# Emotions and Incentives*

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We introduce a concept of emotions that emerge when workers compare their own performance with a given standard or with the performances of co-workers. Assuming heterogeneity among the workers the interplay of emotions and incentives is analyzed by focusing on three incentive schemes that are frequently used in practice: tournaments, bonuses and piece rates. We identify certain conditions under which emotions lead to additional incentives and under which the employer benefits from emotional workers. Furthermore, the concept of emotions is used to explain puzzling results from laboratory and field experiments. Finally, the results provide some insights on an employer's possible preferences in favor of heterogeneous instead of homogeneous work groups.

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# 1 Introduction

Emotions are a natural ingredient of human beings. In particular, when evaluating possible consequences of their decisions people take emotions like anger, frustration, joy or pride into account. Hence, an economic decision maker should also incorporate possible emotions into his objective function. Moreover, the experimental findings of Bosman and van Winden (2002) and van Winden (2001) on emotional hazard point out that emotions play an important role in real decision making. However, as Elster (1996, 1998) and Loewenstein (2000) complain, economists – with some exceptions[1] – do not pay attention to emotions when modelling economic behavior although introducing emotions may "help us explain behavior for which good explanations seem to be lacking" (Elster 1998, p. 489).

In this paper, emotions are introduced into incentive theory while focusing on three incentive schemes that are frequently used in practice – tournaments, bonus schemes and piece-rate systems. The aim of the paper is threefold: First, it will be emphasized that emotions are not always detrimental as pointed out by the experiments on emotional hazard and the model by Mui (1995) on envy. We can show under which conditions emotions are beneficial for a profit maximizing employer and enhance overall welfare. In particular, the employer may even benefit from "negative emotions" of his workers like frustration or anger. As one example, tournament incentive schemes will be considered. It seems somewhat natural that contestants compare themselves with their co-workers who compete in the same tournament, and that a worker feels anger (pride) when losing (winning) against a weaker (predominant) opponent. Standard tournament results show that asymmetric

[1] See, e.g., Hirshleifer (1987) on emotions as guarantors of threats and promises, Kandel and Lazear (1992) on shame and guilt in the context of peer pressure, Mui (1995) on envy.

tournaments between heterogeneous agents are never optimal (e.g., Lazear and Rosen 1981). However, when introducing emotions into tournaments this general result no longer holds. On the contrary, equilibrium efforts may even increase in the ability difference of the competitors.

Second, the paper seizes the suggestion made by Elster and utilizes emotions to explain empirical findings that contradict standard economic theory. For example, there exist diverse experimental findings on asymmetric tournaments which are puzzling as they show that players significantly oversupply effort compared to equilibrium effort levels (Bull et al. 1987, Weigelt et al. 1989, Schotter and Weigelt 1992). By using the concept of emotions these results can be easily explained. Furthermore, the field experiments by Falk and Ichino (2003) document the existence of peer effects within work groups. They show that the pure existence of co-workers enhances incentives. These findings can be explained if we assume that workers compare their performances with those of their co-workers and that workers feel emotions like pride or frustration in case of relative success or failure, respectively.

Third, we assume that emotions that emerge when comparing one's own performance with the performance of co-workers will be stronger if the workers are heterogeneous, since it will be more difficult to beat a more able co-worker than an equally or less talented one. By combining emotions with heterogeneity among workers, we can derive conditions under which an employer prefers heterogeneous departments to homogeneous ones and vice versa.

The paper is organized as follows. In the next section, the basic model is introduced without considering specific incentive schemes. Section 3 deals with emotions in so-called "unfair" tournaments in which a less able worker – the underdog – competes against a more able one – the favorite. Depending on whether tournament prizes are exogenously given or endogenously chosen by the employer under unlimited or limited liability of the workers, emotions as anger and pride may lead to extra incentives and are beneficial from the employer's viewpoint. Section 4 then focuses on individualistic incentive schemes like bonus systems and piece-rate schemes. Again, the incentive effects under exogenous incentive parameters and optimally chosen incentive schemes under unlimited and limited liability are considered. Section 5 concludes.

## 2 The Basic Model

We consider a firm which consists of one risk neutral employer and four risk neutral workers.[2] Each worker's verifiable performance or output can be described by the production function $q_i = e_i + a_i + \varepsilon_i$ ($i = 1, 2, 3, 4$). $e_i$ denotes endogenous effort which is chosen by worker $i$, $a_i$ worker $i$'s exogenous ability and $\varepsilon_i$ individual noise which is also assumed to be exogenous. The noise variables $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ are identically and independently distributed with density $g(\cdot)$ and cumulative distribution function $G(\cdot)$. Let $f(\cdot)$ denote the density and $F(\cdot)$ the cumulative distribution function of the composed random variable $\varepsilon_j - \varepsilon_i$ of each pair of two workers. It is assumed that $f(\cdot)$ has a unique mode at zero.[3] The employer can only observe realized output $q_i$ but none of its components. Hence, a standard moral hazard problem is considered. Exerting effort entails costs on a worker which are described by

[2] Most of the assumptions follow the standard tournament model by Lazear and Rosen (1981).

[3] For example, if $\varepsilon_i$ and $\varepsilon_j$ are uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$ (normally distributed), the convolution $f(\cdot)$ will be a triangular distribution over $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ (normal distribution) with mean zero.

the function $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$ and $c''(e_i) > 0$. The reservation value of each worker is $\bar{u} \geq 0$.

Two of the workers – the so-called "underdogs" – are characterized by low ability $a_U$, whereas the two other workers – the "favorites" – have a high ability $a_F$ with $a_F > a_U$.[4] Let $\Delta a := a_F - a_U > 0$ denote the ability difference between favorites and underdogs. The respective type $U$ or $F$ of each worker is common knowledge.

It is assumed that the firm consists of two departments and that the employer has to choose the composition of the departments. He can either choose a homogeneous design ($D = HOM$) under which one department contains the two underdogs and the other one the two favorites, or a heterogeneous design ($D = HET$) which is characterized by two heterogeneous departments each consisting of one underdog and one favorite.

In the following, different incentive schemes are considered which are frequently used in practice. For simplicity, the type of incentive scheme is assumed to be the same for each department.[5] Under any scheme, each

[4] Of course, heterogeneity between workers can be modelled in different ways. Here we take the additive model of Meyer and Vickers (1997), Holmström (1999), Höffler and Sliwka (2003), Kräkel (2004), for example. Alternatively, heterogeneity can be introduced via the workers' cost functions (or, very similar, by a multiplicative connection of effort and ability). Concerning the tournament literature, the former modelling used in this paper refers to "unfair" contests, whereas the latter one leads to "uneven" contests in the terminology of O'Keefe, Viscusi and Zeckhauser (1984). This distinction and its implications will be discussed in Section 3 in more details.

[5] This restriction is not very important. For $D = HET$, the two departments are identical. For $D = HOM$, the same type of incentive scheme (e.g., tournament, piece rates) leads to similar results in both departments because of the additive production function. The assumption only rules out centralized incentive schemes which include all four workers. However, the peer effects analyzed in this paper become clearer when making use of the assumption.

worker wants to maximize expected wages minus effort costs. However, the employer's objective function depends on the given situation. We differentiate between a situation in which the *parameters of the incentive scheme are exogenously given* (e.g., as the outcome of a bargaining process between the union and the employer which is not modelled here) and a situation where the *employer endogenously chooses the optimal incentive parameters*. In the former case, the employer wants to maximize the sum of the four efforts for a given incentive scheme and, therefore, for given labor costs. In the latter case, he maximizes expected net profits, i.e. expected outputs minus wages.

For any incentive scheme that will be considered in the following the timing of the game is the same: In the situation with *exogenously* given incentive parameters, we have to solve a two-stage game where, at the first stage, the employer decides on the design of the firm, $D$, and thereafter the four workers choose their efforts $e_i$ at the second stage. However, there is a three-stage game in the situation with *endogenously* chosen incentive parameters: Again, at the first stage, the employer chooses $D$. At the second stage he chooses the optimal incentive parameters. At the third stage, for a given design $D$ and given incentive parameters the four workers decide on their efforts.

In the following sections two types of incentive schemes are considered that can be frequently observed in real firms. Section 3 deals with a collective incentive scheme that is based on relative performance evaluation – a tournament or contest scheme. Section 4 focuses on individualistic incentive schemes – piece rates and a bonus scheme.

# 3 On the Optimality of Unfair Tournaments

In a (rank-order) tournament, at least two workers compete against each other for given prizes. The worker with the best performance receives the winner prize, the second best worker gets the second highest prize and so on. There exist many examples for tournaments in economics.[6] They can be observed between salesmen (e.g., Mantrala et al. 2000), in broiler production (Knoeber and Thurman 1994) and also in hierarchical firms when people compete for job promotion (e.g., Baker et al. 1994, Eriksson 1999, Bognanno 2001). Basically, corporate tournaments will always be created if relative performance evaluation is linked to monetary consequences for the employees. Hence, forced-ranking or forced-distribution systems, in which supervisors have to rate their subordinates according to a given number of different grades, also belong to the class of tournament incentive schemes (see, for example, Murphy 1992 on forced ranking at Merck). Boyle (2001) reports that about 25 per cent of the so-called Fortune 500 companies utilize forced-ranking systems to tie pay to performance (e.g., Cisco Systems, Intel, General Electric).

In the given context of departmental tournaments, two workers $i$ and $j$ compete for the monetary prizes $w_H$ and $w_L$ with $w_H > w_L$ in each tournament. If $q_i > q_j$, worker $i$ will receive the high winner prize $w_H$, whereas worker $j$ will get the loser prize $w_L$. This paper departs from the standard tournament literature by assuming that workers have *perceived prizes* which may differ from the monetary tournament prizes $w_H$ and $w_L$. In particular, we can imagine that on the one hand a favorite feels anger or shame when losing against an underdog. This would mean that a favorite's subjectively

[6] For a theoretical analysis of tournaments see Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), Rosen (1986).

perceived loser prize under $D = HET$ is lower than his monetary one, i.e. he gets $w_L - \delta$ in case of losing with $\delta > 0$, whereas the underdog's perceived loser prize is identical with his monetary one. On the other hand, an underdog might feel joy or pride when winning against a favorite. This would imply that under $D = HET$ an underdog has a higher perceived winner prize $w_H + \gamma$ with $\gamma > 0$ compared to his monetary one whereas the favorite's perceived and monetary winner prizes are the same. These two scenarios catch the typical notion that often the subjective prize of a workers also depends on the strength of his opponent.[7] When winning (losing) against a mighty (weak) opponent a worker realizes an extra utility (disutility) compared to a situation in which he wins or loses against an equally able player. Hence, under $D = HOM$ all subjectively perceived prizes are identical to the monetary prizes.[8]

Recall that at the first stage of the game the employer decides on $D$. He can either choose two *fair* tournaments ($D = HOM$) in which two underdogs and two favorites compete against each other, respectively, or two *unfair* tournaments ($D = HET$) each consisting of an underdog and a favorite.[9] We begin the analysis by considering the case of fair tournaments.

[7] For example, there are parallels to the status motive in competition; see, e.g., Frank and Cook (1996), pp. 112-114.

[8] Note that the pure event of winning (losing) may lead to an extra utility (disutility) for a worker even in the case of $D = HOM$. However, then all prizes are subjectively perceived prizes which exceed the monetary ones. In this case, $w_H$ and $w_L$ must be redefined.

[9] O'Keefe, Viscusi and Zeckhauser (1984) introduced the notion of an "unfair tournament" in which the favorite has a lead $\Delta a$. For optimal seeding in a dynamic context see Rosen (1986) and Groh et al. (2003).

