Labor market institutions and the divide of schooling investment between general and specialized skills*

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This version: November 2009

Abstract: This paper examines the impact of labor market institutions on the divide of schooling investment between general and specialized skills. We offer a matching model of unemployment in which individuals determine the scope and intensity of their skills prior to entering the labor market. We point out two effects. According to the Rosen effect, LMIs increase market frictions, which motivates the acquisition of general skills. According to the rent-capture effect, LMIs lower match surplus, which raises the return to specialized skills. We examine how these effects interact in the case of unemployment compensation, minimum wage and firing costs. The rent-capture effect is likely to dominate, which is in line with the empirical evidence relating LMIs to the skill divide.

Keywords: Matching frictions; Education; Adaptability skills; Labor market institutions

J.E.L. classification: I21; J24

*This paper has benefited from the comments of participants at the AFSE conference in Paris, and the ASSET conference in Istanbul. We also wish to thank seminar participants at GREQAM. We are indebted to Ophélie Cerdan, Elvira Kazbakova, Paul Maarek, and Etienne Wasmer. Bruno Decreuse also thanks the research grant ACI "Institutions et générations sur le marché du travail" for financial support. The usual disclaimer applies.

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1 Introduction

Labor market institutions (LMIs) and the divide of schooling investment between general and specialized skills have been put forward to explain the relatively low performance of a number of European labor markets since the end of the 1970s. On the one hand, the minimum wage, the generosity of unemployment insurance, and the strictness of employment protection legislation would favor the persistence of high unemployment rates while slowing down the job reallocation process necessary to sustain high productivity growth (see Ljungqvist and Sargent, 1998, Mortensen and Pissarides, 1999, Nickell et al, 2005). On the other hand, vocationnally-oriented European schooling systems would alter workers' between-sector mobility (see Krueger and Kumar, 2004). These two lines of arguments are generally advanced separately. What about their interaction?

This paper analyzes the impacts of LMIs on the type of skills acquired during education. Consider the following situation. You are a student and you have the choice between two types of skills. Specialized skills that are very productive and that you can only use in a specific sector, and general skills that are less productive, but that you can use everywhere. Which skills will you acquire? Rosen (1983) provides a simple answer. He argues that the incentives to specialization are very related to skill use: "the return to investment in a particular skill is increasing in its subsequent rate of utilization". So, if your chances of having a particular job are high, choose the specialized skills. Otherwise, if job opportunities are scarce, invest in general skills. This powerful idea has a puzzling implication: on the basis that more regulated labor market are characterized by lower job-finding rates, LMIs should promote general skills rather than specialized skills.

We introduce a theoretical model tailored to thinking about the effects of LMIs on the skill divide. This model features job creation, matching frictions, and multi-dimensional skills. The abrupt distinction between general and specific human capital cannot be directly used in the study of educational investment. At the time of educational choice, individuals are not well informed of the identity of the firms they will meet. Human capital cannot be purely specific in the traditional sense: workers would have no chance of using such a kind of human capital, and, consequently, the whole investment would be spent in general human capital. Our paper builds on Charlot et al (2005). In this model, education jointly determines the scope – or adaptability – and the intensity – or productivity – of skills. The scope of skills governs the fraction of jobs on which the worker can operate, while the intensity of skills refers to worker's productivity on these jobs. In this framework, vocational schooling differs from general schooling as it puts more weight on productivity skills, and, as a result, less weight on adaptability skills. Of course, both types of education provide with adaptability and productivity skills. But an individual who follows a vocational education ends up with fewer adaptability skills than an individual who attends a general programme.

In Charlot et al, the divide of investment between adaptability and productivity is fixed. We complete that paper by endogenizing the mix of adaptability and productivity skills. We thus assume that the total investment in education is given. Students allocate this investment between the two types of skills. The scope of one's skills and the intensity of these skills evolve in opposite directions: an individual devoting a larger part of her resources to vocational education will be more productive once in the job, but her skills will be worthwhile in fewer jobs (less general). In the competitive environment – the limit case where frictions disappear –, adaptability skills are useless because it is very easy to contact any type of job. Therefore, individuals devote the main part of their investment to productivity skills. Conversely, very specialized skills become much less attractive when contacting a proper job takes a lot of time/resources. Thus, matching frictions originate incentives to acquire more general skills[1].

We use this framework to analyze changes in LMIs. We proceed in three steps.

In a first step, we present the model and interpret worker's bargaining power as a measure of LMIs. We examine its impact on the skill divide and highlight two effects. On the one hand, there is a Rosen-type effect: the employer's share of match surplus goes down and firms post fewer jobs. This increases the severity of market frictions, thereby raising the returns to adaptability skills. On the other hand, there is a rent-capture effect that goes in the opposite direction: worker's share of match surplus goes up, which lowers the returns to adaptability skills once in the job. We show that the rent-capture effect dominates for low values of the bargaining power, while it is dominated by the Rosen effect for higher values. Overall, the model predicts a U-shaped relationship between bargaining power/LMIs and the proportion of educational investment spent into adaptability skills. The minimum of the curve is reached when bargaining is socially efficient (when the Hosios condition is satisfied).

In a second step, we turn to the empirical evidence and show that there is a (weak) negative correlation between a proxy for the proportion of educational investment spent in general skills and LMIs in the cross-section of OECD countries. We consider the proportion of upper-secondary graduates with a general education. LMIs are captured by standard OECD indices. We use union density, the ratio of minimum wage to median wage, an index of unemployment insurance generosity, and an employment protection legislation index. The partial correlations are either negative or nil, but we do not find evidence of a positive relationship between LMIs and general skill investment. This motivates the last step.

In a third step, we focus on each institution separately. We examine unemployment benefits (UB), the minimum wage (MW), and employment protection (EPL). UB and associated wage taxation lower match surplus and further motivates the acquisition of specialized skills. The MW distorts investments towards productive skills among the workers who get paid the MW, while it motivates the acquisition of adaptability skills among the other workers. Similarly, EPL lowers match surplus. The marginal return to adaptability skills falls as a result. Overall, these various extensions to the basic

[1] In a different setting, Gould, Moav and Weinberg (2001) argue that unemployment creates educational incentives, because it originates a demand for precautionary education from risk averse individuals. Unlike Gould et al, individuals are risk neutral in our paper, and education can offer both general and specific skills.

model suggest that the rent-capture effect is stronger than initially expected. LMIs direct schooling investments towards specialized skills, which is line with the evidence reported in the second step.

This paper is related to different strands of literature. There is a substantial theoretical literature on the relationships between matching frictions and the magnitude of educational investment (see e.g. Acemoglu, 1996, Moen, 1999, Burdett and Smith, 2002, Charlot and Decreuse, 2005). We complement this literature by focusing on the type of skills rather than on the skill level. Mukoyama and Sahin (2006) examine the impact of unemployment compensation on the incentives to specialization. However, there is no trade-off between general and specialized skills: a worker who invests more can perform more tasks with unchanged productivity. Mukoyama and Sahin are more interested in the level than in the composition of educational investment. In addition, there are no general equilibrium effects: contact rates are exogenous.

In a contribution devoted to on-the-job training, Wasmer (2006) examines the decision to invest in general vs specific skills. Unlike specific skills, general skills improve outside options at the time of wage bargaining. Matching frictions and layoff taxes favor the acquisition of specific skills. Indeed, the return to general skills increases with the matching probability, which goes down with market frictions. Wasmer uses an elegant metaphor to explain his result: "in an economy made up of far-spread islands, it is better to learn the technology of the island on which one lives." Our paper complements Wasmer's when one considers human capital investments before the labor market entry, rather than once in the job. Compared to Wasmer's, our framework can be interpreted as follows: students are sailing the ocean, waiting for an island to inhabit. As the density of islands goes down, the knowledge of one peculiar technology becomes less and less useful. Nevertheless, both papers share the view that firing costs are detrimental to general skills.

This paper is also related to the literature that emphasizes the role of industry-specific skills in labor markets where workers are imperfectly mobile between sectors. Stevens (1994) introduces the notion of transferable skills. These skills can only be used in a proportion of the different available jobs. Stevens argues that there is an underprovision of transferable skills by employers. Smits (2007) distinguishes industry-specific skills from generic skills (that have a higher value elsewhere in the economy). He argues that workers want more generic skills than is socially optimal, while firms prefer industry-specific skills. Our paper complements this literature by focusing on educational investments rather than on-the-job training.

There is a growing literature that analyses the role of LMIs on the incentives for firms to fund general training investment. Unions may encourage training because they reduce labor turnover (Booth and Chatterji, 1998). Wage compression induced by a minimum wage increase may have a positive effect on the incentives to train the less skilled workers to improve their productivity (Acemoglu and Pischke, 1999, 2003). Fella (2004) predicts a positive correlation between investment in general training and the strictness of employment protection rules. The present paper complements this literature mainly by analyzing the role of LMIs on the schooling allocation between general and specialized skills.

The trade-off between adaptability and productivity borrows from the notions of marketability and specialization highlighted in the literature on money and search (see e.g. Kiyotaki and Wright, 1993, and Shi, 1997). The main idea in these papers is that each producer faces a trade-off between specialization and marketability. Specializing in the production of a given commodity allows better productivity (or, equivalently, saves on production costs), but at the expense of reducing the proportion of consumers interested in purchasing the good, *i.e.* marketability is smaller. Typically, money plays a crucial role in this approach as it enlarges the size of the market and therefore allows producers to specialize.

The rest of the paper is organized as follows. Section 2 introduces our model. Section 3 presents some empirical evidence. Section 4 analyzes the impacts of three different institutions: unemployment benefits, minimum wage, and employment protection. Section 5 concludes.

All proofs are in the Appendix.

2 The skill divide with market frictions

In this section, we propose a model of educational investment that features a trade-off between general and specialized skills and matching frictions on the labor market. We proceed in two steps. First, we examine the skill divide in partial equilibrium and highlights the Rosen and rent-capture effects. Then, we endogenize job creation and show that the model predicts a U-shaped relationship between the proportion of investment in general skills and workers' bargaining power.

2.1 The Rosen effect and the rent-capture effect

We are interested in the schooling investment of an infinite lifetime individual living in a stationary environment. She is risk neutral, and discounts time at rate r. Her total human capital investment I is given. It can be viewed either as the exogenous schooling duration, or total spending in education. The individual must divide this investment between adaptability skills and productivity skills. Let g denote the amount of adaptability skills, while $s = I - g$ denote the amount of productivity skills.

The notions of adaptability and productivity skills rely on the technological side of the economy. There are a continuum of sectors, each producing a final good entering preferences symmetrically. Sectors are of mass one. Each sector is associated to a particular technology. While dividing human capital, the worker chooses the scope and the intensity of her skills. Adaptability skills increase the share of technologies the worker can operate, while productivity skills raise the productivity in each known technology. Formally, the proportion of technologies the worker knows is $H(g)$, with $H(0) = 0$, $H(I) \leq 1$, $H'(g) > 0$, $H''(g) < 0$. The intensity of her skills is $f(s)$, with $f(0) = 0$, $f'(s) > 0$, $f''(s) < 0$.

A worker who increases adaptability skills g can work on more jobs at lower productivity. Human capital is more general as a result. For this reason, we shall sometimes refer to adaptability skills as general skills, and to productivity skills as specialized skills.

