Understanding Migration Responses to Local Shocks*

Kirill Borusyak†, Rafael Dix-Carneiro‡ and Brian K. Kovak§

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Abstract

We examine how to interpret estimates from a commonly used migration regression relating changes in local population or employment to exogenous local labor demand shocks. Using a simple model of local labor markets with mobility costs, we find that common conclusions drawn from migration regression estimates are likely to be substantially misleading. Intuitively, the conventional migration regression is misspecified due to the bilateral nature of location choices. Workers choose where to live based not only on the shock to their current location, but also on the shocks to potential alternative locations, which are omitted from the regression. Analytical results and simulations based on Brazilian data show that conventional migration regression estimates are inaccurate for the within-sample effects of observed shocks on local populations and uninformative on the effects of counterfactual shocks to local labor demand. These problems are particularly acute when workers face industry switching costs in addition to geographic mobility costs.

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*All errors are our own.
†Department of Economics, University College London (k.borusyak@ucl.ac.uk)
‡Department of Economics, Duke University (rafael.dix.carneiro@duke.edu)
§Heinz College, Carnegie Mellon University (bkovak@cmu.edu)
1 Introduction

Regional migration regressions are of central importance to literatures in labor, development, urban, and international economics.\textsuperscript{1} These conventional migration regressions relate changes in local population or employment to observed local labor demand shocks and take the form,

\[ \hat{L}_\ell = \beta_0 + \beta \hat{z}_\ell + \varepsilon_\ell, \]  

(1)

where \( \hat{L}_\ell \) is the proportional change in population or employment in location \( \ell \), \( \hat{z}_\ell \) is an exogenous local labor demand shock, and \( \varepsilon_\ell \) is the error term.

Research papers utilize this specification with various motivations. The welfare effects of a given shock depend upon whether workers can smooth adverse labor market outcomes by migrating (Yagan, 2014). The speed of migration adjustment partly determines how long the impacts of local shocks persist (Topel, 1986; Blanchard and Katz, 1992), and a lack of migration response may help explain long-lasting economic effects of changes in local labor demand (Dix-Carneiro and Kovak, 2017, 2019; Autor et al., 2021). Mobility responses determine how local shocks affect inter-regional inequality (Topalova, 2010; Cadena and Kovak, 2016), and differences in mobility responses across demographic groups can drive between-group inequality as well (Bound and Holzer, 2000; Dix-Carneiro and Kovak, 2015). If migration frictions are particularly large, there may be a role for policy to help workers relocate.\textsuperscript{2} To address these important questions, the estimate of \( \beta \) from (1) must accurately estimate the within-sample effects of the observed labor demand shocks and must be informative regarding the effects of potential counterfactual shocks.

In this paper, we examine what one can and cannot learn from conventional migration regression estimates. We do so using a simple model of local labor markets in which workers face mobility costs and have idiosyncratic preferences for living in different locations. In the context of this model, we find that most common interpretations of \( \beta \) are problematic and that conclusions drawn

\textsuperscript{1}Subsequent revisions will include a more complete set of citations and a thorough discussion of contributions relative to the prior literature.

\textsuperscript{2}For example, the U.S. Trade Adjustment Assistance program includes funding for distant job search activities and an allowance for relocation costs (Hyman, 2018).
from these estimates are likely to be substantially misleading. Analytical results and simulations show that conventional migration regression estimates provide inaccurate estimates of the within-sample effects of observed shocks on local populations and are uninformative regarding the effects of counterfactual shocks to one or many locations.

The central intuition emerging from our analysis is that the conventional migration regression is misspecified due to the bilateral nature of location choices. Workers choose where to live based not only on the shock to their current location, but also on the shocks to potential alternative locations. Because workers face migration costs that differ for each source-destination pair, the most important potential alternative locations will be those with relatively low migration costs and hence larger baseline migration flows. The impact of the misspecification in (1) will then be largest when the shocks used to estimate $\beta$ are correlated across these migrant-connected locations. For example, if a worker’s current location faces similar shocks to relevant alternative locations, there is little incentive to migrate, and the estimated value of $\beta$ will be close to zero. In contrast, when shocks are not correlated across connected regions, or if only a single region faces a shock, the population responses will be much larger. The conventional migration regression therefore reveals little about underlying migration costs or the population changes that might emerge in response to other shocks of similar magnitude but with different spatial correlation.

This problem is exacerbated when, in addition to migration costs, workers face frictions in switching industries. Many papers estimating (1) analyze local labor demand shocks with a shift-share structure in which, for example, $\hat{z}_\ell = \sum_n L^0_\ell n \frac{L^0_\ell n}{L^0_\ell} \hat{z}_n$, where $\hat{z}_n$ is a labor demand shock facing industry $n$, and $L^0_\ell n / L^0_\ell$ is industry $n$’s initial share of employment in location $\ell$. If industry switching costs are large, workers see minimal benefit to moving across locations because they primarily face the shock to their industry no matter where they choose to live. When labor demand shocks have an industry component, the presence of industry switching frictions thus reduces migration, beyond what one would observe in a setting with regional frictions alone. This is true even when spatial migration costs are low and purely regional shocks would drive a substantial migration response. This intuition suggests that although many papers using shift-share shocks in the conventional
migration regression find $\beta \approx 0$, this does not imply that regional mobility costs are prohibitive or that regional populations would be unresponsive to counterfactual shocks.\textsuperscript{3} This intuition is reflected in an extension of our model that incorporates frictional mobility across both regions and industries.

To understand the quantitative importance of these issues, we analyze simulated shocks based on real-world geography and migration patterns. By simulating shocks, we ensure that they are not confounded by unobservable labor supply or labor demand shocks that might arise in the context of real-world shocks.\textsuperscript{4} We use longitudinal administrative data on formally employed workers in Brazil from the Relação Anual de Informações Sociais (RAIS) covering 1994 to 2000. This census of formally employed workers allows us to observe worker transitions across locations and industries at an annual frequency. We simulate labor demand shocks following a variety of different data generating processes and use these shocks and the worker transitions observed in RAIS to generate model-implied regional population changes when workers face migration frictions alone or when facing both migration and industry frictions. The resulting population changes are then used to estimate the conventional migration regression, the results of which we compare to the true model-based effects of the simulated labor demand shocks.

The baseline simulation results suggest that these problems with the conventional migration regression are quantitatively large. Because the migration regression does not use information on the shocks in other migrant-connected regions and omits regional heterogeneity in migration intensity, it fits relatively poorly. This is particularly true when using industry shift-share shocks in the presence of location and industry frictions. The regression-based estimates therefore poorly predict the within-sample effects of the observed shocks themselves. Counterfactual estimates are even less accurate, particularly when the observed shock used to estimate $\beta$ and the counterfactual shocks

\textsuperscript{3}Papers finding $\beta \approx 0$ when using industry shift-share measures in the conventional migration regression include Autor et al. (2013, 2021) (except for age 25-39), Bound and Holzer (2000) (for less educated), Cadena and Kovak (2016) (for less educated native-born), Dix-Carneiro and Kovak (2017, 2019), and Topalova (2010), among many others.

\textsuperscript{4}As with any observational research design, the conventional migration regression will yield inconsistent estimates when the shocks are correlated with unobserved determinants of regional population growth. See, for example, Greenland et al. (2019), who argue that the population regression in Autor et al. (2013) was confounded by pre-existing trends in population growth.
have very different correlations between migrant-connected locations, implying different reductions in migration response.

These baseline simulations assumed that population changes were driven solely by observable labor demand shocks. We perform more realistic simulations including additional random variation from unobservable factors affecting population changes and vary the magnitude of this residual variation. We compare the predictive accuracy of the conventional migration regression against a model-consistent nonlinear least squares (NLLS) procedure that captures the spillovers across locations and industries implied by the worker transitions observed in the RAIS data.\(^5\) For all magnitudes of the unobserved component, the predictive performance of this NLLS procedure far outperforms that of the conventional migration regression, which often performed as poorly as an uninformative prediction of zero migration response. The model-based procedure has the best predictive power when estimated using baseline worker flows and employment changes at the location–industry level, and this relative performance is largest for industry shocks rather than regional shocks. These findings highlight the importance of accounting for observed connections between locations and industries and the value of leveraging data at the location–industry level.

The paper proceeds as follows. Section 2 describes the baseline model of local labor markets in which workers face costs of moving across locations, solves the model, and relates model-based expressions to observable quantities. Section 3 defines and characterizes the conventional migration regression coefficient and discusses problems when using the associated estimates to make within-sample or counterfactual predictions. It also describes the model-consistent NLLS procedure. Section 4 generalizes the model to include frictions in switching both locations and industries. Section 5 presents the RAIS data and descriptive statistics on worker transitions across locations and industries. Section 6 presents the simulation results, and Section 7 concludes.

\(^5\)We do not implement the NLLS procedure in the simulations without additional residual variation because it would fit perfectly by definition.
2 Simple Model of Mobility with Regional Frictions

2.1 Setup

This section presents a stylized model that yields clear and intuitive expressions for how local labor demand shocks affect local populations in the presence of costly mobility. Although this framework omits a number of potentially empirically relevant mechanisms, including housing and capital markets, it highlights important and general issues with standard regression-based approaches to understanding how changes in local populations respond to local shocks.

Consider a small open economy consisting of many sectors indexed by $n$ produced in many locations indexed by $\ell$. Products within a sector are differentiated by location of production. Product markets are frictionless so the price of a sector-$n$ good produced in location $\ell$, $p_{\ell n}$, is constant across locations. Homogeneous labor is the only input, and sector-location productivity is $z_{\ell n}$. Output and labor markets are perfectly competitive, so $p_{\ell n} = w_{\ell} / z_{\ell n}$, where $w_{\ell}$ is the wage in location $\ell$.

Individuals make optimal choices over consumption bundles and their location of employment, but face frictions in moving across locations. We assume that workers may migrate within the country of interest by choosing among locations $\ell \in \Lambda_c$, but not internationally. Each of $L$ workers inelastically supplies one unit of labor in their chosen location, so changes in employment are equivalent to changes in population. Utility from consumption is Cobb-Douglas across sectors and CES across location-specific varieties within sector.

$$
U = \sum_n \alpha_n \ln(Q_n), \quad \text{where } Q_n \equiv \left( \sum_{\ell \in \Lambda_w} x_{\ell n}^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}}
$$

Note that $Q_n$ aggregates over location-specific varieties produced around the world ($\ell \in \Lambda_w$), including those within the country of interest ($\Lambda_c \subset \Lambda_w$). Normalizing national aggregate expenditure
to one, the aggregate optimal demand bundle satisfies

\[ x_{\ell n} = p_{\ell n}^{-\sigma} P_n^{\sigma-1} \alpha_n, \tag{3} \]

where \( P_n \equiv \left( \sum_{\ell \in \Lambda_W} p_{\ell n}^{1-\sigma} \right)^{1-\sigma} \) is the CES exact price index for sector \( n \), including varieties produced globally. The small-country assumption implies that \( P_n \) is set on the world market and is exogenous to developments in the country of interest.

