A Saga of Wage Resilience: Like a Bridge over Troubled Water☆

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Abstract

This paper proposes a dynamic search and matching model adjusted to the Southern European and French based labour markets, where collective bargaining assumes a key role. Using the Portuguese employer-employee matched data for the last two decades, and its institutionally defined categories of workers, we reach a consistent and unified framework where we estimate an average worker bargaining power of 20%; an elasticity of quasi-rents of 0.062; an average passthrough of bargained wages of 44.8%; and a degree of assortative matching of 44.1%. These findings conform with the literature developed in each of these dimensions. Throughout the period, we witness a secular deterioration of worker's bargaining powers at the top and the middle of the wage distribution, while in the bottom we recorded a broad stability. Throughout the Great Recession, these findings are remarkably stable, signalling a significant wage setting resilience. Accordingly, the considerable real wage distribution adjustment throughout the downturn was led by job and firm flows, and for the staying workers through the valuation of the quasi rents of the worker-firm match and of the worker's outside options.

JEL Classification: C55; C61; C62; C78; J31; J51; J53

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1 Introduction

This paper addresses two closely related questions. First, how can the microeconomic wage setting mechanism be structurally described and empirically estimated for a Southern European and French based labour market? Second, how does the resulting bargaining powers evolved overtime, and particularly throughout the Great Recession?

Wage setting mechanisms, and in particular the evolution of bargaining powers, have received a significant recent interest due to its pervasive implications to macroeconomic dynamics. Since the 1980s, a significant strand of papers focused in the U.S. have recorded substantial increases in corporate profitability and firm markup trends, while the labour share has been in consistent decline. Gutiérrez and Philippon (2017), Farhi and Gourio (2018) and De Loecker et al. (2020) have linked these trends to the rise of market power, focusing in the rise of monopoly (or monopsony) power, but concurrent developments have broaden the perspective. Krueger (2018) influentially noted in his *2018 Jackson Hole* address, that the evolution of labour market practices have not only enhanced monopsony power, but has significantly weaken worker bargaining powers. Following his line, Stansbury and Summers (2020) provides a significant case for a relevant (if not leading) role of bargaining power trends in explaining those macroeconomic dynamics, and Lombardi et al. (2020) has theoretically linked a weakening of worker's bargaining powers with an abatement of inflation dynamics and an amplification of employment adjustments over the business cycle.

In a related literature, the wage setting and bargaining powers are also particularly relevant in the context of an imperfectly competitive labour market due to its role on wage inequality. Since the 1980s, cross sectional data unveiled a significant increasing trend of wage inequality in the United States.[1] Then, the skill biased technological change was coined as the leading cause (see Krueger (1993), Berman et al. (1994) and Acemoglu (2002)). Later, in a more microeconomic perspective, Autor et al. (2003) influentially highlighted job polarization, due to the idiosyncratic share of automatable work across occupations (see Acemoglu and Autor, 2011 for a recent overview). However, from the outset of the debate, several developed economies like France, Japan or Germany were not displaying the same wage inequality trend, leading several authors to focus on the role of 'institutions' (Freeman and Katz, 1995).[2] Amidst those, different degrees of unionization, wage setting structures and minimum wage policies lead the most studied.[3]

[1] For the U.S., Katz and Murphy (1992), Levy and Murnane (1992) Bound and Johnson (1992) correspond to a sample of studies that decisively contributed to the consensus.

[2] A strand of the literature has focused on top income inequality (see Piketty and Saez (2003)), and more recently, in the context of the Great Recession, confirmed markedly different trends along those geographical lines (see Piketty and Saez, 2013).

[3] See DiNardo et al. (1996), Lee (1999), Card and DiNardo (2002), Dickens and Manning (2004)

In Southern European and French based labour markets, while each country's labour market has institutional idiosyncrasies, the blueprint is largely analogous, and derived from few common ancestral roots (see Botero et al., 2004). A brief wage setting synopsis would present a strong union coverage despite lower membership rates, significant employment protection, and centralized sector wage bargaining as its institutional cornerstones. Accordingly, among the existing labour markets, we select Portugal for our empirical implementation due to three convenient reasons. Firstly, as reinforced by Visser (2016), Boeri and van Ours (2013) and Card and Cardoso (2021), Portugal is in most wage setting mechanisms a representative of the Southern European and French based labour markets. Secondly, the country was particularly hit by the Great Recession. Finally, we have access to high quality data about the characteristics of the privately employed workers, including detailed information on wages, and of the balance sheet and income statement of firms.[4]

This paper makes four contributions. First, it develops a dynamic search and matching model, which presents a wage setting mechanism accounting for the most relevant Southern European and French based labour market institutions. Second, this paper explores the role of those institutions by resorting to the collective bargaining administrative ranking of workers, which allow us to identify the value of outside options and the value of quasi-rents, while relying on a significantly parsimonious parameter dimensionality when compared with a typical high dimensional fixed effect wage regression. It does not rely on worker mobility patterns across firms, or in general on firm effects. Third, differently from most of the rent-sharing literate we do not need to directly rely on firm-side measures of quasi-rents or value added (see Card et al. (2018) for an overview of these approaches), and we will abstain from following a substantive structural perspective as we will not specify a production function.[5] Fourth, this framework translates into a unified and consistent empirical setting capable to estimate time-varying bargaining powers, quasi-rent elasticities, collective bargaining pass-through in wages, wage variance decompositions, and the degree of assortative matching in the economy, which lie within the ranges of the recent developments in those literatures.

Our proposed model entails: (i) firms composed by a hierarchical occupational structure; (ii) worker-firm multi-layered high dimensional heterogeneity; (iii) the possibility of on-job-search in the classic Roy (1951) model approach; (iv) existence of binding firing taxes as in Boeri (2011); and (v) the adoption of the intra-firm bargaining apparatus of Stole and Zwiebel, 1996*a*,*b*, unfolding between firms and a rep-

and Autor et al. (2008) for evidence on the influence of these institutional settings on wage inequality.

[4]See appendix A1 for a detailed description of the dataset used in this study.

[5]See Mortensen et al. (2010) and Bagger et al. (2014) for representative examples of a more structural approach, with a fully specified production function.

resentative union, which however is unable to cause an *hold-up problem* in case of a bargaining breakdown, as considered in Dobbelaere and Luttens (2016). Altogether, the core modelling infrastructure was crafted in the intersection between Acemoglu and Hawkins (2014), Cahuc et al. (2008), Mortensen (2010), and under small twists in the assumption framework it becomes isomorphic to them.[6]

The empirical implementation of the resulting wage equation resorts to the collective bargaining administrative ranking, which is consistently produced for decades, covering directly around 85 percent of the labour contracts, without any *opt-out* possibility. As noted by Schulten (2016) and Card and Cardoso (2021), rather than a Portuguese idiosyncrasy the existence of these rankings is a Southern European and French based labour markets' cornerstone, which can be equally seen in countries like Belgium, France, Italy, Netherlands and Spain. This ranking corresponds to the core of the legal framework of the labour relationships, defining strictly the job descriptions, the clauses of the contracts, and the bargained tables of wage floors.[7] In detail, the ranking is composed of a granular system of around 30,000 institutionally defined sector-occupation ranks of the workers. We estimate the wage floors compatible with such ranking, and we follow the model's postulate that those coincide with the union and firm belief about the valuation of the workers' outside options, which incidentally do not depend on the employing firm alone.[8] Further, those ranks also provide a framework to identify the *type* of workers, and thus enhance the identification of the marginal products of the match.

Our use of collective bargaining rankings to estimate outside options consists in an alternative to recent developments this literature. Caldwell and Harmon (2019) uses past co-workers job movements as a measure of the value of the social network of the worker, Schubert et al. (2020) resorts to the analysis of worker's job histories, and Caldwell and Danieli (2020) implements a sufficient statistic that assesses the supply of jobs in the area and the worker's flexibility to take them. In a broader perspective, the proper use of rankings of workers and/or firms has received great attention in the mincerian wage equation estimation. Bonhomme et al. (2017), Bonhomme et al. (2019) and Lentz et al. (2018) resort to a two-step algorithm where in the first step either firms or workers (or both) are classified in categories by using a *k-means clustering*

[6] Lise et al. (2016) is in several theoretical aspects close in spirit to the exercise we propose, but focused on the U.S. economy and henceforth without the theoretical and empirical contribution of the European institutional setting. Tschopp (2017) also mostly abstracts the institutional setting, but implements an empirical strategy to identify key parameters in a search and matching model that in spirit is close to ours, however with less degrees of heterogeneity.

[7] See Cardoso and Portugal (2005), Martins (2014) and Addison et al. (2017), Card and Cardoso (2021) for further detail about the referred ranking.

[8] For Italy, Card et al. (2014) uses bargained wages to precisely control for outside options in a rent-sharing analysis, but does so at sector level, without the degree of granularity presented here.

algorithm; Sorkin (2018) resorts to a replica of the *Google's PageRank* algorithm to rank firms based on revealed preference, and thus identify the value of compensating differentials; and Hagedorn et al. (2017) presents the classical *Kemeny-Young rank aggregation* algorithm as a way to rank workers and then firms based on the worker's ranking.[9] Our approach, instead of resorting to indirect measures to estimate the value option options or statistical algorithms to identify types of workers or firms, relies on firms and unions being capable to assess the worker's worth in the market, which translates into a credible and consistent administrative ranking.

In the last two decades, the wage setting mechanism synchronized a notable stability of workers' bargaining positions at the bottom of the wage distribution, with a perennial erosion at the middle and the top. On average, we estimate the worker's bargaining powers at 20 percent for the entire economy, with higher levels for managers, and very identical values for the remainder of the workforce. Those translate into some responsiveness of wages to exogenous shocks in the level of the quasi-rents of the labour relationship, concretely with an average estimated elasticity of 0.062, within the 0.05-0.15 most referred interval in the literature (see Card et al. (2018)). Likewise, we find levels of positive assortative matching on the range 38-48 percent, and an average passthrough of changes in bargained wages into changes of total wages of around 44.8 percent, in line with Card and Cardoso (2021). In respect to wage dispersion, the workplace heterogeneity justifies around 60 percent of the overall wage dispersion, conforming with an imperfectly competitive labour market perspective, while the outside option effectively contributes for a compression of wages relatively to the implied productivity distribution.

The structural slow paced erosion of bargaining powers at the top and middle of the skill distribution, unveil potential future productivity hazards, particularly if amplified by a considerable progressive income taxation. With compressed wage differentials across skill groups and the increasing orthogonality between wages and firm productivity levels, which naturally arises when the worker has a lower take on the quasi-rents of the match, the matching efficiency in the economy may be degraded, the incentives for training may be dulled, and the alignment of worker and firm incentives may be laxed.

The macroeconomic context of our analysis has been particularly turbulent in a significant part of the sample, when the Great Recession emerged. Within the recessionary shock, for those that were capable to remain employed throughout the rough waters, the wage setting laid out a sound bridge, displayed through a significant resilience of bargaining powers, which contained the propagation of the sluggish macroeconomic outlooks to temporary and moderate real wage losses, arising from the

[9] See Kemeny (1959) for the first treatment of this algorithm.

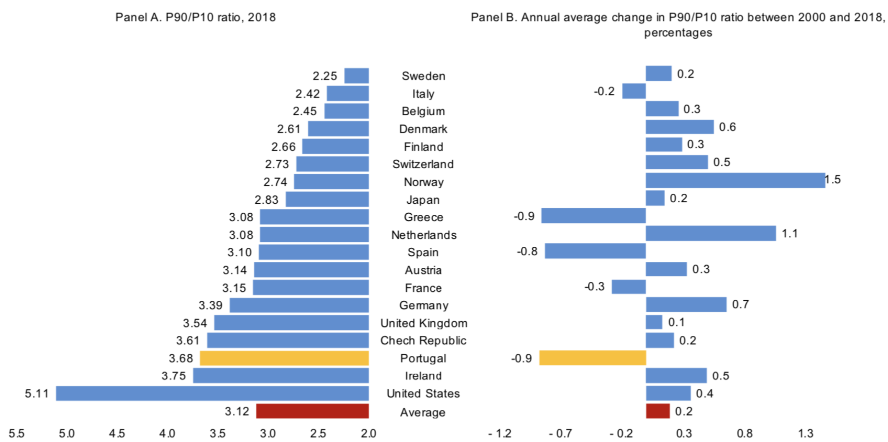
fall of outside option real values and real quasi-rents of the match.

The paper is organized as follows. Section 2 presents the main features of the considered labour market and the way those are subsequently introduced in the model. Section 3 presents the dynamic search and matching model proposed in the paper, and section 4 presents a straightforward way to empirically estimate the resulting wage equation. Section 5 presents the empirical results of the model, the estimates of the level of firm-worker sorting in the economy, of the elasticity of wages to the quasi-rents and of the average passthrough of bargained wages. It further analyses the variance of wages in the proposed wage setting perspective. Section 6 discusses the resilience of the wage setting mechanisms in the wake of the Great Recession. Section 7 concludes.

2 Wage Setting and Labour Market Institutions

The cross-country comparison of labour market institutions literature, classically featuring an *U.S. - Europe comparison*[10], has developed around *countries' institutional clusters*. Indicatively, Boeri (2011) partitions Europe in a *Continental cluster*, a *Southern European cluster*, a *Nordic cluster*, and an *Anglo-Saxon cluster*. Alternatively, Botero et al. (2004) presents a partition of taxonomies based on ancestry, grouping labour markets in the *French*, *German*, *Anglo-Saxon* and *Socialist* systems. Given our modelling emphasis in the wage setting process, we follow a mixed version of those taxonomies, with our analysis being particularly suitable for *Southern European* and *French* based systems. Noteworthy, as presented in figure 1, a primer assessment of recent trends of wage inequality presents divergent paths among OECD countries, with this particular cluster decoupling from others by displaying a reduction of wage inequality.

Figure 1: Inequality of Gross Earnings of Full-Time Dependent Employees



Source: OECD.

[10]See Nickell and Layard (1999), Bertola (1999), Blau and Kahn (1999), Boeri (2011) for chapters of the *Handbook of Labour Economics* featuring a classical analysis.

Institutional Setting and its Dynamics

As presented in table 1, the wage setting has been considerably more centralized and/or coordinated with a leading role of trade unions in the continental European labour markets than in their Anglo-Saxon counterparts. The former is dominated by industry, sector or even national level agreements, whereas the latter unfolds recurrently at firm or even plant level (see Boeri and van Ours (2013)). While such structural institutional differences could have little influence in market outcomes, the literature has strongly suggested otherwise, with these asymmetries suspected to play a role in explaining the degree of wage compression across economies.[11]

Table 1: Average Collective Bargaining Indicators by Decade

Countries:	Level of Bargaining		Bargaining Centralization		Union Density (%)		Union Coverage (%)		Single Employer Bargaining (%)	
	2000's	2010's	2000's	2010's	2000's	2010's	2000's	2010's	2000's	2010's
Portugal	3	3	2.8	2.6	18.4	17.4	80.5	75.5	3	5
Spain	3	3	2.52	2.17	19.3	18.9	70.9	70.5	7.5	5.9
France	3	3	2.4	2.3	10.7	10.9	98	94.2	3.6	3.8
Italy	3	3	2.59	2.41	33.6	35.5	80	80	-	-
Germany	3	3	2.2	2.2	21.7	17.7	65	57.1	8.5	8.1
Austria	3	3	2.29	2.28	33.3	27.6	78.5	78.5	3	3
Netherlands	3.4	3	2.59	2.19	20.4	18.2	82	83.1	10.1	7.6
Belgium	4.5	4.6	4.09	4.14	54.9	53.6	94.6	92	10	-
Denmark	3	3	2.34	2.3	70.4	67.4	77	79.3	22	-
Sweden	3	3	2.49	2.39	74.5	67	92	89.6	8.3	5.6
Norway	3.2	3	2.57	2.39	54.3	52.3	73.7	71	-	-
Finland	3.65	3.67	3.06	3.06	69.2	63.45	86.2	89.6	9	9
Switzerland	3	3	2.49	2.39	74.5	67	91.9	89.6	8.3	5.6
Greece	3.9	2.4	3.6	1.3	27.75	21.07	100	57.1	8.2	15
Poland	1	1	0.96	0.9	17.1	16.9	20.9	17.9	15.4	14.8
Czech Republic	2	2	1.8	1.8	23	13.3	27.9	33.1	27.9	33.1
United Kingdom	1	1	1	1	28.6	24.9	34.6	28.3	29.4	27.5
Ireland	4.6	1	4.05	0.99	33.8	27.2	42.1	34	32.4	-
United States	1	1	1	1	12.2	10.7	13.4	11.9	-	-

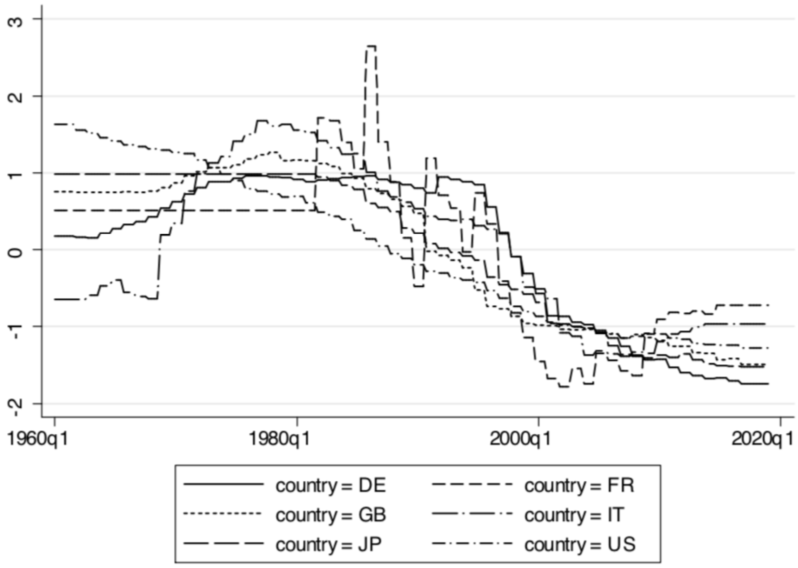
Notes: Level of bargaining takes the values: (5) central or cross-industry level bargaining; (4) alternating between central and industry bargaining; (3) sector or industry bargaining; (2) sector or company bargaining; (1) company bargaining. Bargaining Centralization is a measure created by ICTWSS, ranging between 1 and 5, with 5 being the highest level of centralization. *Source: ICTWSS*, version 6.1, 1960-2018.

In the last decades, the OECD indicators on labour market institutions and regulations points to a detrimental evolution to workers, as presented in the synthetic indicator of Lombardi et al. (2020) in figure 2. While in the Anglo-Saxon markets

[11]The suggestion of different degrees of trade union influencing the wage setting and wage compression has been established in Freeman (1980), Blau and Kahn (1996), Aidt and Tzannatos (2002) and Card et al. (2003), among others.

we have witnessed a co-movement of de-unionization and the fall in collective bargaining coverage, rendering a greater subsidiary role of unions in the wage setting mechanism, that is not the case in the continental European labour markets, where despite the strong fall in membership, union coverage remains remarkably high.[12] As claimed by Booth (2014), this persistence justifies the importance of modelling the behaviour of trade unions when, in general, one assesses the wage setting in these economies.[13] Moreover, employment protection remains relevant in those latitudes (See OECD (2016) for a data overview).

Figure 2: Synthetic indicator of worker's bargaining powers



Note: Each line represents their synthetic indicator of the worker's bargaining power, by using the first principal components of several indicators, including union density, union coverage, employment protection indexes, and coverage of collective agreements.
Source: Lombardi et al. (2020)

[12] See Blanchflower et al. (1996), Blanchflower and Bryson (2004), Bryson et al. (2011), and Visser (2016) which document this process. In the Anglo-Saxon case, several papers, among which DiNardo et al. (1996), Fortin and Lemieux (1997), Firpo et al. (2009), Frandsen (2012) and Farber et al. (2018) correspond to a sample, have studied the relationship between the fall in membership and collective bargaining coverage and the increase in wage inequality.

[13] See Pissarides (1986) for the first inclusion of unions in a search and matching framework, and Bauer and Lingens (2010) and Dobbelaere and Luttens (2016) for recent treatments with an approach closer to ours. Finally, also see Abowd and Lemieux (1993) for a classical and mainstream treatment, and Krusell and Rudanko (2016) which revisits the question in the context of a defined frictional labour market in a macroeconomic perspective.

Modelling the Labour Market Institutional Setting

In our framework we account for a significant employment protection and a pre-eminent union presence both in bargaining wages and ensuring the enforcement of implicit clauses of contracts, in the spirit of Hogan (2001). Consequently, our model considers that firms define employment, and then with the *right-to-manage* union bargain sequentially the contract of every worker first at sector/industry level through an employer's association, and then at local level. Further, the modelled union has universal coverage, but lacks the ability to force a full lockdown of production when a bargaining process breaks down. Instead, we assume the Dobbelaere and Luttens (2016) *gradual collective bargaining* structure.[14] If negotiations break down, the parties reach stalemate until one of the workers in that contract leaves the firm without any firing tax being levied. Then, both sides restart bargaining every contract aiming at unlocking the stalemate, with the process unfolding with the same gradualism until a full simultaneous agreement is reached.[15]

Moreover, we assume bargaining takes place under three additional requirements building the implicit foundation of the wage bargaining contract. Concretely, the sides define: (i) the accurate belief about the level of outside options; (ii) a fair ladder of outside options' beliefs in the market when compared with productivity of the workers, so that no worker is unfairly and consistently degraded; and (iii) that wages are settled under the principle of match stability, implying that both parties bargain over the split of the match surplus assuming ex-ante that neither side will exert their at-will option to dissolve that match.[16] Finally, we complement the bargaining apparatus with an employment protection framework, which notably translates into the existence of firing costs/taxes, in the spirit of Bentolila and Bertola (1990), Bertola and Caballero (1994) and Boeri (2011).[17]

[14] Dobbelaere and Luttens (2016) justifies the discarding of this mainstream prior, due to the almost lack of empirical evidence of such event, with the sole exception of the *Ronald Reagan and the air traffic controllers* case in 1981, which they argue was political. Holden (1988) and Holden (1989) resort to the nordic *peace-clause* to also abstain from a *full-lockdown assumption*.

[15] Under this bargaining protocol, the presence of unions by itself does not deviate the equilibrium wages from the one that would emerge through individual bargaining, as in Cahuc et al. (2008) or Acemoglu and Hawkins (2014). See appendix B1 for further insights about the relationship of our modelling approach and other canonical union-firm models.

[16] The ability of unions to have more accurate information than isolated workers, and to have the ability to coordinate actions of workers to enforce state-contingent actions or implicit clauses in the contract is precisely at the core of Hogan (2001) analysis. As a matter of fact, the dataset on workers' characteristics was created on the purpose that unions could inspect and monitor firm's behaviour. Created by law in 1976, our workers' characteristics dataset - *Quadros de Pessoal* - were mandatorily sent by firms to the Ministry, and posted in a visible place in each establishment, with every relevant employment characteristic, including the wages and the worker's position.

[17] While Elsby and Michaels (2013) presents evidence on the identical fitting properties of a macroeconomic search and matching model with kinked employment adjustment costs on hiring, on firing or on both sides, we opted to follow the modelling option of Bentolila and Bertola (1990) and Bertola

The Wage Setting and Outside Options

The debate about how to model outside options in the context of wage determination has been prolific. The work of Postel-Vinay and Robin (2002) for the French labour market sparked attention to the implications of different modelling choices for the bargaining framework of wages. The authors adopted sequential bargaining where firms bid for worker's services, creating an enforceable link between the prescribed value of the worker's outside option and the history of his past job offers while employed.

This link has been disputed due to the empirical rarity of a sequential bidding in defining wages, and the predictable lack of enforceability of incumbent firm - individual worker promises.[18] Barron et al. (2006) highlights the implications of the Postel-Vinay and Robin (2002) apparatus on the co-workers' contracts, and thus define as theoretically reasonable the existence of at most a selective counter-offer policy. Cullen and Pakzad-Hurson (2021) reinforces the limited use of sequential bidding if there is transparent pay. They empirically find a very moderate decline of wages when transparent pay is introduced in some US locations, which is consistent with an ex-ante limited application of sequential bidding. Concurrently, Di Addario et al. (2021) finds that the impact of previous firm match is moderate in the wage setting, particularly in low-middle skilled jobs, making such empirical results also difficult to conciliate with sequential bidding.