## 3.1 Fair Tournaments

If the employer chooses $D = HOM$, we will have two fair tournaments in the meaning of O'Keefe, Viscusi and Zeckhauser (1984) in which perceived and monetary prizes are identical. In each of these tournaments the agents $i$ and $j$ $(i, j = t;\ t = U, F)$ want to maximize

$$\begin{aligned} EU_i(e_i) &= w_L + \Delta w \cdot \text{prob}\{q_i > q_j\} - c(e_i) \\ &= w_L + \Delta w \cdot F(e_i - e_j) - c(e_i) \end{aligned}$$

and

$$EU_j(e_j) = w_L + \Delta w \cdot [1 - F(e_i - e_j)] - c(e_j),$$

respectively, with $\Delta w = w_H - w_L$. If an equilibrium in pure strategies exists, it will be described by the following first-order conditions:[10]

$$\Delta w f(e_i - e_j) = c'(e_i) \quad \text{and} \quad \Delta w f(e_i - e_j) = c'(e_j). \tag{1}$$

Hence, we have a unique symmetric equilibrium $(e_i, e_j) = (e^*, e^*)$ with

$$\Delta w f(0) = c'(e^*). \tag{2}$$

## 3.2 Anger in Unfair Tournaments

In the case of two unfair tournaments $(D = HET)$ in which the favorite feels anger when losing against an underdog whereas the underdog's perceived and monetary prizes are identical, the underdog's first-order condition for his optimal effort $e_U^*$ is given by

$$\Delta w f(e_U^* - e_F^* - \Delta a) - c'(e_U^*) = 0, \tag{3}$$

[10] To guarantee existence, $f(\cdot)$ has to be sufficiently flat and $c(\cdot)$ sufficiently steep; see Lazear and Rosen (1981), p. 845, Nalebuff and Stiglitz (1983), for example. In the special cases considered below, explicit conditions for existence will be given.

and the favorite's one for $e_F^*$ by

$$\alpha \Delta w f\left(e_U^* - e_F^* - \Delta a\right) - c'(e_F^*) = 0. \tag{4}$$

with $\alpha\Delta w \equiv w_H - (w_L - \delta)$ and $\alpha > 1$. A comparison of (3) and (4) immediately shows that a symmetric equilibrium no longer exists. Because of $\alpha > 1$, the favorite always exerts more effort than the underdog in equilibrium: $e_F^* > e_U^*$. Note that standard preferences with $\alpha = 1$ would again lead to a symmetric equilibrium now being described by

$$\Delta w f(-\Delta a) = c'(\hat{e}^*). \tag{5}$$

The resulting effort $\hat{e}^*$ would be smaller than $e^*$ characterized by (2), since $f(\cdot)$ has a unique mode at zero. The more unfair the tournament (i.e., the higher $\Delta a$), the smaller would be $f(-\Delta a)$ and, therefore, the effort level $\hat{e}^*$.

However, according to (4) incentives will be (partly) restored for the favorite, if he feels anger from losing against his weaker opponent (i.e., $\alpha > 1$). Because of $e_F^* > e_U^*$ we have $e_U^* - e_F^* - \Delta a < 0$. Hence, equilibrium efforts according to (3) and (4) are determined by using the left-hand tail of the density $f(\cdot)$ with $f'(\cdot) < 0$ because of its unique mode at zero. Considering the system of equations (3) and (4), the general implicit-function rule yields:[11]

$$\frac{\partial e_U^*}{\partial \Delta a} = -\frac{\Delta w \bar{f}' c''(e_F)}{|J|} < 0 \tag{6}$$

$$\frac{\partial e_F^*}{\partial \Delta a} = -\frac{\alpha \Delta w \bar{f}' c''(e_U^*)}{|J|} < 0 \tag{7}$$

$$\frac{\partial e_U^*}{\partial \alpha} = -\frac{\Delta w^2 \bar{f}' \bar{f}}{|J|} < 0 \tag{8}$$

$$\frac{\partial e_F^*}{\partial \alpha} = -\frac{\Delta w \bar{f}}{|J|} \cdot \underbrace{\left(\Delta w \bar{f}' - c''(e_U^*)\right)}_{< 0 \text{ due to } \mathrm{SOC_U}} > 0 \tag{9}$$

[11]"SOC$_t$" denotes the second-order condition of the worker of type $t \in \{U, F\}$.

with $\bar{f} := f\left(e_U^* - e_F^* - \Delta a\right)$ and

$$|J| = \underbrace{\left(\Delta w \bar{f}' - c''\left(e_U^*\right)\right)}_{<0 \text{ due to SOC}_\text{U}} \underbrace{\left(-\alpha \Delta w \bar{f}' - c''\left(e_F^*\right)\right)}_{<0 \text{ due to SOC}_\text{F}} + \alpha \Delta w^2 \left[\bar{f}'\right]^2 > 0$$

as the Jacobian determinant. According to (6) and (7), increasing unfairness in form of $\Delta a$ leads to decreasing incentives – as under standard preferences. However, the comparison of (8) and (9) shows that $\frac{\partial e_F^*}{\partial \alpha} > \left|\frac{\partial e_U^*}{\partial \alpha}\right|$, i.e. we have a net positive incentive effect from the favorite feeling anger when losing against an underdog. In other words, the employer strictly gains from the favorite's disutility due to anger. Altogether, for given tournament prizes the employer will prefer unfair ($D = HET$) to fair tournaments ($D = HOM$), if $e_U^* + e_F^* > 2e^*$ where $e^*$ is described by (2). Note that $e^*$ is rather large – it is always larger than $e_U^*$ – since the density $f(\cdot)$ has its peak at zero. However, the effort $e_F^*$ may be larger than $e^*$, if anger is strong enough. The findings can be summarized as follows:

**Proposition 1** *Let tournament prizes be exogenously given. If $\alpha$ is sufficiently large and $\Delta a$ sufficiently small, the employer will prefer $D = HET$ to $D = HOM$.*

The results have shown that the employer benefits from emotions in form of anger when organizing an unfair tournament. If these emotions are strong enough, they will even dominate the incentive loss due to heterogeneity among the workers, and the employer will strictly prefer the design $D = HET$.

In order to check, whether there exist feasible values for $\alpha$ and $\Delta a$ so that unfair tournaments indeed dominate fair ones from the employer's viewpoint, consider the special case of quadratic costs $c(e_i) = \frac{c}{2}e_i^2$ (with $c > 0$) and noise $\varepsilon_i$ being uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The resulting convolution $f(x)$

for $\varepsilon_j - \varepsilon_i$ is triangular with[12]

$$f(x) = \begin{cases} \frac{1}{2\bar{\varepsilon}} + \frac{x}{4\bar{\varepsilon}^2} & \text{if} \quad -2\bar{\varepsilon} \leq x \leq 0 \\ \frac{1}{2\bar{\varepsilon}} - \frac{x}{4\bar{\varepsilon}^2} & \text{if} \quad 0 < x \leq 2\bar{\varepsilon} \\ 0 & \text{otherwise.} \end{cases}$$

as density function and

$$F(x) = \begin{cases} 0 & \text{if } x < -2\bar{\varepsilon} \\ \frac{x}{2\bar{\varepsilon}} + \frac{x^2}{8\bar{\varepsilon}^2} + \frac{1}{2} & \text{if } -2\bar{\varepsilon} \leq x \leq 0 \\ \frac{x}{2\bar{\varepsilon}} - \frac{x^2}{8\bar{\varepsilon}^2} + \frac{1}{2} & \text{if } 0 < x \leq 2\bar{\varepsilon} \\ 1 & \text{if } x > 2\bar{\varepsilon} \end{cases}$$

as corresponding distribution function. As additional assumptions let

$$\Delta w < 4c\bar{\varepsilon}^2 \quad \text{and} \quad \Delta a < 2\bar{\varepsilon}.$$

The first assumption makes the agents' objective functions strictly concave. Without the second assumption, interior pure-strategy solutions cannot exist, because exogenous noise is completely offset by the ability difference. In this case, either the favorite would choose a preemptive effort or there would be an equilibrium in mixed strategies analogously to the case of an all-pay auction with full information. Simple calculations show that

$$e^* = \frac{\Delta w}{2c\bar{\varepsilon}}, \quad \text{and} \tag{10}$$

$$e_U^* = \frac{\Delta w (2\bar{\varepsilon} - \Delta a)}{(\alpha - 1) \Delta w + 4c\bar{\varepsilon}^2} \quad \text{and} \quad e_F^* = \frac{\alpha \Delta w (2\bar{\varepsilon} - \Delta a)}{(\alpha - 1) \Delta w + 4c\bar{\varepsilon}^2}. \tag{11}$$

For given tournament prizes, the employer will prefer unfair to fair tournaments, if $2e^* < e_U^* + e_F^*$. By inserting for the three equilibrium efforts according to (10) and (11) we obtain the following result:

[12]For construction of this convolution see analogously Kräkel (2000).

**Corollary 1** *Let tournament prizes be exogenously given. For quadratic costs and uniformly distributed noise, the employer will prefer* $D = HET$ *to* $D = HOM$*, iff*

$$\Delta w < 2c\bar{\varepsilon}^2 - \frac{1+\alpha}{\alpha-1}\Delta ac\bar{\varepsilon}. \tag{12}$$

The corollary shows that there are feasible parameter constellations, for which the employer strictly benefits from designing heterogeneous departments. In particular, according to condition (12) this preference is more likely the larger the impact of anger (i.e., the higher $\alpha$) and the smaller the ability difference $\Delta a$.

Now we can analyze the three-stage game in which the employer optimally chooses $w_H$ and $w_L$ at the second stage. Here we can differentiate between two subcases. On the one hand, tournament prizes may be chosen by the employer without restriction. In particular, the employer can choose arbitrarily negative loser prizes to extract rents from the workers – in other words, he demands an entrance fee of the workers. On the other hand, workers may be characterized by limited liability so that the loser prize is restricted to nonnegative values ($w_L \geq 0$). The following results can be obtained:

**Proposition 2** *Let tournament prizes be endogenously chosen by the employer. (i) Without restriction on* $w_L$*, the employer strictly prefers* $D = HOM$ *to* $D = HET$*. (ii) If the loser prize is restricted to* $w_L \geq 0$ *(limited liability) and the workers receive positive rents under* $D = HOM$ *in equilibrium, there will exist parameter values for* $\delta$ *and* $\Delta a$ *so that the employer prefers* $D = HET$ *to* $D = HOM$*.*

**Proof.** See the appendix. ■

If no restrictions are imposed on the loser prize (i.e., we have unlimited liability), the employer is always better off by choosing two fair tournaments ($D = HOM$) (result (i)). Under this design, equilibrium efforts are identical functions of the prize spread so that the employer can implement first-best efforts for both workers by using an appropriate value for $\Delta w$. Unlimited liability then ensures that the employer indeed wants to implement this solution, because he can choose an – arbitrarily negative – loser prize $w_L$ in order to extract all rents from the workers. However, under $D = HET$ symmetric equilibria no longer exist at the tournament stage, and the employer is only able to implement first-best effort for at most one worker. Moreover, the worker with the higher expected utility receives a positive rent in equilibrium i.e. full rent extraction is not possible for the employer under $D = HET$. Finally, organizing two unfair tournaments unambiguously leads to a welfare loss amounting to $-\delta$ in each department due to the favorite's anger when losing the tournament. Note that we assumed that the workers' types are common knowledge because otherwise the employer would not be able to choose between $D = HOM$ and $D = HET$. Theoretically the employer could then choose two different pairs of prizes $(w_L^t, w_H^t)$ $(t = U, F)$ in the unfair tournament that depend on the type $t$ of the winner and loser. Now the employer would be able to implement first-best efforts for both workers even under $D = HET$. However, the employer would still prefer $D = HOM$ because of the overall welfare loss $-2\delta$ under $D = HET$.[13]

If the loser prize $w_L$ has to be non-negative (limited liability), the compar-

[13] Moreover, the sum of winner and loser prize that are paid after the tournament are typically different depending on whether the underdog or the favorite wins. However, then the employer would always choose the lower sum of prizes ex post which could distort ex ante incentives. In other words, unfair tournaments would lose their important self-commitment properties that have been highlighted by Malcomson (1984).

ison between the two tournament designs may end differently (result (ii)). Given that workers earn positive rents that are sufficiently high and that anger yields an incentive-enhancing effect as in Proposition 1, the employer will prefer unfair tournaments to fair ones. The rents have to be high enough to fully cover both the disutility $-\delta$ of feeling anger and the higher effort costs imposed on the favorite. In this case, more effort is elicited from the workers by the employer but the latter one does not pay for the extra incentives because they only reduce the workers' rents. Note that the lower the workers' reservation utilities the more likely workers will earn positive rents under limited liability and – given positive rents – the higher are these rents. In other words, low reservation utilities support the possible superiority of unfair tournaments with emotional contestants.

To summarize, the results have shown that emotions in form of anger may be beneficial for the employer although they directly lead to a welfare loss. We found out two kinds of situations in which the employer benefits form anger in unfair tournaments. The first situation assumes exogenously given tournament prizes, the second one limited liability and sufficiently high rents for workers. In both situations, the extra incentives induced by anger do not lead to additional costs for the employer.

