*e.g.*, Bowlus, 1997), but are explained by a different mechanism than has been discussed in the literature.

[5] Low types who keep on searching because of uncertainty could also be seen as involuntarily unemployed in this context, since under perfect information they would immediately opt for nonparticipation.

is likely to be negative.

The model is also relevant for the current debate regarding volatility of unemployment (see Shimer, 2005; Mortensen and Nagypal, 2005), providing an explanation for why unemployment is more volatile in reality than predicted by standard search models, which is based on inflows of discouraged workers.[6]

Our paper relates to several recent theoretical contributions that consider uncertainty in a search framework. Andolfatto *et al.* (2004) consider an environment where individuals are uncertain about their abilities. Their focus is on modelling ego utility, or preference for positive beliefs about the self in a two-period, partial equilibrium model. Our aim is to nest uncertainty about the self in a general equilibrium search framework. This allows us to investigate whether firms' vacancy posting or the wage determination have effects that revoke the results under partial equilibrium, and to provide a behaviorally founded equilibrium search model of non-stationary job search in the spirit of Frijters and van der Klaauw (2006). A different group of papers studies uncertainty and learning in search, but focuses on learning about market conditions, such as the shape of the wage distribution (Morgan, 1985; Burdett and Vishwanath, 1988; Flam and Risa, 2003; Dubra, 2004) or about the aggregate matching efficiency of the market (Gonzalez and Shi, 2005). Our paper is different in that it highlights the role of uncertainty about the self, rather than market conditions. Moreover, we view individual unemployment history as natural candidate for explaining the observed heterogeneity in individuals' behavior. In addition, and in contrast to the other literature, the learning process embedded in our model is micro-founded on the basis of experimental evidence.[7]

The paper is organized as follows. In section 2 we highlight findings from our companion paper that motivate new behavioral assumptions regarding search behavior. In section 3 we develop a general equilibrium search model that incoporates these new assumptions. Section 4 discusses field evidence in light of our model, and 5 concludes.

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[6] The effect is analogous to the added-worker and discouraged-worker effects discussed by Pissarides (2000, chapter 7).

[7] There are previous job search experiments that study search behavior when the wage or price distribution is unknown (*E.g.*, Hey, 1982 and 1987). This uncertainty is imposed rather than related to individual ability, however, and these experiments do not elicit beliefs directly, limiting the possibilities for verifying whether and how individuals update beliefs about wage distributions.

2 Some Evidence on Uncertainty and Search Behavior

In order to motivate our equilibrium search model, we briefly survey some of the main findings from a companion paper, in which we conducted a laboratory experiment on learning and search (Falk, Huffman, and Sunde, 2006). In the experiment we tested whether people are in fact uncertain about their own relative abilities, and studied how this uncertainty affects decisions in a search environment.

In the experiment, subjects first took part in a math test. Subjects were compensated for each correct answer according to a piece rate. After the test, subjects were informed about their score, but not the scores of the other subjects in the room. Subjects then participated in eight rounds of a search process, which involved real stakes. In each round, they were given 80 points, and could decide whether to keep the 80 points or invest them. If they invested the points, they could win 200 or they could win nothing. Importantly, the probability of winning 200 depended on their relative performance on the initial math test. If they had scored above the median they were a high type, with a 60 percent of winning 200. If they had scored below the median they were a low type, with only a 30 percent chance of winning. These parameters were chosen so that a risk neutral individual would always search given certainty of being a high type, and never search if certain of being a low type.

Figure 1 shows that people are uncertain about their relative abilities. Between the math test and the first period of search, we asked subjects how likely they thought it was, in percentage terms, that they had scored better than half of the other subjects in the room. The top panel of Figure 1 shows the distribution of subjective beliefs of all participants. If subjects knew with certainty whether they were a high or a low type, the distribution should be bi-modal, with half of the mass at 0 at half at 1. Instead, the distribution is nearly uniform. Subjects did have some idea of their type, as shown by the bottom two panels: the beliefs of low types and high types are clearly different, in the right direction. There is still substantial uncertainty, however. For instance, some individuals who are absolutely certain that they are of one type are absolutely wrong. Given that the environment in our experiment is relatively simple, involving a known number of competing applicants, and only a single skill dimension, these findings provide a strong empirical basis for assuming that job searchers in the real world are also uncertain of their relative abilities.

The second main finding of the experiment is that subjects change their beliefs about themselves based on search outcomes, in the direction predicted by rational updating. We asked subjects about the likelihood of being the high type after each subsequent round of search. Regressions (not shown) reveal that the most important determinants of an individual's beliefs in period t are the belief in $t-1$ and an indicator for whether search was successful or unsuccessful in $t-1$. Following an unsuccessful round of search, beliefs about the probability of being the high type drop significantly.

The third main finding of the experiment is that belief updating has an impact on subsequent search decisions. We find that beliefs in t are the most important determinant of whether an individual decides to search in period $t+1$, dominating demographic factors and risk preferences.

In summary, the experiment provides an empirical basis for assuming that job searchers are uncertain of their own relative abilities. Furthermore, the data show how this uncertainty plays out in a search environment: people respond to search outcomes by updating beliefs about themselves, and this updating has an impact on subsequent search decisions.

The experiment also provides two additional results, which suggest possible extensions to our model. One is a gender difference in initial priors: women are significantly less confident about being the high type before search begins, even though they perform better on the test than men on average. A second finding suggests that the process of search may be psychologically painful. At the end of the search process, after learning whether they were in the top or bottom half, subjects were asked whether they wanted to know their exact rank on the math test. Although this information was free, roughly 30 percent of individuals declined to learn this information. Overwhelmingly, these were individuals who had scored in the bottom half. This finding suggests that people have a strong aversion to learning that they are one of the worst in terms of ability. At the end of the paper we discuss the implications of both the gender difference and psychological costs of search for the predictions of our model.

3 An Equilibrium Search Model with Type Uncertainty

In this section we construct a general equilibrium model of the labor market. We develop the model incrementally, beginning by describing the behavior of workers and then characterizing the steady state. Without adding the complexity of firm behavior, or wage determination, this portion of the model makes it possible to analyze the implications of our alternative assumptions in a partial equilibrium. We then proceed to add the firm side of the market and wage bargaining, and allow for endogenous wages and vacancy posting to close the model. We conclude the section by characterizing equilibrium in the full model, and performing comparative statics exercises.

3.1 Workers

In our model, workers are either unemployed or employed. Unemployed workers decide in each instant whether to search actively for a job or not. Active search entails a flow cost of c, and with some probability leads to a job offer. Passive, or inactive search is costless but results in a job-finding probability of zero.[8] Employed individuals are assumed not to search while they have a job, and to lose their jobs and re-enter the pool of unemployed with rate δ, reflecting exogenous job destruction due to bankruptcies, plant closings, *etc.*. Regardless of whether they are unemployed or employed, workers leave the labor force at a rate ρ, due to death or retirement. New workers of either type are born, and enter the pool of unemployed at the same rate ρ. Workers discount the future at rate r.

We depart from a standard model by assuming that individual workers do not know their objective job-finding probabilities with certainty. This assumption is motivated by our finding that people are uncertain about their relative ability even in the simple environment implemented in our laboratory experiment. This type of uncertainty is likely to be even more pronounced in real labor market decisions, given that these involve a much larger pool of competing applicants, and multiple skill dimensions rather than only one. Under the reasonable assumption that job-finding probabilities in the labor market depend on relative ability, this uncertainty about the self leads to uncertainty about one's job-finding probability. For simplicity, we assume that there are only two types of workers

[8] Alternatively we can allow for a non-zero job-finding probability for passive searchers, without changing the main insights of the model as is discussed below. Without costs for search, however, individuals would never stop searching.

in the market, high and low, with different job finding rates λ^h and λ^l, respectively. The precise job-finding rates are determined in equilibrium, and are increasing in labor market tightness, θ, as discussed below. However, we impose the restriction $\lambda^h > \lambda^l$, *i.e.*, the high type has a relatively good chance of finding a job while the low type has a relatively poor chance. In fact, we will assume below that search is always worthwhile for a worker who knows with certainty that she is a high type, and never worthwhile for a worker who knows she is a low type. We normalize the labor force to one and assume that a fraction $q^l = q \in (0, 1)$ are low types, with the remaining fraction $q^h = (1 - q)$ consisting of high types. The objective job-finding probabilities for high and low types, and the fraction of high and low types in the population, are both common knowledge to workers and firms.

The key assumption of the model is that workers do not know their true type with certainty. They are assumed to have an initial prior abut the likelihood of being a high type, equal to the fraction of high types in the population, $p^h(0) = q^h$. We assume that firms are also uninformed about a worker's true type, because this simplifies the analysis considerably. In particular, it precludes the possibility that firms exploit an information asymmetry in wage bargaining with workers. We also assume that productivity on the job, and therefore the wage, does not depend on whether a worker is a high or low type. In this case types can still be interpreted as reflecting differences in relative ability, but ability in the search process itself, *e.g.*, in writing resumes or interviewing, rather than in the workplace.[9] As will become clear below, this simplifying assumption does not imply that there is only a single wage in the market. Rather, a worker's wage will depend on the worker's level of confidence about being the high type, through wage bargaining. Thus there will be a distribution of wages across individuals, but due to different confidence levels rather than due to differences in randomly drawn productivity as is typically the case in search models.