Altogether, our modelling choice follows the principle of nonexistence of counter-offers. Rather, we assume unions and firms predict the value of outside options for each type of worker, and those predictions are enforced in the collective bargaining. If a worker receives a beneficial proposal, he leaves the current match.

The Institutional Data on Outside Options

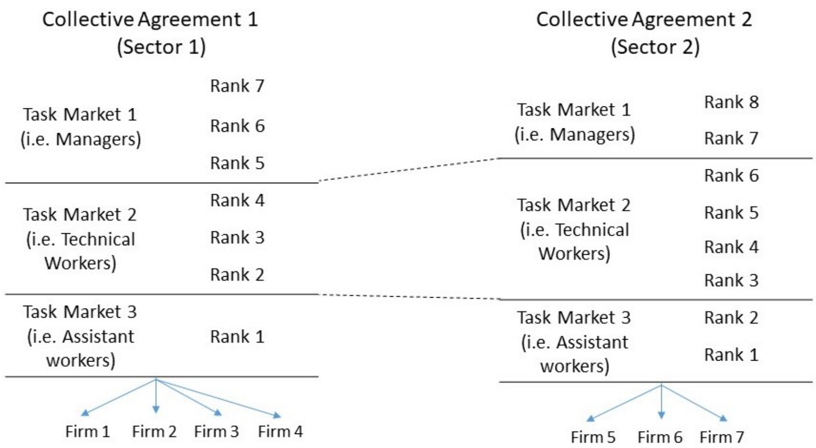
In Portugal, as in the majority of the Southern European and French based labour markets, the centralization of wage bargaining is clear and engraved in roughly 850 existing collective bargaining agreements. Those are signed without any *opt-out* possibility for a firm in a covered industry or sector, and overwhelmingly by unions linked to the two major union confederations. While there exist a fringe of tailored agreements at firm level, those represent less than 4 percent of the workforce, being the sector agreements, negotiated between employers' associations and unions, or the agreements only entangling some employers in a sector, that predominate. Jointly with their subsequent administrative extensions either to other similar sectors, or to

and Caballero (1994), and the evidence presented by Lazear (1990), about the relevance of considering firing costs.

[18] See Pissarides (1994), Mortensen (2005), Shimer (2006), and Dolado et al. (2008) for examples of studies resorting to this set of arguments.

an entire sector when the initial coverage was reduced to some employers, the total coverage of collective bargaining reaches more than 85 percent of the private sector workforce. This quite centralized apparatus provides strengthened coherence to the bargained outcomes, particularly across comparable labour market contexts.[19]

Figure 3: The Collective Bargaining System.



In toto, the enacted agreements set a substantive set of rules on working conditions, and a system of wage floors for detailed categories of workers. As presented in figure 3, those wage floors are defined based on the firm's sector and the worker's career rank, or category, within a given task market (i.e. senior manager, junior manager, and so on). Those ranks are then aggregated in a comparable set of types of workers, or task markets across the sectors of activity (i.e. managers, group leaders, senior technical workers, non-technical workers, and so on). As a matter of example, this labour market functions in resemblance to the organization of the armed forces. You have a hierarchy composed by groups of ranks or task markets (i.e. generals, commissioned officers and unlisted grades), arguably compared across the three branches of the armed forces (i.e. army, air force and navy), and within each task market you have a plethora of ranks (i.e. field marshall, general, brigadier, captain, and so on). In total, the sectoral agreements and their extensions explain the claim that Portugal has no less than 30,000 minimum wages (see Martins (2014)).[20]

[19] Legally, the firm-level agreements signed between an individual company and one or more unions are designated *Acordos de Empresa* or AEs. Those are important in the oil sector and transport and communications. The collective agreements signed by several employers that are not part of an employers' association and one or more trade unions are known as *Acordos Colectivos de Trabalho* or ACTs. Those are significant in the financial sector and utilities. Finally, the industry-level or sectoral agreements, the so-called *Contratos Colectivos de Trabalho* or CCTs, are the ones negotiated between one or more employers' associations and one or more unions. The administrative instruments that extend the agreements are either *Portarias de Extensão* or *Portarias de Condições de Trabalho*. See further details in Addison et al. (2017). See appendix A2 for an example of a representative wage floor table, collected out of the 849 defined in 2016.

[20] We abstract the possibility of several collective agreements covering the same firm, as either they

In our framework, we assume this definition corresponds to an estimate of the worker's outside option value, or *fire-sale*, assuming the match stability principle. Therefore, this estimate corresponds to the worker's value if he becomes suddenly and unexpectedly unemployed. Empirically, we only have access to the ranks of the workers, and therefore we establish the minimum paid regular wage in each identified rank as the proxy of the legally binding minimum. In a nutshell, the system of sector-occupation ranks provides a comparable measure of the outside option value of the workers across sectors, and individual histories. This empirical choice is not at the odds of the literature (see Card et al. (2014) for an identical empirical choice).

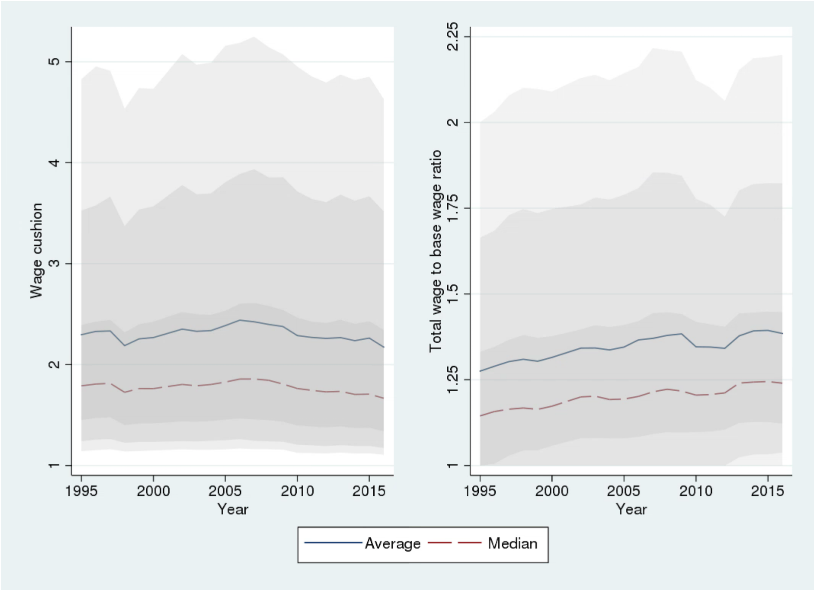
Bargained Wages and Wage Cushion

As presented in figure 4, while this *bargained wage* sets the minimum wage conditions of each labour relationship it does not correspond to the actual wage of the worker, as the latter results from the proper wage bargaining dynamics at firm level. As seen by the *wage cushion* measure, corresponding to the ratio between the total wage and this proxy of the bargained wage, it is extremely common, and a stable feature of the market, to see firms paying above the minimum condition.[21] By the same token, the base wage does not match the total compensation of the workers, as the proper institutional setting often determines the mandatory existence of several supplements, as the meals subsidy or even tenure related regular payments. Moreover, this distinction confers different degrees of future enforceability among types of pay, with some other regular compensation supplements, as shift subsidies or availability supplements being in *de-jure* temporary or partially temporary.

tend to be coincidental, or the workers tend to pick the most advantageous one.

[21] The term *wage cushion* was proposed in Cardoso and Portugal (2005), corresponding to the difference on the levels of the bargained wage and the base wage actually paid to the worker. Noteworthy, while Cardoso and Portugal (2005) and Card and Cardoso (2021) assesses the difference between bargained and base wages of the worker, we assess the difference between bargained and total wages. See Addison et al. (2018) for other analysis resorting to the *wage cushion* concept and unions. Further, this concept differs from the *wage drift* which assesses minimum wage changes versus actual wage changes. See Holden (1988) and Holden (1989) for treatments of this concept for the Nordic countries.

Figure 4: Evolution of the wage cushion and the wage cushion.



Notes: The wage cushion is calculated as the ratio $wage_{cushion} = \frac{wage_{total}}{wage_{bargained}}$. The fading grey shades corresponds, from the darker to the lighter respectively, to (a) 75^{th}-25^{th} percentile range; (b) $90^{th} - 10^{th}$ percentile range; and (c) $95^{th} - 5^{th}$ percentile range. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

Heuristic Reduced Form Approaches to Estimate the Wage Setting

A widely accepted way to portray the wage setting in the context of imperfect labour markets is given by:[22]

$$w_{ift} = Out_{it} + \beta QR_{ift} \tag{1}$$

where Out_{it} corresponds to the outside wage of worker i in moment t, and QR_{ift} corresponds to the non-competitive quasi-rent the worker obtains at firm f in time t.

In this context, the leading way to identify the parameter β, or its related elasticity resides in using proxies of the value of the quasi-rent. This path can be seen in Card et al. (2014), and its extensively reviewed in Card et al. (2018). The potential limitation of this approach resides in the proper availability of credible measures of productivity, as the value added. Also, even when it exists, it often doesn't enable the possibility to adopt an identification that secures within firm heterogeneity of the quasi-rent component.

[22]This style of wage determination equation is for example compatible with: (a) bargaining in single worker firm models as in Pissarides (2000); (b) search with multiple workers and Stole and Zwiebel (1996*a*) bargaining as in Cahuc et al. (2008), Acemoglu and Hawkins (2014) and this paper; (c) efficient unions bargaining as in Abowd and Lemieux (1993); and (d) monopsonistic wage posting as in Manning (2011) and Card et al. (2018).

The advent of matched employer-employee administrative datasets represented a turning point on the study of labour market dynamics. Empirically, the major drive to rely on this data lies on its detailed, and overtime consistent, description of the relationship network established among market actors (i.e. workers, firms, unions, households, among others). Such interest on this type of data became most salient after the seminal contribution of Abowd et al. (1999) – the widely known AKM model. Their model owes its popularity to the pleasant empirical properties of the within estimator, which it naturally extends, the easiness of its empirical implementation, its high explanatory power, the apparent intuitive interpretation of its results in a canonical wage setting model, and even the natural relationship with the traditional schooling and experience frameworks of Mincer (1974) and Griliches (1977). Consistently, Card et al. (2013) highlighted the importance of considering network data to identify unobservables. Contrary to past wage inequality surge episodes, their assessment of the recent rise in inequality for West Germany found that the most relevant drivers were not easily observable worker and firm characteristics. Identically, Bloom et al. (2018) records long-lasting unobservable wage differentials with firm size.

Typically, as presented in Card et al. (2018), a typical AKM approach to estimate the parameter β (or its implied elasticity) would entail:

$$\begin{aligned} ln(w_{ift}) =& ln(Out_{it}) + \tilde{\beta} ln(QR_{ift}) \\ ln(Out_{it}) =& \alpha_i^O + \phi_t^O + \mathbf{X}_{it}^O \gamma^O \\ ln(QR_{ift}) =& \alpha_i^{VA} + \psi_f^{VA} + \phi_t^{VA} + \mathbf{Z}_{ift}^{VA} \gamma^{VA} \end{aligned} \tag{2}$$

which results in a AKM type model as:

$$\begin{aligned} ln(w_{ift}) =& \underbrace{\alpha_i}_{=\alpha_i^O + \tilde{\beta}\alpha_i^{VA}} + \underbrace{\phi_t}_{=\phi_t^O + \tilde{\beta}\phi_t^{VA}} + \underbrace{\psi_f}_{=\tilde{\beta}\psi_f^{VA}} + \mathbf{X}_{it}^O \gamma^O + \mathbf{Z}_{ift}^{VA} \gamma^{VA} \tilde{\beta} + \epsilon_{ift}, \\ \widehat{\psi_f} =& \xi + \tilde{\beta} E_f[ln(QR_{ift})] + v_f, \end{aligned} \tag{3}$$

where the identification lies on the idea that the firm fixed effect only affects the non competitive quasi-rent, and consists in a measure of the link between different compensation policies across firms after partialling out the value of individual specific traits, and time effects.

However, from those outset contributions, the debate acknowledged empirical hazards in both the interpretation and use of the fixed effect estimates, and in its proper estimation, which may suffer from several econometric impairments.[23] Within the latter group, limited mobility bias leads the suspects (see Abowd and Kramarz (2004)

[23] A leading example of an interpretation an use of fixed effects issue concerns the identification of age and experience profiles in the presence of worker fixed effects (see Card et al. (2018)).

for the seminal treatment of the question),[24] endogenous mobility closely trails (see Card et al. (2016) and Card et al. (2018) for classical discussions), and finite-sample bias and measurement error complete the quartet of the most referred potential econometric faults. Even if validity is set aside, the functional form of the canonical AKM setting has been questioned.[25]

Those empirical *crossroads* encouraged researchers to enlarge their strategies. Some attempted to increase the parameter set to account for misspecification, even though it implies even stronger identification restrictions when compared with the canonical AKM (see Woodcock (2015) for a recent discussion). In more encouraging paths, some attempted to devise bias correction methods for the limited mobility issue (see Andrews et al. (2008), Borovicková and Shimer (2017) for two examples), while others defined different estimators theoretically capable to address some of the identified issues without restricting the modelling environment (see Kline et al. (2020) for a leave-out estimator of quadratic form). Alternatively, several studies, among which Bonhomme et al. (2019) is a prime example, tried to perform rankings of workers and/or firms, before the proper estimation of the wage equation, with the aim of reducing the parameter set to be estimated while maintaining the sound statistical explanatory power of AKM.[26]

In the spirit of these last developments, and given the collective bargaining admin-

[24]One of the most relevant consequences of limited mobility bias concerns the validity of assessing the level of sorting of workers and firms through the empirical correlation of worker and firm effects. See Eeckhout and Kircher (2011) and Jochmans and Weidner (2019) for theoretic discussions; and Abowd and Kramarz (2004), Andrews et al. (2008), Lopes de Melo (2008), Andrews et al. (2012), Hagedorn et al. (2017), Borovicková and Shimer (2017), Torres et al. (2018) and Bonhomme et al. (2020) for a sample of relevant studies on this debate.

[25]Allowing the value of the worker-firm match to unveil in more convoluted ways than its additive specification, and considering dynamic mechanisms composing a richer and arguably more robust longitudinal analysis than its static environment have been two leading courses of action (see Bonhomme et al. (2019), Card et al. (2018) and Lachowska et al. (2020) for recent analysis).

[26]In this broad class of papers, one can consider: (a) Bonhomme et al. (2017), Bonhomme et al. (2019) and Lentz et al. (2018) that resort to a two-step algorithm where in the first step either firms or workers (or both) are classified in categories by using a *k-means clustering algorithm*; (b) Sorkin (2018) which resorts to a replica of the *Google's PageRank* algorithm to rank firms based on revealed preference to then identify compensating differentials; and (c) Hagedorn et al. (2017) which, building on theoretic foundations of Shimer and Smith (2000) and Eeckhout and Kircher (2011), presents the classical *Kemeny-Young rank aggregation* algorithm as a way to rank workers and then firms based on the worker's ranking (see Kemeny (1959) for the first treatment of this algorithm).

istrative ranking, a simplified version of our approach will empirically implement:[27]

$$
\begin{aligned}
w_{ift} =& Out_{it} + \beta QR_{ift} \\
QR_{ift} =& TS_{ift} - Out_{it} \\
Out_{it} =& \varphi_t + f(rank_{it}, age_{it}, gender_i) \\
TS_{ft} =& \psi_{ft} + \mathbf{X}_{it}\gamma + \alpha_{it} \\
\mathbf{X}_{it}\gamma + \alpha_{it} =& E[\mathbf{X}_{it}|rank_{it}]\gamma.
\end{aligned}
\tag{4}
$$

where TS_{ft} corresponds to the total surplus of the match worker-firm. Notice that the use of the collective bargaining administrative ranking will allow to identify bargaining powers in a more parsimonious and flexible model as compared with the AKM, and in a more flexible approach than the one usually available to the rent-sharing approach. In the next two sections we will present the model and its implied assumption framework that postulates this presented system of equations.

3 A Search and Matching Model of Labour Tasks

In a nutshell, this model corresponds to a search and matching archetype at workplace level (*id est* firm/occupation/time partition), with gradual collective bargaining, intra-firm wage bargaining, severance payments and on-job search. Under several twists in its assumptions it becomes isomorphic to the models presented in Mortensen (2010), Cahuc et al. (2008), Acemoglu and Hawkins (2014) or Pissarides (1990).

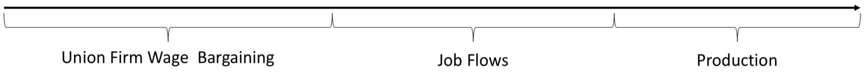
In this section we focus our description of the model in the required ingredients to obtain dynamic equilibrium wages. The treatment of the remainder conditions describing the full dynamic equilibrium, an alternative steady-state equilibrium, and the theoretical links of this model and the literature of search and matching and union-firm bargaining are presented in appendix B.[28]

Labour market structure. The proposed model unfolds in each period t according with figure 5, where each period is decomposed in three hypothetical moments, with wage bargaining happening before the job flow decisions take place.

[27] In the actual empirical implementation we will add a task market dimension - j - as we will subsequently introduce.

[28] In detail, appendix B provides further technical details on the model, namely: (i) the conditions under which the model become isomorphic to several search and matching models in the literature; (ii) the relationship between the gradual collective bargaining apparatus and other canonical union-firm bargaining solutions; (iii) the derivation of the remainder conditions describing the dynamic equilibrium and of equilibrium wages; (iv) the definition and properties of the dynamic equilibrium and the steady state equilibrium.

Figure 5: Structure of the Model in each moment t.



In this context, consider an economy with a *numeraire* good sold under perfectly competitive conditions, and produced by a unit measure of large firms. In each period t, each firm employ multiple workers, from the available pool $i \in \{1, \ldots, \aleph\}$, with each specializing in one of the available labour tasks, $j \in \{1, \ldots, J\}$. Workers sell their tasks exclusively to a single firm. Time is continuous, and workers, the union, and firms discount time at rate $r \geq 0$.

The labour market is assumed to be frictional, as firms are required to post vacancies to hire workers, and pay a cost γ_j per vacancy posted in task market j. Workers, either employed or unemployed, do direct search at task market level, by selecting the task market they are willing to perform search. They incur in the search cost c_j if they search, and then meet firms following a random search process within the task market.

The flow of worker-firm meets in task market j is determined by a typical constant returns to scale aggregate matching function, $M(u_j(t) + e_j(t), \bar{V}_j(t))$, where $u_j(t)$ is the measure of unemployed workers searching for a job in task market j at moment t, $e_j(t)$ is the measure of employed workers searching in market j at time t, and $\bar{V}_j(t)$ is the measure of vacancies in such task market. Correspondingly, the market tightness is given by:

$$\theta_j(t) = \frac{\bar{V}_j(t)}{u_j(t) + e_j(t)}, \tag{5}$$

and

$$\theta_j(t)q(\theta_j(t)) = \frac{M(u_j(t) + e_j(t), \bar{V}_j(t))}{\bar{V}_j(t)} \tag{6}$$

represents the Poisson rate at which a worker, either employed or unemployed meets a firm. Further, $q(\theta_j(t))$ is the Poisson rate at which a firm meets a candidate, per vacancy posted. For notational ease, we often just write θ_j, u_j, e_j or $q(\theta_j)$, omitting its time dependence.

On the other side, matches are dissolved due to one of four reasons namely: (a) a bargaining breakdown in the wage negotiation; (b) a termination exogenous shock, representing reasons beyond the control of workers and firms, which happens with probability $\bar{s}$; (c) a successful on-job-search of a worker; and (d) the decision of the firm to fire the worker at will, which may be triggered after the firm pays a severance payment given by S.

Description of Market Agents. Firms to be productive employ a J dimensional vector of workers, $\mathbf{N}$, resort to an exogenously predetermined capital input, K, whose rental cost, $I(K)$, is considered to be fixed and sunk, and implement the available homogeneous production function $F(\mathbf{N}, K)$.[29] Moreover, firms bargain with a representative union over wages with heterogeneous bargaining strength, with $I - \beta$ representing the firm's bargaining power vector - a $[J \times 1]$ dimensional vector - implying heterogeneous bargaining powers across types of tasks. Altogether, the firm exogenous heterogeneity is captured in the two dimensional tuple $\{K, \beta\}$.[30] The cumulative distribution of the firm's types in each moment is given by $\Gamma(K, \beta)$.

Workers are potentially infinitely lived, but may suffer a death shock with a constant hazard rate δ, and new workers arrive at the market at the same rate. Each worker is exogenously endowed with an initial generic training and ability, whose stock is given by $a(0)$ extracted from the distribution $\Psi_0(a) = N(\mu_0, \Sigma_0)$. Then, the worker develops skill through a stationary and invariant process with the Markov property, so that the transitions are described by the cumulative distribution function

$$\begin{aligned} \Psi(a'|a) &= Prob(a_i(t+1) \leq a'|a_i(t) = a), \\ \psi(a'|a) &= \frac{d}{da'}\Psi(a'|a) \sim N(B_0 a, CC'), \end{aligned} \tag{7}$$

and accordingly, the density over the history of the worker $a^t = [a(t), a(t-1), \ldots, a(0)]$ corresponds to:

$$\psi(a^t) = \psi[a(t)|a(t-1)] \ldots \psi[a(1)|a(0)]\psi_0[a(0)], \tag{8}$$

with the unconditional invariant distribution given by:

$$\psi(a') = \int_a \psi(a'|a)\psi(a)da. \tag{9}$$

In these regards, in each period, firms are required to incur in a operating cost per employed worker dependent on the training and ability of the worker, and the task market, otherwise he will become fully unproductive. The operating cost, corresponds

[29] The firm's production function is continuous at all arguments, concave, with constant returns to scale, infinitely differentiable for all positive arguments. As will be clear in our identification strategy, the adoption of an homogeneous production function is taken for exposition purposes, and do not constraint our empirical environment.

[30] Notice that in describing the model, we present β as a scalar, so that we ease notation burden. When pertinent, we present the implied differences. Further, we are assuming the agents while are forward looking, they assume $\{K, \beta\}$ will be stable, so that any future change in firm's fundamentals is fully unexpected, when bargaining takes place.

to the flow:[31]

$$A(j,a) = \omega_j(a). \tag{11}$$

Conditional on the firm's characteristics, the function $G_{j,t}(a|K,\beta)$ represents the number of workers, employed in task market j in moment t, with at most a as level of operating cost, which is assumed to be, in each moment t, common knowledge.[32] Consistently, the economy pool of workers is given by:

$$\aleph = \int_a d\aleph(a,t)da = \sum_{j=1}^{J}\int_a\int_K\int_\beta dG_{j,t}(a|K,\beta)d\Gamma(K,\beta)da + \int_a dU_t(a)da, \tag{12}$$

where: (a) $U_t(a)$ corresponds to the number of unemployed workers with at most an operating cost of a; and (b) $\aleph(a,t)$ consists in the number of available workers with at most an estimated operated cost a in period t.[33]

The last notable agent in our model is the representative union. We assume the union fully represents the workforce in the wage bargaining, independently of the actual workforce membership status, while employment decisions are left to firms. In representing workers, the objective of this utilitarian union is to maximize the workforce value given by:

$$W_t = \sum_{j=1}^{J}\int_a\int_K\int_\beta \Xi_{j,t}(a|K,\beta)dG_{j,t}(a|K,\beta)d\Gamma(K,\beta)da + \int_a Out(a)dU_t(a)da, \tag{14}$$

where $\Xi_{j,t}(a|K,\beta)$ is the value of a worker of type a conditional on being in a firm of type $\{K,\beta\}$, and $Out(a)$ is the value of the outside option of the worker of type a.

[31]Technically, assume that $\omega_j(x) > \omega_l(x), \forall x \in [0,\bar{A}], \forall j > l$ due to the increasing complexity of task market. Further $\omega_j(x)$ is strictly convex and holds $\lim_{x\to 0^+}\omega(x) = \bar{A}$, $\lim_{x\to\infty}\omega(x) = 0$. The use of a Markov process in this context is classical. Bonhomme et al. (2017) uses a Markovian process to describe earnings directly, whereas we adopt a Markovian process in skill, which allow for dynamics to be treated in a slightly different angle. Jointly, the operating cost function and the skill acquisition can be represented by a linear state-space system, as:

$$\begin{aligned} a(t) =& B_0 a(t-1) + Ce(t) \\ A(j,a) =& \omega_j(a(t)), \end{aligned} \tag{10}$$

with $e(t) \sim N(0,I)$. See Ljungqvist and Sargent (2012) for further details of this process.