### 3.3 Pride in Unfair Tournaments

When considering an unfair tournament with an underdog who feels pride after winning against a favorite, we have to modify the workers' objective functions under $D = HET$. Now the favorite's perceived and monetary prizes are identical, whereas the underdog has a higher perceived winner prize $w_H + \gamma$ with $\gamma > 0$ which leads to a higher perceived prize spread $\beta \Delta w$ with $\beta > 1$ for the underdog. The two workers' first-order conditions for their optimal effort choices are now given by

$$\beta \Delta w f\left(e_U^* - e_F^* - \Delta a\right) - c'(e_U^*) = 0 \tag{13}$$

for the underdog, and

$$\Delta w f\left(e_U^* - e_F^* - \Delta a\right) - c'(e_F^*) = 0 \tag{14}$$

for the favorite. Comparing (13) and (14) shows that again a symmetric equilibrium does not exist. Because of $\beta > 1$, now the underdog always exerts more effort than the favorite in equilibrium. However, now it is no longer clear whether the left-hand side ($e_U^* - e_F^* - \Delta a < 0$) or the right-hand side ($e_U^* - e_F^* - \Delta a > 0$) of the convolution $f(\cdot)$ becomes relevant in equilibrium and, therefore, which type of worker has a higher probability of winning. If the incentive effect outweighs the ability deficit $\Delta a$ of the underdog (i.e., if $e_U^* > e_F^* + \Delta a$), the underdog will have a higher winning probability than the favorite, otherwise the opposite holds. By applying the implicit-function rule to (13) and (14) we obtain – because of the shape of $f(\cdot)$:[14]

$$\frac{\partial e_U^*}{\partial \Delta a} = -\frac{\beta \Delta w \bar{f}' c''(e_F)}{|J|} \begin{cases} > 0, \text{ if } e_U^* > e_F^* + \Delta a \\ < 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{15}$$

$$\frac{\partial e_F^*}{\partial \Delta a} = -\frac{\Delta w \bar{f}' c''(e_U^*)}{|J|} \begin{cases} > 0, \text{ if } e_U^* > e_F^* + \Delta a \\ < 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{16}$$

$$\frac{\partial e_U^*}{\partial \beta} = -\frac{\Delta w \bar{f}}{|J|} \underbrace{\left(-\Delta w \bar{f}' - c''(e_F^*)\right)}_{< 0 \text{ due to SOC}_\text{F}} > 0 \tag{17}$$

$$\frac{\partial e_F^*}{\partial \beta} = \frac{\Delta w^2 \bar{f}' \bar{f}}{|J|} \begin{cases} < 0, \text{ if } e_U^* > e_F^* + \Delta a \\ > 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{18}$$

[14]Again "SOC$_t$" denotes the second-order condition of the worker of type $t \in \{U, F\}$.

with $\bar{f} := f\left(e_U^* - e_F^* - \Delta a\right)$ and

$$|J| = \underbrace{\left(\beta \Delta w \bar{f}' - c''\left(e_U^*\right)\right)}_{<0 \text{ due to SOC}_\text{U}} \underbrace{\left(-\Delta w \bar{f}' - c''\left(e_F^*\right)\right)}_{<0 \text{ due to SOC}_\text{F}} + \beta \Delta w^2 \left[\bar{f}'\right]^2 > 0$$

as the Jacobian determinant. Hence, for both workers a higher ability difference $\Delta a$ has a motivating effect at the positive tail and a discouraging effect at the negative tail of $f(\cdot)$. The motivating effect seems to be curious at first sight, because incentives increase in the unfairness of the tournament which is impossible under standard preferences. However, here a large value of $\beta$ implies an uneven situation $e_U^* > e_F^* + \Delta a$ in favor of the underdog – we are at the positive tail of $f(\cdot)$ – and in this situation an increase of $\Delta a$ leads back to the mode of $f(\cdot)$ (i.e., it makes the tournament less uneven) where incentives are maximal. Intuitively, here the additional incentives due to $\beta$ make the underdog exert a very high effort, but by an increase in the ability difference the favorite would get back into the race. $\partial e_U^* / \partial \beta > 0$ shows that the underdog's incentives always increase in the motivating effect of beating a predominant opponent. However, for the favorite the positive incentive effect only holds at the negative tail of $f(\cdot)$. Note that the net effect is always positive since $\frac{\partial e_U^*}{\partial \beta} > \left|\frac{\partial e_F^*}{\partial \beta}\right|$.

These comparative statics are interesting for at least two reasons. First, they give an explanation for the puzzling experimental findings of Weigelt et al. (1989) and Schotter and Weigelt (1992). They conducted several experiments on unfair tournaments and, according to their data, both types of players significantly oversupply effort. Note that their theoretical benchmark is given by $\hat{e}^*$ (see equation (5)), but by the impact of pride as modelled in this paper we obtain $e_U^* > \hat{e}^*$ and $e_F^* > \hat{e}^*$ in the relevant range (i.e., at the negative tail of $f(\cdot)$) due to the stimulating effect of $\beta$. Second, we can derive the principal's optimal tournament design at the first stage:

**Proposition 3** *Let tournament prizes be exogenously given. If $\beta$ is sufficiently large and $\Delta a \in [e_U^* - e_F^* - \eta, e_U^* - e_F^* + \eta]$ with $\eta > 0$ being sufficiently small, the employer will prefer $D = HET$ to $D = HOM$. Otherwise, he prefers $D = HOM$ to $D = HET$.*

**Proof.** The employer will prefer unfair to fair tournaments, if $2e_U^* + 2e_F^* > 4e^*$ where $e^*$ is given by equation (2), whereas the efforts $e_U^*$ and $e_F^*$ are described by (13) and (14), respectively. The comparative statics have shown that $\frac{\partial e_t^*}{\partial \Delta a} > 0$ $(t = U, F)$ for $\Delta a < e_U^* - e_F^*$, and $\frac{\partial e_t^*}{\partial \Delta a} < 0$ for $\Delta a > e_U^* - e_F^*$. In both cases, in the limit $\Delta a \to (e_U^* - e_F^*)$ implies $\bar{f} \to f(0)$ and, hence, $e_F^* \to e^*$ but – because of $\beta > 1 - e_U^* > e^*$ (compare (2), (13) and (14)). ■

The proof of the proposition shows that if, in the unfair tournament, the ability difference comes arbitrarily close to the difference of the equilibrium efforts, all three effort levels $e^*$, $e_U^*$ and $e_F^*$ will be determined by $f(0)$. However, since we have an extra incentive effect in unfair tournaments, the underdogs will exert higher efforts than the competitors in the fair tournaments and, therefore, unfair tournaments dominate fair ones. If, on the other hand, $\Delta a$ and $e_U^* - e_F^*$ clearly differ, $e_U^* - e_F^* - \Delta a$ will tend to the tails of $f(\cdot)$ so that $\bar{f}$ becomes very small and the employer strictly prefers fair to unfair tournaments.

Of course, the condition of subjectively perceived prizes (i.e., $\beta > 1$) is necessary for unfair tournaments dominating fair ones. However, we can use the framework of Weigelt et al. (1989) and Schotter and Weigelt (1992) – quadratic costs, uniformly distributed noise – in order to show that there are cases in which further restrictions on $\beta$ are not necessary for the dominance of unfair tournaments. Hence, as an example, consider again the case of quadratic costs $c(e_i) = \frac{c}{2}e_i^2$ (with $c > 0$) and noise $\varepsilon_i$ being uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The resulting convolution has been already described in Subsection 3.2. To guarantee existence of pure-strategy equilibria I assume that $(\beta-1)\Delta w < 4c\bar{\varepsilon}^2$ (strict concavity) and $\Delta a < 2\bar{\varepsilon}$. The second assumption ensures that existing noise is not completely offset by the ability difference. Again, symmetric equilibrium efforts in the fair tournament, $e^*$, are described by equation (10) but, by using (13), (14) and the assumptions concerning the cost and the distribution function, equilibrium efforts in the unfair tournament are now given by

$$e_U^* = \frac{\beta\Delta w\,(2\bar{\varepsilon}+\Delta a)}{(\beta-1)\,\Delta w + 4c\bar{\varepsilon}^2} \quad \text{and} \quad e_F^* = \frac{\Delta w\,(2\bar{\varepsilon}+\Delta a)}{(\beta-1)\,\Delta w + 4c\bar{\varepsilon}^2} \tag{19}$$

if $e_U^* - e_F^* > \Delta a$, and

$$e_U^* = \frac{\beta\Delta w\,(2\bar{\varepsilon}-\Delta a)}{4c\bar{\varepsilon}^2 - (\beta-1)\Delta w} \quad \text{and} \quad e_F^* = \frac{\Delta w\,(2\bar{\varepsilon}-\Delta a)}{4c\bar{\varepsilon}^2 - (\beta-1)\Delta w} \tag{20}$$

if $e_U^* - e_F^* < \Delta a$. Note that $e_U^* - e_F^* > \Delta a \iff \Delta a < \frac{(\beta-1)\Delta w}{2c\bar{\varepsilon}}$ and $e_U^* - e_F^* < \Delta a \iff \Delta a > \frac{(\beta-1)\Delta w}{2c\bar{\varepsilon}}$. Calculating $2e_U^* + 2e_F^* > 4e^*$ for both cases yields $\Delta a > \frac{\left(\Delta w - 2c\bar{\varepsilon}^2\right)(\beta-1)}{(\beta+1)c\bar{\varepsilon}} =: \Delta\hat{a}_L$ for $e_U^* - e_F^* > \Delta a$, and $\Delta a < \frac{\left(\Delta w + 2c\bar{\varepsilon}^2\right)(\beta-1)}{(\beta+1)c\bar{\varepsilon}} =: \Delta\hat{a}_H$ for $e_U^* - e_F^* < \Delta a$, which do not contradict the two preceding conditions for any $\beta > 1$.[15] Hence, we obtain the following result:

**Corollary 2** *Let tournament prizes be exogenously given. For quadratic costs and uniformly distributed noise, the employer will prefer $D = HET$ to $D = HOM$, iff $\Delta a \in [\Delta\hat{a}_L, \Delta\hat{a}_H]$.*

The corollary shows that, the employer will choose two unfair tournaments as long as the ability difference lies inside a certain range. Solving $e_U^* - e_F^* = \Delta a$ for the ability difference $\Delta a$, with $e_U^*$ and $e_F^*$ being either given by (19) or (20), leads to the middle of the interval $[\Delta\hat{a}_L, \Delta\hat{a}_H]$, which is given by $(\beta-1)\,\Delta w/\,(2c\bar{\varepsilon})$. Here, the function $e_U^* + e_F^*$ of $\Delta a$ has its maximum, which confirms the findings of Proposition 3.

[15] Note that in each case $e_U^* - e_F^* - \Delta a \in [-2\bar{\varepsilon}, 2\bar{\varepsilon}]$. In addition, note that $\Delta\hat{a}_H < 2\bar{\varepsilon}$.

In analogy to the case of anger, we can finally consider endogenous tournament prizes that are optimally chosen by the employer within the three-stage game. Again we have to differentiate between unlimited liability (i.e., $w_L$ can be arbitrarily negative) and limited liability ($w_L \geq 0$) of the workers. Without restriction on the loser prize, under $D = HOM$ again the employer implements first-best effort for both workers and extracts all rents.[16] Under $D = HET$, as in the anger case, the employer is only able to induce first-best incentives for at most one worker ($e_U^* \neq e_F^*$ according to (13) and (14)), and he has to leave a positive rent to the worker with the higher expected utility. However, there is a crucial difference to the anger case. Under the pride scenario, one of the workers – the underdog – receives an extra utility $\gamma$ with a certain probability. This expected extra utility relaxes the underdog's participation constraint so that the employer is able to induce higher incentives compared to fair tournaments. We can imagine that there exist specifications for the cost function $c(e_i)$ and the distribution $G(\varepsilon_i)$ for which this incentive effect becomes dominant and the employer prefers $D = HET$ to $D = HOM$ (see the proof of Proposition 4 in the appendix).

