Relational Delegation*

Ricardo Alonso[†]
Northwestern University

Niko Matouschek[‡]
Northwestern University

First Draft: June 2004
This Version: December 2004

Abstract

We explore the optimal delegation of decision rights by a principal to a better informed but biased agent. In an infinitely repeated game a long lived principal faces a series of short lived agents. Every period they play a cheap talk game ala Crawford and Sobel (1982) with constant bias, quadratic loss functions and general distributions of the state of the world. We characterize the optimal delegation schemes for all discount rates and show that they resemble organizational arrangements that are commonly observed, including centralization and threshold delegation. For small biases threshold delegation is optimal for almost all distributions. Outsourcing can only be optimal if the principal is sufficiently impatient.

Keywords: delegation, cheap talk, relational contract.

JEL Classification: D23, D82, L23

*We are grateful for discussions with Heski Bar-Isaac, Bob Gibbons, Scott Schaefer and Kathy Spier and for comments from seminar participants at the Kellogg School of Management, Melbourne Business School and the University of New South Wales as well as from participants of the "2004 Summer Camp in Organizational Economics" at MIT Sloan. All remaining errors are our own.

[†]Kellogg School of Management, 2001 Sheridan Road, Evanston, IL 60201, USA. Email: `r-alonso@kellogg.northwestern.edu`.

[‡]Corresponding author. Kellogg School of Management, 2001 Sheridan Road, Evanston, IL 60201, USA. Email: `n-matouschek@kellogg.northwestern.edu`.

1 Introduction

To understand the behavior of firms one must consider their internal allocation of decision rights. While owners have the formal authority to take all decisions on behalf of their firms, they typically delegate at least some important decision rights to their employees. These employees, however, often have consistent biases and can be expected to take different decisions than the owners would (Jensen 1986). An understanding of what determines the internal allocation of decision rights is therefore a prerequisite for understanding, and potentially being able to predict, the decisions that firms take, such as how much to invest and how many workers to hire and fire. In this paper we investigate the optimal allocation of decision rights within firms. In particular, we investigate how the owner of a firm should delegate decision rights to a biased employee.

While the formal authority to take decisions is concentrated at the top of firms, the information needed to make effective use of this authority is often dispersed throughout their ranks. The legal right to decide on the allocation of capital, for instance, resides with the owners of firms but CEOs, division managers, and other employees are often better informed about the profitability of different investment projects. The benefit of delegating decision rights is that it allows the owners to utilize the specific knowledge that their employees might have (Holmström 1977, 1984; Jensen and Meckling 1992).

There are two main difficulties in delegating decision rights, however. First, as mentioned above, there is ample evidence which suggests that employees have consistent biases and are therefore likely to take different decisions than the owners would want them to. Agency costs therefore place a limit on the ability of owners to delegate decision rights (Holmström 1977, 1984; Jensen and Meckling 1992). Second, delegated decision rights are always "loaned, not owned" (Baker, Gibbons, and Murphy 1999, p.56). In other words, while owners can delegate decision rights ex ante they can always overrule the decisions that employees take ex post. Anticipating the possibility of being overruled the employees in turn may act strategically and, as a result, their specific knowledge might not get used efficiently. Imperfect commitment therefore places a second limit on the ability of owners to delegate decision rights (Baker, Gibbons, and Murphy 1999).

Due to the presence of agency costs and the lack of perfect commitment owners rarely engage in complete delegation, that is they rarely delegate decision rights without putting in place rules and regulations that constrain the decisions their employees can take. Consider, for instance, the decision over the allocation of capital which is often delegated to lower level managers and, in particular, to division managers. While in some firms these division managers have almost full discretion in deciding between different investment projects, in most they face a variety of constraints. In some firms, for instance, division managers are allowed to decide on investment projects that affect the daily operation of their divisions but not on those that are deemed to affect the future of the firm as a whole. In other firms division managers can decide on investment projects that do not exceed a certain threshold size and their superiors decide on larger projects.[3] In this paper we show that many of the organizational arrangements that we observe in practice arise optimally in a model in which a principal with imperfect commitment delegates decision rights to a better informed but biased agent.

We develop an infinitely repeated game in which a long lived principal faces a sequence of short lived agents each of whom interacts with the principal only once. In every period a project has to be implemented and the principal has the formal authority to do so. The potential projects differ on one dimension, for instance investment size, and the principal and the agents have different preferences over this dimension. At the beginning of each period the current agent observes the state of the world which determines the identity of his preferred project and that of the principal. The agent then recommends a project after which the principal takes her decision. Finally, payoffs are realized, the state of the world becomes public information and time moves on to the next period.

Although the principal always has the formal authority to decide on the projects, she can engage in many different types of relational delegation. In other words, she can implicitly commit to many different decision rules that map the agents' recommendations into decisions. For instance, she can implicitly engage in *complete delegation* by

[3]A large number of studies have described the capital budgeting rules that firms use. See, for instance, Marsheutz (1985), Taggart (1987) and, in particular, Bower (1970).

committing herself to always rubber-stamp the agents' recommendations. Other possibilities include *threshold delegation* – in which case the principal rubber-stamps the agents' recommendations up to a certain size and implements her preferred project if they recommend a project that is above the threshold – and *menu delegation* – in which case the principal rubber-stamps the agents' recommendations only if they propose one of a discrete number of projects. Of course the principal can also choose to ignore the agents' recommendations altogether and simply implement the project that maximizes her expected payoff given her prior. In other words, she can engage in *centralization*.

Should the principal centralize or delegate? And if she delegates, should she engage in complete delegation, threshold delegation, or some other form of delegation? The key trade-off that the principal faces when she considers the many different organizational arrangements is between the direct cost of biasing her decisions in favor of the agents and the indirect benefit of inducing the agents to reveal more information. We show that in many cases the organizational arrangements that optimize this trade-off are commonly observed in the real world. In particular, we show that centralization, threshold delegation and menu delegation are often optimal and that which one of these arrangements is optimal depends only on the principal's commitment power, on the one hand, and a simple condition on the agents' bias and the distribution of the state space, on the other. Moreover, we show that for small biases threshold delegation is optimal for almost all common distributions. These results are consistent with the pervasive use of threshold delegation in organizations. Having derived our main characterization result we then investigate further implications, including the effects of changes in the bias and the amount of private information on the optimal organizational arrangement. We also show that complete delegation is never optimal and that outsourcing can only be optimal if the principal is sufficiently impatient. Finally, we discuss empirical implications of our analysis.

In the next section we discuss the related literature. In Section 3 we present the model after which we characterize the equilibrium of the stage game, in Section 4, and of the repeated game, in Section 5. We discuss further implications in Section 6, extensions in Section 7 and we conclude in Section 8. All proofs are relegated to the appendix.

2 Related Literature

The stage game in our model is a standard principal-agent problem of the following form. There is a principal and an agent who have different preferences over a decision that has to be taken. The payoffs that the principal and the agent realize depend on the decision and the state of world but the state of the world is only known by the agent.

A large number of papers have analyzed this static problem and they can be categorized in two dimensions: (i.) whether or not they allow for transfers between the principal and the agent and (ii.) the extent of commitment power by the principal.

Our paper contributes to the strand of the literature which argues that in many environments transfers between the principal and the agent are difficult or impossible. Within this strand of the literature one can distinguish between delegation- and cheap talk models. In the cheap talk models that follow Crawford and Sobel (1982) principals cannot commit to arbitrary decision rules, that is they cannot commit to act on the information they receive in a particular way. In contrast in the delegation models that follow Holmström (1977, 1984) the principal can commit to a decision rule.[4] Since we allow for different degrees of commitment by the principal, varying from no commitment all the way to perfect commitment, our paper contributes to the delegation literature which we discuss next.

Holmström (1977, 1984) considers a general version of the set up described above and proves the existence of an optimal delegation set or, equivalently, an optimal decision rule. He does not, however, characterize the optimum.[5] Melumad and Shibano (1991) do solve for the optimal decision rule but restrict attention to the uniform distribution and particular preferences. Dessein (2002) allows the principal to commit to only one type of delegation, namely complete delegation, and shows that for a large number of

[4]Formally, Holmström (1977, 1984) assumes that the principal can commit to a delegation set, i.e. she can commit to a set of decisions from which the agent can choose his preferred one. This is equivalent to letting the agent make a recommendation and assuming that the principal can commit to any decision rule that maps the recommendation into a decision. As Holmström (1984) puts it "delegation of authority to an agent" is equivalent to "asking the agent for information and promising to act on the information in a particular way" (see also Melumad and Shibano 1991).

[5]He restricts the set of feasible delegation sets to intervals. This is equivalent to restricting attention to continuous decision rules.

distributions, the principal does better when she commits to complete delegation than when she cannot commit to any decision. We contribute to the delegation literature in two main ways. First, we characterize the optimal decision rule for general distributions and constant bias without restricting the set of feasible decision rules. Second, instead of making assumptions about what the principal can and cannot commit to, we endogenize her commitment power and characterize the optimal decision rule for any amount of commitment power.

The second strand of the literature that analyzes the principal-agent problem described above does allow for transfers. Ottaviani (2000) and Krishna and Morgan (2004), in particular, both allow for message-contingent transfers but make different assumption about the principal's commitment power. In particular, Krishna and Morgan (2004) focus on the case in which the principal can only commit to a transfer rule while Ottaviani (2000) allows the principal to commit to a transfer- and a decision rule.

Finally, our work is related to several recent papers that investigate the role of relational contracts within and between organizations. Baker, Gibbons, and Murphy (1994, 2002) investigate the use of objective and subjective performance measures and the ownership structures of firms in a repeated setting. Levin (2003) investigates relational incentive contracts in the presence of moral hazard and asymmetric information.

3 The Model

We consider an infinitely repeated game in which the stage game is the Crawford and Sobel (1982) cheap talk game with constant bias and quadratic loss functions. The principal is infinitely long lived and faces a sequence of agents who only interact with her for one period.

Specifically, we consider a model in which time runs from $t = 1, 2, 3 \ldots$. In any period t a firm that consists of one principal and one agent must implement a project. The agent knows the state of the world that determines the payoffs associated with all possible projects and makes a recommendation to the principal who has the formal authority to decide what project is chosen. In particular, at the beginning of period t, the agent observes the state of the world $\theta_t \in \Theta = [0, 1]$, which is drawn from a distribution with a cumulative density function $F(\cdot)$ and is i.i.d. over time, and then sends a message $m_t \in M$ to the principal. Next the principal chooses a project that can be represented by a real number $y_t \in \mathbf{R}$. Although one can interpret y_t as measuring any one dimension on which the projects differ – for instance the number of workers to be hired for a new plant or the size of a new office building – we interpret it as the financial size of an investment. This interpretation facilitates the exposition and allows us to relate our findings to a number of papers that describe the capital budgeting rules which firms use to regulate the internal allocation of capital.[6]

After the project is chosen both players realize their stage game payoffs which are given by $U_P(y_t, \theta_t) = -(y_t - \theta_t)^2$ for the principal and $U_A(y_t, \theta_t, b) = -(y_t - \theta_t - b)^2$ for the agent. The parameter b measures the congruency of the agent's and the principal's preferences. Given these preferences, the principal's preferred project is given by $y_t = \theta_t$ and the agent's is given by $y_t = \theta_t + b$. As mentioned in the introduction there is ample anecdotal evidence that documents the tendency of many managers to engage in empire building, i.e. to invest more than would be optimal from the perspective of their principals (see for instance Jensen 1986). For this reason we assume $b > 0$ so that the agent prefers a larger investment than the principal. The analysis can easily be adapted, however, to allow for negative biases. Since we are interpreting y_t as the financial size of an investment and since the agent's and the principal's preferred project sizes are increasing in the state of the world θ_t, it is natural to think of low realizations of θ_t as bad states of the world in which the business environment is unfavorable to new investments and large realizations of θ_t as good states of the world in which the business environment is more favorable.

After the project is chosen and the payoffs are realized, the state of the world becomes publicly known and the stage game ends.

We assume that the principal is infinitely long lived and faces a single, new agent every period. All the agents have the same preferences, i.e. they all have the same stage

[6] Clearly, focusing on this interpretation is without loss of generality and does not rule out other possible interpretations. For studies describing the capital budgeting rules that firms use see Footnote 3. Theoretical papers seeking to rationalize the observed rules include Harris and Raviv (1996) and Marino and Matsusaka (2004).

game payoff function $U_A(\cdot)$ and the same congruency parameter b. While the agents care only about the payoff they realize in the one period in which they interact with the principal, the principal cares about the present discounted value of her payoff stream and discounts the future at a rate $\delta \in [0, 1)$.

We denote the history of the game up to date t by $\mathbf{h}^t = (\theta_0, m_0, y_0, ..., \theta_{t-1}, m_{t-1}, y_{t-1})$ and the set of all possible date t histories by H^t. A relational contract then specifies for any date t and any history $\mathbf{h}^t \in H^t$, (i.) a communication rule $\mu_t : \Theta \times H^t \to \Delta_t(M)$ which assigns a probability distribution over M for any state of the world θ_t; (ii.) a decision rule $Y_t : M \times H^t \to \mathbf{R}$ which assigns a project y_t for every message m_t; (iii.) a belief function $G_t : M \to \Delta_t(\Theta)$ which assigns a probability distribution over the states θ_t for every message m_t. Note, in particular, that histories are public. The belief function G_t is derived from μ_t using Bayes' rule wherever possible. Such a relational contract is self-enforcing if it describes a subgame perfect equilibrium of the repeated game.