[32]In the process of matching in the labour market, we critically assume that a hiring firm only acquires knowledge about a after the hiring is completed.

[33]Notice that $\aleph(a,t)$ unfolds according with:

$$d\aleph\Big(a,t\Big) = \int_a d\aleph\Big(a,t-\epsilon\Big)\psi(a'|a)da. \tag{13}$$

Further $\aleph$ is exogenous and fixed. Then given $\Gamma(K,\beta)$ and an initial distributions $dG_{j,0}(a|K,\beta)$ and $d\aleph(a,0)$ the distribution $dU_0(a)$ is identified, and given the dynamics of the former distributions the dynamics of the latter is equally identified. The dynamics of $dG_{j,t}(a|K,\beta)$ are described latter.

Value functions.[34] The profit of a firm with fundamentals $\{K, \beta\}$ is assumed to be strictly concave and twice continuously differentiable in employment. It is given by:

$$
\begin{aligned}
r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} &= \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J}\int_a w_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Wage Bill}} \\
&- \underbrace{\sum_{j=1}^{J}\int_a A(j,a)dG_j(a|K,\beta)da}_{\text{Operating Cost Bill}} - \underbrace{\sum_{j=1}^{J}\int_a s_j(a|K,\beta)J_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Cost of Displacement}} - \underbrace{I(K)}_{\text{Sunk Cost}} \\
&- \underbrace{\sum_{j=1}^{J}\int_a \tilde{s}_j(a|K,\beta)SdG_j(a|K,\beta)da}_{\text{Exp. firing tax}} + \underbrace{\sum_{j=1}^{J}\max_{V_j(K,\beta)}\Big\{-\gamma_j V_j(K,\beta) + V_j(K,\beta)q(\theta_j)J_j^R(K,\beta)\Big\}}_{\text{Value of the Hiring Policy}},
\end{aligned}
\tag{15}
$$

where $\tilde{s}_j(a|K,\beta)$ corresponds to the probability that the firm (K,β) fires at will the worker of type a paying in such event a firing tax of S, and $s_j(a|K,\beta)$ corresponds to the probability that the match (a,K,β) is dissolved. The intuition of equation (15) is standard in the models of this type (see Cahuc et al. (2008)). Accordingly, profit of a firm $\{K,\beta\}$ accounts for: (a) the output of the firm; (b) the firm expenditure in the wages of the employed workers; (c) the firm expenditure with operating costs; (d) expected firing taxes; (e) the sunk cost related to the capital input; (f) the firm losses due to the separation shock; and (g) the proceeds of the firm's optimal vacancy posting behaviour (i.e. $V_j(K,\beta)$), considering the probability the firm meets a candidate, the cost of creating a vacancy (i.e. γ_j), and $J_j^R(K,\beta)$ the firm's expectation about the marginal profit obtained with a new hire.

The corresponding HJB equation of the marginal profit of a worker is given by:

$$
\begin{aligned}
rJ_j(a|K,\beta) - \frac{\partial J_j(a|K,\beta)}{\partial t} &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a|K,\beta) - A(j,a) \\
&- \underbrace{\sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da}_{\text{Employment effect on wages of other task markets}} - \underbrace{s_j(a|K,\beta)J_j(a|K,\beta)}_{\substack{\text{Expected value loss of}\\ \text{dissolving the match}}} - \underbrace{\tilde{s}_j(a|K,\beta)rS}_{\text{Firing tax}} + \\
&+ \underbrace{\sum_{l=1}^{J}\left\{y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\right\}\frac{\partial J_j(a,K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact on value of the job of hiring and firing policies of the firm}}.
\end{aligned}
\tag{16}
$$

Altogether, the value function of a filled job in firm (K,β) by a worker a can be

[34] For the sake of simplicity, and with a slight abuse of notation, we drop the time subscript in the remainder of this modelling section and in most of appendix B, excepting when the expression is dynamic.

described as the match marginal productivity discounting the value of the worker's wage, the impact of the marginal hiring on the wages set in the other task markets, the loss inherent to the dissolution of the this match, potentially including a firing tax if the dissolution was a firm at-will decision, and lastly the impact of the firm's hiring and firing decisions in other task markets on the value of the filled job.

Regarding the unemployed worker, we have that his HJB equation is given by:

$$
\begin{aligned}
rOut(a) - \frac{\partial Out(a)}{\partial t} = & \underbrace{b}_{\substack{\text{Unemployment} \\ \text{benefit}}} + \\
& + \underbrace{\sum_{j=1}^{J} \xi_j^o(a) \left\{ \theta_j q(\theta_j) \frac{\int_K \int_\beta \Xi_l(a|K,\beta) V_l(K,\beta) d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta) d\Gamma(K,\beta)} - Out(a) - c_j(a) \right\}}_{\text{Expected value of searching for a job}},
\end{aligned} \tag{17}
$$

where $\xi_j^o(a)$ corresponds to an indicator function being 1 if the unemployed is searching in task market j, and zero otherwise.[35] By the same token, the corresponding value function for the employed worker in the match with fundamentals $\{a, K, \beta\}$ is given by:

$$
\begin{aligned}
r\Xi_j(a|K,\beta) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} = & w_j(a|K,\beta) + \underbrace{\bar{s}\left(Out(a) - \Xi_j(a|K,\beta) \right)}_{\text{Value loss of losing the job}} + \\
& + \underbrace{\sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta) \left\{ \theta_l q(\theta_l) \frac{\int_K \int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)] \Xi_l(a|K,\beta) V_l(K,\beta) d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta) d\Gamma(K,\beta)} - Out(a) - c_j(a) \right\}}_{\text{Value of searching for a job while employed}} \\
& + \underbrace{\sum_{l=1}^{J} \left[y_l(K,\beta) V_l(K,\beta) - s_l(K,\beta) N_l(K,\beta) \right] \frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact of hiring and firing policy of the firm in the value of the employment}},
\end{aligned} \tag{18}
$$

with $\mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]$ representing an indicator function equal to 1 if the value in alternative match is greater than the current, and $\xi_l^{\Xi}(a|K,\beta)$ corresponding to an indicator function being 1 if the employed worker is searching in task market j, and zero otherwise.

Collective Bargaining Protocol. In wage bargaining, which occurs in every period t, the union and firms follow bilateral bargaining protocols, with a system of offers and counter-offers in the spirit of Rubinstein (1982) and Brügemann et al. (2018).[36]

[35] It is assumed that the unemployed worker only searches in the task market that maximizes is expected value of search. The same takes place in the case of the employed worker. Further details in appendix B2. Moreover, the unemployment benefits - b - are independent of a merely for exposition purposes. It will be clear that considering it dependent on a, i.e. $b(a)$, will not affect our identification strategy.

[36] In particular, the insight of the Brügemann et al. (2018) allows for the ordering at which the contracts for each match fundamentals are bargained do not influence the outcome of the bargaining.

Further, as previously described, the sides will firstly bargain binding minimum wages at aggregate level, and subsequently will bargain actual wages at firm level.

In both stages, the union and the firm, or employer's association, bargain wages assuming *the principle of match stability*, which is algebraically translated into:

$$\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a) = 0, \forall l \in \{1,\ldots,J\}, \forall\{a,K,\beta\}. \tag{19}$$

Notice that precisely match stability implies that neither side is ex-ante considering the other will dissolve the match at-will.

At aggregate level, we assume the association of firms and the union bargain the minimum binding wage which is compatible with the lowest surplus viable match, namely the match that generate a zero expected quasi-rent. Notice that the level of the expected quasi-rent of the match of a worker of type a with the average firm in the bargaining corresponds to:

$$\begin{aligned} E_{K,\beta}[QR_j(a,K,\beta)] =& E_{K,\beta}\left[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}\right] - A(j,a) \\ & - E_{K,\beta}\Bigg[\underbrace{\sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages of other task markets}}\Bigg] - rOut(a) - \frac{\partial Out(a)}{\partial t}. \end{aligned} \tag{20}$$

Altogether, the aggregate bargaining solves the axiomatic constrained Nash bargaining, considering the Dobbelaere and Luttens (2016) proposition, and thus that the disagreement points are given by the loss of one match with a worker of type-a without the existence of side payments. Consequently:

$$\begin{aligned} & w_{j,t}^{MIN}(a) = argmax_w \Big\{E_{K,\beta}[\Xi_{j,t}(a|K,\beta)] - Out(a)\Big\}^{\beta}\Big\{E_{K,\beta}[J_{j,t}(a|K,\beta)]\Big\}^{1-\beta} \\ & \text{subject to:} \\ & \tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \quad \textbf{(match stability)} \\ & E_{K,\beta}[QR_j(a,K,\beta)] = 0, \quad \textbf{(No quasi-rent condition)} \\ & \forall l \in \{1,\ldots,J\}, \forall\{a,K,\beta\}. \end{aligned} \tag{21}$$

The solution of the aggregate bargaining problem is given by:

$$w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t}, \tag{22}$$

which precisely defines the minimum wage at aggregate level for type-a worker, i.e.

the bargained wage of type-a worker, as the level of his outside option.[37]

Considering the bargained wages of each type-a worker, the firm level bargaining takes place, with the union and each firm bargaining contracts for each match fundamentals $\{a, K, \beta\}$. In case of a full bargaining breakdown one of the matches, with match fundamentals as $\{a, K, \beta\}$, is expected to be dissolved without the existence of side payments among the involved market actors.

Accordingly, the wage of a match with fundamentals $\{a, K, \beta\}$ is obtainable by solving an axiomatic generalized Nash bargaining as:

$$
\begin{aligned}
& w_{j,t}(a|K,\beta) = argmax_w \Big\{ \Xi_{j,t}(a|K,\beta) - Out(a) \Big\}^{\beta} \Big\{ J_{j,t}(a|K,\beta) \Big\}^{1-\beta} \\
& \text{subject to:} \\
& \tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \quad \textbf{(match stability)} \\
& w_{j,t}(a|K,\beta) \geq w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t} \quad \textbf{(agg. bargaining constraint)} \\
& \forall l \in \{1,\dots,J\}, \forall \{a,K,\beta\}.
\end{aligned} \tag{23}
$$

Equilibrium Wages in the Dynamic Equilibrium. Considering the assumptions referred in the previous sub-section, and the generalization of the bargaining power parameter set to be heterogeneous at task-market-firm level, the unique solution of the equilibrium wages, which is compatible with the dynamic equilibrium presented, is given by:

$$
\begin{aligned}
& w_j(a|K,\beta) = \\
& = \begin{cases}
(1-\beta_j) \underbrace{rOut(a) - \frac{\partial Out(a)}{\partial t}}_{Out^{\star}(a)} + \underbrace{\underbrace{\int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}_{\substack{\textit{Perturbed marginal productivity} \\ \textit{of the worker in the workplace (f,j,t)}}} - \beta_j \underbrace{A(j,a)}_{\text{Op. Cost}}}_{\text{Idiosyncratic Surplus of the Match } \{a,K,\beta\}}, \\
\qquad \text{if } w_j(a|K,\beta) \geq w_{j,t}^{MIN}(a) \\
w_{j,t}^{MIN}(a), \qquad \text{if otherwise.}
\end{cases}
\end{aligned} \tag{24}
$$

[37]The solution of the aggregate bargaining entails that the worker's bargaining power does not directly influence the bargained wage of type a worker, but it influences the outside option through the worker's bargaining powers of the expected potential offers of the worker. Thus aggregate movements of bargaining powers affect the level of the outside option and the bargained wage, while idiosyncratic movements of bargaining powers of the bargained contract do not.

The expression of the idiosyncratic surplus of the match in the interior solution has a *perturbed marginal productivity of the worker in the workplace*, affected by the heterogeneity in the bargaining powers across task markets, which is fully consistent with Cahuc et al. (2008). Critically, notice that this term is invariant within the workplace (f, j, t).[38]

Moreover, in the absence of corner solutions, the difference between the average wage within the workplace, and individual wages, is determined by two fundamental factors, namely: (i) the differences in the level of outside options of the workers in the workplace; and (ii) the heterogeneity in the level of operating costs of the workers. This conclusion is easily reached if one represents equation (20) as a function of the average wage within the workplace. Accordingly, we have that:

$$w_j(a|K,\beta) = w_j(K,\beta) + (1-\beta_j)\underbrace{\left[Out^\star(a) - E[Out^\star(a)|K,\beta]\right]}_{\substack{\text{Diff. in outside options=}\\ \Delta Out^\star(a,K,\beta)}} + \beta_j \underbrace{\left\{E[A(j,a)|K,\beta] - A(j,a)\right\}}_{\text{Diff. in Op. Costs}=\Delta A_j(a,K,\beta)}. \tag{26}$$

Finally, notice that the aggregate bargaining constraint is assumed to not be binding in the dynamic and steady state equilibria.[39] We assume that the worker will pull off at will from any match he is involved into that offers him a wage equal to the flow value of his unemployment. Consequently, in the empirical implementation we will consider just interior solutions.

4 Empirical Implementation

The main objective of the empirical implementation of the model is to estimate the actual wage equation of the worker. According to the presented model, wages are given by:[40]

$$w_j(a_t|K_t,\beta_t) = (1-\beta_{j,t})Out^\star(a_t) + \int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz - \beta_{j,t}A(j,a_t). \tag{27}$$

[38] Technically, we refer to a workplace as the combination of worker-observations that share (f, j, t) dimensions. Intuitively, the workers that in moment t are in firm f in task market j. Additionally, note that the average wage in each workplace in the absence of corner solutions is given by:

$$w_j(K,\beta) = (1-\beta_j)E[Out^\star(a)|K,\beta] + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta_j E[A(j,a)|j,K,\beta]. \tag{25}$$

[39] See Haanwinckel and Soares (2020) for an analysis of binding minimum wages in a wage bargaining setting close to ours, and the consequences for the wage setting. They assume an arbitrary minimum wage is set, and thus may become binding, while we assume the bargaining that results in the minimum wage.

[40] Notice that in the empirical description we recover the index t to avoid any confusion on the dimensions of heterogeneity in the empirical model.

In estimating equation (27), we will take advantage of the information about the workplace definition to infer the average wages, the outside option of each worker, provided by the referred workers' ranks, and the context of the union-firm bargaining, which will inform the behaviour of operating costs.

Empirical Outside Options. Given our theoretical framework, we have an estimate of the outside options of each worker (i.e. $Out^\star(a_t)$ is known), which coincides with the estimate of the wage floor of the rank in which the worker is placed. Thereafter, assuming that: (i) the outside options' valuation within each task market evolve, holding a parallel trends assumption, as $\lambda(j,t)$; and (ii) a first order Taylor approximation of the worker's outside option value around the average outside option of each workplace is reasonable, we have that the outside option can be estimated as:

$$ln[w_{j,t}^{MIN}(a_t)] = ln[Out^\star(a_t)] = \lambda(j,t) + \psi[rank(i,t), t - \tau_0(i), female_i] + v_{i,t}, \quad (28)$$

where λ is a task market and period effect, ψ is the contract characteristics effect. The latter accounts for: (a) the experience of the worker (i.e. $t - \tau_0(i)$), (b) the current rank of the worker (i.e. $rank(i,t)$), and (c) different career prospects by gender.[41]

Empirical Operating Costs To fully estimate equation (27) one needs to consider the functional form of the operating costs. In general, those correspond to the individual characteristics of the workers, which influence the marginal productivity of the match worker-workplace. For estimation purposes, we assume an additive functional form as:

$$A(j,a_t) = \underbrace{\xi(i,t) + \mathbf{X}_{i,t}\zeta_j}_{\text{Worker Characteristics}}, \quad (29)$$

where $\xi(i,t)$ corresponds to time-varying unobservable characteristics of the worker, and $\mathbf{X}_{i,t}$ to a vector of k observable characteristics of the worker. Notice that ξ would translate in a very high dimensional parameter set, which ordinarily is beyond identification capabilities of ordinarily implemented models.

In our approach we do not directly estimate the individual operating costs. Rather, we assume unions and firms have an estimate of these operating costs, and consequently use it to place the worker in the ranks of the firm. Consequently, we have that:

$$A(j,a_t) \approx E[A(j,a_t)|rank(i,t)] = E[\mathbf{X}_{i,t}|rank(i,t)]\zeta_j + E[\xi(i,t)|rank(i,t)]. \quad (30)$$

Equation (30) results from a *revealed preference argument.* Concretely, *if a worker is*

[41] The derivation details of this functional form are provided in appendix C. While in theory we could resort directly to our proxy of $Out^\star(a)$, we use the insight of Pei et al. (2018) to minimize the potential impact of measurement error arising from the definition of the proxy.

placed in a given rank within the firm, it can only be because the union and the firm agree he has a level of operating cost that is compatible with that rank.

Moreover, we assume that the expected value of the unobserved characteristics of operating costs is the same within each task-market. Thus:

$$E[\xi(i,t)|rank(i,t)] = E\left[E[\xi(i,t)|rank(i,t)]\middle|j,t\right], \forall\ rank \in j. \tag{31}$$

This assumption implies that idiosyncratic characteristics of the workers, either observable or unobservable, contribute to their career path, i.e. their placement on a given rank, as the measure of operating costs of each worker is assumed to be equal to the workers' average of operating cost in the rank he is enlisted on - $E[A(j,a)|rank(i,t)]$. Moreover, the unobserved component of this average operating cost, $E[\xi(i,t)|rank(i,t)]$, is common to every rank in the corresponding task market of the relevant collective agreement. Intuitively, this assumption implies that the ranks are differentiated based on observable characteristics of their respective workforces, which we find a natural assumption given the bargaining takes place between unions and firms.[42]

Given the equation (31), and our interest in the difference in operating cost between the average worker in workplace and the worker's rank, we have that:

$$\begin{aligned}\Delta A_j(a_t, K_t, \beta_t) =& \underbrace{\left\{E\left[E\left[\mathbf{X}_{i,t}\middle|rank(i,t)\right]\middle|K_t,\beta_t\right] - E\left[\mathbf{X}_{i,t}\middle|rank(i,t)\right]\right\}}_{\text{Diff. in workplace versus rank on time-varying observables}}\zeta_j \\ =& \Delta E[\mathbf{X}_{i,t}|rank(i,t), K_t, \beta_t]\zeta_j,\end{aligned} \tag{32}$$

as the unobservable components cancel out. Notice that the *ceteris paribus* interpretation of empirical marginal effects of any of the variables in matrix $E[\mathbf{X}_{i,t}|rank(i,t)]$ is equivalent to the interpretation of a change in $\mathbf{X}_{i,t}$ in terms of wage change. Mechanically, one expects that the change in operating costs changes the rank of the worker, implying that the worker gets promoted. Altogether, equation (32) holds for that same worker, but on a different rank the worker was then assigned to.[43]

The logarithm of actual wages. Considering equation (26), and the described behaviour of outside options and operating costs, we have that the log of actual wages

[42] Notice that we are not fully excluding idiosyncratic pay to a given characteristic of the worker *vis-à-vis* the remuneration in the corresponding rank, as long as such payment is performed by resorting to bonuses, or irregular compensation policies.

[43] Noteworthy, implicit on the ceteris paribus analysis, we are assuming that the change in operating costs is not affecting the outside option, which may be an unrealistic assumption. To fully study *general equilibrium* marginal effects, one would have to estimate the impact of the change in that observable characteristic both on operating costs and outside options.

corresponds to:

$$\begin{aligned}ln[w_j(a_t|K_t,\beta_{j,t})] =&ln\Bigg[w_j(K_t,\beta_{j,t}) + (1-\beta_{j,t})\Delta Out^\star(a_t,K_t,\beta_t)\\ &+\beta_{j,t}\Delta E\Bigg(\mathbf{X}_{i,t}|rank(i,t),(f,j,t)\Bigg)\zeta_j\Bigg] + \epsilon_{i,f,j,t},\end{aligned} \tag{33}$$

where $\epsilon_{i,f,j,t}$ corresponds to a disturbance. Accordingly, the logarithm of the wage is decomposed into: (i) the logarithm of average wages in the workplace of the worker; (ii) the differences in the outside options, properly weighted by the worker's bargaining power; and (iii) the differences in the operating cost observables and the average wage in the workplace, weighted by the firm's bargaining power.

The estimation procedure. The first step deals with potential measurement error in our proxy of outside option values, and consists in resorting to a high dimensional heterogeneous slope model as:

$$ln[Out^\star(a_t)] = \lambda(j,t) + \psi[rank(i,t), t-\tau_0(i)] + v_{i,t}. \tag{$\mathbf{1^{st}\ Step}$}$$

While this first step estimates a large number of parameters, due to ψ term, it is significantly more parsimonious than a model that resorts to worker and/or firm effects.

Then we resort to the predicted outside option value - $\widehat{Out^\star(a_t)}$ to feed the estimation of the actual wage empirical model, which corresponds to:

$$\begin{aligned}ln[w_j(a_t|K_t,\beta_{j,t})] =&ln\Bigg[w_j(K_t,\beta_{j,t}) + (1-\beta_{j,t})\Delta \widehat{Out^\star(a_t,}K,\beta)+\\ &+\beta_{j,t}\zeta_j\Delta E[\mathbf{X}_{i,t}|rank(i,t),(f,j,t)]\Bigg] + \epsilon_{i,f,j,t},\end{aligned} \tag{$\mathbf{2^{nd}\ Step}$}$$

we use a non linear least squares, as:

$$\hat{\Theta} = argmax_{\theta\in\Theta}\sum_i^N\Bigg\{ln[w_j(a_t|K_t,\beta_{j,t})] - f\Bigg(\theta,\mathbf{X}_{i,t},w_j(K_t,\beta_{j,t}),\Delta\widehat{Out^\star(a_t,}K,\beta)\Bigg)\Bigg\}^2, \tag{34}$$

where $\theta = \theta(\beta_{j,t},\zeta_j)$ is the parameter vector.

Conjointly, the workings of section 3 and section 4 presented an empirically implementable search and matching model, with the sufficient ingredients to present a credible environment for empirical implementation. While not every ingredient, particularly on job flows, is strictly necessary to establish the identification of equilibrium wage equations as presented, their inclusion allows to understand how the modelled collective bargaining environment interacts with them, in a setting of a multi-employee and multi-occupational firm. In appendix E, we present a Toy model version of the sections 3 and 4 presenting the minimal components considered in a search and matching model capable to deliver the empirical identification presented.

5 Empirical Results

Highlights from a Parsimonious Wage Setting Equation

The outset of our empirical analysis adopts the most parsimonious and static version of our model. For this purpose, over the presented framework we assume that the workers' bargaining powers are constant overtime. In this stylized version, the model preserves a within-workplace structural approach, accounting for workplace time-varying effects and workers' observed and unobserved time-varying characteristics. The analogous reduced-form implementation would correspond to:

$$ln[w_{ifjt}] = ln\left[\phi_{f,j,t} + (1-\beta_j)\widehat{Out(a)} - \beta_j[\xi(i,t) + \mathbf{X}_{i,t}\zeta_j]\right] + \epsilon_{i,f,j,t}, \tag{35}$$

where $E[\mathbf{X}_{i,t}|rank(i,t)] = \xi(i,t) + \mathbf{X}_{i,t}\zeta_j$.

While apparently straightforward, this reduced form would frame a conundrum in the absence of the model design of data usage. Either from a fixed or random effects perspectives, the estimation of parameters and/or implied distributional features for $\phi_{f,j,t}$, $\xi_{i,t}$ and $Out(a)$ would become computationally unattainable; would be largely unidentifiable given any largest connected set requirement; would imply strictly unrealistic orthogonality conditions; or even would result in a preposterous overparameterized model. The suggested relationship between workplace hourly wage and worker's hourly wage; the use of bargained wages; and the presented structure of individual and rank operational costs pave the way out of the riddle.