The labor market is frictional. Matching frictions have two important consequences. First, there is only a probability of contacting a job per period. Let μ denote the flow probability that a worker receives a job offer from a particular sector. Thus μ^{-1} measures the severity of frictions. Given that $H(g)$ is the proportion of jobs the worker can occupy, $\mu H(g)$ is the rate of acceptable job offer. It is increasing in g, and decreasing in the severity of frictions. Second, each match is associated to a match surplus that the employer and the worker must share. We follow the literature and assume that there is wage bargaining over the match surplus.

Let $U = U(s, g)$ denote the utility of an unemployed, and $W = W(s, g)$ the utility of an employed worker. We have:

$$rU = \mu H(g)[W - U] \tag{1}$$

$$rW = w + q[U - W] \tag{2}$$

where q is the (exogenous) rate of job destruction and w is the wage. Symmetrically, $J = J(s, g)$ is the value of a filled job. We have:

$$rJ = f(I - g) - w + q[V - J] \tag{3}$$

where V the value of a vacancy is given. The wage splits the match surplus $S = W - U + J - V$ according to

$$W - U = \beta S = \frac{\beta}{1-\beta}(J - V) \tag{4}$$

It follows that the match surplus is:

$$S(s, g) = \frac{f(s) - rV}{r + q + \beta\mu H(g)} \tag{5}$$

At the time of investment, the individual does not know which firm will hire her. As a consequence, she maximizes the value of her future search. From the different equations, we have

$$rU(s, g) = \beta\mu H(g) S(s, g) \tag{6}$$

The optimal allocation of educational investment between general and specialized skills maximizes the contact surplus $H(g) S(s, g)$. The skill divide results from

$$\frac{H'(g)}{H(g)} + \frac{\partial S(I - g, g)/\partial g}{S(I - g, g)} = \frac{\partial S(I - g, g)/\partial s}{S(I - g, g)} \tag{7}$$

where

$$\frac{\partial S(s, g)/\partial g}{S(s, g)} = -\frac{H'(g)}{H(g)} \frac{\beta\mu H(g)}{r + q + \beta\mu H(g)} \tag{8}$$

$$\frac{\partial S(s, g)/\partial s}{S(s, g)} = \frac{f'(s)}{f(s) - rV} \tag{9}$$

The optimal skill divide balances the marginal returns to general and specialized skills. General skills increase the contact surplus by raising the probability that such a contact gives birth to an employment relationship. However, general skills reduce the match surplus because they improve the chance of contacting an adequate employer, thereby making the economic position of the unemployed closer to that of an employed worker. The size of the latter effect increases with the product $\beta\mu$, that is with the chance of contacting a vacancy times the share of contact surplus obtained in such a case. Specialized skills do not alter the matching probability, yet they raise match output, thereby increasing match surplus.

It follows that:

$$\frac{H'(g)}{H(g)} \frac{f(I-g) - rV}{r + q + \beta\mu H(g)} = \frac{f'(I-g)}{r+q} \tag{10}$$

Proposition 1 Matching frictions, bargaining power, and the skill divide

Let V be sufficiently small. Then,

(i) Rosen effect: $d\widehat{g}/d\mu^{-1} > 0$, that is general skill investment increases with matching frictions;

(ii) Rent-capture effect: $d\widehat{g}/d\beta < 0$, that is general skill investment decreases with workers' bargaining power

The divide between general and specialized skills responds to alterations in their respective marginal returns. The severity of frictions μ^{-1} originates incentives to acquire general skills rather than specialized skills. This is the Rosen effect. Intuitively, the purpose of general skills is to improve the ability of receiving job offers, thereby raising worker's share of match surplus. The proportion of investment accruing to such skills thus goes up with match surplus. Hence, the severity of frictions motivates the acquisition of general skills because frictions increase the size of match surplus. General skills are useless when it is very easy to get matched with a proper vacancy. In the walrasian environment, μ tends to infinity, which implies that unemployment spells are arbitrarily short, and contacting any type of alternative employer is immediate. Match surplus is nil, and there is no need to speed up job search. The whole investment is then devoted to the acquisition of specialized skills, i.e. $g = 0$. Conversely, market frictions reduce the interest of very specialized skills, which become much more difficult to trade.

General skills decrease with bargaining power β. This is the rent-capture effect. Worker's bargaining power reduces current match surplus as it allows the job-seeker to capture a larger part of alternative match surpluses. General skills allow the worker to enhance the value of outside options once in the job. The worker benefits from such outside options through a better wage. This effect is all the higher than worker's bargaining power is low. As the bargaining power increases, the need to raise outside options decreases, and so does the investment in general skills.

The rent-capture effect is not specific to workers' bargaining power. Any institution that affects the match surplus modifies the incentives to acquire adaptability skills. Meanwhile, changes in match surplus should alters firms' incentives to create jobs, which also modifies the educational trade-off through the Rosen effect. The rent-capture effect and the Rosen effect should work in opposite directions, so that the global effect of LMIs is indeterminate. We discuss this issue in the next subsection.

2.2 Equilibrium unemployment and the composition of human capital

In this sub-section, we examine the equilibrium relationship between the skill divide and the bargaining power. We incorporate the framework of the previous sub-section into an equilibrium matching model of the labor market. The main result is that there is a ∪-shaped relationship between the bargaining power and the share of schooling investment spent in general skills.

To close the model, we need an explicit matching market with a matching technology for heterogenous jobs and heterogenous skills. We assume that there is a unique search place for all workers and vacant jobs, i.e. search is undirected[2]. Let θ be the labor market tightness, that is the ratio of vacancies to unemployed. The rate of contacting a vacancy is thus $\mu = \mu(\theta)$, while the rate of contacting a worker is $\mu(\theta)/\theta$. The function μ is such that $\mu'(\theta) > 0$, $\mu''(\theta) < 0$, and $\mu(0) = \mu(\infty)^{-1} = 0$. The elasticity of the contact rate with respect to market tightness is $\alpha(\theta) = \theta\mu'(\theta)/\mu(\theta)$.

We also need agents who make schooling decisions at each instant. We assume that new cohorts enter the economy at rate $n > 0$. Given constant returns to scale in the matching technology, parameter n will only affect the unemployment rate through changes in the inflow rate.

Equilibrium tightness is derived from a zero-profit condition. Assume that all workers have the same amount of general and specific skills. Let c be the flow cost of posting a vacancy. The value of a vacancy is defined as follows:

$$rV = -c + \frac{\mu(\theta)}{\theta}(1-\beta)H(g)S(\theta, s, g, \beta) \tag{11}$$

In equilibrium, $V = 0$ and

$$c\frac{\theta}{\mu(\theta)} = (1-\beta)H(g)S(\theta, s, g, \beta) \tag{12}$$

This equation defines tightness as an increasing function of contact surplus $H(g)S(\theta, s, g, \beta)$, where the dependence of match surplus vis-à-vis θ and β has been highlighted. As discussed in the previous section, general skills improve the probability to match with an adequate worker, but deteriorate match surplus. Specialized skills raise output, thereby increasing match surplus. It follows that tightness is increasing in s. Finally, tightness is decreasing in workers' bargaining power β, which lowers firms' profitability.

[2] Alternatively, the search market could be segmented by technology, and workers would participate in all the submarkets they know the underlying technology. The results would be the same.

Equilibrium tightness θ^* and general skill investment g^* jointly solve

$$\frac{H'(g)}{H(g)} S(\theta, I-g, g, \beta) = \frac{f'(I-g)}{r+q} \tag{SD}$$

$$(1-\beta) H(g) S(\theta, I-g, g, \beta) = c\frac{\theta}{\mu(\theta)} \tag{MT}$$

Proposition 2 Bargaining power and the skill divide

(i) There exists a unique equilibrium

(ii) $d\theta^*/d\beta < 0$, that is tightness strictly decreases with workers' bargaining power

(iii) $dg^*/d\beta > 0$ iff $\beta > 1-\alpha(\theta^*)$, that is there is a U-shaped curve between general skill investment and bargaining power

Figure 1 depicts the equilibrium. The skill divide equation defines the curve (SD). Along the lines of Proposition 1, it features a decreasing relationship between the investment in general skills and the labor market tightness. The market tightness equation defines the curve (MT). This curve is bell-shaped. Indeed, market tightness increases with the contact surplus $H(g) S(\theta, g, I-g)$. In turn, this contact surplus first increases and then decreases with general skill investment. The two curves intersect once at the maximum of (MT). This property results from the fact that optimal general skill investment maximizes the contact surplus. It follows that there is a unique equilibrium.

[Figure 1: Existence and uniqueness of equilibrium]

Proposition 2 shows that tightness is strictly decreasing in bargaining power. This relationship between tightness and bargaining power broadly captures the notion that more rigid labor markets are characterized by fewer job opportunities and, therefore, lower job-finding rates. Consequently, the bargaining power affects the skill divide in two different ways. Following the rent-capture effect, it directly decreases match surplus, which reduces the return to general skills. Owing to the Rosen effect, it reduces tightness, thereby increasing match surplus and raising the return to general skills. Overall,

$$\frac{dg^*}{d\beta} \overset{sign}{=} \beta - (1-\alpha(\theta^*)) \tag{13}$$

The proportion of investment spent in general skills follows a $\cup$-shaped curve as β goes from 0 to 1. It reaches a minimum when the Hosios condition is met (Hosios, 1990), that is when $\beta = 1-\alpha$. This property results from the fact that, other things equal, the optimal investment in general skills is increasing in match surplus, that is in the size of rents accruing to employed individuals. Such a match surplus is minimized when the Hosios condition holds. The upper bound on general skill investment is reached for $\beta = 0$ and $\beta = 1$. Using (SD), this gives $g_0 < I$ such that $H\prime(g_0)/H(g_0) = f'(I-g_0)/f(I-g_0)$.

Our model predicts that the rent-capture effect dominates for low values of the bargaining power, while the Rosen effect dominates for higher values.

3 Empirical evidence

In this section, we present some empirical evidence at the aggregate level. We focus on one index of the skill divide, namely the proportion of upper-secondary graduates with a general education. We show that this index is negatively correlated with various measures of LMIs in the cross-section of OECD countries.

We use ISCED data, which organize an horizontal differentiation of educational attainments. These data rank educational attainments into six levels (1 to 6), that go from pre-primary schooling to research. At each schooling level, there are three different types of education: from A (general) to C (vocational). Both vocational and general schooling provide with adaptability and productivity skills. Our presumption is that general schooling (type A) is more turned towards the acquisition of adaptability skills than vocational schooling (types B and C).

We restrict our attention to upper-secondary schooling. Krueger and Kumar (2004) and Mukoyama and Sahin (2006) compute the enrolment rate in general education among the students at upper-secondary level. However, the enrolment rate may be misleading. Individuals who decide to follow a general secondary education may then decide to follow a vocational tertiary education. From that perspective, the enrolment rate not only captures the skill divide, but also the access to tertiary education.[3] This is why we focus on a different variable, the proportion of individuals with a general education among the graduates with an upper-secondary education. Focusing on graduates is a priori a better strategy, as, by definition, graduates have already completed their education. Unfortunately, the variable is not very well documented. In several cases, the proportion is one, while the enrolment rate in the corresponding program is lower than one. We choose to impute the value taken by the enrolment rate in such a case. The variable is computed for the year 2003 (only a few consecutive years are available).