We solve for the 'optimal' relational contract that maximizes the principal's present discounted payoff. In doing so we assume that the most severe punishment that can be implemented off the equilibrium path calls for the agents and the principal to revert to statically optimizing behavior. In other words, in the punishment phase the principal and the agents play the strategies that maximize their expected stage game payoffs. This assumption captures our belief that, when relational contracts break down, members of the same firm are likely to coordinate on the equilibrium that maximizes their respective payoffs in the absence of trust.[7] It should be noted, however, that qualitatively our results are not sensitive to this assumption.

As discussed in Section 2, we follow the cheap talk and delegation literatures by ruling out transfers. We relax this assumption in Section 7 where we allow for wage payments. For models with contingent transfers see Ottaviani (2000) and Morgan and Krishna (2004). In Section 7 we also discuss the implications of allowing for commitment by the agents which is ruled out in the set up described above.

[7] Baker, Gibbons, Murphy (1994) make a similar assumption for the same reason.

4 Stage Game

We start by considering the static equilibria of the stage game. An equilibrium of the stage game is characterized by (i.) a family of communication rules $\mu(m \mid \theta)$ for the agent, where $\mu(m \mid \theta)$ is the probability of sending message m conditional on the agent observing state θ, (ii.) a decision rule $y(m)$ for the principal that maps messages $m \in M$ into actions $y \in Y$, (iii.) a belief function $g(\theta \mid m)$ for the principal, where $g(\theta \mid m)$ is the probability of state θ conditional on receiving message m, such that (i.) for each $\theta \in \Theta$, if m is in the support of $\mu(m \mid \theta)$, then it maximizes the expected payoff of the agent given the principal's decision rule $y(m)$, (ii.) for each $m \in M$, $y(m)$ maximizes the expected payoff of the principal given her beliefs and (iii.) the belief function $g(\theta \mid m)$ is derived from $\mu(m \mid \theta)$ using Bayes' rule whenever possible.

Crawford and Sobel (1982) show that all equilibria are interval equilibria in which the agent only communicates the interval that the state of the world lies in. In this sense the agent's communication is noisy and information is lost. Having learned what interval the state of the world lies in, the principal implements the project that maximizes her expected payoff, given her updated beliefs.

To describe these interval equilibria, let $a \equiv (a_0, ..., a_N)$ denote the partition of $[0, 1]$ into N steps and dividing points between steps $0 \equiv a_0 < a_1 < ... < a_N \equiv 1$. Define for all $a_{i-1},\ a_i \in [0, 1]$, $\widehat{y}_i \equiv \arg\max_y \int_{a_{i-1}}^{a_i} U_P(y, \theta)\mathrm{d}F(\theta)/(F(a_i) - F(a_{i-1}))$. Finally, let y_i denote the project that the principal implements if she receives a signal from interval i, i.e. $y_i \equiv y(m)$ for $m \in (a_{i-1}, a_i)$. We can now state the following proposition which follows immediately from Theorem 1 in Crawford and Sobel (1982).

PROPOSITION 1 (Crawford and Sobel). *If $b > 0$, then there exists a positive integer $N(b)$ such that for every N with $1 \leq N \leq N(b)$, there exists at least one equilibrium $(\mu(\cdot)\ , y(\cdot),\ g(\cdot))$, where*

$$\begin{array}{ll}
\mu(m \mid \theta) \textit{ is uniform, supported on } [a_{i-1}, a_i] & \textit{if } \theta \in (a_{i-1},\ a_i), \\
y_i = \widehat{y}_i & \textit{if } m \in (a_{i-1},\ a_i), \\
g(\theta \mid m) = f(\theta)/(F(a_i) - F(a_{i-1})) & \textit{if } m \in (a_{i-1},\ a_i), \\
a_i = \frac{1}{2}\left(\widehat{y}_i + \widehat{y}_{i+1} - 2b\right) & \textit{for } i = 1, ...,\ N-1.
\end{array}$$

The expression for the a_i's is derived from the indifference condition $U_A(\widehat{y}_i, a_i) = U_A(\widehat{y}_{i+1}, a_i)$ which ensures that in state of the world a_i the agent is indifferent between projects y_i and y_{i+1}. An implication of this condition is that the length of successive intervals grows. In this sense less information gets communicated, the larger the state of the world. Intuitively, since the agent always prefers larger projects than the principal, his proposals are less credible the larger the projects that he recommends.

Crawford and Sobel (1982) also provide sufficient conditions under which the expected payoffs of the principal and the agent are increasing in the number of intervals N. When these conditions are satisfied, as they are in our specification, one may therefore expect the players to coordinate on the equilibrium in which the number of intervals is maximized, i.e. in which $N = N(b)$. We denote this equilibrium by $(\mu^{CS}, y^{CS}, g^{CS})$ and the corresponding payoffs by U_A^{CS} and U_P^{CS}, where the superscript 'CS' stands for 'Crawford and Sobel.'

In this paper we interpret interval equilibria of the type described in the first proposition as a form of 'menu delegation,' as defined next.

DEFINITION 1 (Menu Delegation). *Under 'menu delegation' agents reveal the interval that the state of the world lies in and the project that the principal implements only depends on the reported interval. Formally,* $[0, 1]$ *is partitioned into* $N \geq 1$ *intervals with dividing points* $0 = a_0 < a_1 < ... < a_N = 1$. *The communication rule* $\mu(m \mid \theta)$ *is uniform, supported on* $[a_{i-1}, a_i]$ *if* $\theta \in (a_{i-1}, a_i)$. *The decision rule is given by* $y(m) = y_i$ *for all* $m \in (a_{i-1}, a_i]$ *and* $i = 1, ..., N$.

We can think of interval equilibria as menu delegation schemes since in any such equilibrium the principal essentially offers a menu with a discrete number of projects and the agents then choose between these different projects. In the static game the projects on the menu have to be chosen such that implementing any one of them maximizes the principal's stage game payoff, given her updated beliefs after receiving the agents' messages. In contrast, in a repeated setting the principal can commit to a menu in which the projects do not maximize her stage game payoff. As we will see below, committing to such a menu can be optimal since it allows her to elicit more information from the agents.

5 Relational Delegation

We now analyze the repeated game and characterize the optimal relational contract that maximizes the discounted payoff stream for the principal. We start by showing that, without loss of generality, we can restrict attention to '*stationary*' and '*monotonic*' relational contracts.

DEFINITION 2 (Stationarity and Monotonicity). *(i.) A relational contract is 'stationary' if (a.) on-the-equilibrium path $\mu_t(\cdot) = \overline{\mu}(\cdot)$ and $y_t(\cdot) = \overline{y}(\cdot)$ for every date t, where $\overline{\mu}(\cdot)$ is some communication rule and $\overline{y}(\cdot)$ is some decision rule and (b.) off the equilibrium path $\mu_t(\cdot) = \underline{\mu}(\cdot)$ and $y_t(\cdot) = \underline{y}(\cdot)$ for every date t, where $\underline{\mu}(\cdot)$ is some communication rule and $\underline{y}(\cdot)$ is some decision rule.*

(ii.) A relational contract is 'monotonic' if for any period t and for any two states θ_t^1 and $\theta_t^2 > \theta_t^1$ the chosen projects satisfy $y_t(\theta_t^2) \geq y_t(\theta_t^1)$.

Thus, in a monotonic relational contract the implemented projects are weakly increasing in the state of the world. We can now establish the following proposition.

PROPOSITION 2 (Stationarity and Monotonicity). *There always exists an optimal relational contract that is stationary and monotonic.*

It then follows that in the optimal relational contract a deviation by the principal in period t, i.e. the use of a decision rule $y_t(m_t) \neq \overline{y}(m_t)$, leads to the best static equilibrium $(\mu^{CS}(\cdot)\ , y^{CS}(\cdot),\ g^{CS}(\cdot))$ in every subsequent period. The optimal on-the-equilibrium path communication and decision rules are then given by

$$(\overline{\mu}(\cdot), \overline{y}(\cdot)) \in \arg \max_{y(m),\mu(\theta)} \frac{1}{1-\delta} \mathrm{E}_\theta \left[U_P(y(m), \theta)\right] \tag{1}$$

subject to

$$\mu(\theta) = \arg\max U_A(y(m), \theta) \tag{2}$$

$$\Delta y(m)^2 \leq \frac{\delta}{1-\delta} \mathrm{E}_\theta \left[U_P(y(m), \theta) - U_P^{CS}\right], \tag{3}$$

where $\Delta y(m)$ is the difference between the on-the-equilibrium path decision and the decision that maximizes the reneging payoff, i.e. $\Delta y(m) \equiv y(m) - \widehat{y}(m)$ and $\widehat{y}(m) \equiv \arg\max \mathrm{E}_\theta \left[U_P(y, \theta) \mid m\right]$.

The first constraint states that the communication rule maximizes the agent's stage game payoff given the decision rule $y(m)$ and the second constraint ensures that the principal has no incentive to renege. The RHS of the reneging constraint is the future loss from reneging, namely the appropriately discounted difference between the principal's expected on- and off-the-equilbrium path payoffs. The LHS is the expected one period benefit from reneging $\mathrm{E}_\theta\left[U_P(\widehat{y}(m),\theta) - U_P(y(m),\theta) \mid m\right]$ which, given the quadratic loss function, simplifies to $\Delta y(m)^2$.

To characterize the solution of the contracting problem, we first parameterize the reneging constraint by replacing (3) with

$$\Delta y(m)^2 \leq q^2, \tag{4}$$

where $q \in [0, \infty)$ is an exogenously given constant. We then solve the parameterized problem (1) subject to (2) and (4) for all q. The solution to this problem for a given q is equivalent to the solution of the original problem for the unique discount rate δ that solves

$$\frac{\delta}{1-\delta}\mathrm{E}_\theta\left[U_P(q) - U_P^{CS}\right] = q,$$

where $U_P(q)$ is the principal's stage game payoff under the optimal contract. Thus, solving the original problem for all $\delta \in [0, 1)$ is equivalent to solving the parameterized problem for all $q \in [0, \infty)$ as we do below.

The parameter q can be interpreted as the amount of 'relational capital' or 'commitment power' that the principal has. In the next subsection we characterize the solution to the contracting problem when the principal has a high level of relational capital, in the sense that $q \geq b$, where b is the agents' bias. In Subsection 5.2 we then characterize the solution when the principal has a low level of relational capital, in the sense that $q < b$. As will become clear as we proceed, $q = b$ is a natural cut-off level since the agents can be induced to reveal the true states of the world for some subset $\Theta' \subseteq \Theta$ if and only if $q \geq b$.

5.1 High Relational Capital

In this subsection we characterize the solution to the contracting problem (1) subject to (2) and (4) for $q \geq b$. We will show that in many cases commonly observed organizational arrangements are optimal. In particular, we will show that often the optimal relational contract takes the form of either '*centralization*' or '*threshold delegation*,' as defined next.

DEFINITION 3 (Centralization). *Under 'centralization' agents do not communicate any information and the principal implements the project that she expects to maximize her stage game payoff, given the limited information that she has. Formally, the communication rule* $\mu(\theta)$ *is uniform, supported on* $[0,1]$ *for all* $\theta \in [0,1]$*. The decision rule is given by* $y(m) = \widehat{y}(m) = \mathrm{E}[\theta]$ *for all* $m \in [0,1]$*.*

Under centralization the principal disregards the agents information and simply implements the project that she expects to maximize her stage game payoff, given her prior. The agents in turn do not communicate any information.

DEFINITION 4 (Threshold Delegation). *Under 'threshold delegation' agents reveal the state of the world up to a threshold and pool in a single interval above the threshold. The principal implements the agents' preferred project below the threshold and her own preferred project above the threshold. Formally, the communication rule is given by* $\mu(\theta) = \theta$ *for all* $\theta \in [0, a_1)$ *and* $\mu(\theta)$ *is uniform, supported on* $[a_1, 1]$ *for all* $\theta \in [a_1, 1]$, where $a_1 \in [0,1)$*. The decision rule is given by* $y(m) = m + b$ *for all* $m \in [0, a_1)$ *and* $\widehat{y}(m) = a_1 + b$ *for all* $m \in [a_1, 1]$*.*

A graphical illustration of threshold delegation is given in Figure 1. The lower diagonal line plots the principal's preferred project θ for any state of the world and the higher diagonal line $\theta + b$ plots the preferred projects for the agent. The bold line graphs the implemented projects as a function of the state of the world. Essentially, under threshold delegation the principal rubber-stamps the agents' recommendations up to some threshold and implements her preferred project above this threshold. Threshold delegation of this type is widely observed in organizations and, in particular, capital budgeting rules often take this form. Threshold delegation is also consistent with the observation in Ross (1986) that in many firms lower level managers can decide on small investments while senior managers can decide on larger investments.

The next proposition shows that in many cases threshold delegation is in fact the optimal relational contract.

PROPOSITION 3 (Threshold Delegation). *Suppose that* $q \geq b$ *and that* $G(\theta) \equiv F(\theta) + bf(\theta)$ *is strictly increasing in* θ *for all* $\theta \in \Theta$*. Then threshold delegation is optimal.*

The distributional assumption stated in the proposition is satisfied for a large number of distributions and a wide range of biases. It is, for instance, always satisfied by the uniform distribution and by any unimodal distribution as long as the variance is large relative to the bias. Also, for any distribution that is continuously differentiable there exists a strictly positive value b' such that the condition is satisfied for all $b \leq b'$. Thus, it is satisfied for most common distributions when the bias is small.