The estimation results are presented in table 2. Forthwith, the goodness-of-fit of the model is convincing, even in the context of a very favourable number of estimated parameters versus number of observations trade-off, particularly if one uses the AKM standpoint. Notice that in the first and second stages, the model resorts to 606,759 and 10 parameters, respectively, while the number of observations are above 29 million in each stage, with an average number of workers per year of around 1.4 million.[44]

Among the presented results, while the operating cost coefficients will be discussed at a later stage of this section, we promptly highlight the worker's bargaining power. Our estimate for the entire economy is around 20%, which is consistent with several other studies in the literature. For instance, for the Veneto region of Italy, Card et al. (2014) find a reduced form coefficient of the outside option (i.e. $(1-\beta)$) of 80%, when using sector minimum wage as the proxy of the outside option, in both OLS and IV within spell models of rent sharing. For France, Cahuc et al. (2006) estimate bargaining powers mostly in the range between 0 and 38% depending on

[44] In our sample, a typical worker-firm-time AKM specification entangles 3,660,238 worker fixed effects, 127,333 firm effects and 21 yearly dummies. So we estimate around 16 percent of the number of parameters a typical AKM would use in the first stage.

the task-market.[45] For Germany, Hirsch and Schnabel (2011) implement a right-to-manage model and estimate yearly bargaining powers between 11%-18%, for the years 1992-2009. For Denmark, resorting to a structural model with some commonalities with our theoretical approach, Bagger et al. (2014) estimates an average workers' bargaining power of around 30 percent, and while Mortensen et al. (2010) matches that empirical estimate for the same dataset it further presents sectoral heterogeneity, ranging from 7-61 percent. Discordantly, Dumont et al. (2012) presents higher workers' bargaining power estimates for Belgium, between 45-71 percent depending on the sector in analysis.

Table 2: Non-linear Least Squares with Common Slopes.

Panel A: First-stage on Outside Option

Heterogeneous Slopes and Intercepts	Number of Parameters
Rank - 3^{rd} order polynomial age function - Female	606,696
Task Market - Year	63
Adjusted R^2	0.9371
N. Obs.	29,586,448

Panel B: Second-stage NLS Estimation

Variable	Coefficient (s.e.)		
	Managers	Skilled workers	Unskilled workers
β	0.3547***	0.1761***	0.1733***
	(0.0021)	(0.0002)	(0.0013)
$\zeta_{\text{tenure}}/10$	-0.0037	-0.0640***	-0.2681***
	(0.0130)	(0.0055)	(0.0045)
$\zeta_{\text{tenure}^2}/100$	0.1457***	0.0159***	0.0555***
	(0.0092)	(0.0046)	(0.0036)
$\zeta_{\text{tenure}^3}/1000$	-0.0290***	-0.0040***	-0.0006
	(0.0018)	(0.0009)	(0.0008)
$\zeta_{\text{age}}/10$	2.291***	-1.1753***	-1.4556***
	(0.6120)	(0.0413)	(0.0230)
$\zeta_{\text{age}^2}/100$	-0.6283***	0.2289***	0.3414***
	(0.1531)	(0.0106)	(0.0061)
$\zeta_{\text{age}^3}/1000$	0.0477***	-0.0086***	-0.0258***
	(0.0124)	(0.0009)	(0.0005)
$\zeta_{\text{education}}$	-0.0312***	0.0916***	-0.0452***
	(0.0028)	(0.0015)	(0.0012)
$\zeta_{\text{education}^2}$	-0.0040***	-0.0139***	0.0009***
	(0.0002)	(0.0001)	(0.0001)
ζ_{female}	0.0453***	-0.3189***	-0.2211***
	(0.0054)	(0.0025)	(0.0023)
Adjusted R^2	0.9034	0.8567	0.7288
Obs.	4,797,735	14,937,457	9,851,256

Notes: Robust clustered workplace standard errors used in first-stage. Robust standard errors use in the second-stage. ***, **, * denote significance at 0.01, 0.05, and 0.10 levels, respectively. *Sources:* Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2016.

[45] The sole exception of that range is 98% for managers in the construction sector. Their partition of the task markets is identical to ours, but they have 4 categories. Their two top categories (i.e. 1 and 2) are condensed in our 1^{st} category.

A breviary of the empirical results of the supra-cited studies and of table 2 highlights the importance of bargaining power and operating cost heterogeneity over the time and sector dimensions. Accordingly, we consider the preceding dimensions of heterogeneity in our empirical analysis.

The Prime Empirical Wage Setting Equation

In our benchmark empirical setting, we explicitly model an idiosyncratic time evolution of bargaining powers for each task market, through two alternative functional form specifications, expressing different smoothness degrees ex-ante assumed for such time progression. Ergo, in a more involving specification, we relax any smoothness prerequisite and estimate bargaining powers resorting to a vector of year-task market dummies, while in a more frugal alternative, we impose a type-task specific third order degree polynomial functional form of the time trend. Figure 6 presents the resulting workers' bargaining powers estimates.[46]

At first glance, figure 6 reaffirms the existence of relevant bargaining power heterogeneity at task market level, and confirms the relevance of accounting for time evolution. Technically, even though the polynomial specification doesn't fully match the curves resulting from the best fitting polynomial trend of the corresponding dummies' series, it provides a close approximation, and thus consists in a reliable modelling option particularly in the presence of smaller samples. Moreover, both sets of estimates are consistent with the results from table 2, and consequently confirm higher bargaining power levels for managers, and very identical levels for skilled and unskilled workers. While some studies present an higher bargaining power at the bottom of the wage table (for example Dumont et al. (2012)), others present either broad monotonicity between wage tables and bargaining powers, or even very identical, U-shaped, or mixed results depending on the sector of activity under analysis (see Cahuc et al. (2006), Mortensen et al. (2010) or even Bagger et al. (2014)).

[46] In the estimation, we adopted robust standard errors instead of clustering at any dimension. Abadie et al. (2017) advocate the absence of clustering in the presence of a fixed effect specification when there is homogeneous treatment effects within the cluster formed at the level of the fixed effect. We assume such homogeneity by design as the workplace heterogeneity arises solely from the heterogeneity in worker's characteristics and not from the valuation of their characteristics. Moreover, the use of average real hourly wage, i.e. $w(K, \beta)$, approximates our setting to the fixed effect setting.

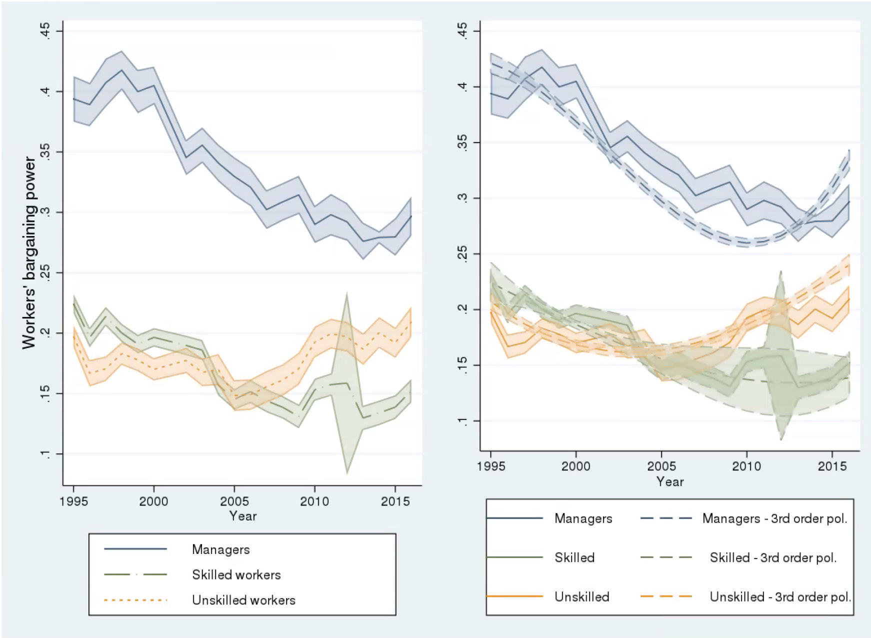
The time variation of bargaining powers will be modelled in two alternative specifications. Firstly, we will consider

$$\beta_{j,t} = \mathbf{D}'\tilde{\beta}, \tag{36}$$

where $\mathbf{D}$ is a $[(J \times T) \times 1]$ vector of year-task market dummies, and $\tilde{\beta}$ is the corresponding vector of parameters. Alternatively, we will assume sufficient smoothness of the time series of bargaining powers, and consequently fit a polynomial approximation as:

$$\beta_{j,t} \approx b_0 + b_{1,j} \times t + b_{2,j} \times t^2 + b_{3,j} \times t^3. \tag{37}$$

Figure 6: Estimated workers' bargaining power per task market.



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval, using robust standard errors. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

Table 3: Average predicted change in bargaining powers implied by best fitting 3rd order polynomial trend to the dummy series.

Period	Dates	Ratio $\beta_{final}/\beta_{initial} - 1$		
		Managers	Skilled workers	Unskilled workers
The Boom	1995-2000	-4.44%	-12.96%	-11.95%
The Slump	2000-2008	-17.98%	-20.58%	0.39%
Financial Crisis	2008-2011	-7.69%	-6.23%	5.77%
Euro Crisis	2011-2014	-3.43%	-0.89%	8.00%
Timid Recovery	2014-2016	1.78%	3.54%	6.04%
Overall	1995-2016	-28.89%	-33.48%	7.08%
β_{1995}		40.30%	21.76%	18.99%
β_{2016}		28.66%	14.48%	20.34%

Notes: The 3rd order polynomial best fitting curve to the dummy series of bargaining powers is used to avoid the over-influence of any transitory fluctuation. The used periods are collected from Blanchard and Portugal (2017) which outlines a detailed macroeconomic analysis of the Portuguese Economy. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

On the time dimension, figure 6 records a downward trend in the bargaining powers unfolding in-tandem for skilled workers and managers, broadly up to the Great Recession, while for the unskilled workers we record a broad stability. These findings are consistent with the estimates of Hirsch and Schnabel (2011) for the entire German economy in the 2002-2009 period, where the authors resorting to a right-to-manage model record a one-third decline, which compares with 19 percent fall for managers, 20 percent for skilled workers and rough stability for the unskilled workers, as displayed in table 3. For the preceding period, i.e. 1995-2000, the cited authors find a stable environment whereas we record a decreasing trend in every task market. Moreover, our findings also broadly concurs with the evolution of the synthetic indicator of collective bargaining of Lombardi et al. (2020) in figure 2.[47]

At the outbreak of the Great Recession, we estimate that bargaining powers were around historical minimums. As presented in both table 3 and table 6, the path of bargaining powers was thereafter markedly idiosyncratic to each task market. For managers and skilled workers, throughout the crisis we record a consistent fall in bargaining powers, but at a less than halved pace relatively to the previous period. For the unskilled workers the bargaining drainage is halted, and we witnessed a substantial re-surge, which visibly commenced in the 2007-2010 period, coinciding with the most sizable surge in the minimum wage for the entire time-frame of the study, of around 18 percent.[48] In the post recession period, namely marked by the end of the adjustment program in July 2014, every task market observed an increase in the bargaining powers which was more pronounced the lower the position of the task markets at the wage tables. By the finale of our analysis, while for bottom of the distribution the bargaining drainage in the earlier period was fully recovered, for the managers and skilled workers the bargaining bygones are estimated of around 30 percent. Such abatement is a consequence of a long-lasting trend which has unfolded with notable stability.

The erosion of bargaining powers, for managers and skilled workers, particularly if amplified by a significant progressive income taxation, may spark a plethora of effects. Namely, the reduction in the wage differentials across skill groups and the increasing decoupling of the worker's wage from the firm productivity levels may degrade the incentives for skill acquisition and on job training, and the alignment of worker and firm incentives may be reduced.

Elasticity of Wages to Exogenous Changes in the idiosyncratic Quasi-Rents

One of the most pursued measures when studying wage setting mechanisms consists in the elasticity of wages to exogenous changes in the idiosyncratic quasi-rents of the worker-firm match, or in general the response of wages to exogenous productivity shocks.[49] As noted by Card et al. (2018), such measures provide a succinct description

[47]See appendix D for an analysis of the remaining parameters of the model, namely the estimates of operating costs.

[48]The evolution of the monthly minimum wage is presented in figure 10.

[49]See Garin and Silvério (2019) for a compatible theory review of the difference in changes in quasi-rents which are: (a) idiosyncratic to the match; and (b) general to the relevant portions of the

of the link between productivity heterogeneity and wage inequality and it constitutes a cornerstone indicator in the rent-sharing literature. Beyond its own merits, the estimation of this elasticity, implied by the results of our model, contributes for the external validity of our results given the proliferation of studies with different empirical techniques, inclusively resorting to the Portuguese labour market.

In detail, the average elasticity of wages to exogenous changes in the quasi-rents is given by:[50]

$$\epsilon_t^{QR}(K,\beta) = \frac{\partial w_j(a|K,\beta)}{\partial QR_j(K,\beta)} \frac{QR_j(K,\beta)}{w_j(a|K,\beta)} = \beta_{j,t} \frac{QR_j(K,\beta)}{w_j(a|K,\beta)}. \tag{39}$$

Moreover, we implement the insight of Card et al. (2014) and compute the quasi-rents estimate as:

$$E_{ft}[QR_j(K,\beta)] = VAB_{ft}(K,\beta) - 0.1K - Out(a), \tag{40}$$

where $VAB_{ft}(K,\beta)$ is the value added per hour, K corresponds to the level of assets presented in the balance sheet of each firm, and 10% the considered costs of capital, as in Card et al. (2014). Given equations (39) and (40), we resort to a two-step GMM, with the logarithm of sales at firm level as instrument (i.e. $ln[F(K,\beta)]$), to estimate the average of the referred elasticity of the economy per year.

Table 4: Estimates of the elasticity of wages to exogenous changes in the idiosyncratic quasi-rents for Portugal.

Study	Dates	Elasticity (s.e.)
Our study	2005-2016	0.062 (0.0090)
Card et al. (2018)	2005-2009	0.056 (0.016)
Card et al. (2016)	2002-2009	Males - 0.14-0.16 Females - 0.04-0.05
Cardoso and Portela (2009)	1991-2000	0.00 (transitory shock) 0.09 (permanent shock)
Martins (2009)	1993-1995	0.03-0.05
Garin and Silvério (2019)	2005-2013	0.15 (0.066)

Sources: Quadros de Pessoal and Relatório Único, 1995-2016; SCIE 2005-2016; and the referred articles.

Table 4 presents our average result, 0.062, and a sample of comparable studies about the Portuguese labour market, which highlights the consistency of our findings

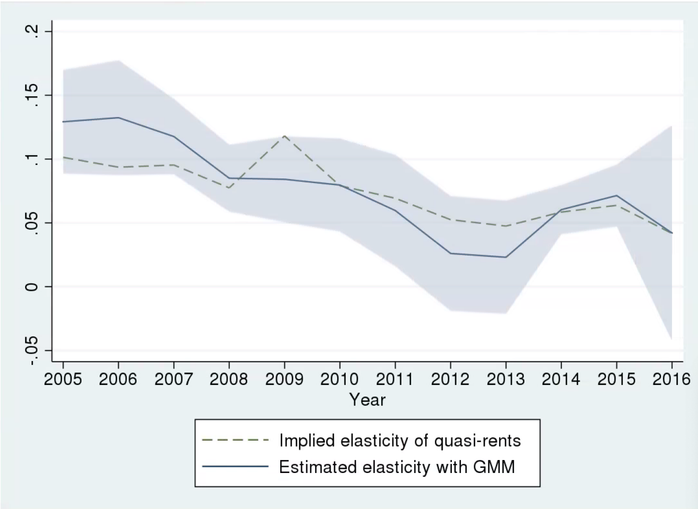
labour market. The major difference is due to the presence of feedback effects on the outside options. We analyze option (a) in this section.

[50] The structure of wages relates with the quasi-rents implied by our model as:

$$w_j(a|K,\beta) = Out(a) + \beta_{j,t} QR_t(K,\beta). \tag{38}$$

with a significant branch in the literature that locates this elasticity estimates in the range between 0.05-0.15.[51] Regarding the time evolution, as presented in figure 7, the average elasticity is generally within the 0.05-0.15 bounds, and present a downward trend as in the bargaining powers, with a particular fall during the *Euro Crisis*.

Figure 7: Estimated Average Elasticity of Wages to an Exogenous Change in the Quasi-Rents.



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval range, using clustered standard errors at collective bargaining level. The implied elasticity of quasi-rents corresponds to the raw measure of equations (39) and (40). *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

Passthrough Rate of Bargained Wages

In the analysis of the wage setting in the continental European labour markets, the passthrough rate of wage floors constitutes a particularly relevant market measure. It displays how the settlement of bargained wages translates to total wages, thus broadly displaying the influence of collective bargaining wage setting on the total wages. In our setting, the average passthrough per year and task market, i.e. the average elasticity of the total wage to the wage floor, is given by:

$$\tau(j,t) = E\left[(1-\beta_{j,t})\frac{Out(a)}{w(a|K,\beta)}\right]. \tag{41}$$

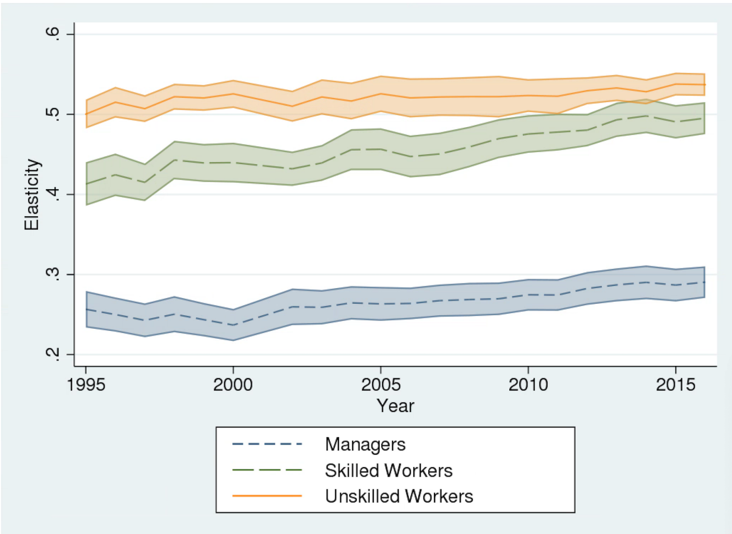
Our results support a classical continental European wage setting feature, where the raise of bargained wages correspond to a shrinkage of the wage cushion, due to an imperfect passthrough. We estimate the average passthrough at 44.8 percent, implying

[51]See Card et al. (2018) for a more extended review of the literature covering 22 different studies including for several European countries and the U.S.

that a 10 percent increase of bargained wages translates into a 4.5 percent increase on total wages. Our findings concur with Card and Cardoso (2021) which estimates an average passthrough of about 50 percent.

Moreover, our finding displays the relevance of collective bargaining, as changes in wage floors are associated to meaningful changes in total wages. According to our results in figure 8, the link between wage floors and total wages is ordered, with the lowest task markets displaying the strongest link, in the relationship significantly stable overtime. This finding is also consistent with Card and Cardoso (2021) which presents evidence of an ordered passthrough rates with skill groups.

Figure 8: Estimated Average Passthrough.



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval range, using clustered standard errors at collective bargaining level. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

An Analysis of Heterogeneity

A classical heuristic method to assess wage differentials and the contribution of its constituents to mounting inequalities is a covariance decomposition assessment. This section assesses the contribution of each of the components of our primal wage setting equation, accounting for the adopted formulation of the operating costs. Consequently, we have:

$$ln[w_j(a_t|K_t,\beta_t)\times \text{hours}_{it}] = ln[\text{hours}_{it}] + ln\Bigg[\underbrace{(1-\beta_{j,t})Out^{\star}(a_t)}_{\text{Out. option component}} + \underbrace{\int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}}\frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})}dz}_{\text{Real productivity per hour of the match}}$$

$$-\underbrace{\beta_{j,t}\mathbf{X}_{i,t}\zeta_j}_{\text{Obs. operating costs comp.}} + \underbrace{\beta_{j,t}[E[E(\xi(i,t)|rank(i,t))|j,t]] - \beta_{j,t}\xi(i,t)}_{\text{Unobservable operating costs comp.}} + \underbrace{\epsilon_{i,f,j,t}}_{\text{residual}}\Bigg]. \tag{42}$$

with the covariance decomposition components obtained as:

$$\mathbf{\Gamma}(n) = \frac{cov\Big(ln[w_j(a_t|K_t,\beta_t)\times \text{hours}_{it}];\Gamma_{i,f,j,t}(n)\Big)}{var(ln[w_j(a_t|K_t,\beta_t)\times \text{hours}_{it}])}, \quad \text{with} \sum_{n=1}^{6}\mathbf{\Gamma}(n)=1 \tag{43}$$

with $\Gamma_{i,f,j,t}(n)$ representing each of the components of equation (42). Inspired on the same structural view of decomposing the components an equivalent mean decomposition is performed. While we could perform an yearly decomposition, we present the results for the average decomposition given we find a broad stability of the contributes and of the overall wage inequality over the period of analysis.[52] Differently, Card et al. (2014) and Song et al. (2019) link the evolution of heterogeneity with perceived growing inequality in the US and Germany, which in their case is noticeable.

Our average covariance and mean decomposition results are presented in table 5. Our first finding corresponds to the peripheral role of the hours worked in wage dispersion, as it even reduces the magnitude of the heterogeneity of monthly wages *vis-á-vis* the hourly counterpart, implying a negative correlation between hourly wages and hours worked. Within the hourly wages, the major drivers of wage differentials resides at workplace level, which explains around 63 percent of the overall heterogeneity in monthly wages, while the components attributed to the worker, namely the level of the outside options and operating cost components, contribute around 22 percent.

Table 5: Average Covariance and Mean Decompositions of Monthly Wages, over the period 1995-2016.

Panel A: Average Variance Decomposition of Monthly Wages.

Dimension	Detailed Components	Contributions	
Extensive Margin	Hours worked		-2.66%
	Outside Option	6.92%	
Worker	Observed Operating Costs	4.95%	21.84%
	Unobserved Operating Costs	9.97%	
Workplace	Workplace Real Productivity		62.61%
Residual			18.21%

Panel B: Average Mean Decomposition of Monthly Wages.

Dimension	Detailed Components	Contributions	
	Outside Option	40.67%	
Worker	Observed Operating Costs	-3.71%	29.5%
	Unobserved Operating Costs	-7.46%	
Workplace	Workplace Real Productivity		71.29%
Residual			-0.8%

Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

[52]See figure 10 for the evolution of the percentiles of the wage distribution.

Within the later dimension, it is noteworthy the different weight of the outside option in the covariance and mean decompositions of table 5. While the outside option component only weights around 7 percent of the variability of wages, it represents around 41 percent of the average wage paid in the economy. This finding highlights the property of the outside option, and the role of labour market institutions, in compressing wages relatively to the implied distribution of the real productivity of the match.

Holistically, table 5 indicates the workplace level productivity as the core engine of wage dispersion, and consequently for the prevalence of heterogeneous labour market histories of otherwise identical workers, as measured by the level of their outside options and operating costs. In an experimental approach, this analysis signals that two unemployed workers with identical ex-ante valuations for their work may end up with significantly different wages based on the way they (re-)enter the labour market. This finding stresses the importance of studying the mechanisms, and their underlying efficiency, by which workers pair with a given level of occupation, and subsequently a given firm. It gives credit to the literature branch that verses on the imperfectly competitive nature of the labour market, either from a more monopsony perspective as in Manning (2011) and Card et al. (2018), or a bargaining in a frictional labour market as we adopted.

In general, the literature has seen a sizable proliferation of variance decompositions of wages, particularly since the advent of fixed effects spurred in the Abowd et al. (1999) model (i.e. AKM). Typically, those, as in Card et al. (2014) for Germany, Torres et al. (2018) for Portugal, and Song et al. (2019) for the US, attributes a leading role of the worker dimension, which can be perceived at odds with our findings. However that might not be the case. Firstly, note that we perform covariance decompositions and not variance-covariance decompositions.[53] Secondly, our central results from table 5 are not based on firm dimension, but workplace dimension which is substantially different and more atomistic, as it combines firm and task market. Thirdly, the studies resorting to ranking algorithms or new estimator designs capable of solving the limited mobility bias issue present mixed results. Borovicková and Shimer (2017) for Austria indicates that wage heterogeneity is attributable more to the firm side than the worker side, while Bonhomme et al. (2019) for Sweden, Bonhomme et al. (2020) for U.S. and several European countries, and Kline et al. (2020) for Italy point for a leading role of the worker dimension with a much marginal role for firms relatively to the AKM designs. Finally, in a more structural and rent-sharing perspective, Mortensen et al. (2010) for Denmark conforms with our findings by attributing a leading role

[53] The results could be comparable by considering our covariances correspond to the relevant variance plus half of the covariance term where the relevant term is displayed.

to the *rent-sharing* component of wages *vis-á-vis* the labour heterogeneity. As noted by both Borovicková and Shimer (2017) and Bonhomme et al. (2020), the different specifications for modelling earnings and the processes of worker and firm heterogeneity can be a leading cause for the disparity of results.

