Should low skilled work be subsidized?

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Preliminary, comments welcome

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Abstract

A number of countries have recently implemented variants of a negative income tax, to push the less skilled members of the economy into work, or to make work pay in comparison with welfare benefits. In most cases, these measures have resulted for the concerned groups in a decrease of the tax rates, that remain positive, rather than in a subsidy, in conformity with the recommendations of the current theory of optimal taxation. Indeed in the Mirrlees setup (continuous labor supply or intensive margin, unobserved productivity, utilitarian planner) the marginal tax rate is non negative at the optimum.

The purpose of the paper is to question this result of the theory. We study economies where it is optimal to have people in the economy work more than in the laissez-faire. We provide an example in the intensive setup. The utilitarian optima in the extensive model seem to exhibit this property quite generally. We hope that these results help towards providing some theoretical foundations for low skilled work subsidy, and extending the scope of welfare to work programs.

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1 Introduction

A number of countries have recently implemented variants of a negative income tax, to push the less skilled members of the economy into work, or to make work pay in comparison with welfare benefits. In most cases, these measures have resulted for the concerned groups in a decrease of the tax rates, that remain positive, rather than in a subsidy (see e.g. CBO (2000) for the United States). This is in conformity with the current recommendations of the theory of optimal taxation. Indeed it is now well established in the Mirrlees setup (continuous labor supply or intensive margin, unobserved productivity, utilitarian planner) that the marginal tax rate is non negative at the optimum[1] (Seade (1977), Seade (1982), Hellwig (2005), Werning (2000)). The purpose of the present paper is to revisit this theoretical result, to question its robustness when there are multiple dimensions of heterogeneity, and to draw its implications for labor market distortions. This is done through two examples, one in the intensive setup, the other in the extensive framework.

In fact, early on Diamond (1980), more recently Saez (2002), Beaudry and Blackorby (2004), Boone and Bovenberg (2004), Boone and Bovenberg (2006), Choné and Laroque (2005) and Laroque (2005) have described setups where the positive tax rate result does not hold. A common feature of the (rather different) models used in these works is that labor supply decisions involve a zero-one component, an extensive margin. Furthermore there are typically several (sometimes implicit) dimensions of heterogeneity. These studies exhibit cases where negative tax rates can occur at an optimum. But it is fair to say that their theoretical foundations remain unclear as well as their practical relevance. Also it is important to note that the implications of negative tax rates are quite different in an extensive model and in an intensive model. In the intensive model, they imply that labor supply is distorted upwards compared with the laissez-faire. In the extensive model negative tax rates are mostly related to the shape of the distribution of agents in the economy, and to the best of our knowledge, the previous literature has not studied the extent of labor supply distortions in this setup.

We use a framework which contains as limit cases both the intensive and the extensive models, and allow for multiple dimensions of heterogeneity. We take a very simple separable specification for the agents tastes, in fact much simpler than the standard Mirrlees specification: utility is linear in commodity and for the participating agents labor supply has a constant elasticity with respect to wages. Technically, our line of approach is to look first for properties of all the second best optimal allocations, then restricting the attention to those that are consistent with a utilitarian criterion.

The study of the intensive model follows on the steps of Sandmo (1993), but

[1] However Mirrlees (1976) in its Section 4 indicates, along a line that will be pursued further here, that the sign of the marginal tax rate cannot be predicted when the agents in the economy differ along several dimensions of heterogeneity.

we allow for a general non-linear tax. There are two dimensions of heterogeneity, productivity and a variable opportunity cost of work. The specification however makes it possible to subsume these two dimensions into a single one[2]. We are able to completely characterize the set of second best allocations, including the ones that involve pooling, in line with the general analysis of Jullien (2000). Heterogeneity comes into play in the measurement of the agents' utilities, which increase with productivity and may either decrease or increase with the variable work opportunity cost. It is likely to decrease when the cost is associated with poor living conditions (i.e. a handicap); it increases when the cost reflects opportunities outside the legal market (such as gardening at home or black market activities). We find that the Mirrlees result, of positive marginal tax rates, extends here whenever the distribution of opportunity costs is independent of that of productivities, whatever the impact of these costs on the agents utilities. We give an example of optimal negative marginal rates in an economy where agents with low productivities exhibit a large spectrum of opportunity costs, and are better off, the larger their costs. The negative tax rate serves to screen out the agents with large costs, who anyway benefit from working at home or on the black market, in the spirit of the imperfect screening literature (e.g. Akerlof (1978) or Salanié (2002)).

The extensive model has built in several dimensions of heterogeneity, since both differences in productivity and in fixed opportunity cost to work are an essential feature of the model. It also presents technical difficulties because of its intrinsic non convexity. We specifically study the shape of the second best allocations that are consistent with a utilitarian criterion. For simplicity, and for comparison with the intensive case, we restrict our attention to the situation where work opportunity costs have a log-concave distribution and are distributed independently of productivity in the population. To our surprise, we find that *all the utilitarian optima involve upwards labor supply distortions for low productivity workers.* The optimal financial incentives to work involve a subsidy: low productivity workers are paid more than their productivity at the optimal allocation. The argument is as follows. In the absence of income effects, the marginal cost of public funds, say c, is equal to 1, the social value of transferring 1$ per head to everyone in the population (the population size having been normalized to one). Consider then a small change in the tax schedule in favor of the working agents of (low) productivity ω, sufficiently small not to modify the situation of the other agents, of productivity different from ω. It has two effects: it gives more money to the agents that are already working, and it brings into the labor force some pivotal agents previously unemployed. Under utilitarianism (and not full redistribution!), the social value of a marginal transfer to the working

[2]With a similar aim as ours, Beaudry and Blackorby (2004) have studied a model with several 'true' dimensions of heterogeneity. This makes the study of the optimal taxes much more complicated.

agents of productivity ω is larger than that of a transfer to the whole population, and therefore has a social value larger than c per dollar transferred. The pivotal agents that enter the labor force are essentially indifferent between working and not working, and their contribution is the difference between their productivity ω and their pay. For the first order condition to hold, this difference must be negative: pay has to be larger than productivity. The result appears to hold in a number of cases, and it would be of interest to study more precisely its domain of validity.

To summarize, non negative optimal marginal tax rates, which obtain under utilitarianism in the Mirrlees model, appear to be non robust to the presence of heterogeneity, apart from that affecting productivity, in the economy. Then upwards distortions in labor supply may be useful for screening purposes. In our simple intensive model, this occurs in a rather special case, when the low income people are thought to be well off agents who shirk. In the extensive model, under utilitarianism, the less skilled workers have typically their work subsidized: they work more than in the laissez-faire, and the utilitarian optimal allocations have more 'working poors' than the competitive equilibrium. All this should the subject of further research.

2 The setup

2.1 The model

We consider an economy with a continuum of agents of measure 1. The agents supply labor, in quantity h, $h \geq 0$, to produce an undifferentiated commodity in quantity $\omega h = y$. Here ω, $\omega \geq 0$, denotes the idiosyncratic productivity of the agent, and y her before tax income.

After government transfers, the after tax income of the agent is denoted $R(y)$, where the non linear function $R : \mathbb{R}_+ \to \mathbb{R}$ summarizes the action of the tax authority. The tax function T corresponding to R is defined by

$$T(y) = y - R(y),$$

so that a negative marginal tax rate corresponds to a derivative $R'(y)$ larger than one.

Faced with the function R, the typical agent choses her labor supply by maximizing a *choice index*

$$u(R; \alpha, \beta, \omega) = \max_{h \geq 0} \begin{cases} R(0) + \alpha & \text{if } h = 0 \\ R(\omega h) - \beta v(h) & \text{if } h > 0 \end{cases} \tag{1}$$

We say that an agent *participates* in the labor market when she choses a positive labor supply, so that her choice index is given by the bottom line of the formula.

This specification is adopted for convenience, but is in line with a number of works in the literature. The choice index of the agent is linear in commodity (labor supply does not depend on the income level). The penibility of labor is described with the function $v(h)$, which we specify as

$$v(h) = \frac{h^{1+\frac{1}{e}}}{1+\frac{1}{e}}.$$

The parameter e, $e \geq 0$, common to all the agents in the economy, is the elasticity of the labor supply of the participating agents with respect to wage. In the limiting case $e = 0$, when R is non decreasing, everyone supplies one unit of labor when participating: we obtain the extensive model.

On top of her productivity ω, an agent is characterized by the non negative idiosyncratic parameters α and β. The former, α, the *fixed opportunity cost of work*, represents the gain of being at home, not doing any work at all. When α is equal to zero, we fall back on the intensive model. The latter, β, the *variable opportunity cost of work*, scales the penibility of labor. We note $\theta = (\alpha, \beta, \omega)$. The distribution of agents' characteristics has support Θ in $\mathbb{R}^3_+$ and is known to the government. The cumulative distribution function is H.