Of course, these data have shortcomings. Our paper focuses on the individual trade-off between general and specialized skills at the time of educational investment. Namely, we want to understand how LMIs alter the skill divide *at the margin*. However, the aggregate data we use concern people who either completed a vocational education, or a general education. Our assumption is that changes in the proportion of people who choose a vocational education are correlated with changes in the individual proportion of educational investment that is invested in specialized skill acquisition.

We consider four LMIs: union density, unemployment insurance, the minimum wage, and employment protection. Union density is the 2000 ratio of union members to total number of employees. Unemployment insurance is proxied by the OECD measure of benefit entitlements averaged over the period 1999-2001. The measure is defined as the

[3] Krueger and Kumar (2004) as well as Mukoyama and Sahin (2006) know that fact. In their view, general education is associated to longer studies. It is implicit in Krueger and Kumar, who also consider cross-country differences in the entry rate into universities, "where general education is primarily imparted" (their words). It is explicit in Mukoyama and Sahin in which choosing a general education means paying more than choosing a vocational education.

average of the gross unemployment benefit replacement rates for two earnings levels, three family situations and three durations of unemployment. The minimum wage variable is the 2000 ratio of the minimum wage to median wage for full-time workers. Employment protection is proxied by the weighted mean of two OECD indices, the strictness of employment protection legislation on regular jobs and the strictness of EPL for collective dismissals. The weights correspond to the weights used for the computation of the overall OECD index (that also captures the availability of temporary contracts): 5/7 for regular jobs and 2/7 for collective dismissals. The index is computed for the end of the 1990s.

The panel of Figures 2 depicts the correlations between the four LMIs and our proxy for the proportion of investment spent in adaptability skills. The panel features negative correlations. Where LMIs are strong, people tend to invest more in specialized skills. These correlations are weak, highlighting the poor quality of the data and the need for time-varying data. However, they are all negative, suggesting that the Rosen effect is always dominated by the rent-capture effect.

[Panel of Figures 2]

Negative correlations may also reflect reverse causality. The composition of skills of the workforce may affect the social demand for LMIs. For instance, more specialized skills may generate a desire for unemployment insurance. In turn, private contracts cannot always address this insurance demand, which motivates the extent of the public coverage, or the strictness of employment protection legislation. Meanwhile, workers with very specialized skills are more likely to suffer from employer's monopsony power given the small size of their outside options. They may favor collective bargaining or minimum wage laws as a result. Nevertheless, Botero et al (2004) show that LMIs are strongly determined by the legal origins of the judicial system. They distinguish five different legal origins: English, French, German, Scandinavian, and Socialist. They argue that countries with common law legal systems (English law) regulate much less their labor market than countries with civil law legal systems (other laws). Legal origins are predetermined with respect to the skill divide among graduates. Table 1 computes the mean value by legal origins of our proxy for the skill divide. It shows that common law countries invest more than civil law countries in general skills.

Law	English	French	German	Scandinavian	Socialist	civil
Mean index	0.7	0.62	0.33	0.41	0.48	0.48
No countries	7	8	5	5	4	22

Table 1: Legal origins and the skill divide in education

Reading: the mean value of the index for the skill divide is 0.33 in the five countries with a German law legal system. 'civil' is the weighted mean of French, German, Scandinavian, and Socialist

Overall, this section suggests that LMIs tend to distort educational investments towards specialized skills. This result is consistent with the theoretical model provided that workers' bargaining power is generally too weak with respect to the Hosios condition. However, inefficient bargaining is a very strong assumption in light of the recurrent debate about the strictness of LMIs in several European countries. In the next section, we analyze the impact of each institution more carefully. We then argue that the rent-capture effect is larger than the basic model with bargaining power suggests.

4 Labor market institutions and the composition of educational investment

In this section, we examine the role played by different labor market institutions on the divide of educational investment between adaptability and productivity skills. We examine unemployment compensation, the minimum wage, and firing costs. The three institutions tend to reduce the incentives to acquire adaptability skills.

4.1 Unemployment compensation

It has been argued that unemployment insurance allows the workers to invest in specific skills (see e.g. Grossman and Shapiro, 1982, Estevez et al, 2001). In this sub-section, we revisit this prediction. Using our model, we show that unemployment compensation and wage taxation are detrimental to adaptability skills.

Let b denote unemployment benefits. For simplicity, there are no time limit to UB, and eligibility to unemployment insurance is obtained with the first job. UB are financed by a payroll tax on wages. Employers' tax rate is t_e, while workers' tax rate is t_w.

We must distinguish U_0 the intertemporal utility of a newcomer on the labor market from U the intertemporal utility of an unemployed who is eligible to unemployment insurance. We have:

$$rU_0 = \mu H(g)[W - U_0] \quad (14)$$

$$rU = b + \mu H(g)[W - U] \quad (15)$$

Other value functions write

$$rW = w(1 - t_w) + q(U - W) \quad (16)$$

$$rJ = f - w(1 + t_e) + q(V - J) \quad (17)$$

Nash bargaining over match surplus S yields

$$W - U = \gamma S \quad (18)$$

$$J - V = (1 - \gamma) S \quad (19)$$

where $\gamma = \beta(1-t_w)/[\beta(1-t_w)+(1-\beta)(1+t_e)]$. Equations (15) to (19) jointly define the match surplus:

$$S = \frac{\beta(1-t_w)+(1-\beta)(1+t_e)}{1+t_e}\frac{f - b\frac{1+t_e}{1-t_w} - rV}{r+q+\beta\mu H} \tag{20}$$

Match surplus strictly decreases with UB and payroll taxes. UB shorten the wealth difference between employment and unemployment. Wage taxation implies that payroll taxes flow out of the employment relationship, thereby reducing match surplus.

The skill divide results from the maximization of the return to search:

$$rU_0(s,g) = \gamma\mu(\theta)H(g)S(s,g) + \frac{\mu H(g)}{r+\mu H(g)}b \tag{21}$$

The return to search is equal to the contact rate times the proportion γ of contact surplus HS, plus a term that corresponds to the permanent gain achieved once the first job is obtained. This gain increases with general skills.

The f.o.c. writes:

$$\frac{H'(g)}{H(g)}\frac{f(I-g) - b\frac{1+t_e}{1-t_w}}{r+q+\beta\mu(\theta)H(g)} + \beta^{-1}\frac{H'(g)}{H(g)}\frac{b\frac{1+t_e}{1-t_w}}{r+q}\frac{r[r+q+\beta\mu(\theta)H(g)]}{[r+\mu(\theta)H(g)]^2} = \frac{f'(I-g)}{r+q} \tag{22}$$

Equation (22) features an additional return to general skills on its left-hand side. To benefit from the permanent increase in human wealth due to the first job, individuals set general skills above the point that maximizes the contact surplus HS.

Job creation results from

$$\frac{c\theta}{\mu(\theta)} = (1-\gamma)H(g)S(\theta, I-g, g) \tag{23}$$

An equilibrium is a pair (θ^*, g^*) that solves equations (22) and (23). The following result summarizes the impacts of UB and wage taxation on the skill divide.

Proposition 3 Unemployment compensation and the skill divide

Assume that $f(I) > b$. Provided that the discount rate r is sufficiently low,

(i) an increase in UB lowers market tightness and general skill investment, i.e. $d\theta^*/db < 0$ and $dg^*/db < 0$;

(ii) an increase in employer's or in employee's tax rate lowers market tightness and general skill investment, i.e. $d\theta^*/dt_w < 0$, $d\theta^*/dt_e < 0$, $dg^*/dt_w < 0$, and $dg^*/dt_e < 0$.

UB have three different effects. First, there is a rent-capture effect as UB decrease match surplus. This effect tends to raise the investment in productivity skills. Second, there is a Rosen effect. As match surplus decreases, firms post fewer jobs, which reduces the contact rate. This effect motivates the acquisition of adaptability skills. Third, there is an entitlement effect that further raises the incentives to invest in general skills. Like

UB tend to increase search effort/decrease search choosiness among non-entitled workers, UB also tend to favor general skills that speed-up job-finding.

For a sufficiently low discount rate, the entitlement effect can be neglected. Formally, the second term in the left-hand side of equation (22) vanishes. Then, the Rosen effect reveals dominated by the rent-capture effect. According to equation (22), adaptability skills increase with match surplus. If the Rosen effect were dominating the rent-capture effect, match surplus would increase with UB. In turn, equation (23) would imply that market tightness goes up, a contradiction. The reason why small discounting plays against the entitlement effect can be understood from the extreme situation where individuals do not discount time. In such a case, they are only interested in their long-run situation. As non-entitlement is only temporary, they neglect this first period of working life.

Qualitatively, wage taxation has similar effects to UB. Wage taxation lowers equilibrium match surplus, which reduces the incentives to acquire adaptability skills. Meanwhile, wage taxation decreases the returns to finding a job, thereby increasing the relative return due to benefit entitlement. This second effect favors adaptability skills. It also vanishes when the discount rate is sufficiently small.

How small must the discount rate be? To answer that question, we turn to numerical simulations. The matching technology is Cobb-Douglas, with $\mu(\theta) = M_0\theta^{\alpha}$. Output is $f(s) = 1 - \exp(-\nu s)$. The proportion of jobs that the worker can operate is $H(g) = 1 - \exp(-\kappa g)$. The parameter I is set to one. Consequently, g^* is directly the proportion of educational investment spent in adaptability skills. Parameters α and β have been set equal to 1/2. It is a rather consensual value for the elasticity of the matching technology. Furthermore, bargaining is efficient in the absence of UB/wage taxation.

Let $b = \rho(1 - t_w) w$, where ρ denote the replacement rate over net wage. Let u^o denote the number of unemployed who already had a job. Balanced budget requires that $(1 - u)(t_w + t_e) w = u^o b$. In steady state, u^o is equal to overall unemployment $u = (n + q) / (n + q + \mu H)$ minus the number u^y of young workers who seek their first job, with $u^y = n/(n + \mu H)$. It follows that $t_w + t_e = \rho q/(n + \mu H)$.

The other parameters have been chosen to (i) broadly replicate the upper-secondary segment of the French labor market in 2000, and (ii) to target our proxy for the proportion of educational investment spent in adaptability skills. (i) The mean OECD replacement rate was 40%, and the unemployment rate of upper-secondary graduates was about 9%. As the proportion of upper-secondary graduates goes down among younger cohorts, the mean unemployment rate slightly undervalues the stationary rate. The mean job destruction rate q was about 10%. As $u = (n + q) / (n + q + \mu(\theta^*) H(g^*))$ and n is at most 0.5%, the job-finding rate is about one. This corresponds to a mean unemployment duration of about one year. (ii) Meanwhile, the proportion of upper-secondary graduates with a general education was 25% in 2003. We choose to consider, somewhat arbitrarily, that it indicates that 25% of the schooling investment was directed towards adaptability skills.

Table 2 presents the parameters of the baseline simulation. The discount rate has been set to 5%.

Parameters	α	β	n	q	M_0	c	I	ν	κ	r	ρ
Values	0.5	0.5	0.005	0.1	1.0	0.15	1.0	2.0	2.0	0.05	0.4

Table 2: Parameters

The corresponding stationary values are about $g^* = 22.8\%$ and $u = 9.9\%$. As $H(g^*) = 0.36$, an unemployed worker needs to contact three vacancies on average before being hired. To illustrate the results of Proposition 3, we examine the sensitivity of g^* vis-à-vis changes in ρ and r. Figure 3 depicts five curves in the (ρ, g^*) plane. Each curve is associated with a particular value of the discount rate r, from 0 to 20%. When the discount rate is 0, the entitlement effect is nil, and the curve is strictly decreasing. In the other cases, the curve is U-shaped. As the magnitude of the entitlement effect increases with the discount rate, the minimum of the curve decreases with the discount rate. The main message of Figure 3 is that the entitlement effect is dominated for reasonable values of the replacement rate and the discount rate.