The indirect utility of choosing destination location \( d \) for individual \( i \) currently living in location \( \ell \) is

\[ V_{i\ell d} = \ln \left( \frac{w_{d}}{P} \right) - \ln \tau_{\ell d} + \frac{1}{\theta} \epsilon_{id}, \tag{4} \]

where \( P \equiv \Pi_n P_n^{\alpha_n} \) is the Cobb-Douglas exact price index across sectors, \( \ln \tau_{\ell d} \) is the utility cost of moving from \( \ell \) to \( d \), and \( \epsilon_{id} \) is an i.i.d. type-I extreme-value taste shock. The parameter \( \theta \) determines the strength of location preferences relative to the log real wage, with smaller values of \( \theta \) implying stronger location preference and larger frictions across locations.

### 2.2 Regional Labor Supply

Given the assumptions in (4), the probability an individual in location \( \ell \) chooses to live in location \( d \) is given by \( \pi_{\ell d} \).

\[ \pi_{\ell d} = \frac{(w_{d}/\tau_{\ell d})^\theta}{\sum_{d' \in \Lambda_c} (w_{d'}/\tau_{d'})^{\theta}} \tag{5} \]

We refer to these location-choice probabilities as “out-migration shares.” Labor supply to location \( d \) is determined by these out-migration shares and the initial distribution of workers across locations, \( L^0_\ell \).

\[ L_d = \sum_{\ell \in \Lambda_c} M_{\ell d} = \sum_{\ell \in \Lambda_c} \pi_{\ell d} L^0_\ell \tag{6} \]

where \( M_{\ell d} \equiv \pi_{\ell d} L^0_\ell \) is the number of people choosing to migrate from location \( \ell \) to location \( d \).

Now consider an economic shock leading to small changes in wages across (potentially all) locations while holding moving costs \( \tau_{\ell d} \) fixed. Let hats represent proportional changes relative to
a no-shock counterfactual, and assume small changes so \( \dot{x} \equiv dx/x = d\ln x \). Totally differentiating (6) yields

\[
\dot{L}_d = \theta \left( \hat{w}_d - \sum_{\ell \in \Lambda_e} \gamma_{ld} \sum_{d' \in \Lambda_e} \pi_{ld'} \hat{w}_{d'} \right). \tag{7}
\]

Note that \( \gamma_{ld} \equiv M_{ld}/L_d \) is the share of those in location \( d \) who arrived from location \( \ell \), which we refer to as the “in-migration share.” This expression describes how the quantity of labor supplied in location \( d \) responds to wage changes across all destinations and can be written in matrix notation as

\[
\dot{L} = \theta (I - \Gamma'\Pi) \dot{\mathbf{w}}. \tag{8}
\]

### 2.3 Regional Labor Demand

Given perfectly competitive markets, \( p_{\ell n} = w_{\ell}/z_{\ell n} \), and revenue to sector-\( n \) producers in location \( \ell \) is

\[
R_{\ell n} = \alpha_n \left( \frac{w_{\ell}/z_{\ell n}}{P_n} \right)^{1-\sigma}. \tag{9}
\]

Perfect competition implies that the regional wagebill equals total regional revenue across sectors.

\[
w_{\ell}L_{\ell} = \sum_n R_{\ell n} \tag{10}
\]

Plugging in (9) and rearranging yields regional labor demand.

\[
L_{\ell} = D_{\ell}w_{\ell}^{-\sigma}, \text{ where } D_{\ell} \equiv \sum_n \alpha_n (z_{\ell n}P_n)^{\sigma-1} \tag{11}
\]

The term \( D_{\ell} \) is a regional demand shifter affected by changes in productivity \( z_{\ell n} \) or industry price indexes \( P_n \).

Consider an economic shock that affects this demand shifter across locations in the country of
The total derivative of (11) is

\[ \hat{L}_\ell = \hat{D}_\ell - \sigma \hat{w}_\ell. \]  

(12)

This expression describes labor demand in location \( l \) and can be expressed in matrix notation as

\[ \hat{L} = \hat{D} - \sigma \hat{w}. \]  

(13)

### 2.4 Equilibrium Response to Shocks

Equating the labor supply and labor demand expressions in (8) and (13) and solving yields the equilibrium effects of regional labor demand shocks on regional population and wages.

\[ \hat{L} = (I - \Psi^{-1}) \hat{D} \]  

(14)

\[ \hat{w} = \frac{1}{\sigma} \Psi^{-1} \hat{D} \]  

(15)

where

\[ \Psi \equiv \left( 1 + \frac{\theta}{\sigma} \right) I - \frac{\theta}{\sigma} \Gamma \Pi. \]

(16)

Equation (14) shows the relationship between population changes and local demand shocks, i.e. the relationship whose empirical investigation we seek to inform. Two practical implications are immediately evident. First, the population change in a given location depends upon the shocks to all other locations. In fact, if all locations face the same shock so \( \hat{D}_\ell = \hat{D} \ \forall \ell \), then population in each location remains unchanged. The goal is to show that \( (I - \Psi^{-1}) \mathbf{1} \hat{D} = 0 \), where \( \mathbf{1} \) is a column vector of ones and \( \hat{D} \) is a scalar reflecting the identical shock in all locations. The preceding expression is equivalent to \( \Psi^{-1} \mathbf{1} = \mathbf{1} \) which in turn implies \( \Psi \mathbf{1} = \mathbf{1} \). We
depends upon migrant connections between the focal location and other locations, as reflected in \( \Gamma' \) and \( \Pi \) in \( \Psi \).

To gain intuition for the ways in which these migrant connections influence the effects of shocks on connected regions, consider an approximation in which mobility is low enough that the effects of indirect connections become unimportant. Specifically, write \( \Gamma = I + \Delta \Gamma \) and \( \Pi = I + \Delta \Pi \) and assume that \( \Delta \Gamma' \Delta \Pi \approx \Delta \Gamma' \Delta \Gamma \approx \Delta \Pi \Delta \Pi \approx 0 \). Plugging this into (16) and then (14) yields

\[
\hat{L}_\ell \approx \frac{\theta}{\sigma} \left( \sum_{o \in \Lambda_c} \gamma_{o\ell} (\hat{D}_\ell - \hat{D}_o) + \sum_{d \in \Lambda_c} \pi_{\ell d} (\hat{D}_\ell - \hat{D}_d) \right)
\]  

(17)

The local shock’s effect on local population depends upon how its value compares to shocks facing all other locations, with more weight placed upon shocks in the location’s typical migrant sources (captured by \( \gamma_{o\ell} \)) and its typical migrant destinations (captured by \( \pi_{\ell d} \)). This expression also shows that when mobility frictions are smaller (\( \theta \) is larger), more individuals choose to migrate in response to a given vector of shocks. Similarly, when location-specific product varieties are less substitutable (\( \sigma \) is smaller) and therefore regional labor demand is less elastic, a given vector of demand shifts leads to larger population changes.

We next must find \( \Psi^{-1} \). Let \( A \equiv \frac{\theta}{\sigma} (\Delta \Gamma' + \Delta \Pi) \) such that \( \Psi \approx I - A \). Then define the infinite series \( S = I - A + A^2 - A^3 \ldots \). Under the low-mobility assumption, \( \Psi \approx S \) since \( \Delta \Gamma' \Delta \Pi \approx \Delta \Gamma' \Delta \Gamma \approx \Delta \Pi \Delta \Pi \approx 0 \). Since \( S + SA = S(I + A) = I \), we have \( S^{-1} = I + A \) and in turn \( \Psi^{-1} \approx I + A \), which is equivalent to

\[
\Psi^{-1} \approx I + \frac{\theta}{\sigma} (\Delta \Gamma' + \Delta \Pi) .
\]

Plugging this into (14) and simplifying yields

\[
\hat{L} = \frac{\theta}{\sigma} (I - \Gamma' + I - \Pi) \hat{D} ,
\]

which is (17) in matrix notation.
2.5 Linking to Observables

The changes shown in the preceding expressions represent counterfactual differences between situations with and without the labor demand shocks $\hat{D}$. The migration shares in $\Gamma'$ and $\Pi$ are also based on migration flows that would have been observed in the absence of shocks. To make these unobserved counterfactuals equivalent to observable quantities, we assume the economy is initially at steady-state. Specifically, we assume the population in each location was initially stable in period 0, prior to the shock.

$$L^0_\ell = \sum_{o \in \Lambda} L^0_o \pi^0_{od}$$  \hspace{1cm} (18)

This assumption implies that in the absence of shocks, the population in each location would remain constant, so the counterfactual difference $\hat{L}_\ell$ corresponds to the observed change over time. It also implies that the counterfactual migration shares in the absence of shocks equal the observed migration shares from the pre-shock period, i.e. $\pi_{od}^0 = M^0_{od}/L^0_o$ and $\gamma_{od}^0 = M^0_{od}/L^0_d$.

We also assume that we do not directly observe the overall labor demand shocks $\hat{D}$, but instead observe a demand shifter $\hat{z}$ such that

$$\hat{D}_\ell = \hat{z}_\ell + \zeta_{1\ell},$$  \hspace{1cm} (19)

where $\zeta_{1\ell}$ is an error term satisfying $\mathbb{E}[\hat{z}_\ell | \zeta_1] = \mu$ for a constant $\mu$. Combining this formulation with the steady state assumption, the local population change in (14) becomes

$$\hat{L} = \left( I - (\Psi^0)^{-1} \right) \hat{z} + \zeta_2$$  \hspace{1cm} (20)

where $\zeta_2$ is a random error term determined by $\zeta_1$, and $\Psi^0$ is calculated using the pre-shock steady-state values of $\Gamma^0$ and $\Pi^0$.

We can similarly express the low-mobility approximation (17) in terms of observed shocks and pre-shock migration patterns. To aid in proving Theorem 1 below, it is helpful to represent the approximation using a slightly different form than that shown in (17). Letting $M^0_\ell$ be the steady-
state number of migrants into or out of location $\ell$, i.e. $M_{\ell}^0 \equiv \sum_{k \neq \ell} M_{k\ell}^0 = \sum_{k \neq \ell} M_{k\ell}^0$, we have

$$\hat{L}_{\ell} \approx 2 \frac{\theta}{\sigma} \frac{M_{\ell}^0}{L_{\ell}^0} (\hat{z}_{\ell} - \hat{z}_{-\ell}) + \zeta_{3\ell},$$

(21)

where $\hat{z}_{-\ell} \equiv \sum_{k \neq \ell} \frac{1}{2} \left( \frac{M_{k\ell}^0 + M_{\ell k}^0}{M_{\ell}^0} \right) \hat{z}_{k}$, where $\zeta_3$ is a random residual determined by $\zeta_1$. This expression highlights a number of features that determine how a given vector of shocks affects local population, each of which will be important as we interpret the results of conventional migration regressions in the next section. As discussed above, population responds more strongly when mobility frictions are smaller and labor demand is more elastic, i.e. when $\theta/\sigma$ is larger. Regions with more steady-state migration, i.e. larger $M_{\ell}^0/L_{\ell}^0$, respond more strongly as well. Finally, population responses are larger when the local shock, $\hat{z}_{\ell}$, is larger in comparison to a migration-weighted average of shocks in other regions, $\hat{z}_{-\ell}$.