2.2 Second best optimality and utilitarianism

Given a function R, an allocation $\mathbf{y}_R$ is a function from Θ into $\mathbb{R}_+$ such that, for all θ, $\mathbf{y}_R(\theta) = \omega h$ for some h that maximizes the program (1) of the agent of characteristics θ. In this paper, all allocations are associated in this way with some function R. To alleviate notations, we shall drop the index R when this does not create ambiguity. The allocation $\mathbf{y}_R$, and the associated function R are *feasible* when

$$\int_\Theta [\mathbf{y}_R(\theta) - R(\mathbf{y}_R(\theta))] \,\mathrm{d}H(\theta) = 0.$$

An allocation $\mathbf{y}_{R^*}$ and the associated transfer function R^* are *second best optimal* when there does not exist another feasible allocation which gives at least as much utility to everyone in the economy and strictly more to a subgroup of agents of positive measure. By definition, R^* is second best optimal if and only if the program

$$\begin{cases} \max_R \int_\Theta [\mathbf{y}_R(\theta) - R(\mathbf{y}_R(\theta))] \,\mathrm{d}H(\theta) \\ u(R;\theta) \geq u(R^*;\theta) \quad \text{for all } \theta \text{ in } \Theta \end{cases} \tag{2}$$

has solution R^* and value 0. It follows that to any second best allocation there is associated a non negative measure Π on Θ, such that the function R^* is a local maximum of the Lagrangian

$$\mathcal{L} = \int_\Theta [u(R;\theta) \,\mathrm{d}\Pi(\theta) + (\mathbf{y}_R(\theta) - R(\mathbf{y}_R(\theta)) \,\mathrm{d}H(\theta)]. \tag{3}$$

Note that by quasi-linearity of the utilities, the solution to the program where a constant a is added to R^* is $a + R^*$, with y_{R^*} equal to y_{a+R^*} . Therefore

$$\int_\Theta \mathrm{d}\Pi(\theta) = 1,$$

and Π is a probability measure. Furthermore, when looking for all the second best allocations, it will be convenient to ignore the feasibility condition, which essentially fixes the intercept of the function R^*, and choose a simpler normalization condition, such as $\inf_{\theta\in\Theta} u(R^*, \theta) = 0$.

To simplify the presentation, in most of the paper, we shall work under the assumption that the measure Π is absolutely continuous with respect to the distribution of the agents characteristics. Then, for any measurable set A,

$$\Pi(A) = \int_A \pi(\theta)\, \mathrm{d}H(\theta),$$

and $\pi(\theta)$ is interpreted as the *social weight* of the agents of characteristics θ. In fact, the results that we obtain are typically valid for a general measure, possibly with discrete masses: they cover in particular the Rawlsian optimum, which corresponds to a unit mass on the agents with the lowest utility level.

Second best optimality is an ordinal concept, which does not depend on the particular representation of the agents' utilities, up to an increasing transformation. For comparison with the literature, we also study the subset of allocations that obtain under utilitarianism, a cardinal notion[3]. Let $\Psi(u(R, \theta), \theta)$ be the utility that society assigns to the agent θ when her choice index is $u(R, \theta)$. The function Ψ is non decreasing and concave in its first argument (a requirement of consistency with private values). The social weight of agent θ, $\Psi'_u(u(R, \theta), \theta)$, depends in an arbitrary way on its second argument: for instance society may dislike the agents who like staying at home (decreasing in α), or would like to compensate people with a large penibility of labor (increasing in β). The second best allocation is *consistent with utilitarianism* when the associated weights are proportional to the marginal social utility for some admissible function Ψ

$$\pi(\theta) = \frac{\Psi'_u(u(R, \theta), \theta)}{\int_\Theta \Psi'_u(u(R, \theta), \theta)\, \mathrm{d}H(\theta)}.$$

When Ψ is allowed to vary with the parameter θ, it is easy to see that any second best optimal allocation can be supported with a well chosen Ψ: consistency with utilitarianism is not a binding restriction.

[3]It may be worth emphasizing that we stick here to a purely welfarist viewpoint. We do not consider situations where the social objective includes moral considerations other than the effects of policies on individual utilities, as discussed in Sen (1982) and Kaplow and Shavell (2001).

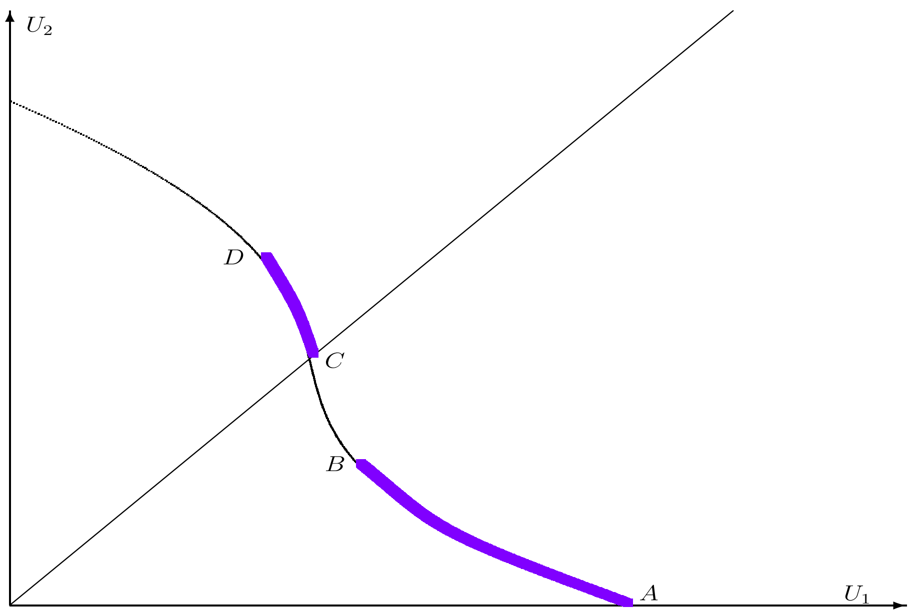

Figure 1: Second best optimality and utilitarianism

When the function Ψ does not depend on its second argument, the standard situation studied in the optimal tax literature, the condition binds and can be written as

$$\pi(\theta_1) > \pi(\theta_2) \text{ if and only if } u(R, \theta_1) < u(R, \theta_2).$$

This is illustrated on the stylized Figure 1, which sketches an hypothetical economy with two types of agents in the plan of their choice indices (U_1, U_2). The second best frontier is the black curve $ABCD$, while the subset of the frontier that is consistent with utilitarianism is made of the union of AB and CD, where B and D are the points where the frontier has slope -1: it must have a tangent of slope larger than 1 in absolute value below the 45 degree line, and larger than 1 above the 45 degree line.

Our purpose is to find properties of the second best optimal functions R, in particular when the social weights are consistent with utilitarianism.

3 The pure intensive case

This situation obtains when the fixed opportunity cost of labour α is equal to zero for all the agents in the economy.

3.1 A change of variable

It turns out that there is a convenient reformulation of the problem, introducing the choice index of the participating agents as a variable, instead of the function R. Indeed, in general when there are several dimensions of heterogeneity (productivity, penibility of labor) and the government has only one dimension of observation (income), a major difficulty is to identify the set of idiosyncratic shocks that are associated with a given level of income, which typically depends on the announced transfer function. Here, the specification of the choice index and of the way shocks enter the model allow to reduce the problem to a single dimension of heterogeneity from the start, independently of the function R.

Proposition 1. *1. Consider a function $R : \mathbb{R}_+ \to \mathbb{R}$. Let*

$$V(\gamma) = \max_{y \geq 0} R(y) - \gamma \frac{y^{1+\frac{1}{e}}}{1+\frac{1}{e}},$$

where

$$\gamma = \frac{\beta}{\omega^{1+\frac{1}{e}}}.$$

V is a convex nonincreasing function, which satisfies

$$V'(\gamma) = -v(\mathbf{y}_R(\gamma)),$$

whenever it is differentiable, so that $R(\mathbf{y}_R(\gamma)) = V(\gamma) - \gamma V'(\gamma)$.

2. Conversely, to any convex nonincreasing function V corresponds a real function $\tilde{R} : \mathbb{R}_+ \to \mathbb{R}_+$ through

$$\tilde{R}(q) = \min_{\gamma \geq 0} V(\gamma) + \gamma q.$$

$\tilde{R}(.)$ is concave non decreasing in its argument. If V has been derived from a function R as in 1., $\tilde{R}(.)$ coincides with the function $R \circ v^{-1}$ when $R \circ v^{-1}$ itself is concave, which implies that R itself is non decreasing.

We shall denote Γ the support of the distribution of γ, with $\underline{\gamma} \geq 0$ and $\overline{\gamma}$ its lower and upper (possibly infinite) bounds. From the point of view of the agents the only thing that matters is the level $V(\gamma)$ of their choice index, and Proposition 1 shows that without loss of generality we can consider any convex nonincreasing function. Also, without loss of generality, the government can restrict the R functions to be non decreasing and such that $R \circ v^{-1}$ be concave.

3.2 Optimal tax

Using the change of variable, the Lagrangian (3) becomes

$$\mathcal{L} = \int V \,\mathrm{d}\Pi(\theta) + \int [y - (V - \gamma V')] \,\mathrm{d}G(\gamma).$$

Since V depends only on γ, the Lagrangian depends only on the marginal distribution of $\mathrm{d}\Pi$ in the dimension γ, which will be denoted $\mathrm{d}\Pi(\gamma)$, with a slight abuse of notation, and $\Pi(\gamma)$ is its cumulative distribution. We can write

$$\mathcal{L} = \int V \,\mathrm{d}\Pi(\gamma) + \int [y - (V - \gamma V')] \,\mathrm{d}G(\gamma)$$

By integration by parts (apply Lemma A.1. in Appendix with $F = V$, $f = V'$, $y = \Pi$, $y(\underline{\theta} = 0)$ and $y(\overline{\theta}) = 1$) , the Lagrangian becomes

$$\mathcal{L} = \int (y + \gamma V') \,\mathrm{d}G(\gamma) + \int V'(G - \Pi) \,\mathrm{d}\gamma,$$

where $V' = -v(y)$. Note that it depends only on the allocation y, i.e. the derivative of the choice index (and *not* on its level V).