A worker's type plays a crucial role in her choice of whether to search actively or not, because it affects the expected value of search. In the model, the only information that a worker can use to infer something about her true type is the cumulative time, t, she has spent unemployed so far. We assume that workers use Bayes' rule to update beliefs, so that their subjective belief about the likelihood of being the high type is decreasing

[9] This assumption eliminates the complication of different, potentially non-stationary reservation wage policies on the part of workers. For a model considering this type of non-stationarity, see Burdett and Vishwanath (1988).

in the length of unsuccessful search t. This assumption is in line with the results of our experiment, in the sense that people were observed to update beliefs in a way that is qualitatively consistent with Bayes' rule.[10] Below, we will also impose the restriction that the initial prior is high enough that all individuals find it worthwhile to search in the initial period, regardless of their type. This ensures that some portion of the population finds it optimal to search. For simplicity we do not consider any informational content of the number of previous employment spells n of an individual with cumulative unemployment duration t. While the model could be extended to account for this, it would complicate the analysis considerably, because the optimal policies would be conditional on both t and n.[11] We also assume perfect recall, namely that when an individual loses her job due to job destruction she recalls the duration of unsuccessful search t that she had right before finding her job. This is not essential for the basic insights of the model; we could alternatively assume that the individual starts over with $t = 0$ each time she loses her job.

Formally, the worker's subjective belief about the probability of being the high type, conditional on t, is given by the continuous time version of Bayes' rule

$$p^h(t, \theta) = \frac{e^{-\lambda^h(\theta) \cdot t} \cdot (1-q)}{e^{-\lambda^h(\theta) \cdot t} \cdot (1-q) + e^{-\lambda^l(\theta) \cdot t} \cdot q}, \tag{1}$$

where $p^h(0) = 1 - q$ is the prior belief about the probability of being a high type. A longer cumulative unemployment duration t unambiguously implies a lower subjective belief of being a high type.[12] Importantly, beliefs need not converge to the truth, at least over the short run. It takes some time for a low type with a high initial prior to become certain that she is the low type, and similarly, a run of bad luck can convince a high type that she is a low type. Given her subjective beliefs about the probability of being a high type, the worker's subjective job-finding probability is simply the weighted average of the

[10] In fact, people were somewhat too conservative in their updating in our experiment. For simplicity we maintain the assumption of fully-rational updating, but in principle it is possible to modify the model, replacing Bayes' rule with a more conservative updating rule, in which case the main results are unchanged except that updating occurs more slowly.

[11] The equilibrium would have to be characterized by optimal policies for all combinations t and n and the respective consistent distributions, without altering the main insights of the model.

[12] By taking partial derivatives, we have

$$\frac{\partial p^h(t, \theta)}{\partial t} = \frac{\left(\lambda^l(\theta) - \lambda^h(\theta)\right) q(1-q) e^{-(\lambda^h(\theta) + \lambda^l(\theta))t}}{\left(e^{-\lambda^h(\theta) \cdot t} \cdot (1-q) + e^{-\lambda^l(\theta) \cdot t} \cdot q\right)^2} < 0$$

since $\lambda^l(\theta) < \lambda^h(\theta)$ for any θ.

job-finding probabilities of high and low types

$$\widetilde{p}\,(t,\theta) = \lambda^{l}(\theta) + p^{h}\,(t,\theta)\cdot\Big(\lambda^{h}(\theta) - \lambda^{l}(\theta)\Big)\ . \tag{2}$$

This subjective job-finding probability is clearly also decreasing in t.

We can now characterize the unemployed worker's decision problem formally. Let $U\,(t)$ denote the expected discounted value of being unemployed for a worker with unemployment duration t, and let $W\,(t)$ denote the expected discounted value of a newly found job, which also depends on t through subsequent search behavior in the case that the job is destroyed. When unemployed, the worker receives an unemployment benefit, b, which is independent of time and unemployment duration. The benefit level is assumed to be sufficient to cover the worker's basic needs, but to be sufficiently low for her to accept any job offer she might receive, i.e. $W\,(t) - U\,(t) > 0$. The value of unemployment for a worker with history t can then be characterized as

$$(r+\rho)U(t) = b + \max\left\{\widetilde{p}(t,\theta)\cdot(W(t) - U(t)) - c + \dot{U}(t), 0\right\}\ , \tag{3}$$

where the expressions in curly brackets give the value of searching actively and searching passively, respectively. The last expression, $\dot{U}\,(t)$, is the derivative with respect to unemployment duration t and reflects the decline in utility associated with another instant of unsuccessful search, as discussed in detail below.

Clearly, the unemployed worker will choose to search actively as long as the expected benefit from doing so exceeds the utility of passive search. This implies that there exists a critical subjective job-finding probability such that the individual is just indifferent between searching and not searching. We assume that high types always find it worthwhile to search, given full information about their type, and that low types never find it worthwhile to search, which is equivalent to $\lambda^{h}(\theta) > \widetilde{p}\,(T,\theta) > \lambda^{l}(\theta)$, where the critical probability $\widetilde{p}\,(T,\theta)$ is given by

$$\widetilde{p}\,(T,\theta) = \frac{c}{W\,(T) - U\,(T)}. \tag{4}$$

For each individual this probability is associated with a threshold duration of unemployment, $T < \infty$, sufficient to reduce their confidence about being the high type to the point that their subjective job-finding probability is equal to $\widetilde{p}\,(T,\theta)$. We will refer to unemployed workers with $t > T$ as discouraged workers, *i.e.*, workers whose personal search history has lead them to revise their subjective beliefs downwards, to the point where they no longer see a point in searching.[13] Note that high types should always search, by assumption, but will become discouraged and stop search if they are sufficiently unlucky to remain unemployed for more than T. Low types should never search, but will search until they reach T, assuming that they were sufficiently uncertain about their type to justify search initially.

To show that T exists and is unique for each individual, we use the fact that workers process no new information once they become discouraged and stop search. This implies that they have stationary values for $U(t) = \underline{U}$, and $W(t) = \underline{W}$, where $\underline{U}$ satisfies

$$(r + \rho)\, \underline{U} = b\,, \tag{5}$$

and $\underline{W}$ is discussed below. Active searchers with durations $t \leq T$, on the other hand, face a value of being unemployed of

$$(r + \rho)\, U\,(t) = b - c + \widetilde{p}\,(t, \theta)\,(W\,(t) - U\,(t)) + \dot{U}\,(t)\ . \tag{6}$$

Optimality of T implies $U(T) = \underline{U}$ and $W(T) = \underline{W}$, which can be used to rewrite condition (4) as

$$\widetilde{p}\,(T, \theta) = \frac{c}{\underline{W} - \underline{U}}\ . \tag{7}$$

In a later section, we discuss the process of wage formation in detail, which is based on Nash bargaining between workers and firms. Bargaining occurs after a firm and a worker have met, and all characteristics of the match have been revealed, in particular t for the worker. For the moment, let $w(t, \theta)$ denote the wage of a worker that has an unemployment history of cumulative duration t, given an aggregate labor market characterized by θ. The value of being employed for a worker with cumulative unemployment duration $t \leq T$ can then be characterized as

$$(r + \rho)\, W\,(t) = w(t, \theta) + \delta\,(U\,(t) - W\,(t)) + \dot{W}(t)\,, \tag{8}$$

while the value for discouraged workers with $t > T$ is given by

$$(r + \rho)\, \underline{W} = w(T, \theta) + \delta\,(\underline{U} - \underline{W})\ . \tag{9}$$

Using the value of being unemployed for discouraged workers given by (5), one obtains

$$\underline{W} - \underline{U} = \frac{w(T, \theta) - b}{r + \rho + \delta}\,, \tag{10}$$

[13] Note that once individuals become discouraged they stop search and thus receive no new information. This implies $\dot{U}(t) = 0$ for all $t \geq T$.

which then makes it possible to express the critical condition for the subjective job offer arrival rate as

$$\widetilde{p}\,(T,\theta) = \frac{c}{w(T,\theta) - b}(r + \rho + \delta)\,. \tag{11}$$

This condition is crucial in the model, implicitly determining the maximum cumulative unemployment duration T after which individuals stop searching actively for employment. In partial equilibrium, i.e. taking the wages as exogenously given, existence follows directly from condition (11) and the fact that $\widetilde{p}\,(t,\theta)$ is strictly falling in t. In general equilibrium, the threshold also exists and is unique, since, as shown below, equilibrium wages depend negatively on unemployment duration and therefore on T. This implies that the right hand side is strictly increasing in T, while the left hand side falls in T.

In summary, we assume that the job-finding probability depends on the worker's true type, but that workers are uncertain of their type. Workers use search outcomes to update their beliefs, where the outcome is the cumulative duration of unsuccessful search. This updating in turn determines search decisions. Once the duration of unemployment reaches a critical threshold T, the likelihood of being a hight type, and thus the subjective job-finding probability, is perceived as being too low to justify further search. Importantly, uncertainty causes low types to search too long in the model, and can cause high types to stop search too early.

3.2 Steady State

Having defined the behavior of workers, we next characterize the conditions required to hold in a steady state. This yields a sufficiently developed framework for a partial equilibrium analysis focusing on worker behavior, without worrying about firms or how job-finding probabilities and wages are determined.