If we restrict the loser prize to non-negative values (limited liability) and the workers earn sufficiently large rents, again $D = HET$ may be beneficial for the employer. The reasoning is the same as for the anger scenario: Pride of the underdog leads to additional incentives for at least one of the workers, and the net incentive effect for both workers is always positive (see equations (17) and (18)). Hence, if the workers receive large rents under $D = HOM$, the employer can induce higher incentives to them under $D = HET$ without paying for the additional effort costs, since they only reduce the workers' rents. Note that such situations are even more likely in the pride case than

[16] See the proof of Proposition 2.

in the anger case since, with pride, the underdog receives the extra utility $\gamma$ whereas in the anger scenario the favorite suffers from an extra disutility $\delta$. Therefore, the positive rents have to cover $\delta$ as well as the additional effort costs of the favorite who feels anger, but in the case in which the underdog feels pride the additional effort costs are partly covered by the expected extra utility $\gamma F\left(e_U^* - e_F^* - \Delta a\right)$. The findings can be summarized in the following proposition:

**Proposition 4** *Let tournament prizes be endogenously chosen by the employer. (i) Under unlimited liability of the workers, there exist cost functions $c(e_i)$ and distributions $G(\varepsilon_i)$ for which the employer prefers $D = HET$ to $D = HOM$. (ii) If, under limited liability, the workers receive sufficiently large rents under $D = HOM$ in equilibrium, $D = HET$ may dominate $D = HOM$ from the employer's viewpoint.*

**Proof.** See the appendix. ■

## 3.4 Discussion

The results above have shown that, in unfair tournaments, emotions as anger and pride effect both overall welfare and the employer's expected profits. The effects on expected profits have been analyzed in detail: The comparative statics have shown that the net effect of emotions on both workers' efforts is always positive. If emotions create additional incentives compared to fair tournaments and if the employer need not pay for the enhanced incentives – since (1) tournament prizes are exogenous or (2) the underdog's participation constraint is sufficiently relaxed by expected pride or (3) workers receive sufficiently high rents –, the employer will benefit from emotional incentives due to unfair tournaments. Consider, for example, an unfair tournament in which both the underdog and the favorite may feel emotions – the underdog pride and the favorite anger. Then according to equations (4) and (13) the workers' first-order conditions are given by

$$(\Delta w + \gamma) f(e_U^* - e_F^* - \Delta a) - c'(e_U^*) = 0 \quad (21)$$

$$\text{and} \quad (\Delta w + \delta) f(e_U^* - e_F^* - \Delta a) - c'(e_F^*) = 0 \quad (22)$$

with $\gamma, \delta > 0$. If in this situation the employer need not pay for emotional incentives, he will have the following preferences (see Propositions 1 and 3): If $\gamma > \delta$ (i.e., $e_U^* > e_F^*$), then $\gamma$ and $\delta$ should be large and $\Delta a$ close to $e_U^* - e_F^*$. If $\gamma < \delta$ (i.e., $e_U^* < e_F^*$), then $\gamma$ and $\delta$ should be large and close together, whereas $\Delta a$ should be close to zero.

However, the welfare effects of emotions are not quite clear. For example, if pride (anger) is extremely important so that the underdog (favorite) realizes a very large extra utility (disutility) $\gamma$ $(-\delta)$ in case of winning (losing) the unfair tournament, then it will be always (never) efficient to choose $D = HET$ instead of $D = HOM$, since the workers' monetary incomes and the employer's expected profits will only play a marginal role in this situation. If we restrict the welfare analysis to *monetary values* and do not count emotional gains or losses, we will obtain a much clearer result. Recall from equation (A3) from the proof of Proposition 2 that first-best effort $e^{FB}$ which equalizes marginal revenue and marginal costs is implicitly described by

$$1 = c'\left(e^{FB}\right). \quad (23)$$

Hence, monetary welfare is maximized when implementing effort $e^{FB}$ for both workers. The proof of Proposition 2 has shown that under unlimited liability first-best effort is always induced by the employer to both workers in a fair tournament, whereas he cannot implement $e^{FB}$ for both workers in an unfair one if workers feel either anger or pride. However, we can show that even under the most promising circumstances – (a) workers feel anger as well as pride with $\gamma = \delta$ in equations (21) and (22), and (b) workers are characterized by unlimited liability – the employer does not want to implement first-best effort for both workers. Let $EU_t(e_t^*)$ denote the expected utility of the worker of type $t$ $(= U, F)$ in equilibrium. Then we obtain the following result:

**Proposition 5** *Let the employer choose prizes endogenously under unlimited liability in an unfair tournament with both anger and pride. If both emotions have the same impact (i.e., $\gamma = \delta$ in (21) and (22)), then we will have a symmetric equilibrium $e_U^* = e_F^* = \tilde{e}^*$ at the tournament stage with*

$$\tilde{e}^* \begin{cases} > e^{FB}, & \text{if } EU_F(\tilde{e}^*) = \bar{u} \\ < e^{FB}, & \text{if } EU_U(\tilde{e}^*) = \bar{u}. \end{cases}$$

**Proof.** See the appendix. ■

Proposition 5 shows that "symmetric emotions" allow for a symmetric equilibrium at the tournament stage so that the employer is able to implement first-best efforts for both workers. However, the employer will never do so. He either induces excessively high efforts so that expected anger leads to a binding participation constraint for the favorite, or he chooses less than efficient effort so that expected pride makes the underdog's participation constraint bind. The intuition for this result is the following: Note that in equilibrium each worker exerts effort according to

$$(\Delta w + \gamma) f(-\Delta a) = c'(\tilde{e}^*).$$

Hence, the lower the ability difference, $\Delta a$, and the higher the impact of emotions, $\gamma$, the higher will be the effort level $\tilde{e}^*$.[17] In the case of $\tilde{e}^* > e^{FB}$, the underdog's expected utility must exceed the expected utility of the

[17] Recall that the convolution $f(\cdot)$ has a unique mode at zero.

favorite, i.e.

$$\begin{aligned}(\Delta w+\gamma)\,F(-\Delta a) &> -\gamma+(\Delta w+\gamma)\,[1-F(-\Delta a)] \Leftrightarrow \\ \gamma &> \left(\frac{1}{2F(-\Delta a)}-1\right)\Delta w.\end{aligned}$$

In other words, for an excessively high effort level the emotional influences have to be sufficiently high and the ability difference sufficiently low.

The tournament considered here is modelled as a one-shot game. In a dynamic setting (e.g., in a career-concerns framework), perhaps alternative interpretations can be given for $\gamma$ and $\delta$. From a dynamic perspective, both parameters may be interpreted as reputation effects if the labor market is uncertain about the true abilities of the workers. Then if a presumable favorite loses against a presumable underdog, the former one will realize an extra disutility because the labor market adjusts its ability expectations downward whereas the latter one receives an extra utility due to Bayesian updating. Of course, the model considered in this paper is static with abilities being common knowledge and ignores aspects of career concerns, but there are dynamic tournament models which particularly focus on these aspects (see Zabojnik and Bernhardt 2001, Koch and Peyrache 2003).

As mentioned above the distinction between fair and unfair tournaments was introduced in the literature by O'Keefe et al. (1984). We can also apply the concept of emotions and subjectively perceived prizes to "uneven tournaments" in the terminology of O'Keefe et al. In those tournaments, again a favorite competes against an underdog, but now the underdog is characterized by a steeper cost function compared to the favorite. The experimental findings of Bull et al. (1987) and Schotter and Weigelt (1992) on uneven tournaments show that only the underdogs exert significantly more effort than theoretically predicted. The concept introduced in this paper can explain these findings: If pride leads to additional incentives, the underdog will always choose more than the equilibrium effort of a worker with standard preferences. If the impact of pride is (a) larger than that of anger and (b) sufficiently high to compensate for the steeper cost function, the underdog may even choose higher effort than the favorite.

Finally, note that there exist a few papers that also deal with non-standard preferences in tournaments. Kräkel (2000) applies the concept of relative deprivation on workers' behavior in tournaments. Here a worker will feel relatively deprived if he (e.g., a tournament loser) earns less than the members of a certain reference group (e.g., the tournament winners). The results show that workers with relative deprivation exert more effort than workers with standard preferences when tournament prizes are exogenously given. However, if prizes are endogenously chosen and workers do not face limited liability, first-best efforts will be implemented even under relative deprivation. Demougin and Fluet (2003) and Grund and Sliwka (2004) apply inequity aversion – as defined by Fehr and Schmidt (1999) – to tournament competition. Similar to the concept of emotions considered in this paper, the employer will benefit from inequity aversion, if tournament prizes are exogenously given or if workers earn positive rents. However, contrary to the influence of emotions, under unlimited liability the employer always suffers from inequity aversion since he has to pay for the workers' inequity costs via their binding participation constraints. It will also be straightforward to apply prospect theory of Kahneman and Tversky (1979) to tournament competition. If workers are homogeneous we will have a symmetric outcome of the contest and the expected tournament prize $(w_L + w_H)/2$ should be a natural reference point for the value function of both workers. Then the tournament loser suffers from loss aversion ex post, which should lead to higher incentives for both workers ex ante.

# 4 Emotions in Individualistic Incentive Schemes

Emotions based on success or failure when comparing oneself with a more or less able colleague seem to be obvious for compensation systems that use relative performance evaluation like tournaments. However, such emotions may be relevant even for pay methods that only focus on individual output. In this case, workers are compensated independently – i.e. there is no compensation game between the workers –, but either a worker feels joy/frustration when meeting/non-meeting a certain target, or the pure existence of co-workers and their success may influence the behavior of other workers at the same workplace (peer effects).

The field experiments by Falk and Ichino (2003) empirically support the existence of such peer effects: In their experiments, subjects either have to work alone (single treatment) or as pairs consisting of two subjects (pair treatment). Each subject earns a fixed payment. The empirical findings show that the average output in the pair treatment significantly exceeds the output in the single treatment. Hence, observing the performance of co-workers leads to positive peer effects that raise overall productivity.

In this section, two individualistic pay methods should be discussed which are frequently used in practice. The first one is a so-called *bonus system* (Subsection 4.1). Here the employer sets a certain performance standard and the worker receives a high bonus when beating the standard, whereas he gets a low bonus if he does not meet the given target. There are a lot of examples for such bonus schemes – see, for example, Otley (1992) on bonus payments at United Bank, Merchant and Riccaboni (1992) on bonuses at the Fiat Group, and Engellandt and Riphahn (2004) on bonus systems at a large multinational company. The second compensation scheme is well-known as *piece-rate system* in the literature (Subsection 4.2). Under a piece-rate scheme, a worker's remuneration consists of a fixed payment and a certain percentage – the piece rate – of the worker's realized output in monetary terms. There are also lots of examples for piece-rate schemes in practice: see, among many others, Lazear (2000) on the introduction of piece rates at the Safelite Glass Corporations, and Freeman and Kleiner (1998) on the use of piece-rate systems in the American shoe industry.

I will consider the bonus system in order to analyze the implications of emotions when workers try to beat a certain performance standard. Piece rates will be analyzed to get more insights into the consequences of peer effects (as observed by Falk and Ichino) when workers are paid on the basis of individual performance. The central question will be whether emotions (or peer effects) either enhance or deteriorate induced work incentives and are, therefore, either advantageous or detrimental from the employer's viewpoint.

## 4.1 Frustration and Joy in Bonus Systems

Consider again the model of Section 2. We assume now that a worker $i$ ($i = 1, 2, 3, 4$) is compensated according to a bonus system that consists of three ingredients. First, there is a performance standard $\bar{q}$ which the worker has to beat. If he is successful (with probability $\text{prob}\{e_i + a_i + \varepsilon_i > \bar{q}\} = 1 - G(\bar{q} - e_i - a_i)$), he will get a high bonus $b_H$. In case of a failure (with probability $\text{prob}\{e_i + a_i + \varepsilon_i < \bar{q}\} = G(\bar{q} - e_i - a_i)$), he only receives a low bonus $b_L$ ($< b_H$).[18] In order to introduce the influence of emotions, each

[18] Intuitively, the assumption $b_L < b_H$ makes sense. However, as can be seen below we do not really need this restriction since the worker realizes an extra utility (disutility) from beating (not beating) the given standard.

worker is assumed to feel joy or pride, $\gamma$, in case of success, but frustration or anger, $\delta$, when not beating the standard. Hence, worker $i$'s expected utility can be written as

$$\begin{aligned} EU_i(e_i) &= (b_H+\gamma)[1-G(\bar{q}-e_i-a_i)] + (b_L-\delta)G(\bar{q}-e_i-a_i) - c(e_i) \\ &= b_H+\gamma - G(\bar{q}-e_i-a_i)(b_H-b_L+\gamma+\delta) - c(e_i). \end{aligned}$$

The first-order condition for the worker's optimal effort choice $e_i^*$ is given by[19]

$$g(\bar{q}-e_i^*-a_i)(b_H-b_L+\gamma+\delta) - c'(e_i) = 0.$$

By using the implicit-function rule we obtain

$$\frac{\partial e_i^*}{\partial(\gamma+\delta)} = -\frac{g(\bar{q}-e_i^*-a_i)}{-g'(\bar{q}-e_i^*-a_i)(b_H-b_L+\gamma+\delta) - c''(e_i^*)},$$

which is strictly positive since the denominator describes the second derivative of the worker's objective function which has to be negative in optimum. Therefore, simple comparative statics point out that, for a given performance standard and exogenously given bonuses, both frustration and joy have a positive effect on work incentives. In the literature, we often find the claim that frustration is detrimental for incentives because it leads to demoralization. However, the economic logic is a little bit more complicated. Ex ante, each worker tries to avoid to feel frustration and, therefore, exerts more effort. In analogy, each worker wants to feel joy which motivates him ex ante. Of course, ex post, after the workers have been paid, all incentives are gone, but this holds for any compensation system and independently of the type of experienced emotion.