The problem is to maximize $\mathcal{L}$ on the set of nonincreasing and positive functions y, or, equivalently, on the set of nondecreasing and negative functions V'. The Lagrangian is strictly concave in V'. It is maximized on a convex set. It follows that it has a unique maximum.

Lemma 1. *An allocation y is second best optimal if and only if there exists a nondecreasing function $\Pi : [\underline{\gamma}, \overline{\gamma}] \to [0, 1]$ such that y is the solution to*

$$\max \mathcal{L} = \int (v^{-1}(-V') + \gamma V') \, dG(\gamma) + \int V'(G - \Pi) \, d\gamma,$$

on the set of nondecreasing and negative function V'.

The set of second best optimal allocations is easy to describe when the distribution of heterogeneity is continuous, i.e.

Assumption 1 (Continuous distribution)**.** *The parameter γ is distributed in the economy with the c.d.f. G of support $[\underline{\gamma}, \overline{\gamma}]$, $0 < \underline{\gamma} < \overline{\gamma} < \infty$. Furthermore G has a continuous positive density g.*

We have

Proposition 2. *Suppose that Assumption 1 holds. A non negative decreasing function $y(\gamma)$ defined on Γ is a second best allocation if and only if the function*

$$\Pi(\gamma) = \begin{cases} G(\gamma) - g(\gamma)\left[\dfrac{1}{v'(y(\gamma))} - \gamma\right] & \text{for } \gamma \text{ in } [\underline{\gamma}, \overline{\gamma}) \\ 1 & \text{for } \gamma = \overline{\gamma} \end{cases}$$

is non negative and non decreasing.

Then both $y(\gamma)$ and $\Pi(\gamma)$ are continuous on $(\underline{\gamma}, \overline{\gamma})$. There is no distortion at the top when Π is continuous at $\overline{\gamma}$: $\overline{\gamma} v'(y(\overline{\gamma})) = 1$. There is no distortion at the bottom when $\Pi(\underline{\gamma}) = 0$: $\underline{\gamma} v'(y(\underline{\gamma})) = 1$. The social weights $\pi(\gamma)$ associated with this allocation are the (Stieltjes) derivative of $\Pi(\gamma)$.

Proof: I) Necessity. Since y is increasing, V' is strictly positive and a necessary condition for optimality is that the pointwise derivative of the Lagrangian in Lemma 1 be equal to zero. This yields the condition of the Proposition.

Continuity is proved as follows. Since $y(\gamma)$ is decreasing, any discontinuity has to be downwards. That creates a downwards discontinuity for $-1/v'(y)$ and therefore for Π, a contradiction with the fact that Π is non decreasing. The no distortion properties are straightforward consequences of the first order condition.

II) Sufficiency. The measure Π defined in the proposition is an adequate multiplier for the second best program. The function

$$V(\gamma) = \int_{\gamma}^{\overline{\gamma}} v'(y(\gamma))\, \mathrm{d}G(\gamma)$$

is convex non increasing. It maximizes the Lagrangian of Lemma 1 since its derivative is a pointwise maximum of a concave function of V'. ∎

Remark: Here is a general version of Proposition 2 with proof in the Appendix, which allows for pooling (i.e. y may be constant on some interval). In what follows, a pooling interval is a maximal interval where y is constant.

Proposition 3. *Suppose that Assumption 1 holds. A nonnegative nonincreasing function $y(\gamma)$ defined on Γ is a second best allocation if and only if there exists a nonnegative and nondecreasing function $\Pi(\gamma)$ with values in $[0, 1]$ such that*

$$\int_{\underline{\gamma}}^{\gamma} \left\{ G(\tilde{\gamma}) - g(\tilde{\gamma})\left[\frac{1}{v'(y(\gamma))} - \tilde{\gamma}\right]\right\} d\tilde{\gamma} \geq \int_{\underline{\gamma}}^{\gamma} \Pi(\tilde{\gamma})\, d\tilde{\gamma} \qquad (4)$$

for all γ, and (4) is an equality at any γ where y is decreasing.

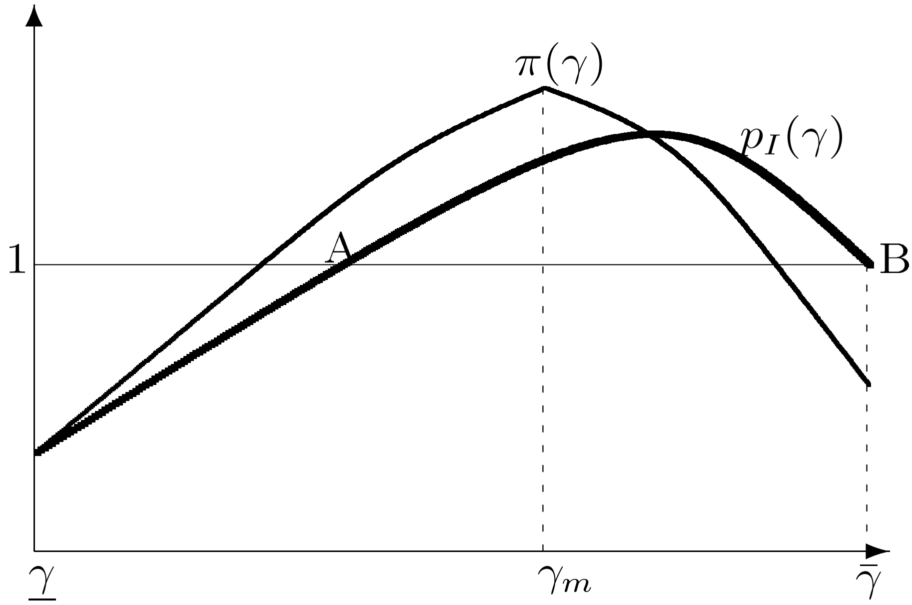

Figure 2: Social weights and negative marginal tax rates

3.3 Utilitarianism and marginal tax rates

The program of the typical consumer yields the first order condition

$$R'(y) = \gamma v'(y),$$

or, using the equality $R' = 1 - T'$

$$\frac{T'(y)}{1 - T'(y)} = \frac{1}{\gamma v'(y)} - 1.$$

Let $p_I(\gamma)$ be the average value of the social weights of all the agents with idiosyncratic characteristics smaller than γ:

$$p_I(\gamma) = \frac{\Pi(\gamma)}{G(\gamma)} = \frac{1}{G(\gamma)} \int_{\underline{\gamma}}^{\gamma} \pi(x)\, \mathrm{d}G(x).$$

Using Proposition 2, we get an expression of the optimal tax rate as a function of the distribution of the heterogeneity in the population and of the social weights:

$$\frac{T'(y(\gamma))}{1 - T'(y(\gamma))} = \frac{G(\gamma)}{\gamma g(\gamma)} \left(1 - p_I(\gamma)\right). \tag{5}$$

Under Assumption 1, G/g is well defined and positive for all γ larger than $\underline{\gamma}$, and the marginal tax rate has the same sign as $(1 - p_I(\gamma))$.

Consider the standard Mirrlees case where β is constant across the population, and ω has a continuous distribution on $[\underline{\omega}, \overline{\omega}]$. Then

$$\underline{\gamma} = \frac{\beta}{\overline{\omega}^{1+\frac{1}{e}}} \quad \overline{\gamma} = \frac{\beta}{\underline{\omega}^{1+\frac{1}{e}}},$$

and productivity, as well as utility, decreases with γ. Utilitarianism is equivalent to have social weights which increase with γ, which in turn implies that $p_I(\gamma)$ increases with γ. Since $p_I(\overline{\gamma}) = 1$, $p_I(\gamma) < 1$ for all $\gamma < \overline{\gamma}$, and we (fortunately) get the standard result: the marginal tax rate is always positive, but for the boundaries of the domain where it is equal to zero.

The situation can change when there are other dimensions of heterogeneity, which non trivially act on the agents utility levels. Suppose as an illustration that the utility is of the shape $\Psi[V(\gamma), \beta]$, with Ψ concave in its first argument, i.e. Ψ'_V decreasing in V. When Ψ'_V does not depend on β, the standard argument applies and optimal marginal tax rates are non negative. But Ψ'_V can be decreasing in β: this is the case when the utility of the agent is a concave transformation of $[V(\gamma) + \beta H]$, where $H > 0$ and the additive term βH stands for the 'home' production of the agent supposed to increase with her variable cost to work on the market. It can also be increasing in β, when a negative H in the above formula stands for a handicap: larger β's are associated with a lower quality of life, on top of the direct market effects. Let

$$\tilde{\pi}(\gamma, \beta) = \frac{\Psi'_V[V(\gamma), \beta]}{\int \Psi'_V[V(\gamma), \beta] \, \mathrm{d}H(\theta)},$$

so that the weights of interest to characterize the optimal allocation and tax schedule are

$$\pi(\gamma) = \int \tilde{\pi}(\gamma, \beta) \, \mathrm{d}G(\beta|\gamma),$$

where $G(\beta|\gamma)$ is the distribution of β conditional on the parameter γ. There are a variety of situations where tax rates are non negative:

Proposition 4. *Assume that the social weight $\Psi'_V[V, \beta]$ is decreasing in V, increasing (resp. decreasing) in β and that the distribution of β, conditional on γ, is first order stochastically increasing (resp. decreasing) in γ.*

Then the weights $\pi(\gamma)$ are increasing and marginal tax rates are non negative.