Note that there are, in principle, four groups of workers in the model. For characterizing the steady state it is important to keep track of all of these. For each type $j \in \{l, h\}$ we denote the proportion of unemployed that search actively by a_u^j, and let a_e^j be the proportion of employed workers who would search actively if they lost their job. Let d_u^j be the proportion of unemployed of each type that are discouraged, that is, have experienced a cumulative unemployment duration exceeding T. Finally, let d_e^j be the proportion of employed of type j that would be discouraged if they became unemployed. By definition, these proportions satisfy

$$a_u^h + d_u^h + a_u^l + d_u^l \;=\; 1\,, \quad \text{and} \tag{12}$$

$$a_e^h + d_e^h + a_e^l + d_e^l \;=\; 1\,. \tag{13}$$

We denote the proportion of the labor force that is unemployed by u. Given that the fraction of low (high) types in the labor force is q $(1-q)$ at any point in time, the following must also hold

$$\left(a_u^l + d_u^l\right) u + \left(a_e^l + d_e^l\right)(1-u) \;=\; q\,, \quad \text{and} \tag{14}$$

$$\left(a_u^h + d_u^h\right) u + \left(a_e^h + d_e^h\right)(1-u) \;=\; 1-q\,. \tag{15}$$

Let $f_u^j(T)$ be the density of actively searching unemployed, of type j, who become discouraged at any point in time, because their cumulative unemployment duration reaches T. This density differs across types because of the difference in job finding rates. Note that workers cannot become discouraged while being employed.

With this notation in hand we can now characterize the steady state, which is reached when all of the following are stationary: $a_u^j, d_u^j, a_e^j, d_e^j$, the unemployment rate, u, and the density of unemployed individuals becoming discouraged in each moment of time, $f_u^j(T)$. In other words, steady state is characterized by a collection of balancing conditions, such that the flows in and out of each of these states are equal, for both types of workers, $j \in \{l, h\}$. Using such conditions, one can derive an expression for the discouragement density $f_u^j(T) = \gamma^j \frac{e^{-\gamma^j T}}{1-e^{-\gamma^j T}}$, with $\gamma^j = \rho \frac{\rho+\delta+\lambda^j}{\rho+\delta}$, $j = l, h$.[14] This density is strictly decreasing in T. Solving the steady state conditions for the fractions of active and discouraged workers in and out of employment, and using the expression for $f_u^j(T)$ one obtains

$$u a_u^j \;=\; \frac{\rho+\delta}{\rho+\delta+\lambda^j}\left(1-e^{-\gamma^j T}\right) q^j\,, \tag{16}$$

$$(1-u)\, a_e^j \;=\; \frac{\lambda^j}{\rho+\delta+\lambda^j}\left(1-e^{-\gamma^j T}\right) q^j\,, \quad \text{and} \tag{17}$$

$$u d_u^j \;=\; e^{-\gamma^j T} q^j\,. \tag{18}$$

These conditions express the masses of different groups of employed and unemployed workers in terms of exogenous parameters.[15] One can easily verify that restrictions (14)

[14] The full derivation of the steady state conditions can be found in Appendix A.1.

[15] Note that in steady state the mass of employed workers who would immediately discouraged upon losing their job is zero.

and (15) are satisfied. This completes the characterization of the steady state, treating the job finding probabilities and the wage schedule as given. Note that the masses of (potential) active searchers $(1-u)\,a_e^j$ and ua_u^j are strictly increasing in T, whereas the masses of discouraged are strictly decreasing in T, as one would expect.

3.3 Labor Market Tightness and Job Arrival Rates

In this section we move beyond a partial equilibrium framework, defining the process that matches workers and firms. This matching process is assumed to be a function of labor market tightness, θ, which is given by the effective number of unemployed searchers in the market divided by the number of job vacancies available. Through the matching process, θ determines the equilibrium job-arrival rates for workers and the applicant arrival-rate for firms.

One component of labor market tightness is the effective number of unemployed workers searching for jobs. A formal investigation of the equilibrium requires an explicit treatment of the effective number of unemployed that determines the effective labor market tightness faced by all agents on the labor market. This effective mass of searching unemployed workers can be characterized in terms of efficiency units as

$$\tilde{u} = \left(a_u^h + \frac{\lambda^l}{\lambda^h} a_u^l\right) u \,,$$

where the search efficiency of actively searching high types is normalized to one, low types have a strictly lower relative search efficiency since $\lambda^l < \lambda^h$, and the search efficiency of inactive unemployed workers is zero. The second component of labor market tightness is the number of vacancies, v, available in the market. Firms looking for workers are assumed to post vacancies according to their hiring policy, which is described in the next section.

The labor market exhibits search frictions, so productive matches do not form instantaneously, but require a costly search process by both firms and unemployed workers. Matches between searching firms and searching unemployed workers are assumed to arise randomly, search is undirected. The flow of successful job matches arising per unit of time is generated by a matching function m such that

$$m = m\left(\tilde{u}, v\right) \,, \tag{19}$$

where m is increasing and concave in both arguments, exhibits constant returns to scale and satisfies $m(0,v) = m(\tilde{u},0) = 0$.

Given this matching function, the rate of job-arrivals per effective searcher is $m(\tilde{u}, v)/\tilde{u} = m(\theta)$, where $\theta = \frac{v}{\tilde{u}}$ is market tightness in terms of search efficiency units. At some points the discussion below will be facilitated by making specific assumptions about the relationship between the job finding probabilities of high and low types. Since we assume that $\lambda^l < \lambda^h$, a natural assumption to ensure this is that low types have a discretely lower job-arrival rate. The job finding probabilities for high and low types are then given by

$$\lambda^h = m(\theta), \text{ and } \lambda^l = \max\{m(\theta) - \phi, 0\}, \tag{20}$$

respectively, with $\partial\lambda^l/\partial\theta > 0$ and $\partial\lambda^h/\partial\theta > 0$.[16] We will discuss the role of this assumption and the effects of alternative assumptions where needed. For firms, the analogue is the applicant arrival rate, or number of matches per vacancy, given by $m(\tilde{u}, v)/v = m(\theta)/\theta$, which is decreasing in θ.

3.4 Vacancy Posting

We now specify the hiring policy of firms in the model, which determines the number of vacancies in the market. We assume that jobs are created by small firms, offering only one job at a time, where v denotes the total number of vacancies posted in the economy. The productivity of a newly hired worker does not to depend on the worker's unemployment history t. Nevertheless, as mentioned earlier, t will affect the value of the match to the firm, through its impact on the wage that the firm has to pay. We assume that firms have to post a vacancy before it is possible to encounter a match. As long as a job is vacant, the firm incurs a flow cost κ for maintaining the vacancy. Therefore, firms decide whether to post a vacancy or not by comparing the expected cost of the vacancy to the expected wage that will have to be paid to the worker. Free entry of firms implies that the expected vacancy cost has to be equal to the expected return from the job for the firm, $EJ > 0$, where E denotes the expectations operator over worker types. Importantly, this must be true *ex ante*, that is before a particular match has been formed and the unemployment duration t of the respective worker is known. Free entry (with positive job creation) implies the following[17]

$$\kappa = \frac{m(\theta)}{\theta} EJ. \tag{21}$$

[16] The maximum operator is needed to rule out negative job finding probabilities.

[17] Note that we assume that q, T etc. are common knowledge and therefore known to firms.

Given the assumptions on $m(\tilde{u}, v)$, this condition determines the equilibrium value of market tightness θ, as will be shown below.

With free entry of firms, the value of a vacancy is zero in equilibrium. As soon as a match between a vacancy and an unemployed applicant is realized, however, information about the worker's cumulated unemployment duration t is revealed. This t determines the worker's threat point and thus has an impact on wage bargaining. What is important from the firm's point of view is that a filled job generates an expected payoff that depends on the particular wage that has to be paid to the worker with state variable t. The value of a filled job to the firm is then given by

$$(r + \rho + \delta)J = y - w(t, \theta) . \tag{22}$$

3.5 Wage Determination and Search Decisions

Wages are a key determinant of vacancy posting and job creation behavior on the side of firms. At the same time, wages determine the search decisions of workers. The set-up of the model implies that wages are determined once a match has been formed and all match characteristics, such as the worker's cumulative unemployment duration at the time of hiring, have materialized. We assume that the surplus S that a new match generates is divided according to the generalized Nash solution to the bargaining problem. Let β denote the workers' bargaining power. Then, the solution to the bargaining problem $\max_{W(t)-U(t),J(t)}(W(t) - U(t))^{\beta}(J(t))^{1-\beta}$ s.t. $S(t) = W(t) - U(t) + J(t)$ can be characterized as

$$\frac{W(t) - U(t)}{\beta} = S(t) = \frac{J(t)}{(1 - \beta)} \tag{23}$$

as long as the surplus $S(t)$ is non-negative. The key issue to note is that a worker's threat point in the wage negotiations depends on his subjectively perceived job finding probability, $\tilde{p}(t, \theta)$, and, in particular, his subjective probability of being a high type, $p^h(t, \theta)$, which determines the value of being unemployed. Once a match is realized and firm and worker begin bargaining, the unemployment 'clock' is stopped, and the workers threat point does not decrease during the negotiations. This rules out that any party can change the surplus by delaying.[18] Using (3), (8) and (22), and eliminating $(r+\rho+\delta)$, the

[18] In this sense, the problem differs from a non-stationary bargaining problem where the possible payoffs change over time even during the negotiations, as in the environment studied by Coles and Muthoo (2003).

wage can be expressed as

$$w(t,\theta) = \beta y + (1-\beta)(b-c) + (1-\beta)\tilde{p}(t,\theta)\left(W(t) - U(t)\right) + (1-\beta)\left(\dot{U}(t) - \dot{W}(t)\right) . \quad (24)$$