A sketch of the formal proof of the proposition can be provided in three steps. The first part of the proof shows that it cannot be optimal to have two pooling intervals next to each other. In other words, it can never be optimal for the principal to implement a project y_i if $\theta \in [a_{i-1}, a_i]$ and another project $y_{i+1} \neq y_i$ if $\theta \in (a_i, a_{i+1}]$. The second part of the proof shows that it cannot be optimal to have a pooling interval to the left of separation. In other words, it cannot be the case that the principal implements a project y_i if $\theta \in [a_{i-1}, a_i]$ and projects $y = \theta + b$ if $\theta \in [a_i, a_{i+1}]$. Together the first and the second part imply that the optimal delegation scheme is characterized by separation for low states of the world and a single pooling interval for high states. Finally, the first order condition for the optimal threshold implies that it is chosen so as to ensure that the principal's preferred project is implemented above the threshold.

To get an intuition for why among the very many possible organizational arrangements threshold delegation often does best for the principal, we first need to think about the trade-off that she faces when deciding what projects to implement. The key question for the principal is how much she should bias her decisions in favor of the agents. On the one hand, the principal clearly incurs a direct cost when she biases her decisions in favor of the agents by implementing projects that are larger than the ones she expects to maximize her payoff. On the other hand, however, the agents are more willing to give precise recommendations, the more they expect their interests to be taken into account by the principal. Thus, the key trade-off that the principal faces is between the direct cost of biased decision making and the indirect benefit of better information. A feature of threshold delegation is that, conditional on the information the principal receives, decision making is biased entirely in favor of the agents when the state of the world is below the threshold a_1 and it is biased entirely in favor of the principal when the state of the world is above the threshold. To see this, note that when the principal gets a message $m = \theta \leq a_1$ she knows exactly the state of the world but instead of using this information to implement her preferred project θ she uses it to implement the agent's preferred project $\theta + b$. In contrast, when the principal gets a message $m = \theta > a_1$ she does not know the exact state of the world and only knows that it is above the threshold. In this case it is optimal for her to implement the project $\mathrm{E}(\theta \mid \theta \geq a_1)$ that maximizes her expected payoff and not bias the decision at all in favor of the agents. As a result of this decision rule, agents are willing to communicate all information when the state of the world is below the threshold and very limited information when it is above the threshold.

To get an intuition for Proposition 2 it is therefore key to understand why it is optimal to bias the decisions in favor of the agents for low states of the world and in favor of the principal for high states of the world. For this purpose, it is instructive to compare threshold delegation to two benchmarks. In the first benchmark the principal always implements her preferred projects and in the second she always implements the agents' preferred projects.

When the principal always implements her preferred projects, the agents are not willing to reveal the states of the world and instead only reveal the intervals that they lie in. An example of such an equilibrium is illustrated in Figure 2a in which the agents only reveal whether the state of the world is below a threshold a_1 or above it and the principal implements her respective preferred projects $\widehat{y}_1 \equiv \mathrm{E}(\theta \mid \theta \leq a_1)$ and $\widehat{y}_2 \equiv \mathrm{E}(\theta \mid \theta \geq a_1)$. If $G(\theta)$ is increasing in θ, the principal can do better by implementing the agents' preferred project $y = \theta + b$ for $\theta \in [0, \widehat{y}_2 - b]$ and $\widehat{y}_2$ for $\theta \in [\widehat{y}_2 - b, 1]$, i.e. she can do better by entirely biasing her decisions in favor of the agents for low states of the world. On the one hand, doing so is costly for the principal since she implements projects that are worse for her if $\theta \in [0, a_1]$. This loss is indicated by triangle A in Figure 2b. On the other hand, however, precisely because she is implementing projects that are worse for her if $\theta \in [0, a_1]$ she is able to implement projects that are better for her if $\theta \in [0, a_1]$. This gain is indicated in Figure 2b by triangle B. Essentially, biasing decisions in favor of the agents for low states of the world relaxes the incentive constraint for higher states which in turn allows the principal to implement projects that are better for her. As long as the probability of being in the loss making interval $[0, a_1]$ is not too large compared to the probability of being in profiting interval $[a_1, \widehat{y}_2 - b]$, the gain of biasing the decisions in favor of the agents outweighs the costs and the principal is made better off. The condition that $G(\theta)$ is always increasing ensures that this is indeed the case.

In the second benchmark, the principal biases her decisions entirely in favor of the agents who in turn always reveal the state of the world. This case is illustrated in Figure 3a. While this arrangement allows the principal to elicit all available information, it also commits her to implement projects $y > 1$ that cannot be optimal for her in any state of the world. This suggests an alternative arrangement in which the principal implements the agents' preferred projects below a threshold $a_1 \leq 1$ and implements a single project $a_1 + b$ above the threshold, as illustrated in Figure 3b. If a_1 is sufficiently high the principal is made better off under the alternative scheme since she can realize the benefit of less biased decision making, indicated by triangle A in Figure 3b, without the cost of tightening the incentive constraint for any higher states of the world.

A key questions we are interested in is what form delegation takes when a principal's ability to commit is limited. From our analysis above it follows that the optimal threshold delegation scheme can be implemented for any $q \geq b$ and not just as $q \to \infty$. This is the case since, under threshold delegation, the principal never biases her decision by more than b and thus never faces a reneging temptation of more than b^2. Thus, when $G(\theta)$ is everywhere increasing, a principal with high relational capital $q' \geq b$ behaves in exactly the same way as a principal with very high relational capital $q'' > q'$.

Proposition 3 has shown that in many cases threshold delegation is optimal. In the next proposition we show that when the conditions of that proposition are not satisfied, it is often optimal for the principal to centralize, that is to implement the project $y = \mathrm{E}(\theta)$ that she expects to maximize her payoff, given her prior.

PROPOSITION 4 (Centralization). *Suppose that $q \geq b$ and that $G(\theta) \equiv F(\theta) + bf(\theta)$ is strictly decreasing in θ for all $\theta \in \Theta$. Then centralization is optimal.*

A necessary condition for $G(\theta)$ to be decreasing for all $\theta \in \Theta$ is that $f(\theta)$ is everywhere decreasing. In this sense, the condition is satisfied if bad states of the world are more likely than better states of the world. As we will show in an example below, this condition is satisfied, for instance, for exponential distributions with sufficiently low means.

The formal proof of this proposition has two key parts. The first shows that separation can never be optimal, that is it can never be optimal to induce the agents to reveal the true state of the world. For the intuition consider Figure 4a which illustrates an equilibrium in which the principal implements a project y_1 if θ is below a threshold a_1, project y_2 if θ is above another threshold a_2 and the agents' preferred project $y = \theta + b$ if θ is between a_1 and a_2. If $G(\theta)$ is decreasing in θ, the principal can do better by only implementing projects y_1 and y_2 as illustrated in Figure 4b. On the one hand, doing so makes the principal worse off if $\theta \in [\frac{1}{2}(a_1 + a_2), a_2]$. This loss is indicated by triangle A in the figure. On the other hand, however, it makes her better off if $\theta \in [a_1, \frac{1}{2}(a_1 + a_2)]$, as indicated by triangle B in the figure. As long as the probability of being in the loss making interval $[\frac{1}{2}(a_1 + a_2), a_2]$ is not too large compared to the probability of being in profiting interval $[a_1, \frac{1}{2}(a_1 + a_2)]$, the gain of biasing the decisions in favor of the agents outweighs the costs and the principal is made better off. The condition that $G(\theta)$ is always decreasing ensures that this is indeed the case. The first part of the proof therefore establishes that under the stated conditions only a discrete number of projects get implemented. The second part of the proof shows that if $G(\theta)$ is always decreasing, centralization dominates any menu delegation scheme that offers two or more projects.

The proposition shows that in the absence of sophisticated monetary incentive schemes, it is often optimal for a principal to forgo the information that her agents possess and to simply impose a decision on the firm. Essentially, when the principal is limited to delegation schemes, the cost of extracting information from the agents can easily be so high that the principal is better off taking an ignorant but unbiased decision than to try to bias decisions in favor of her subordinates to elicit more information. Business history and newspapers are abound with descriptions of monolithic firms in which bureaucratic rules and regulations stifle the creativity and flexibility of their employees.[8] The proposition suggests that such bureaucracy may simply be a symptom of the firms' optimal responses to the agency problems they face.

We have seen above that when $G(\theta)$ is everywhere increasing, a principal with limited ability to commit implements the same delegation scheme as a principal with unlimited commitment power. The same is true when $G(\theta)$ is everywhere decreasing. This is so since the principal is always able to implement centralization, independent of the amount of relational capital q that she possesses.

From the two previous propositions it is clear that the key condition that determines the optimal relational contract when relational capital is high is whether $G(\theta)$ is increasing or decreasing. To get a better sense for this condition and its implications we next consider an example. In particular, suppose that θ is drawn from a truncated exponential distribution with cumulative density

$$F(\theta) = \frac{1}{1 - e^{-1/\beta}} \left(1 - e^{-\theta/\beta}\right),$$

where $\beta > 0$ is the scale parameter. An increase in β is a first order stochastic increase of the distribution and thus increases the mean $\mathrm{E}(\theta)$. Moreover, as $\beta \to \infty$, the distribution approaches the uniform distribution. It can be verified that for this exponential distribution $G(\theta)$ is everywhere increasing if $b \leq \beta$ and it is everywhere decreasing otherwise. Thus, if the bias is smaller than the scale parameter threshold delegation is optimal and if the bias is larger than the scale parameter centralization is optimal, as illustrated in Figure 5. To get some sense for the comparative statics, which we analyze more generally in Section 6, take a point above the diagonal in Figure 5 and consider the effect of an increase in the bias. Initially, such an increase leads to a reduction of threshold below which the principal rubber-stamps the agents' recommen-

[8] For a colorful historical example see the case of The Hudson Bay Company in Milgrom and Roberts (1992, pp. 6-9).

dations. Eventually, $b > \beta$ and the principal centralizes, i.e. she simply implements $\mathrm{E}(\theta)$. At this point further increases in the bias do not affect the optimal relational contract or the decision that is taken. Similarly, take a point below the diagonal in Figure 5 and consider the effect of an increase in β. Such an increase moves probability mass from low- to high states of the world, making it less and less costly for the principal to implement the agents' preferred projects when their recommendations are small. When β is sufficiently high, i.e. when $\beta \geq b$, it then becomes optimal for the principal to switch to threshold delegation and implement the agents' preferred projects for low states of the world. Further increases in β then simply increase the threshold up to the maximum value of $a_1 = 1 - 2b$.

While for any exponential- and many other distributions, $G(\theta)$ is either everywhere increasing or decreasing, this is, of course, not always the case. For instance, for normal distributions with a sufficiently small variance, $G(\theta)$ is first increasing and then decreasing. For such distributions we can use the same proof strategy as described above by dividing the support of this distributions into intervals in which $G(\theta)$ is monotonic. For an analysis of such distributions in the full commitment limit see Alonso and Matouschek (2004).

5.2 Low Relational Capital

In this subsection we characterize the solution to the contracting problem (1) subject to (2) and (4) for $q < b$. The key difference between the high- and the low relational capital cases is that in the former the principal can credibly commit to decision rules that induce the agents to reveal the true state of world for some $\Theta' \subseteq \Theta$ while in the latter this is not possible. In other words, separation can be supported when $q \geq b$ but it cannot be supported when $q < b$. Together with the fact that optimal contracts are monotonic, as established in Proposition 2, this implies that when relational capital is low, the optimal contract takes the form of menu delegation, as defined in Definition 1 above. We make this point formally in the next proposition.

PROPOSITION 5 (Menu Delegation). *Suppose that $q < b$. Then menu delegation is optimal.*

Thus, when relational capital is low, the principal cannot do better than to let the agents choose between a discrete number of projects. We believe that menu delegation is a widespread organizational arrangement, just like centralization and threshold delegation. Consider for instance a business school that tries to hire junior and/or senior faculty members. In most cases the Dean will delegate this decision to the relevant department. When she does so, however, she may well restrict the department members to making either two junior or one senior offer and she will not allow them to make three junior offers if the junior search is more successful than the senior search. In other words, she may not allow the department members to fine-tune their decision to the job market conditions. Note that such fine-tuning would be possible if the Dean instead engaged in threshold delegation, that is if she allowed the department members to make any combination of offers that together do not cost more than some threshold amount. The above proposition shows that allowing for such fine-tuning is not optimal for the Dean if her relational capital is below some threshold.

Having established that for $q < b$ threshold delegation is optimal, the only remaining question is what projects the principal should put on the menu. To address this question it is useful to restate the original contracting problem (1) subject to (2) and (4) as

$$\max_{N,y_1,...,y_N} \mathrm{E}_\theta\left[U_P\right] = \frac{-1}{1-\delta}\sum_{i=1}^{N}\int_{a_{i-1}}^{a_i}(y_i-\theta)^2\mathrm{d}F(\theta) \tag{5}$$

subject to $a_0 = 0$, $a_N = 1$,

$$a_i = \frac{1}{2}\left(y_i + y_{i+1} - 2b\right) \text{ for } i = 1, ..., N-1 \tag{6}$$

and

$$\Delta y_i^2 \leq q^2 \text{ for } i = 1, ..., N, \tag{7}$$

where $\Delta y_i \equiv y_i - \widehat{y}_i$ is the difference between the project y_i that the principal is committed to implement when the state of the world is reported to lie in interval i and project $\widehat{y}_i$, the project that maximizes her expected stage game payoff in this case. In this formulation the incentive constraint (6) is derived from the indifference conditions $U_A(y_i, a_i) = U_A(y_{i+1}, a_i)$ for $i = 1, ..., N-1$ which ensure that in states of the world $a_1, ..., a_{N-1}$ the agents are indifferent between projects y_i and y_{i+1}.