Proof: Let

$$f(a, b) = \int \tilde{\pi}(a, \beta) \, \mathrm{d}G(\beta|b).$$

f is increasing in a, since π, proportional to $\Psi'_V[V(a), \beta]$, is. It is increasing in b by first order stochastic dominance. It follows that $\pi(\gamma) = f(\gamma, \gamma)$ is also increasing in its argument. ■

Since $\beta = \gamma/\omega^{1+1/e}$, it is plausible that $G(\beta|\gamma)$ be first order stochastically increasing in γ. Then if Ψ'_V is increasing in β, i.e. larger opportunity costs are due to a handicap, the optimal marginal tax rates are non negative.

As a counterpart to the above proposition, it is easy to build examples with negative marginal tax rates, say when Ψ'_V decreases with β while the conditional distribution of β given γ increases. Consider the following economy. At the lowest wage rate $\underline{\omega}$, there are a variety of β's, a continuous distribution on $[\underline{\beta}, \overline{\beta}]$. For all the wage rates above the minimum, a continuous distribution on $(\underline{\omega}, \overline{\omega}]$, there is a unique value of β, equal to $\underline{\beta}$. In terms of γ's, we have:

$$\underline{\gamma} = \frac{\underline{\beta}}{\overline{\omega}^{1+\frac{1}{e}}} \quad \gamma_m = \frac{\underline{\beta}}{\underline{\omega}^{1+\frac{1}{e}}} \quad \overline{\gamma} = \frac{\overline{\beta}}{\underline{\omega}^{1+\frac{1}{e}}}.$$

The agent $\underline{\gamma}$ is the most productive with the smallest opportunity cost to work. All the agents of the segment $[\underline{\gamma}, \gamma_m]$ differ only by their productivities. All the agents in $[\gamma_m, \overline{\gamma}]$ have the same low productivity $\underline{\omega}$, but have different, increasing, opportunity costs. Figure 2 represents in a stylized way a possible profile of $\pi(\gamma)$, when the social weights are decreasing in β. Following standard utilitarianism, π is increasing on $[\underline{\gamma}, \gamma_m]$; it is supposed to decrease further on, the home production effect more than compensating the mechanical increase in γ as β rises. The agent with the largest social weight is the person with lowest productivity and opportunity cost to work. The associated function $p_I(\gamma)$, which measures the average height of $\pi(x)$ for x smaller than γ, is also represented: $p_I(\gamma)$ increases whenever it lies under the graph of π, decreases when it is above the graph, and has an horizontal tangent when it crosses the π curve. Also, we know that $p_I(\overline{\gamma}) = 1$. In the situation depicted on Figure 2, all the agents in the segment AB face negative tax rates. As noted by Saez (2002), page 1054, negative marginal tax rates at the bottom of the wage distribution as here can only occur if the social weight of the $\overline{\gamma}$ agent, smallest productivity, largest work opportunity cost, is smaller than the average social weight[4].

4 The extensive model

We now turn to the study of the second best optimal allocations in the extensive model.

4.1 Optimal taxes

The extensive model obtains as a limit case of model (1) when the elasticity e tends to zero: then the function v tends to zero for all h smaller than 1, and to $+\infty$

[4]Indeed the function p_I has to decrease towards one, and therefore must lie above the graph of π.

for all h larger than 1. If the agent participates, she is indifferent supplying any quantity of labor smaller than 1, since the variable opportunity cost $\beta v(h)$ then is equal to zero. It follows that the after tax income schedule $R(y)$ can be taken to be non decreasing without loss of generality[5]. Then, when she participates, the agent supplies one unit of labor and her before tax income y is equal to ω. As a consequence, before tax income can take any value in the support Ω of productivity, as well as the value 0. The function R has to be defined on $\{0\} \cup \Omega$.

Let $D(y) = R(y) - R(0)$ denote the financial incentive to work for an income y. The choice index of the typical agent, taken from (1), is

$$u(R;\theta) = R(0) + \max[\alpha, D(\omega)].$$

An agent works whenever α is less than or equal[6] to $D(\omega)$. This implies

$$\int_\Theta [\mathbf{y}_R(\theta) - R(\mathbf{y}_R(\theta))]\,\mathrm{d}H(\theta) = \int_{\alpha \le D(\omega)} [\omega - D(\omega)]\,\mathrm{d}H(\theta) - R(0),$$

and the Lagrangian (3) becomes

$$\mathcal{L} = \int_\Theta \left\{ \max[\alpha, D(\omega)]\,\mathrm{d}\Pi(\theta) + [\omega - D(\omega)]\,\mathbb{1}_{\alpha \le D(\omega)}\,\mathrm{d}H(\theta) \right\}.$$

It is convenient to work under the

Assumption 2. *For all ω, the distribution of opportunity costs of work α, conditional on ω, is continuous with support $[\underline{\alpha}(\omega), \overline{\alpha}(\omega)]$, $\underline{\alpha}(\omega) \ge 0$, and cumulative distribution function $F(\alpha|\omega)$. Its probability distribution function $f(\alpha|\omega)$ is positive everywhere on its support.*

After simple manipulations, the objective becomes

$$\begin{aligned}\mathcal{L} \;=\; & \int_\Theta \pi(\theta)\alpha\,\mathrm{d}H(\theta) + \\ & \int_\omega \int_{\alpha=\underline{\alpha}(\omega)}^{D(\omega)} \mathbb{1}_{\underline{\alpha}(\omega) \le D(\omega)}\,\mathrm{d}G(\omega)\left\{[D(\omega) - \alpha]\,\mathrm{d}\Pi(\theta) + [\omega - D(\omega)]\,\mathrm{d}H(\theta)\right\}.\end{aligned}$$

We have shown

Lemma 2. *An income tax schedule $R(.)$ is second best optimal if and only if there is a probability measure of cdf $\Pi(\theta)$ such that the incentive schedule $D(\omega) = R(\omega) - R(0)$ maximizes*

$$\int_\omega \left\{ [\omega - D(\omega)]F(D(\omega)|\omega)\,dG(\omega) + \int_{\underline{\alpha}(\omega)}^{D(\omega)} [D(\omega) - \alpha]\,d\Pi(\theta) \right\} \mathbb{1}_{\underline{\alpha}(\omega) \le D(\omega)}$$

---

[5] Take any, possibly sometimes decreasing, function $\tilde{R}(y)$. Let $R(y) = \max_{y \ge z} \tilde{R}(z)$. The agents have the same behavior under R and $\tilde{R}$.

[6] For efficiency, since productivity is positive, the agents that are indifferent between working and not working are supposed to be working.

on the set of non decreasing functions $D(.)$, such that $D(0) = 0$. When $\Pi(\theta)$ is absolutely continuous with respect to $H(\theta)$, with pdf $\pi(\theta)$ the criterion can be rewritten as

$$\int_{\omega} \left\{ [\omega - D(\omega)]F(D(\omega)|\omega) + \int_{\underline{\alpha}(\omega)}^{D(\omega)} [D(\omega) - \alpha]\pi(\theta)\, dF(\alpha|\omega) \right\} \mathbb{1}_{\underline{\alpha}(\omega) \leq D(\omega)}\, dG(\omega) \tag{6}$$

Let

$$L(D; \omega) = [\omega - D]F(D|\omega) + \int_{\underline{\alpha}(\omega)}^{D} [D - \alpha]\pi(\theta)\, \mathrm{d}F(\alpha|\omega).$$

Since under Assumption (2), $L(\underline{\alpha}(\omega); \omega)$ is equal to zero, the program can be restricted to the domain $D(\omega) \geq \underline{\alpha}(\omega)$. Whenever at the optimum $L(D(\omega); \omega) = 0$, $D(\omega)$ is indeterminate and can take any value less than or equal to $\underline{\alpha}(\omega)$, without changing the objective: the condition $D(0) = 0$ can always be satisfied.

We therefore have to maximize (6) on the set of non decreasing functions $D(.)$ which satisfy $D(\omega) \geq \underline{\alpha}(\omega)$. Unfortunately, contrary to the intensive case, the function L is not a concave function of D. Nevertheless, at any point ω where the solution is strictly increasing and larger than $\underline{\alpha}(\omega)$, it satisfies the first order condition for a pointwise maximum[7]

$$\frac{\partial L}{\partial D} = [\omega - D]f(D|\omega) - F(D|\omega)[1 - p_E(D|\omega)] = 0,$$

where $p_E(D|\omega)$ is the average social weight of the agents of productivity ω and of work opportunity cost smaller than D

$$p_E(D|\omega) = \frac{1}{F(D|\omega)} \int_{\underline{\alpha}(\omega)}^{D} \pi(\theta)\, \mathrm{d}F(\alpha|\omega). \tag{7}$$

The expression of $\partial L/\partial D$ has a direct economic interpretation: the first term $[\omega - D]f(D)$ is the gain in government income obtained from the new $f(D)$ workers that participate because of the increase in D; the second term $F(D)[1 - p_E(D)]$ is the loss on the existing workers $F(D)$ which depends on their social weights (and indeed is a social gain for those of weights larger than 1).