The last term reflects the fact that another instant of unsuccessful search alters (more precisely, decreases) the worker's self-evaluation, and therefore the values of being unemployed and employed, respectively.[19] In the current context, these terms can not simply be dismissed since they affect the worker's position in the bargaining, and therefore imply a discount on the wage, as we now show. From the sharing rule in equation (23), one can solve for $U(t) - W(t) = \frac{\beta}{1-\beta}\frac{1}{r+\rho+\delta}\left(w(t) - y\right)$. Taking derivatives with respect to t, one obtains an expression for $\left(\dot{U}(t) - \dot{W}(t)\right)$ that can be substituted into the wage expression. In this case, the wage function is characterized by a non-homogeneous, non-autonomous ordinary differential equation that can be written as

$$w(t,\theta) = \frac{r+\rho+\delta+\tilde{p}(t,\theta)}{r+\rho+\delta+\beta\tilde{p}(t,\theta)}\beta y + \frac{(1-\beta)(r+\rho+\delta)}{r+\rho+\delta+\tilde{p}(t,\theta)}(b-c) + \frac{\beta}{r+\rho+\delta+\beta\tilde{p}(t,\theta)}\dot{w}(t,\theta) . \quad (25)$$

A general solution to this ordinary differential equation exists and can be obtained by applying an integrating factor method.[20] To see how the logic of discouragement shows up in the wage distribution, consider the wage of an individual that is just about to give up search at $t = T$. Using the fact that no new information arrives in the next instant when search is abandoned, $\dot{w}(T,\theta) = 0$, it turns out that the wage in the last instant of search is given by the respective share of the surplus without taking into account any option value of future job offers.[21] Using condition (7) and simplifying, the wage expression (25) can be simplified to yield

$$w(T,\theta) = \beta y + (1-\beta)(b-c) + (1-\beta)\tilde{p}(T,\theta)\left(\underline{W} - \underline{U}\right) = w(T,\theta) = b + \beta(y-b) . \quad (26)$$

Solving the differential equation yields a unique solution for the wage function conditional on T, θ, and u.

Given the solution to the wage function, we can now also explicitly state the assumptions mentioned in the discussion of worker behavior before, which ensure that search is

[19] It is important to note, however, that the time derivatives are derivatives with respect to unemployment duration, i.e. $\dot{U}(t) = \partial U(t)/\partial t$, not with respect to actual (calendar) time.

[20] See Appendix A.2 for details.

[21] Put differently, this condition implies a terminal condition that the differential equation (25) has to solve, see Appendix A.2.

optimal for a subset of the unemployed population. Without loss of generality and for the sake of illustration, in the following we assume first that active search is always preferred to inactivity when individuals first enter the labor market with their initial (flat) prior. Secondly, we assume that inactivity is better than active search if an individual is certain to be a low type. Formally, these assumptions imply

$$\tilde{p}(0,\theta) > \frac{(r+\rho+\delta)c}{\beta(y-b)} ,$$
$$\text{and} \quad \tilde{p}(p^h = 0,\theta) < \frac{(r+\rho+\delta)c}{\beta(y-b)} .$$

Taken together, these conditions imply that search tends to be better than inactivity for small t, while for large t the opposite is the case because individuals become certain that they are the low type.

We now turn to the determination of the labor market equilibrium.

3.6 Labor Market Equilibrium

Intuitively, a labor market equilibrium is given by a search threshold T^*, a labor market tightness θ^* and a level of unemployment u^*, such that individuals stop search optimally given their beliefs about themselves, firms post vacancies optimally with free entry ensuring the value of a vacancy being zero, and the level of unemployment as well as the composition of active and discouraged unemployed is stationary. In this section we specify the equilibrium condition for each of these three endogenous variables. As we go along, we derive the properties of each condition that are needed to show that the equilibrium exists and is unique.

For determining the equilibrium, note that the wage function is strictly decreasing in unemployment duration over the entire support. That is, for a given labor market tightness θ, the wage decreases as the cumulative unemployment duration t of an individual increases, $\dot{w}(t,\theta) < 0$ for all searchers with $t \in [0,T)$.[22] Intuitively, individuals with low cumulated unemployment history t have a high subjective probability of being a high type, and thus can credibly extract higher wages in equilibrium than individuals with a long unemployment record, because the outside option is better with a low unemployment duration. Recalling the implicit condition (11), and the fact that $\tilde{p}(t,\theta)$ is strictly

[22] The result is shown in Appendix A.3.

decreasing in t, this implies that, for each θ, a search threshold T exists and is unique. By feeding the wage $w(T,\theta)$ back into the threshold condition (11), the condition that implicitly determines T in equilibrium is given by

$$G \equiv \tilde{p}(t,\theta) = \phi m(\theta) + (1-\phi)m(\theta)p^h(T,\theta) = \frac{c(r+\rho+\delta)}{\beta(y-b)} \,. \tag{27}$$

This implies that, for any state of aggregate labor market conditions, θ and u, there exists a unique search threshold, T, such that condition (27) holds. Given the assumptions on the job finding rates made in (20), one can rewrite $p^h(t,\theta) = \frac{1}{1+\frac{q}{1-q}e^{\phi t}} = p^h(t)$ and $\tilde{p}(t,\theta) = -\phi + m(\theta)(1+p^h(t)$. With this, and using the threshold conditions (11) and (26), the threshold can be explicitly written as function of equilibrium labor market tightness θ^*,

$$T(\theta) = \frac{1}{\phi}\ln\left[\left(\frac{m(\theta)\beta(y-b)}{c(r+\rho+\delta)+\phi\beta(y-b)}\right)\frac{1-q}{q}\right] \,. \tag{28}$$

Hence, in equilibrium, the search threshold T is strictly increasing in labor market tightness θ. This implies that the higher θ in equilibrium, the larger is the unemployment duration T before people get discouraged in equilibrium.[23]

Note that, although there is a unique discouragement threshold T^*, there is a distribution of wages along the dimension of cumulative unemployment duration. More precisely, in equilibrium wages are distributed according to the density $\frac{a_e^l f_e^l(t)+a_e^h f_e^h(t)}{a_e^l+a_e^h}$ on a support that is bounded by the range of subjective probabilities $p^h(t)$ that sustain active search behavior (i.e. in equilibrium, wages only exist for individuals of unemployment duration $t \in [0, T^*]$).

The second equilibrium condition characterizes the optimal vacancy posting behavior of firms, and thus the equilibrium value of θ. In equilibrium the costs of posting a vacancy must be exactly outweighed by the expected value of a job, which depends crucially on

[23] This positive relationship follows from the assumption in (20) concerning the different job finding probabilities for high and low types. Allowing for a positive job finding probability $\underline{\lambda}$ even under inactivity would leave the results unaltered if it is unrelated to labor market conditions θ. With a $\underline{\lambda}$ that increases in θ just as the job finding probabilities of active searchers of either type, however, the relationship between θ and T would be negative. In this case inactivity is relatively more attractive if labor market conditions are better. Finally, assuming that low types have a proportionally lower job finding probability than high types, i.e. $\lambda^l = \varphi\lambda^h$, $0 < \varphi < 1$, the sign of $\frac{\partial T}{\partial \theta}$ would be *a priori* ambiguous. As long as the direct, positive effect on the labor market outweighs the indirect, negative effect through self perception in that case, the subjective job finding probability $\tilde{p}(t,\theta)$ would be higher for a labor market with more vacancies per searcher, and thus $\frac{\partial T}{\partial \theta} > 0$, leading to the same results as in the presented scenario. In this context it is worth noting that the model can also be solved for a negative relationship between T and θ. In this case better labor market conditions imply a faster discouragement of unemployed. While changing the comparative statics results below, this would provide different, but potentially interesting implications.

the expected wage. Note that the firm has no way to influence the type of worker in terms of t that it encounters in the frictional labor market. From the characterization of the wage function before, wages are strictly decreasing in the t of the worker the firm is eventually matched with.[24] Note also that the subjective probability of being a high type at T defines the lower bound for the wage distribution, while the upper bound is given by the prior probability of being a high type that one presumes (e.g. $(1-q)$ with a flat prior). The expected wage therefore depends on the expected unemployment duration of the available workers that the firm can be matched with. Hence, in equilibrium it must hold that

$$\kappa = \frac{m(\theta)}{\theta} EJ = \frac{m(\theta)}{\theta} \frac{y - Ew(t,\theta)}{r+\rho+\delta} = \frac{m(\theta)}{\theta} \frac{y - w\left(E(t|T^*,\theta),\theta\right)}{r+\rho+\delta} \equiv H \,. \tag{29}$$

Because the probability to be matched with an applicant decreases with θ, $\frac{\partial(m(\theta)/\theta)}{\partial\theta} < 0$, we have that the right hand side of the condition is strictly falling in θ, $\frac{\partial H}{\partial \theta} < 0$.[25]

To evaluate the behavior of the function H with respect to T^*, we again use the fact that the wage is decreasing in the unemployment duration of a worker that is hired, which implies that the wage is also decreasing in the expected duration. But the expected duration is a strictly monotonic increasing function of the search threshold T^*, $\frac{\partial E(t|T^*,\theta)}{\partial T^*} > 0$, and so the right hand side of condition (29) increases in T^*, $\frac{\partial H}{\partial T^*} > 0$.[26] This implies a one-to-one relationship between the optimal search threshold T^* and firms' vacancy posting, which determines θ. In particular, the higher the search threshold T^*, the more it pays off for firms to post vacancies, and therefore the higher is θ.