If the bonus scheme is endogenous, the employer will choose $(\bar{q}, b_H, b_L)$ to maximize expected output minus expected bonus payments. Under unlimited

[19] The cost function is assumed to be sufficiently convex so that we have a strictly concave objective function and can concentrate on interior solutions.

liability and, for simplicity, $E[\varepsilon_i] = 0$ the employer's Lagrangian can be written as

$$\begin{aligned} L(e_i, \bar{q}, b_H, b_L) &= e_i + a_i - b_H [1 - G(\bar{q} - e_i - a_i)] - b_L G(\bar{q} - e_i - a_i) \\ &\quad +\lambda_1 \cdot [g(\bar{q} - e_i - a_i)(b_H - b_L + \gamma + \delta) - c'(e_i)] \\ &\quad +\lambda_2 \cdot [b_H + \gamma - G(\bar{q} - e_i - a_i)(b_H - b_L + \gamma + \delta) - c(e_i) - \bar{u}] \end{aligned}$$

with $\lambda_1 \geq 0$ as multiplier for the worker's incentive constraint and $\lambda_2 \geq 0$ as multiplier for his participation constraint. In optimum, the following conditions must hold:

$$\begin{aligned} \frac{\partial L}{\partial e_i} &= 1 - (b_H - b_L)\bar{g} + \lambda_1 [-\bar{g}' \cdot (b_H - b_L + \gamma + \delta) - c''(e_i)] \\ &\quad +\lambda_2 [\bar{g} \cdot (b_H - b_L + \gamma + \delta) - c'(e_i)] = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{\partial L}{\partial \bar{q}} &= (b_H - b_L)\bar{g} + \lambda_1 \bar{g}' \cdot (b_H - b_L + \gamma + \delta) \\ &\quad -\lambda_2 \bar{g} \cdot (b_H - b_L + \gamma + \delta) = 0 \end{aligned} \tag{25}$$

$$\frac{\partial L}{\partial b_H} = -\left[1 - \bar{G}\right] + \lambda_1 \bar{g} + \lambda_2 - \lambda_2 \bar{G} = 0 \tag{26}$$

$$\frac{\partial L}{\partial b_L} = -\bar{G} - \lambda_1 \bar{g} + \lambda_2 \bar{G} = 0 \tag{27}$$

with $\bar{g} := g(\bar{q} - e_i - a_i)$ and $\bar{G} := G(\bar{q} - e_i - a_i)$. Conditions (26) and (27) together yield $\lambda_2 = 1$ and $\lambda_1 = 0$. Hence, the worker's participation constraint is binding in optimum. Conditions (24) and (25) together with $\lambda_1 = 0$ then lead to

$$1 = c'(e_i). \tag{28}$$

Comparing (28) and (23) immediately shows that the employer implements first-best incentives.

It would be interesting to check whether the labor costs for implementing first-best will be larger or smaller when workers feel emotions like frustration and joy. Combining (28) with the worker's incentive constraint yields

$$(b_H - b_L + \gamma + \delta) = \frac{1}{g\left(\bar{q} - e^{FB} - a_i\right)}. \tag{29}$$

By inserting into the binding participation constraint we obtain

$$b_H^* = \bar{u} + c\left(e^{FB}\right) + \frac{G\left(\bar{q} - e^{FB} - a_i\right)}{g\left(\bar{q} - e^{FB} - a_i\right)} - \gamma \tag{30}$$

for the optimal high bonus. Hence, (29) gives the expression

$$b_L^* = \bar{u} + c\left(e^{FB}\right) - \frac{1 - G\left(\bar{q} - e^{FB} - a_i\right)}{g\left(\bar{q} - e^{FB} - a_i\right)} + \delta \tag{31}$$

for the optimal low bonus. According to (30) and (31), labor costs decrease in joy $\gamma$ and increase in the level of frustration $\delta$. Independent of beating or not beating the standard the worker is compensated for his effort costs, $c\left(e^{FB}\right)$, and his foregone income from his best alternative job offer, $\bar{u}$. As the employer extracts all rents from the worker, he charges him a fee $\gamma$ in case of feeling joy, but compensates the worker for the frustration $\delta$ when he fails to meet the target $\bar{q}$. In other words, joy relaxes the worker's participation constraint and, hence, increases the employer's profits, while frustration aggravates the constraint. Expected labor costs amount to

$$\begin{aligned} b_H^*\left[1 - G\left(\bar{q} - e^{FB} - a_i\right)\right] + \quad & b_L^* G\left(\bar{q} - e^{FB} - a_i\right) = \\ \bar{u} + c\left(e^{FB}\right) \quad & -\gamma\left[1 - G\left(\bar{q} - e^{FB} - a_i\right)\right] + \delta G\left(\bar{q} - e^{FB} - a_i\right). \end{aligned}$$

To sum up, emotions will be beneficial for the employer, iff

$$\frac{G\left(\bar{q} - e^{FB} - a_i\right)}{1 - G\left(\bar{q} - e^{FB} - a_i\right)} < \frac{\gamma}{\delta}. \tag{32}$$

Condition (32) shows that – besides the effects of $\gamma$ and $\delta$ just mentioned – emotions are more likely to increase profits

- the lower is the standard $\bar{q}$ (since the cumulative distribution function $G(\cdot)$ is monotonically increasing)

- the less convex the worker's cost function $c(e_i)$ (since first-best effort $e^{FB}$ is larger the flatter the marginal costs $c'(\cdot)$)

- the higher the ability of the worker; hence, an emotional favorite is more likely to lower expected labor costs than an emotional underdog $(a_F > a_U)$.

Finally, we can consider the optimal bonus contract for the employer if workers are characterized by limited liability so that both bonuses have to be non-negative, i.e. $b_H \geq 0$ and $b_L \geq 0$.[20] Hence, the Lagrangian $L(e_i, \bar{q}, b_H, b_L)$ above has to be completed by "$+\lambda_3 b_H + \lambda_4 b_L$" with $\lambda_3, \lambda_4 \geq 0$. By these two limited-liability constraints we have to add "$+\lambda_3$" to the left-hand side of condition (26) and "$+\lambda_4$" to the left-hand side of (27), whereas conditions (24) and (25) remain unchanged. The modified conditions (26) and (27) now lead to

$$\lambda_2 + \lambda_3 + \lambda_4 = 1. \tag{33}$$

Therefore, at least the participation constraint or one of the limited-liability constraints have to be binding in optimum. Combining conditions (24) and (25) yields

$$1 - \lambda_2 c'(e_i) - \lambda_1 c''(e_i) = 0. \tag{34}$$

Now we can check the seven possible cases for the multipliers $\lambda_2$, $\lambda_3$ and $\lambda_4$ that satisfy condition (33). By using the modified conditions (26) and (27) we obtain the following results: (1) $\lambda_2 = 1$ together with $\lambda_3 = \lambda_4 = 0$ (i.e., both limited-liability constraints do not bind) again leads to the above result under unlimited liability. (2) $\lambda_3 = 1$ together with $\lambda_2 = \lambda_4 = 0$ implies $G(\bar{q} - e_i - a_i) + \lambda_1 g(\bar{q} - e_i - a_i) = 0$ which cannot hold. (3) $\lambda_4 = 1$ together

[20] Note again that the high bonus need not be larger than the low bonus because of the emotions the workers feel (see (30) and (31)).

with $\lambda_2 = \lambda_3 = 0$ implies $\lambda_1 = \frac{1-G(\bar{q}-e_i-a_i)}{g(\bar{q}-e_i-a_i)}$ and (because of (34)) $c''(e_i) = \frac{g(\bar{q}-e_i-a_i)}{1-G(\bar{q}-e_i-a_i)}$. (4) $\lambda_2 = 0$ and $\lambda_3, \lambda_4 > 0$ imply $c''(e_i) = \frac{g(\bar{q}-e_i-a_i)}{\lambda_4-G(\bar{q}-e_i-a_i)}$. (5) $\lambda_3 = 0$ and $\lambda_2, \lambda_4 > 0$ yield $1 = \lambda_2 c'(e_i) + \frac{(1-\lambda_2)(1-G(\bar{q}-e_i-a_i))}{g(\bar{q}-e_i-a_i)} c''(e_i)$. (6) $\lambda_4 = 0$ and $\lambda_2, \lambda_3 > 0$ result into $G(\bar{q}-e_i-a_i)(1-\lambda_2) + \lambda_1 g(\bar{q}-e_i-a_i) = 0$ which is not possible. (7) $\lambda_2, \lambda_3, \lambda_4 > 0$ mean that $g(\bar{q}-e_i-a_i)(\gamma+\delta) = c'(e_i)$ and $\gamma - G(\bar{q}-e_i-a_i)(\gamma+\delta) = c(e_i) + \bar{u}$ have to hold at the same time. To sum up, if at least one limited-liability constraint is binding, either the remaining conditions will lead to a contradiction (cases (2) and (6)) or the implemented effort level will not be first best in general.

The findings for the bonus system with emotional workers can be summarized in the following proposition:

**Proposition 6** *(i) If the bonus scheme is exogenously given, both joy $\gamma$ and frustration $\delta$ will enhance incentives. (ii) If the employer optimally chooses the bonus scheme and the workers have unlimited liability, first-best efforts are implemented. Emotions will be beneficial for the employer, if and only if*

$$\frac{G\left(\bar{q}-e^{FB}-a_i\right)}{1-G\left(\bar{q}-e^{FB}-a_i\right)} < \frac{\gamma}{\delta}.$$

*(iii) If the employer optimally chooses the bonus scheme but the workers are restricted by limited liability, first-best efforts are not implemented in general.*

## 4.2 Peer Effects in Piece-Rate Systems

Now we turn to the piece-rate system and to the former question whether homogeneous ($D = HOM$) or heterogeneous ($D = HET$) departments are advantageous from the employer's viewpoint. I assume that workers compare with their co-workers of the same department and may feel emotions when being more or less successful than the respective co-worker. In particular, such emotions are assumed to exist in heterogeneous departments but not in homogeneous ones.[21] Each worker $i$ is compensated according to the typical linear piece-rate formula

$$w_i = x_i + y_i q_i.$$

Hence, each worker receives a wage $w_i$ that consists of a fixed payment $x_i$ and a percentage $y_i$ (piece rate) of his realized output $y_i$. The assumptions of the basic model in Section 2 should still hold. For simplicity, let $E[\varepsilon_i] = 0$.

Under $D = HOM$, emotions do not play any role by definition. If workers are characterized by unlimited liability, we will obtain the standard solution of principal-agent models with a risk neutral agent: By appropriately choosing the fixed payment $x_i$, the employer extracts all rents so that on average each worker exactly earns his reservation utility $\bar{u}$. In addition, the employer chooses $y_i = 1$ ("selling the firm") and, therefore, implements first-best effort levels for all workers.

Under $D = HET$, however, the workers' objective functions have to be modified to include the emotional effects. As in Section 3, the favorite realizes a disutility (he feels anger or shame $-\delta < 0$) when being less successful than the underdog (i.e., $q_F < q_U$), whereas the underdog receives an extra utility (he feels pride or joy $\gamma > 0$) in this case. Since the probability for this event is given by $F(e_U - e_F - \Delta a)$, the underdog's expected utility becomes

$$EU_U(e_U) = x_U + y_U a_U + y_U e_U + \gamma F(e_U - e_F - \Delta a) - c(e_U)$$

with $\Delta a > 0$ again denoting the workers' ability difference, and the favorite's one

$$EU_F(e_F) = x_F + y_F a_F + y_F e_F - \delta F(e_U - e_F - \Delta a) - c(e_F).$$

[21] Of course, workers may also feel emotions under $D = HOM$. Hence, in the following we focus on the *extra* emotions due to heterogeneity. Moreover, for the following theoretical results it is irrelevant whether workers are equally talented or not as long as workers' emotions differ.

For the first-order conditions we obtain:[22]

$$y_U + \gamma f\left(e_U^* - e_F^* - \Delta a\right) - c'\left(e_U^*\right) = 0 \tag{35}$$

$$\text{and} \quad y_F + \delta f\left(e_U^* - e_F^* - \Delta a\right) - c'\left(e_F^*\right) = 0. \tag{36}$$

Conditions (35) and (36) emphasize that by introducing emotions the workers' reaction functions to the employer's compensation now interconnect. In other words, now we have a game between the two workers which leads to peer effects, whereas their compensation schemes are completely independent without emotions. As we will see below, this new compensation game which looks like a kind of contest – see Section 3 – substantively changes the standard solution of piece-rate schemes.