The tax supported by the workers of productivity ω is $T(\omega) = \omega - D(\omega) - R(0)$, so that the first order condition can be rewritten as

$$\omega - D(\omega) = R(0) + T(\omega) = \frac{F[D(\omega)|\omega]}{f[D(\omega)|\omega]}[1 - p_E(D(\omega)|\omega))]. \tag{8}$$

[7] The second order condition is

$$\frac{\partial^2 L}{\partial D^2} = [\omega - D]f'(D|\omega) - (2 - \pi(D, \omega))f(D|\omega) < 0.$$

In general, there may exist several solutions to the first order condition, corresponding to local maxima or minima. Furthermore, as in the intensive case, the optimum may involve pooling, with regions where D stays constant because of the monotonicity condition.

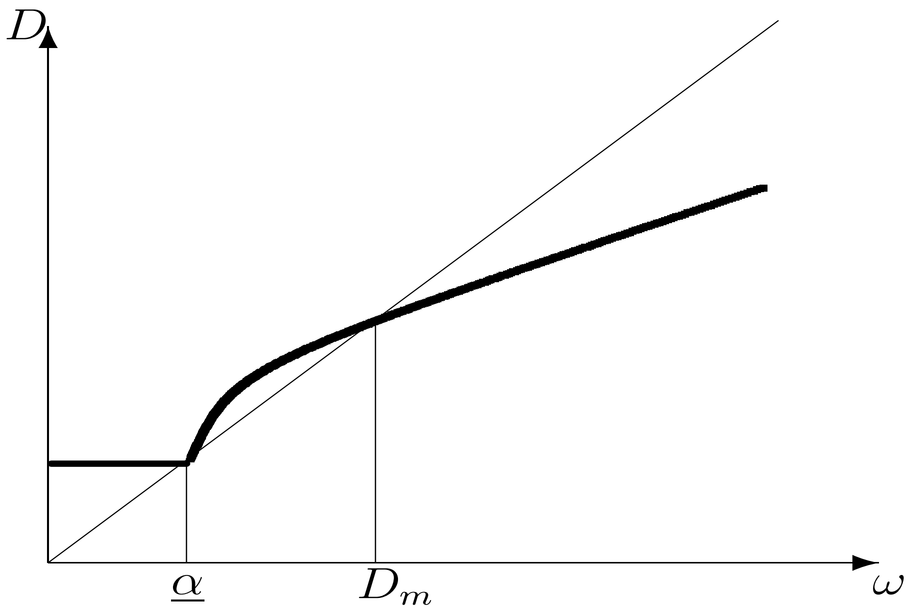

Figure 3: The extensive model: 'well behaved' optimal financial incentives to work

This equation is strikingly similar to (5), which describes optimal taxes in the intensive model. However, the formal similarity hides important differences. In the intensive model this is the *marginal* rate of taxation that appears on the left hand side, while here it is the *level* of tax. The right hand sides look the same, but again this is deceptive. The average weight here, $p_E(D(\omega)|\omega))$, is that of the subset of the employed ($\alpha \leq D(\omega)$) agents of productivity equal to ω. In the intensive model it is the average weight of the agents of parameter $\beta/\omega^{1+1/e}$ smaller than the current γ, i.e. of larger productivity or smaller opportunity cost to work. Social weights larger than 1, corresponding to a group of people whose average social weight is larger than that of society as a whole, which are associated with negative rates in the intensive model, here correspond to a financial incentive $D(\omega)$ larger than ω. In both models they distort labor supply upwards, compared with laissez-faire.

4.2 Comparison with the standard intensive literature

The extensive model has imbedded at its heart two dimensions of heterogeneity, which cannot be reduced to one. This gives a lot of leeway to get results of the type 'any kind of tax function can occur' manipulating (8): one can play with the distribution $F(\alpha|\omega)$, as in Choné and Laroque (2005) for a Rawlsian planner, or with the distribution of weights, see Laroque (2005) for all second best allocations. For comparison with the literature on the intensive model, we restrict these degrees of freedom.

The main restriction bears on the distribution of the agents characteristics. First, we do not want to play with the correlation between productivity and the opportunity cost of work to play a role, and in the main analysis we assume independence of the two distributions: $F(\alpha|\omega)$ does not depend on ω. It simplifies the exposition to suppose that the lower bound of productivity is not larger than the lower bound of the opportunity cost to work. Also we suppose that the distribution of heterogeneity is well behaved. Formally, in this section, on top of Assumption 2, we assume

Assumption 3. *The cdf $F(\alpha|\omega)$ of the work opportunity cost α is independent of productivity. Furthermore $\ln(F(\alpha))$ is concave on its support $(\underline{\alpha}, \overline{\alpha}]$, and $\underline{\alpha} \geq \underline{\omega}$.*

Of course, we require consistency with utilitarianism: the social weights should be a decreasing function of the appropriately defined utility level of the agents. Here, in the *benchmark model*, the parameter α is considered as an incidental cost of work (and *not* as a benefit in case of not working). The utility of the typical agent is

$$\Psi[u(R,\theta) - \alpha] = \Psi[R(0) + \max(0, D(\omega) - \alpha)].$$

Then the unemployed agents are the worse off agents in the economy and, given $R(0)$, the social weights consistent with utilitarianism are of the form $\tilde{\pi}(D - \alpha)$, with $\tilde{\pi}$ non increasing independent of ω, and $\tilde{\pi}(x) = \tilde{\pi}(0)$ for all negative x.

It turns out that a number of the properties proved below are satisfied for a large class of economies and social welfare functions, apart form the benchmark case. They rely on two aspects of the behavior of the average social weight of the employed agents, defined in (7), formalized in the following assumption and the next property.

Assumption 4. *The average weight of the employed agents $p_E(D|\omega)$*

$$p_E(D|\omega) = \begin{cases} \pi(\underline{\alpha}(\omega), \omega) & \text{for } \alpha = \underline{\alpha}(\omega) \\ \dfrac{1}{F(D)} \displaystyle\int_{\underline{\alpha}(\omega)}^{D} \pi(\alpha, \omega)\, dF(\alpha|\omega) & \text{for } D > \underline{\alpha}(\omega) \end{cases}$$

is a function $p_E(D)$ independent of productivity. It is continuously differentiable and non increasing.

Assumption 4 is satisfied in the benchmark model. The fact that the function decreases seems natural to a utilitarist, but in fact it depends both on the welfare criterion and on the shape of the distribution of α. A simple differentiation yields

$$p_E'(D) = \frac{f(D)}{F(D)}[\tilde{\pi}(0) - p_E(D)] + \frac{1}{F(D)} \int_{\underline{\alpha}}^{D} \tilde{\pi}'(D - \alpha)\, \mathrm{d}F(\alpha).$$

An increase in D increases the wealth of all the already employed agents, and therefore decreases their average social weights (the second term), but it brings into employment new blood, formerly unemployed with a high social weight (the first term). In the benchmark model, the first effect dominates:

Lemma 3. *Under Assumptions 2 and 3, Assumption 4 in the benchmark model.*

Proof: The continuous differentiability of $p_E(D)$ is straightforward. We first show that it is decreasing. We have

$$\begin{aligned} F(D)^2 p_E'(D) &= \left[\tilde{\pi}(0) f(D) + \int_{\underline{\alpha}}^{D} \tilde{\pi}'(D-\alpha) f(\alpha)\,\mathrm{d}\alpha\right] F(D) \\ &- f(D) \int_{\underline{\alpha}}^{D} \tilde{\pi}(D-\alpha) f(\alpha)\,\mathrm{d}\alpha. \end{aligned}$$

So $p_E' \leq 0$ is equivalent to

$$\tilde{\pi}(0) + \frac{1}{f(D)} \int_{\underline{\alpha}}^{D} \tilde{\pi}'(D-\alpha) f(\alpha)\,\mathrm{d}\alpha \leq \frac{1}{F(D)} \int_{\underline{\alpha}}^{D} \tilde{\pi}(D-\alpha) f(\alpha)\,\mathrm{d}\alpha. \tag{9}$$

For $\alpha \leq D$, we have, thanks to the log-concavity of F

$$\frac{f(\alpha)}{f(D)} \geq \frac{F(\alpha)}{F(D)}.$$

Since $\tilde{\pi}' \leq 0$, we have

$$\begin{aligned} \tilde{\pi}(0) + \frac{1}{f(D)} \int_{\underline{\alpha}}^{D} \tilde{\pi}'(D-\alpha) f(\alpha)\,\mathrm{d}\alpha &\leq \tilde{\pi}(0) + \frac{1}{F(D)} \int_{\underline{\alpha}}^{D} \tilde{\pi}'(D-\alpha) F(\alpha)\,\mathrm{d}\alpha \\ &= \frac{1}{F(D)} \int_{\underline{\alpha}}^{D} \tilde{\pi}(D-\alpha) f(\alpha)\,\mathrm{d}\alpha \end{aligned}$$

which gives (9). ■

An important threshold for the financial incentive to work is the one that makes the average social weight of all the employed agents equal to the marginal cost of public funds, here 1. Let D_m be such that $p_E(D_m) = 1$, or when $p_E(D)$ is smaller than 1 for all D, $D_m = +\infty$. Then

Proposition 5. *Consider the benchmark model under Assumptions 2 and 3.*

When some agents in the economy work (i.e. have $D(\omega) > \underline{\alpha}$), $p_E(D_m) = 1$ for some finite D_m, $D_m > \underline{\alpha}$.