The third equilibrium condition concerns the level of unemployment. The first thing to note is that for each level of unemployment u, there is a unique level of effectively searching unemployed $\tilde{u}$, so one can restrict attention to either of these variables in the determination of the equilibrium.[27] In the following, we consider the equilibrium level of aggregate unemployment u.

[24] This is shown formally in Appendix A.3.

[25] Under mild assumptions on ϕ, one can also show that the wage is strictly increasing in θ for any t, see Appendix A.3.

[26] This result follows from applying the Leibniz rule to both densities f_u^j, $j = h, l$, since the expected unemployment duration $E(t|T,\theta)$ is an average of the expected unemployment durations of both types weighted by their respective share of the total pool of unemployed. Because $E(t^j|T,\theta) = \int_0^T f_u^j(t) t dt = \int_0^T \gamma^j \frac{e^{-\gamma^j t}}{1-e^{-\gamma^j T}} t dt$, we have $\frac{\partial E(t^j|T,\theta)}{\partial T} = \int_0^T (\gamma^j)^2 \frac{e^{-\gamma^j t} e^{-\gamma^j T}}{\left(1-e^{-g^j T}\right)^2} t dt + \int_0^T \gamma^j \frac{e^{-\gamma^j t}}{\left(1-e^{-g^j T}\right)} dt + \gamma^j \frac{e^{-\gamma^j T}}{1-e^{-\gamma^j T}} > 0$.

[27] The proof of the injective relationship between u and $\tilde{u}$ depends on the assumptions concerning the relative search effectiveness of the two types in terms of λ^l and λ^h. Given the assumptions in (20),

The level of unemployment only affects the level of vacancy posting through its effect on labor market tightness, and does not affect the structure of wages, which is entirely determined by the pattern of unemployment duration t. For consistency between expected wages and the wages that firms actually have to pay, we assumed before that firms cannot strategically target unemployed with certain unemployment histories. Using conditions (16) and (17) and the steady state condition for unemployment (35), the equilibrium unemployment rate can be expressed as

$$u = 1 - \frac{\lambda^h q}{\rho+\delta+\lambda^h}\left(1 - e^{-\rho\frac{\rho+\delta+\lambda^h}{\rho+\delta}T}\right) - \frac{\lambda^l(1-q)}{\rho+\delta+\lambda^l}\left(1 - e^{-\rho\frac{\rho+\delta+\lambda^l}{\rho+\delta}T}\right), \qquad (30)$$

using the implicit notation of before.[28] Taking derivatives shows that unemployment is strictly falling in the horizon until unemployed become discouraged, *i.e.*, $\frac{\partial u}{\partial T}|_\theta < 0$.[29] Likewise, unemployment is lower if labor market tightness is higher, *i.e.*, $\frac{\partial u}{\partial \theta}|_T < 0$.[30]

The steady state equilibrium is fully characterized by a vector $\{T^*, \theta^*, u^*\}$ for which conditions (27), (29) and (30) hold simultaneously, and by a composition of the population a_u^j, a_e^j and d_u^j such that the steady state conditions (16), (17) and (18) hold as well. Hence, together with the steady state conditions, the equilibrium is characterized by the unique solution of a system of six equations in six unknowns. Apart from the unemployment rate u^*, all these variables are jump variables. The properties of the equilibrium conditions, identified above, imply that conditional on the prior the equilibrium exists and is unique.[31] The results so far are sufficient to illustrate the working of the model.[32]

$\frac{\tilde{u}}{u} = a_u^h + a_u^l \frac{\max\{m(\theta)-\phi\}}{m(\theta)}$. The claim then follows, because the left-hand side is strictly increasing in $\tilde{u}$, while the right hand side is strictly decreasing in $\tilde{u}$. This can be seen since the right hand side increases in $m(\theta)$ while $\partial m/\partial\theta > 0$ and $\partial\theta/\partial\tilde{u} < 0$. For alternative assumptions like a proportional mark-off in search efficiency of low types φ mentioned in footnote 23, an analogous proof applies.

[28] Given the assumptions on the job finding probabilities λ^j, this condition can be explicitly written as

$$u = 1 - \frac{m(\theta)q}{\rho+\delta+m(\theta)}\left(1 - e^{-\rho\frac{\rho+\delta+m(\theta)}{\rho+\delta}T}\right) - \frac{(m(\theta)-\phi)(1-q)}{\rho+\delta+(m(\theta)-\phi)}\left(1 - e^{-\rho\frac{\rho+\delta+(m(\theta)-\phi)}{\rho+\delta}T}\right).$$

[29] This result can be seen by the fact that $\partial(1-e^{-\gamma^j T})/\partial T > 0$.

[30] The result follows since $\partial(1-e^{-\gamma^j T})/\partial\theta > 0$ and $\partial(\frac{m(\theta)(1-q)}{\rho+\delta+m(\theta)})/\partial\theta > 0$.

[31] See also appendix B. In case of type specific priors, there are two type specific search thresholds in equilibrium. More generally, if priors are dispersed, the equilibrium is characterized by a distribution of threshold durations that correspond to the priors. A detailed analysis of these cases is beyond the scope of this paper.

[32] The main results of this section do not depend on the assumption of wage determination through Nash bargaining. In a wage posting environment, a qualitatively similar wage distribution would arise. The problem faced by firms in this case would be very similar to the model by Albrecht and Vroman (2005), where individuals' unemployment benefits b vary, and can fall to a lower level $s < b$ according to a

To summarize, in equilibrium the outflows from unemployment into employment decline with cumulative unemployment duration, and become zero for durations above T^*. On the other hand, outflows into the inactive state of discouragement increase with unemployment duration, in the sense of a discrete jump at T^*. The extent of this negative duration dependence of the hazard into employment, and the positive duration dependence of hazards into discouragement is endogenously determined in equilibrium and characterized by T^* and the associated distributions f_u^i. Finally, wages decline with unemployment history in terms of t, reflecting the lower self-confidence in terms of subjective job finding probabilities that eventually determines the bargaining position of the unemployed. Overall, fewer individuals with high unemployment duration are employed.

3.7 Some Comparative Static Results

Having characterized equilibrium in the model, we can now perform comparative statics. We illustrate by discussing the impact of a change in the income stream from unemployment, b, and a change in productivity, y. Comparative statics with respect to other parameters follow directly, based on analogous arguments.

An unexpected, exogenous increase in the income stream from unemployment, $b' > b$, leads to workers becoming discouraged earlier, and increases the level of unemployment. To see this, note that an increase in b improves all workers' payoff from inactivity. From condition (27) this leads to an increase in the subjective probability that makes an unemployed worker indifferent between active search and discouragement. In other words, there is a new equilibrium unemployment duration T', such that the unemployed leave the active search pool earlier, i.e. $T' < T^*$. This, in turn, leads to an increase in the wages that firms expect to pay to newly hired workers, as implied by the discussion of condition (29), and therefore reduces the job creation activity of firms. A lower labor market tightness $\theta' < \theta^*$ and an earlier discouragement threshold $T' < T^*$, lead, in turn, to a higher

random process. With type uncertainty, it is not the value of unemployment that is changing along an unemployment spell, but the value of employment, due to the decreasing expected wage. In equilibrium, firms would post wages corresponding to the respective worker's reservation wage, and vacancies would depend on the reservation wage of the expected worker, i.e. the expected unemployment duration t. Firms would pay the reservation wage ex post, since offers that are below a worker's reservation wage are rejected. The main complication in such a model is the continuum of different reservation wages determined by the unemployed's beliefs about their type and characterized by their t. The determination of a continuous wage distribution with unknown types under wage posting is beyond the scope of this paper.

equilibrium unemployment level, $u' > u^*$, as can be seen from condition (28). We obtain the same comparative static result for an exogenous increase in the search cost, c, by an analogous argument. Higher search costs imply faster discouragement, and a higher level of unemployment.

Following an unexpected, exogenous increase in productivity to $y' > y$, the time it takes for workers to become discouraged increases, and unemployment decreases. This follows from the fact that, on the one hand, an increase in y implies that search is more profitable. This induces the unemployed to search longer, i.e. it must be that $T' > T^*$ for (27) to be satisfied because $\partial\tilde{p}(T^*, \theta)/\partial T^* < 0$. On the other hand, higher productivity makes vacancies more attractive, and thus for (29) to be satisfied, labor market tightness must increase. Consequently, the steady state is characterized by lower unemployment $u' <^* u$ from condition (28). Similar effects emerge for an unexpected exogenous decrease in vacancy costs κ. On the other hand, higher rates of r, ρ or δ have precisely the opposite effect of an increase in productivity, since they make vacancy posting and search more expensive. They lead to a lower θ and faster discouragement $T' < T^*$ in equilibrium.

As an aside, note that a policy enforcing active search, that is, increasing T^* exogenously regardless of whether this is optimal or not, unambiguously decreases unemployment since $u(T^* = \infty) < u(T^* = 0)$. Hence, if unemployed could be forced to search, not surprisingly, unemployment would be lower in steady state.