Table 6: Comparison of Covariance Decomposition between AKM and our methodology of the logarithm of real hourly wages.

		Our methodology		AKM		
Dimension	Components	Aggregate	Detailed	Workplace Worker	Firm Worker Task market Year	Firm Worker Year
Worker	Worker attributes	21.27%	10.60%	34.37%	38.06%	41.42%
	Task market FE		6.06%	-	-	-
	Generic time FE		2.49%	-	-	-
	Within residual		2.12%	-	-	-
Workplace	Firm FE	60.99%	37.59%	58.16%	35.60%	39.02%
	Task market FE		9.35%		-	-
	Generic time FE		2.68%		-	-
	Within residual		11.37%		-	-
Year effects FE		-	-	-	6.91%	7.68%
Task market FE		-	-	-	7.93%	-
Residual		17.74%	17.74%	7.47%	11.46%	11.88%

Notes: The detailed decomposition consists in resorting to AKM models to decompose the worker and workplace components, namely: $(1-\beta_{j,t})Out^{*}(a_t) - \beta_{j,t}[\mathbf{X}_{i,t}\zeta_j + \xi(i,t)] + \beta_{j,t}E[E(\xi(i,t)|rank(i,t))|j,t] = \mathbf{X}_{i,t}\tilde{\zeta} + \alpha_i + \delta_j + \chi_t + \epsilon_t$, and $\int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz - \beta_{j,t}E[E(\xi(i,t)|rank(i,t))|j,t] = \alpha_i + \gamma_f + \delta_j + \chi_t + \epsilon_t$, respectively. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

Amidst this debate, we partial out the main components of table 5 decomposition into worker, task market, firm and time dimensions. This procedure approaches our decomposition to an AKM setting, given the intuition of equations 2 and 3 of section 2. Then, using the same dataset, we resort to a typical covariance decomposition of 3 different AKM based strategies, which correspond to: (a) a firm-worker-year fixed effect formulation, in the spirit of Card et al. (2014) and Song et al. (2019); (b) a firm, worker, year and task market decomposition in the spirit of Torres et al. (2018); and (c) a workplace-worker decomposition, where in the workplace fixed effect, measured as firm-task-market-year cells, those three dimensions and their complementarities are captured.

In this exercise, we could perform bias correction for the covariances estimated in the AKM, following recent trends in the literature. However, we take the approach of Song et al. (2019), which abstains from going in this direction by noticing: (a) the computational cost of the solution of Kline et al. (2020); and (b) the additional assumption framework required in Andrews et al. (2012) and Borovicková and Shimer (2017). If anything the bias is likely to affect equally the AKM decompositions and our AKM detailed decomposition, thus establishing a comparable ground between our model and the most canonical empirical approach. Noteworthy, our aggregate decomposition in table 5 does not suffer from limited mobility bias, given it is based on a very small set of parameters.

The findings reported in table 6, particularly the comparison of the workplace-worker AKM and our methodology presents a bridge between both approaches, as the workplace dimension weights roughly around 60 percent of the entire variability of real hourly wages in both methodologies. The differences between both lie on the worker and residual components, with the latter sizably reduced in the AKM approach, and as a consequence the formed fully absorbing the proceeds of the higher fit of the model. As one moves towards more time invariant formulations of the firm side, and account for a partition of the labour market into types of tasks, the worker fixed effect is largely capable to absorb the *leftovers* of the variation which was previously enclosed in the workplace definition, with the types of tasks acquiring a peripheral role.

Altogether, beyond modelling differences in earnings and in the processes of worker and firm heterogeneity, this analysis raises the question of the boundaries of the worker effect realm in such estimations as to precisely which components of wages are encompassed in such set of parameters. Further, it also reinforces the importance to further assess to what extend such worker's dimension primary role could be a data based idiosyncrasy, a mechanical consequence of the relative dimensionality of the worker's parameter set versus the firm, the consequence of any statistical shrinkage of such dimensionality to conform workers and firms into classes, and/or the biases generated from the proper mobility patterns of the workers - known to particularly affect AKM designs. In this context, our approach provides a novel decomposition which links each covariance decomposition component to the structural component of the equilibrium wage equation. While, a comparative scrutiny of the referred empirical methods would be interesting, it is beyond the scope of this paper.

Assortative Matching

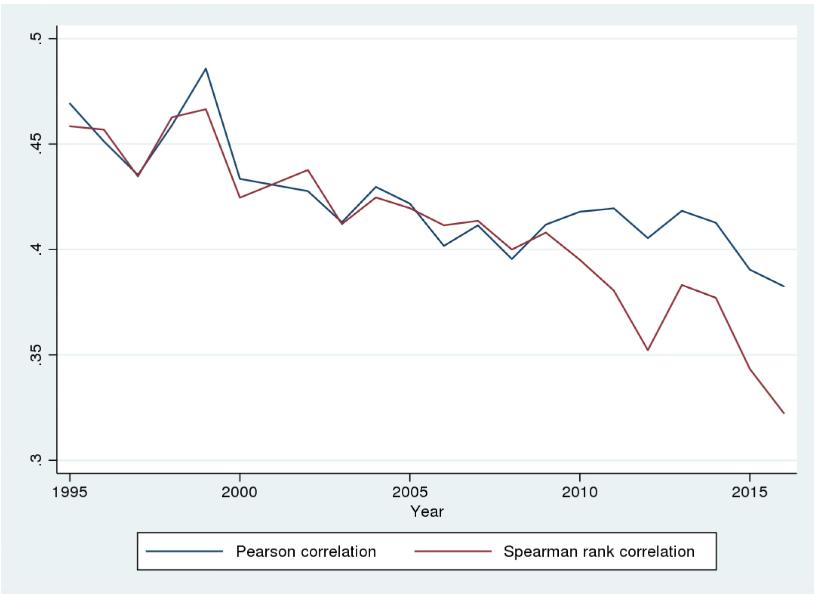
In another but related domain, the literature has embodied the proliferation of studies with measures of assortative matching, canonically defined as the correlation between the worker and firm components of wages. This empirical turf signalled a plausible defect of the fixed effect designs based on Abowd et al. (1999), as it consistently estimated unrealistically small or even negative correlations. These unheralded results soon crystallized as one of the most ad rem *puzzles* of the referred approach. In our setting, a measure of this class is given by:

$$AM_t = corr\left\{ \underbrace{ln\left(E\left[\frac{1}{\beta_{j,t}} \int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz \middle| f,t \right] - E[A(j,a_t)|f,t] \right)}_{\text{firm component}};\ \underbrace{ln[Out^{\star}(a_t)]}_{\text{worker component}} \right\}, \tag{44}$$

where intuitively the worker component is given by his outside option value, while the firm component is composed by the average real productivity in the firm.

The empirical estimates are translated in figure 9, where we resort to both the Pearson and Spearman correlations to estimate equation (44). Our findings locate the Pearson correlation in the range 0.38-0.48, with an average over the period of 44.14%, which is broadly consistent with the Spearman rank correlation, despite its moderate decline in the latter years of the sample. In detail, our time evolution do not present the upward trend of Card et al. (2014), which the authors link with the increase of the wage inequality in Germany; it does not record the significant fall as in Torres et al. (2018); and it conforms with a broad stability as observed in Lentz et al. (2018) for Denmark.

Figure 9: Measure of Assortative Measure.



Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

Regarding the level, which has been the hotspot of scrutiny for the AKM approach, our findings are consistent with the most recent developments on this specific literature, which are resumed in table 7. Noteworthy, our approach seems not to suffer from the downward bias that limited mobility allegedly causes to AKM, and conforms with the findings of Bonhomme et al. (2019) for Sweden, Borovicková and Shimer (2017) for Austria, and are somewhat higher than Kline et al. (2020) for Veneto, Bonhomme et al. (2020) for a range of European countries and the US, and Lentz et al. (2018) for Denmark.

Table 7: Breviary on recent results on Assortative Matching.

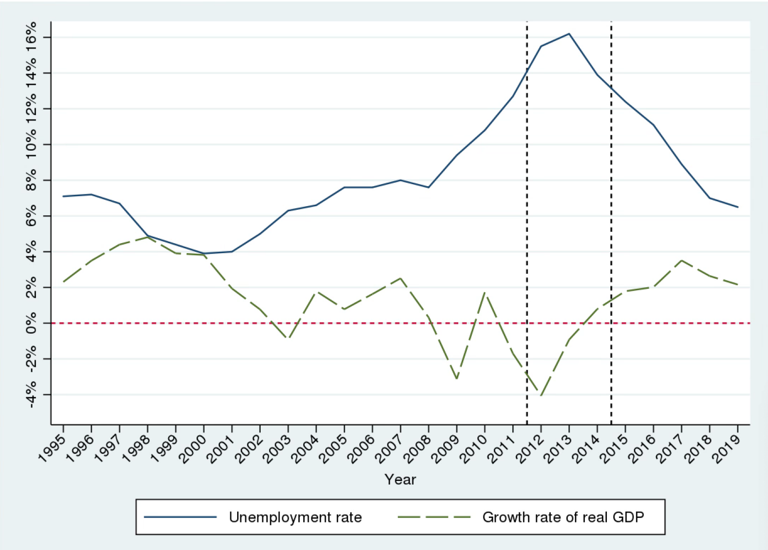
Study	Dates	Country	Method	Estimate
Our study	1995-2016	PT	Our methodology AKM	0.4414 0.1797
Torres et al. (2018)	1986-2013	PT	Firm Measures AKM	0.308 0.08
Card et al. (2016)	2002-2009	PT	AKM	0.152-0.162
Kline et al. (2020)	1984-2001	IT (Veneto)	Leave-out	0.283
Borovicková and Shimer (2017)	1972-2007	AUS	Non-parametric measure	Within 0.4-0.6
Hagedorn et al. (2017)	1993-2007	GER	Ranking Algorithm AKM	0.75 0.055
Card et al. (2013)	1985-2009	GER	AKM	0.034-0.249
Andrews et al. (2012)	1998-2007	GER	Turnover correction	0.2-0.3
Andrews et al. (2008)	1993-1997	GER	High turnover plants	0.224
Bonhomme et al. (2019)	1997-2008	SWE	Bonhomme et al. (2017)	0.4913 (static) 0.419 (dynamic)
Lentz et al. (2018)	1985-2013	DEN	Bonhomme et al. (2017)	0.28
Lachowska et al. (2020)	2002-2014	Washington	AKM Leave-out	-0.03-0.1 0.1-0.2
Abowd et al. (2019)	1999-2003	US	AKM Structural	0.1 0.0
Woodcock (2015)	1990-1999	US	Match effects	-0.1
Bonhomme et al. (2020)	1996-2015	US, IT, AUS, NOR, SWE	Woodcock (2015) Bonhomme et al. (2019)	0.24-0.34
Lopes de Melo (2008)	1972-2007	BRA	Co-workers AKM	0.3 -0.08

6 The Resilience of the Wage Setting Mechanisms: Like a Bridge over Troubled Water

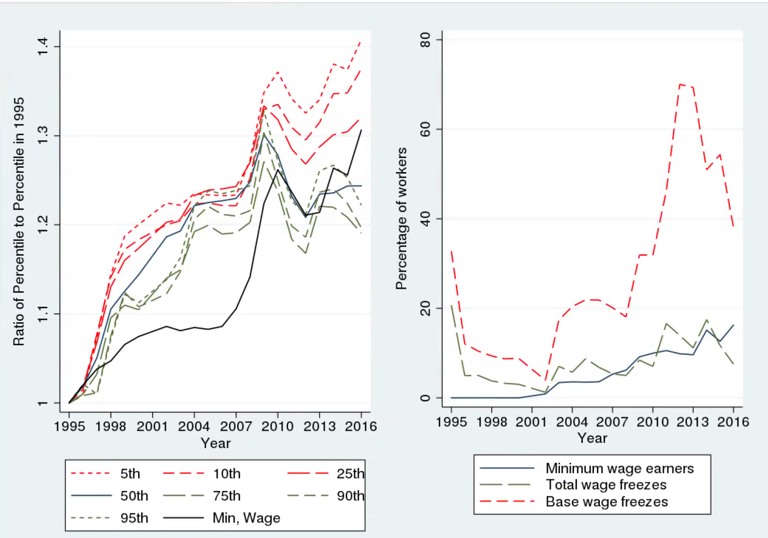
One of the advantages of studying Portugal in the last years resides precisely in the severe magnitude of the Great Recession shock witnessed by this continental European labour market. Moreover, the mishaps of the economic performance of the country, which records the last period of sizable growth ending by 2001, being followed by a long slump, two geminated crisis, and a modest recovery from 2014 onwards, presents an economy and a labour market in a severe frail standing (see figure 10 and Blanchard (2007) and Blanchard and Portugal (2017) for two analysis of the Portuguese economy over the timeline of our study). Altogether, it presents the framework of assessing a Southern European and French based labour market in distress.

Figure 10: The Labour Market Dynamics.

Panel A: The the real GDP growth and the unemployment rate.

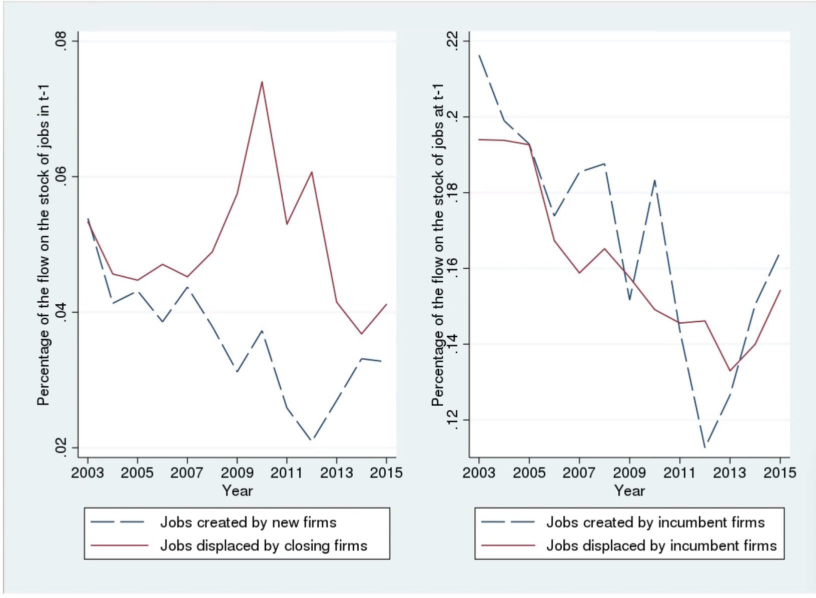


Panel B: The evolution of the wage distribution in the labour market.



Notes: In the graph the two vertical black dashed lines delimit the period of *Financial Assistance Program* with the ECB, the IMF and the European Commission, namely between 7th April 2011 and 30 June 2014. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016; Pordata website.

Figure 11: The evolution of the job flows in the labour market.



Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

Within these troubled times, the two crisis, most notably the European debt crisis, decisively stroke the Portuguese economy, sparking the need for a significant loan from the European Institutions and IMF to avoid an immediate sudden stop by 2011, backed by significant economic conditionality under a *Financial Assistance Program.* On the labour market side, while the collective bargaining apparatus kept its kinetics in the the wake of the financial crisis shock, potentially even witnessing a more wage prone policy, it came to a complete stall during the European debt crisis.[54]

Despite the regulatory stall, the labour market entropy was by 2013 evident, with the unemployment rate skyrocketing for record highs, in a phenomena coined by Carneiro et al. (2014) as *a catastrophic job destruction.* As noted in figure 11 and confirmed by Carneiro et al. (2014), the dramatic surge in unemployment was largely fuelled by very low job creation dynamics, and a stunning and unique surge of bankruptcies and firms exiting the market.

Simultaneously, as noted in figure 10, the wages endured the shock with same signs of downward nominal wage rigidity, evidenced by a sizable proportion of base wages, around 70 percent in our sample for 2013, frozen, while the distribution of real total wages shifted rightwards, in a more sizable drift in the higher percentiles of the distribution, thus evidencing a downward real adjustment capable of preserving, or even shrinking the pre-crisis wage inequality level.

[54] As presented in figure 10, the financial crisis coincided with the most sizable surge on the minimum wage in our timeframe, as a policy intended to boost internal demand in the wake of the financial crisis.

By the end of the adjustment program in 2014, and at the outbreak of the recovery path, the capability of the labour market to follow the recovery dynamics of the economy, and the timespan required for it to recoup pre-crisis levels, both on wages and unemployment was the quintessential question. Alongside, the labour attachment of the newly created employment, the potential regime shifts on the wage setting mechanisms, and the conceivable alterations on the mechanisms of labour market matching, capable of shifting the quality of newly created matches were pertaining queries.

Regarding the core question, figure 10 briefly provide the answer. In the first two years of recovery, the labour market exhibited a strong and steady recovery path, affirming its capability to absorb large chunks of unemployment in a relatively short period of time, while keeping moderate real wage evolution, particularly at the top of the distribution.

Precisely, regarding the wage dynamics, our paper provides answers for the latter two questions, pertaining the wage setting mechanisms and the assortativeness of the economy. Throughout the entire timeframe of analysis, and particularly during the European debt crisis, the resilience of the wage setting mechanisms and of the matching quality in the labour market is remarkable. For those incumbent workers that were spared from unemployment, a freeze of base wages, coupled with at most a 10 percent fall in real wages was momentarily imposed, but the underlying collective bargaining mechanisms of wage setting were not destroyed, thus hastening the wage recovery once the crisis fade out. Beyond significant doubts, those workers that were capable to sail out the crisis while employed faced a solid bridge over troubled waters, particularly at the bottom of the wage distribution. Their real wage losses broadly represented the depreciation of their real outside option value and the real value of quasi-rents. For those that faced the prospects of unemployment or tried to enter the tormented market, they encountered dire prospects of employability during the crisis, but witnessed a swift strong recovery, that in large proportions absorbed them into employment. In toto, the assortativeness of the market, if something, recorded a moderate decline and the bargaining powers kept their trends virtually unchanged.

7 Final Remarks

The analysis of the wage setting mechanisms in a typical Southern European and French based labour market is the focal point of this paper. For this purpose, we develop an empirically implementable microeconomic search and matching model with a collective bargaining apparatus, and we implement it in the Portuguese labour market using data from 1995 until 2016.

The proposed model has the convenience of discipline the use of data about the characteristics of the placement of the worker-firm match on the collective bargaining wage tables - the most perennial and comparable characterization of the labour relationships. Consequently, our empirical identification do not rely on the mobility of workers across firms, or on the definition and estimation of a production function or marginal product. Despite such flexibility, the framework provided allows for the estimation of bargaining powers, elasticities of quasi-rents, the passthrough rate of bargained wages, assortative matching and variance decomposition of wages in a fully unified framework.

The macroeconomic context of our analysis has been particularly turbulent, which underscores the displayed resilience of the wage setting mechanism, particularly at the bottom of the wage distributions, as a noteworthy feature of the market. For those that were capable to remain employed throughout the rough waters, the wage setting laid out a sound bridge, where the potential impact of the macroeconomic outlooks was circumscribed to temporary and moderate wage losses, particularly at the top of the wage distribution. Those reflected the temporary decline in the valuation of the real quasi rents of the worker-firm match and of the workerâs real outside options.

Finally, the structural slow paced continuous erosion of bargaining powers, for managers and skilled workers, highlights potential future productivity hazards, particularly if amplified by a significant progressive income taxation. In the absence of wage differentials across skill groups, and the increasing decoupling of wages from firm productivity levels, the sorting in the economy may be degraded, the incentives for skill acquisition and on job training may be abated, and the alignment of worker and firm incentives may be reduced. Any of these endanger future productivity levels.

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Appendices

A - The Dataset and Collective Bargaining Structure in Portugal

A1 - Quadros de Pessoal, Relatório Único, and Sistema de Contas Integradas das Empresas (SCIE).

The data sources of this study comprises information about the balance sheet and income statement of the firms obtained from SCIE - *Sistema de Contas Integradas das Empresas*, for the years 2005-2016, and information about the characteristics of the workers and of their employment relationship, provided in *Quadros de Pessoal* (Personnel Tables) for the years 1986-2009, and *Relatório Único* (Single Report), for the years 2010-2016.[55]

The three datasets are fully matched for the common years of the data. Consequently, for each year from 1986 to 2016, we have a longitudinal matched employee-employer-contract-job-title database, which from 2005 is supplemented with matched information at firm level about the balance sheet and income statement of those firms.

The *Quadros de Pessoal* and the *Relatório Único* datasets are recorded by the Ministry of Employment and Social Security and correspond to a mandatory survey on an annual basis for all establishments with at least one wage earner. In this survey all workers employed in October of each year are reported, although civil servants and workers in domestic service are not covered. Therefore, the dataset covers the entire population of workers of private-sector firms in manufacturing and services. Further, the long-lived requirement of the information to be published at establishment level, ordinarily at the door of the establishment ensures greater validity.

The dataset reports the firm's location, industry, employment, sales, ownership, and legal basis. Worker information includes gender, age, skill, occupation, schooling completed, starting date at the firm, earnings, and working hours. In addition, the survey also records the collective bargaining arrangement and the specific job-title held by the worker under collective agreement, which is of particular importance for this study.

In these datasets the following restrictions were applied: (a) we only consider full-time employers in receipt of what is contractually defined for the reporting month; (b) we exclude workers from agriculture, fisheries, and energy products/extraction sectors; (c) we exclude workers aged less than 18 years or greater than 65 years; and (d) we exclude workers earning less than 80 percent of the minimum wage[56]. A significant array of descriptive statistics on this matched dataset is provided in table A1 and A2.

[55]For the year 1990 and 2001, the survey was either not administered, or not digitized.

[56]Corresponding to the lowest admissible wage in the case of apprendiceships.

As a matter of empirical implementation, we do not have the wage floor for each job-title, and we just have the identification of each category for each worker. However, we are aware that each job-title is subjected to a minimum base wage. Therefore we estimate the minimum base wage for each job-title as the minimum hourly wage reported for the workers working with that category in the entire economy. Implicitly, we are assuming that the minimum base wage for each job-title is at least binding for one worker.

Then we combine the established matched dataset with the SCIE dataset, with a coverage of the match above 97 percent. The SCIE dataset is managed by National Statistics Institute, which provides a unified survey system. Its reporting is mandatory for the universe of registered firms operating in Portugal, including those with no employees. This dataset has a vast array of accounting information, namely with detailed information about every entry of the balance sheet and income statement legally required for accounting purposes under the *SNC - Sistema Normalização de Contas*. Among the information provided, one has access to the level of assets, liabilities and equity, and its typical accounting partitions, as well as profits, output value, value-added, payroll, purchase of intermediate goods, investment levels, service of debt among others.

A2 - Example of a Wage Table of a Collective Agreement of the Portuguese Labour Market

In the Portuguese Labour Market, each collective agreement and its wage table is published in the *Boletim do Trabalho e Emprego* (i.g. Work and Employment Bulletin). As a matter of example, table A3 presents one of the wage tables existing in 2017. It corresponds to a CCT, or sector agreement, between *AHRESP* (i.g. the association of employers of hospitality and similar), and *SITESE* (i.g. the union of workers and technicians of services, commerce and hospitality), signed in April 22, 2017.[57]

Interestingly, and common to the vast majority of the agreements in place, the presented agreement is incredibly detailed with more than 60 clauses and a vast number of appendices. For example, there are specific rules for: (a) working conditions; (b) minimum payments for meal subsidy among other side payments; (c) specific rules that even includes clothing; and (d) the definition of minimum ranks based on observables, for example education and tenure.