Just as in the case with high relational capital, the key trade-off that the principal faces is between the extent to which decision making is biased in favor of the agents, given her information, and the amount of information that is communicated by the agents. To see this note that the incentive constraints (6) imply that

$$(a_{i+1} - a_i) = (a_i - a_{i-1}) + (4b - 2\Delta y_{i+1} - 2\Delta y_i) \,. \tag{8}$$

The lengths of the intervals therefore increase by $4b - 2\Delta y_{i+1} - 2\Delta y_i > 0$ as i increases. Thus, just as in the static equilibrium, less information gets communicated by the agents, the larger their recommendation. The above expression, however, shows that in a repeated setting the principal can reduce the loss of information by committing to bias her decisions in favor of the agents, i.e. by setting $\Delta y_i > 0$ for $i = 1, ..., N-1$. Intuitively, agents are more willing to communicate information if the principal is committed to take the agents' interests into account when making a decision. It is because of the improved communication that the principal may be willing to incur the direct cost of biasing her decisions in favor of the agents.

The solution to the above contracting problem again depends crucially on the distribution of θ and the bias b. The next proposition shows that when $G(\theta) \equiv F(\theta)+bf(\theta)$ is decreasing in θ then, just as in the high relational capital case, centralization is optimal. In other words, under this condition, the principal only puts one project on the menu from which the agents can 'choose.'

PROPOSITION 6 (Centralization with Low Relational Capital). *Suppose that $q < b$ and that $G(\theta) \equiv F(\theta)+bf(\theta)$ is decreasing for all $\theta \in \Theta$. Then centralization is optimal.*

This proposition follows immediately from Proposition 4 since, under centralization, the temptation to renege is equal to zero and can therefore be implemented for any level of relational capital q. Together Propositions 4 and 6 imply that when $G(\theta)$ is decreasing for all $\theta \in \Theta$, centralization is *always* optimal, independent of the amount of relational capital that the principal possesses. Put differently, when $G(\theta)$ is everywhere decreasing, commitment power does not matter at all, the principal behaves the same whether she has no relational capital, an infinite amount of it, or anything in between.

When $G(\theta)$ is not everywhere decreasing, the optimal menu delegation scheme does depend on the amount of relational capital q. To get a better understanding of how changes in q affect the optimal menu delegation scheme in this case, the next proposition provides a characterization for an example in which θ is uniformly distributed on $[0, 1]$.

PROPOSITION 7 (Uniform Example). *Suppose that $q < b$ and that $F(\theta) = \theta$. Then there exists a $\overline{q} \in (0, b)$* such that

i. for all $q \leq \overline{q}$, $\Delta y_i = q$ for all i and the number of intervals N is maximized.

ii. for all $q > \overline{q}$, $\Delta y_1 \leq q$, $\Delta y_i = q$ for $i = 2, ..., N-1$, and $\Delta y_N \leq q$.

Thus, when the principal has very little relational capital, i.e. when $q \leq \overline{q}$, her desire for better information is so large that the benefit of biasing decisions dominates the costs. As a result, it is optimal for her to bias her decisions up to the maximum credible level. Note that in this case the number of intervals is maximized and that intervals grow by $4(b - q)$, as can been from (8). Thus, the amount of information that is being communicated is exactly the same as the one that would be communicated in the best Crawford and Sobel equilibrium of the static game when the agent has a bias of $(b - q)$. In terms of information transmission, therefore, relational capital is a *perfect substitute* for a reduction in the agents' bias.

When the amount of relational capital grows beyond the threshold $\overline{q}$, it is still the case that the principal wants to extract more information by biasing all intermediate decisions $y_2, ...y_{N-1}$ as much as possible. However, it can now be optimal to reduce Δy_1 and Δy_N so as to economize on the cost of biased decision making. In fact, we know from Proposition 3 that as $q = b$, the bias of the last and largest interval is optimally set to zero. Thus, although the principal could extract as much information as in a static game with bias $(b - q)$, it is not always optimal for her to do so when $q \geq b/4$.

In summary, the analysis thus far has shown that commonly observed organizational arrangements are often optimal in our model. Moreover, we have seen that exactly what arrangement is optimal depends crucially on two factors, namely the amount of relational capital and the interplay between the bias and the distribution of the state of the world, as summarized in the simple condition $G(\theta) = F(\theta) + bf(\theta)$. In particular, Table 1, which summarizes some of the key results that we derived so far, shows that when $G(\theta)$ is always increasing, threshold delegation is optimal when the amount of relational capital is high and menu delegation is optimal when the amount of relational capital is low while centralization is always optimal when $G(\theta)$ is decreasing. Also, we have seen that in many cases changes in the amount of relational capital do not affect the optimal delegation scheme. In particular, when either $G(\theta)$ is decreasing or $G(\theta)$ is increasing and relational capital is high, increases in q have no effect on the optimal delegation scheme. Only when relational capital is small and $G(\theta)$ is not everywhere decreasing can changes in q lead to changes in the optimal delegation scheme.

6 Implications

In this section we explore further implications of our analysis of optimal relational delegation schemes.

Relational Delegation for Small Biases In the previous section we have seen that in many cases three commonly observed organizational arrangements are optimal. It turns out that for small biases the set of potentially optimal arrangements is even smaller. Specifically, as the next proposition shows, threshold delegation is optimal for almost all common distributions when the bias is sufficiently small.

PROPOSITION 8 (Threshold Delegation for Small Biases). *Suppose that $f(\theta)$ is twice continuously differentiable. Then threshold delegation is optimal for sufficiently small biases.*

Recall that when $f(\theta)$ is continuously differentiable, $G(\theta)$ is increasing for a sufficiently small bias b. To prove Proposition 8 we therefore only need to show that threshold delegation can be credibly implemented when b is small enough. To see that this is indeed the case, consider the reneging constraint $b^2 \leq \delta/(1-\delta)\mathrm{E}_\theta\left[U_P^{TD} - U_P^{CS}\right]$, where the LHS is the maximum reneging temptation under threshold delegation, the RHS is the punishment for reneging and U_P^{TD} is the principal's stage game payoff under the optimal threshold delegation scheme. Note that a reduction in the bias increases the payoff U_P^{CS} the principal can realize in the absence of a relational contract. Thus, a reduction in the bias not only reduces the benefit of reneging – the LHS of the inequality – but also, potentially, the punishment of doing so – the RHS. It is therefore not immediate that a reduction in the bias makes the reneging constraint less binding. In the formal proof we show, however, that when $f(\theta)$ is twice continuously differentiable, then, as b goes to zero, the benefit of reneging goes to zero faster than the punishment. Thus, for sufficiently small b the reneging constraint is satisfied and threshold delegation can be credibly implemented.

The Effects of Changes in the Bias and the Amount of Private Information
Since threshold delegation and centralization play such prominent roles in our model we next investigate how they are affected by changes in the economic environment.

Suppose first that threshold delegation is optimal and consider the maximization problem that determines the optimal threshold a_1 below which the principal implements the agents' preferred project and above which she implements her own preferred project:

$$\max_{a_1} \frac{1}{1-\delta} \mathrm{E}_\theta \left[U_P\right] = -\frac{1}{1-\delta}\left(\int_0^{a_1} b^2 \mathrm{d}F(\theta) + \int_{a_1}^{1} (a_1 + b - \theta)^2 \mathrm{d}F(\theta)\right).$$

The optimal threshold level then solves the necessary first order condition

$$(\mathrm{E}(\theta \mid \theta \geq a_1) - (a_1 + b)) \quad \begin{cases} \leq 0 & \text{if } a_1 = 0 \\ = 0 & \text{if } a_1 > 0. \end{cases}$$

Comparative statics can now be easily performed using the graphical representation of the first order condition in Figure 6.

For instance, suppose that threshold delegation is optimal for a given b and consider the effect of a reduction in the bias. Note that if $G(\theta)$ is increasing for a given b then it is also increasing for any $b' < b$; thus threshold delegation remains optimal after the reduction in the bias. It can be seen in Figure 6 that a reduction in b shifts down $(a_1 + b)$ but does not affect $\mathrm{E}(\theta \mid \theta \geq a_1)$. Thus, a reduction in the bias increases the optimal threshold, i.e. it leads to more delegation. This result is in line with Jensen and Meckling (1992) who argue that a reduction in agency costs should generally lead to more delegation.

Suppose next that threshold delegation is optimal for a given distribution and consider the effect of an increase in the amount of private information, as formalized by a mean preserving spread of the distribution. At first glance one may think that such a change makes the agents' information 'more important' and should thus lead to more delegation. In our model, however, there are two reasons why this is not necessarily the case. First, a mean preserving spread can affect the sign of $G(\theta)$. Thus, it is quite possible that after an increase in private information threshold delegation is no longer optimal. Second, even if $G(\theta)$ is still increasing after the mean preserving spread, it has an ambiguous effect on the optimal threshold a_1. To see this, consider Figure 6 and note that while a mean preserving spread does not affect $(a_1 + b)$, it has an ambiguous effect on conditional mean $\mathrm{E}(\theta \mid \theta \geq a_1)$. Thus, in our model, a change in the amount of private information can lead to more or less delegation, depending on the exact parameter values and distributional assumptions.

Finally, consider the effects of changes in the economic environment on centralization. Suppose that centralization is optimal for a given bias b' and consider the effect an increase in the bias to $b'' > b'$. If $G(\theta)$ is everywhere decreasing for b' then it is also everywhere decreasing for $b'' > b'$. Thus, after the increase in the bias centralization is still optimal. Moreover, since an increase in the bias does not affect $\mathrm{E}(\theta)$, the principal implements the same decision after the increase in b as she did before the increase. While the effect of an increase in the bias on centralization is unambiguous, the effect of an increase in the amount of private information is less clear-cut. This is again the case since a mean preserving spread can change the sign of $G(\theta)$ so that centralization may no longer be optimal after the increase in private information. If it does not change the sign of $G(\theta)$ an increase in the amount of private information does not affect the optimal delegation scheme and the decision that is implemented by the principal remains $\mathrm{E}(\theta)$.

Threshold Delegation and Investment Inefficiencies Whenever the principal chooses threshold delegation, she optimally induces overinvestment in low states of the world and underinvestment in high states.

COROLLARY 1 (Investment Inefficiencies). *Under the conditions in Proposition 3, it is optimal for the principal to induce underinvestment if* $\theta \geq \mathrm{E}(\theta \mid \theta \geq a_1)$ *and to induce overinvestment otherwise.*

To see this, consider Figure 1 which gives an example of a threshold delegation scheme. From the principal's perspective, the efficient investment level in state θ is simply θ. In Figure 1, however, it can be seen that this efficient investment level is almost never achieved. Instead, it is optimal for the principal to induce investments that are larger than θ when the states of the world are low, i.e. below $\mathrm{E}(\theta \mid \theta \geq a_1)$, and to induce investments that are smaller than θ when states of the world are high. In other words, given the informational asymmetry, the principal cannot do better than to allow the agents to spend too much on small projects and too little on large projects.

Why Complete Delegation Is Never Optimal An organizational arrangement that has received a lot of attention in the literature (see in particular Dessein (2002)) and is notably absent from our discussion up to this point is '*complete delegation*,' as defined next.

DEFINITION 5 (Complete Delegation). *Under 'complete delegation' agents always reveal the state of the world and the principal always implements the agents' preferred project. Formally, the communication rule is given by $\mu(\theta) = \theta$ for all $\theta \in [0, 1]$ and the decision rule is given by $y(m) = m + b$ for all $m \in [0, 1]$.*

Thus, under complete delegation the principal always rubber-stamps the agents' proposals and the agents, in turn, always recommend their preferred projects. The next proposition establishes that complete delegation is never optimal in our model.

PROPOSITION 9 (Complete Delegation). *Complete delegation is never optimal.*

To see this suppose that the principal does engage in complete delegation, as illustrated in Figure 3a. Note that to do so she must be able to resist a maximum reneging temptation of b^2. When she has enough relational capital to implement complete delegation, however, she also has enough relational capital to implement an alternative scheme in which she rubber-stamps the agents' proposals when they are small and implements a threshold project when they are large. As can be seen from Figure 3b such a scheme increases the principal's expected payoff but does not increase the maximum reneging temptation, which remains to be b^2. Thus, whenever complete delegation is feasible, it is not payoff maximizing for the principal.

This proposition is in contrast to the key result in Dessein (2002). He considers a game that is very similar to our stage game and compares complete delegation to 'communication,' i.e. the best equilibrium without any commitment. The important result in his paper is that in many cases the principal is better off committing to complete delegation than to rely on communication. Our analysis shows that although complete delegation often dominates communication it is itself always dominated by other delegation schemes.

Outsourcing So far we have ruled out the possibility of outsourcing, by which we mean the transfer of formal authority to the agents. In some contexts, however, it may be possible for the principal to outsource. The next proposition shows that outsourcing can only be optimal if the principal's relational capital is sufficiently small.

PROPOSITION 10 (Outsourcing). *There exists a critical level of relational capital $q' < b$ such that outsourcing does better than relational delegation only if $q < q'$.*

When the principal outsources, the agents always choose their preferred project. Thus, outsourcing implements the same decision rule as complete delegation. In contrast to complete delegation, however, it does not require any commitment power by the principal. We have seen above that when relational capital is high, the principal can implement complete delegation but does not find it optimal to do so. It is then immediate that outsourcing cannot be optimal for a principal with high relational capital. However, a principal with low relational capital cannot implement complete delegation and may find it optimal to outsource since doing so allows her to credibly commit to having the agents' preferred projects being implemented.