### 3 Conventional Regression and Model-Consistent Procedures

The model in the preceding section describes how local populations change in response to labor demand shocks across locations. In this section, we investigate how to interpret the results of conventional migration regressions when the data generating process follows the model just described.

#### 3.1 The Conventional Migration Regression: Defining $\beta$

We consider a conventional approach to estimating the impact of labor demand shocks on local population, via Ordinary Least Squares (OLS) regression

$$\hat{L}_{\ell} = \beta_0 + \beta \hat{z}_{\ell} + \varepsilon_{\ell},$$

(22)
with observations weighted by the pre-period population \( L_0^\ell \). The corresponding OLS estimator is

\[
\hat{\beta} = \frac{\sum_\ell L_0^\ell \hat{L}_\ell (\hat{z}_\ell - \bar{z})}{\sum_\ell L_0^\ell (\hat{z}_\ell - \bar{z})^2},
\]

(23)

where \( \bar{z} = \sum_\ell L_0^\ell \hat{z}_\ell / \sum_\ell L_0^\ell \).\(^{10}\)

We do not assume that equation (22) is correctly specified. Instead, we assume that the data on \( \hat{L}_\ell \) are generated according to the model of Section 2: equation (20) or its approximation (21). We also maintain the assumption that \( \{\hat{z}_\ell\}_{\ell \in \Lambda_c} \) are as-good-as-randomly assigned to regions with respect to all unobserved shocks to labor demand and supply, although they may be mutually correlated (as with shift-share variables).

Since \( \hat{\beta} \) has random variation due to unobserved shocks, we focus on the *estimand* of regression (22), which we define as

\[
\beta = \frac{\mathbb{E} \left[ \sum_\ell L_0^\ell \hat{L}_\ell (\hat{z}_\ell - \bar{z}) \right]}{\mathbb{E} \left[ \sum_\ell L_0^\ell (\hat{z}_\ell - \bar{z})^2 \right]},
\]

replacing the numerator and denominator of \( \hat{\beta} \) with their expectations taken over observed and unobserved shocks (while viewing the period-0 equilibrium as fixed). This formulation applies in finite samples of regions and thus allows us to avoid random sampling assumptions, which are inappropriate in typical applications, which include all regions of the country. Yet, \( \beta \) can also be viewed as approximating the probability limit of \( \hat{\beta} \) in an asymptotic sequence of economies with growing numbers of regions, under appropriate regularity conditions (see Appendix A.1 for a formalization). That is, in large samples where the impact of the unobserved shocks on \( \hat{\beta} \) vanishes, we expect \( \hat{\beta} \approx \beta \).

### 3.2 Characterizing \( \beta \)

We now characterize \( \beta \) when the data were generated by the model described in Section 2. To make the results intuitively interpretable, we focus throughout on the low-mobility approximation to \( \hat{L}_\ell \), shown in (21).

\(^{10}\)Similar, although more cumbersome, results can be obtained for unweighted regressions used in some studies.
To understand our characterization, it is helpful to define some notation. Let $M^0 \equiv \sum_{o\neq d} M^0_{od}$ be the steady-state total number of migrants in the country of interest, observed in the pre-shock period, so $M^0/L^0$ is the steady-state migrant share of the population. We then define similar quantities that would prevail if there were no migration costs, i.e. if $\tau_{od} = 0 \forall o,d$. Without migration costs, the steady-state probability of moving to any destination will equal the destination’s share of national population, regardless of the origin. Define $\tilde{M}^0_{od} \equiv L^0_o \cdot \tilde{L}^0_d L^0_o$ as the number of migrants from $o$ to $d$ and $\tilde{M}^0 \equiv \sum_{o\neq d} \tilde{M}^0_{od}$ as the total number of migrants in this costless-migration setting. The following theorem then characterizes $\beta$ estimated with data generated by the model.

**Theorem 1.** Suppose the data are generated by the low-mobility approximation (21) and the shocks are homoskedastic, i.e. $\text{Var} [\hat{z}_\ell]$ is the same for all $\ell$. Then

$$\beta = \frac{2\theta}{\sigma} \cdot \frac{M^0/L^0}{\tilde{M}^0/L^0} \cdot \frac{1 - \rho}{1 - \tilde{\rho}}, \tag{24}$$

where $\rho = \sum_{o\neq d} \frac{M^0_{od}}{M^0} \text{Corr} [\hat{z}_o, \hat{z}_d]$ is the average correlation between shocks to pairs of distinct regions weighted by the migration flows and $\tilde{\rho} = \sum_{o\neq d} \frac{\tilde{M}^0_{od}}{\tilde{M}^0} \text{Corr} [\hat{z}_o, \hat{z}_d]$ is a similar shock correlation weighted by the flows in the no-migration-cost scenario.

Appendix A.2 proves this result, along with its generalization to heteroskedastic shocks (equation (36)).

Equation (24) shows that $\beta$ can be viewed as a product of several factors with distinct economic meanings. First, $\beta$ depends on the structural elasticities: as discussed above, true migration responses are larger when $\theta$ is large or $\sigma$ is small, and $\beta$ inherits those relationships. Second, $\beta$ increases with the national migration share $M^0/L^0$. Third, $\beta$ is smaller when $\rho > \tilde{\rho}$; that is, if shocks are mutually correlated and particularly so between regions with high migration flows, relative to the sizes of both regions—for instance, if migration is more likely at close geographic distances and shocks exhibit spatial correlation. We call the term $\frac{1 - \rho}{1 - \tilde{\rho}}$ the *attenuation factor*: it reflects how the mutual correlation of shocks along the migration network attenuates the OLS coefficient. The final
term, $\tilde{M}^0/L^0$, is generally close to one and can therefore be ignored in practice. \footnote{Specifically, it is easy to verify that $1 - \tilde{M}^0/L^0$ equals the Herfindahl index of population across regions, which is small when the number of regions is sufficiently large and population is not too concentrated in a small number of them, as in our data.}

### 3.3 $\beta$ Is Not Informative about Effects of Counterfactual or Observed Shocks

The expression for $\beta$ in Theorem 1 informs us about how one can and cannot interpret estimates from conventional migration regressions. Here, we will consider various potential interpretations and compare them to the correct interpretation in light of the model.

**Counterfactual shock to a single region.** One potential interpretation of $\beta$ is the effect of a labor demand shock to a single location on that location’s population. Specifically, one might interpret $\beta$ as the change in population in location $\ell$ resulting from a unit shock in that location while shocks in other locations are zero. Equation (21) shows that, according to the model (with low mobility and homoskedastic shocks), when $\hat{z}_\ell = 1$ and $\hat{z}_{-\ell} = 0$ expected population growth in $\ell$ is $2^\theta \sigma M^0_{\ell} L^0_{\ell}$. The estimate of $\beta$ in (24) differs from this model-implied population growth in two important ways. First, it misses location-specific heterogeneity in the effect of a local shock; the model-implied population growth depends on the location-specific migration share $M^0_{\ell} L^0_{\ell}$, while the regression estimate depends on the national migration share $M^0_{\ell}$. As we will see in Section 5.2, local migration shares often vary substantially in practice, so this heterogeneity can be practically important.\footnote{This problem would not arise if the outcome variable of the regression was the net population change normalized by the initial gross migration flow, rather than initial population.}

Second, the estimate of $\beta$ includes the attenuation factor $\frac{1}{1-\rho}$ reflecting the correlation in shocks among locations with strong migrant connections. The intuition behind the attenuation factor is straightforward. Strongly migrant-connected locations have relatively low migration costs. If these locations faced similar shocks in the data, individuals had minimal incentive to migrate. We then observe minimal migration response and estimate a small value of $\beta$. Yet, this estimate will...
understate the response that would be observed when shocking only a single location, because there will be a substantial incentive to migrate between locations with relatively small migration costs. Because of this attenuation factor, a researcher may estimate a small value of $\beta$ that is statistically indistinguishable from zero even in a setting where a shock to any single location would actually lead to a substantial migration response. As we will see in Section 6, substantial attenuation is quite likely in realistic empirical settings.

**Observed shock used in estimation.** Although the population regression estimate is not informative regarding the effect of a counterfactual shock to a single location, one might hope that it is informative about the effects of observed shocks. Put differently, perhaps one can use $\beta$ to make *internally valid* inferences regarding the effects of observed shocks even when *externally valid* predictions regarding counterfactual shocks are not possible. Unfortunately, even this much more modest objective is unlikely to be achieved using the conventional migration regression.

First consider estimating the effect of the observed shocks on the population change in location $\ell$ using the fitted values from the conventional regression, $\beta \hat{z}_\ell$ where $\hat{z}$ is the observed set of shocks used to estimate $\beta$. This approach has a clear shortcoming: it is likely to imply changes in the national population. For example, if all of the shocks have the same sign, as is often the case in practice, the analysis would predict either population growth in all locations or population decline in all locations.

A more sensible approach attributes to the shock only the effect relative to the mean, $\beta (\hat{z}_\ell - \bar{z})$, consistent with the inclusion of an intercept term $\beta_0$ in the conventional regression (22). This approach yields the best (mean squared error (MSE) minimizing) prediction of the expected population growth in location $\ell$ among linear functions of $\hat{z}_\ell - \bar{z}$. However, using $\hat{z}_\ell - \bar{z}$ alone to predict local population growth is likely to yield very poor predictions. To see why, use the decomposition

\[ \beta (\hat{z}_\ell - \bar{z}) \]

This approach parallels a difference-in-differences interpretation in which the *difference* in population growth between two locations $k$ and $\ell$ is estimated as $\beta (\hat{z}_\ell - \hat{z}_k)$. This difference could involve comparing realized shocks facing two actual locations or comparing hypothetical locations facing shock values representing different points in the shock distribution, e.g. the standard deviation or interquartile range. The shortcomings described here apply to this difference-in-differences interpretation as well.

\[ \beta (\hat{z}_\ell - \bar{z}) \]  

We do not assume *iid* shocks across locations, which makes our results applicable to shift-share research designs. See Appendix A.3 for a proof that $\beta (\hat{z}_\ell - \bar{z})$ yields the MSE-minimizing predicted population growth in this setting.
of $\beta$ in (24) to write

$$
\beta (\hat{z}_\ell - \bar{z}) = \frac{2\theta}{\sigma} \cdot \frac{M^0/L^0}{\hat{M}^0/L^0} \left( (\hat{z}_\ell - \bar{z}) - \frac{\rho - \bar{\rho}}{1 - \bar{\rho}} \cdot (\hat{z}_\ell - \bar{z}) \right).
$$

(25)

Compare this prediction to (21), the model’s expected population growth in $\ell$ when facing the observed vector of shocks: $\frac{2\theta}{\sigma} \cdot \frac{M^0/L^0}{L^0} ((\hat{z}_\ell - \bar{z}) - (\hat{z}_{-\ell} - \bar{z}))$. The estimate $\beta (\hat{z}_\ell - \bar{z})$ omits two key sources of variation. As before, it omits heterogeneity based on the local migration intensity, $\frac{M^0}{L^\ell}$. Even more importantly, it replaces the shocks to other migration-connected locations, $\hat{z}_{-\ell} - \bar{z}$, with the scaled local shock $\frac{\rho - \bar{\rho}}{1 - \bar{\rho}} (\hat{z}_\ell - \bar{z})$. Shocks to other locations are important drivers of local population change according to the model and can differ substantially among locations facing similar direct shocks, but this information is omitted from (25). Because the fitted values from the population regression only use information on the direct shocks to the location, they cannot account for the extent to which other migrant sources and destinations faced similar or different shocks. As we will see in Section 5.2, the omission of these cross-location spillover effects can be quantitatively important.