Our claim in the text is that each of these agreements correspond to a branch, each ranking the workers, in this case with 13 ranks, and classifying the workers, within the ranks, according to their job titles. These classifications often exist in more than one

[57]The translations presented are freely derived by the authors, and thought as faithful to the original meaning in Portuguese. This corresponds to a revision of a previous sector agreement between the parties as it is often the case in these agreements.

agreement. It is noteworthy the sector nature of these agreements. While it may be perceived as a breach in the ranking, in our interpretation it provides adjustment of the outside options to sector preferences and sector labour market outcomes governing the outside option. Such argument is reinforced by the proper structure of unions, which are aggregated in two major confederations (i.e. UGT and CGTP) - clearly a centralized system.

Our interpretation is reinforced by the idea that those wages are pledged and legally binding for the future, and tie the firm to pay those values given the worker's rank, whom cannot be demoted. Simultaneously, the firm and the union may still at firm level bargain, with the latter pushing for better working conditions and compensation, which for the former will be more flexible to withdraw in the presence of adverse economic conditions. It is this reasoning that lead us to consider that those bargained wages are indeed the level of the immediate outside option of the worker.

Table A1: Descriptive Statistics on the Dataset Dimmensions

Variables:	Percentiles			Mean	Standard	Skweness	Kurtosis	Total
	5	50	95		Deviation			Number
Observations	-	-	-	-	-	-	-	29,115,656
Per Year	1,305,735	1,391,557	1,474,118	1,386,460	-	-	-	-
Workers characteristics								
Age (years)	22	38	57	38.46	10.87	0.26	2.23	-
Education (years)	4	9	16	8.56	3.97	0.38	2.01	-
Tenure in the Firm (years)	0	6	23	8.7	9.09	1.26	4	-
Female (perc./year)	39.91	41.89	46.04	42.4	0.03	0.25	1.73	-
Mover to the Firm (perc./year)	15.96	22.77	27.08	22.81	4.89	1.20	5.96	-
Duration of Spells (years)								
Workers	1	4	14	6.04	5.29	1.03	3.07	4,816,652
Worker-Firm	1	2	10	3.92	4.09	1.86	6.16	7,436,538
Worker-Task Market	1	2	10	3.88	3.91	1.80	6.01	7,513,257
Worker-Rank	1	1	3	1.27	0.68	2.14	5.59	22,946,436
Firms								
Workforce Size	6	13	55	33.84	187.13	53.28	4,488.52	1,139,819
Workplace Size	1	3	15	8.9	58.36	70.69	7,894.81	4,309,996
Collective agreements								
Agreements-Year	477	536	764	574	108	1.02	2.46	-
Task Markets-Year	3	7	8	6.11	2.12	-0.93	2.70	-
Ranks in Agreement-Year	2	25	113	48.43	75.59	5.07	47.31	-
Firms-Year	1	6	164	82.55	350.89	12.54	224.98	-
Workers-Year	9	260	4,599	2,455.2	9,073.7	8.34	93.09	-
Years	1	5	21	8.47	7.45	1.26	3.64	-

Notes: There are 21 years in the dataset. The year 2001 is missing from the dataset as it was not recorded. *Source: Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2016.*

Table A2: Descriptive Statistics on Worker's Wages

Variables:	Percentiles					Mean	Standard	Skweness	Kurtosis
	5	25	50	75	95		Deviation		
Log of Nominal Monthly Wages									
Total Wages	5.30	6.07	6.48	6.93	7.76	6.51	0.72	0.29	3.63
Base Wages	5.20	5.92	6.26	6.63	7.44	6.29	0.65	0.41	3.99
Bargained Wages	4.78	5.52	5.92	6.18	6.71	5.85	0.56	-0.02	3.99
Wage cushion $\left(\frac{w_{total}}{w_{base\|barg.}}\right)$	1	1.20	1.42	1.85	3.49	1.77	1.61	57.34	11,407.95
Base Wage Ratio $\left(\frac{w_{base}}{w_{total}}\right)$	0.44	0.72	0.85	0.95	1	0.81	0.19	-1.98	36.16
Bargained Wage Earners (%)	0.32	0.58	0.64	0.86	0.98	0.68	0.23	-0.20	2.05
Hours Worked	152	168	173	194	193	171.8	11.62	-0.25	4.32

Source: Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2016.

Table A3: Representative Wage Table, with Bargained Wages.

Rank	Job Description	Minimum Base Wage
13	General Manager	1,515
12	Board Assistant; Commercial Manager; Service Manager; Human Resources Manager; Technical Manager	1,240
11	Head of Department; Head of Division; Head of Services; Nutrition Technician 1st Class	1,018
10	Head of section (office); Head of Sales; Inspector; Board Secretariat officer; Nutrition Technician 2nd Class	898
9	Administrative; Head of Cafeteria; Head of Purchases; Head of Kitchen; Head of Pastries; Head of Storage; Head of Dinning Room; Inspector of Sales	808
8	Cashier; Head of Preparation Room; Controller; Cook of 1st Class; Sub-Head of Dinning Room; Administrative Assistant; Pastry Cook; Sales Technician	771
7	Driver of Heavy Vehicles; Storage Keeper; Polyvalent Worker	716
6	Driver of Non-heavy Vehicles; Administrative Assistant 2nd Class; Pastry Cook 2nd Class; Sub-Head of Dinning Room 2nd Class; Sales Representative	700
5	Cook 2nd Class; Controller of Balcony; Controller of Bar; Controller of Storage and packing; Admin. Assistant 3rd Class	629
4	Head of Copa; Cook of 3rd Class; Packing worker; Storage Worker	582.50
3	Controller cashier; Storage worker; Bar Worker; Balcony Worker 1st Class; Distribution Handler;	570
2	Balcony Worker 2nd Class; Admin. Intern; Hospitality Assist.	562
1	Driver Assistant; Distribution Assistant; Barman Intern (1 year); Cook Intern (1 year); Pastry Intern (1 year); Cleaning Worker; Dining room Employer	557

Notes: Amounts in euros, and corresponding to monthly payment. Table extracted from the Sector agreement signed on the 22^{nd} of April 2017, between *AHRESP* (e.g. the association of employers of hospitality and similar) and *SITESE* (e.g. the union of workers and technicians of services, commerce and hospitality). *Source: Boletim do Trabalho e Emprego, 2017.*

B - Technical Details on the Model and Links with the Literature

In this appendix we present: (i) the relationship of the model with the search and matching literature and the union-firm bargaining literature, in part B1; (ii) the derivation of additional relevant conditions to fully describe both the dynamic and the steady state equilibria, in part B2; (iii) the derivation of equilibrium wages for both types of equilibria, in part B3; and (iv) the description of the dynamic and steady state equilibria and their properties, concretely regarding existence and uniqueness, in part B4.

B1 - The Links between the Model and the Search and Matching Literature and the Firm-Union Bargaining Literature

Bridging the Model of Labour Tasks and some Canonicals of the Search and Matching Literature. The apparatus of our model was constructed so it can be easily reshaped as several canonical models in the literature of search, mostly resorting to: (b) a wise choice of parameters, and (c) variations of the modelling of the choice of capital.

Firstly, the mutation of our model into the model of Acemoglu and Hawkins (2014) critically requires that: (a) we set $J = 1$, as they abstain from modelling different types of tasks; (b) $\xi^{\Xi}(a|K, \beta) = 0, \forall\{a, K, \beta\}$, as they don't consider a model with on job search; (c) $\{K\}$ to be relabelled as z their idiosyncratic firm's productivity parameter; (d) $\beta = \bar{\beta}$ so there is homogeneous bargaining power in the economy; (e) $a = \bar{a}$ and known by workers and firms; and (f) $\tilde{s}(a|K, \beta) = 0$ as there is no firing at-will.

Further, in the model of Acemoglu and Hawkins (2014) consider $\gamma(V)$ to be linear and not strictly convex as presented. The fundamental implication of this deviation is that instead of having a growth path of each firm, which is pivotal in the author's analysis, we assume a firm can immediately attain its optimal scale without incurring in further costs due to simultaneous hiring.[58] Closely related with this deviation, we do not model entry and exit of firms, and thus in their model consider $FC_t = \infty$ - no entry of new firms, and $\delta = 0$ - no exit of firms.[59]

Secondly, one can also adapt this model to resemble Cahuc et al. (2008), which models a representative firm. For this purpose, consider: (a) $\xi^{\Xi}(a|K, \beta) = 0, \forall\{a, K, \beta\}$, so there is no on job search; (c) $A(j, a) = 0, \forall j, a$ so that operating costs are fully

[58] The extension to allow the incorporation of their class of vacancy costs increases the complexity of the model by some degrees, given one is required to keep track of the history of firms.

[59] This deviation allows for notation simplicity, and given the intention to focus on optimal scale, comes without further implication.

neglectful; (d) there is no firing at will so $\tilde{s}(a|K,\beta) = 0, \forall\{a, K, \beta\}$; and either (e1) $\{K, \beta\} = \{k, \bar{\beta}\}, \forall\{K, \beta\}$, where k is a given constant, and β is homogeneous across firms, but potentially different across task markets, representing a $J \times 1$ vector. Therefore, firms do not have any heterogeneity arising from capital or bargaining powers, and we follow the most restrictive version of their model, without capital; or (e2) consider that capital is also chosen optimally *ex-ante* to the task decisions and thus add the following condition to our equilibrium:[60]

$$\frac{\partial F(\mathbf{N}, K)}{\partial K} = r + d + \int_0^1 \sum_{j=1}^{J} N_j \pi^{\frac{1-\beta_j}{\beta_j}} \frac{\partial^2 \tilde{F}(\mathbf{N}\mathbf{A_j}(\pi), K)}{\partial N_j \partial K} d\pi, \tag{1}$$

where d is the depreciation rate, which we have abstracted in our model formulation, and $\mathbf{A_j}(\pi)$ is identical to equation (41) of appendix B2.

Thirdly, resorting to the insights of Cahuc and Wasmer (2001), the model can also be translated to the large firm version of the matching model of Pissarides (1990). For that purpose, we have: either (a1) $J = 1$ so that there is only one type of tasks; or (a2) $J > 1$, but the types of tasks are perfect substitutes; (c) $\xi^{\Xi}(a|K, \beta) = 0, \forall\{a, K, \beta\}$, so no on job search; (d) $a = \bar{a}, \forall j, a$, so that the worker heterogeneity is fully neglectful for equilibrium purposes; and (e) perfect capital markets, and simultaneous decision of labour and capital so that the capital stock of the firm becomes a function of employment ($K_t(\mathbf{N})$), and the following condition hold:[61]

$$\frac{\partial F(\mathbf{\Phi}_t, K_t)}{\partial K_t} = r + d. \tag{2}$$

Fourthly, the model would mimic Mortensen (2010) if we consider: (a) there is not entry or exit of firms (i.e. $\delta = 0$ and $FC_f = \infty$); (b) $J = 1$ so that there is only one type of tasks; (c) $a = \bar{a}, \forall j, a$ so that operating costs are fully neglectful; (d) K is constant overtime, and represents the idiosyncratic productivity of the firms, presented in the paper as $p(x)$, and (e) $\beta = \bar{\beta}$ is constant for every firm and in every task market.

In a nutshell, our model is isomorphic to a wide range of standard search and matching models. Mainly that is attainable by: (a) sufficiently restricting the parameterization of the model (i.g. heterogeneity); (b) considering alterations of the capital allocation mechanism, whose implications for our modelling objectives are fairly minor; and (c) considering the dynamics of vacancy costs and entry and exit of firms.

[60]See Cahuc et al. (2008) to the details on how to obtain this expression from our model under this set of assumptions. Further notice that if one considers optimal choice of capital, *ex-ante* to task allocation, and fully neglects the firm heterogeneity arising from bargaining powers, then we are in an environment of a representative firm.

[61]See Cahuc and Wasmer (2001) for the specific details about this equivalence.

Finally, Krause and Lubik (2007) provides relevant insight about the macroeconomic implications for the empirical fitting of a search and matching model with intra-firm bargaining in the spirit of Stole and Zwiebel (1996*a*). Firstly, intra-firm bargaining causes firms to expand employment in order to bolster their bargaining position relatively to workers, as analyzed by Cahuc et al. (2008). Theoretically, such movement causes not only an expansion in employment, but also an increase in wages, due to the lower unemployment and higher vacancies posted, which raises outside option values in a general equilibrium framework. While this could be perceived as an important effect, whose implications are worth to study, Krause and Lubik (2007) provides evidence of a meaningless effect in the fitting of a proper macroeconomic model. Our choice of modelling is thus rooted not by claims it increases the macroeconomic fitting, but by the microeconomic appeal it has in fitting with the workings of the market.

The Relationship of the model with some Canonical Firm-Union Bargaining Models. Particularly since the 1970's and 1980's, there was a growing interest in developing and assessing different approaches to incorporate trade unions in regular labour economics models in general, and in the search and matching context in particular.[62]

From this debate four stylized approaches have settled, concretely: (i) the monopoly model of Dunlop (1944); (ii) the right-to-manage model of Nickell and Andrews (1983); (iii) the efficient bargaining model of McDonald and Solow (1981); and (iv) the sequential models of wage-employment bargaining of Manning (1987). The fundamental theoretic divergences among those classes of models lie on the structure of the bargaining and the *bargainable* variables, rather than the profit maximization objective of the firm, and the members' welfare maximization of unions. Those objective functions are presented in equations 15 and 14 in section 3.

While there is a *consensus* on the objectives of the parties, the notion of union's members has been debated. Some approaches consider as union members the employed workforce, while others consider the entire available labour force as *potential* union members.

Regarding the structure of bargaining, each of the four presented categories of models have different implications, which however can be compared. The monopoly model considers that the union maximizes its welfare by choosing the wage, and firms

[62] See Johnson (1975), Parsley (1980), Oswald (1985) and Manning (1992) for comprehensive surveys of the literature of theory models of union and firm wage and employment setting. Further, see Pissarides (1986) which corresponds to the first paper to introduce a union in a search and matching model).

are left to define the employment. Consequently we would have:

$$\begin{aligned} \max_{w(a|K,\beta)} \quad & W_t \\ \text{s. to} \quad & J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)} \end{aligned} \tag{3}$$

assuming an interior solution in equilibrium.

The right to manage model is a generalization of the monopoly model, where instead of unions defining wages, union and the firm bargain over the wage, resorting to an axiomatic generalized bargaining as in Nash (1950), or a game theoretic approach as in Rubinstein (1982) and Binmore et al. (1986). In our model that would translate into:

$$\begin{aligned} \max_{w(a|K,\beta)} \quad & \beta ln\Big(W_t - \bar{W}_t\Big) + (1-\beta)\Big(\Pi(K,\beta) - \bar{\Pi}\Big) \\ \text{s. to} \quad & J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}, \end{aligned} \tag{4}$$

where $\bar{W}_t$ and $\bar{\Pi}$ would correspond to union's members welfare and profits in the case of a full bargaining breakdown, respectively.

In a different paradigm, in the efficient bargain model, unions and firms bargain resorts to the propositions of either Nash (1950) or Rubinstein (1982) and Binmore et al. (1986), but the parties bargain simultaneously about wages and employment. Consequently, we have:

$$\max_{w(a|K,\beta),\mathbf{N}(\mathbf{K},\beta)} \quad \beta ln\Big(W_t - \bar{W}_t\Big) + (1-\beta)\Big(\Pi(K,\beta) - \bar{\Pi}\Big). \tag{5}$$

While apparently different, the work of Manning (1987) bridges the previously presented models, by considering a sequential bargaining, where first unions and firms bargain over wages, and subsequently they bargain over employment. Thus solving by backwards, we have that the second stage corresponds to:

$$\max_{\mathbf{N}(\mathbf{K},\beta)} \quad \beta ln\Big(W_t - \bar{W}_t\Big) + (1-\beta)\Big(\Pi(K,\beta) - \bar{\Pi}\Big). \tag{6}$$

and the first stage to:

$$\max_{w(a|K,\beta)} \quad \tilde{\beta} ln\Big(W_t(\mathbf{N}^\star(\mathbf{K},\beta)) - \bar{W}_t\Big) + (1-\tilde{\beta})\Big(\Pi(K,\beta,\mathbf{N}^\star(\mathbf{K},\beta)) - \bar{\Pi}\Big). \tag{7}$$

Noteworthy, this model boils into: (i) the monopoly model if $\tilde{\beta} = 1$ and $\beta = 0$; (ii) the right to manage model if $\beta = 0$; and (iii) the efficient bargain model if $\tilde{\beta} = \beta$. Moreover, Manning (1987) shows that if employment is decided before wages the sequential model always yield the efficient bargain model solution, regardless of the relative strength of bargaining powers. Altogether, in a the canonical firm-union environment the models' equilibria can be graphically compared, as in figure B1.

The way the representative union is integrated in our model relates to this debate, subjected to the remainder assumption framework of the model. Firstly, given in continental Europe collective bargaining coverage is extremely high, independently of membership, we adopt a fully representative union, which therefore considers the welfare of the entire available labour force, as presented in equation 14. Secondly, we consider the insight of the right-to-manage and monopoly models by considering the union bargains over wages alone. Thirdly, in the dynamic formulation of our model, we adopt a sequential modelling where firms set employment and then firms and the union bargain over wages alone.

Finally, we supplement our model with another instrumental assumption. Most of the models in the literature assume that if there is a bargaining breakdown the entire workforce is displaced at least temporarily. In our setting, we adopt a gradual collective bargaining setting as in Dobbelaere and Luttens (2016), where the union bargains sequentially in the name of each worker, and does not impose a full lockdown in case of bargaining reaching a standstill. Rather, in the specific contract that originated the stalemate, a worker leaves the firm without being entitled to a severance payment. Then, the parties restart the bargain again, for every contract. This process unfolds until the deadlock may be surpassed.

Altogether, our *kind* of union will determine that the outcome of the bargaining corresponds to the competitive environment, as in other search and matching models with individual intra-firm bargaining, whose equilibrium conditions perfectly match the ones of our model (for example Cahuc et al. (2008) and Acemoglu and Hawkins (2014)). In a more stylized model would therefore be identifiable as point C in figure B1.

Alternatively, one could consider the proposition of Bauer and Lingens (2010), where in a setting close to ours the authors consider a union with power to impose lockdowns, and firms with the ability to displace the entire workforce. The major change actually comprises equation (23) of the model. Instead of bargaining sequentially for each worker, the union bargains for the entire mass of workers simultaneously. Thus, we would have:

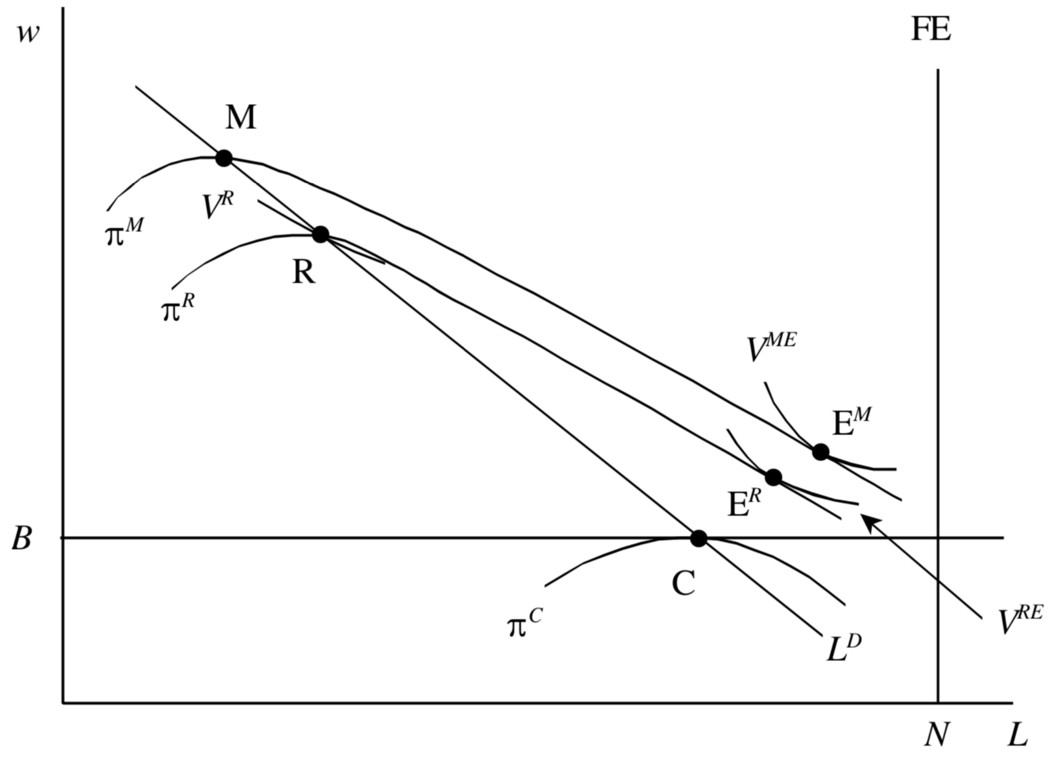
$$w_{j,t}(a|K,\beta) = \\ argmax_w \Big\{ \sum^{J} \int_{\beta} \int_{K} \int_{a} [\Xi_{j,t}(a|K,\beta) - Out(a)] dG_j(a|K,\beta) dK d\beta da \Big\}^{\beta} \Big\{ \Pi(K,\beta) \Big\}^{1-\beta} \tag{8}$$

s. to: same constraints.

In this case we would end up with a solution somewhere in the locus of points $[C, M]$.

Our modelling choice relies on the lack of empirical evidence that a bargaining breakdown leads to either: (i) the dismissal of the entire workforce of a firm, or (ii) a prolonged strike that locks down the firm. Regarding the former, the only known case of an entire dismissal of workers following the breakdown of negotiations happened in the *Ronald Reagan and the air traffic controllers case* in 1981, which as presented by Dobbelaere and Luttens (2016) was predominantly political. Regarding the latter, Boeri and van Ours (2013) presents evidence that strikes are relatively rare, and prolonged strikes even more in Continental European labour markets. For the years 2000-2003, Spain had the most lost working days to strikes among continental European countries, representing 0.6 working days per year and worker. For the same sample, the average duration of strikes was at most 5.1 days, recorded for Ireland and Poland.

Figure B1: Graphical Analysis of the Union's Models Solutions.



Notes: The M point represents the Monopoly model solution, and C point represents the competitive solution, where in the canonical environment $\beta = 0$. The right to manage model solution takes place anywhere on the demand curve (L^D), between points M and C, as it entails bargaining with an intermediate bargaining power of unions, compared to the presented extremes. Then point E^R represents the efficient bargain solution, when β would be compatible with a right-to-manage solution in point R.

B2 - Complementary Definitions on Job-Flows.