Empirical Implications Our model is quite stylized and abstracts from many factors that are likely to influence delegation schemes in the real world. Nevertheless, the model does offer a number of predictions that differentiate it from other models of delegation and that are, in principle, testable. Below we provide a short list of our main empirical implications, focusing on two of the three choice variables in our model – the type of delegation schemes that is being used and the projects that get implemented. We do not discuss the third choice variable – the information that is transmitted – since in most contexts in cannot be observed. In discussing empirical implications we concentrate on capital budgeting rules since this is an area in which both delegation schemes and project choices are often observable. The main empirical implications are the following:

- A variety of delegation schemes such as, for instance, centralization and threshold delegation should be observed. Complete delegation, however, should never be observed.[9]

- A larger variety of delegation schemes should be observed in firms in which agency problems are severe than in firms in which they are limited. In the latter case threshold delegation should be pervasive.

- Firms that operate in booming markets, i.e. in markets in which good states of the world are more likely than bad states, should engage in more threshold delegation and less centralized decision making than firms that operate in depressed markets, i.e. in markets in which bad states of the world are more likely than good states.

- The size distribution of investments should by 'lumpier' in firms with severe agency problems than in firms in which they are limited. In other words, investments of many different sizes should be observed in firms in which agency problems are small whereas investments of a limited number of different sizes should be observed in firms in which agency problems are large.

- The size distribution of investments should by lumpier in firms in which the top management has a low discount factor – for instance because the firm operates in an industry with high exit rates or is characterized by high labor turnover of top managers – than in firms in which top management in which a high discount factor.

[9] This is in contrast to Dessein (2002). His analysis suggests that complete delegation should be observed often.

- The size distribution of investments should be lumpier in firms that operate in booming markets than in firms that operate in depressed markets.

All these implications follow immediately from the results derived above, we therefore do not provide proofs for them.

7 Extensions

So far we have not allowed for wage payments and for any commitment power by the agents. In this section we discuss the implications of relaxing these two assumptions.

Wages Suppose that at the beginning of each period the principal makes the agent a take-it-or-leave-it wage offer w and that the agent's outside option is x. It is immediate that the principal will always offer a wage such that the agent is indifferent between accepting the offer and realizing x. The optimal on-the-equilibrium path communication and decision rules are then given by

$$(\overline{\mu}(\cdot), \overline{y}(\cdot)) \in \arg \max_{y(m), \mu(\theta)} \frac{1}{1-\delta} \mathrm{E}_\theta \left[U_P(y(m), \theta) + U_A(y(m), \theta) - x\right]$$

subject to

$$\mu(\theta) = \arg\max U_A(y(m), \theta)$$

$$\Delta y(m)^2 \leq \frac{\delta}{1-\delta} \mathrm{E}_\theta \left[U_P(y(m), \theta) - \left(U_P^{CS} + U_A^{CS} - x\right)\right].$$

In contrast to the contracting problem analyzed above, the principal now aims to maximize the net joint surplus $\mathrm{E}_\theta \left[U_P(y(m), \theta) + U_A(y(m), \theta) - x\right]$ rather than her stage game payoff $\mathrm{E}_\theta \left[U_P(y(m), \theta)\right]$. Using the same type of arguments that we made above, it can be shown that all of our results continue as long as one replaces $G(\theta) \equiv F(\theta) + bf(\theta)$ with $\widehat{G}(\theta) \equiv 2F(\theta) + bf(\theta)$. Thus, threshold delegation is more likely to be optimal once one allows for wages, in the sense that the set of distributions for which threshold delegation is optimal if one allows for wages contains the set of distributions for which threshold delegation is optimal if one does not allow for wages. In the same sense centralization is less likely once one allows for wages. Thus, the nature of our results is not sensitive to the possibility of wage payments.

Agent Commitment In our model the principal has some commitment power but the agents do not. While we believe that these are the right assumptions to make when one analyzes delegation, there are clearly other contexts in which one wants to allow for commitment power by the agents and not the principals. Consider, for instance, the relationship between the British civil service and Her Majesty's governments. In contrast to the US system, most British civil servants are not replaced when a new government comes to power. One can therefore think of British civil servants as long run agents who advice a series of short run and opportunistic governments that have the formal authority to take decisions on behalf of the state.

We leave the full analysis of a version of model in which a single, long lived agent faces a sequence of short lived principals for future work. Here we just make a number of informal observations about this case. It is straightforward to show that if the long run agent is patient enough, it is optimal for him to exactly reveal the state of the world and let the receivers take their preferred action. Thus, a patient enough agent does not have the incentive to manipulate the information that he transmits so as to influence the principals' actions. Essentially, in equilibrium the loss of information that must arise from doing so always outweighs any gain in control. Note that this contrasts with our model in which complete revelation of information is never optimal. Note also that this implies that as long as the agent is patient enough, the principals achieve their most preferred outcome: they learn all the information and can implement their most preferred project. Moreover, as the bias becomes very small, complete truth revelation is optimal for any strictly positive discount rate. Applied to the question of whether governments should rely on their own advisors or on life time civil servants, this suggests a trade-off between the potentially smaller bias of the own advisors and the stronger reputational concerns of life time civil servants. Even if the government and life time civil servants have very different policy preferences, the government can only gain by bringing in their own advisors if the preferences of these advisors are exactly the same as those of the government.

8 Conclusion

In this paper we investigated the allocation of decision rights within firms. In particular, we analyzed a principal-agent problem in which an uninformed principal can elicit information from an informed agent by implicitly committing herself to act on the information she receives in a particular manner. We derived the optimal relational delegation schemes and showed that they often resemble organizational arrangements that are observed in practice. Specifically, we showed that centralization, threshold delegation and menu delegation are often optimal. Which one of these organizational arrangements is optimal depends only on the principal's commitment power, on the one hand, and a simple condition on the agents' bias and the distribution of the state space, on the other. Moreover, we showed that for small biases threshold delegation is optimal for almost all common distributions. Finally, we showed that complete delegation is never optimal and that outsourcing can only be optimal if the principal is sufficiently impatient.

In ongoing work we use the techniques developed in this paper to address further issues related to delegation. First, in Alonso and Matouschek (2004) we consider a static model and characterize the optimal delegation sets for general distributions and generalized quadratic preferences with arbitrary state-contingent biases. Second, in Alonso and Matouschek (2004b) we consider a model with endogenous information acquisition and investigate how a principal should delegate decision rights if she does not only wants to elicit information from agents but also motivate them to acquire information in the first place.

9 Appendix

This appendix contains the proofs of all Propositions in the paper.

9.1 Proof of Propositions 1 and 2

Proof of Proposition 1: Follows from Theorem 1 in Crawford and Sobel (1982). ■

The proof of Proposition 2 will follow from the following two lemmas. The first lemma establishes the existence of an optimal stationary relational contract for any discount factor $\delta \in [0, 1)$.

LEMMA 1 *For any optimal relational contract there is a stationary optimal relational contract.*

Proof: Consider an optimal relational contract $(H_t, \mu_t(\cdot), y_t(\cdot))$, $t \in \{0, 1, ...\}$ and let $V_R(h_t) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \mathrm{E}_{\theta_\tau} [U_P(y_\tau(\mu_\tau(\theta_\tau)), \theta_\tau)]$ be the receiver's expected discounted utility at time t after history h_t with $V_R(h_0) = v$. Now let $\Psi = \{\theta_0 : V_R(\{\theta_0, \mu_0(\theta_0), y_0(\mu_0(\theta_0))\}) < v\}$ be the set of states in the first period which generate continuation utilities less than v. If $\Pr[\theta_0 \in \Psi] > 0$ we can construct a new relational contract that after the first period history $\{\theta_0, \mu_0(\theta_0), y_0(\mu_0(\theta_0))\}, \theta_0 \in \Psi$ calls for play of the original relational contract. To see that this new contract is subgame perfect note that since histories are common knowledge the first period choice $y_0(\mu_0(\theta_0)), \theta_0 \notin \Psi$ remains optimal for the receiver and she obtains a higher continuation utility by playing $y_0(\mu_0(\theta_0))$ $\theta_0 \in \Psi$. Finally, since $V_R(h_0) = \delta \mathrm{E}_{\theta_0} [U_P(y_0(\mu_0(\theta_0)), \theta_0)] + (1-\delta)\mathrm{E}_{\theta_0} [V_R(h_1))] = v$ it follows that $\mathrm{E}_{\theta_0} [U_P(y_0(\mu_0(\theta_0)), \theta_0)] = v$.

Now consider a stationary contract $(H_t, \mu_t'(\cdot), y_t'(\cdot))$, $t \in \{0, 1, ...\}$ where along the equilibrium path $\mu_t'(\cdot) = \mu_0(\cdot), y_t'(\cdot) = y_0(\cdot)$. Since on and off the equilibrium path the principal obtains the same continuation utility this new contract is subgame perfect. ■

The next lemma establishes monotonicity of the optimal relational contract.

LEMMA 2 (Melumad and Shibano 1991) *An incentive compatible $y(\mu(\theta))$ must satisfy the following: (i.) $y(\mu(\theta))$ is weakly increasing, (ii.) If $y(\mu(\theta))$ is strictly increasing and continuous in (θ_1, θ_2), then $y(\mu(\theta)) = \theta + b$ on (θ_1, θ_2), (iii.) if $y(\mu(\theta))$*

is discontinuous at $\widehat{\theta}$, then the discontinuity must be a jump discontinuity that satisfies: (a.) $\left(y^-(\mu\left(\widehat{\theta}\right)) + y^+(\mu\left(\widehat{\theta}\right))\right)/2 = \widehat{\theta} + b$, *(b.)* $y(\theta) = y^-(\widehat{\theta})$ *for* $\theta \in [\max\left\{0, y^-(\mu\left(\widehat{\theta}\right)) - b\right\}, \widehat{\theta})$ *and* $y(\theta) = y^+(\widehat{\theta})$ *for* $\theta \in (\widehat{\theta}, \min\left\{1, y^+((\mu\left(\widehat{\theta}\right)) - b\right\}]$ *and (c.)* $y((\mu\left(\widehat{\theta}\right))) \in \left\{y^-((\mu\left(\widehat{\theta}\right))), y^+((\mu\left(\widehat{\theta}\right)))\right\}$.

This lemma corresponds to Proposition 1 in Melumad and Shibano (1991) and we refer to their proof.

Proof of Proposition 2: Follows immediately from Lemmas 1 and 2. ■

9.2 Proofs of Propositions 3 and 4

For the proofs of Propositions 3 and 4 it is useful to introduce some new notation. In particular, let $\widetilde{F}(\theta) \equiv 1 - F(\theta)$, $S(\theta) \equiv \widetilde{F}(\theta)\left[(\theta + b) - \mathrm{E}\left[s\,|s \geq \theta\right]\right]$ and $T(\theta) \equiv F(\theta)\left[(\theta + b) - \mathrm{E}\left[s\,|s \leq \theta\right]\right]$. It is also useful to introduce two lemmas. To do so, let

$$\Delta(p,t) \equiv -\int_{p-b-t}^{p-b+t} b^2 \mathrm{d}F(\theta) - \left(-\int_{p-b-t}^{p-b} [p-t-\theta]^2 \mathrm{d}F(\theta) - \int_{p-b}^{p-b+t} [p+t-\theta]^2 \mathrm{d}F(\theta)\right),$$

where $(p-b-t)$ and $(p-b+t)$ belong to $[0,1]$ and $t \geq 0$. To understand the economic meaning of this function, suppose that there are two decision rules which only differ in the projects that are induced for $\theta \in [p-b-t, p+t-b]$. In particular, the first decision rule induces $y = \theta + b$ for $\theta \in [p-b-t, p+t-b]$ while the second decision rule implements $y = p - t$ for all $\theta \in [p-b-t, p-b)$ and $y = p + t$ for all $\theta \in [p-b, p+t-b]$. The function $\Delta(p,t)$ captures the difference in the principal's expected stage game payoff from these decision rules.

LEMMA 3. *Suppose that $G(\theta)$ is strictly monotone in $[\underline{\theta}, \overline{\theta}] \subset [0,1]$. If $G(\theta)$ is strictly increasing in $[\underline{\theta}, \overline{\theta}]$ then $\Delta(p,t) > 0$. If $G(\theta)$ is strictly decreasing in $[\underline{\theta}, \overline{\theta}]$ then $\Delta(p,t) <$* 0, *with* $p = \left(\underline{\theta} + \overline{\theta}\right)/2 + b$ *and* $t = \left(\underline{\theta} - \overline{\theta}\right)/2$.

Proof: We first note that,

$$\frac{\partial \Delta(p,t)}{\partial t} = \int_{p-b-t}^{p-b} 2\left[-p+\theta\right] \mathrm{d}F(\theta) + \int_{p-b}^{p-b+t} 2\left[p-\theta\right] \mathrm{d}F(\theta) + 2t[F(p-b+t) - F(p-b-t)]$$

and

$$\frac{\partial^2 \Delta(p,t)}{\partial t^2} = 2\left[F(p-b+t) + bf(p-b+t) - F(p-b-t) - bf(p-b-t)\right].$$

Thus, if $G(\theta)$ is strictly increasing in $\theta \in [\underline{\theta}, \overline{\theta}]$, then $\partial^2\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2+b,t)/\,\partial t^2 > 0$ for all $0 < t \le \left(\underline{\theta}-\overline{\theta}\right)/2$. Since $\partial\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2+b,0)/\partial t = 0$ it follows that for all $t > 0$, $\partial\Delta(p,t)/\,\partial t > 0$ and $\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2+b, \left(\underline{\theta}+\overline{\theta}\right)/2) = \int\limits_0^{\left(\underline{\theta}+\overline{\theta}\right)/2} \frac{\partial\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2+b,t')}{\partial t}\mathrm{d}t' > 0$. By a similar reasoning, if $G(\theta)$ is decreasing in θ, we have that $\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2+b, \left(\underline{\theta}+\overline{\theta}\right)/2) < 0$. ■

LEMMA 4. *Suppose that $G(\theta)$ is strictly monotone in $[0,1]$, then (i.) if $G(\theta)$ is strictly decreasing in $[0,1]$ then $\mathrm{E}\left[\theta\,|\theta \ge a_1\right] < a_1 + b$ for all $a_1 \in [0,1]$ (ii.) if $G(\theta)$ is strictly increasing in $[0,1]$ then the equation $\mathrm{E}\left[\theta\,|\theta \ge a_1\right] = a_1 + b$, $a_1 \in (0,1)$ has a solution if and only if $\mathrm{E}\left[\theta\right] > b$. Furthermore, this solution is unique.*

Proof: (i.) Since $G(\theta)$ is strictly decreasing we have that $1-F(\theta) < 1-F(1) + b\left(f(\theta)-f(1)\right) = b\left(f(\theta)-f(1)\right)$. Integrating both sides between a_1 and 1 yields the inequality $\int\limits_{a_1}^{1} \theta f(\theta)\mathrm{d}\theta - \widetilde{F}(a_1)a_1 < \widetilde{F}(a_1)b - bf(1)(1-a_1) < \widetilde{F}(a_1)b$. It then follows that $\mathrm{E}\left[\theta\,|\theta \ge a_1\right] < a_1 + b$.