[Figure 3: Equilibrium skill divide as a function of UB replacement rate]

4.2 Minimum wage

Changes in the minimum wage replicate changes in bargaining power at given educational investment. However, the fact that workers may get paid independently from their skills crucially alter the skill divide. In this sub-section, we introduce worker heterogeneity with respect to schooling level, and emphasize the situation where low-educated workers receive the MW. We show that an increase in the MW implies that low-educated workers direct their investment towards more specialized skills, while high-educated workers acquire more general skills.

There are two types of workers indexed by $i = 1, 2$. Type-i workers lie in proportion $p_i \in (0, 1)$, $p_1 = 1 - p_2$. The schooling level of each type is I_i, with $I_1 > I_2$. The level of educational attainment I_2 that we discuss is the upper-secondary level, while I_1 corresponds to early stages of tertiary education.

The minimum wage $w_{\min}$ is introduced as follows. Suppose that type-i workers produce more than the minimum wage, i.e. $f(s_i) \geq w_{\min}$. Then, their wage is

$$w_i = \max\left\{\arg\max_w \left[W_i - U_i\right]^\beta \left[J_i - V\right]^{1-\beta}, w_{\min}\right\} \tag{24}$$

The wage is either the bargained wage or the minimum wage. This wage rule results in the following utility profiles:

$$W_i - U_i = \max\left\{\beta S_i, \frac{w_{\min}}{r + q + \mu H(g_i)}\right\} \tag{25}$$

$$J_i - V = \min\left\{(1-\beta) S_i, \frac{f(I_i - g_i) + qV - w_{\min}}{r + q}\right\} \tag{26}$$

where $S_i = \left(f\left(I_i - g_i\right) - rV\right) / \left(r + q + \beta \mu H\left(g_i\right)\right)$ is the match surplus.

Individuals set their skills so as to maximize the returns to search. Doing so, they must account for their employability, that is they must produce at least what they will be paid. Thus,

$$\widehat{g}_i = \arg\max_g \left\langle rU_i = \mu H\left(g\right) \max\left\{\beta S_i\left(I_i - g, g\right), \frac{w_{\min}}{r + q + \mu H\left(g\right)}\right\}\right\rangle \tag{27}$$

subject to the constraint $f\left(I_i - g\right) \geq w_{\min}$.

Firms create jobs on the basis of their expectations on the distribution of skills among the job-seekers. Assuming free entry, $V = 0$, tightness results from

$$\frac{c\theta}{\mu\left(\theta\right)} = \textstyle\sum_i \pi_i \min\left\{\left(1 - \beta\right) S_i, \frac{f\left(I_i - g_i\right) + qV - w_{\min}}{r + q}\right\} \tag{28}$$

where the composition of skills is given by

$$\pi_1 \equiv \pi \equiv 1 - \pi_2 = \frac{p_1 q / \left(q + \mu H\left(g_1\right)\right)}{p_1 q / \left(q + \mu H\left(g_1\right)\right) + p_2 q / \left(q + \mu H\left(g_2\right)\right)} \tag{29}$$

An equilibrium is a tuple $\left(g_1^*, g_2^*, \theta^*\right)$ that satisfies optimal schooling (27) and job creation decisions (28).

We focus on an equilibrium where the MW binds for low-educated workers. Such workers set the skill divide so that $f\left(I_2 - g_2^*\right) = w_{\min}$. High-educated workers set the skill divide so that

$$\frac{H'\left(g_1\right)}{H\left(g_1\right)} S\left(\theta, I_1 - g_1, g_1\right) = \frac{f'\left(I_1 - g_1\right)}{r + q} \tag{30}$$

Finally, tightness is given by

$$\frac{c\theta}{\mu\left(\theta\right)} = \pi\left(\theta, g_1, g_2\right)\left(1 - \beta\right) S\left(\theta, I_1 - g_1, g_1\right) \tag{31}$$

Only matches with high-educated workers allow the firms to capture part of the rents that finance job creation.

Proposition 4 Minimum wage and the skill divide

> *Assume that the MW is binding for low-educated workers. The MW reduces the labor market tightness, i.e.* $d\theta^*/dw_{\min} < 0$, *distorts the skill divide towards specialized skills for the low-educated, i.e.* $dg_2^*/dw_{\min} < 0$, *and towards general skills for the high-educated, i.e.* $dg_1^*/dw_{\min} > 0$

Following a MW increase, the less educated workers raise the share of investment devoted to productivity skills. The idea according to which the minimum wage may create incentives to skill acquisition has already been put forward by different studies (see e.g. Cahuc and Michel, 1996, Agell and Lommerud, 1997, Acemoglu and Pischke, 2005). The key prediction of our model relies on the opportunity cost of the increase in specialized skills: workers have to reduce their investment in adaptability skills, which lowers their employment prospects[4]. This effect rationalizes the negative correlation reported in the panel of Figures 2 between the MW and general skill investment at upper-secondary education level.

As g_2^* goes down, the unemployment rate of low-educated workers increases. The proportion π of high-educated workers among the job-seekers decreases as a result. It follows that tightness θ^* decreases with the MW. In turn, the fall in tightness lowers the job offer rate for all, which incites high-educated workers to change the skill allocation. However, unlike the low-educated workers, the high educated increase the proportion of investment spent in general skills rather than in specialized skills.

4.3 Employment protection

Wasmer (2006) argues that employment protection distorts on-the-job skill investments towards specific rather than general skills. In this sub-section, we show that Wasmer's main message also holds at the time of education: firing costs incite individuals to allocate a larger proportion of their educational investment in specialized skills. Specialized skills become more attractive because match surplus goes down with employment protection, and this is so whether the model accounts for job creation decisions or not.

The modelling aspects of EPL and job destruction closely follows Mortensen and Pissarides (1994) and Wasmer (2006). The productivity of a job depends on specialized skills s and on a firm-specific component ε as follows: $y = f(s) + \varepsilon$. The firm component is random. It evolves according to a Poisson process with intensity λ and is drawn from a density function $g(\varepsilon)$ with c.d.f. $G(\varepsilon)$. The density has support $[\varepsilon^-, \varepsilon_0]$ and ε_0 is also the initial value of ε at the time of match formation. Before a new shock arrives, separation takes place at no cost. After the shock, the firm must pay the administrative firing cost T in case of separation. We assume that $\varepsilon_0 > \lambda T$.

We distinguish match surplus according to whether the job never experienced any productivity shock – $S^0(s,g)$ – or a productivity shock already occurred – $S(s,g,\varepsilon)$. We define value functions accordingly, i.e. worker's and firm's value functions are denoted by $W^0(s,g)$, $W(s,g,\varepsilon)$, $J^0(s,g)$ and $J(s,g,\varepsilon)$. Match surpluses are defined as follows:

$$S^0(s,g) = W^0(s,g) - U(s,g) + J^0(s,g) - V \quad (32)$$

$$S(s,g,\varepsilon) = W(s,g,\varepsilon) - U(s,g) + J(s,g,\varepsilon) - V + T \quad (33)$$

Match surplus is still split between the firm and the worker so that $W^0(s,g) = \beta S^0(s,g)$ and $W(s,g,\varepsilon) = \beta S(s,g,\varepsilon)$. This rule has two well-known implications. First, it leads to efficient job separation: job destruction occurs whenever match surplus becomes negative. Let $\varepsilon^d \equiv \varepsilon^d(s,g)$ denote the reservation productivity such that match surplus is equal to zero, i.e. $S\left(s,g,\varepsilon^d\right) = 0$. Second, $S^0(s,g) = S(s,g,\varepsilon_0) - T$.

[4] Becker (1964) also argues that the minimum wage tends to reduce skill acquisition. His argument relies on the fact that the minimum wage prevents from wage cuts used by firms to finance on-the-job training.

After usual computations, and setting V to zero, we obtain

$$S^0(s,g) = \frac{\varepsilon_0 - \varepsilon^d(s,g) - (r+\lambda)T}{r+\lambda} \tag{34}$$

$$\varepsilon^d(s,g) = \beta\mu H(g) S^0(s,g) - f(s) - rT - \frac{\lambda}{r+\lambda}\int_{\varepsilon^d(s,g)}^{\varepsilon_0} [1 - G(\widetilde{\varepsilon})]\, d\widetilde{\varepsilon} \tag{35}$$

Changes in educational mix (s,g), tightness θ or bargaining power β only transit through changes in ε^d. Importantly, initial match surplus decreases with firing costs T.

The optimal divide of schooling investment between adaptability and productivity skills results from

$$\max_g \left\{ rU(I-g,g) = \beta\mu H(g) S^0(I-g,g) \right\} \tag{36}$$

The f.o.c. writes

$$\frac{H'(g)}{H(g)} S^0(I-g,g) = \frac{f'(I-g)}{r+\lambda G(\varepsilon^d(I-g,g))} \tag{37}$$

The left-hand side is the marginal return to adaptability skills, while the right-hand side is the marginal return to productivity skills. An increase in firing costs distorts investments towards more specialized skills. On the one hand, initial match surplus S^0 goes down, which deteriorates the returns to general skills. On the other hand, ε^d decreases and jobs last longer, which raises the returns to specialized skills.

To close the model, consider tightness determination

$$c\frac{\theta}{\mu(\theta)} = (1-\beta) H(g) S^0(\theta, I-g, g) \tag{38}$$

where the dependence vis-à-vis θ has been highlighted.

Equilibrium tightness θ^* and general skills g^* solve equations (34), (35), (37), and (38). The following result summarizes the impact of job protection on the skill divide.

Proposition 5 Employment Protection and the skill divide

Firing costs lower the labor market tightness, i.e. $d\theta^*/dT < 0$*, and distort the skill divide towards specialized skills, i.e.* $dg^*/dT < 0$.

Dismissal costs have two different effects. At given tightness, they lower initial match surplus S^0 and favor specialized skills. However, the decrease in initial match surplus deteriorates job creation and tightness θ^* falls as a result – the usual Rosen effect. The fall in tightness lowers the decline in initial match surplus, thereby reducing the direct effect of job protection on schooling investment allocation. The latter effect being a second-order effect (the fall in tightness must occur because initial match surplus goes down), job protection promotes specialized skills. Accounting for endogenous tightness does not alter the reasoning made at given contact rate: firing costs lowers the relative returns to general skills at the time of educational investment.

5 Conclusion

This paper examines the impact of labor market institutions on the divide of schooling investment between general and specialized skills. We offer a matching model of unemployment in which individuals determine the scope and intensity of their skills prior to entering the labor market. We point out two effects. According to the Rosen effect, LMIs increase market frictions, which motivates the acquisition of general skills. According to the rent-capture effect, LMIs lower match surplus, which raises the return to specialized skills. We examine how these effects interact in the case of unemployment compensation, minimum wage and firing costs. We argue that the rent-capture effect is likely to dominate, which is in line with the empirical evidence relating LMIs to the skill divide.