Firm's Hiring Policy. The profit of the firm with fundamentals $\{K, \beta\}$ is assumed to be strictly concave and twice continuously differentiable in employment. It is given, as in equation (15) of the text, by:

$$
\begin{aligned}
r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} &= \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J}\int_a w_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Wage Bill}} \\
&- \underbrace{\sum_{j=1}^{J}\int_a A(j,a)dG_j(a|K,\beta)da}_{\text{Operating Cost Bill}} - \underbrace{\sum_{j=1}^{J}\int_a s_j(a|K,\beta)J_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Cost of Displacement}} \\
&- \underbrace{\sum_{j=1}^{J}\int_a \tilde{s}_j(a|K,\beta)SdG_j(a|K,\beta)da}_{\text{Exp. firing tax}} - \underbrace{I(K)}_{\text{Sunk Cost}} + \\
&+ \underbrace{\sum_{j=1}^{J}\max_{V_j(K,\beta)}\Big\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta)q(\theta_j)J_j^R(K,\beta)\Big\}}_{\text{Value of the Hiring Policy}}.
\end{aligned} \tag{9}
$$

Solving for the optimal vacancy policy, we obtain first order conditions as:[63]

$$
J_j^R(K,\beta) = \begin{cases} \frac{\gamma_j}{q(\theta_j)} & \text{if } V_j(K,\beta) > 0 \\ \\ \left[\underline{J}_j^R(K,\beta); \frac{\gamma_j}{q(\theta_j)}\right] & \text{if } V_j(K,\beta) = 0 \end{cases}. \tag{10}
$$

Firm's Firing Policy. Apart from the hiring policy, firms also define their firing policy. Firms, may fire at-will any worker, as long as they pay a corresponding exogenous

[63] The considered corner solution exists due to the impossibility of costless firing at will, as motivated in Bertola and Caballero (1994). Further we assume $\underline{J}_j^R(K,\beta) > -\infty$, so it is bounded from below.

firing tax, given as S. Therefore, the decision of fire at-will is given by:[64]

$$\tilde{s}_j(a|K,\beta) = argmax_{\tilde{s}_j(a|K,\beta)\in\{0,1\}} \sum_{j=1}^{J} \int_a \Bigg[\underbrace{\Big(1-\tilde{s}_j(a|K,\beta)\Big)J(a,K,\beta)}_{\text{Value of not firing}} +$$

$$\underbrace{+\,\tilde{s}_j(a|K,\beta)\Big(-J_j(a,K,\beta) - S + \underbrace{\max\Big\{-e^{-r\Delta t}\gamma + e^{-r\Delta t}q(\theta_j)J_j^R(K,\beta);0\Big\}}_{\text{Optimally decide if replace or not the fired worker}}\Big)}_{\text{Value of Firing the worker}}\Bigg]\times \quad (11)$$

$$\times\, dG_j(a|K,\beta)da,$$

where the firm balances the option of keeping the worker, against the options to firing him, and subsequently either replace or not replace him in the following period. Notice the firm considers a consistent policy for its entire workforce due to the linkages between the marginal values of the jobs across workers. The maximand function $\tilde{s}_j(a|K,\beta)$ corresponds to a threshold function as:

$$\tilde{s}_j(a|K,\beta) = \begin{cases} 1 & \text{if } a \leq a_j(K,\beta) \\ 0 & \text{if } a > a_j(K,\beta) \end{cases}. \tag{12}$$

Beyond a potential bargaining breakdown with the union, and its decision of firing a given worker, the firm is also subjected to displacement due to an exogenous shock, for example due to the death of a worker, which happens with probability $\bar{s}$, and a successful outcome of the on-job-search of its employed worker, given by the function $m_j(a|K,\beta)$. The displacement rate function is given by:

$$s_j(a|K,\beta) = \begin{cases} 1 & \text{if } a \leq a_j(K,\beta) \\ \bar{s} + m_j(a|K,\beta) & \text{if } a > a_j(K,\beta). \end{cases} \tag{13}$$

Altogether, we have defined the behaviour of firm's (K,β) type in managing its workforce.

Job Search. Synchronously, both workers and the unemployed have the option to search for jobs. For that purpose, when they decide to search, they do so in the task market which yields the most expected return, and in searching they incur in a cost given by $c_j(a)$, dependent on the task market they intend to search for, and their type

[64] The operator Δt represents a time lag. $-e^{-r\Delta t}$ represents a discount factor, where (1) $\lim_{\Delta t\to 0^+} -e^{-r\Delta t} = 1$, and (2) $\lim_{\Delta t\to\infty} -e^{-r\Delta t} = 0$.

a. For an employed worker, he solves his search problem as:

$$\xi_j^{\Xi}(a|K,\beta) = argmax_{\xi_l^{\Xi}\in\{0,1\},\sum_{l=1}^{J}=\{0,1\}} \sum_{l=1}^{J}\Bigg[\xi_l^{\Xi}(a|K,\beta)\Bigg\{\theta_l q(\theta_l)\times \times\left(\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|x,y)>\Xi_j(a|K,\beta)]\Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - \Xi_j(a|K,\beta)\right) - c_l(a)\Bigg\}\Bigg], \tag{14}$$

where $\mathbf{1}[\Xi_l(a|x,y)>\Xi_j(a|K,\beta)]$ is an indicator function being 1 if the potential offer $\Xi_l(a|x,y)$ provides an higher value than $\Xi_j(a|K,\beta)$.

The unemployed solves a similar problem as:

$$\xi^u(a) = argmax_{\xi_l^{u}\in\{0,1\},\sum_{l=1}^{J}=1} \sum_{l=1}^{J}\Bigg[\xi_l^{o}(a)\Bigg\{\theta_l q(\theta_l)\times \times\left(\frac{\int_K\int_\beta \Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - Out(a)\right) - c_l(a)\Bigg\}\Bigg],, \tag{15}$$

where $Out(a)$ is the unemployed value, given his type, and ξ are indicator functions, displaying the selected task market of the worker, given his context. We assume that: (i) $\theta_j q(\theta_j)\int_K\int_\beta \Xi_j(a|x,y)V_l(x,y)d\Gamma(x,y) - Out(a) - c_j(a) > 0$ for at least one task market j, so that the unemployed always search in some market; (ii) $\Xi_j(a|x,y) > Out(a)\ \forall\{x,y\}$, so that after they decide to search and after paying the search cost, which becomes sunk, an unemployed worker will always prefer to work;[65] (iii) $c_j(a)$ is differentiable, convex, strictly decreasing in a, holds $\lim_{x\to 0} c_j(x) = \bar{C} > 0$ and $\lim_{x\to\infty} c_j(x) = 0$; and (iv) $c_j(a) > c_l(a)\ \forall l > j$. Altogether, the optimal search behaviour for the unemployed follows a system of threshold rules. Thus, the vector of search choices becomes:

$$\xi^u(a) = \Bigg[1(l=1, a>a_1), \ldots, 1(l=j, a_{l-1}>a>a_l), \ldots, 1(j=J, a<a_J)\Bigg] \tag{17}$$

implying that unemployed perfectly segment across the task markets accordingly to their type. Furthermore, given the search behaviour of an unemployed, we have that:

$$\xi_j^{\Xi}(a|K,\beta) \in \{\mathbf{0}, \xi^u(a)\}, \tag{18}$$

so that an employed worker if he searches, he does so in the task market he would search if he was unemployed. Concretely, given $\Xi_l(a|K,\beta) > Out(a), \forall l \in \{0,\ldots,J\}$,

[65] Further, we also assume that:

$$\lim_{N_j\to\infty} \frac{\partial F(\mathbf{N},K)}{\partial N_j} < b, \tag{16}$$

where b stands for the unemployment benefit, so that eventually a firm shall not grow indefinitely.

the worker may eventually decide not to search, when he would do so if unemployed. Therefore, the worker's type, a, is a sufficient statistic of task market choice in on-job search, conditional on searching.

As a reference, this behaviour of the agents regarding employment flows and selection reproduces the behaviour of the classical selection model of Roy (1951).[66]

Probability of successful on-job-search. Following this structure, the probability of a success on-job-search for a worker with match fundamentals (a, K, β) in task market j is given by:

$$m_j(a|K,\beta) = \sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta)\, \underbrace{\theta_l q(\theta_l) \Big[1 - D_l(\Xi_j(a|K,\beta))\Big]}_{\text{Prob. the worker accepts a job in } j} \tag{19}$$

where $D_l(w(a|K,\beta))$ is the distribution of wage offers in task market j.[67] The expectation of the marginal profit of a new hire to be given by:

$$\begin{aligned} J_j^R(K,\beta) = & \underbrace{\frac{1}{u_j}\int_a \xi_j^u(a) J(a|K,\beta) dU(a)}_{\substack{\text{Expected value of a job} \\ \text{when unemployed is hired}}} + \\ & + \underbrace{\int_a \frac{\sum_{l=1}^{J}\int_K\int_\beta \xi_j^{\Xi}(a|x,y) 1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big) J_j(a|K,\beta) dG_l(a|x,y) d\Gamma(x,y)}{\sum_{l=1}^{J}\int_K\int_\beta \xi_j^{\Xi}(a|x,y) dG_l(a|x,y) d\Gamma(x,y)} da}_{\text{Expected value of a job when a employed searching at task } j \text{ is hired}}. \end{aligned} \tag{20}$$

Market tightness and vacancy yield. Given the search behaviour of workers and unemployed, the labour market tightness of task market j is also given by:

$$\theta_j = \frac{\int_K \int_\beta V_j(K,\beta) d\Gamma(K,\beta)}{\int_a \xi_j^u(a) dU(a) da + \sum_{l=1}^{J}\int_a\int_K\int_\beta \xi_j^{\Xi}(a|K,\beta) dG_l(a|K,\beta) d\Gamma(K,\beta) da}, \tag{21}$$

where: (i) $e_j = \sum_{l=1}^{J}\int_a\int_K\int_\beta \xi_j^{\Xi}(a|K,\beta) dG_l(a|K,\beta) d\Gamma(K,\beta) da$ represents the number of workers performing on-job-search in task market j; (ii) $u_j = \int_a \xi_j^u(a) dU(a) da$ gives

[66] Notice that in this framework, theoretically our results will not be plagued by endogenous mobility conditional on the described behaviour. A different outcome would potentially be achieved if for instance one would consider a generalized Roy model (see Heckman and Vytlacil (2007) for an example). For modelling ease we do not consider such generalization, as we do not empirically explore the potential selection model that would emerge from this behaviour.

[67] The presented distribution is given by:

$$D_j(\Xi_j(a|K,\beta)) = \frac{\int_K\int_\beta V_l(x,y) 1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big) d\Gamma(x,y)}{\int_K\int_\beta V_l(K,\beta) d\Gamma(x,y)}.$$

the number of unemployed searching for a job in the task market j; and (iii) the number of vacancies are obtained as $\int_K \int_\beta V_j(K,\beta)d\Gamma(K,\beta)$.

Given this market structure, the probability of a firm of type $\{K,\beta\}$ to find a worker of type a in task market j corresponds to:

$$y_j(a|K,\beta) = q(\theta_j)\frac{\frac{\partial u_j}{\partial a} + \frac{\partial e_j}{\partial a}X_j^-(\Xi_j(a|K,\beta)}{\frac{\partial uj}{\partial a} + \frac{\partial e_j}{\partial a}}, \tag{22}$$

where $X_j^-(w(a|K,\beta)) = lim_{x\uparrow w(a|K,\beta)}X_j(x)$ is the distribution of wages that employed workers which are searching in task market j are receiving. Notice, as typical in these type of models we assume workers do not move to a worse paying match.[68] Accordingly, the vacancy yield of a firm of type (K,β), i.e. the probability of firm of type $\{K,\beta\}$ to hire a worker, is given by:

$$y_j(K,\beta) = \int_a y_j(a|K,\beta)da. \tag{23}$$

Evolution of workforce composition. Finally, the expected evolution of the workforce composition of a firm of type (K,β) is then given by:[69]

$$\begin{aligned}\frac{\partial dG_{j,t}(a|K,\beta)}{\partial t} = & \underbrace{-dG_{j,t-\epsilon}(a|K,\beta)}_{\text{Workers of type } a \text{ at period } t-\epsilon} + \underbrace{y_{j,t}(a|K,\beta)V_{j,t}(K,\beta)}_{\text{Prob. hiring worker of type } a} \\ & + \underbrace{\int_{a''}\left(1 - s_{j,t-\epsilon}(a''|K,\beta)\right)\psi(a|a'')dG_{j,t-\epsilon}(a''|K,\beta)da''}_{\text{Prob. that incumbent workforce at firm develops into workforce of type } a},\end{aligned} \tag{24}$$

where: (i) the probability that a worker to keep his operational cost fixed from a period to another is zero so the workforce in a at period $t-\epsilon$ will not be in a at period t; (ii) $y_{j,t}(a|K,\beta)V_{j,t}(K,\beta)$ represents the probability of hiring a worker of precisely operating cost a per vacancy posted (i.e. $V_j(K,\beta)$; and (iii) the third term consider, from the workers that have not left the firm of type $\{K,\beta\}$, those whose skill acquisition process leave them precisely at operating cost level a, where $\psi(a|a'')$ is the probability distribution function of the random component of the skill acquisition process, from previous period a'' to current period a.

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[68] The distribution presented is given by: $X_j(w) = \frac{\sum_{l=1}^J \int_K \int_\beta \xi_j^\Xi(a|x,y)1\left(\Xi_j(a|K,\beta)>\Xi_l(a|x,y)\right)dG_l(a|x,y)d\Gamma(x,y)}{\sum_{l=1}^J \int_K \int_\beta \xi_j^\Xi(a|x,y)dG_l(a|x,y)d\Gamma(x,y)}$.

[69] For notation clarity, in this equation, we refer to a'' as the skill stock in the previous period, and a as the skill process in the current period.

B3 - Derivation of Equilibrium Wages

On the heterogeneity of bargaining powers. The derivation of the interior solution of the dynamic equilibrium wages follows closely the steps considered in Acemoglu and Hawkins (2014) and Cahuc et al. (2008). In this derivation, we will assume that β is a vector of bargaining powers, implying instead of a common bargaining power for every task market within the firm, the existence of heterogeneous bargaining powers per task market, i.e. $\beta = [\beta_1, \ldots, \beta_j, \ldots, \beta_J]$.

System of differential equations for equilibrium wages. Consider the equation (18) of the text:

$$\begin{aligned} r\Xi_j(a|K,\beta) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} &= w_j(a|K,\beta) + \underbrace{\bar{s}\Big(Out(a) - \Xi_j(a|K,\beta)\Big)}_{\text{Value loss of losing the job}} + \\ &+ \underbrace{\sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta)\left\{\theta_l q(\theta_l)\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j\right\}}_{\text{Value of searching for a job while employed}} \\ &+ \underbrace{\sum_{l=1}^{J}\Big[y_l(K,\beta)V_l(K,\beta) - s_l(K,\beta)N_l(K,\beta)\Big]\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact of hiring and firing policy of the firm in the value of the employment}}, \end{aligned} \tag{1}$$

At this stage we impose the assumption that both parties bargain under the assumption of *match stability*, i.e. no party, at-will, will dissolve the match, implying the parties believe, for wage bargaining purposes, that $\xi_l^{\Xi}(a|K,\beta) = 0, \forall l \in \{1,\ldots,J\}$, and $\tilde{s} = 0$. Thus equation (1) becomes:

$$\begin{aligned} \Big(r + \bar{s}\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) - \left[\frac{\partial \Xi_j(a|K,\beta)}{\partial t}\right] &= w_j(a|K,\beta) - rOut(a) + \\ + \sum_{l=1}^{J}\left[y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\right]\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}. \end{aligned} \tag{2}$$

Given the bargaining arrangement, expressed in equation (23) of the text, assuming the match stability condition, and that the aggregate bargaining constraint is not binding, we have:

$$\Big(1 - \beta_j\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) = \beta_j\Big(J_j(a|K,\beta)\Big). \tag{3}$$

In addition, considering that the outside option bargained between the parties is not affected by changes in firm's employment, given the presence of a large number of firms, i.e.:

$$\frac{\partial Out(a)}{\partial N_j(K,\beta)} = 0, \;\; \forall j \in [1,\ldots,J], \tag{4}$$

we have that:

$$\Big(1-\beta_j\Big)\Big(\frac{\partial \Xi_j(a|K,\beta)}{\partial t}-\frac{\partial Out(a)}{\partial t}\Big)=\beta_j\Big(\frac{\partial J_j(a|K,\beta)}{\partial t}\Big), \tag{5}$$

$$\Big(1-\beta_j\Big)\Big(\frac{\partial \Xi_j(a|K,\beta)}{\partial N_j(K,\beta)}\Big)=\beta_j\Big(\frac{\partial J_j(a|K,\beta)}{\partial N_j(K,\beta)}\Big). \tag{6}$$

Using the result of equation (6) with equation (1), we have:

$$\begin{aligned}
&\sum_{l=1}^{J}\Bigg[\Bigg\{y_l(K,\beta)V_l(K,\beta)-\int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\Bigg\}\frac{\partial J_j(a|K,\beta)}{\partial N_l(K,\beta)}\Bigg]=\\
&=\frac{1-\beta_j}{\beta_j}\sum_{l=1}^{J}\Bigg[\Bigg\{y_l(K,\beta)V_l(K,\beta)-\int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\Bigg\}\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}\Bigg]=\\
&=\frac{1-\beta_j}{\beta_j}\Bigg[\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta)-Out(a)\Big)-\frac{\partial \Xi_j(a|K,\beta)}{\partial t}-w_j(a|K,\beta)+rOut(a)\Bigg].
\end{aligned} \tag{7}$$

Moreover, resorting to equation (16) of the text, under the assumption of match stability, we have:

$$\begin{aligned}
&rJ_j(a|K,\beta)-\frac{\partial J_j(a|K,\beta)}{\partial t}=\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}-w_j(a|K,\beta)\\
&-\sum_{l=1,l\neq j}^{J}\int_a\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da-A(j,a)\\
&-\bar{s}J_j(a|K,\beta)+\sum_{l=1}^{J}\Bigg\{y(l|K,\beta)V(l|K,\beta)-\int_a s_l(a|K,\beta)G_l(a|K,\beta)da\Bigg\}\frac{\partial J_j(a,K,\beta)}{\partial N_l(K,\beta)}.
\end{aligned} \tag{8}$$

and together with equation (7), one obtains:

$$\begin{aligned}
&\beta_j(r+\bar{s})J_j(a|K,\beta)-\beta_j\frac{\partial J_j(a|K,\beta)}{\partial t}=\beta_j\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}-\beta_j w_j(a|K,\beta)\\
&-\beta_j\sum_{l=1,l\neq j}^{J}\int_a\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da-\beta_jA(j,a)+\\
&+(1-\beta_j)\Bigg[\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta)-Out(a)\Big)-\frac{\partial \Xi_j(a|K,\beta)}{\partial t}-w_j(a|K,\beta)+rOut(a)\Bigg].
\end{aligned} \tag{9}$$

Incorporating equations (3) and (5), and simplifying the resulting equation, we obtain a system of differential equations governing the equilibrium wages for each match with fundamentals $\{a,K,\beta\}$. Such system is given by:

$$\begin{aligned}
&w_j(a|K,\beta)=(1-\beta_j)\Bigg(rOut(a)-\frac{\partial Out(a)}{\partial t}\Bigg)\\
&+\beta_j\Bigg(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}-\sum_{l=1,l\neq j}^{J}\int_a\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da-A(j,a)\Bigg).
\end{aligned} \tag{10}$$

Further, we have that the average wage per workplace - $w_j(K,\beta)$ - is given by:

$$w_j(K,\beta) = \frac{1}{N_j(K,\beta)} \int_a w_j(a|K,\beta) dG_j(a|K,\beta) da, \tag{11}$$

and consistently with equation (10), becomes:

$$\begin{aligned} w_j(K,\beta) &= (1-\beta_j) E\left(rOut(a) - \frac{\partial Out(a)}{\partial t} \middle| K,\beta \right) + \\ &+ \beta_j \left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J} \frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)} N_l(K,\beta) - E(A(j,a)|K,\beta) \right). \end{aligned} \tag{12}$$

Further, given equation (11), the proper wage for the match fundamentals $\{a,K,\beta\}$ is given by:

$$\begin{aligned} w_j(a|K,\beta) &= (1-\beta_j) \left(rOut(a) - \frac{\partial Out(a)}{\partial t} \right) + \\ &+ \beta_j \left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J} \frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)} N_l(K,\beta) - A(j,a) \right). \end{aligned} \tag{13}$$

Solving the system of differential equations for the partial equilibrium wages To solve the system of differential equations of equation (12), we follow the insight of Cahuc et al. (2008). Thus, take the partial derivative of the average wages, $w_j(K,\beta)$, with respect to employment in another task market $N_l(K,\beta)$, $l \neq j$, given the difference between any $w_j(a|K,\beta)$ and $w_j(K,\beta)$ is based, in the moment of the bargaining of prices within the firm, on the exogenous values, i.d. (a) $Out(a)$ versus $E[Out(a)|K,\beta]$; and (b) $E[A(j,a)|K,\beta]$ and $A(j,a)$. Thus:

$$\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} + \beta_j \frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)} = \beta_j \left[\frac{\partial^2 F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta) \partial N_l(K,\beta)} - \sum_{k=1}^{J} \frac{\partial^2 w_k(K,\beta)}{\partial N_j(K,\beta) \partial N_l(K,\beta)} N_k(K,\beta) \right] \tag{14}$$

which yields second-order differential equation as:

$$(1-\beta_j) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \beta_j \frac{\partial^2}{\partial N_j(K,\beta) \partial N_l(K,\beta)} \left[F(\mathbf{N}(K,\beta),K) - \sum_{j=1}^{J} w_k(K,\beta) N_k(K,\beta) \right]. \tag{15}$$

Further, given the equality of second-order cross derivatives, one can also infer that:

$$\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \underbrace{\frac{\beta_j}{1-\beta_j} \frac{1-\beta_l}{\beta_l}}_{\chi_{jl}} \frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}, \tag{16}$$

and:

$$\sum_{j=1}^{J} N_j(K,\beta)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \sum_{j=1}^{J} \chi_{lj} N_j(K,\beta)\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}. \tag{17}$$

Jointly, this allows to write equation (12) as:

$$\begin{aligned} w_j(K,\beta) &= (1-\beta_j)E\left(rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right) + \\ &+ \beta_j\left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J} \chi_{j,l}\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} N_l(K,\beta) - E(A(j,a)|K,\beta)\right). \end{aligned} \tag{18}$$

The case of homogeneous β at firm level. At this stage let us first assume that $\beta_j = \beta$ - homogeneous bargaining power at firm level so that $\chi_{jl} = 1, \forall\{l,j\}$. Considering the generalized spherical coordinates $\iota, \omega_1, \ldots, \omega_{J-1}$, where ι is the distance to the origin such that $\sum_{j=1}^{J} N_j(K,\beta)^2 = \iota^2$, where ω_j are angles of projection in different subplanes, one can write:

$$\begin{aligned} N_1(K,\beta) &= \iota cos\omega_1 \ldots cos_{J-2}cos_{J-1} \\ N_2(K,\beta) &= \iota cos\omega_1 \ldots cos_{J-3}sin_{J-2} \\ N_2(K,\beta) &= \iota cos\omega_1 \ldots cos_{J-2}sin_{J-3} \\ &\ldots \\ N_{J-1}(K,\beta) &= \iota cos\omega_{f1t}sin\omega_{f2t} \\ N_J(K,\beta) &= \iota sin\omega_{f1t}, \end{aligned} \tag{19}$$

and with such coordinates, using the notation $\omega = (\omega_1, \ldots, \omega_J)$, one writes:

$$\sum_{l=1}^{J} N_l(K,\beta)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \iota\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial \iota}, \tag{20}$$

where ι is the scale of use of labour tasks, and ω reflects the proportions in which the different types of labour tasks are used. $\omega = (0, \ldots, 0)$ means that firm only employ workers in the first task market. Then equation (12) reads as:

$$\begin{aligned} \beta\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial \iota} + w_j(\iota,\omega,K,\beta) &= (1-\beta)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right] + \\ &+\beta\left[\frac{\partial F(\iota,\omega,K,\beta)}{\partial N_j(K,\beta)}\right] - \beta E[A(j,a)|K,\beta]. \end{aligned} \tag{21}$$

Notice that given the exogeneity of: (1) $(1-\beta)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right]$; and (2) $A(j,a)$, we can drop it and thus we have:

$$\frac{dw_j(\iota,\omega,K,\beta)}{d\iota}+\frac{w_j(\iota,\omega,K,\beta)}{\beta\iota}-\frac{\partial F(\iota,\omega,K_{ft})}{\partial\phi_{fjt}}\frac{1}{\iota}=0. \tag{22}$$

Notice that the solution of the homogeneous equation $\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial\iota}+\frac{w_j}{\beta\iota}=0$ is given by:

$$w_j(\iota,\omega,K,\beta)=C\iota^{-\frac{1}{\beta}} \tag{23}$$

and thus derivating it towards ι, while assuming C depends on ι, one obtains:

$$\frac{dw_j(\iota,\omega,K,\beta)}{d\iota}=\frac{dC}{d\iota}\iota^{\frac{-1}{\beta}}-\frac{1}{\beta}C\iota^{-1-\frac{1}{\beta}} \tag{24}$$

which plugging back (24) and (23) in equation (22), one obtains:

$$\frac{dC}{d\iota}\iota^{\frac{-1}{\beta}}-\frac{1}{\beta}C\iota^{-1-\frac{1}{\beta}}+\frac{C\iota^{-\frac{1}{\beta}}}{\beta\iota}-\frac{\partial F(\iota,\omega,K,\beta)}{\partial N_j(K,\beta)}\frac{1}{\iota}=0 \tag{25}$$

and simplifying one obtains:

$$\frac{dC}{d\iota}=\iota^{\frac{1-\beta}{\beta}}\frac{\partial F(\iota,\omega,K,\beta)}{\partial N_j(K,\beta)}, \tag{26}$$

and through integration one gets:

$$C_j(\omega,K,\beta)=\int_0^{\iota}z^{\frac{1-\beta}{\beta}}\frac{\partial F(z,\omega,K,\beta)}{\partial N_j(K,\beta)}dz+D, \tag{27}$$

where D is the constant of integration. Given the property that $\lim_{\iota\to0^+}\iota w_j(\iota,\omega,K,\beta)=0$, we have that the constant D is identically equal to zero. Therefore the solution to equation (21) satisfies:

$$\begin{aligned}w_j(\iota,\omega,K,\beta)=&(1-\beta)E\left[rOut(a)-\frac{\partial Out(a)}{\partial t}\middle|K,\beta\right]+\\&+\iota^{\frac{-1}{\beta}}\left(\int_0^{\iota}z^{\frac{1-\beta}{\beta}}\frac{\partial F(z,\omega,K,\beta)}{\partial\phi_{fjt}}dz\right)-\beta E[A(j,a)|K,\beta].\end{aligned} \tag{28}$$

Further, notice that if $\mathbf{N}(K,\beta)=(\iota,\omega)$, then $(z\iota,\omega)=[zN_1(K,\beta),zN_2(K,\beta),\dots,zN_J(K,\beta)]=z\mathbf{N}(K,\beta)$, one can turn equation (28) in:

$$\begin{aligned}w_j(K,\beta)=&(1-\beta)E\left[rOut(a)-\frac{\partial Out(a)}{\partial t}\middle|K,\beta\right]+\\&+\int_0^1 z^{\frac{1-\beta}{\beta}}\frac{\partial F(z\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz-\beta E[A(j,a)|K,\beta],\end{aligned} \tag{29}$$

and by doing so fully eliminating the spherical coordinates, which results in the solution of the system of differential equations in equation (12).