(ii.) Recall the definition of $S(\theta)$ and note that $\mathrm{d}S(\theta)/\mathrm{d}\theta = 1 - G(\theta)$ for $\theta \in (0,1)$. Thus, $S(\theta)$ is strictly concave from the assumptions on $G(\theta)$. Since $S(1) = 0$, strict concavity implies that there can be at most one point $\theta \in [0,1)$ at which $S(\theta) = 0$. The existence of this point in turn requires $S(0)$ to be non-positive, i.e. $S(0) = b - \mathrm{E}\left[\theta\right] \le 0$ and thus establishes necessity. Now suppose that $\mathrm{E}\left[\theta\right] > b$. Since for $0 < \varepsilon < b$ we have that $(1-\varepsilon+b) - \mathrm{E}\left[s\,|s \ge 1-\varepsilon\right] \ge b - \varepsilon > 0$. Therefore $S(0) < 0$ and $S(1-\varepsilon) > 0$ which guarantees the existence of a solution to $\mathrm{E}\left[s\,|s \ge \theta\right] = \theta + b$ thus proving sufficiency. ■

9.2.1 Proof of Proposition 3

To establish Proposition 3 we need to introduce two more lemmas.

LEMMA 5. *Suppose that $G(\theta)$ is strictly increasing in $[0,1]$. Then if $y(\cdot)$ and $\mu(\cdot)$ is an optimal delegation scheme there cannot be two consecutive pooling regions, i.e. there cannot be two intervals $[\theta_i, \theta_{i+1}]$ and $[\theta_{i+1}, \theta_{i+2}]$ with $0 \le \theta_i < \theta_{i+1} < \theta_{i+2} \le 1$ such that $y(\mu(\theta)) = y_i$ for all $\theta \in (\theta_i, \theta_{i+1})$ and $y(\mu(\theta)) = y_{i+1}\,(\ne y_i)$ for all $\theta \in (\theta_{i+1}, \theta_{i+2})$.*

Proof: We consider three cases: (i.) both consecutive projects $y_i, y_{i+1} \in [b, 1+b]$, (ii.) $y_i < b$ and $b < y_{i+1} < 1+b$ (iii.) $b < y_i < 1+b$ and $y_{i+1} > 1+b$.

CASE I: Let $[\theta_i, \theta_{i+1}]$ and $[\theta_{i+1}, \theta_{i+2}]$ be the two pooling regions that, for all interior points, induce the projects y_i and y_{i+1}, respectively. Then from incentive compatibility, it must be that $y_i - b \in [\theta_i, \theta_{i+1}]$ and $y_{i+1} - b \in [\theta_{i+1}, \theta_{i+2}]$. Now consider an alternative $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that $\widehat{y}(\widehat{\mu}(\theta)) = \theta + b$ if $\theta \in [y_i - b, y_{i+1} - b]$ and $y(\mu(\theta))$ otherwise. It is immediate that $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is incentive compatible if $y(\cdot)$ and $\mu(\cdot)$ is. Since $G(\theta)$ is strictly increasing in $[0,1]$ by Lemma 3 we infer that $\Delta((y_i + y_{i+1})/2 - b, (y_{i+1} - y_i)/2) > 0$ and therefore the principal strictly prefers $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ to $(y(\cdot), \mu(\cdot))$. Hence $(y(\cdot), \mu(\cdot))$ cannot be optimal.

CASE II: If both y_i and $y_{i+1}$are to be induced with positive probability it must be that $b < (y_i + y_{i+1})/2$. From incentive compatibility of the agent, y_i is induced in $[0, (y_i + y_{i+1})/2 - b)$ and for $((y_i + y_{i+1})/2 - b, y_{i+1} - b]$ we have $y(\mu(\theta)) = y_{i+1}$. Now consider the alternative incentive compatible $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that

$$\widehat{y}(\widehat{\mu}(\theta)) = \begin{cases} b & \text{if} \quad \theta \in [0, (y_i + y_{i+1})/2 - b] \\ y_i + y_{i+1} - b & \text{if} \quad \theta \in ((y_i + y_{i+1})/2 - b, y_i + y_{i+1} - 2b] \\ \theta + b & \text{if} \quad \theta \in [y_i + y_{i+1} - 2b, y_{i+1} - b] \\ y(\mu(\theta)) & \text{otherwise.} \end{cases}$$

Let $a \equiv (y_i + y_{i+1})/2 - b$. Then the increment in the principal's expected payoff of switching from $(y(\cdot), \mu(\cdot))$to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\begin{aligned} \Delta \quad &\equiv \quad \int_0^a \left([y_i - \theta]^2 - [b - \theta]^2\right) \mathrm{d}F(\theta) + \int_a^{2a} \left(\left[y_{i+1} - \theta\right]^2 - [y_i + y_{i+1} - b - \theta]^2\right) \mathrm{d}F(\theta) \\ &\quad + \int_{2a}^{y_{i+1}-b} \left(\left[y_{i+1} - \theta\right]^2 - b^2\right) \mathrm{d}F(\theta). \end{aligned}$$

Note that

$$\Delta > \int_0^a 2\left([y_i - b]\left[\frac{y_i + b}{2} - \theta\right]\right) \mathrm{d}F(\theta) + \int_a^{2a} 2\left([b - y_i]\left[\frac{y_i + 2y_{i+1} - b}{2} - \theta\right]\right) \mathrm{d}F(\theta).$$

Using $T(\theta)$ as defined at the beginning of this section, the above inequality can then be rewritten as $\Delta > [b - y_i][2T(2a) - 4T(a)] + [b - y_i]^2 F(2a)$. Since $G(\theta)$ is strictly increasing, $T(\theta)$ is strictly convex which in turn implies that $T(2a) > 2T(a)$. This establishes that $\Delta > 0$. Thus, $(y(\cdot), \mu(\cdot))$ cannot be optimal.

CASE III: Suppose $y_{i+1} > 1+b, b < y_i < 1+b$. In this case if both y_i and $y_{i+1}$are to be induced with positive probability it must be that $(y_i + y_{i+1})/2 < 1+b$. From incentive compatibility of the agent, y_i is induced in $[y_i - b, (y_i + y_{i+1})/2 - b)$ and for $((y_i + y_{i+1})/2 - b, 1]$ we have $y(\mu(\theta)) = y_{i+1}$. Now consider the alternative incentive compatible $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that $\widehat{y}(\widehat{\mu}(\theta)) = y_i$ if $\theta \in [y_i - b, 1]$ and $y(\mu(\theta))$ otherwise. Note that $(y(\cdot), \mu(\cdot))$ and $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ only differ for $\theta \in ((y_i + y_{i+1})/2 - b, 1]$. Thus, the increment in the principal's expected payoff of switching to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\Delta \equiv \int_{(y_i+y_{i+1})/2-b}^{1} \left(\left[y_{i+1} - \theta \right]^2 - \left[y_i - \theta \right]^2 \right) \mathrm{d}F(\theta) = 2 \left[y_{i+1} - y_i \right] S((y_i + y_{i+1})/2 - b).$$

If $\mathrm{E}[\theta] < b$, then by the proof of Lemma 4 (ii.) we have $S(\theta) > 0$ for $\theta \in [0,1)$. Hence, $\Delta > 0$. If $\mathrm{E}[\theta] \geq b$, then let θ^* be the unique solution to $\mathrm{E}[s \,| s \geq \theta] = \theta + b$. If $(y_i + y_{i+1})/2 - b > \theta^*$ then again $S((y_i + y_{i+1})/2 - b) > 0$ and $\Delta > 0$.

For the case that $\mathrm{E}[\theta] \geq b$ and $(y_i + y_{i+1})/2 - b < \theta^*$ consider the alternative incentive compatible $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ which is derived from $(y(\cdot), \mu(\cdot))$ by replacing the project y_{i+1} with $\widehat{y}$, such that $y_i < \widehat{y} < y_{i+1}$. Then $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is defined by $\widehat{y}(\widehat{\mu}(\theta)) = y_i$ if $\theta \in [y_i - b, (y_i + \widehat{y})/2 - b)$, $\widehat{y}(\widehat{\mu}(\theta)) = \widehat{y}$ if $\theta \in [(y_i + \widehat{y})/2 - b, 1]$ and $y(\mu(\theta))$ otherwise. Defining $a = (y_i + \widehat{y})/2 - b$ and $c = (y_i + y_{i+1})/2 - b$ the increment in the principal's expected payoff of switching to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\Delta \equiv \int_a^c \left([y_i - \theta]^2 - [\widehat{y} - \theta]^2 \right) \mathrm{d}F(\theta) + \int_c^1 \left(\left[y_{i+1} - \theta \right]^2 - [\widehat{y} - \theta]^2 \right) \mathrm{d}F(\theta).$$

Noting that $\int_a^c \left([y_i - \theta]^2 - [\widehat{y} - \theta]^2 \right) \mathrm{d}F(\theta) = 2(y_i - \widehat{y})[S(a) - S(c) + ((y_{i+1} - \widehat{y})/2) \widetilde{F}(c)]$ and $\int_c^1 \left(\left[y_{i+1} - \theta \right]^2 - [\widehat{y} - \theta]^2 \right) \mathrm{d}F(\theta) = 2(y_{i+1} - \widehat{y})[S(c) - ((y_i - \widehat{y})/2) \widetilde{F}(c)]$ we can express the increment Δ as

$$\Delta \equiv 2(y_i - \widehat{y})S(a) + 2(y_{i+1} - \widehat{y})S(c) \tag{9}$$

From $a < c < \theta^*$ and Lemma 4 (ii.) we have that $S(a) < 0$, which implies that the first term on the RHS of (9) is positive and the second term is negative. By selecting a project $\widehat{y}$ close enough to y_{i+1} we have that $\Delta > 0$ and $(y(\cdot), \mu(\cdot))$ cannot be optimal. ■

LEMMA 6. *Suppose that $G(\theta)$ is strictly increasing in $[0,1]$. Then if $y(\cdot)$ and $\mu(\cdot)$ is an optimal delegation scheme it must be that (i.) if $\mathrm{E}[\theta] < b$ then $y(\mu(\theta)) = \mathrm{E}[\theta]$ for all $\theta \in [0,1]$ (ii.) if $\mathrm{E}[\theta] > b$ then there exists a threshold level $\overline{\theta}$, such that $y(\mu(\theta)) = \theta + b$ if $\theta \in [0, \overline{\theta}]$ and $y(\mu(\theta)) = \overline{\theta} + b$ if $\theta \in [\overline{\theta}, 1]$. Moreover the threshold level $\overline{\theta}$ satisfies $\overline{\theta} + b = \mathrm{E}\left[\theta \left| \theta \geq \overline{\theta}\right.\right]$.*

Proof: From the previous lemma the optimal delegation scheme is characterized by two threshold levels $\{\underline{\theta}, \overline{\theta}\}$ with (a.)

$$y(\mu(\theta)) = \begin{cases} \underline{\theta} + b & \text{if} \quad \theta \in [0, \underline{\theta}) \\ \theta + b & \text{if} \quad \theta \in [\underline{\theta}, \overline{\theta}] \\ \overline{\theta} + b & \text{if} \quad \theta \in (\underline{\theta}, 1]. \end{cases}$$

if $0 < \underline{\theta} < \overline{\theta} < 1$, and (b.) $y(\mu(\theta))$ constant over $[0,1]$ if $\underline{\theta} = \overline{\theta}$. The expected utility of the principal is given by

$$-\int_0^{\underline{\theta}} [\underline{\theta} + b - \theta]^2 \, \mathrm{d}F(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} b^2 \mathrm{d}F(\theta) - \int_{\overline{\theta}}^1 \left[\overline{\theta} + b - \theta\right]^2 \mathrm{d}F(\theta)$$

Optimality of $y(\cdot)$ and $\mu(\cdot)$ requires that $\underline{\theta}$ and $\overline{\theta}$ satisfy the first order necessary conditions $2F(\underline{\theta})\left[(\underline{\theta} + b) - \mathrm{E}\left[\theta \left| \theta \leq \underline{\theta}\right.\right]\right] = \lambda - \nu$ and $2\widetilde{F}(\overline{\theta})\left[(\overline{\theta} + b) - \mathrm{E}\left[\theta \left| \theta \geq \overline{\theta}\right.\right]\right] = \nu$, where λ, ν are nonnegative multipliers associated with the constraints $\underline{\theta} \geq 0$, and $\overline{\theta} \geq \underline{\theta}$, respectively. First we establish that that $\underline{\theta} = 0$ is necessary. If at an optimum $\underline{\theta} > 0$ then, since $\underline{\theta} + b > \mathrm{E}\left[\theta \left| \theta \leq \underline{\theta}\right.\right]$, we must have $\lambda - \nu > 0$. This necessarily implies that $\lambda > 0$ and the constraint $\underline{\theta} \geq 0$ binds, reaching a contradiction.