Proof: Since some agents are working and the utilitarian criterion is concave, the weight of the least favored agents, $\tilde{\pi}(0) = p_E(\underline{\alpha})$, is larger than 1. The weights at an optimum sum up to 1:

$$\int_{\Omega} [F(D(\omega))p_E(D(\omega)) + (1 - F(D(\omega)))\tilde{\pi}(0)] \, \mathrm{d}G(\omega) = 1.$$

It follows that, for some $D(\omega)$, $p_E(D(\omega)) < 1$. By continuity, there exists D_m, $D_m > \underline{\alpha}$, with $p_E(D_m) = 1$. ∎

We are now in a position to describe the qualitative properties of the optimal tax schedule. The first proposition deals with all non pooling equilibria, the next ones give sufficient conditions where there is no pooling at the equilibrium and provide a more precise characterization of the optimum.

Proposition 6. *Consider an economy satisfying Assumptions 2 to 4. Suppose that the optimum $D(\omega)$ is strictly increasing at all points where $D(\omega) > \underline{\alpha}$ (no pooling). Then:*

1. *For $\omega \geq D_m$, the financial incentive to work $D(\omega)$ is smaller than before tax income ω: labor supply is distorted downwards compared to laissez-faire. Furthermore the marginal tax rate is nonnegative.*
2. *For $\underline{\alpha} \leq \omega \leq D_m$, the financial incentive to work $D(\omega)$ is larger than before tax income ω: labor supply is distorted upwards compared to laissez-faire.*

Proof: Since by assumption the optimal schedule is (strictly) increasing, the first order condition (8) for a pointwise maximization holds everywhere

$$\omega - D(\omega) = \frac{F[D(\omega)]}{f[D(\omega)]}[1 - p_E(D(\omega))].$$

Then $\omega \geq D_m$ if and only if $1 \geq p_E(D_m)$.

When D is larger than D_m, using Assumption 4, the right hand side of the above equation, $[1 - p_E(D)]F(D)/f(D)$, is increasing as the product of two non negative increasing functions. This implies that $\omega - D(\omega)$ is an increasing function of ω: the marginal tax rate is non negative. ∎

A possible shape of the optimal incentive schedule is drawn on Figure 3, which obtains in the cases described in the following proposition.

Proposition 7. *Consider an economy that satisfies Assumptions 2 to 4.*

Assume that

$$M(D) = D + \frac{F(D)}{f(D)}[1 - p_E(D)]$$

is strictly increasing on $[\underline{\alpha}, D_m]$.

Then there is no pooling at the optimum. The optimal incentives $D(\omega)$ *are uniquely defined for all* ω *larger than* $\underline{\alpha}$ *and satisfy the equation*

$$M(D(\omega)) = \omega,$$

on $[\underline{\alpha}, \overline{\alpha}]$.

Furthermore $D(\omega)$ *is an increasing function of* ω *on* $[\underline{\alpha}, \overline{\alpha}]$ *which satisfies*

$$D(\underline{\alpha}) = \underline{\alpha},$$

$$D(\omega) \gtrless \omega \quad \textit{whenever} \quad D \lessgtr D_m.$$

Proof: We first show that $D(\omega)$, defined through the equality $M(D(\omega)) = \omega$, maximizes $L(D;\omega)$ on the set $D \geq \underline{\alpha}$ for all ω. We have

$$\frac{\partial L}{\partial D}(D;\omega) = f(D)[\omega - M(D)].$$

By construction, under the monotonicity assumption, $D(\omega)$ is the unique solution of the first order condition: we have to check that it yields a global maximum on $[\underline{\alpha}, \infty)$. Now, at the lower end of the domain,

$$\frac{\partial L}{\partial D}(\underline{\alpha}) = (\omega - \underline{\alpha})f(\underline{\alpha}) \geq 0.$$

Also, for D larger than $\max(\omega, D_m)$,

$$\frac{\partial L}{\partial D} = [\omega - D]f(D) - F(D)[1 - p_E(D)]$$

is negative as the sum of two negative terms. $D(\omega)$ is the unique zero of the derivative which goes from non negative at $\underline{\alpha}$ to negative for large D: it is the unique maximum of the function $L(D;\omega)$.

Now note that $M(D)$ is increasing for $D > D_m$ as the sum of two increasing functions. Since M is increasing everywhere, $D(\omega)$ is increasing too, and satisfies the monotonicity constraint. It therefore is the global optimum. Finally, the location of $D(\omega)$ with respect to the 45 degree line is a straightforward consequence of the shape of $M(D)$.

■

Figure 3 illustrates the two foregoing propositions in the 'well-behaved' situation[8]. The financial incentives to work are a continuous increasing function of

[8] It is similar to Figure IIa in Saez (2002), who discusses from a more applied perspective the occurrence of negative marginal tax rates.

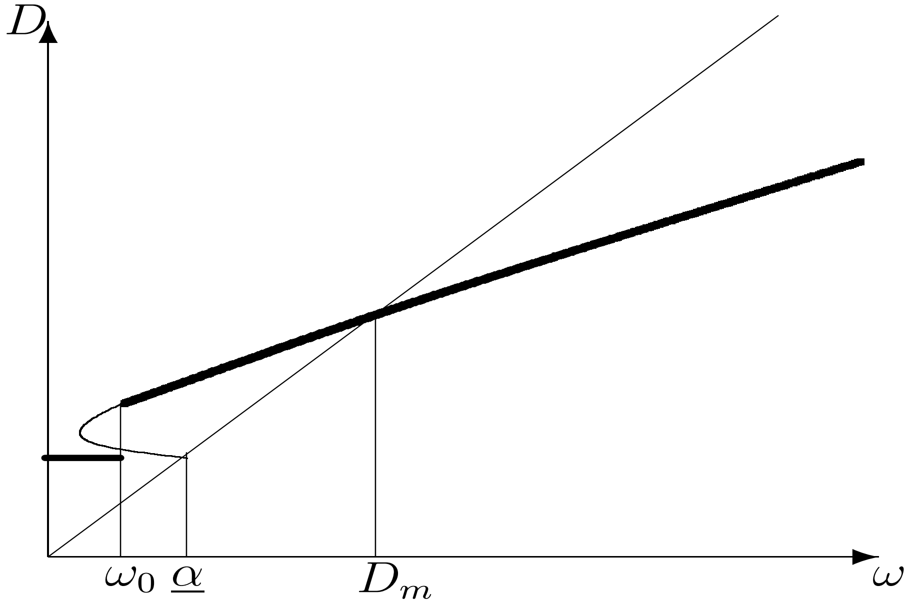

Figure 4: The extensive model: a possible shape of optimal financial incentives to work in the uniform case

productivity. Under utilitarianism, there is a low skilled region, $\underline{\alpha} \leq \omega \leq D_m$, where labor supply is distorted upwards, while for higher productivities labor is taxed and the marginal tax rate is positive. In the more restricted case of Proposition 7, the marginal tax rate is negative for low enough productivities (indeed, since $D(\underline{\alpha}) = \underline{\alpha}$ and $D(\omega) > \omega$ in a neighborhood, D' has to be larger than one in the region).

More generally, the function $M(D)$ may very well be non increasing for $D < D_m$, in which case the first order condition $\omega = M(D)$ typically has several solutions. The proof of Proposition 7 goes through by selecting the solution $D(\omega)$ associated with the *global* maximum of $L(D;\omega)$, provided this selection is increasing in ω. The shape of the incentive curve in the region $\underline{\alpha} \leq D \leq D_m$ then could look quite different, for instance starting at a point $D(\underline{\alpha}) > \underline{\alpha}$ and possibly exhibiting upward discontinuities at solution switches. This is illustrated in the following example:

Proposition 8. *Consider a benchmark economy satisfying Assumptions 2 and 3. Suppose that the opportunity cost α is uniformly distributed on $[\underline{\alpha}, \overline{\alpha}]$.*

Then $D(\omega)$ is increasing and concave whenever some agents of productivity ω work, i.e. on the set $\{\omega | D(\omega) > \underline{\alpha}\}$. Moreover:

1. *If $\tilde{\pi}_0 \leq 2$, the conditions of Proposition 7 are verified, $D(\underline{\alpha}) = \underline{\alpha}$ and $D'_+(\underline{\alpha}) = 1/(2-\tilde{\pi}_0) > 1$. At the optimum, none of the agents of productivity smaller than $\underline{\alpha}$ work.*

2. *If $\tilde{\pi}_0 > 2$, there exists ω_0, $\underline{\omega} \leq \omega_0 < \underline{\alpha}$, such that $D(\omega) > \underline{\alpha}$ for all $\omega \geq \omega_0$ and $D(\omega) \leq \underline{\alpha}$ for productivities smaller than ω_0. There is an upwards discontinuity in the incentives to work at ω_0.*

The situation where the social weights of the unemployed are high ($\tilde{\pi}(0) > 2$) is shown on Figure 4. None of the agents with very low productivities, $\omega < \omega_0$ work. But for all ω larger than or equal to ω_0, a fraction of the agents do some work. In fact the upwards distorsion to labor supply here is particularly strong: some agents with productivity smaller than the minimal cost of going to work participate in the labor force. The piece of horizontal parabola on the Figure describes the roots of the first order condition. There is a single root, corresponding to a global maximum of L for ω larger than $\underline{\alpha}$, but there are two roots in a part of the low productivity region. The bold line describes the solution. The curve is concave, implying a progressive tax system. It is not always the case that there are negative marginal tax rates at the beginning of the curve, close to ω_0, contrary to the situation when $\tilde{\pi}_0 < 2$ of Figure 3. But there is an upwards discontinuity in the tax schedule at ω_0, indeed an infinite negative marginal tax rate.