**Counterfactual shocks to multiple regions.** Estimating the effect of counterfactual shocks combines the problems just discussed. As when estimating the effect of the observed shock, predictions based on $\beta$ from the conventional migration regression omit heterogeneity based on the local migration share and fail to incorporate information on shocks to other migrant-connected locations under the counterfactual of interest. However, the latter problem is more severe with counterfactual shocks. Because the spatial correlation structures of the observed and counterfactual shocks may differ, it is likely that the two sets of shocks imply different values of $\rho$. Thus, in addition to the problems faced when estimating the effects of the observed shocks, $\beta (z_\ell - \bar{z})$ is no longer the best linear predictor of local population changes among linear functions of $z_\ell - \bar{z}$, because the value of $\rho$ implicit in $\beta$ is incorrect under the counterfactual shocks. This difference in the spatial correlation of shocks can attenuate or amplify the estimated effect of counterfactual shocks relative to the model-predicted effect.
In none of these cases is the conventional regression likely to yield credible estimates of the effects of regional shocks on local population growth. A given empirical analysis may find an estimate of $\beta$ that is close to zero even in contexts where the observed shock actually had important effects on migration and where counterfactual shocks would drive large population changes.\footnote{Some analyses estimate the conventional migration regression in (22) not to draw conclusions about the effects of shocks on local populations, but to assess whether interregional migration is likely to substantially affect estimates of the effect of local shocks on local economic outcomes such as wages. We discuss this through the lens of the model in Appendix A.4. The analysis finds that a conventional regression estimate of $\beta$ that is close to zero does suggest that wage regression estimates will experience minimal confounding, regardless of the mechanism driving the small estimate of $\beta$.} We now describe an alternative empirical approach that integrates information on regional shocks and pre-shock migration patterns to estimate migration responses in a way that is consistent with the model.

### 3.4 A Model-Consistent Specification

The model-implied relationship between local population changes and the vector of local labor demand shocks is given by equation (20) above, rearranged slightly here.

$$\hat{L} = \left(I - \left(I + \frac{\theta}{\sigma} (I - \Gamma^0 \Pi^0) \right)^{-1}\right) \hat{z} + \zeta_2$$

We observe the population change $\hat{L}$, the pre-shock migration matrices $\Gamma^0$ and $\Pi^0$, and the shocks $\hat{z}$, so we can estimate the parameter $\theta/\sigma$ using non-linear least squares (NLLS). Formally, in Appendix A.5 we show that $\frac{\theta}{\sigma}$ uniquely solves the NLLS problem\footnote{Borusyak and Hull (2021, Appendix D.5) show in a similar setting that a “recentering” adjustment is generally necessary for the appropriate moment condition to hold under the as-good-as-random assignment of the shocks, $\mathbb{E} [\hat{\zeta}_t \mid \zeta_t] = \mu$. That issue does not arise in our setting because NLLS estimates are invariant to $\mu$: adding a constant shock has no implications for population changes under any $\theta/\sigma$. Recentering would be necessary, however, if the shocks were conditionally as-good-as-randomly assigned: e.g. to manufacturing industries only, as in many empirical settings.}

$$\min_{\eta} \mathbb{E} \left[ \left( \hat{L} - (I - \Psi^0(\eta)^{-1}) \hat{z} \right) \left( \hat{L} - (I - \Psi^0(\eta)^{-1}) \hat{z} \right)^{\prime} \right],$$

where $\Psi^0(\eta) \equiv (I + \eta (I - \Gamma^0 \Pi^0))$.}
Given the resulting estimate of $\theta/\sigma$ we can calculate shock-induced population changes in each location using the same relationship in (26), omitting the error term. This process can be used to estimate internally valid effects of the observed shocks used to estimate $\theta/\sigma$ in (26) and externally valid effects of counterfactual shocks.

Two caveats should be kept in mind here. First, we assume that the researcher has available a set of exogenous shocks to local labor demand (defined in (19)) with which to estimate $\theta/\sigma$. Any endogeneity in the shocks will lead to inconsistency in both the proposed NLLS procedure and the conventional migration regression. Second, our model is purposefully stylized in order to focus on the conventional migration regression’s failure to account for shocks to other migrant-connected locations. The model does not include other mechanisms, e.g. housing and capital markets. Such model misspecification may lead to inconsistent estimates of the effects of labor demand shocks on local populations even in the proposed NLLS procedure. We nonetheless find our relatively parsimonious model enlightening regarding the cross-location spillover problem facing conventional migration regressions and expect estimates following (26) to substantially resolve this problem, if not all others.

4 Extension to Frictional Labor Mobility across Industries

The model described in the preceding section allows for frictions in moving across locations, but assumes costless mobility across industries within a given location. Here, we extend the model to allow for costly mobility across both industries and locations by applying the same analysis at the location-industry level. For example, the indirect utility in (4) could be written identically, but the subscript $\ell$ would refer to the worker’s current location-industry pair and $d$ would refer to the location-industry pair that the worker might choose. It will be helpful to make this distinction explicit by defining the initial location and industry as $\ell$ and $n$ and the new location and industry as $d$ and $q$. To clarify the distinction between prior and current location-industry information, we separate the subscripts with a comma. Workers face a moving cost $\tau_{\ell n,dq}$ when switching from
location-industry pair $\ell n$ to pair $dq$, and $\theta$ is the wage elasticity of switching from one location-industry pair to another. Note that, for simplicity, this setup assumes that the same parameter $\theta$ applies to both the location and industry dimensions.\footnote{As discussed in footnote 27, available estimates suggest that $\theta$ is similar across locations and industries, supporting the restriction of a single parameter in both dimensions.}

The model extends directly to the location-industry context, and we again make use of the low-mobility approximation to derive an intuitive expression relating labor demand shocks to population changes at the location-industry level.\footnote{Note that the low-mobility approximation may perform more poorly in the location-industry context, since there may be substantial mobility across industries within location.} Applying (17) and (19) in that setting yields

$$
\hat{L}_{\ell n} \approx \frac{\theta}{\sigma} \left( \sum_{o \in \Lambda_c} \sum_{p} \gamma_{op,\ell n}^0 (\hat{z}_{\ell n} - \hat{z}_{op}) + \sum_{d \in \Lambda_c} \sum_{q} \pi_{\ell n,dq}^0 (\hat{z}_{\ell n} - \hat{z}_{dq}) \right) + \zeta_{\ell n},
$$

where $\gamma_{op,\ell n}^0 = M_{op,\ell n}^0 / L_{\ell n}^0$ is the share of workers in location-industry cell $\ell n$ who came from cell $op$ in the pre-shock steady state, $\pi_{\ell n,dq}^0 = M_{\ell n,dq}^0 / L_{\ell n}^0$ is the share of workers in cell $\ell n$ who went to cell $dq$, the $\hat{z}_{\ell n}$ are observable labor demand shocks in cell $\ell n$, and $\zeta_{\ell n}$ is an error term such that $\hat{z}_{\ell n}$ is mean independent of $\zeta$, as above.

This model allows us to interpret the conventional location-level migration regression when the data reflect a setting with frictions across both industries and locations. The conventional regression remains identical to equation (22), where we aggregate across industries to yield the location’s change in population $\hat{L}_\ell = \sum_n L_{\ell n}^0 \hat{L}_{\ell n}$ and the location’s average labor demand shock $\hat{z}_\ell = \sum_n \frac{L_{\ell n}^0}{L_{\ell}^0} \hat{z}_{\ell n}$.

To illustrate the regional population responses in this more general model, we contrast scenarios in which the shocks are purely regional vs. purely industry-level. In the case of purely regional shocks, i.e. when $\hat{z}_{\ell n} = \hat{z}_\ell \forall \ell, n$, we find population responses that are identical to those in the model with regional frictions alone, and therefore the same problems arise. Simplifying equation...
(28) yields

\[
\hat{L}_\ell \approx \frac{\theta}{\sigma} \left( \sum_n \sum_{o \in \Lambda_c} \sum_p \frac{M_{op,\ell n}}{L^0_\ell} (\hat{z}_\ell - \hat{z}_o) + \sum_n \sum_{d \in \Lambda_c} \sum_q \frac{M_{\ell n,dq}}{L^0_\ell} (\hat{z}_\ell - \hat{z}_d) \right) + \zeta_\ell
\]  

(29)

where \(\zeta_\ell \equiv \sum_n \frac{L^0_{\ell n}}{L^0_\ell} \zeta_{\ell n}\). This expression is identical to (17) in the model with location frictions only (expressed using the observable component of the labor demand shocks \(\hat{z}\) rather than the overall shocks \(\hat{D}\)). Because the two situations imply the same population changes (given the low-mobility approximation), they imply the same values for \(\beta\) in the conventional migration regression. In other words, when the shock is purely regional, the presence of industry frictions does not alter migration behavior, and all of the problems of interpretation discussed in Section 3.3 apply to the model with location and industry frictions: the conventional migration regression estimates do not accurately reflect the effects of the observed shocks used to estimate \(\beta\) nor are they informative about the effects of counterfactual shocks.\(^{19}\)

While purely regional shocks yield identical migration behavior in the models with and without industry switching frictions, the two models yield very different results with purely industry-level labor demand shocks, i.e. when \(\hat{z}_{\ell n} = \hat{z}_n \forall \ell, n\). In a regional migration analysis, industry-level shocks imply a shift-share structure, in which the average shock facing workers in location \(\ell\) is an employment-weighted average of industry shocks: \(\hat{z}_\ell = \sum_n \frac{L^0_{\ell n}}{L^0_\ell} \hat{z}_n\). When switching industries is costless, as in the baseline model of Section 2, this shift-share measure captures the regional labor demand shock facing all workers in location \(\ell\), i.e. the shift-share measure is the regional labor demand shock. In contrast, when industry switching frictions are present, workers in the same location but in different industries experience different labor demand changes and have different incentives to migrate. We therefore find distinct migration behavior in models with and without industry frictions when facing industry-level labor demand shocks.