Next suppose that $\mathrm{E}[\theta] < b$. By Lemma 4 (ii.) there is no interior point at which $S(\theta) = \widetilde{F}(\theta)\left[\theta + b - \mathrm{E}\left[s \left| s \geq \theta\right.\right]\right]$ and therefore $\nu > 0$. But then $\overline{\theta} \geq \underline{\theta}$ binds, i.e. $\overline{\theta} = \underline{\theta} = 0$. In this case the principal selects a single project for all states of the world. Since no information from the agent is used in this case it must be that the principal implements $y(\mu(\theta)) = \mathrm{E}[\theta]$ for all $\theta \in [0,1]$.

Now suppose that $\mathrm{E}[\theta] > b$. Then by Lemma 4 (ii.) there is a unique interior point $\overline{\theta} \in (0,1)$ at which $S(\overline{\theta}) = 0$. Then the quadruple $\{0, \overline{\theta}, 0, 0\}$ satisfies the necessary conditions where $\overline{\theta}$ solves $\overline{\theta} + b = \mathrm{E}\left[\theta \left| \theta \geq \overline{\theta}\right.\right]$. ■

Proof of Proposition 3: Follows from Lemmas 3-6. ■

9.2.2 Proof of Proposition 4

The proof of Proposition 4 is carried out through a sequence of lemmas that gradually reduce the class of potential optimal delegation schemes when $G(\theta)$ is strictly decreasing in $[0,1]$.

LEMMA 7. *Suppose that $G(\theta)$ is strictly decreasing in $[0,1]$. Then if $(y(\cdot),\mu(\cdot))$ is an optimal delegation scheme, there cannot be a non-degenerate interval $[\underline{\theta},\overline{\theta}] \subset [0,1]$ where $y(\mu(\theta)) = \theta + b$ for $\theta \in [\underline{\theta},\overline{\theta}]$.*

Proof: Suppose on the contrary that $y(\mu(\theta)) = \theta + b$ for $\theta \in [\underline{\theta},\overline{\theta}]$. Now consider the alternative $(\widehat{y}(\cdot),\widehat{\mu}(\cdot))$ such that

$$\widehat{y}(\widehat{\mu}(\theta)) = \begin{cases} \underline{\theta} + b & \text{if} \qquad \theta \in [\underline{\theta}, \left(\underline{\theta}+\overline{\theta}\right)/2] \\ \overline{\theta} + b & \text{if} \qquad \theta \in (\left(\underline{\theta}+\overline{\theta}\right)/2, \overline{\theta}] \\ y(\mu(\theta)) & \text{otherwise.} \end{cases}$$

It is immediate that $\widehat{y}(\widehat{\mu}(\cdot))$ is incentive compatible if $y(\mu(\cdot))$ is. Furthermore, under $\widehat{y}(\widehat{\mu}(\theta))$ only the extreme projects $\left\{\underline{\theta}+b, \overline{\theta}+b\right\}$ are implemented in $[\underline{\theta},\overline{\theta}]$. Since $G(\theta)$ is strictly decreasing in $[0,1]$ by Lemma 1 we infer that $\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2, \left(\underline{\theta}-\overline{\theta}\right)/2) < 0$ and therefore the principal strictly prefers $(\widehat{y}(\cdot),\widehat{\mu}(\cdot))$ to $(y(\cdot),\mu(\cdot))$. Hence $y(\cdot)$ and $\mu(\cdot)$ cannot be optimal. ■

LEMMA 8. *Suppose that $G(\theta)$ is strictly decreasing in $[0,1]$. Then if $(y(\cdot),\mu(\cdot))$ is an optimal menu delegation scheme then $y(\mu(\cdot))$ induces at most two projects, i.e. $y(\mu(\theta)) \in \{y_1, y_2\}$ for $\theta \in [0,1]$.*

Proof: Let $D_A = \{y \in \mathbb{R} : y(\mu(\theta)) = y, \theta \in [0,1]\}$ be the set of projects induced by $y(\cdot)$ and $\mu(\cdot)$ and suppose that D_A contains more than two projects. We consider in turn two cases: (i.) there are three projects $y_1 < y_2 < y_3$, $\{y_1, y_2, y_3\} \subset D_A$, that are consecutive in the sense that no other project is induced between them (i.e. such that $(y_1, y_2) \cap D_A = (y_2, y_3) \cap D_A = \varnothing$), (ii.) there do not exist three consecutive projects in D_A.

i.) Consider then three consecutive projects $y_1 < y_2 < y_3$. Incentive compatibility implies that (a.) $y(\mu(\theta)) = y_1$ for $\theta \in [y_1 - b, (y_1+y_2)/2 - b)$, (b.) $y(\mu(\theta)) = y_2$ for

$\theta \in ((y_1+y_2)/2-b, (y_2+y_3)/2-b)$, and (c.) $y(\mu(\theta)) = y_3$ for $\theta \in ((y_2+y_3)/2-b, y_3-b]$. We now propose an alternative delegation scheme in which the project y_2 is not implemented by the principal, i.e. consider $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that

$$\widehat{y}(\widehat{\mu}(\theta)) = \begin{cases} y_1 & \text{if} \quad \theta \in [y_1-b, (y_1+y_3)/2-b] \\ y_3 & \text{if} \quad \theta \in ((y_1+y_3)/2-b, y_3-b] \\ y(\mu(\theta)) & \text{otherwise.} \end{cases}$$

Suppose that $y_2 \leq (y_1+y_3)/2$ (the analysis if $y_2 \geq (y_1+y_3)/2$ would follow the same argument). Letting $r = (y_1+y_2)/2-b$, $s = (y_2+y_3)/2-b$, $t = (y_2+y_3)/2-b$, the increment in the principal's expected payoff when switching to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\begin{aligned} \Delta U &\equiv \int_r^s \left([y_2-\theta]^2 - [y_1-\theta]^2\right) \mathrm{d}F(\theta) + \int_s^t \left([y_2-\theta]^2 - [y_3-\theta]^2\right) \mathrm{d}F(\theta) \\ &= 2(y_2-y_1)\int_r^s \left[\frac{y_2+y_1}{2} - \theta\right] \mathrm{d}F(\theta) + 2(y_2-y_3)\int_s^t \left[\frac{y_2+y_3}{2} - \theta\right] \mathrm{d}F(\theta) \end{aligned}$$

Making use of $T(\cdot)$ as defined above, we obtain

$$\Delta U = 2\left[(y_3-y_1)T(s) - (y_2-y_1)T(r) - (y_3-y_2)T(t)\right].$$

If we express $s = (y_2+y_3)/2-b$ as a convex combination of r and t, $s = \lambda r + (1-\lambda)t$, and noting that $y_3 - y_2 = (1-\lambda)(y_3-y_1)$ and $y_2 - y_1 = \lambda(y_3-y_1)$, we can write ΔU in the more transparent form $\Delta U = 2(y_3-y_1)\left[T(\lambda r + (1-\lambda)t) - \lambda T(r) - (1-\lambda)T(t)\right]$.

Since $G(\theta)$ is strictly decreasing, $T(\theta)$ is strictly concave and hence $\Delta U > 0$. This establishes that the original delegation scheme $(y(\cdot), \mu(\cdot))$ where more than two projects are implemented cannot be optimal.

ii.) Suppose that D_A does not contain three consecutive projects. From Lemma 2 $y(\mu(\theta))$ is weakly increasing and therefore continuous except in a countable set of points $\{\alpha_i\}, i \in \mathbb{N}$. [10] We will now introduce some notation pertinent to this part of the proof. Let $\overline{d} = \max D_A$ and $\underline{d} = \min D_A$ be the maximum and minimum projects induced under $y(\mu(\theta))$ and, for each $i \in \mathbb{N}$, let $y_i^+ = \lim_{\theta \to \alpha_i^+} y(\mu(\theta))$ and $y_i^- = \lim_{\theta \to \alpha_i^-} y(\mu(\theta))$ be the two projects induced to the left and to the right of the point of discontinuity

[10] We note that since D_A does not contain three consecutive projects and does not contain any nondegenerate interval the set of discontinuity points of $y(\mu(\theta))$ must indeed be infinite.

α_i, respectively . Finally let $A_i = \left\{\theta \in [0,1] : y(\mu(\theta)) \in \left\{y_i^+, y_i^-\right\}\right\}$ be the set of states under which the projects $\left\{y_i^+, y_i^-\right\}$ are induced. For convenience we will order the set of discontinuity points $\{\alpha_i\}, i \in \mathbb{N}$ in such a way that the probability of the projects y_i^+ or y_i^- being induced is (weakly) larger than the probability of inducing y_{i+1}^+ or y_{i+1}^-, i.e. such that $\text{Prob}[\theta \in A_i] \geq \text{Prob}[\theta \in A_{i+1}]$. By the assumptions on $y(\mu(\theta))$ we have that $\lim_{i\to\infty} \text{Prob}[\theta \in \cup_1^i A_j] = 1$.

Consider now a sequence of incentive compatible delegation schemes $(y_i(\cdot),\mu_i(\cdot))$ $i = 0, 1, 2....$to be defined momentarily. Denoting by $D_A^i = \{y \in \mathbb{R} : y_i(\mu_i(\theta)) = y, \theta \in [0,1]\}$ the set of projects induced by $(y_i(\cdot),\mu_i(\cdot))$, we define D_A^i inductively as follows: $D_A^0 = \left\{\overline{d}, \underline{d}\right\}$, $D_A^{i+1} = D_A^i \cup \left\{y_{i+1}^+, y_{i+1}^-\right\}$. We note that this scheme fully identifies $y_i(\mu_i(\cdot))$ except possibly at its points of discontinuity. For completeness we define $y_i(\cdot), \mu_i(\cdot)$ such that $y_i(\mu_i(\theta))$ is left continuous at its points of discontinuity.

Since $G(\theta)$ is strictly decreasing, the analysis of the case with three consecutive projects establishes that $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right] > \text{E}_\theta\left[U_P(y_{i+1}(\mu_{i+1}(\theta)), \theta)\right]$. Next we show that $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right]$ converges to $\text{E}_\theta[U_P(y(\mu(\theta)), \theta)]$ as $i \to \infty$.For $\varepsilon > 0$ there exist an i such that $\text{Prob}[\theta \in \cup_0^i A_j] > 1 - \frac{\varepsilon}{2(\overline{d}-\underline{d})\overline{d}}$. Therefore we have that for $k > i$, with $S_k = \left(\cup_1^k A_j\right)^c$.

$$|\text{E}_\theta\left[U_P(y_k(\mu_k(\theta)), \theta)\right] - \text{E}_\theta[U_P(y(\mu(\theta)), \theta)]| < \int_{S_k} \left|[y(\mu(\theta)) - \theta]^2 - [y_i(\mu_i(\theta)) - \theta]^2\right| \text{d}F(\theta) =$$

$$= \int_{S_k} 2\left|y(\mu(\theta)) - y_i(\mu_i(\theta))\right| \left|\frac{y(\mu(\theta)) + y_i(\mu_i(\theta)}{2} - \theta\right| \text{d}F(\theta) \leq 2\left(\overline{d} - \underline{d}\right)\overline{d}\,\text{Pr ob}\left[\theta \in S_k\right] < \varepsilon.$$

Thus $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right] \longrightarrow \text{E}_\theta[U_P(y(\mu(\theta)), \theta)]$ for $i \to \infty$. Therefore $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right] > \text{E}_\theta[U_P(y(\mu(\theta)), \theta)]$ for all i which implies that $y(\cdot)$ and $\mu(\cdot)$ cannot be optimal. ■

LEMMA 9. *Suppose that $G(\theta)$ is strictly decreasing in $[0,1]$. Then any two-project delegation scheme is dominated by centralization, i.e. by a decision rule that implements* $\text{E}[\theta]$ *for all messages the agent selects.*

Proof: Let $y(\mu(\theta)) \in \{y_1, y_2\}$ be an optimal two-project equilibrium with $y_1 < y_2$. Since both projects are selected with positive probability there must exist a state of the world a_1, with $0 < a_1 < 1$, at which the agent is indifferent between y_1 and y_2, i.e. $(y_1 + y_2)/2 = a_1 + b$. Since, for fixed a_1, the projects $\{y_1, y_2\}$ are optimal they must satisfy the first order condition $(y_1 - \mathrm{E}[\theta\,|\theta \le a_1])\,F(a_1) = (y_2 - \mathrm{E}[\theta\,|\theta \ge a_1])\,\widetilde{F}(a_1)$. Equivalently, these can be expressed as

$$S(a_1) - T(a_1) + \frac{y_2 - y_1}{2} = 0 \tag{10}$$

Now consider the difference in expected utility to the principal between the best centralized decision $\mathrm{E}[\theta]$ and $y(\mu(\theta))$

$$\Delta U = \int_0^1 -(E[\theta] - \theta)^2 \mathrm{d}F(\theta) + \int_0^{a_1} (y_1 - \theta)^2 \mathrm{d}F(\theta) + \int_{a_1}^1 (y_2 - \theta)^2 \mathrm{d}F(\theta)$$