However, before, we can do some simple comparative statics in order to show how emotions will influence the workers' incentives, if the compensation formulas for $w_U$ and $w_F$ are exogenously given. As in Section 3, let $\bar{f} := f\left(e_U^* - e_F^* - \Delta a\right)$. By implicitly differentiating the system of equations (35) and (36) we get[23]

$$\frac{\partial e_U^*}{\partial \gamma} = \frac{-\bar{f} \cdot EU_F''\left(e_F^*\right)}{|J|} > 0 \tag{37}$$

$$\frac{\partial e_U^*}{\partial \delta} = \frac{-\bar{f} \cdot \gamma \cdot \bar{f}'}{|J|} \begin{cases} > 0, \text{ if } e_U^* > e_F^* + \Delta a \\ < 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{38}$$

$$\frac{\partial e_F^*}{\partial \gamma} = \frac{\bar{f} \cdot \delta \cdot \bar{f}'}{|J|} \begin{cases} < 0, \text{ if } e_U^* > e_F^* + \Delta a \\ > 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{39}$$

$$\frac{\partial e_F^*}{\partial \delta} = \frac{-\bar{f} \cdot EU_U''\left(e_U^*\right)}{|J|} > 0, \tag{40}$$

[22] Again, the cost function is assumed so be sufficiently convex so that the second-order condition holds.

[23] Recall that the second-order conditions $EU_U''\left(e_U^*\right) < 0$ and $EU_F''\left(e_F^*\right) < 0$ have been assumed to hold.

where

$$|J| := EU_U''(e_U^*) \cdot EU_F''(e_F^*) + \gamma\delta\left[\bar{f}'\right]^2 > 0$$

denotes the Jacobian determinant. The comparative static results clearly show that a worker's own emotions – anger felt by the favorite as well as pride felt by the underdog – lead to higher incentives ex ante (see conditions (37) and (40)). The intuition for this result is just the same as mentioned in Subsection 4.1 when discussing the bonus system. In addition, we have also an impact of the respective co-worker's emotions due to the game-theoretic context into which the workers of the same department are put. As (38) and (39) show, these effects crucially depend on the fact which worker has the higher probability of being more successful than the other one in equilibrium. The additional incentive effects will only be positive, if the influenced worker is more likely to beat the other worker than vice versa: If $e_U^* > e_F^* + \Delta a$, the favorite's anger $\delta$ will also increase the underdog's incentives, whereas for $e_U^* < e_F^* + \Delta a$ it will decrease them. Similarly, if $e_U^* < e_F^* + \Delta a$, the underdog's pride enhances the favorite's effort choice, whereas for $e_U^* > e_F^* + \Delta a$ effort decreases. Interestingly, a similar effect holds for unfair tournaments with anger (see (8); there we always have $e_U^* < e_F^* + \Delta a$) and pride (see (18)). However, the net incentive effect of emotions is not always positive under the piece-rate system: The net incentive effect of pride, $\frac{\partial e_U^*}{\partial \gamma} + \frac{\partial e_F^*}{\partial \gamma}$, will be positive if $e_U^* < e_F^* + \Delta a$; otherwise it will be positive if $2\delta\left|\bar{f}'\right| < c''(e_F^*)$, i.e. if the favorite's anger is not too high. Similarly, the net incentive effect of anger, $\frac{\partial e_U^*}{\partial \delta} + \frac{\partial e_F^*}{\partial \delta}$, will be positive if $e_U^* > e_F^* + \Delta a$; otherwise it will be positive if $2\gamma\bar{f}' < c''(e_U^*)$, i.e. if the underdog's pride is not too high.

Now we can derive the employer's optimal piece-rate scheme that maximizes his expected profits $E[q_U] + E[q_F] - E[w_U] - E[w_F]$ under the two workers' participation constraints $EU_U(e_U) \geq \bar{u}$ and $EU_F(e_F) \geq \bar{u}$ , and the two incentive constraints (35) and (36). Let the workers have unlimited liability.[24] Then the Lagrangian is given by

$$\begin{aligned} L \quad & (e_U, e_F, x_U, y_U, x_F, y_F) = \sum_{i \in \{U,F\}} [(1 - y_i)(e_i + a_i) - x_i] \\ & +\lambda_1 \cdot [x_U + y_U a_U + y_U e_U + \gamma F(e_U - e_F - \Delta a) - c(e_U) - \bar{u}] \\ & +\lambda_2 \cdot [x_F + y_F a_F + y_F e_F - \delta F(e_U - e_F - \Delta a) - c(e_F) - \bar{u}] \\ & +\lambda_3 \cdot [y_U + \gamma f(e_U - e_F - \Delta a) - c'(e_U)] \\ & +\lambda_4 \cdot [y_F + \delta f(e_U - e_F - \Delta a) - c'(e_F)] \end{aligned}$$

with $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$ as multipliers. Let again, for brevity, $\bar{f} := f(e_U - e_F - \Delta a)$. Then, we have the following optimality conditions:

$$\begin{aligned} \frac{\partial L}{\partial e_U} &= 1 - y_U + \lambda_1 \left[y_U + \gamma \bar{f} - c'(e_U)\right] \\ &\quad -\lambda_2 \delta \bar{f} + \lambda_3 \left[\gamma \bar{f}' - c''(e_U)\right] + \lambda_4 \delta \bar{f} = 0 \end{aligned} \tag{41}$$

$$\begin{aligned} \frac{\partial L}{\partial e_F} &= 1 - y_F - \lambda_1 \gamma \bar{f} + \lambda_2 \left[y_F + \delta \bar{f} - c'(e_F)\right] \\ &\quad -\lambda_3 \gamma \bar{f}' + \lambda_4 \left[-\delta \bar{f}' - c''(e_F)\right] = 0 \end{aligned} \tag{42}$$

$$\frac{\partial L}{\partial x_U} = -1 + \lambda_1 = 0 \quad \text{and} \quad \frac{\partial L}{\partial x_F} = -1 + \lambda_2 = 0 \tag{43}$$

$$\frac{\partial L}{\partial y_U} = -a_U - e_U + \lambda_1 a_U + \lambda_1 e_U + \lambda_3 = 0 \tag{44}$$

$$\frac{\partial L}{\partial y_F} = -a_F - e_F + \lambda_2 a_F + \lambda_2 e_F + \lambda_4 = 0. \tag{45}$$

Condition (43) shows that, not surprisingly, the employer will choose the compensation variables $(x_i, y_i)$ $(i = U, F)$ so that both workers' participation constraints are just binding: As under $D = HOM$ the employer sets the fixed payments $x_U$ and $x_F$ in order to fully extract the rents from the workers. Inserting $\lambda_1 = 1$ and $\lambda_2 = 1$ into (44) and (45) yields $\lambda_3 = \lambda_4 = 0$. Using

[24] The case of limited liability is omitted in this subsection since it adds no new insights.

the four values of the Lagrangian multipliers to simplify conditions (41) and (42) yields for the optimal effort levels $e_U^*$ and $e_F^*$:

$$1 + (\gamma - \delta) f (e_U^* - e_F^* - \Delta a) = c' (e_U^*) \quad (46)$$

$$1 - (\gamma - \delta) f (e_U^* - e_F^* - \Delta a) = c' (e_F^*) . \quad (47)$$

Comparing (46) and (47) with expression (23) for the first-best effort $e^{FB}$, immediately gives the following results:

**Proposition 7** *Let the employer choose $(x_i, y_i)$ $(i = U, F)$ for $D = HET$ with both anger and pride under unlimited liability of the workers. If both emotions have the same intensity – i.e., $\gamma = \delta$ –, the employer will implement $e_U^* = e_F^* = e^{FB}$; otherwise we have*

$$e_U^* > e^{FB} \text{ and } e_F^* < e^{FB}, \text{ if } \gamma > \delta,$$

$$\text{and } e_U^* < e^{FB} \text{ and } e_F^* > e^{FB}, \text{ if } \gamma < \delta.$$

Interestingly, the results of the proposition point out that – contrary to piece rates under $D = HOM$, and contrary to the bonus system under $D = HET$ – the employer does not implement first-best efforts for the two workers in general, although the workers are risk neutral and not restricted by limited liability. Therefore, emotions are always detrimental for piece-rate systems under a monetary-welfare perspective. The intuition for this result comes from the game between the two workers that is created by the emotions – we have no longer two separate optimization problems for the workers from the employer's viewpoint. Only if both workers' emotions lead to completely symmetric behavior $(\gamma = \delta)$ the game-theoretic context cancels out in equilibrium and the employer implements first-best incentives. Otherwise, he utilizes the game between the workers to generate extra incentives due to emotions. For example, if pride dominates anger $(\gamma > \delta)$, it will be optimal to induce higher than first-best incentives to the underdog at the expense of less than first-best incentives for the favorite. However, the employer is strictly better off than implementing $e^{FB}$ for both workers.

Finally, whether homogeneous ($D = HOM$) or heterogeneous ($D = HET$) departments are advantageous for the employer, crucially depends on the magnitudes of $\gamma$ and $\delta$. Let, for example, effort costs be quadratic: $c(e_i) = \frac{c}{2}e_i^2$ with $c > 0$. In this situation, first-best effort amounts to $e^{FB} = \frac{1}{c}$. From (46) and (47) we obtain $e_U^* + e_F^* = \frac{2}{c} = 2e^{FB}$. Hence, under both $D = HOM$ and $D = HET$ the employer implements the same collective effort and extracts all rents from the workers (i.e., each worker's participation constraint is always binding under both designs). In this case, the employer prefers the design $D$ which implements $2e^{FB}$ at lowest costs. By inspection of the Lagrangian $L(e_U, e_F, x_U, y_U, x_F, y_F)$ above, it becomes obvious that the employer will prefer $D = HET$ to $D = HOM$, if $\gamma$ is sufficiently large compared to $\delta$. In this case, the underdog's participation constraint is significantly relaxed while the favorite's participation constraint is only weakly aggravated so that the employer can save labor costs when choosing heterogeneous instead of homogeneous departments. However, the employer will choose $D = HOM$ if $\gamma$ small and $\delta$ large.

# 5 Conclusion

In this paper, the impact of emotions on workers' incentives and the employer's profits is considered. For this purpose, we differentiate between three incentive systems that are often used in practice – tournaments, bonus payments and piece-rate systems. In tournaments, the net effect of both anger and pride on the two workers' efforts is always positive. Furthermore, the employer will benefit from emotional incentives, if he need not directly pay for the enhanced incentives, when tournament prizes are exogenous or the workers' participation constraints are sufficiently relaxed by expected pride or workers receive sufficiently high rents. Under the bonus scheme, feeling both joy and frustration when meeting/non-meeting a given standard enhances incentives. In particular, emotions will be beneficial from the employer's viewpoint, if joy has a higher impact than frustration, the given standard is not too high, the workers' cost function is not too steep and workers have a high ability. Under the piece-rate system, emotions concerning the performance of co-workers in the same department lead to peer effects. A worker's own emotions – anger felt by a more able worker as well as pride felt by a less able one – lead to higher incentives, but the spillover effects on co-workers depend on the magnitude of the respective emotions. If the employer can optimally design the piece-rate system, he will typically not induce first-best efforts in order to benefit from utilizing the emotional game between the workers for incentive purposes.

The concept of emotions introduced in this paper has a special focus. Here, we have concentrated on emotions that emerge when comparing one's own performance with the performance of heterogeneous co-workers or with a given target. By this concept, the interplay of emotions and incentives can be analyzed in detail. Moreover, results can be derived concerning the optimal design of departments from the employer's viewpoint. Finally, the concept is used in order to explain experimental findings – oversupply of effort in tournaments and peer effects in work groups – that contradict standard economic theory. Of course, the analysis of emotions can be extended in several directions. For example, this paper focuses on the impact of emotions on incentives. Perhaps, there are also matching effects concerning different types of workers with different emotional attitudes. Considering such weak factors like the "chemistry" between co-workers may be important when deciding about the composition of departments and work groups. As another example, it may be interesting to discuss emotions in a dynamic setting. Over time there may be reinforcement effects concerning such emotions like anger or frustration and, hence, the existence of certain threshold levels may be decisive for workers' actions. Furthermore, in a dynamic context evolutionary aspects concerning the emergence or disappearance of certain emotional attitudes in work groups can be analyzed.