Remark: Proposition 6 does not apply to a Rawlsian planner that puts a Dirac mass on the least favored agent in the economy. This case can be dealt with here by letting $p_E(D)$ equal to zero for all D greater than $\underline{\alpha}$: all the employed agents are better off than the unemployed. Then, when there is no pooling, D_m is equal to $\underline{\alpha}$ and the optimal incentive satisfies 1. of Proposition 6: it is everywhere smaller than productivity and the marginal tax rate is always positive. This is in line with the results of Choné and Laroque (2005).

Remark: Theorem 6 of Choné and Laroque (2005) applies here: all the utilitarian optimal allocations correspond to incentive schemes located above the Rawlsian (Laffer) curve. Theorem 3 of Laroque (2005) also applies: any incentive scheme above the Laffer curve which does not overtax and such that $D(\omega) \leq \omega$ corresponds to a second best optimal allocation. Note that in a benchmark model, from the above results, none of these allocations satisfy a utilitarian criterion. All the utilitarian optimal allocations are such that $D(\omega) > \omega$ for some ω's, a property discussed in Remark 2.3 of Laroque (2005).

Remark: A number of the qualitative features of the solution carry over to the more general model where the utilities of the agents take the form

$$R(0) + \alpha_u + \max(0, D(\omega) - \alpha)),$$

where $\alpha = \alpha_u + \alpha_c$ is the opportunity cost of working, which separates into two terms, α_u the utility of staying at home, and α_v a pure sunk cost of going to work. The social weight of an agent is therefore of the form $\tilde{\pi}(\alpha_u + \max(0, D(\omega) - \alpha))$ with, under utilitarianism, $\tilde{\pi}$ a decreasing function of its argument. As in the intensive example, society puts a low weight on the shirkers who enjoy staying unemployed (high α_u's). The average weight of the workers who have a financial incentive equal to D can then be written

$$p_E(D) = \frac{1}{F(D)} \int\int_{\alpha_u+\alpha_c \le D} \tilde{\pi}(D-\alpha_c)\, \mathrm{d}F(\alpha_c, \alpha_u).$$

The polar case where $\alpha_c = 0$ is easy to handle. It yields $p_E(D) = \tilde{\pi}(D)$ which is decreasing and the previous arguments carry over to this situation. The economy then is quite different from our real world: here the unemployed agents have a higher utility than the employed with the same productivity, and therefore smaller social weights. It follows that, for small D, there is a zone of subsidy where $p_E(D)$ is larger than 1; it is not sure that $p_E(D)$ becomes smaller than 1 for large enough D (D_m may be equal to $+\infty$). The utilitarian criterion typically subsidizes the workers through a lump sum tax on everyone, $R(0) < 0$.

More generally, a sufficient condition (proved in the Appendix) for $p_E(D)$ to be a decreasing function of D is that α_u, conditional on α, first order stochastically increases with α:

Lemma 4. *Let α_u and α_c be nonnegative random variables and $\alpha = \alpha_u + \alpha_c$. We suppose that F, the c.d.f. of α, is log-concave and that α_u, conditional on α, first-order stochastically increases with α. Then*

$$p_E(D) = \frac{1}{F(D)} \int\int_{\alpha_u+\alpha_c \le D} \tilde{\pi}(D-\alpha_c)\, dF(\alpha_c, \alpha_u)$$

is nonincreasing with D.

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A Appendix

Lemma 5. *Let f be in $L^1(\underline{\theta}, \overline{\theta})$ and F be given by $F(\theta) = \int_{\underline{\theta}}^{\theta} f(t)\, dt$. Let y be a nondecreasing and bounded function on $[\underline{\theta}, \overline{\theta}]$.*

Then the following integration by parts formula holds

$$\int_{\underline{\theta}}^{\overline{\theta}} f(\theta)y(\theta)\, d\theta = F(\overline{\theta})y(\overline{\theta}) - F(\underline{\theta})y(\underline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} F\, dy, \tag{10}$$

where $\int_{\underline{\theta}}^{\overline{\theta}} F\, dy$ is defined as a Riemann-Stieltjes integral, that is, as the limit of

$$S = \sum_{i=0}^{n} F(t_i)\ [y(\theta_{i+1}) - y(\theta_i)]$$

for any mesh $(\theta_0 = \underline{\theta}, \theta_1, ..., \theta_n, \theta_{n+1} = \overline{\theta})$ and any $t_i \in (\theta_i, \theta_{i+1})$, when the mesh size $\max_i |\theta_{i+1} - \theta_i|$ tends to zero.

Proof of Lemma 5

First note that the left hand side of Eq. (10) is well defined since the function fy is Lebesgue integrable. Note also that the function F is continuous and almost everywhere differentiable with $F' = f$ a.e.

A simple computation shows that

$$\begin{aligned} S &= -F(t_0)y(\underline{\theta}) - y(\theta_1)[F(t_1) - F(t_0)] - ... - y(\theta_n)[F(t_n) - F(t_{n-1})] + F(t_n)y(\overline{\theta}) \\ &= -F(t_0)y(\underline{\theta}) + F(t_n)y(\overline{\theta}) - \sum_{i=1}^{n} y(\theta_i) \int_{t_{i-1}}^{t_i} f(t)\, \mathrm{d}t. \end{aligned}$$

By the Lebesgue Theorem, the last sum tends to $\int_{\underline{\theta}}^{\overline{\theta}} f(\theta)y(\theta)\, \mathrm{d}\theta$ when the mesh size tends to zero, which (together with the continuity of F) gives (10). ■

Proof of Proposition 3

Suppose first that y is second best optimal. The derivative of the Lagrangian is

$$< \mathrm{d}\mathcal{L}, H >= \int \left[-\frac{1}{v'(y)} + \gamma \right] \dot{H}\, \mathrm{d}G(\gamma) + \int \dot{H}(G - \Pi)\, \mathrm{d}\gamma.$$

Since the problem is concave, a function V is the solution if and only if

$$< \mathrm{d}\mathcal{L}, H > \leq 0$$

for all admissible variations $\dot{H}$ (ie, for all functions $\dot{H}$ such that $\dot{V} + \varepsilon \dot{H}$ is negative and non decreasing for ε small enough).

When y is strictly decreasing, $<\mathrm{d}\mathcal{L}, H>$ must be zero for all $\dot{H}$ (since, in that case, $\dot{V}$ and $\dot{V}+\varepsilon\dot{H}$ are increasing for small ε). It follows that we have in the no pooling region

$$\Pi(\gamma) = G(\gamma) - g(\gamma)\left[\frac{1}{v'(y)} - \gamma\right].$$

In a pooling interval $[\underline{\gamma}_i, \overline{\gamma}_i]$, the functions y and $\dot{V}$ are constant and any H such that $\dot{H}$ is decreasing is not an admissible test function (since $\dot{V}+\varepsilon\dot{H}$ is decreasing in $[\underline{\gamma}_i, \overline{\gamma}_i]$).

It is easy to check that if H satisfies

$$\dot{H} = \begin{cases} 1 & \text{in } [\underline{\gamma}_i, \overline{\gamma}_i] \\ 0 & \text{otherwise.,} \end{cases} \tag{11}$$

then H and $-H$ are admissible variations, so we must have: $<\mathrm{d}\mathcal{L}, H>=0$. It follows that

$$\int_{\underline{\gamma}_i}^{\overline{\gamma}_i} \left\{G(\tilde{\gamma}) - g(\tilde{\gamma})\left[\frac{1}{v'(y)} - \gamma\right]\right\} \mathrm{d}\gamma = \int_{\underline{\gamma}_i}^{\overline{\gamma}_i} \Pi(\tilde{\gamma})\,\mathrm{d}\tilde{\gamma}. \tag{12}$$

Now if H satisfies

$$\dot{H}(\tilde{\gamma}) = \begin{cases} -1 & \text{for } \tilde{\gamma} < \gamma \text{ in } [\underline{\gamma}_i, \overline{\gamma}_i] \\ 0 & \text{for } \tilde{\gamma} > \gamma \text{ in } [\underline{\gamma}_i, \overline{\gamma}_i] \end{cases} \tag{13}$$

for some $\gamma \in [\underline{\gamma}_i, \overline{\gamma}_i]$, then H is admissible (but $-H$ is not) and we must have: $<\mathrm{d}\mathcal{L}, H>\leq 0$. It follows that

$$\int_{\underline{\gamma}_i}^{\gamma} \left\{G - g\left[\frac{1}{v'(y_i)} - \tilde{\gamma}\right]\right\} \mathrm{d}\tilde{\gamma} \geq \int_{\underline{\gamma}_i}^{\gamma} \Pi(\tilde{\gamma})\,\mathrm{d}\tilde{\gamma}. \tag{14}$$

The conditions (12) and(14) are equivalent to the first statement of the proposition. The last statement (geometrical interpretation) follows from the convexity of the function $\int_{\underline{\gamma}}^{\gamma} \Pi(\tilde{\gamma})\,\mathrm{d}\tilde{\gamma}$.