Beyond these comparative statics results, the presence of type uncertainty has important implications for the dynamics of the labor market. After an unexpected increase in productivity y, individuals' subjective beliefs about their type, and hence about their job finding probability, are unchanged. This is because the change in labor market conditions does not affect the information value of their previous search history in the old environment, and therefore does not affect their current subjective self-evaluation.[33] However, the threshold probability needed to justify search decreases with an increase in y. This induces individuals at the discouragement threshold to resume active search. In the model presented so far, there is a mass point of discouraged workers with unemployment duration T^*, so the response to an increase in y in terms of renewed search is substan-

[33] Recall that under the assumption (20) it follows that $p^h(t, \theta) = p^h(t) = \frac{1}{1+\frac{q}{1-q}e^{\phi t}}$. This claim would be also true for any change in labor market conditions under any other parametric specification of the difference in job finding probabilities, however, since there would be no reason to adjust the subjective belief retroactively.

tial. In fact, immediately after the increase, all workers in the economy will be searching actively, and will continue to do so until they reach the new unemployment threshold T'. This implies that type uncertainty leads to an amplification of the effects of changes in productivity on unemployment and vacancies, compared to a standard search model. The increase in y raises θ^* as jobs become more valuable for firms, but the attendant increase in T^* induces more individuals to search, which makes it even easier for firms to meet an applicant as labor market tightness is moderated by more searchers. This facilitates job creation, amplifying the firms' increase in vacancy posting. In a model with a non-degenerate distribution of unemployment durations above T^*, the increase in the pool of active searchers would be less pronounced but still existent. Thus, in comparison to a standard model without uncertainty, unemployment decreases more in response to an increase in y.

4 Explaining Search Behavior in the Field

In this section we turn to a discussion of field evidence. We show that the model developed above provides a new, unifying explanation for a collection of important stylized facts regarding search behavior and labor markets.

4.1 Duration Dependence, Discouragement, and Volatility

Field studies frequently document the existence of duration dependence in hazard rates. In particular, the evidence suggests that exit rates out of unemployment tend to decrease, and hazards into non-participation tend to increase, as unemployment duration lengthens (see Machin and Manning, 1999 for a review; for recent evidence see Frijters and van der Klaauw, 2006). One explanation discussed in the literature is that skills, or human capital may deteriorate as unemployment duration lengthens. This depreciation would reduce productivity and thus decrease the objective chances that an employer will want to hire the worker (Ljungqvist and Sargent, 1998, Sen, 1997). Another possibility is that employers cannot observe the current state of applicants' human capital, but use a long unemployment duration as a negative signal about productivity (see, *e.g.*, Berkovitch, 1985, Vishwanath, 1989; Blanchard and Diamond, 1994). Although both explanations are plausible, there is little direct evidence for either (Machin and Manning, 1999), partly because surveys typically do not measure objective skills of an individual over time, or the strategies of potential employers.

Our model highlights a different mechanism, uncertainty about the self, which has not been discussed in the literature but has a strong empirical basis in light of our laboratory experiment. Our experiment shows that uncertainty about the self, and updating, are relevant for search even in a relatively simple environment. The experimental evidence has the additional advantage that it rules out other possible explanations, such as human capital depreciation or stigma, by design. Incorporating these findings, our model predicts behavior that is consistent with the field evidence: as unemployment duration lengthens, individuals lose self-confidence and are increasingly likely to stop search, even though their objective job-finding chances are unchanged. We do not claim that human capital depreciation and stigma are irrelevant for explaining the evidence on exit rates, but the message from our model and our experimental findings is that declining self-confidence is likely to be an important factor as well. Moreover, declining self-confidence provides a realistic behavioral channel for the negative effects of unemployment duration for human capital investments and motivation, and consequential statistical discrimination of long-term unemployed.

Another application is explaining evidence on the state of non-participation. Our model offers a different perspective what it means to drop out of the labor force. The subjective aspect of job search highlighted in our model leads to a different interpretation of the non-participation choice, because search outcomes are only a noisy signal of ability, and thus some people who stop search and drop out of the labor force may in fact be types with high ability who have simply been unlucky. In this case these individuals are properly considered part of the problem of involuntary unemployment, in contrast to measures of unemployment based on the criteria of active search. Notably, explanations for non-participation such as human capital depreciation lead to a different interpretation because they imply non-participants are objectively unsuited for employment.

Looking at job search from the perspective of our model also sheds light on a long-standing puzzle regarding the behavior of a particular group of non-participants, those labelled as discouraged workers. Since the 1960's the Bureau of Labor Statistics has included questions in US labor force surveys that identify discouraged workers: workers who report wanting a job but nevertheless are not searching for one, because they believe their job-finding chances are too low. The discouraged worker phenomenon is typically hypothesized to be the result of the business cycle, *e.g.*, a recession causes a drop in objective job-finding probabilities and thus leads individuals at the margin to stop search. This notion has been an important source of policy debate for decades, as it implies that unemployment may be flawed as a welfare measure.[34] If some of those who do not search in fact want a job, they experience the same or worse hardship as the regular unemployed and should be counted as part of the social problem (Norwood, 1988). The puzzling finding is that discouraged workers do not respond strongly to aggregate economic conditions (see, *e.g.*, McElhattan, 1980). One interpretation of this finding is that discouraged workers do not actually want to work, even though they say that they do. However, Jones and Riddell (1999) show that discouraged workers have a significantly higher exit rate into employment than other non-participants, although not as great as for the unemployed. This suggests that the stated desire to work does convey meaningful information. The question, then, is why these individuals do not respond to economic conditions?

Our model provides one possible answer, because it shows how discouragement can be the outcome of an unsuccessful personal search history. An improvement in aggregate conditions would lead to an exogenous increase in an individual's job-finding probability in our model, and could cause a nonparticipant to restart search. However, it is not clear empirically how much weight individuals give to aggregate economic indicators when forming beliefs about job-finding chances, as opposed to information from previous search experiences. Also, even if an improvement in aggregate conditions leads the individual to resume search, in our model it may take only a short duration of unsuccessful search to push subjective beliefs back below the critical threshold. Thus, our model does not rule out that participation decisions respond to economic conditions, but suggests reasons why individuals may respond only weakly and perhaps temporarily to such indicators, even though they in fact desire to work.

In line with the evidence provided by Jones and Riddell (1999), our model provides a rationale that is consistent with permanent, structural flows between the states of unemployment and (marginally attached) non-participation. Outflows from unemployment into non-participation occur because of discouragement after reaching the threshold T^*.

[34] Discouraged workers have also been a source of controversy because of implications for the unemployment rate. Pro-cyclical participation decisions imply that unemployment rates are underestimated during recessions and overestimated during booms.

An extension of the model taking into account cyclical productivity shocks or likewise other shocks affecting individuals' incentives to search for employment, e.g. arising from private issues like marriage or child birth, can generate inflows from non-participation into (active) unemployment. Extending the model to allow for positive job finding probabilities even in the state of no active search would enable the model to generate flows from the state of discouragement back to employment.

Finally, workers' uncertainty has an impact on the dynamics of the labor market in our model, with implications for the current debate on volatility of unemployment. Empirically, unemployment is substantially more volatile in response to cyclical fluctuations than can be explained by standard models (Shimer, 2005; Mortensen and Nagypal, 2005). Our model generates higher variability of unemployment in response to productivity shocks than in standard search models. As discussed above, type uncertainty generates a higher variability of unemployment because of endogenous variations in the search threshold T^*. As labor market prospects become better, the critical unemployment duration beyond which individuals stop searching increases, thereby keeping more unemployed in the market searching. These improved labor supply conditions facilitate job creation by firms, and lead to a lower unemployment rate than without the additional feedback caused by uncertainty.

4.2 Extensions and Further Empirical Implications

The experimental findings from our companion paper suggest two possible extensions to the model. Incorporating these changes formally is beyond the scope of the current paper, but in this section we discuss how these changes would be likely affect the predictions and possible applications for the model.

One extension would be to allow for a gender difference in the initial prior about types, with women being less confident. This is in line with findings from our experiment, and with previous evidence suggesting that women are less confident given the same objective information, due to the presence of negative stereotypes about female ability (Spencer, Steele, and Quinn, 1999; Gneezy, Niederle and Rustichini, 2003). In this case the model implies shorter search duration and greater rates of non-participation for women. Lower confidence would also lead to lower wages for women, holding productivity constant, through its impact on the threat point in wage bargaining. Thus the model could help explain the well-known gender wage gap and greater non-participation rates for women, but based on the role of confidence in the search process, a mechanism that has not been discussed in the literature. This explanation is complementary to Bowlus (1997), who finds that differences in search behavior explain up to 30 percent of wage differences between men and women in the US, and concludes that a large part of these differences are driven by differences in reservation wages. By suggesting that subjective beliefs about the self differ between men and women, our model provides a rationale why reservation wages might differ sharply between men and women with observationally identical characteristics.

Another possible extension would be to incorporate a psychological cost of unsuccessful search. Such a cost could arise if people lose utility "ego utility" (Köszegi, 2001 and 2006) from receiving negative information about the self. Evidence from our experiment suggests such a cost does exists, and that it leads people to avoid further negative information about the self as a coping strategy. Modelling preferences over beliefs is beyond the scope of this paper, but the resulting psychological costs of unsuccessful search would presumably lead to lower willingness to search and faster discouragement. Incorporating ego utility would also provide a link between our model and the decline in self-esteem and mental health associated with increasing unemployment duration, documented in psychology. Unemployment has been shown to be associated with depression, lower health status, higher risk of mortality, and lower subjective well-being (see, *e.g.*, Gerdtham and Johannesson, 2003; Bjorklund and Eriksson, 1998; Mathers and Schofield, 1998; and Winkelmann and Winkelmann, 1998). There is also a large literature in psychology linking low self-esteem to depression and poor mental health.