6 References

Acemoglu, D.T., 1996. A microfoundation for social increasing returns to human capital accumulation. Quarterly Journal of Economics 61, 779-804

Acemoglu, D., Pischke, J.-S., 2003. Minimum wages and on-the-job training. Research in Labor Economics 22, 159-202

Agell, J., Lommerud, K., 1997. Minimum wages and the incentives for skill formation. Journal of Public Economics 64, 25-40

Booth, A, Chatterji, M , 1998. Unions and efficient training. Economic Journal 108, 328-343

Botero, J.C., Djankov, S., La Porta, R., Lopez-de-Silanes, F., Shleifer, A., 2004. The regulation of labor. Quarterly Journal of Economics, 1339-1382

Burdett, K., Smith, E., 2002. The low skill trap. European Economic Review 46, 1439-1451

Cahuc, P., Michel, Ph., 1996. Minimum wage, unemployment and growth. European Economic Review 40, 1463-1482

Charlot, O., Decreuse, B., 2005. Self-selection in education with matching frictions. Labour Economics 12, 251-267

Charlot, O., Decreuse, B., Granier, P., 2005. Adaptability, productivity, and the returns to schooling in a matching model. European Economic Review 49, 1007-1032

Estevez, M., Iversen, T., Soskice, D., 2001. Social protection and the formation of skills: a reinterpretation of the welfare state. In Peter Hall, and David Soskice, (eds), Varieties of Capitalism: the Institutional Foundations of Comparative Advantage

Fella, G , 2005. Termination restrictions and investment in general training. European Economic Review 49, 1479-99

Gould, E., Moav, O., Weinberg, B., 2001. Precautionary demand for education, inequality and technological progress. Journal of Labor Economics 6, 285-316

Grossman, S., Shapiro, C., 1982. A theory of factor mobility. Journal of Political Economy 90, 1054-1069

Hassler, J., Rodriguez Mora, J.V., Storesletten, K., Zilibotti, F., 2005. A positive theory of geographical mobility and social insurance. International Economic Review 46, 263-303

Hosios, A., 1990. On the efficiency of matching and related models of search and unemployment. Review of Economic Studies 57, 279-298

Krueger, D., Kumar, K., 2004. Skill-specific rather than general education: a reason for US-Europe growth differences? Journal of Economic Growth 9, 167-207

Kiyotaki, N., Wright, R., 1993. A search-theoretic approach to monetary Economics. American Economic Review 83, 63-77

Laing, D., Palivos, T., Wang, P., 1995. Learning, matching and growth. Review of Economic Studies 62, 115-129

Ljungqvist, L., Sargent, T., 1998. The European unemployment dilemma. Journal of Political Economy 106, 514-550

Moen, E.R., 1999. Education, ranking, and competition for jobs. Journal of Labor Economics 17, 694-723

Mortensen, D.T., 1977. Unemployment insurance and job search decisions. Industrial and Labor Relations Review 30, 505-517

Mortensen, D.T., Pissarides, C., 1994. Job creation and destruction in the theory of unemployment. Review of Economic Studies 61, 397-415

Mortensen, D.T., Pissarides, C., 1999. Unemployment responses to skill-biased technology shocks: the role of labour market policy. Economic Journal 109, 242-65

Mukoyama, T , Sahin, A , 2006. Specialization and human capital in search equilibrium. Journal of the European Economic Association 4, 503-512

Nickell, S., 1997. Unemployment and labor market rigidities: Europe vs North America. Journal of Economic Perspectives 11, 55-74

Rosen, S., 1983. Specialization and human capital. Journal of Labor Economics 1, 43-49

Shi, S., 1997. Money and specialization. Economic Theory 10, 99-133

Stevens, M., 1994. A theoretical model of on-the-job training with imperfect competition. Oxford Economic Papers 46, 537-562

Smits, W., 2007. Industry-specific or generic skills? Conflicting interests of firms and workers. Labour Economics 14, 653-663

Wasmer, E., 2006. General versus specific skills in labor markets with search frictions and firing costs. American Economic Review 96, 811-831

7 Appendix

In this Appendix, we prove Propositions 1 to 5.

7.1 Proof of Proposition 1

The assumption that V is sufficiently small guarantees that $f(I-\widehat{g}) > rV$. The result follows from the implicit function theorem. Indeed, $d\widehat{g}/d\mu^{-1}$ has the sign of $\partial S/\partial\mu^{-1} > 0$, and $d\widehat{g}/d\beta$ has the sign of $\partial S/\partial\beta < 0$.

7.2 Proof of Proposition 2

Part *(i)*. Equation (SD) defines an implicit function $g = \mathbf{g}(\theta,\beta)$ that is strictly decreasing in both arguments. In addition, $\lim_{\theta\to 0}\mathbf{g}(\theta,\beta) = g_0 < I$ such that $H'(g_0)/H(g_0) = f'(I-g_0)/f(I-g_0)$, and $\lim_{\theta\to\infty}\mathbf{g}(\theta,\beta) = 0$. Equation (MT) defines an implicit function $\theta = \boldsymbol{\theta}(g,\beta)$. It is strictly decreasing in β, while it is non-monotonic in g, increasing at first, reaching a maximum and then decreasing. In addition, $\lim_{g\to 0}\boldsymbol{\theta}(g,\beta) = \lim_{g\to 0}\boldsymbol{\theta}(g,\beta) = 0$. Those properties imply that there exists an equilibrium. In any such equilibrium, $\mathbf{g}_\theta(\theta^*,\beta) < 0$. Moreover, $\boldsymbol{\theta}_g(g^*,\beta)$ has the sign of $(\partial/\partial g)\left[H(g^*)\,S(\theta^*, I-g^*, g^*,\beta)\right]$. As $\mathbf{g}(\theta,\beta)$ results from $(\partial/\partial g)\left[H(g)\,S(\theta, I-g, g,\beta)\right] = 0$, we have $\boldsymbol{\theta}_g(g^*,\beta) = 0$. It follows that (SD) and (MT) cut once and only once.

Parts *(ii)* and *(iii)*. As $\partial\boldsymbol{\theta}(g^*,\beta)/\partial g = 0$, we have

$$\frac{d\theta^*}{d\beta} = \frac{\partial\boldsymbol{\theta}(g^*,\beta)}{\partial\beta} < 0 \tag{39}$$

$$\frac{dg^*}{d\beta} = \frac{\partial\mathbf{g}(\theta^*,\beta)}{\partial\beta} + \frac{\partial\mathbf{g}(\theta^*,\beta)}{\partial\theta}\frac{d\theta^*}{d\beta} \tag{40}$$

Therefore,

$$\frac{dg^*}{d\beta} \overset{sign}{=} \frac{\partial S(\theta^*, I-g^*, g^*,\beta)}{\partial\beta} + \frac{\partial S(\theta^*, I-g^*, g^*,\beta)}{\partial\theta}\frac{d\theta^*}{d\beta} \tag{41}$$

The result follows.

7.3 Proof of Proposition 3

As r tends to 0, the solving reduces to finding (g^*,θ^*) such that

$$\frac{H'(g)}{H(g)}S(\theta, I-g, g) = \frac{\beta(1-t_w)+(1-\beta)(1+t_e)}{1+t_e}\frac{f'(I-g)}{r+q} \tag{42}$$

$$\frac{c\theta}{\mu(\theta)} = (1-\gamma)\,H(g)\,S(\theta, I-g, g) \tag{43}$$

Following the proof of Proposition 2, equations (42) and (43) define two loci in the plane (g,θ). The (SD) locus is strictly decreasing, while the (MT) locus is $\cap$-shaped. The two loci intersect in the maximum of (MT). This property implies that $d\theta^*/db$ has the sign of $\partial S/\partial b < 0$. Using (42) and (43), one obtains

$$\frac{H'(g)}{H(g)}\frac{c\theta}{\mu(\theta)(1-\beta)H(g)} = \frac{f'(I-g)}{r+q} \tag{44}$$

In turn, equation (44) implies that dg^*/db has the sign of $d\theta^*/db < 0$.

Similarly, $d\theta^*/dt_i$, $i = e, w$, has the sign of $-\gamma'(t_i) HS + (1-\gamma) H\partial S/\partial t_i$. Overall, this term is negative. Then, equation (44) implies that $dg^*/dt_i < 0$ since θ^* goes down with t_i.

7.4 Proof of Proposition 4

An equilibrium is a tuple (θ^*, g_1^*, g_2^*) that solves

$$f(I_2 - g_2) = w_{\min} \tag{45}$$

$$\frac{H'(g_1)}{H(g_1)} \frac{f(I-g_1)}{r+q+\beta\mu H(g_1)} = \frac{f'(I-g_1)}{r+q} \tag{46}$$

$$c\frac{\theta}{\mu(\theta)} = \pi(\theta, g_1, g_2)(1-\beta) H(g_1) S(\theta, I_1 - g_1, g_1) \tag{47}$$

Prior to solving, we establish the properties of function π. We have $\pi(0, g_1, g_2) = \pi_1$ and $\pi(\infty, g_1, g_2) = \pi_1/(\pi_1 + (1-\pi_1) H_1/H_2) < \pi_1$. In addition, $\pi(\theta, g, g) = \pi_1$. Computing the partial derivatives, we get

$$\pi_\theta = \frac{\partial \pi}{\partial \theta} = -\frac{\mu' q (H_1 - H_2)}{(q+\mu H_1)(q+\mu H_2)} \pi(1-\pi) < 0 \tag{48}$$

$$\pi_{g_1} = \frac{\partial \pi}{\partial g_1} = -\frac{H_1'}{H_1}\frac{\mu H_1}{q+\mu H_1} \pi(1-\pi) < 0 \tag{49}$$

$$\pi_{g_2} = \frac{\partial \pi}{\partial g_2} = \frac{H_2'}{H_2}\frac{\mu H_2}{q+\mu H_2} \pi(1-\pi) > 0 \tag{50}$$

Using equation (45), we obtain $g_2^* = I_2 - f^{-1}(w_{\min})$. The solving reduces to finding a pair (θ^*, g_1^*) such that (46) and (47) hold. We already know that (46) defines the function $\mathbf{g}_1(\theta)$, with $\mathbf{g}_1(0) = g_0 < I_1$, $\mathbf{g}_1(\infty) = 0$, and $\mathbf{g}_1(\theta) > 0$ for all $\theta \geq 0$. In addition, (47) defines $\boldsymbol{\theta}(g_1, g_2)$ such that $\boldsymbol{\theta}(0, g_2) = 0 = \boldsymbol{\theta}(I_1, g_2)$ and $\boldsymbol{\theta}(g_1, g_2) > 0$ for all $g_1 \in (0, I_1)$. It follows that there exists an equilibrium. We now show that this equilibrium is unique. We compute $\boldsymbol{\theta}_{g_1}(g_1^*)$ and $\mathbf{g}_1'(\theta^*)$ and show that $\boldsymbol{\theta}_{g_1}(g_1^*, g_2^*) < [\mathbf{g}_1'(\theta^*)]^{-1}$.