Using the fact that $S(a_1) + T(a_1) = (y_1 + y_2)/2 - \mathrm{E}[\theta]$ this expression can be rewritten as

$$\Delta U = 2\,(y_2 - \mathrm{E}[\theta])\,S(a_1) + (\mathrm{E}[\theta] - y_1)\left(S(a_1) - T(a_1) + \frac{y_2 - y_1}{2}\right). \tag{11}$$

Using Lemma 4 (i.) we have that $S(a_1) > 0$ for $0 \le a_1 < 1$, and, in particular, $S(0) = b - \mathrm{E}[\theta] > 0$. Since $y_2 > b$ the first term in (11) is positive and the second term is zero from the first order condition (10) implying that $\Delta U > 0$. Thus any two-project optimal delegation scheme is dominated by centralization. ■

Proof of Proposition 4: Follows from Lemmas 3,4 and 7-9. ■

9.3 Proofs of Propositions 5-10

Proof of Proposition 5: Follows from the discussion in the text. ■

Proof of Proposition 6: Follows from Proposition 4. ■

Proof of Proposition 7: We first show that for given N, the solution to (5) subject to (6) and (7) satisfies $\Delta y_i = q$ for $i = 2, ..., N-1$. To see this, note first that the reneging constraint (7) can be stated as $\Delta y_1 = (3y_1 - y_2 + 2b)/4 \le q$, $\Delta y_N = (3y_N - y_{N-1} - 2(1-b)) \le q$ and

$$\frac{1}{4}\,(2y_i - y_{i-1} - y_{i+1} + 4b) \le q \quad \text{for } i = 2, ..., N-1.$$

Thus, an increase in y_j relaxes the reneging constraint for all $i \neq j$. Note second that

$$\frac{\mathrm{d}\mathrm{E}_\theta[U_P]}{\mathrm{d}y_i} = \frac{1}{4}(y_{i+1} - y_{i-1})(y_{i+1} + y_{i-1} - 2y_i) \quad \text{for } i = 2, ..., N-1.$$

If $\Delta y_j < q$ for any $j \in \{2, ..., N-1\}$, then, from the above two equations, $\mathrm{dE}_\theta [U_P] / \mathrm{d}y_j > 0$. Since an increase in y_j relaxes all the other reneging constraints it follows that $\Delta y_j < q$ cannot be a solution. Thus, the solution must satisfy $\Delta y_i = q$ for $i = 2, ..., N-1$.

We can now prove part (i.) of the proposition. We first prove that if $q \leq b/4$, then $\Delta y_i = q$ for $i = 1, N$. Solving $\Delta y_i = q$ for $i = 2, ..., N-1$ we obtain

$$y_i = \frac{N-i}{N-1} y_1 + \frac{i-1}{N-1} y_N - 2(i-1)(N-i)(b-q) \text{ for } i = 2, ..., N-1.$$

Substituting into (5) and differentiating then gives

$$\frac{\mathrm{dE}_\theta [U_P]}{\mathrm{d}y_1} = b (y_2 - y_1) - 2\Delta y_1 a_1 + \sum_{i=2}^{N-1} \frac{\partial \mathrm{E}_\theta [U_P]}{\partial y_i} \frac{\mathrm{d}y_i}{\mathrm{d}y_1}.$$

Note that the third term on the RHS is positive and that $(y_2 - y_1) > a_1$. Thus, $q \leq b/2$ is a sufficient condition for $\Delta y_1 = q$.

Suppose that $q \leq b/2$ and $\Delta y_1 = q$. Note that $\Delta y_N \leq q$ if and only if $y_N \leq (1 + b - (2N-1)(b-q)) \equiv \widetilde{y}_N$ and that $\widetilde{y}_N < 1$. Next, differentiating $\mathrm{E}_\theta [U_P]$ twice gives

$$\frac{\mathrm{d}^2 \mathrm{E}_\theta [U_P]}{\mathrm{d}y_N^2} = \frac{-2}{(2N-1) N} (1 + 4N (N-1) (1 - y_N)).$$

Note that the second derivative is negative for all $y_N \leq \widetilde{y}_N$. Thus, it is optimal to set $y_N = \widetilde{y}_N$, and thus $\Delta y_N = q$, if and only if

$$\left. \frac{\mathrm{dE}_\theta [U_P]}{\mathrm{d}y_N} \right|_{y_N = \widetilde{y}_N} \geq 0.$$

Differentiating $\mathrm{E}_\theta [U_P]$ once shows that this inequality is satisfied if and only if

$$q \leq \frac{(N-1)(2N(N-2)(b-q)+3)(b-q)}{3(1+2N(N-1)(b-q))} \equiv \widetilde{q}(N)$$

Since $\widetilde{q}(N)$ is increasing in N, $\Delta y_N = q$ for all $N \geq 2$ if and only if $q \leq \widetilde{q}(2) \equiv \overline{q}$. It is straightforward to verify that $\overline{q} \in (0, 1)$.

We now know that for given N, the solution satisfies $\Delta y_i = q$ for $i = 1, ..., N$ if $q \leq \overline{q}$. We next argue that the optimal number of intervals is given by the maximum number of intervals $\widetilde{N}$ that can be supported in equilibrium. We do so in two steps. First, we show that $\widetilde{N}$ is increasing in Δy_i. Second, we show that if $\Delta y_i = q$ for $i = 1, ..., N$, then $\mathrm{E}_\theta\left[U_P\right]$ is increasing in N.

>From (6) it follows that

$$(a_i - a_{i-1}) = a_1 + 4(i-1)b - 2\Delta y_1 - 2\Delta y_i - 4\sum_{j=2}^{i-1} \Delta y_j \text{ for all } i = 2, ..., N. \qquad (12)$$

In any equilibrium the intervals must add up to one, i.e. $a_1 + \sum_{i=2}^{N}(a_i - a_{i-1}) = 1$. Since it must be that $a_1 \geq 0$, $\widetilde{N}$ is given by the largest integer for which $\sum_{i=2}^{N}(a_i - a_{i-1}) \leq 1$. From (12) it then follows that $\widetilde{N}$ is increasing in Δy_i for $i = 1, ..., N$.

Next, suppose that $\Delta y_i = q$ for $i = 1, ..., N$. Then

$$\mathrm{E}_\theta\left[U_P\right] = -\left(\left(4(b-q)^2 N^2 \left(N-1\right)\left(N+1\right) + 1\right) + q^2\right) / \left(12N^2\right).$$

The expression on the RHS is increasing in N for all $N \leq \widetilde{N}$. This proves part (i.).

For part (ii.) note that from the above $\Delta y_i = q$ for $i = 2, ..., N-1$ for any $q < b$. ■

Proof of Proposition 8: Note that since $f(\theta)$ is continuously differentiable, there exists a $b' > 0$ such that for all $b \leq b'$, $G(\theta)$ is increasing in θ for all $\theta \in \Theta$. Thus, for sufficiently small b threshold delegation is optimal if it does not violate the reneging constraint. To see that for sufficiently small b threshold delegation does not violate the reneging constraint, let δ_{TD} be the δ for which the reneging constraint is strictly satisfied under threshold delegation, i.e. $b^2 = \delta_{TD}/(1-\delta_{TD})\mathrm{E}_\theta\left[U_P^{TD} - U_P^{CS}\right]$. Similarly, let $\delta_{CD} \equiv -b^2/\mathrm{E}_\theta\left[U_P^{CS}\right]$ be the δ for which the reneging constraint is strictly satisfied under complete delegation. >From Proposition 3 in Dessein (2002) it follows that if $f(\theta)$ is twice continuously differentiable, then $\lim_{b\to 0}\delta_{CD} = 0$. Note next that since $\mathrm{E}_\theta\left[U_P^{TD}\right] \geq \mathrm{E}_\theta\left[U_P^{CD}\right]$, $\delta_{CD} \geq \delta_{TD}$. Thus, $\lim_{b\to 0}\delta_{CD} = 0 \geq \lim_{b\to 0}\delta_{TD}$. ■

Proof of Proposition 9: Follows immediately from the discussion in the text. ■

Proof of Proposition 10: Follows immediately from the discussion in the text. ■

References

[1] Alonso, R. and N. Matouschek (2004), "Optimal Delegation," mimeo, Northwestern.

[2] ____ and ____ (2004b), "Motivating Experts," mimeo, Northwestern.

[3] Baker, G., Gibbons, R. and K. Murphy (1994), "Subjective Performance Measures in Optimal Incentive Contracts," *Quarterly Journal of Economics*, 109, 1125-1156.

[4] ____ , ____ and ____ (1999), "Informal Authority in Organizations," *Journal of Law, Economics, and Organization*, 15, 56-73.

[5] ____ , ____ and ____ (2002), "Relational Contracts and the Theory of the Firm," *Quarterly Journal of Economics*, 117, 39-84.

[6] Bower, J. (1970), *Managing the Resource Allocation Process: A Study of Corporate Planning and Investment*, Boston, Division of Research, Graduate School of Business Administration, Harvard.

[7] Crawford, V. and J. Sobel (1982), "Strategic Information Transmission," *Econometrica*, 50, 1431-1452.

[8] Dessein, W. (2002), "Authority and Communication in Organizations," *Review of Economic Studies*, 69, 811-838.

[9] Harris, M. and a. Raviv (1996), "The Capital Budgeting Process, Incentives and Information," *Journal of Finance*, 51, 1139-1174.

[10] Holmström, B. (1977), "On Incentives and Control in Organizations," Ph.D. Dissertation, Stanford.

[11] ____ (1984), "On the Theory of Delegation," in M. Boyer and R. Kihlstrom (eds.) *Bayesian Models in Economic Theory*, New York: North-Holland.

[12] JENSEN, M. (1986), "Agency Costs of Free Cash Flow, Corporate finance, and Takeovers," *American Economic Review*, 76, 323-329.

[13] ____ AND W. MECKLING (1992), "Specific and General Knowledge and Organizational Structure," in L. Werin and H. Wijkander (eds.) *Contract Economics*, Oxford: Blackwell.

[14] KRISHNA, V. AND J. MORGAN (2004), "Contracting for Information under Imperfect Commitment," mimeo, Penn State and UC Berkeley.

[15] LEVIN, J. (2003), "Relational Incentive Contracts," *American Economic Review*, 93, 835-847.

[16] MARINO, A. AND J. MATSUSAKA (2004), "Decision Processes, Agency Problems, and Information: An Economic Analysis of Capital Budgeting Procedures," *Review of Financial Studies*, forthcoming.

[17] MARSHUETZ, R. (1985), "How American Can Allocates Capital," *Harvard Business Review*, 63, 82-91.

[18] MELUMAD, N. AND T. SHIBANO (1991), "Communication in Settings with No Transfers," *Rand Journal of Economics*, 22, 173-198.

[19] MILGROM, P. AND J. ROBERTS (1992), *Economics, Organization & Management*, Englewood Cliffs: Prentice-Hall.

[20] OTTAVIANI, M. (2000), "Economics of Advice," mimeo, London Business School.

[21] ROSS, M. (1986), "Capital Budgeting Practices of Twelve Large Manufacturers," *Financial Management*, 15, 15-22.

[22] TAGGART, R. (1987), "Allocating Capital Among a Firm's Divisions: Hurdle Rages vs. Budgets," *Journal of Financial Research*, 10, 177-190.

	$G(\theta)$ increasing	$G(\theta)$ decreasing
High Relational Capital	Threshold Delegation	Centralization
Low Relational Capital	Menu Delegation	Centralization

Table 1

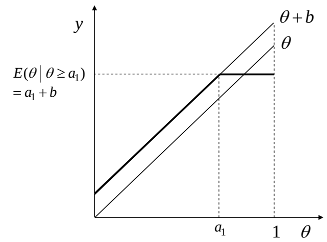

Figure 1

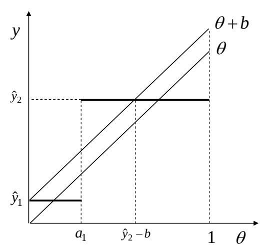

Figure 2a

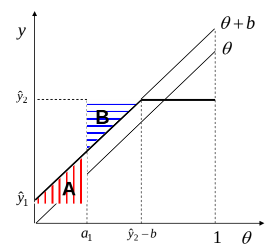

Figure 2b

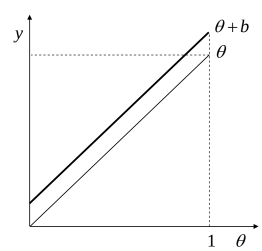

Figure 3a

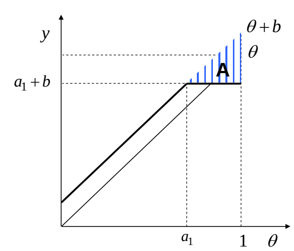

Figure 3b

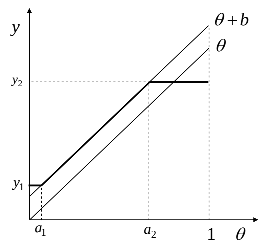

Figure 4a

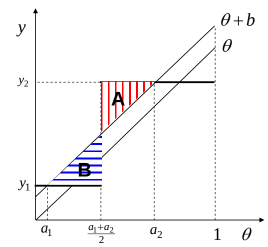

Figure 4b

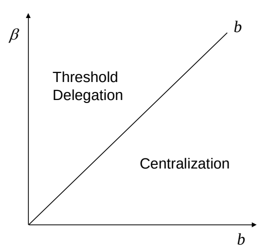

Figure 5

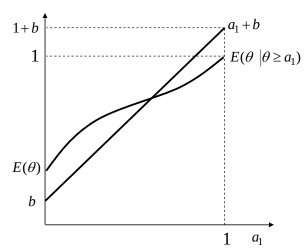

Figure 6