5 Concluding remarks

This paper explores a simple idea: what happens when people are uncertain of their abilities, and thus search without knowing their objective chances of success? We develop a general equilibrium model of the labor market in which workers are uncertain of their type and update beliefs about their subjective job-finding chances based on the duration of unsuccessful search. The model offers a new perspective on several aspects of job-search behavior and unemployment, including falling hazards out of unemployment and lower wages for individuals with long unemployment history, the phenomenon of discouraged workers, the volatility of unemployment, gender differences in search behavior, and the damaging effects of unemployment on mental health.

The mechanism highlighted in the model is also relevant for search more generally. In most types of search, individuals are probably uncertain of their own abilities, and uncertain of their abilities relative to others. As a result, the process of updating and loss of self-confidence described in our model is likely to be relevant for these search behaviors as well. For example, a young researcher trying unsuccessfully to publish papers in top tier journals may eventually lose confidence in his or her own abilities, and begin to aim lower or even switch to another career. Likewise may a lengthy period of unsuccessful search for a partner in life induce a person to conclude that he is unattractive and therefore quit search for a mate.

Evidence supporting our assumptions comes from a laboratory experiment. In principle, it should also be possible to design surveys that to elicit individuals' beliefs about their relative abilities and job-finding chances, and their certainty about these beliefs. Analogous to our experiment, respondents could be asked how certain they are that they job finding chance in the next month is higher than 60 percent. Questions like this would allow an investigation of how duration of unsuccessful search affects confidence and future search decisions. It would also provide an indication of how confidence and updating varies according to different personal characteristics in the field. We believe that this is a fruitful direction for future research on explaining individual search behavior.

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A Appendix: Proof of Claims about the Wage Function

A.1 Derivation of Steady State Conditions

The steady state conditions stated in the text can be derived from equating inflows and outflows into the four respective states. Starting with workers who are employed but will be discouraged if they lose their job, $d_e^j(1-u)$, the inflows must offset the outflows implied by job loss or retirement. But there are no inflows back into employment for discouraged workers, because the job offer arrival rate for inactive unemployed is assumed to be zero, as is the probability of getting a job offer in the instant T. Thus, in steady state the mass of employed workers who would be immediately discouraged upon losing their job is zero.[35] This leads to a first steady state condition

$$d_e^j(1-u) = 0\,, \tag{31}$$

for $j = l, h$. Among the discouraged unemployed, $d_u^j u$, inflows due to newly discouraged unemployed must equal the outflows into employment or retirement.[36] Thus the following must hold

$$ua_u^j f_u^j(T) = \rho u d_u^j\,. \tag{32}$$

For actively searching unemployed, $a_u^j u$, inflows from new births and job losses must equal outflows due to discouragement, new jobs, and retirement

$$\rho q^j + \delta(1-u)a_e^j = ua_u^j f_u^j(T) + \left(\rho + \lambda^j\right) ua_u^j\,. \tag{33}$$

Inflows into the pool of workers $a_e^j(1-u)$, those who are employed but would search actively if they had to, result from new hires. These must equal outflows due to job destruction or death

$$\lambda^j ua_u^j = (\rho+\delta)(1-u)\,a_e^j\,. \tag{34}$$

If the unemployment rate is also to be stationary, inflows into unemployment due to new births and job loss must be exactly offset by outflows, which arise either due to active searchers finding new employment or due to death

$$\rho + \delta(1-u) = \lambda^h ua_u^h + \lambda^l ua_u^l + \rho u\,. \tag{35}$$

To complete the characterization of the steady state we need an expression for $f_u^j(T)$, the density of unemployed individuals of type j who have just reached T. In order to derive this expression we first characterize two cumulative distribution functions. Let $F_u^j(t)$ denote the cumulative distribution of actively searching unemployed over all unemployment durations t. Let $F_e^j(t)$ denote the corresponding cumulative distribution for workers who are employed but would search actively upon losing their jobs. To derive expressions for $F_u^j(t)$ and $F_e^j(t)$, consider a discrete time version of the model for small time increments Δ, and without loss of generality, consider just one type of worker.[37] The

[35] The model can be extended to allow for some low probability that inactive unemployed receive a job offer such that $(1-u)d_e^j > 0$.

[36] Note that from condition (31) the inflows from job destruction are zero.

[37] For simplicity, we suppress indices for types in the following derivation.

mass of 'duration type t' unemployed workers in time period s, denoted $x_u(t,s)$, evolves according to

$$x_u(t,s+\Delta) = x_u(t,s) - \Delta\lambda x_u(t,s) + \Delta\delta x_e(t,s) - \Delta\rho x_u(t,s) \ .$$

The first term on the right hand side reflects the fact that, for some unemployed, duration simply increases without a change in their status. The second term reflects outflows into employment, the third captures inflows due to job loss, and the final term reflects exit due to death. Likewise, the mass of 'duration type t' employed workers in time period s, denoted $x_e(t,s)$, evolves according to

$$x_e(t,s+\Delta) = x_e(t,s) + \Delta\lambda x_u(t,s) - \Delta\delta x_e(t,s) - \Delta\rho x_e(t,s) \ .$$

The next step is to impose the steady state conditions $x_k(t,s+\Delta) - x_k(t,s) = 0$, where $k = u, e$, and let $\Delta \to 0$. Dropping the time index, rearranging, and using $x_u(t) = a_u u f_u(t)$ and $x_e(t) = a_e(1-u) f_e(t)$ one arrives at

$$\begin{aligned} 0 &= -a_u u \frac{d}{dt} f_u(t) - \lambda a_u u f_u(t) + \delta a_e (1-u) f_e(t) - \rho a_u u f_u(t) \\ 0 &= \lambda a_u u f_u(t) - \delta a_e (1-u) f_e(t) - \rho a_e (1-u) f_e(t) \end{aligned}$$

The solution for $j \in \{l,h\}$ is then

$$f_e^j(t) = \tfrac{\lambda^j a_u}{(\rho+\delta)a_e} \tfrac{u}{1-u} f_u^j(t) \ , \quad \text{and} \quad \frac{d}{dt} f_u^j(t) = -\gamma^j f_u^j(t) \ ,$$

where $\gamma^j = \rho \frac{\rho+\delta+\lambda^j}{\rho+\delta}$. The solution of this system of differential equations can be shown to be

$$F_u^j(t) = \frac{1-e^{-\gamma^j t}}{1-e^{-\gamma^j T}} \quad \text{and} \quad f_u^j(t) = \gamma^j \frac{e^{-\gamma^j t}}{1-e^{-\gamma^j T}} \ . \tag{36}$$

Substituting T into (36) we arrive at the necessary expression for completing the steady state: $f_u^j(T) = \gamma^j \frac{e^{-\gamma^j T}}{1-e^{-\gamma^j T}}$, which is strictly decreasing in T.[38] The conditions (16) and (17) in the text can be derived by solving (33) and (34) and using the expression for $f_u^j(T)$. Finally, combining conditions (16) and (17) with (32), one obtains (18).

A.2 General Solution of the Wage Function

After some substitution, the wage function given by equation (25) can be expressed as

$$\begin{aligned} \dot{w}(t,\theta) \quad &- \quad \left[\frac{r+\rho+\delta}{\beta} + \tilde{p}(t,\theta)\right] w(t,\theta) = \\ &- \quad \left[(r+\rho+\delta+\tilde{p}(t,\theta))\, y + (r+\rho+\delta)\frac{1-\beta}{\beta}(b-c)\right] \ , \end{aligned}$$

which is equivalent to

$$\dot{w}(t,\theta) + P(t,\theta) \cdot w(t,\theta) = Q(t,\theta) \ , \tag{37}$$

[38] It is straightforward to show that $\frac{\partial}{\partial t} f_u^j(t) = -\gamma^j f_u^j(t)$, that $\frac{\partial}{\partial t} F_u^j(t) = f_u^j(t)$ and that $F_u^j(0) = 0$ and $F_u^j(T) = 1$.

where

$$\begin{aligned} P(t,\theta) &= -\left[\frac{r+\rho+\delta}{\beta} + \tilde{p}(t,\theta)\right] \\ Q(t,\theta) &= -\left[(r+\rho+\delta+\tilde{p}(t,\theta))\,y + (r+\rho+\delta)\frac{1-\beta}{\beta}(b-c)\right]. \end{aligned}$$

Using $e^{\int P(t,\theta)dt}$ as integrating factor (see e.g. Simon and Blume, 1994, pp. 637ff) the solution can be written as a wage function in terms of unemployment duration,

$$w(t,\theta) = \left[C + \int Q(t,\theta)e^{\int P(t,\theta)dt}dt\right] e^{-\int P(t,\theta)dt}\,, \tag{38}$$

where C is a constant still to be determined. Condition (26) provides a terminal condition that the differential equation (37) has to solve because wages are only defined for $t \in [0,T]$. Using this, one can determine a unique particular solution for the wage function by setting

$$C = \frac{Q(T,\theta)}{P(T,\theta)}e^{\int P(T,\theta)dt} - \int Q(T,\theta)e^{\int P(T,\theta)dt}dt\,. \tag{39}$$

Substituting this expression into (38) yields a unique solution for the wage function conditional on T, θ, and u.