Differentiating equation (47), it comes

$$\begin{aligned} c\frac{\mu - \theta\mu'}{\mu^2} d\theta &= [\pi_\theta (1-\beta) H_1 S_1 + \pi(1-\beta) H_1 S_{1\theta}] \, d\theta \\ &\quad + [\pi_{g_1}(1-\beta) H_1 S_1 + \pi(1-\beta)\partial(H_1 S_1)/\partial g_1] \, dg_1 \\ &\quad + \pi_{g_2}(1-\beta) H_1 S_1 dg_2 \end{aligned}$$

Using (47) and $\alpha = \theta\mu'/\mu$, we have

$$\left[\frac{1-\alpha}{\theta} - \frac{\pi_\theta}{\pi} - \frac{S_{1\theta}}{S_1}\right] d\theta = \left[\frac{\pi_{g_1}}{\pi} + \frac{\partial(H_1 S_1)/\partial g_1}{H_1 S_1}\right] dg_1 + \frac{\pi_{g_2}}{\pi} dg_2 \tag{51}$$

In equilibrium, $\partial(H_1 S_1)/\partial g_1 = 0$ and $dg_2 = 0$, and we obtain

$$\boldsymbol{\theta}_{g_1}(g_1^*, g_2^*) = \frac{\pi_{g_1}/\pi}{\frac{1-\alpha}{\theta} - \frac{\pi_\theta}{\pi} - \frac{S_{1\theta}}{S_1}} \tag{52}$$

Differentiating equation (46), we get

$$[H'' S_1 + 2H' S_{1g_1} + H S_{1g_1g_1}]\, dg_1 = -[H' S_{1\theta} + H S_{1g_1\theta}]\, d\theta \tag{53}$$

The next step consists in evaluating the various terms S_{1g_1}, $S_{1g_1g_1}$, $S_{1\theta}$, and $S_{1g_1\theta}$. After simple computations[5], we obtain

$$S_{1g_1} = -\lambda_H S_1 < 0 \tag{54}$$

$$S_{1\theta} = -\frac{\alpha}{\theta}\frac{\beta\mu H_1}{r+q+\beta\mu H_1} S_1 < 0 \tag{55}$$

$$S_{1\theta g_1} = \frac{\alpha}{\theta}\lambda_H \left(\frac{\beta\mu H_1}{r+q+\beta\mu H_1}\right)^2 S_1 > 0 \tag{56}$$

$$S_{1g_1g_1} = -S_1\left[\lambda_f' + \lambda_H' \frac{\beta\mu H_1}{r+q+\beta\mu H_1}\right] - S_1\lambda_H - S_1\lambda_H^2 \frac{\beta\mu H_1(r+q)}{(r+q+\beta\mu H_1)^2} \tag{57}$$

where $\lambda_H = H'/H$ and $\lambda_f = f'/f$.

Using equations (48), (49), (55), and (52), we obtain

$$\boldsymbol{\theta}_{g_1}(g_1^*, g_2^*) = -\frac{\theta\lambda_H \frac{\mu H_1}{q+\mu H_1}(1-\pi)}{1-\alpha+\frac{\alpha q\mu(H_1-H_2)}{(q+\mu H_1)(q+\mu H_2)}(1-\pi)+\alpha\frac{\beta\mu H_1}{r+q+\beta\mu H_1}} < 0 \tag{58}$$

Using equations (54), (55), (56), (57), and (53) we get

$$\mathbf{g}_\theta(\theta^*) = -\frac{\alpha}{\theta}\frac{\lambda_H \frac{\beta\mu H_1(r+q)}{(r+q+\beta\mu H_1)^2}}{\lambda_H^2\left[3-2\frac{\beta\mu H_1}{r+q+\beta\mu H_1}\right]+\lambda_H - \frac{H''}{H}\frac{r+q}{r+q+\beta\mu H_1} - \frac{f''}{f}} < 0 \tag{59}$$

Therefore,

$$\begin{aligned}
\boldsymbol{\theta}_{g_1}(g_1^*, g_2^*)\,\mathbf{g}_\theta(\theta^*) &= \frac{\theta\lambda_H \frac{\mu H_1}{q+\mu H_1}(1-\pi)}{1-\alpha+\frac{\alpha q\mu(H_1-H_2)}{(q+\mu H_1)(q+\mu H_2)}(1-\pi)+\alpha\frac{\beta\mu H_1}{r+q+\beta\mu H_1}} \\
&\quad \times \frac{\alpha}{\theta}\frac{\lambda_H \frac{\beta\mu H_1(r+q)}{(r+q+\beta\mu H_1)^2}}{\lambda_H^2\left[3-2\frac{\beta\mu H_1}{r+q+\beta\mu H_1}\right]+\lambda_H - \frac{H''}{H}\frac{r+q}{r+q+\beta\mu H_1} - \frac{f''}{f}} \\
&< \frac{\lambda_H \frac{\mu H_1}{q+\mu H_1}(1-\pi)}{\alpha\frac{\beta\mu H_1}{r+q+\beta\mu H_1}}\alpha\frac{\lambda_H \frac{\beta\mu H_1(r+q)}{(r+q+\beta\mu H_1)^2}}{\lambda_H^2\left[3-2\frac{\beta\mu H_1}{r+q+\beta\mu H_1}\right]} \\
&< (1-\pi)\frac{\mu H_1}{q+\mu H_1}\frac{r+q}{r+q+\beta\mu H_1} < 1
\end{aligned}$$

It follows that the equilibrium is unique.

The marginal impact of the MW on the labor market tightness results from the fact that $\boldsymbol{\theta}_{g_2}(g_1^*, g_2^*) < 0$. The marginal impact on g_1^* is due to $\mathbf{g}_\theta(\theta^*) < 0$.

[5] Remind that the various partial derivatives are evaluated in equilibrium.

7.5 Proof of Proposition 5

The equilibrium is a duple (θ^*, g^*) satisfying the following conditions:

(i) Initial match surplus and threshold productivity value

$$S^0(\theta, I-g, g, T) = \frac{\varepsilon_0 - \varepsilon^d(\theta, I-g, g, T) - (r+\lambda)T}{r+\lambda} \tag{60}$$

$$\begin{aligned} \varepsilon^d(\theta, I-g, g, T) &= \beta\mu(\theta) H(g) S^0(\theta, I-g, g, T) \\ &\quad - f(I-g) - rT - \frac{\lambda}{r+\lambda}\int_{\varepsilon^d(\theta,I-g,g,T)}^{\varepsilon_0} [1 - G(\widetilde{\varepsilon})]\, d\widetilde{\varepsilon} \end{aligned} \tag{61}$$

(ii) Optimal skill divide

$$\frac{H'(g)}{H(g)} S^0(\theta, I-g, g, T) = \frac{f'(I-g)}{r + \lambda G(\varepsilon^d(\theta, I-g, g, T))} \tag{62}$$

(iii) Equilibrium tightness

$$c\frac{\theta}{\mu(\theta)} = (1-\beta) H(g) S^0(\theta, I-g, g, T) \tag{63}$$

Equations (60) and (61) define initial match surplus and threshold productivity as functions of tightness, general skills, and firing costs. After computations, it comes

$$\frac{\partial S^0(\theta, I-g, g, T)/\partial\theta}{S^0(\theta, I-g, g, T)} = -\frac{\alpha(\theta)}{\theta}\frac{\beta\mu(\theta) H(g)}{r + \beta\mu(\theta) H(g) + \lambda G(\varepsilon^d(I-g, g))} < 0 \tag{64}$$

$$\frac{\partial \varepsilon^d(\theta, I-g, g, T)/\partial\theta}{S^0(\theta, I-g, g, T)} = -(r+\lambda)\frac{\lambda G\left(\varepsilon^d(I-g, g)\right)}{r + \beta\mu(\theta) H(g) + \lambda G(\varepsilon^d(I-g, g))} < 0 \tag{65}$$

$$\frac{\partial S^0(\theta, I-g, g, T)}{\partial T} = -\frac{\lambda G\left(\varepsilon^d(I-g, g)\right)}{r + \beta\mu(\theta) H(g) + \lambda G(\varepsilon^d(I-g, g))} < 0 \tag{66}$$

$$\frac{\partial \varepsilon^d(\theta, I-g, g, T)}{\partial T} = -(r+\lambda)\frac{r + \beta\mu(\theta) H(g)}{r + \beta\mu(\theta) H(g) + \lambda G(\varepsilon^d(I-g, g))} < 0 \tag{67}$$

Equations (62) and (63) jointly determine the equilibrium pair (g^*, θ^*). As in the proof of Proposition 2, uniqueness derives from the fact that g maximizes the contact surplus HS. Consider the following functions

$$\phi_1(g, \theta, T) = \frac{H'(g)}{H(g)} S^0(\theta, I-g, g, T) - \frac{f'(I-g)}{r + \lambda G(\varepsilon^d(\theta, I-g, g, T))} \tag{68}$$

$$\phi_2(g, \theta, T) = c\frac{\theta}{\mu(\theta)} - (1-\beta) H(g) S^0(\theta, I-g, g, T) \tag{69}$$

Let $\mathbf{J}$ denote the Jacobian matrix of function $\Phi \equiv (\phi_1, \phi_2)$ evaluated in equilibrium.

$$\mathbf{J} = \begin{bmatrix} \partial\phi_1/\partial g & \partial\phi_1/\partial\theta \\ \partial\phi_2/\partial g & \partial\phi_2/\partial\theta \end{bmatrix} \tag{70}$$

where partial derivatives are computed by use of equations (64) and (65). It can be shown that

$$\begin{aligned}
\partial\phi_1/\partial g &< 0 && (71)\\
\frac{H'}{H}\frac{\partial S^0}{\partial\theta} &< \partial\phi_1/\partial\theta < 0 && (72)\\
\partial\phi_2/\partial g &\equiv 0 && (73)\\
\partial\phi_2/\partial\theta &\equiv (1-\beta)\,\frac{HS^0}{\theta}\frac{(1-\alpha)\,(r+\lambda G)+\beta\mu H}{r+\beta\mu H+\lambda G} > 0 && (74)
\end{aligned}$$

By the implicit function theorem,

$$\left[\begin{array}{c} dg^*/dT \\ d\theta^*/dT \end{array}\right] = -\mathbf{J}^{-1}\left[\begin{array}{c} \partial\phi_1/\partial T \\ \partial\phi_2/\partial T \end{array}\right] \tag{75}$$

where

$$\begin{aligned}
\partial\phi_1/\partial T &< \frac{H'}{H}\frac{\partial S^0}{\partial T} < 0 && (76)\\
\partial\phi_2/\partial T &\equiv -(1-\beta)\,H\frac{\partial S^0}{\partial T} > 0 && (77)
\end{aligned}$$

and

$$\mathbf{J}^{-1} = \frac{1}{\det\mathbf{J}}\left[\begin{array}{cc} \partial\phi_2/\partial\theta & -\partial\phi_1/\partial\theta \\ -\partial\phi_2/\partial g & \partial\phi_1/\partial g \end{array}\right] \tag{78}$$

with $\det\mathbf{J} = (\partial\phi_1/\partial g)\,(\partial\phi_2/\partial\theta) < 0$. Therefore,

$$\frac{d\theta^*}{dT} \overset{sign}{=} (\partial\phi_1/\partial g)\,(\partial\phi_2/\partial T) < 0 \tag{79}$$

Similarly,

$$\begin{aligned}
\tfrac{dg^*}{dT} \overset{sign}{=} & (\partial\phi_2/\partial\theta)\,(\partial\phi_1/\partial T) - (\partial\phi_1/\partial\theta)\,(\partial\phi_2/\partial T)\\
& < (1-\beta)\,\tfrac{H'}{H}\tfrac{\partial S^0}{\partial T}\tfrac{HS^0}{\theta}\tfrac{(1-\alpha)(r+\lambda G)+\beta\mu H}{r+\beta\mu H+\lambda G} + (1-\beta)\,\tfrac{H'}{H}\tfrac{\partial S^0}{\partial T}H\tfrac{\partial S^0}{\partial\theta}\\
& = (1-\beta)\,H'\tfrac{\partial S^0}{\partial T}S^0\tfrac{1-\alpha}{\theta} < 0
\end{aligned} \tag{80}$$

8 Side calculations – Not to be published

In this technical Appendix, we provide further details to the discussions in Section 4.