**Appendix**

*Proof of Proposition 2:*

(i) In the case of two fair tournaments ($D = HOM$), for each department the employer chooses tournament prizes in order to maximize

$$\pi = 2e^{*}(\Delta w) + 2a_t - \Delta w - 2w_L \qquad (t = U, F) \tag{A1}$$

subject to the workers' individual participation constraint

$$\frac{\Delta w + 2w_L}{2} - c\left(e^{*}(\Delta w)\right) \geq \bar{u} \tag{A2}$$

with $e^{*}(\Delta w)$ being described by the incentive constraint (2). Note that first-best effort $e^{FB}$ is defined by

$$e^{FB} = \arg\max_{e_t} \{q_t - c(e_t)\} \qquad (t = U, F),$$

which leads to

$$1 = c'\left(e^{FB}\right). \tag{A3}$$

Since the loser prize $w_L$ decreases the employer's objective function, he chooses $w_L$ so that (A2) is binding, i.e. the employer extracts all rents from the workers and wants to maximize overall welfare by implementing first-best efforts. Hence, the employer chooses

$$w_H = c\left(e^{FB}\right) + \bar{u} + \frac{1}{2f(0)} \quad \text{and} \quad w_L = c\left(e^{FB}\right) + \bar{u} - \frac{1}{2f(0)}.$$

In an unfair tournament ($D = HET$), the employer wants to maximize

$$\pi = e_U^{*}(\Delta w) + e_F^{*}(\Delta w) + a_U + a_F - \Delta w - 2w_L \tag{A4}$$

subject to the workers' participation constraints

$$w_L + \Delta w F\left(e_U^{*}(\Delta w) - e_F^{*}(\Delta w) - \Delta a\right) - c\left(e_U^{*}(\Delta w)\right) \geq \bar{u} \tag{A5}$$

$$w_L - \delta + (\Delta w + \delta)\left[1 - F\left(e_U^{*}(\Delta w) - e_F^{*}(\Delta w) - \Delta a\right)\right] - c\left(e_F^{*}(\Delta w)\right) \geq \bar{u} \tag{A6}$$

with $e_U^*(\Delta w)$ and $e_F^*(\Delta w)$ being implicitly defined by (3) and (4). To save labor costs, the employer chooses $w_L$ to make the participation constraint of the worker with the lower expected utility just bind, whereas the other worker receives a positive rent. However, recall that $e_F^*(\Delta w) > e_U^*(\Delta w)$ which implies $F(e_U^*(\Delta w) - e_F^*(\Delta w) - \Delta a) < 0.5$ but also $c(e_F^*(\Delta w)) > c(e_U^*(\Delta w))$. Hence without further specifying the distribution and the cost function it is not clear whether the left-hand side of (A5) is larger than the left-hand side of (A6) or vice versa. Anyway, since the incentive-enhancing effect of $\delta$ is irrelevant here – incentives can be continuously adjusted by appropriately choosing $\Delta w$, whereas $w_L$ solely serves for transferring wealth between the employer and the workers –, disutility $\delta$ yields a welfare loss, and the employer cannot implement $e^{FB}$ for both workers, $D = HOM$ unambiguously dominates $D = HET$ from the employer's viewpoint.

(ii) As a starting point look at the participation constraint (A2) under $D = HOM$ and let (A2) be non-binding in equilibrium, i.e. workers earn positive rents. If we now switch to $D = HET$ with $\Delta a$ being arbitrarily close to zero and with $\delta$ fulfilling $e_U^* + e_F^* > 2e^*$ for given tournament prizes according to Proposition 1, then the employer may prefer $D = HET$ to $D = HOM$: Overall efforts are higher in the unfair tournament but the employer does not have to pay for the large effort costs, $c(e_F^*(\Delta w))$, which only reduce agent $F$'s rent. Of course, according to (A6) the workers' rents have to be sufficiently large so that they are still positive after the switch to $D = HET$ despite the additional disutility $\delta$ and the higher effort costs $c(e_F^*(\Delta w))$.

In order to illustrate that such scenarios indeed exist for feasible values of $\delta$ and $\Delta a$, consider the following example: Let again $\varepsilon_i$ $(i = A, B)$ be uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. Effort costs are described by $c(e_i) = \frac{c}{3}e_i^3$.

Let, for simplicity, $c = \bar{\varepsilon} = \delta = 1$, $\Delta a = 0.1$ and $\bar{u} = 0$. Hence, we can use the triangular convolution above with range $[-2, 2]$ and $f(0) = \frac{1}{2\bar{\varepsilon}} = \frac{1}{2}$. According to (A3), first-best effort is given by $e^{FB} = 1$, and the optimal loser prize $w_L$ for implementing $e^{FB}$ under $D = HOM$ by $w_L = \frac{1}{3}(1)^3 - \frac{2 \cdot 1}{2} = \frac{1}{3} - 1 < 0$ , which is not feasible under limited liability. The optimal solution under $D = HOM$ can be calculated as follows: The workers' incentive constraint (2) simplifies to

$$e^* = \sqrt{\frac{\Delta w}{2}}.$$

Hence, the employer wants to maximize

$$\pi_{HOM} = 2\sqrt{\frac{\Delta w}{2}} + 2a_t - \Delta w - 2w_L \qquad (t = U, F)$$

subject to

$$\frac{\Delta w + 2w_L}{2} - \frac{1}{3}\left(\sqrt{\frac{\Delta w}{2}}\right)^3 \geq 0 \quad \text{and} \quad w_L \geq 0.$$

The employer optimally chooses $\Delta w^* = \frac{1}{2}$ and $w_L^* = 0$ which yields overall profits $2\pi_{HOM}^* = 1 + 2a_L + 2a_H$ from both fair tournaments, whereas each worker receives a positive rent $\frac{5}{24} = 0.20833$.

Under $D = HET$, we know from (3) and (4) and the left-hand side of the triangular convolution that workers behave according to

$$\Delta w \left(\frac{1}{2} + \frac{e_U^* - e_F^* - 0.1}{4}\right) = e_U^{*2} \tag{A7}$$

$$\text{and} \quad (\Delta w + \delta)\left(\frac{1}{2} + \frac{e_U^* - e_F^* - 0.1}{4}\right) = e_F^{*2} \tag{A8}$$

which implies

$$e_F^* = \sqrt{\frac{\Delta w + \delta}{\Delta w}} e_U^*.$$

Inserting into the first-order condition (A7) and solving for $e_U^*$ gives

$$\begin{aligned} e_U^* &= \frac{1}{40}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + (25\delta + 760)\,\Delta w}\right) \\ &= \frac{1}{40}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + 785\Delta w}\right) \\ e_F^* &= \frac{1}{40}\sqrt{\frac{\Delta w + 1}{\Delta w}}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + 785\Delta w}\right) \end{aligned}$$

with $\Omega := \left(1 - \sqrt{\frac{\Delta w+\delta}{\Delta w}}\right) = \left(1 - \sqrt{\frac{\Delta w+1}{\Delta w}}\right)$. The employer's expected profits for organizing an unfair tournament are

$$\begin{aligned} \pi_{HET} &= e_U^* + e_F^* + a_L + a_H - \Delta w - 2w_L \\ &= \left(1 + \sqrt{\frac{\Delta w + 1}{\Delta w}}\right) e_U^* + a_L + a_H - \Delta w - 2w_L \\ &= \left(1 + \sqrt{\frac{\Delta w + 1}{\Delta w}}\right) \frac{1}{40}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + 785\Delta w}\right) \\ &\quad + a_L + a_H - \Delta w - 2w_L. \end{aligned}$$

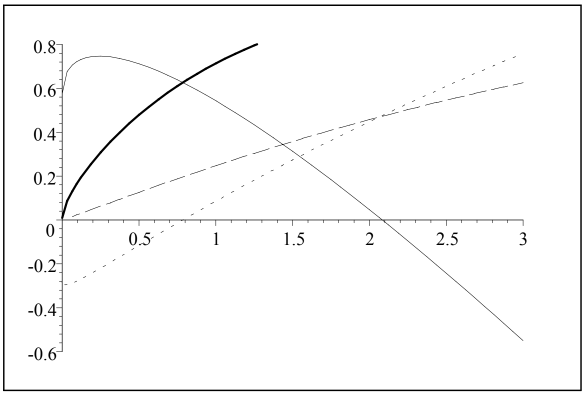
The workers' expected utilities can be written as

$$EU_U\left(e_U^*\right) = w_L + \Delta w\left(\frac{\left(\Omega e_U^* - 0.1\right)}{2} + \frac{\left(\Omega e_U^* - 0.1\right)^2}{8} + \frac{1}{2}\right) - \frac{1}{3}e_U^{*3}$$

and

$$\begin{aligned} EU_F\left(e_F^*\right) &= (\Delta w + 1)\left(1 - \left(\frac{\left(\Omega e_U^* - 0.1\right)}{2} + \frac{\left(\Omega e_U^* - 0.1\right)^2}{8} + \frac{1}{2}\right)\right) \\ &\quad + w_L - 1 - \frac{1}{3}\left(\sqrt{\frac{\Delta w + 1}{\Delta w}} e_U^*\right)^3. \end{aligned}$$

Plotting $\pi_{HET}$, $EU_U$ and $EU_F$ as functions of $\Delta w$ with $a_L = a_H = 0$ gives the following figure:




(independent variable at the abscissa: $\Delta w$; solid thin line: $\pi_{HET}$ under $w_L = 0$; dashed line: $EU_U$ under $w_L = 0$; dotted line: $EU_F$ under $w_L = 0$; solid bold line: $\pi_{HET}$ under $EU_F = 0$)

Note that all but the solid bold line hold for $w_L = 0$. Since the objective functions (function) of both workers (the employer) strictly increase (decreases) in $w_L$, only values between the maximum of the $\pi_{HET}$ graph (= solid thin line) and the intersection between the $EU_F$ graph (= dotted line) and the abscissa are relevant for the optimal $\Delta w$. Note also that $EU_U\left(e_U^*\right) > EU_F\left(e_F^*\right)$ in the relevant parameter range for $\Delta w$. Hence, the employer chooses $\Delta w$ and $w_L$ to maximize $\pi_{HET}$ subject to $EU_F \geq 0$ and $w_L \geq 0$. Since $\pi_{HET}$ strictly decreases in $w_L$ but both restrictions, $EU_F \geq 0$ and $w_L \geq 0$, relax with increasing $w_L$, at least one of the two constraints is binding in equilibrium. In the figure above with $w_L = 0$, the employer would choose $\Delta w$ so that $EU_F$ just intersects the abscissa. This happens at $\Delta w = 0.78525$ where the employer receives profits $\pi_{HET} = 0.62644 + a_L + a_H$. If otherwise $w_L > 0$, the employer would choose $w_L$ to make the favorite's participation constraint just bind which implies

$$\begin{aligned} w_L^* &= 1+\frac{1}{3}\left(\sqrt{\frac{\Delta w+1}{\Delta w}}e_U^*\right)^3 \\ &\quad -(\Delta w+1)\left(1-\left(\frac{(\Omega e_U^*-0.1)}{2}+\frac{(\Omega e_U^*-0.1)^2}{8}+\frac{1}{2}\right)\right). \end{aligned}$$

When inserting into $\pi_{HET}$ we obtain

$$\begin{aligned} \pi_{HET} &= \left(1+\sqrt{\frac{\Delta w+1}{\Delta w}}\right)e_U^*-\frac{2}{3}\left(\sqrt{\frac{\Delta w+1}{\Delta w}}e_U^*\right)^3 \\ &\quad +a_L+a_H-1-2(\Delta w+1)\left(\frac{(\Omega e_U^*-0.1)}{2}+\frac{(\Omega e_U^*-0.1)^2}{8}\right). \end{aligned}$$

which is described by the solid bold line in the figure above. We can easily see that in this case the employer would choose the corner solution $\Delta w = 0.78525$. Altogether, when organizing two unfair tournaments the employer's overall profits are $2\pi_{HET}^* = 1.2529 + 2a_L + 2a_H > 2\pi_{HOM}^* = 1 + 2a_L + 2a_H$.