\(^{19}\)This conclusion applies to any location-specific shock irrespective of its source or level of aggregation. This includes state-level shocks that are common to many locations.
We formalize this intuition by plugging the assumption \( \hat{z}_{\ell n} = \hat{z}_n \) \( \forall \ell, n \) into (28) and simplifying.

\[
\hat{L}_\ell \approx \frac{\theta}{\sigma} \left( \sum_n \sum_{o \in \Lambda_c} \sum_p \frac{M^0_{o p, \ell n}}{L^0_{\ell}} (\hat{z}_n - \hat{z}_p) + \sum_n \sum_{d \in \Lambda_c} \sum_q \frac{M^0_{\ell n, dq}}{L^0_{\ell}} (\hat{z}_n - \hat{z}_q) \right) + \zeta_\ell \quad (31)
\]

\[
= 2 \frac{\theta}{\sigma} \left( \hat{z}_\ell - \frac{1}{2} \sum_p \frac{M^0_{p, \ell}}{L^0_{\ell}} \hat{z}_p - \frac{1}{2} \sum_q \frac{M^0_{\ell, q}}{L^0_{\ell}} \hat{z}_q \right) + \zeta_\ell, \quad (32)
\]

where \( M^0_{p, \ell} \) is the number of workers in location \( \ell \) who previously worked in industry \( p \), and \( M^0_{\ell, q} \) is the number of workers previously in location \( \ell \) currently working in industry \( q \). In this case, migration is determined by the difference between the shift-share shock facing location \( \ell \), \( \hat{z}_\ell \), and the average shocks facing the former industries of workers in that location, \( \sum_p \frac{M^0_{p, \ell}}{L^0_{\ell}} \hat{z}_p \), and those facing the current industries of workers who were previously in that location, \( \sum_q \frac{M^0_{\ell, q}}{L^0_{\ell}} \hat{z}_q \).

To illustrate the implications of (31) for migration behavior, consider an extreme scenario in which workers can not switch industries, formalized by \( \tau_{\ell n, dm} = \infty \) for all \( \ell, d, \) and \( n \neq m \). In that case, although regional shocks will induce migration as in (29), pure industry shocks will not. Because workers can not change industries, those migrating to or from \( \ell \) in steady-state will have the same industry mix as those in \( \ell \), so the terms on the right hand side of (31) cancel out, and \( \hat{L}_\ell = 0 \) \( \forall \ell \). Thus, in this scenario, \( \beta \) will be positive for regional shocks but zero for industry shocks. This extreme example illustrates the more general point that industry-switching frictions reduce migration responses when labor demand shocks have an industry-specific component. Intuitively, if switching industries is difficult, workers see less benefit in moving to a location with a more favorably affected industry mix because they would still face much of their original industry’s shock in the new location.

Given the migration behavior implied by (31), how can we interpret the results of the conventional migration regression? Consider interpreting \( \beta \) as the effect of a counterfactual unit labor demand shock in a single location on that location’s population. As just discussed, the regression estimate will be even further attenuated than in the purely regional context by the presence of industry-switching costs, since industry frictions disincentivize regional mobility when facing industry-level shocks. Using \( \beta \) to estimate the effect of the observed shocks on regional populations...
is similarly problematic for reasons that parallel those discussed in Section 3.3. The conventional migration regression uses the regional shift-share measure $\hat{x}_l$ alone to estimate its direct effect in location $l$ and the spillover effect of the complex averages of shocks in the former and subsequent industries of the location’s workers shown in (31). As the simulations in Section 6 will reveal, these estimates will be even less accurate than those in a context with regional frictions alone (and, therefore, regional shocks).

This discussion reveals how industry frictions undermine the intuitive appeal of measuring local labor demand shocks using shift-share measures. Only when industry frictions are absent does the industry shift-share represent a true regional labor demand shock facing all local workers. With industry frictions, workers in different industries face different shocks and have different outside options, leading to complex spillovers of the effects of shocks across industries and regions. Yet, all of these complexities are omitted by the conventional migration regression, exacerbating the biases examined in Section 3.3.

5 Data and Descriptive Statistics

To assess the practical importance of the conceptual problems with the conventional migration regression that we have discussed in Sections 2 and 4, we now examine these issues empirically using data on observed worker transitions across locations and industries.

5.1 Data

The NLLS approach described in Section 3.4 and the model simulations presented in Section 6 require information on worker transitions across locations or across location-industry pairs, allowing us calculate the steady-state transition matrices $\Gamma^0$ and $\Pi^0$. We do so using administrative panel data covering all formally employed workers in Brazil, allowing us to observe these detailed transitions across locations and industries. Subsequent revisions will present results using similar data in other countries.
We utilize data from the Relação Anual de Informações Sociais (RAIS) covering 1994 to 2000. This administrative dataset is a census of the Brazilian formal labor market that allows us to follow all formally employed workers across jobs in different industries and locations (De Negri et al., 2001; Saboia and Tolipan, 1985). Locations in our analysis are based on the “microregion” definition of the Brazilian Statistical Agency (IBGE), which combines economically integrated contiguous municipalities (counties) with similar geographic and productive characteristics (Instituto Brasileiro de Geografia e Estatística, 2002). We aggregate microregions slightly to ensure consistent boundaries over time, following Dix-Carneiro and Kovak (2017), yielding 486 time-consistent microregion locations in Brazil’s 27 states. Industries are based on the “Subsetor IBGE” classification the RAIS dataset uses to identify the industry associated with each employer. This classification identifies 25 distinct industries including 12 manufacturing industries.

Our sample includes individuals age 18 to 64 with positive earnings in December of the relevant year. We also drop individuals with missing or inconsistent information, including “other/ignored” industries, contradictory education levels across jobs in the same year, holding jobs in very distant geographic areas, etc. If a worker holds multiple jobs, we select the job with the highest December earnings and use it to assign the worker’s location and industry of employment. When examining differences by skill level, we define skilled workers as those with a high school degree or more and unskilled workers as those without a high school degree.

5.2 Descriptive Evidence

We use the RAIS data to calculate yearly worker flows between location-industry pairs. Because we use detailed geographic and industry information, workers may transition between $12,150 = 486 \times 25$ Formality is defined as having a signed work card (carteira assinada), giving workers the right to benefits and protections of the legal employment system. The restriction to formal workers is important in the Brazilian context, as informality rates measured in the Brazilian Census exceed 50% during our sample period (Dix-Carneiro and Kovak, 2019).

As Dix-Carneiro and Kovak (2017) report using Census data, only 3.4 percent of individuals lived and worked in different microregions in the year 2000, so the microregion of employment is a quite accurate measure of both employment and residence location.

RAIS reports earnings for December and average monthly earnings during employed months in the reference year. We use December earnings to ensure that our results are not influenced by seasonal variation or month-to-month inflation.
Table 1: Average Mobility Rates, %

<table>
<thead>
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<th></th>
<th>Across</th>
<th>All Workers</th>
<th>High-Skilled</th>
<th>Low-Skilled</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4.2</td>
<td>4.2</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>States</td>
<td>0.9</td>
<td>0.8</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Industries</td>
<td>8.0</td>
<td>8.3</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>Locations and Industries</td>
<td>1.3</td>
<td>1.3</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Percent of individuals with yearly transitions between the cells defined in the first column, calculated from average yearly flows between location-industry cells in RAIS data from 1994 to 2000 as described in footnote 23. The last row measures the fraction of workers who change both location and industry in the same year. High-skilled is defined as having a high-school degree or more and low-skilled as less than high school.

location-industry cells. Consistent with the steady-state assumption and to reduce noise in estimating transitions between small cells, we average the observed yearly flows between each location-industry cell from 1994 to 2000 and use these averages to calculate migration flows, employment levels, and employment changes both at the location and location-industry levels. Because RAIS only covers formal employment, we do not observe workers who are non-employed or are informally employed. In these cases, the workers are omitted from the flow calculations when they are not observed in adjacent years.

Table 1 reports average yearly mobility rates between locations (microregions), between the 27 Brazilian states, between industries, and between locations and industries. 4.2 percent of workers migrated across locations in a given year on average. Industry transitions are more common than migration events, and workers are much more likely to move across locations within states than across states. Low-skilled workers are slightly more likely to migrate than are high-skilled workers, but are less likely to switch industries. We also find substantial heterogeneity in migration rates across locations. Figure 1 shows a histogram of \(1 - \frac{n^0_{\ell t}}{n_{\ell t}}\), the share of individuals in \(\ell\) who moved to a different location (weighted by the region’s initial employment). As discussed in Section 3.3, this heterogeneity in baseline migration intensity is important because it drives heterogeneity in the true effect of local labor demand shocks on local population (as in (21)), but this heterogeneity is

\[23\] Specifically, define \(flow_{t, t+1}^{\ell n, dq}\) as the observed number of workers moving from cell \(\ell n\) to cell \(dq\) between year \(t\) and \(t + 1\). Then calculate the average across years as \(\overline{flow}_{t, t+1}^{\ell n, dq} = \frac{1}{6} \sum_{t = 1994}^{1999} flow_{t, t+1}^{\ell n, dq}\). These average flows are then aggregated to measure the initial employment levels, local industry compositions, and transition matrices.
Figure 1: Geographic Mobility Rates Across Regions

Histogram of Mobility Rates Across Regions $(1 - \pi_{tt})$

Notes: Percent of individuals migrating across locations (microregions), calculated from average yearly flows between location-industry cells in RAIS data from 1994 to 2000 as described in footnote 23. The histogram weighted by each location’s initial population.

omitted from the estimate of $\beta$ in the conventional migration regression (as in (24)).

As discussed in Section 2.5, we assume that the labor market is in steady-state, so observed transitions across locations and industries reflect the relevant counterfactual transitions absent shocks. To assess the validity of this assumption in the Brazilian context, we estimate the ergodic distribution of employment across locations by computing $(\Pi^0)^\infty$ and comparing it against the observed spatial distributions of employment. We also do the same for the matrix of industry transitions.

Figure 2 shows scatter plots comparing the observed and ergodic log shares of employment at the location (panel a) and industry (panel b) levels. In both cases, the observed employment shares closely match the ergodic shares implied by the transition matrix. This similarity supports the assumption that the Brazilian labor market was close to steady state across regions and industries.

The analyses in Sections 3.3 and 4 emphasized the role of spillovers between connected locations.

---

24 We also find substantial heterogeneity across industries in the probability of switching industries, with rates ranging from close to zero up to 20%.