8.1 Unemployment benefits

We proceed in two steps. (i) We derive the equation for the match surplus given in the text. (ii) We provide further details about the numerical simulations.

(i) The wage results from Nash bargaining

$$\max_{w} \{\beta \ln (W - U) + (1-\beta) \ln (J - V)\} \tag{81}$$

The f.o.c yields

$$\frac{\beta (1-t_w)}{W-U} = \frac{(1-\beta)(1+t_e)}{J-V} \tag{82}$$

Using equations (15) to (17), we obtain equations (18) and (19) in the text, which we reproduce here for convenience:

$$W - U = \gamma S \tag{83}$$

$$J - V = (1-\gamma) S \tag{84}$$

where $\gamma = \beta (1-t_w) / [\beta (1-t_w) + (1-\beta)(1+t_e)]$. Then, we use the fact that $rS = r(W-U) + r(J-V)$. Together with (15) to (17), we get

$$(r + q + \gamma \mu H) S = f - (t_w + t_e) w - b - rV \tag{85}$$

This gives a first equation linking match surplus S to wage w. The second equation comes from $r(W-U) = \gamma r S$. Using (15) and (16), we obtain

$$(r + q + \mu H) \gamma S = w (1-t_w) - b \tag{86}$$

Solving in S, we get equation (20) in the text, which we reproduce here:

$$S = \frac{\beta (1-t_w) + (1-\beta)(1+t_e)}{1+t_e} \frac{f - b\frac{1+t_e}{1-t_w} - rV}{r + q + \beta \mu H} \tag{87}$$

(ii) The equilibrium pair (θ^*, g^*) solves

$$\frac{H'(g)}{H(g)} \frac{f(I-g) - b\frac{1+t_e}{1-t_w}}{r + q + \beta\mu(\theta) H(g)} + \beta^{-1} \frac{H'(g)}{H(g)} \frac{b\frac{1+t_e}{1-t_w}}{r+q} \frac{r[r + q + \beta\mu(\theta) H(g)]}{[r + \mu(\theta) H(g)]^2} = \frac{f'(I-g)}{r+q} \tag{88}$$

$$\frac{c\theta}{\mu(\theta)} = (1-\gamma) H(g) S(\theta, I-g, g) \tag{89}$$

$$= (1-\beta) H(g) \frac{f(I-g) - b\frac{1+t_e}{1-t_w}}{r + q + \beta\mu(\theta) H(g)} \tag{90}$$

UB respond to $b = \rho w (1 - t_w)$. Using the wage equation (86) and the match surplus equation (87), we obtain

$$w (1 - t_w) = \frac{\beta (r + q + \mu H)}{(r + q) (1 - (1 - \beta) \rho) + \beta \mu H} \frac{1 - t_w}{1 + t_e} f \tag{91}$$

Replacing b by $\rho w (1 - t_w)$ in equations (88) and (90), we get

$$\begin{aligned} & \frac{H' (g)}{H (g)} \frac{(1 - \rho) f (I - g)}{(r + q) B + \beta \mu H} \\ & + \frac{H' (g)}{H (g)} \frac{r + q + \mu H}{(r + q) B + \beta \mu H} \frac{\rho f}{r + q} \frac{r [r + q + \beta \mu (\theta) H (g)]}{[r + \mu (\theta) H (g)]^2} \\ & - \frac{f' (I - g)}{r + q} \\ = \; & 0 \end{aligned} \tag{92}$$

$$\frac{c\theta}{\mu (\theta)} = (1 - \beta) H (g) \frac{(1 - \rho) f (I - g)}{(r + q) B + \beta \mu H} \tag{93}$$

with $B = 1 - (1 - \beta) \rho$.

With $\mu (\theta) = M_0 \theta^{1/2}$, equation (90) is equivalent to

$$\beta H c \theta + \frac{c}{M_0} (r + q) B \theta^{1/2} - (1 - \beta) H (1 - \rho) f = 0 \tag{94}$$

This equation can be solved in θ. This gives

$$\mu (\theta (g)) = \frac{- (r + q) B + \left[(r + q)^2 B^2 + 4 \beta (1 - \beta) (1 - \rho) M_0^2 H f / c \right]^{1/2}}{2 \beta H} \tag{95}$$

The solving simplifies to finding g^* such that

$$\begin{aligned} \phi (g) = \; & \lambda_H (g) (1 - \rho) & (96) \\ & + \frac{r}{r + q} \frac{(r + q + \mu (\theta (g))) (r + q + \beta \mu (\theta (g)))}{[r + \mu (\theta (g)) H (g)]^2} \rho & (97) \\ & - \frac{(r + q) B + \beta \mu (\theta (g)) H (g)}{r + q} \lambda_f (I - g) \\ = \; & 0 \end{aligned}$$

with $\lambda_H (g) = H' (g) / H (g)$ and $\lambda_f (s) = f' (s) / f (s)$. The functions indicated in the text are such that $H (g) = 1 - \exp (-\kappa g)$ and $f (s) = 1 - \exp (-\nu s)$. This implies that

$$\lambda_H (g) = \frac{\kappa \exp (-\kappa g)}{1 - \exp (-\kappa g)} \tag{98}$$

$$\lambda_y (I - g) = \frac{\nu \exp (-\nu (I - g))}{1 - \exp (-\nu (I - g))} \tag{99}$$

We use Matlab to find the numerical solution of (96) as a function of ρ for various values of r.

8.2 Minimum wage

An equilibrium is a tuple (g_1^*, g_2^*, θ^*) that satisfies

$$\widehat{g}_i = \arg\max_g \left\langle rU_i = \mu H(g_i) \max\left\{\beta S_i(I_i - g, g), \frac{w_{\min}}{r + q + \mu H(g)}\right\}\right\rangle \tag{100}$$

subject to the constraint $f(I_i - g) \geq w_{\min}$.

$$\frac{c\theta}{\mu(\theta)} = \sum_i \pi_i \min\left\{(1-\beta) S_i, \frac{f(I_i - g_i) + qV - w_{\min}}{r+q}\right\} \tag{101}$$

$$\pi_1 \equiv \pi \equiv 1 - \pi_2 = \frac{p_1 q / (q + \mu H(g_1))}{p_1 q / (q + \mu H(g_1)) + p_2 q / (q + \mu H(g_2))} \tag{102}$$

There can be three types of equilibrium. Either (i) the MW does not bind, or (ii) it only binds for the less educated, or (iii) it binds for both groups of workers. In the text, we only focus on the case where the MW only binds for the less educated. We now briefly discuss the two other cases.

(i) When the MW does not bind, the equilibrium solves

$$\frac{H'(g_i)}{H(g_i)} S(\theta, I_i - g_i, g_i) = \frac{f'(I_i - g_i)}{r+q} \tag{103}$$

$$\frac{c\theta}{\mu(\theta)} = \sum_i \pi_i \min(1-\beta) S_i \tag{104}$$

$$\pi_1 \equiv \pi \equiv 1 - \pi_2 = \frac{p_1 q / (q + \mu H(g_1))}{p_1 q / (q + \mu H(g_1)) + p_2 q / (q + \mu H(g_2))} \tag{105}$$

We denote by θ_{nb}, g_{1nb}, and g_{2nb} the equilibrium variables – nb stands for non-binding. As $I_1 > I_2$, we have $s_{1nb} > s_{2nb}$ and $g_{1nb} > g_{2nb}$. The high-educated are both more productive and more adaptable than the less educated. The non-binding equilibrium prevails iff

$$w_{\min} < \beta\left[r + q + \mu(\theta_{nb}) H(g_{2nb})\right] S(\theta_{nb}, I_2 - g_{2nb}, g_{2nb}) \tag{106}$$

(ii) When the MW only binds for the less educated, the equilibrium solves

$$f(I_2 - g_2) = w_{\min} \tag{107}$$

$$\frac{H'(g_1)}{H(g_1)} S(\theta, I_1 - g_1, g_1) = \frac{f'(I_1 - g_{1i})}{r+q} \tag{108}$$

$$\frac{c\theta}{\mu(\theta)} = \pi(\theta, g_1, g_2)(1-\beta) S(\theta, I_1 - g_1, g_1) \tag{109}$$

We denote by θ_{pb}, g_{1nb}, and g_{2nb} the equilibrium variables – pb stands for partially-binding. This equilibrium prevails if and only if

$$\beta\left[r + q + \mu(\theta_{nb}) H(g_{2nb})\right] S(\theta_{nb}, I_2 - g_{2nb}, g_{2nb}) < w_{\min} < \beta\left[r + q + \mu(\theta_{pb}) H(g_{1pb})\right] S(\theta_{pb}, I_1 - g_{1pb}, g_{1pb}) \tag{110}$$

This condition requires that I_1 is sufficiently larger than I_2.

(iii) When the MW binds for both groups, $f(I_1 - g_1) = w_{\min} = f(I_2 - g_2)$. It follows that match surplus is nil, and market tightness is zero as well.

8.3 Employment protection

We derive the various equations given in the text.

Workers' value functions are

$$\begin{aligned} rW(s,g,\varepsilon) &= w(s,g,\varepsilon) && (111)\\ &+\lambda\left[\int_{\varepsilon^d(s,g)}^{\varepsilon_0} W(s,g,\widetilde{\varepsilon})dG(\widetilde{\varepsilon}) + G(\varepsilon^d(s,g))U(s) - W(s,g,\varepsilon)\right] \\ rW^0(s,g) &= w^0(s,g) && (112)\\ &+\lambda\left[\int_{\varepsilon^d(s,g)}^{\varepsilon_0} W(s,g,\widetilde{\varepsilon})dG(\widetilde{\varepsilon}) + G(\varepsilon^d(s,g))U(s) - W^0(s,g,\varepsilon)\right] \\ rU(s,g) &= \mu H(g)\left[W^0(s,g) - U(s,g)\right] && (113) \end{aligned}$$

Firms' value functions are

$$\begin{aligned} rJ(s,g,\varepsilon) &= f(s) + \varepsilon - w(s,g,\varepsilon) && (114)\\ &+\lambda\left[\int_{\varepsilon^d(s,g)}^{\varepsilon_0} J(s,g,\widetilde{\varepsilon})\,dG(\widetilde{\varepsilon}) - (T-V)\,G\left(\varepsilon^d(s,g)\right) - J(s,g,\varepsilon)\right] \\ rJ^0(s,g) &= f(s) + \varepsilon - w^0(s,g,\varepsilon) && (115)\\ &+\lambda\left[\int_{\varepsilon^d(s,g)}^{\varepsilon_0} J(s,g,\widetilde{\varepsilon})\,dG(\widetilde{\varepsilon}) - (T-V)\,G\left(\varepsilon^d(s,g)\right) - J^0(s,g)\right] \end{aligned}$$

Match surpluses are given in the text. We reproduce them here:

$$\begin{aligned} S^0(s,g) &= W^0(s,g) - U(s,g) + J^0(s,g) - V && (116)\\ S(s,g,\varepsilon) &= W(s,g,\varepsilon) - U(s,g) + J(s,g,\varepsilon) - V + T && (117) \end{aligned}$$