A.3 Characterization of the Wage Function

Claim 1. *The wage is strictly decreasing in unemployment duration, $\dot{w}(t,\theta) < 0\ \forall\ t \in [0,T)$.*

Proof. To show the claim, first use conditions (6) and (8) to obtain

$$(r+\rho+\delta+\tilde{p}(t,\theta))\,(W(t)-U(t)) = w(t,\theta)-(b-c)+\left(\dot{W}(t)-\dot{U}(t)\right).$$

Taking time derivatives we have that

$$(r+\rho+\delta+\tilde{p}(t,\theta))\left(\dot{W}(t)-\dot{U}(t)\right) + (W(t)-U(t))\,\dot{\tilde{p}}(t,\theta) = \dot{w}(t,\theta)$$

applying an approximation with $\left(\ddot{W}(t)-\ddot{U}(t)\right) \approx 0$. From the solution of the bargaining problem, $(W(t)-U(t)) = \frac{\beta}{(1-\beta)}J(t) = \frac{\beta}{(1-\beta)}\frac{y-w(t,\theta)}{r+\rho+\delta}$, and $\left(\dot{W}(t)-\dot{U}(t)\right) = \frac{\beta}{(1-\beta)}\dot{J}(t) = -\frac{\beta}{(1-\beta)}\frac{\dot{w}(t,\theta)}{r+\rho+\delta}$. Substituting and collecting terms, one can express the wage increment as

$$\dot{w}(t,\theta) = \frac{\beta(y-w(t,\theta))\dot{\tilde{p}}(t,\theta)}{r+\rho+\delta+\beta\tilde{p}(t,\theta)}\,. \tag{40}$$

The claim follows since $(W(t)-U(t)) > 0$, or alternatively $y-w(t,\theta) > 0$, and $\dot{\tilde{p}}(t,\theta) < 0$ for any admissible t on the support, excluding the last instant of active search when $\dot{\tilde{p}}(T,\theta) = 0$. Hence, $\dot{w}(t,\theta) < 0\ \forall\ t \in [0,T)$ and $\dot{w}(T,\theta) = 0$, which proves the claim. □

Claim 2. *By making the difference in search effectiveness between high and low types ϕ is sufficiently large, one can ensure that the wage is strictly increasing in labor market tightness, $\frac{\partial w(t,\theta)}{\partial\theta} > 0\ \forall\ t \in [0,T^*]$.*

Proof. First note that, by using the sharing rule given by (23) to eliminate $W(t) - U(t)$ and substituting, the wage function (24) can be rewritten as

$$w(t,\theta) = \frac{(\beta + \beta\tilde{p}(t,\theta))\,y + (1-\beta)(b-c) + \frac{\beta}{r+\rho+\delta}\dot{w}(t,\theta)}{(1+\beta\tilde{p}(t,\theta))}\,.$$

Taking derivatives with respect to θ, one obtains that

$$\frac{\partial w(t,\theta)}{\partial\theta} = \frac{\left[\beta(1-\beta)(y-(b-c)) - \frac{\beta^2}{r+\rho+\delta}\dot{w}(t,\theta)\right]\frac{\partial\tilde{p}(t,\theta)}{\partial\theta} + \frac{\beta}{r+\rho+\delta}\left(1+\beta\tilde{p}(t,\theta)\right)\frac{\partial\dot{w}(t,\theta)}{\partial\theta}}{\left(1+\beta\tilde{p}(t,\theta)\right)^2}\,.$$

But $\frac{\partial\tilde{p}(t,\theta)}{\partial\theta} > 0$ and $\dot{w}(t,\theta) < 0$. One can also show that $\phi\beta > r+\rho+\delta$ is a sufficient condition for $\frac{\partial\dot{w}(t,\theta)}{\partial\theta} > 0$. But then we have that $\frac{\partial w(t,\theta)}{\partial\theta} > 0$ for any admissible t. Note, however, that $\frac{\partial w(t,\theta)}{\partial\theta} > 0$ requires even weaker conditions because of the positive first term in the numerator, and the fact that the parametric condition merely ensures positiveness of the second term, which is a second-order effect. □

B Existence and Uniqueness of Equilibrium

Existence of an interior equilibrium can be inferred from an analysis of the loci implied by G as given by condition (27) and H as given by condition (29) in the $\theta - T$-space. In fact, given the results in the text and that $\frac{\partial\tilde{p}(T,\theta)}{\partial\theta} > 0$, both loci are upward sloping.[39] For a stable equilibrium to exist, it suffices to show that

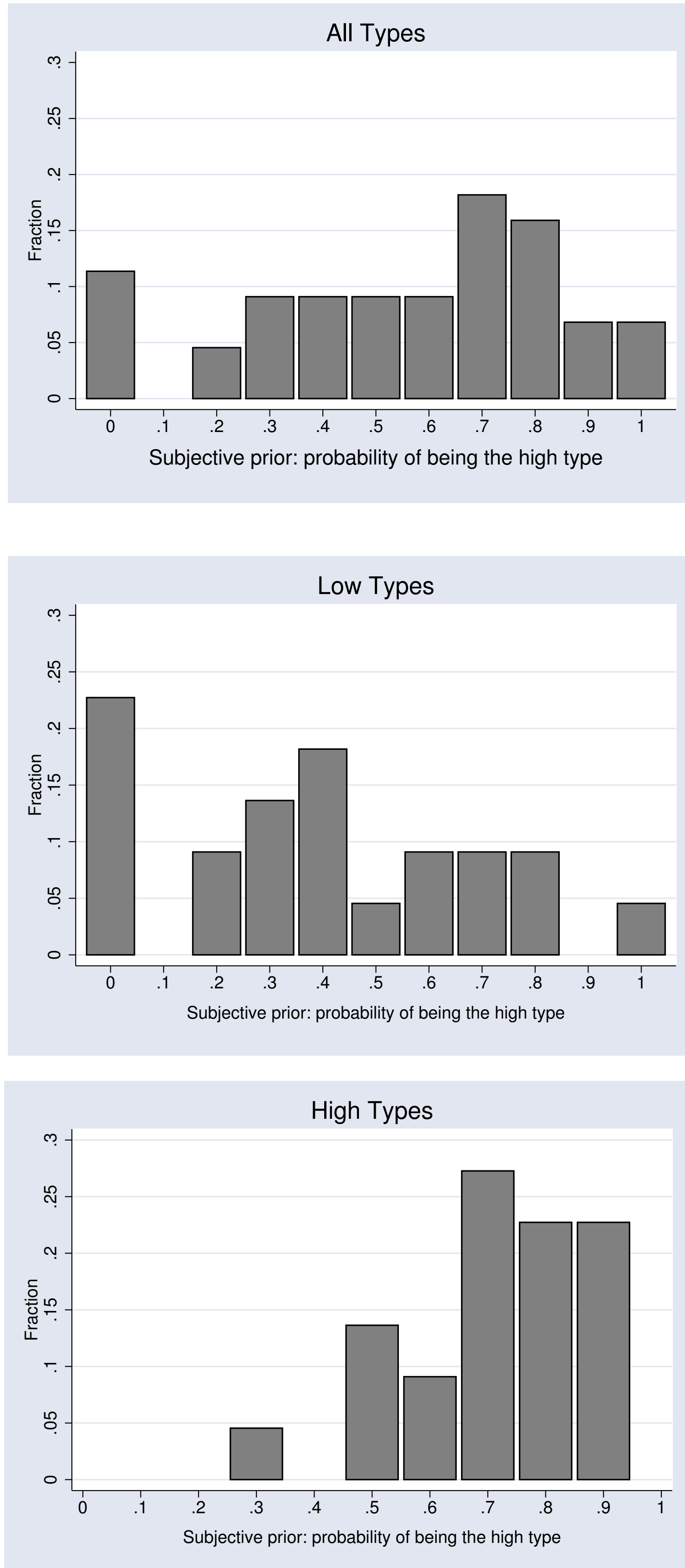
$$\left.\frac{d\theta^*}{dT^*}\right|_G > \left.\frac{d\theta^*}{dT^*}\right|_H \Leftrightarrow -\frac{\partial G/\partial T}{\partial G/\partial\theta} > -\frac{\partial H/\partial T}{\partial H/\partial\theta}\,.$$

But note that as a consequence of rent sharing under Nash bargaining and by the fact that $E(t|T \to \infty, \theta)$ is finite implying that the wage is bounded from below, we have that $\left|\frac{\partial G}{\partial T}\right| > \left|\frac{\partial H}{\partial T}\right|$. Thus, the direct (negative) effect of a longer unemployment spell on the subjective job finding probability $\tilde{p}(T,\theta)$ is larger in absolute terms than the indirect effect of a lower expected wage as consequence of a delayed inactivity threshold. Note also that, $\left|\frac{\partial G}{\partial\theta}\right| < \left|\frac{\partial H}{\partial\theta}\right|$, because of the negative effects of labor market tightness on the value of vacancies through congestion and wages that are larger in absolute terms that the effect on subjective job finding probabilities that is mitigated by a (negative) self-perception effect. This can always be ensured by an appropriate assumption on ϕ. The existence of a unique stable equilibrium can be verified by analyzing the remaining conditions studying the loci implied by G as given by condition (27) and u given by condition (16) in the $u - T$-space and the loci of H and u implied by conditions (29) and (16) in the $\theta - u$-space.

C Figures

[39] With a downward sloping locus G as implied by the alternative assumptions discussed in footnote 23, the existence of a unique interior equilibrium follows immediately.

Figure 1: Uncertainty About the Self



Notes: Subjects were assigned a high job finding probability in the search experiment (high type) if they scored higher than the median on an initial math test. After being informed of their own test score, but not the scores of others, subjects were asked: how likely do you think it is, in percentage terms, that you answered more questions correctly than half of the other subjects in the room today?