25 To be precise, we calculate $\Pi^0$ to the power 10,000 where $\Pi^0$ is the observed matrix of location or industry transitions. We do not repeat this analysis by location-industry cells because of the large number of small cells, for which the relative difference between the observed and ergodic sizes is noisy and uninformative.
Figure 2: Actual vs. Ergodic Employment Distribution

Panel (a): Across Locations

Panel (b): Across Industries

Notes: Panel (a) compares the observed log share of national employment in each location to the ergodic distribution implied by the observed transition matrix $\Pi^0$. Panel (b) makes the same comparison across industries. 45-degree lines are shown in red. The close agreement between observed and ergodic distributions supports the steady-state assumption.

and location-industry cells in driving migration responses to labor demand shocks. These spillovers will be particularly important in practice if these connections differ substantially for workers in different locations or industries, e.g. due to migration costs increasing in distance. To assess these differences we calculate the Herfindahl–Hirschman index ($HHI$) measuring the average concentration of destinations and origins among those who migrated to or from a given region:

$$HHI_\ell = \sum_{k \neq \ell} \left( \frac{1}{2} \frac{(\pi^0_{\ell k} + \gamma^0_{k \ell})}{1 - \frac{1}{2} (\pi^0_{\ell \ell} + \gamma^0_{\ell \ell})} \right)^2 .$$ (33)

We also calculate a parallel expression for those who switched to or from industry $n$, $HHI_n$. The distributions of these concentration measures are shown in Figure 3. The average $HHI_\ell$ across locations is 0.165 (s.d. 0.174), which is equivalent to equally-weighted connections to only 6 other locations out of 485 possibilities. Industry transitions are more dispersed, with an average average $HHI_n$ across industries 0.141 (s.d. 0.050), which is equivalent to equally-weighted connections to 7 other industries out of 24 possibilities. Figure 3 also makes clear that there is substantial variation in these concentration measures for both locations and industries.
Figure 3: Concentration of Locations and Industries Among Switchers

Panel (a): Across Locations

Panel (b): Across Industries

Notes: Panel (a) shows the distribution of HHI indexes measuring the concentration of sources and destinations for those switching locations, as defined in (33), weighted by the initial regional employment. Panel (b) similarly shows the distribution of HHI indexes across industries.

6 Simulation-Based Evidence

In this section we quantitatively examine the predictive accuracy of the conventional migration regression when the data are generated by our model in response to simulated labor demand shocks that follow a variety of different data generating processes. We further compare the conventional regression with the model-based NLLS procedure. We use simulated shocks to be certain they meet the exogeneity requirements required for our analysis. Ensuring that the shocks are exogenous allows us to focus attention on the practical significance of the problems with the conventional regression discussed in detail in previous sections.

6.1 Simulation Procedure and Calibration

We consider three different data generating processes for shocks. The first set of shocks are independent standard normal draws across locations, i.e. $\hat{z}_t \sim N(0, 1)$, and are referred to as “iid across locations.” The second set of shocks introduces spatial correlation by assigning the same shock to all locations (microregions) in the same Brazilian state. The state-level shocks are independently
and normally distributed, such that \( \hat{x}_s \sim N(0, \sigma^2_2) \) and \( \hat{z}_\ell = \hat{x}_{s(\ell)} \), where \( s(\ell) \) is the state containing location \( \ell \). We refer to these shocks as “iid across states.” Finally, we generate independently and normally distributed shocks across industries, such that \( \hat{x}_n \sim N(0, \sigma^2_3) \) and the regional shift-share measure is \( \hat{z}_\ell = \sum_n \frac{L^0_n}{L^0_\ell} \hat{x}_n \). We refer to these shocks as “iid across industries.” To ensure comparable magnitudes across the three types of shocks, we choose \( \sigma_2 \) and \( \sigma_3 \) to ensure that the variances of the iid-across-state and iid-across-industry shocks equal one across locations.\(^{26}\)

Given the relevant vector of shocks, we generate the change in employment based on \( \hat{\mathbf{L}} = \left( \mathbf{I} - (\Psi^0)^{-1} \right) \hat{\mathbf{z}} \) (i.e. (20) without the error term, which we return to below). Recall that \( \Psi^0 \) depends on the observed flow matrices \( \Gamma^0 \) and \( \Pi^0 \), and the parameter ratio \( \frac{\sigma}{\sigma} \). We draw on the literature to determine a realistic calibration of \( \frac{\theta}{\sigma} \). Caliendo et al. (2019) estimate \( \frac{\theta}{\sigma} = 2.02 \) across U.S. states, so we set \( \theta = 0.5 \). In the absence of estimates of the Armington substitution elasticity across sub-national locations, we use standard estimates of the cross-country elasticity from Broda and Weinstein (2006) and Feenstra et al. (2018), both of which find \( \sigma \approx 4 \). We therefore set \( \frac{\theta}{\sigma} = \frac{1}{8} \) when calculating the \( \hat{\mathbf{L}} \). To investigate the effects of industry switching frictions, we generate versions of \( \hat{\mathbf{L}} \) from both the model with regional frictions alone (Section 2) and the more general version with industry-region frictions (Section 4), with the same structural parameters.\(^{27}\)

We then estimate the conventional location-level migration regression using the model-implied employment changes and the associated regional shocks, and weighting by initial population, which yields an estimate of \( \beta \) and \( R^2 \) for a given simulation. We then repeat the entire process 500 times and report averages across the simulations.

6.2 Baseline Simulation Results

Table 2 presents the baseline simulation results. To interpret the magnitudes of the estimates in the model with location frictions only, reported in the first column, we rely on the characterization of \( \beta \) in Theorem 1. In our calibration, \( 2\frac{\theta}{\sigma} = \frac{1}{4} \). Table 1 reports the average mobility rate \( \frac{M^0_{\ell\ell}}{L^0} = 0.042,\)

\(^{26}\)Specifically, \( \sigma_2 = 1.076 \) and \( \sigma_3 = 5.595 \) (when using flows for all workers regardless of skill).

\(^{27}\)Artuç et al. (2010) estimate \( \frac{\theta}{\sigma} = 1.88 \) across 6 U.S. industries, which supports the restriction in our model with location and industry frictions that \( \theta \) is common to both dimensions.
Table 2: Conventional Migration Regression Estimates – Comparison Across Models

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Location Frictions</td>
</tr>
<tr>
<td>iid across locations:</td>
<td>100 × ( \beta )</td>
</tr>
<tr>
<td></td>
<td>( R^2 )</td>
</tr>
<tr>
<td>iid across states:</td>
<td>100 × ( \beta )</td>
</tr>
<tr>
<td></td>
<td>( R^2 )</td>
</tr>
<tr>
<td>iid across industries:</td>
<td>100 × ( \beta )</td>
</tr>
<tr>
<td></td>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

Notes: Average of regression estimates \( \beta \) and its associated \( R^2 \) across 500 simulations. See text for definitions of shock processes. Column “Location Frictions” simulates local employment changes using the model with cross-location frictions only (Section 2), while column “Location-Industry Frictions” simulates employment changes using the model with location-industry frictions (Section 4).

and the mobility rate that would prevail without mobility frictions is \( \frac{\bar{M}^0}{L} \approx 1 \). In the first row of Table 2, when shocks are iid across locations, there will be no attenuation, so \( \rho \approx 0 \) and \( 1 - \rho \approx 1 \).

Combining these, we expect 100 × \( \beta \) \( \approx 1.05 \), which is very similar to the realized estimate of 1.045 in the model with only location frictions, confirming our theoretical predictions.\(^{28}\) The average \( R^2 \) in these simulations is only 0.577 in spite of the fact that the simulations do not include any idiosyncratic noise. This poor predictive performance reflects the fact that \( \beta \) omits location-specific migration intensity \( \frac{M^0}{T^\ell} \) and predicts the spillover effects from migrant-connected locations, \( \hat{z}_{-\ell} \), using only the direct shock facing the location, \( \hat{z}_{\ell} \).

When we introduce spatial correlation to the shocks in the second row of Table 2, the \( \beta \) estimate falls sharply. The strong correlation in shocks across migrant-connected locations increases \( \rho \) and therefore decreases the attenuation factor from approximately 1 in the first row to 0.232 in the second row. Industry shift-share shocks in the third row similarly induce spatial correlation, but much less so than with state-level shocks, so the \( \beta \) estimate is only moderately below that for the iid location shocks.

Turning to simulations from the model with location-industry frictions, when shocks are iid

---

\(^{28}\)The Table 1 coefficient could diverge from the prediction of Theorem 1 for two reasons, even with a very large number of simulation draws: the theorem uses the low-mobility approximation and characterizes the estimand of the regression, which is not the same as the estimator mean (because the ratio of expectations does not equal the expectation of the ratio in (23)). Neither of these differences turn out to be quantitatively important.
Figure 4: Conventional Regression Predictions vs. True Responses for Observed Shocks

Panel (a): iid across locations

Panel (b): iid across industries

Notes: Scatter plots compare the predicted population response to the labor demand shocks against the true model-based response for a single representative simulation. Each point represents a location. The y-axis is the predicted population response based on the conventional migration regression estimate, $\beta$. Specifically, this prediction is $\beta(\hat{z}_L - \bar{z})$, accounting for the intercept term in the migration regression. The x-axis is the true model-based population change in the model with location-industry frictions (Section 4). The left panel shows the result for iid shocks across locations, and the right panel shows the result for iid shocks across industries. 45-degree lines are shown in red. To maintain readability, we exclude the top and bottom 1% of regions based on the true model-based population change.

across locations the regression estimate of 0.982 is very similar to the model with location frictions only, as is the associated $R^2$. This similarity reflects the finding in (29) that the two models are identical under the low-mobility approximation. However, when we introduce industry-level shocks in the last panel, the estimate is much smaller in the model with location-industry frictions than in the model with location frictions alone. This again confirms our findings in (31) that industry frictions reduce mobility responses to industry-level shocks relative to settings with regional frictions alone. Also note that the $R^2$ value for industry shocks with location-industry frictions is extremely low.

Figure 4 presents these results visually, showing scatter plots comparing predicted population changes based on the conventional migration regression estimates, $\beta(\hat{z}_L - \bar{z})$, against the true population change based on the model with industry-region frictions for one example simulation draw. This exercise assesses the extent to which the regression is informative regarding the effects of the observed shocks. Consistent with the $R^2$ values in Table 2, the regression-based predictions are quite inaccurate in many regions under both models, but the predictions are clearly quite a bit
Notes: Scatter plots compare the predicted population response to a counterfactual unit shock to a single location against the true model-based response for a single representative simulation. Each point represents a location. The y-axis is the predicted population response based on the conventional migration regression estimate, $\beta$. The x-axis is the true model-based population change in the model with location-industry frictions (Section 4) when the relevant location faces a unit shock. The left panel shows the result when $\beta$ is estimated using iid shocks across locations, and the right panel shows the result when $\beta$ is estimated using iid shocks across industries. 45-degree lines are shown in red.

worse when facing industry-level shocks. In Panel (b), the regression predicts much less variation in population responses than the model with location and industry frictions, and the predictions are minimally related to the true values.