The sufficient part follows from the fact that conditions (12) and(14) are equivalent to $<\mathrm{d}\mathcal{L}, H>\leq 0$ for all admissible variations H, since the set of non-increasing functions $\dot{H}$ on $[\underline{\gamma}_i, \overline{\gamma}_i]$ is generated by the set of functions H satisfying (11) and (13). ■

Proposition 3 has a geometric interpretation, shown on Figure 5. Let Y be defined by

$$Y(\gamma) = \int_{\underline{\gamma}}^{\gamma} \left\{G(\tilde{\gamma}) - g(\tilde{\gamma})\left[\frac{1}{v'(y(\gamma))} - \tilde{\gamma}\right]\right\} \mathrm{d}\tilde{\gamma},$$

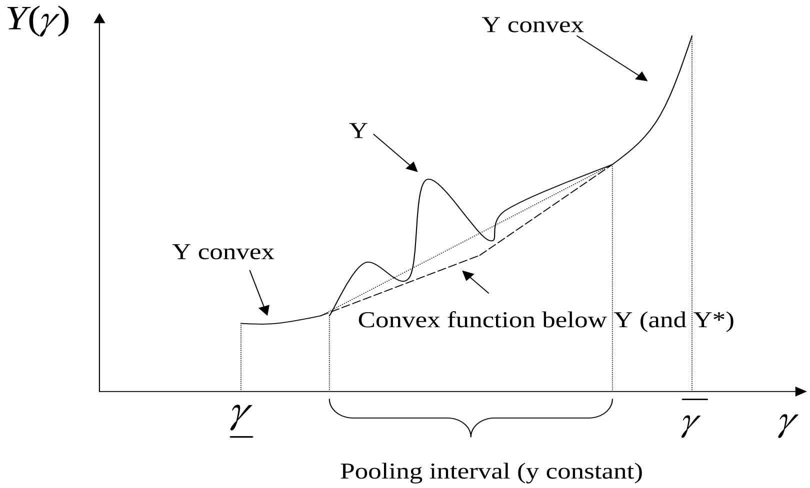

Figure 5: Pooling in the intensive case

and $Y^\star$ be the convex hull of Y. Then y is second best optimal if and only if the slope of $Y^\star$ is in $[0,1]$ and $Y = Y^\star$ outside the pooling intervals.

The derivative of $Y^\star$ is the c.d.f. of a social weight distribution for which the allocation y is optimal. The distribution of social weights Π is unique outside pooling intervals, but it is not unique in the pooling intervals (Π can be the derivative of any convex function below Y which coincides with Y outside the pooling intervals).

Proof of Lemma 4

We note $F_c(\alpha_c|\alpha)$ the cdf of the distribution of α_c conditional on α, and similarly, with a subscript u that of α_u conditional on α. Let

$$\begin{aligned} K(\alpha) &= \int_{\underline{\alpha}}^{\alpha} \tilde{\pi}(D-\alpha_c)\,\mathrm{d}F_c(\alpha_c|\alpha) \\ &= \tilde{\pi}(D-\alpha) + \int_0^{\alpha} \tilde{\pi}'(D-\alpha_c)F_c(\alpha_c|\alpha)\,\mathrm{d}\alpha_c, \end{aligned} \tag{15}$$

where we have used $F_c(\alpha|\alpha) = 1$.

It is easy to check that $p_E(D)$ nonincreasing is equivalent to

$$K(D) + \frac{1}{f(D)} \int_0^D \int_0^{\alpha} \tilde{\pi}'(D-\alpha_c)\,\mathrm{d}F_c(\alpha_c|\alpha)\,\mathrm{d}F(\alpha) \leq \frac{1}{F(D)} \int_0^D K(\alpha)\,\mathrm{d}F(\alpha). \tag{16}$$

By log-concavity of F, we have (using $\tilde{\pi}' \leq 0$)

$$\frac{1}{f(D)}\int_0^D\int_0^\alpha \tilde{\pi}'(D-\alpha_c)\,\mathrm{d}F_c(\alpha_c|\alpha)\,\mathrm{d}F(\alpha) \leq \frac{1}{F(D)}\int_0^D\int_0^\alpha \tilde{\pi}'(D-\alpha_c)\,\mathrm{d}F_c(\alpha_c|\alpha)F(\alpha)\,\mathrm{d}\alpha$$

By integration by parts

$$\frac{1}{F(D)}\int_0^D K(\alpha)\,\mathrm{d}F(\alpha) = K(D) - \frac{1}{F(D)}\int_0^D K'(\alpha)F(\alpha)\,\mathrm{d}\alpha.$$

It follows that (16) is implied by

$$\int_0^D\int_0^\alpha \tilde{\pi}'(D-\alpha_c)\,\mathrm{d}F_c(\alpha_c|\alpha)F(\alpha)\,\mathrm{d}\alpha \leq -\int_0^D K'(\alpha)F(\alpha)\,\mathrm{d}\alpha. \qquad (17)$$

By (15), we get

$$K'(\alpha) = \int_0^\alpha \tilde{\pi}'(D-\alpha_c)\frac{\partial F_c}{\partial \alpha}\,\mathrm{d}\alpha_c.$$

It follows that (17) is equivalent to

$$\int_0^D\int_0^\alpha \tilde{\pi}'(D-\alpha_c)\left[f_c(\alpha_c|\alpha) + \frac{\partial F_c}{\partial \alpha}\right]\,\mathrm{d}\alpha_c.F(\alpha)\,\mathrm{d}\alpha \leq 0. \qquad (18)$$

which is satisfied when

$$f_c(\alpha_c|\alpha) + \frac{\partial F_c}{\partial \alpha} = \frac{\partial F_c}{\partial \alpha_c} + \frac{\partial F_c}{\partial \alpha} = -\frac{\partial F_u}{\partial \alpha} \geq 0$$

that is, when α_u first-order stochastically increases with α.

■

Proof of Proposition 8: Let

$$\lambda = \int_\Theta \Psi'[R(0) + \max(0, D(\omega) - \alpha)]\,\mathrm{d}H(\theta).$$

Then

$$p_E(D) = \frac{1}{\lambda}\int_{\underline{\alpha}}^D \Psi'[R(0) + D - \alpha]\frac{\mathrm{d}\alpha}{\overline{\alpha} - \underline{\alpha}}.$$

Integrating by parts and substituting yields

$$M(D) = 2D - \underline{\alpha} - \frac{1}{\lambda}[\Psi(R(0) + D - \underline{\alpha}) - \Psi(R(0))].$$

The function $M(D)$ is strictly convex in D and $M'(\underline{\alpha}) = 2 - \tilde{\pi}(0)$.

1) Case $\tilde{\pi}_0 \leq 2$. $M(D)$ is strictly increasing and Proposition 7 applies. The convexity of $M(D)$ implies the concavity of $D(\omega)$.

2) Case $\tilde{\pi}(0) > 2$. As in the proof of Proposition 7, we consider the pointwise maximum of $L(D;\omega)$ for $D \geq \underline{\alpha}$. Since it is increasing in ω, it satisfies the monotonicity condition and is the optimum.

Recall that $L(\underline{\alpha}, \omega) = 0$. Now,

$$\frac{\partial L}{\partial D}(D;\omega) = (\omega - M(D))f(D) = \frac{1}{\overline{\alpha} - \underline{\alpha}}\,(\omega - M(D))$$

for $\underline{\alpha} \leq D \leq \overline{\alpha}$ is a concave function of D which becomes negative for large enough D. We consider three cases:

a. For $\omega > \underline{\alpha}$, $\partial L/\partial D(\underline{\alpha};\omega)$ is positive. There is a single zero $D(\omega)$ of the derivative, solution to $\omega = M(D)$, which maximizes $L(D,\omega)$.

b. For $\omega = \underline{\alpha}$, $\partial L/\partial D(\underline{\alpha};\omega)$ is equal to zero. $\partial^2 L/\partial D^2(\underline{\alpha};\omega) = (\tilde{\pi}(0) - 2)f(\underline{\alpha})$ is positive, so that there is another root $D(\underline{\alpha})$, larger than $\underline{\alpha}$ ($D = \underline{\alpha}$ is a local minimum of L). Recall that $L(\underline{\alpha}, \omega)$ is equal to zero for all ω: the maximum is positive.

c. Finally consider $\omega < \underline{\alpha}$. The function $\partial L/\partial D(\cdot;\omega)$ is linear increasing in ω: when ω decreases from $\underline{\alpha}$, its smallest root increases, its largest root (a local maximum of L), say $\Delta(\omega)$, decreases, until eventually they both disappear, say at ω_1. Note that $L(\Delta(\omega), \omega)$ is an increasing function of ω. Since $L(\underline{\alpha}, \underline{\alpha}) = 0$, $L(\Delta(\omega_1, \omega_1)$ is negative. Let ω_2, $\omega_2 > \omega_1$, be such that $L(\Delta(\omega_2, \omega_2)$ is equal to zero. Define $\omega_0 = \max(\underline{\omega}, \omega_2$, $D(\omega) = \Delta(\omega)$ for $\omega_0 \leq \omega \leq \underline{\alpha}$, and $D(\omega) = \underline{\alpha}$ for ω smaller than ω_0.

It is easy to check that the $D(\omega)$ function thus defined indeed is the solution of the problem. ∎