Nash bargaining implies that

$$\begin{aligned} W^0(s,g) &= \beta S^0(s,g) && (118)\\ W(s,g,\varepsilon) &= \beta S(s,g,\varepsilon) && (119) \end{aligned}$$

Finally, the productivity threshold derives from

$$S\left(s,g,\varepsilon^d\right) = 0 \qquad (120)$$

Mixing the different conditions leads to the following equation for match surplus

$$(r+\lambda)\,S(s,g,\varepsilon) = f(s) + \varepsilon - r\,[U(s,g) - T] + \frac{\lambda}{r+\lambda}\int_{\varepsilon^d(s,g)}^{\varepsilon_0} [1 - G(\widetilde{\varepsilon})]\,d\widetilde{\varepsilon} \qquad (121)$$

Using (i) $S^0(s,g) = S(s,g,\varepsilon_0) - T$, (ii) equations (113), (118) and (119), we finally obtain the equations given in the text, that is

$$\begin{aligned} S^0(s,g) &= \frac{\varepsilon_0 - \varepsilon^d(s,g) - (r+\lambda)T}{r+\lambda} && (122)\\ \varepsilon^d(s,g) &= \beta\mu H(g)\,S^0(s,g) - f(s) - rT - \frac{\lambda}{r+\lambda}\int_{\varepsilon^d(s,g)}^{\varepsilon_0} [1 - G(\widetilde{\varepsilon})]\,d\widetilde{\varepsilon} && (123) \end{aligned}$$

The equation defining initial match surplus is very standard (see for instance the analogous equation in Wasmer, 2006). The second equation is also standard, even though we have written it a bit differently to get a more compact equation. These equations can also be written in a less elegant way:

$$S^0(s,g) = \frac{f(s) + \varepsilon_0 - \lambda T + \frac{\lambda}{r+\lambda}\int_{\varepsilon^d(s,g)}^{\varepsilon_0}[1 - G(\widetilde{\varepsilon})]\,d\widetilde{\varepsilon}}{r + \lambda + \beta\mu H(g)} \tag{124}$$

$$\begin{aligned} \varepsilon^d(s,g)\left[r + \lambda + \beta\mu(\theta)H(g)\right] + \lambda\int_{\varepsilon^d(s,g)}^{\varepsilon_0}[1 - G(\widetilde{\varepsilon})]\,d\widetilde{\varepsilon} &= \beta\mu H(g)\varepsilon_0 \\ &\quad - (r+\lambda)\left\{f(s) + T\left[r + \beta\mu(\theta)H(g)\right]\right\} \end{aligned} \tag{125}$$

Both initial match surplus and threshold productivity level go down with firing costs T.

The f.o.c. to the maximization program writes down:

$$H'(g)\,S^0(I-g,g) = H(g)\left[\frac{\partial S^0(I-g,g)}{\partial s} + \frac{\partial S^0(I-g,g)}{\partial g}\right] \tag{126}$$

Using the facts that

$$\frac{\partial S^0(s,g)}{\partial s} = \frac{f'(s)}{r + \beta\mu H(g) + \lambda G(\varepsilon^d(s,g))} \tag{127}$$

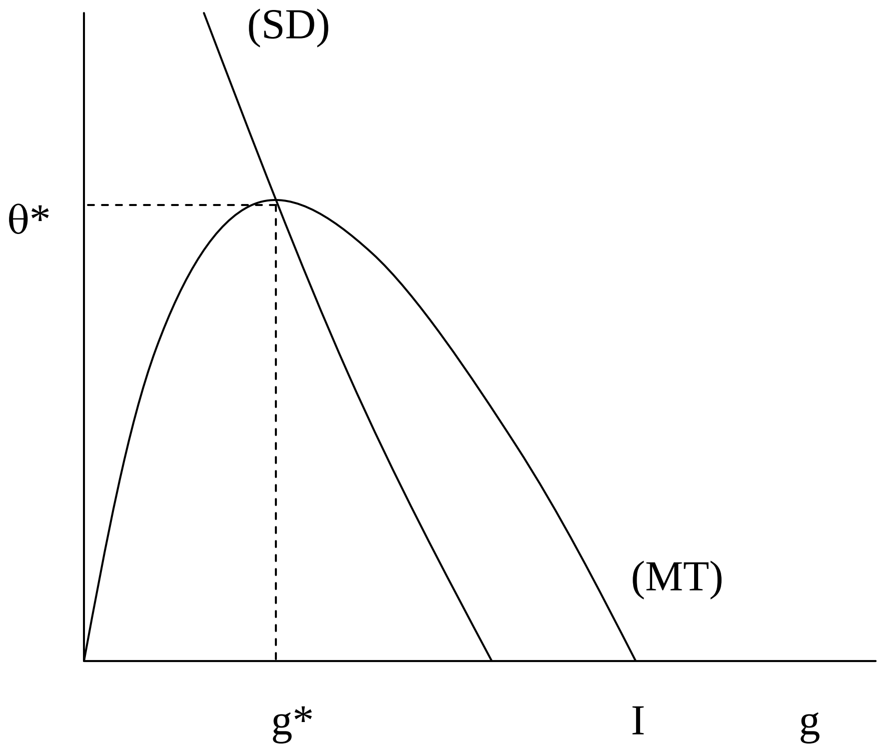
$$\frac{\partial S^0(s,g)/\partial g}{S^0(s,g)} = -\frac{H'(g)}{H(g)}\frac{\beta\mu H(g)}{r + \beta\mu H(g) + \lambda G(\varepsilon^d(s,g))} \tag{128}$$

We finally get the equation given in text:

$$\frac{H'(g)}{H(g)}S^0(I-g,g) = \frac{f'(I-g)}{r + \lambda G(\varepsilon^d(I-g,g))} \tag{129}$$

Figure

Fig.1 : Existence and uniqueness of equilibrium. The loci (MT) and (SD) intersect once in the maximum of (MT).



Figure

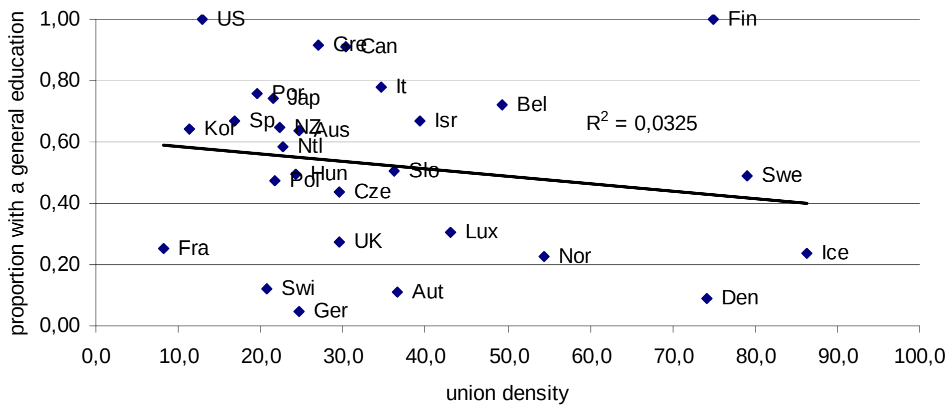


Fig.2a : Union density and proportion of upper-secondary educated with a general education

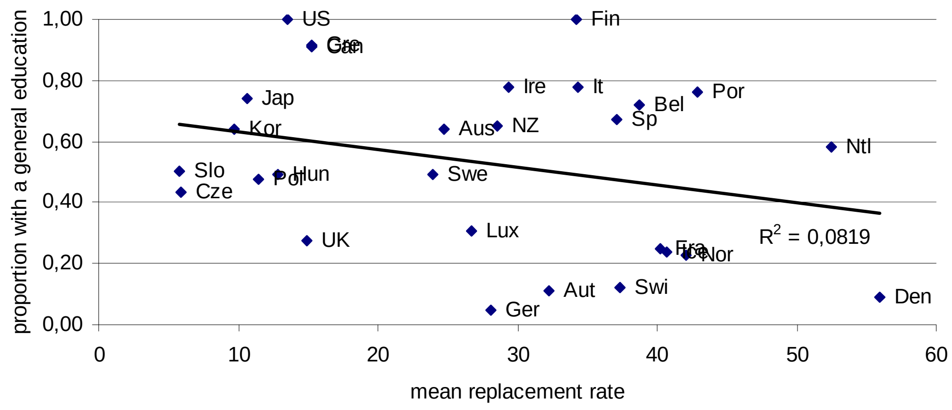


Fig.2b : Unemployment compensation and proportion of upper-secondary educated with a general education

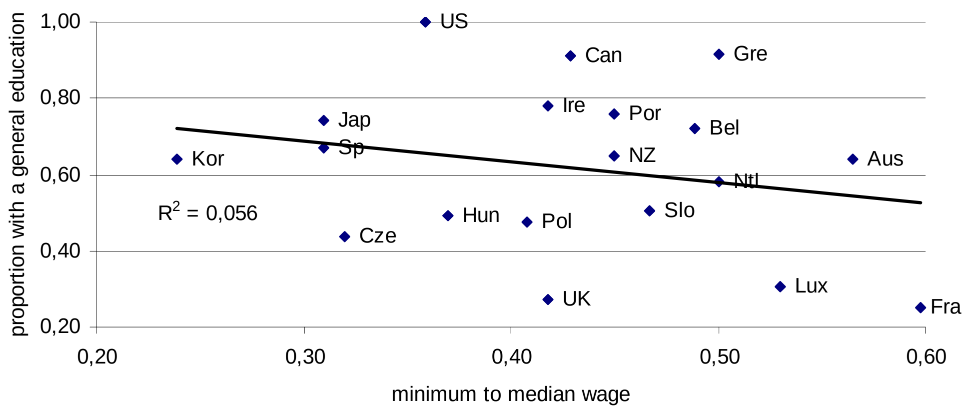


Fig.2c : Minimum wage and proportion of upper-secondary educated with a general education

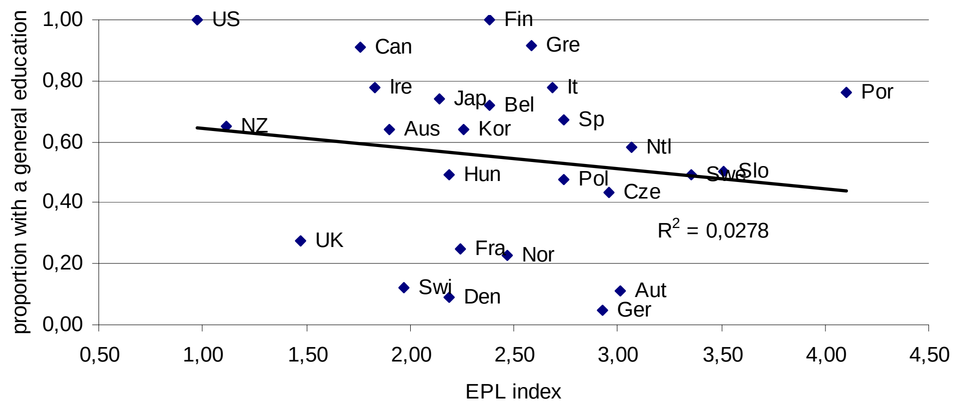


Fig.2d : EPL and proportion of upper-secondary educated with a general education

Figure

Fig.3: Equilibrium skill divide as a function of UB replacement rate