Together, the results in Table 2 and Figure 4 confirm the patterns predicted in our theoretical analysis and reveal substantial quantitative attenuation when shocks are correlated across migrant-connected locations. The low $R^2$ values also imply that the conventional regression yields quite poor predictions of the effects of the observed shocks used to estimate $\beta$ even in a setting where migration is determined by the labor demand shocks alone, without additional noise. In other words, the conventional migration regression performs poorly in generating internally valid estimates of the effects of observed shocks.

Now consider interpreting the estimate $\beta$ as the effect of a counterfactual unit labor demand shock to a single location on that location’s population. The regression-based prediction for this counterfactual is $\beta \cdot 1$, so the counterfactual prediction is the constant $\beta$ for all locations. We calculate the true model-based population change when individually shocking each location and
plot the responses against the estimate $\beta$ in Figure 5. Panel (a) considers the average $\beta$ estimate using *iid* regional shocks in the model with location-industry frictions, so $\beta = 0.982$ (see Table 2). While the regression estimate based on *iid* regional shocks matches the mean response it omits significant heterogeneity in the magnitude of the true counterfactual responses in each location. The situation is even worse when $\beta$ is estimated using a regional shift-share measure based on *iid* industry shocks in Panel (b). Not only does the estimate omit all heterogeneity, but it is subject to substantial attenuation due to the shock correlation across locations and the industry-switching frictions. In this case, the regression-based counterfactual estimate substantially understates the true effect in nearly every location.

### 6.3 Simulations by Skill Level

The simulation results thus far make clear that estimates from the conventional migration regression do not accurately reflect the effects of historical shocks on affected locations, nor can they be interpreted as the effect of counterfactual shocks to individual locations. We now investigate conventional migration estimates separately by skill level. Prior work has emphasized that less skilled workers’ location choices are less responsive to local labor demand shocks than those of more highly skilled workers (Bound and Holzer, 2000; Cadena and Kovak, 2016). Our goal is to understand the extent to which the various problems identified above may explain these commonly-observed differences.

We implement the simulation procedure described in the preceding subsection separately for high-skilled and low-skilled workers. We assign the same value of $\frac{\theta}{\sigma} = \frac{1}{8}$ to both education groups and note that the two groups have similar average mobility rates, though low-skilled workers are a bit more likely to migrate across locations and less likely to switch industries (Table 1). Table 3 shows the conventional migration estimates for all workers (replicating the last column of Table 2) and then by education group for the three different shock processes in the model with location-industry frictions. The $\beta$ estimates are quite similar across skill groups when shocks are *iid* across locations.
Table 3: Conventional Migration Regression Estimates – Comparison Across Skill Levels with Location-Industry Frictions

<table>
<thead>
<tr>
<th>Shocks</th>
<th>All Workers</th>
<th>High-Skilled</th>
<th>Low-Skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>iid across locations:</td>
<td>100 × β</td>
<td>0.982</td>
<td>1.014</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>0.608</td>
<td>0.539</td>
</tr>
<tr>
<td>iid across states:</td>
<td>100 × β</td>
<td>0.247</td>
<td>0.285</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>0.713</td>
<td>0.703</td>
</tr>
<tr>
<td>iid across industries:</td>
<td>100 × β</td>
<td>0.283</td>
<td>0.560</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>0.104</td>
<td>0.139</td>
</tr>
</tbody>
</table>

Notes: Average of regression estimates β and R² across 500 simulations. See text for details of shock processes. All simulations are based on the model with location-industry frictions (Section 4). High-skilled workers are those with at least a high-school degree, and Low-skilled workers are those with less than a high-school degree.

reflecting our imposition of the same structural parameters on both groups and the groups’ similar average migration rates. Both groups also experience roughly similar attenuation of β when facing spatially correlated state-level shocks. However, the two groups exhibit very different estimates when facing iid shocks across industries. The β estimate for low-skilled workers is less than half the magnitude of that for high-skilled workers. This difference suggests that low-skilled workers may appear to respond less to local industry shift-share shocks not because of fundamental differences in regional mobility but because they face different shift-share shocks or because they have larger costs of switching industries than high-skilled workers. These distinctions are essential for determining what kinds of policies will be most effective at addressing regional disparities in unemployment among low-skilled workers and will be addressed in more detail in subsequent revisions.

6.4 Introducing Noise

The preceding simulations determine the true population changes based on the model’s structure and simulated labor demand shocks alone. In that setting, the model fits the data perfectly by construction, so we were unable to assess the quality of estimates based on the proposed NLLS procedure introduced in Section 3.4. Here, we add noise from unobserved changes in labor demand or supply (assumed orthogonal to the observed labor demand shocks to avoid introducing endogeneity bias), to assess the relative performance of the conventional migration regression and the proposed
NLLS procedure and to better mimic the data observed in a typical study of migration responses to local shocks.

Specifically, for a shock vector \( \hat{z} \), we calculate the change in population as \( \hat{L} = \hat{L}^{\text{due to } \hat{z}} + \zeta \) where \( \hat{L}^{\text{due to } \hat{z}} = (I - (\Psi^0)^{-1}) \hat{z} \) and \( \Psi^0 \) is computed using the true parameter \( \theta/\sigma = 1/8 \). We focus on the model with location-industry frictions, so each element of the vectors in this expression is a location-industry combination. We allow the unobserved shocks in \( \zeta_{\ell n} \) to consist of additive location, industry, and location-industry components, such that

\[
\zeta_{\ell n} = \epsilon_{\ell} + \epsilon_{n} + \epsilon_{\ell n}.
\]

To ensure a realistic scale for this noise term, we take the observed changes in employment in each location-industry cell and regress them on location fixed effects and industry fixed effects, which allows us to measure the location-level variation, industry-level variation, and residual location-industry variation, which we use to calibrate the variance of each noise component.\(^{30}\)

We compare the in-sample predictive quality of four methods for estimating the population effects of the simulated labor demand shocks. We first estimate \( \beta \) from the conventional regression (22) and, like before, take \( \hat{\beta}(\hat{z}_{\ell} - \overline{z}) \) as the prediction. We then estimate \( \theta/\sigma \) with two versions of the NLLS specification (26) based on the correctly specified model which allows for location-industry mobility frictions. First, we perform the estimation on location-industry observations, with the simulated \( \hat{L}_{\ell n} \) as the dependent variable and the model’s prediction for it as a function of \( \hat{z} \) and \( \theta/\sigma \) on the right-hand side. The second version is analogous, except estimation is done with regional observations, in parallel to the conventional regression. That is, we use the simulated \( \hat{L}_{\ell} \) as the dependent variable and the model’s prediction for it, which aggregates predicted \( \hat{L}_{\ell n} \) across industries for each region, on the right-hand side. Since both of these approaches are based on the

\(^{30}\)From historical data, we estimate \( \hat{L}_{\ell n} = \nu_{\ell} + \nu_{n} + \varepsilon_{\ell n} \), where \( \nu_{\ell} \) are location fixed effects, \( \nu_{n} \) are industry fixed effects, and \( \varepsilon_{\ell n} \) is the residual. The employment-weighted standard deviation of the estimated location terms is 0.0158, of the estimate industry terms is 0.0139, and of the estimated residual is 0.0512. We then simulate normally-distributed noise components as \( \epsilon_{\ell} \sim \mathcal{N}(0, 0.0158^2) \), \( \epsilon_{n} \sim \mathcal{N}(0, 0.0139^2) \), and \( \epsilon_{\ell n} \sim \mathcal{N}(0, 0.0512^2) \). Finally, we demean the resulting overall noise term \( \zeta_{\ell n} \) so \( \sum_{\ell, n} \frac{\partial}{\partial \theta} \zeta_{\ell n} = 0 \).
correctly specified model, their prediction is only imperfect because of the noisy estimates of $\theta/\sigma$. Finally, we consider NLLS estimation of (26) where $\Psi^0(\theta/\sigma)$ is constructed from the misspecified baseline model that does not allow industry mobility frictions, and again using $\hat{L}_\ell$ as the dependent variable. The advantage of this specification is that it is feasible without data on mobility across location-industry cells; it only requires regional flows, which are much more readily available in many countries.

We use the Root Mean Square Error (RMSE) as the measure of in-sample predictive quality for regional population changes $\hat{L}_\ell$. Specifically, given predictions $\hat{L}_{\ell,s}^{\text{predicted}}$ from one of the four methods in simulation $s$, we compute

$$RMSE = \sqrt{\frac{1}{S} \sum_{s=1}^{S} \left[ \sum_{\ell=1}^{L} \frac{L^0_{\ell}}{L^0} \left( \hat{L}_{\ell,s}^{\text{due to } \hat{z}} - \hat{L}_{\ell,s}^{\text{predicted}} \right)^2 \right]}, \quad (34)$$

which averages squared prediction errors both across locations (with population weights) and across $S = 500$ simulations. We rescale this RMSE measure relative to its value for the uninformed prediction, $\hat{L}_{\ell,s}^{\text{predicted}} = 0$, i.e. by the square root of the average cross-simulation variance of the component of population changes due to the shock:

$$RMSE_{\text{uninformative}} = \sqrt{\frac{1}{S} \sum_{s=1}^{S} \left[ \sum_{\ell=1}^{L} \frac{L^0_{\ell}}{L^0} (\hat{L}_{\ell,s}^{\text{due to } \hat{z}})^2 \right]}. \quad (35)$$

Table 4 reports the results for our four methods and for $iid$ location and $iid$ industry shock processes. The first row confirms that predictions by the conventional regression are very poor: it outperforms the uninformative prediction by less than 30% when the shocks are location-specific and performs 2% worse than the uninformative prediction for industry shocks. The second and third rows show that NLLS performs well when taking into account mobility frictions across both regions and industries. When estimated at the location-industry level, its RMSE is only 17% of that of the uninformative prediction with $iid$ location shocks, and as little as 3% for $iid$ industry shocks. For $iid$ location shocks, estimation at the more aggregated regional level increases the
Table 4: RMSE of Conventional Regression and NLLS Predictions

<table>
<thead>
<tr>
<th>Prediction method</th>
<th>RMSE, relative to uninformative prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>iid location shocks</td>
</tr>
<tr>
<td>Conventional regression</td>
<td>0.719</td>
</tr>
<tr>
<td>NLLS, region-industry level</td>
<td>0.171</td>
</tr>
<tr>
<td>NLLS, regional level</td>
<td>0.250</td>
</tr>
<tr>
<td>NLLS, misspecified regional model</td>
<td>0.262</td>
</tr>
</tbody>
</table>

Notes: Each row corresponds to a method of predicting the effect of observed shocks on regional population, while each column defines the shock process; see the text for details. Each cell reports the ratio of the root mean squared error (RMSE) for a given prediction method to the RMSE from the uninformative prediction, computed via (34) and (35), respectively. All simulations are based on the model with location-industry frictions (Section 4).

estimation error only slightly, from 17% to 25% of the uninformative prediction RMSE. However, for iid industry shocks, data aggregation raises the RMSE from 3% to 34% — although still three times lower than with the conventional regression. Finally, the fourth row of Table 4 reports the predictive quality of the misspecified model without industry frictions. For location shocks, the RMSE is almost identical to the previous row, at 26%. Indeed, as discussed before, the predictions of the benchmark and full models would be identical if the low-mobility approximation was precise. However, with iid industry shocks the misspecified model provides very poor predictions, with a RMSE of 92% relative to the uninformative prediction and only slightly ahead of the conventional regression.