*Proof of Proposition 4:*

Since result (ii) proceeds analogously to result (ii) of Proposition 2, it remains to show that under unlimited liability of the workers there exist cost functions $c(e_i)$ and distributions $G(\varepsilon_i)$ for which the employer prefers $D = HET$ *to* $D = HOM$. The employer's optimization problem can be characterized by the Lagrangian

$$\begin{aligned} L \quad & (e_U, e_F, \Delta w, w_L) = e_U + e_F + a_U + a_F - \Delta w - 2w_L \\ & +\lambda_1 \cdot [(\Delta w+\gamma) f(e_U - e_F - \Delta a) - c'(e_U)] \\ & +\lambda_2 \cdot [\Delta w f(e_U - e_F - \Delta a) - c'(e_F)] \\ & +\lambda_3 \cdot [w_L + (\Delta w+\gamma) F(e_U - e_F - \Delta a) - c(e_U) - \bar{u}] \\ & +\lambda_4 \cdot [w_L + \Delta w [1 - F(e_U - e_F - \Delta a)] - c(e_F) - \bar{u}] \end{aligned}$$

with $\lambda_1, \lambda_2 \geq 0$ as multipliers for the workers' incentive constraints (13) and (14), and $\lambda_3, \lambda_4 \geq 0$ as multipliers for the workers' participation constraints. In optimum, we must have that

$$\begin{aligned} \frac{\partial L}{\partial e_U} &= 1 + \lambda_1 \left[ (\Delta w + \gamma) \bar{f}' - c''(e_U) \right] + \lambda_2 \Delta w \bar{f}' \\ &\quad + \lambda_3 \left[ (\Delta w + \gamma) \bar{f} - c'(e_U) \right] - \lambda_4 \Delta w \bar{f} = 0 \end{aligned} \tag{A9}$$

$$\begin{aligned} \frac{\partial L}{\partial e_F} &= 1 - \lambda_1 (\Delta w + \gamma) \bar{f}' + \lambda_2 \left[ -\Delta w \bar{f}' - c''(e_F) \right] \\ &\quad - \lambda_3 (\Delta w + \gamma) \bar{f} + \lambda_4 \left[ \Delta w \bar{f} - c'(e_F) \right] = 0 \end{aligned} \tag{A10}$$

$$\frac{\partial L}{\partial \Delta w} = -1 + (\lambda_1 + \lambda_2) \bar{f} + (\lambda_3 - \lambda_4) \bar{F} + \lambda_4 = 0 \tag{A11}$$

$$\frac{\partial L}{\partial w_L} = -2 + \lambda_3 + \lambda_4 = 0 \tag{A12}$$

with $\bar{f} := f(e_U - e_F - \Delta a)$ and $\bar{F} := F(e_U - e_F - \Delta a)$. Condition (A12) shows that at least one participation constraint is binding in equilibrium. Typically, exactly one participation constraint will be binding: Since the loser prize $w_L$ only serves to transfer wealth between the employer and the workers and because this prize can be arbitrarily negative, the employer chooses it so that the worker with the lower expected utility just receives $\bar{u}$ in expected terms. Combining (A9) and (A10) gives

$$2 - \lambda_1 c''(e_U) - \lambda_2 c''(e_F) - \lambda_3 c'(e_U) - \lambda_4 c'(e_F) = 0. \tag{A13}$$

The two incentive constraints together yield

$$\frac{c'(e_U)}{\Delta w + \gamma} = \frac{c'(e_F)}{\Delta w}. \tag{A14}$$

Of course, without further specifying the cost function and the probability distribution no clear results can be derived. Hence, Proposition 4(i) only claims that for certain specifications the employer prefers $D = HET$ to

$D = HOM$. Consider, for example, the case of quadratic costs $c(e_i) = \frac{c}{2} e_i^2$ and uniformly distributed noise $\varepsilon_i \in [-\bar{\varepsilon}, \bar{\varepsilon}]$ so that $\varepsilon_j - \varepsilon_i$ is triangularly distributed – as in the Corollaries 1 and 2. In order to guarantee a strictly concave objective function for both workers and the existence of pure-strategy equilibria, let

$$\Delta w + \gamma < 4c\bar{\varepsilon}^2 \tag{A15}$$

and

$$\Delta a < 2\bar{\varepsilon}. \tag{A16}$$

Furthermore, let the favorite's participation constraint be binding so that we have $\lambda_3 = 0$ and $\lambda_4 = 2$ (see (A12)). In this case, (A13) can be rewritten as

$$\lambda_1 + \lambda_2 = \frac{2}{c} - 2e_F.$$

Inserting into (A11) (together with $\lambda_3 = 0$ and $\lambda_4 = 2$) leads to

$$\left(\frac{2}{c} - 2e_F\right) f(e_U - e_F - \Delta a) - 2F(e_U - e_F - \Delta a) + 1 = 0.$$

By substituting for the triangular distribution and assuming $e_U - e_F - \Delta a < 0$ (hence, later on we have to check whether this condition indeed holds) we can rearrange the last condition to

$$\left(\frac{4\bar{\varepsilon}}{c} - 4\bar{\varepsilon}e_F\right) + \left(\frac{2}{c} - 2e_F - 4\varepsilon\right)(e_U - e_F - \Delta a) - (e_U - e_F - \Delta a)^2 = 0. \tag{A17}$$

For quadratic costs, (A14) simplifies to

$$\frac{e_U}{(\Delta w + \gamma)} = \frac{e_F}{\Delta w} \tag{A18}$$

and the favorite's participation constraint to

$$\Delta w \left(\frac{1}{2\bar{\varepsilon}} + \frac{e_U - e_F - \Delta a}{4\bar{\varepsilon}^2}\right) = ce_F. \tag{A19}$$

Solving the system of equations (A17)–(A19) for $e_U$, $e_F$ and $\Delta w$ yields

$$e_U^* = \frac{\gamma^2 + 4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 2\gamma c\Delta a\left(4\bar{\varepsilon} - \Delta a\right)}{2c\left(4c\bar{\varepsilon}^2 - \gamma\right)\left(2\bar{\varepsilon} - \Delta a\right)}$$

$$e_F^* = \frac{\gamma^2 + 4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{2c\left(4c\bar{\varepsilon}^2 - \gamma\right)\left(2\bar{\varepsilon} - \Delta a\right)}$$

$$\Delta w^* = \frac{\gamma^2 + 4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2}.$$

At last, the favorite's binding participation constraint

$$w_L + \Delta w\left[1 - F\left(e_U^* - e_F^* - \Delta a\right)\right] - \frac{c}{2}e_F^{*2} = \bar{u}$$

leads to the optimal loser prize

$$\begin{aligned} w_L^* &= \bar{u} - \left(2\gamma\left(2\bar{\varepsilon} - \Delta a\right) + 3\gamma\left(\gamma - 8c\bar{\varepsilon}^2\right) + 4c\bar{\varepsilon}^2\left(8c\bar{\varepsilon}^2 + c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(\Delta a - 2\bar{\varepsilon}\right)\right)\right) \times \\ &\quad \frac{\gamma^2 + 4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{8c\left(4c\bar{\varepsilon}^2 - \gamma\right)^2\left(2\bar{\varepsilon} - \Delta a\right)^2}. \end{aligned}$$

The employer's expected profits from organizing an unfair tournament are, therefore,

$$\begin{aligned} \pi_{HET} &= e_U^* + e_F^* + a_U + a_F - \Delta w^* - 2w_L^* \\ &= a_U + a_F - 2w_L^* \\ &\quad + \frac{\gamma\left(8c\bar{\varepsilon}^2 - \gamma\right) - 2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right)\left(\left(c\bar{\varepsilon} - 1\right)2\bar{\varepsilon} + \Delta a\right)}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2} \\ &= a_U + a_F - 2\bar{u} \\ &\quad + \frac{\gamma\left(8c\bar{\varepsilon}^2 - \gamma\right) - 2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right)\left(\left(c\bar{\varepsilon} - 1\right)2\bar{\varepsilon} + \Delta a\right)}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2} \\ &\quad + \left(2\gamma\left(2\bar{\varepsilon} - \Delta a\right) + 3\gamma\left(\gamma - 8c\bar{\varepsilon}^2\right) + 4c\bar{\varepsilon}^2\left(8c\bar{\varepsilon}^2 + c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(\Delta a - 2\bar{\varepsilon}\right)\right)\right) \times \\ &\quad \frac{\gamma^2 + 4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{4c\left(4c\bar{\varepsilon}^2 - \gamma\right)^2\left(2\bar{\varepsilon} - \Delta a\right)^2} \end{aligned}$$

However, when organizing a fair tournament the employer's expected profits amount to

$$\begin{aligned}\pi_{HOM} &= 2e^{FB} - 2c\left(e^{FB}\right) + a_U + a_F - 2\bar{u} \\ &= \frac{2}{c} - 2\frac{c}{2}\left(\frac{1}{c}\right)^2 + a_U + a_F - 2\bar{u} \\ &= \frac{1}{c} + a_U + a_F - 2\bar{u}.\end{aligned}$$

The comparison

$$\frac{\gamma\left(8c\bar{\varepsilon}^2 - \gamma\right) - 2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right)\left(\left(c\bar{\varepsilon} - 1\right)2\bar{\varepsilon} + \Delta a\right)}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2}$$
$$+\left(2\gamma\left(2\bar{\varepsilon} - \Delta a\right) + 3\gamma\left(\gamma - 8c\bar{\varepsilon}^2\right) + 4c\bar{\varepsilon}^2\left(8c\bar{\varepsilon}^2 + c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(\Delta a - 2\bar{\varepsilon}\right)\right)\right) \times$$
$$\frac{\gamma^2 + 4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{4c\left(4c\bar{\varepsilon}^2 - \gamma\right)^2\left(2\bar{\varepsilon} - \Delta a\right)^2} > \frac{1}{c}.$$

can be simplified to

$$\begin{aligned}&\gamma^4 - 4\gamma^3\left(2\bar{\varepsilon}\left(2c\bar{\varepsilon} + 1\right) - \Delta a\right) + 4c\gamma^2\left(8\bar{\varepsilon}^2 + 4\Delta a\bar{\varepsilon} - \Delta a^2\right)\left(2\bar{\varepsilon}\left(c\bar{\varepsilon} + 1\right) - \Delta a\right) \\ &-32\bar{\varepsilon}^2\gamma c^2\left(2\Delta a\bar{\varepsilon}^2\left(4c\bar{\varepsilon} + 3\right) - 2\bar{\varepsilon}\Delta a^2\left(3 + c\bar{\varepsilon}\right) + \Delta a^3 + 4\bar{\varepsilon}^3\right) \\ &+16\Delta a c^3\bar{\varepsilon}^4\left(4\bar{\varepsilon} - \Delta a\right)\left(4\bar{\varepsilon}\left(c\Delta a + 2\right) - \Delta a\left(c\Delta a + 4\right)\right) > 0. \qquad \text{(A20)}\end{aligned}$$

According to Proposition 4(i), we have only to show that inequality (A20) holds for at least one feasible parameter constellation. It can easily be checked that $c = \bar{\varepsilon} = 1$ and $\Delta a = \gamma = 0.5$ satisfy (A20). Moreover, we obtain

$$\begin{aligned}e_F^* &= 1.3095 > 1 = e^{FB} \\ e_U^* &= 1.5238 > 1 = e^{FB}\end{aligned}$$

so that (A15), (A16) and $e_U^* - e_F^* - \Delta a < 0$ hold. Hence, under the given specifications it is optimal for the employer to induce higher than first-best efforts to both workers.

*Proof of Proposition 5:*

Suppose we have $\gamma = \delta$ in (21) and (22). This yields a symmetric solution for the tournament stage, $\tilde{e}^* = \tilde{e}^*(\Delta w)$, implicitly defined by

$$(\Delta w + \gamma) f(-\Delta a) = c'(\tilde{e}^*).$$

The employer's Lagrangian at the second stage of the three-stage game (with $D = HET$ at the first stage) can be written as

$$\begin{aligned} L(\Delta w, w_L) &= 2\tilde{e}^*(\Delta w) + a_U + a_F - \Delta w - 2w_L \\ &\quad + \lambda_1[w_L + (\Delta w + \gamma) F(-\Delta a) - c(\tilde{e}^*(\Delta w)) - \bar{u}] \\ &\quad + \lambda_2[w_L - \gamma + (\Delta w + \gamma)[1 - F(-\Delta a)] - c(\tilde{e}^*(\Delta w)) - \bar{u}] \end{aligned}$$

with $\lambda_1, \lambda_2 \geq 0$ as multipliers. In optimum, we must have

$$\begin{aligned} \frac{\partial L}{\partial \Delta w} &= 2\frac{\partial \tilde{e}^*}{\partial \Delta w} - 1 + \lambda_1 F(-\Delta a) - \lambda_1 c'(\tilde{e}^*) \frac{\partial \tilde{e}^*}{\partial \Delta w} \\ &\quad + \lambda_2[1 - F(-\Delta a)] - \lambda_2 c'(\tilde{e}^*) \frac{\partial \tilde{e}^*}{\partial \Delta w} = 0 \end{aligned} \tag{A21}$$

and

$$\frac{\partial L}{\partial w_L} = -2 + \lambda_1 + \lambda_2 = 0. \tag{A22}$$

Hence, according to (A22) at least one participation constraint is binding in equilibrium. In general, we have $(\Delta w + \gamma) F(-\Delta a) \neq -\gamma + (\Delta w + \gamma)[1 - F(-\Delta a)]$ and the employer chooses $w_L$ to make the participation constraint of the worker with the lower expected utility bind. If, therefore, $\lambda_1 = 0$ and $\lambda_2 = 2$, equation (A21) yields

$$2\frac{\partial \tilde{e}^*}{\partial \Delta w}(1 - c'(\tilde{e}^*)) + 1 - 2F(-\Delta a) = 0.$$

Since $F(-\Delta a) < \frac{1}{2}$ because of the symmetry of the convolution, we must have $c'(\tilde{e}^*) > 1$. Comparing with (A3) gives $\tilde{e}^* > e^{FB}$ because marginal costs are strictly increasing due to the convexity of the cost function. If, however, $\lambda_1 = 2$ and $\lambda_2 = 0$, equation (A21) leads to

$$2\frac{\partial \tilde{e}^*}{\partial \Delta w}\left(1 - c'\left(\tilde{e}^*\right)\right) - 1 + 2F\left(-\Delta a\right) = 0$$

and, hence, to $\tilde{e}^* < e^{FB}$.

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