The absolute and, possibly, relative performance of different methods may depend on how much useful variation the observed shocks provide relative to the noise in the data — a parameter which may vary across empirical settings. To check robustness of our findings, we therefore repeat the Table 4 analysis varying the relative variance of $\hat{\mathbf{z}}$ and $\zeta$, specifically by multiplying $\zeta$ by a factor $\delta \in [0, 4]$, with $\delta = 1$ corresponding to the Table 4 results. Appendix Figure A1 shows that the results are robust.32

31This gap in performance can be linked to how much of the variation of the regional vs. region-industry levels the observed shocks capture. With iid industry shocks, 83.1% of the $L_{\ell n}$ variation is due to the observed shocks, compared to only 10.0% for $L_{\ell}$ (measured by the employment-weighted $R^2$ on average across simulations). This difference is much smaller, and in the opposite direction, for iid regional shocks: they on average explain 7.7% of the variation in $L_{ln}$, but a higher 21.2% of the variation in $L_{\ell}$.

32The only exception is NLLS estimated from iid industry shocks at the level of regions, which is sensitive to $\delta$: it performs well for small $\delta$ (although uniformly worse than NLLS estimation by region-industry) but not for large amounts of noise.
Taken together, these results yield two lessons. First, accounting for the spillovers via the model-consistent NLLS procedure allows one to capture the population responses of local labor demand shocks even in the presence of noise in the data, which the conventional regression is not capable of. Second, ignoring the industry dimension of the data — in either the unit of observations or the model employed — is relatively innocuous when the shocks vary by region. However, when the shocks are industry-driven, it is crucial to take the structure of industry mobility into account, in using the appropriate model as well as estimating the structural parameter at the disaggregated location-industry level.

7 Conclusion

This paper has examined how to interpret conventional migration regression estimates relating changes in local population or employment to observed local labor demand shocks. Using a simple model of local labor markets with mobility costs, we find that common conclusions drawn from migration regression estimates are likely to be substantially misleading. Analytical results and quantitative simulations show that conventional migration regression estimates provide inaccurate estimates of the within-sample effects of observed shocks on local populations and are uninformative regarding the effects of counterfactual shocks to one or many locations.

Conventional migration regression estimates are therefore not informative regarding the magnitude of migration frictions or the potential benefits of policies aimed at helping workers relocate to stronger labor markets. Nor should the estimates be used to justify omitting inter-regional migration from quantitative economic models of local labor markets. Instead, researchers should adopt methods accounting for spillovers of shocks between connected locations, as in our proposed NLLS procedure. Yet, to implement that and related analyses, the data demands are significant. Our analysis using Brazilian data shows the substantial practical benefit of observing transitions between relatively detailed location-industry cells to successfully capture the interrelationships in a large and diverse economy.
References


A Theory Appendix

A.1 $\beta$ as Probability Limit of $\hat{\beta}$

Proof coming soon.

A.2 Proof of Theorem 1

We prove a generalization of Theorem 1 which does not require homoskedasticity of shocks:

$$\beta = 2\frac{\sigma}{\theta} \frac{\sum \ell v_\ell M_\ell (1 - \beta_\ell)}{\sum \ell v_\ell M_\ell^* (1 - \beta_\ell^*)},$$  \hspace{1cm} (36)

where $v_\ell = \text{Var}[\hat{z}_\ell]$, $\beta_\ell = \text{Cov}[\hat{z}_{-\ell}, \hat{z}_\ell] / v_\ell$, $\hat{z}_{-\ell} = \sum_{d \neq \ell} (M_d + M_{d\ell})/2 \hat{z}_d$, $\beta_\ell^* = \text{Cov}[\hat{z}_{-\ell}^*, \hat{z}_\ell] / v_\ell$, and $\hat{z}_{-\ell}^* = \sum_{d \neq \ell} (M_d^* + M_{d\ell}^*)/2 \hat{z}_d$. At the end of this section we show that (36) implies the Theorem 1 statement under homoskedasticity. Given that regression (22) includes the constant and our interest is in the slope parameter, we assume without loss of generality that the shocks have zero expectation, $\mu = 0$.

We first note that in (21)

$$\sum_o \gamma^0_o(\hat{z}_\ell - \hat{z}_o) + \sum_d \pi^0_d(\hat{z}_\ell - \hat{z}_d) = 2\frac{M^0_\ell}{L^0_0} (\hat{z}_\ell - \hat{z}_{-\ell})$$

where $\hat{z}_{-\ell}$ is the average shock to regions $k$ other than $\ell$ weighted by the gross migration flows in both directions between $k$ and $\ell$.

$$\hat{z}_{-\ell} = \sum_{k \neq \ell} M^0_{k\ell} + M^0_{\ell k} \hat{z}_k.$$  \hspace{1cm} (37)

Thus the numerator of $\beta$ equals

$$\mathbb{E}\left[\sum \ell L^0_\ell \hat{L}_\ell (\hat{z}_\ell - \bar{z})\right] = \mathbb{E}\left[\sum \ell L^0_\ell \hat{L}_\ell \hat{z}_\ell\right] = \mathbb{E}\left[\frac{2\theta}{\sigma} \sum \ell M^0_\ell \hat{z}_\ell (\hat{z}_\ell - \hat{z}_{-\ell})\right] = \frac{2\theta}{\sigma} \sum \ell M^0_\ell v_\ell (1 - \beta_\ell).$$

Here the first line used the fact that $\sum \ell L^0_\ell \hat{L}_\ell \bar{z} = \bar{z} \sum \ell L^0_\ell \hat{L}_\ell = \bar{z} \hat{L} = 0$ because total national population is not allowed to change in the model. The second line plugged in (21), and the last line used the definitions of $v_\ell$ and $\beta_\ell$.

Now turn to the denominator of $\beta$. Since $\bar{z} = L^0_0 \hat{z}_\ell + \left(1 - \frac{L^0_0}{L^0}\right) \hat{z}_{-\ell}^*$ for any $\ell$, we have

$$\hat{z}_\ell - \bar{z} = \left(1 - \frac{L^0_0}{L^0}\right) (\hat{z}_\ell - \hat{z}_{-\ell}^*).$$  \hspace{1cm} (38)
Thus,

\[
E \left[ \sum_\ell L^0_\ell (\hat{z}_\ell - \bar{z}) \right] = E \left[ \sum_\ell L^0_\ell \hat{z}_\ell (\hat{z}_\ell - \bar{z}) \right] = E \left[ \sum_\ell L^0_\ell \left( 1 - \frac{L^0_\ell}{L^0} \right) \hat{z}_\ell (\hat{z}_\ell - \hat{z}^*_\ell) \right] = E \left[ \sum_\ell M^0_\ell \hat{z}_\ell (\hat{z}_\ell - \hat{z}^*_\ell) \right] = \sum_\ell M^0_\ell v_\ell (1 - \beta^*_\ell). \tag{39}
\]

Here the first line used \( \sum_\ell L^0_\ell \hat{z} (\hat{z}_\ell - \bar{z}) = \bar{z} \sum_\ell L^0_\ell (\hat{z}_\ell - \bar{z}) = 0 \) by definition of \( \bar{z} \). The second line used (38). The third line used the definition of \( M^0_\ell \), and the final line used the definition of \( \beta^*_\ell \).

Combining (37) and (39) directly yields (36).

Finally, suppose shocks are homoskedastic. Then, the numerator of \( \beta \) equals

\[
\frac{2\theta}{\sigma} \sum_\ell M^0_\ell v_\ell (1 - \beta_\ell) = \frac{2\theta}{\sigma} \sum_\ell \left( M^0_\ell v - \sum_{k \neq \ell} \frac{M^0_{k\ell} + M^0_{k\ell}}{2} \text{Cov}[\hat{z}_k, \hat{z}_\ell] \right) = \frac{2\theta}{\sigma} \sum_\ell \left( M^0_\ell - \sum_{k \neq \ell} \frac{M^0_{k\ell} + M^0_{k\ell}}{2} \text{Corr}[\hat{z}_k, \hat{z}_\ell] \right) = \frac{2\theta}{\sigma} v M^0 \left( 1 - \sum_{k \neq \ell} \frac{M^0_{k\ell} + M^0_{k\ell}}{2} \text{Corr}[\hat{z}_k, \hat{z}_\ell] \right) = \frac{2\theta}{\sigma} v M^0 (1 - \rho).
\]

The denominator of \( \beta \) analogously equals \( v M^{*0} (1 - \rho^*) \), and the statement of Theorem 1 follows.

### A.3 Proof \( \beta(\hat{z}_\ell - \bar{z}) \) Yields MSE-Minimizing Predicted Population Growth

**Proposition.** \( \beta(z_\ell - \bar{z}) \) is the best linear predictor of \( \hat{L}_\ell \) among all \( b(z_\ell - \bar{z}) \), i.e. after demeaning the shocks, in the sense that it solves

\[
\min_b \quad \mathbb{E} \left[ \sum_\ell L_\ell \left( \hat{L}_\ell - b(z_\ell - \bar{z}) \right)^2 \right]. \tag{40}
\]
Proof. The first order condition for (40) yields

$$\mathbb{E} \left[ \sum_{\ell} L_{\ell} \left( \hat{L}_{\ell} - b (z_{\ell} - \bar{z}) \right) \cdot (z_{\ell} - \bar{z}) \right] = 0$$

$$\mathbb{E} \left[ \sum_{\ell} L_{\ell} \hat{L}_{\ell} (z_{\ell} - \bar{z}) \right] = b \cdot \mathbb{E} \left[ \sum_{\ell} L_{\ell} (z_{\ell} - \bar{z})^2 \right],$$

which implies $b = \beta$. \qed

A.4 Relationship Between $\beta \approx 0$ and Local Economic Outcomes

Coming soon.

A.5 Non-Linear Least Squares

Coming soon.

Appendix Figures and Tables

Figure A1: Robustness of RMSE Measures to Noise Scaling

Panel (a): iid across locations

Panel (b): iid across industries

Notes: This figure repeats the analysis of Table 4 varying the noise scaling factor $\delta$, which multiplies the error term $\zeta$ in the simulations. Panel (a) reports the root mean-squared error (RMSE) of the four methods relative to the RMSE of the uninformative prediction for iid location shocks, while Panel (b) uses iid industry shocks. See notes to Table 4 for details.