Further, from equation (29), one can infer that the equilibrium wages, and the solution of the system of differential equations in equation (13) is given by:

$$\begin{aligned} w_j(a|K,\beta) =& (1-\beta)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right] + \\ &+ \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(z\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta A(j,a), \end{aligned} \tag{30}$$

The case of heterogeneous β at firm level. It is helpful to consider a new variable, as does Cahuc et al. (2008). Accordingly, define $\mathbf{M}_j(K,\beta) = \{M_{j,1}(K,\beta), M_{j,2}(K,\beta), \ldots, M_{j,J}(K,\beta)\}$, such that:

$$\sum_{l=1}^{J} M_{j,l} \frac{\partial v_l(\mathbf{M}_j, K)}{\partial M_{j,l}} = \sum_{l=1}^{J} \chi_{jl} N_l(K,\beta) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}, \tag{31}$$

with $v_j[M_{j,j}(K,\beta), K] = w_j(K,\beta)$. Also, we assume it holds:

1. $G(\mathbf{M}_j, K) = F(\mathbf{N}(K,\beta), K)$;
2. $M_{j,l} = M_{j,l}(N_l(K,\beta))$;
3. $\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \frac{\partial v_l(\mathbf{M}_j)}{\partial M_{j,l}} \frac{dM_{j,l}}{dN_l(K,\beta)}$.

For equation (31) to hold it suffices that the following equation to hold:

$$M_{j,l} \frac{\partial v_l(\mathbf{M}_j, K)}{\partial M_{j,l}} = \chi_{jl} N_l(K,\beta) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}. \tag{32}$$

Given property (3), one obtains a differential equation for $M_{j,l}$, which is given by:

$$M_{j,l} = \chi_{jl} N_l(K,\beta) \frac{dM_{j,l}}{dN_l(K,\beta)}. \tag{33}$$

One feasible solution, not necessarily the only one, corresponds to:

$$M_{j,l} = N_l(K,\beta)^{\frac{1}{\chi_{jl}}} = N_l(K,\beta)^{\chi_{lj}} \tag{34}$$

given $\chi_{lj} = \frac{1}{\chi_{jl}}$. Considering that the mapping between notations, and properties (1) and (2), we have:

$$\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_l(K,\beta)} = \chi_{lj} N_l(K,\beta)^{\chi_{lj}-1} \frac{\partial G(\mathbf{M}_j, K)}{\partial M_{j,l}}, \tag{35}$$

and concretely,

$$\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} = \chi_{jj} N_j(K,\beta)^{\chi_{jj}-1} \frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,j}} = \frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,j}}, \tag{36}$$

since $\chi_{jj} = 1$. The system in equation (12) can be expressed as:

$$\begin{aligned} v_j(\mathbf{M}_j,K) = (1-\beta_j)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right] + \\ + \beta_j\left(\frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,j}} - \sum_{l=1}^{J} M_{j,l}\frac{\partial v_j(\mathbf{M}_j,K)}{\partial M_{j,l}}\right) - \beta_j E[A(j,a)|K,\beta], \end{aligned} \tag{37}$$

which is identical to equation (18). Therefore, following the procedure explained for identical β's, one obtains:

$$\begin{aligned} v_j(\mathbf{M}_j,K) = (1-\beta_j)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right] + \\ + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial G(z\mathbf{M}_j,K)}{\partial M_{j,j}}, K)dz - \beta_j E[A(j,a)|K,\beta] \end{aligned} \tag{38}$$

and translating the transformed variables in the initial notation variables, one realizes equation (38) becomes:

$$\begin{aligned} w_j(K,\beta) =& (1-\beta_j)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right] + \\ &+ \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta} dz - \beta_j E[A(j,a)|K,\beta] \end{aligned} \tag{39}$$

Then, given the definition of the wages for each match fundamentals $\{a,K,\beta\}$, and namely that the heterogeneity arises in operating costs and outside options only, one realize that:

$$\begin{aligned} w_j(a|K,\beta) =& (1-\beta_j)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right] + \\ &+ \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta_j A(j,a), \end{aligned} \tag{40}$$

where the matrix $\mathbf{Q}_j(z)$ is a diagonal matrix of the shape:

$$\mathbf{Q}_j(z) = \begin{bmatrix} z^{\frac{\beta_{f1t}}{1-\beta_{f1t}}\frac{1-\beta_{fjt}}{\beta_{fjt}}} & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 \\ 0 & \dots & z^{\frac{\beta_{flt}}{1-\beta_{flt}}\frac{1-\beta_{fjt}}{\beta_{fjt}}} & 0 \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & z^{\frac{\beta_{fJt}}{1-\beta_{fJt}}\frac{1-\beta_{fJt}}{\beta_{fjt}}} \end{bmatrix}. \tag{41}$$

As one can notice, considering heterogeneous β fundamentally change the calculus of the relevant marginal product of labour, and the portion of the idiosyncratic surplus that the worker is capable to extract.

General equilibrium wages in a dynamic equilibrium with heterogeneous β. To obtain the equilibrium wages in a dynamic equilibrium, we need to consider the HJB equation of the unemployed worker in equation (17) of the main text. Thus, we have that equation (40) becomes:

$$\begin{aligned} w_j(a|K,\beta) =&(1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\Bigg\{b+\theta_j q(\theta_j)\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|K,\beta)>Out(a)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} \\ &-Out(a)\Bigg)-c_l\Bigg\}+\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz-\beta A(j,a), \end{aligned} \tag{42}$$

where $\mathbf{1}[\Xi_l(a|K,\beta)>Out(a)]$ is an indicator function being 1 if the offer has a higher value than the outside option. Notice that this result confirms the intuition that under the assumption of *match stability* the outside option do not depend on the current match of the worker. Moreover, notice that by assumption we have that $\Xi_l(a|K,\beta)>Out(a)\forall\{a,K,\beta\}$, so that a worker is not unemployed by choice. Thus we have that equation (42) becomes:

$$\begin{aligned} w_j(a|K,\beta) =&(1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\Bigg\{b+\theta_j q(\theta_j)\frac{\int_K\int_\beta \Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} \\ &-Out(a)\Bigg)-c_l\Bigg\}+\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz-\beta A(j,a), \end{aligned} \tag{43}$$

and the average wage per workplace becomes:

$$\begin{aligned} w_j(K,\beta) =&(1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\Bigg\{b+\theta_j q(\theta_j)E_a\Bigg[\frac{\int_K\int_\beta \Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)}\Bigg] \\ &-Out(a)\Bigg)-c_l\Bigg\}+\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz-\beta A(j,a), \end{aligned} \tag{44}$$

Given the assumptions of: (1) bounded expression; (2) smoothness of production function; and (3) match stability in bargaining, the unique solution for wages in the dynamic equilibrium is given by equation (41) and (43). This expression is identical to wage expressions using intra-firm bargaining of Stole and Zwiebel (1996*a*). In that stream of studies, one can refer to Cahuc et al. (2008), Bauer and Lingens (2010), Elsby and Michaels (2013), Acemoglu and Hawkins (2014) and Dobbelaere and Luttens (2016).

The solution of the aggregate bargaining. Consider the solution of the firm level bargaining, as provided in equation (24) of the text. Accordingly, the average wage for a type a worker is given by:

$$E_{K,\beta}[w_j(a|K,\beta)] = (1-\beta_j)\left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right) + \beta_j\left[E_{K,\beta}\left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\right) - A(j,a)\right], \tag{45}$$

and therefore, we have that:

$$E_{K,\beta}[w_j(a|K,\beta)] = \left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right) + \beta_j\left[E_{K,\beta}\left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\right) - A(j,a) - \left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right)\right], \tag{46}$$

$$E_{K,\beta}[w_j(a|K,\beta)] = \left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right) + \beta_j E_{K,\beta}[QR(a)], \tag{47}$$

and consequently, once we impose the constraint of zero expected quasi rents, we have:

$$w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t}. \tag{48}$$

Therefore, the solution of the aggregate bargaining in equation (21) of the text is equation (48). Notice, that the key to identify this solution is to realize that an identical derivation to the one presented in equations (1) to (10) of this appendix can be easily computed for the average firm in the market. Indeed if equation (10) holds for every single firm, it also holds on average for each type a worker. Then one needs just to consider the additional constraint presented in the aggregate bargaining versus the firm bargaining, namely the absence of quasi rents.

B4 - The Dynamic and Steady State Equilibria.

Additional Assumptions for dynamic equilibrium. At this stage we consider two additional technical assumptions, as follows:

- **Bounded expression** - $\lim_{N_j \to} w_j(a|K,\beta)N_j = 0$;
- **Smoothness of Production Function** - $F(\mathbf{N}(K,\beta),K)$ is continuous for all $N_j > 0$, and infinitely differentiable for all $N_j > 0$. Further, $N_j \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j}$, and the quantity $N_1^{m_1} \dots N_J^{m_J} \frac{\partial^2 F}{\partial N_1^{m_1} \partial \dots} N_J^{m_J}$, or simply $\mathbf{N^m F^{(m)} N}(N,\beta)$, with $\bar{m} \sum_{j=1}^{J} m_j$ is continuous at zero.

These assumptions are fairly technical to ensure there exists an equilibrium wage function that is smooth in all $N_j > 0$, and unique. Altogether, the dynamic equilibrium of the model is defined as follows:

Theorem 1 (Dynamic Equilibrium)**.** *A tuple*

$$\Big\{\theta_j(t), Out(a), G(a|K,\beta), dG(a|K,\beta), J(a|K,\beta), \Xi(a|K,\beta), w_j(a|K,\beta), \xi_j^u(a), \\ \xi^{\Xi}(a|K,\beta), s_j(a|K,\beta), m_j(a|K,\beta), y_j(a|K,\beta), V_j(K,\beta), d\aleph(a)\Big\} \tag{1}$$

is a dynamic equilibrium if for all t, the following statements are jointly satisfied:

- *$J(\cdot)$, $Out(\cdot)$ and $\Xi(\cdot)$ satisfy HJB equations (16), (17) and (18) of the text;*
- *Vacancy Posting is optimal so it holds equation (9) and equation (20) of appendix B2;*
- *$G(a|K,\beta)$ has a density $dG(a|K,\beta)$ satisfying equation (24) of appendix B2;*
- *Job search is optimal so it solves the problems in equations (14) and (15) of appendix B2;*
- *$s_j(a|K,\beta)$ holds equation (13) and $m_j(a|K,\beta)$ holds equation (19) of appendix B2;*
- *The vacancy yield holds equations (22) and (23) of appendix B2;*
- *The market tightness hold equation (21) of appendix B2, and equation (5) of the text;*
- *The unemployed distribution $dU(a)$ and the distribution of workers $d\aleph(a)$ follow equations (12) of the text;*

- *The equilibrium wage satisfies the problem in equation (21) and equation (23) of the text.*

Core definitions and assumptions of the steady state equilibrium. In the model one can define a steady state equilibrium where all aggregate variables are constant over time, and where wages and the vacancy-posting strategies of firms depend only on firm's fundamentals (K, β). Let us define a level a^R such that:

$$J_j^R(K,\beta) = J_j(a^R|K,\beta), \tag{2}$$

so that it is the level of skill that is compatible with the expected marginal profit profit of the firm with fundamentals (K, β). Thus consistent with equation (16) of the text, we have:

$$\begin{aligned} rJ_j(a^R|K,\beta) - \frac{\partial J_j(a^R|K,\beta)}{\partial t} &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a^R|K,\beta) - A(j,a^R) \\ &- \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da - s_j(a^R|K,\beta)J_j(a^R|K,\beta) + \\ &+ \sum_{l=1}^{J} \left\{ y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da \right\} \frac{\partial J_j(a^R,K,\beta)}{\partial N_l(K,\beta)}. \end{aligned} \tag{3}$$

Given the steady state equilibrium imposes stability of aggregate variables, there is stability of the workforce, namely:

$$y_l(K,\beta)V_l(K,\beta) = \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da, \tag{4}$$

and, also consider that:

$$\frac{\partial J_j(a^R|K,\beta)}{\partial t} = \frac{\partial \Xi_j(a^R|K,\beta)}{\partial t} = \frac{\partial Out(a^R)}{\partial t} = 0. \tag{5}$$

Through a process identically presented in *The system of Differential equations for equilibrium wages* part of appendix B3, we have:

$$\begin{aligned} (r+\bar{s})J_j(a^R|K,\beta) &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a^R|K,\beta) \\ &- \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da - A(j,a^R). \end{aligned} \tag{6}$$

The Vacancy Curve. Given by assumption $\bar{s} > 0$, then we have that $V(K, \beta) > 0$

in a steady state equilibrium. Thus, given equation (10) of appendix B2, we have:

$$J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}. \tag{7}$$

Consequently:

$$\begin{aligned}
\underbrace{\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R)}_{\text{Marginal Product net of op. costs}} = \underbrace{w_j(a^R|K,\beta)}_{\text{Wage}} + \underbrace{(r+\bar{s})\frac{\gamma_j}{q(\theta_j)}}_{\text{Turnover Costs}} + \\
+ \underbrace{\sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages}}.
\end{aligned} \tag{8}$$

This result is typical in steady-state equilibria of search and matching models, and intuitively entails that the expected marginal worker produces on the margin the value of the cost of hiring such worker.[70] Following similar steps to the ones presented to solve this system, we have:

$$\underbrace{\frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}{\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}}}_{\text{Overemployment Effect - } OE_j(K,\beta)} \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R) = \underbrace{w_j(a^R|K,\beta) + (r+\bar{s})\frac{\gamma_j}{q(\theta_j)}}_{\text{Labour costs}}$$

$$OE_j(K,\beta)\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R) = w_j(a^R|K,\beta) + (r+\bar{s})\frac{\gamma_j}{q(\theta_j)}. \tag{9}$$

Considering the wage equation with the assumptions identified in equations (5) and (6), and the definition of employment effect, we have:

$$\begin{aligned}
w_j(a^R|K,\beta) &= (1-\beta_j)\sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\} \\
&\quad + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta A(j,a^R) \\
w_j(a^R|K,\beta) &= (1-\beta_j)\sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\} \\
&\quad + \beta OE_j(K,\beta)\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta A(j,a^R).
\end{aligned} \tag{10}$$

[70] For instance, Cahuc et al. (2008) finds an identical equation in their equation (9).

Joining equations (9) and (10) so that one eliminates wages, we have:

$$OE_j(K,\beta)\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R) =$$
$$\sum_{j=1}^{J} \xi_j^u(a)\left\{ b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l \right\}, \tag{$VC_j(K,\beta)$}$$

and equivalently:

$$\int_0^1 \frac{1}{\beta_j} z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - A(j,a^R) =$$
$$\sum_{j=1}^{J} \xi_j^u(a)\left\{ b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l \right\}. \tag{$VC_j(K,\beta)$}$$

Those equations corresponds to the vacancy curves of the firm with fundamentals (K,β) - $VC_j(K,\beta)$.[71] Notice that the right-hand side of the vacancy curve is unambiguously increasing in θ.

Lemma on Profit of firms and wages. The flow profit

$$r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} = \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J}\int_a w_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Wage Bill}}$$
$$- \underbrace{\sum_{j=1}^{J}\int_a A(j,a)dG_j(a|K,\beta)da}_{\text{Operating Cost Bill}} - \underbrace{I(K)}_{\text{Sunk Cost}} - \underbrace{\sum_{j=1}^{J}\int_a s_j(a|K,\beta)J_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Cost of Displacement}} +$$
$$+ \underbrace{\sum_{j=1}^{J}\max_{V_j(K,\beta)}\left\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta)q(\theta_j)J_j^R(K,\beta)\right\}}_{\text{Value of the Hiring Policy}}. \tag{12}$$

is continuous, strictly concave and satisfies:

$$\lim_{\mathbf{N}\to\mathbf{0}^+} \Pi(K,\beta) = 0. \tag{13}$$

Then, given:

$$\lim_{N_j(K,\beta)\to\infty} \frac{\partial F(\mathbf{N}(K,\beta))}{\partial N_j(K,\beta)} < b, \tag{14}$$

[71] Notice that as typically we have:

$$\begin{aligned} &\lim_{x\to 0^+} q(x) = \infty; \quad \lim_{x\to\infty} q(x) = 0; \quad \frac{\partial q(\theta)}{\partial \theta} < 0 \\ &\lim_{x\to 0^+} xq(x) = 0; \quad \lim_{x\to\infty} xq(x) = \infty; \quad \frac{\partial \theta q(\theta)}{\partial \theta} > 0. \end{aligned} \tag{11}$$

assumed in the model, it implies that:

$$\lim_{N_j(K,\beta)\to\infty} \Pi(K,\beta) = -\infty, \tag{15}$$

implying that the optimal workforce size vector $\mathbf{N}(K,\beta)$ is finite in every task market for every firm. Further notice that

$$\begin{aligned}
\lim_{N_j(K,\beta)\to\infty} w_j(a^R|K,\beta) &= \sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+ \\
&+ \lim_{N_j(K,\beta)\to\infty} \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta_j A(j,a^R) \\
\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) &= \sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+ \\
&+ \lim_{N_j(K,\beta)\to 0^+} \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta_j A(j,a^R)
\end{aligned} \tag{16}$$

is identical to

$$\begin{aligned}
\lim_{N_j(K,\beta)\to\infty} w_j(a^R|K,\beta) &= \sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+ \\
&+ \lim_{N_j(K,\beta)\to\infty} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j A(j,a^R) \\
\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) &= \sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+ \\
&+ \lim_{N_j(K,\beta)\to 0^+} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j A(j,a^R).
\end{aligned} \tag{17}$$

Therefore, given the production function is strictly concave and displaying decreasing returns to scale, one concludes, given

$$\sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\} > 0, \tag{18}$$

by assumption, that wages are strictly positive and strictly decreasing with firm size. As noted by Acemoglu and Hawkins (2014), given

$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = +\infty \tag{19}$$

the level of employment that maximizes the flow profit is strictly positive.

Considerations on steady-state equilibrium. Given the shape of the profit function of a firm, precisely: (a) $\lim_{\mathbf{N}\to\mathbf{0}^+} \Pi(K,\beta) = 0$; (b) $\Pi(K,\beta)$ is strictly concave on employment; and (c) $J_j^R(K,\beta)$ is strictly decreasing in employment, and given

$$J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}, \tag{20}$$

then equation (20) has a unique vector of employment $\mathbf{N}(K,\beta)$ conditional on the values of the endogenous variables.

The steady-state condition The steady state equilibrium concept offers a greater simplification to our framework, which arises from imposing stability of aggregate flows at firm level. Thus, the stability of aggregate flows, equation (24) of appendix B2, and equation (5) jointly yield:

$$\begin{aligned} 0 = &- dG_j(a|K,\beta) + y_j(a|K,\beta)V_j(K,\beta) + \\ &+ \int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da'' \end{aligned} \tag{21}$$

which after further simplification becomes:

$$dG_j(a|K,\beta) = \frac{\int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da''}{1 - s_j(a|K,\beta)} \tag{22}$$

Accordingly, in the steady state equilibrium we hold that:

$$G_j(a|K,\beta) = \int_{-\infty}^{a} \frac{\int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da''}{1 - s_j(a|K,\beta)} da. \tag{23}$$

Given the stability of the workforce in each workplace economy-wide, we therefore also can hold that:

$$\int_K \int_\beta y_j(K,\beta)V_j(K|\beta)dKd\beta = \int_K \int_\beta \int_a s_j(a|K,\beta)dG_j(a|K,\beta)dadKd\beta. \tag{24}$$

Steady state equilibrium description. Therefore, the steady-state equilibrium is a specialization of the dynamic equilibrium presented with the following properties:

Theorem 2 (Steady-State Equilibrium)**.** *A tuple*

$$\begin{aligned} &\Big\{\theta_j(t), Out(a), G(a|K,\beta), dG(a|K,\beta), J(a|K,\beta), \Xi(a|K,\beta), w_j(a|K,\beta), \xi_j^u(a), \\ &\xi^{\Xi}(a|K,\beta), s_j(a|K,\beta), m_j(a|K,\beta), y_j(a|K,\beta), V_j(K,\beta), d\aleph(a)\Big\} \end{aligned} \tag{25}$$

is a steady state equilibrium if for $q(\theta) > 0$ and $\bar{s} > 0$, the following statements are jointly satisfied:

- *$J(\cdot)$, $Out(\cdot)$ and $\Xi(\cdot)$ satisfy HJB equations (16), (17) and (18) of the text;*

- *Vacancy Posting is optimal so it holds equation (9) and equation (20) of appendix B2;*
- $G(a|K,\beta)$ *has a density* $dG(a|K,\beta)$ *satisfying equation (24) of appendix B2, and equation (23) of this appendix;*
- *Job search is optimal so it solves the problems in equations (14) and (15) appendix B2;*
- $s_j(a|K,\beta)$ *holds equation (13), and* $m_j(a|K,\beta)$ *holds equation (19) of appendix B2;*
- *The vacancy yield holds equations (22), and (23) of appendix B2;*
- *The market tightness hold equation (21) of appendix B2, and equation (5) of the text;*
- *The unemployed distribution* $dU(a)$ *and the distribution of workers* $d\aleph(a)$ *follow equations (12) of the text;*
- *The equilibrium wage satisfies the problem in equation (21) and (23) of the text;*
- *The steady state conditions of equation (4) and equation (24);*
- *The stability of expectations of HJB functions in equation (5).*

Note that the the distribution of skill within workplaces satisfies an ergodicity condition and thus $G_j(a|K,\beta)$ is unique. So there is no loss of generality to apply such distribution which is assumed to be continuously differentiable. Note that $\bar{s} > 0$, which is partially justified by the death and birth shocks d. Note, that to ease the technical explanation on ergodicity, one can reason such shocks as a massive destructive shock on skill, which leads the worker to have his skill extracted from $\Psi_0(a)$, rather than having the worker dying and a new worker entering the market. Thus, we arive at the uniqueness of the invariant distribution through Theorem 11.9 of Stokey et al. (1989).[72]

[72] An identical argument is used in Acemoglu and Hawkins (2014).

C - Derivation of the Empirical Implementation Expressions

C1 - First order Taylor approximation of Outside Options.

The equilibrium wage expression with heterogeneous bargaining powers is given by:

$$w_j(a|K,\beta) = (1-\beta_j)Out^\star(a) + \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K}{\partial N_j(K,\beta)} dz - \beta_j A(j,a). \tag{1}$$

Let us focus in the dynamic behaviour of $Out^\star(a)$ function.

The time effect on outside Options. Notice that we have that the outside option is given by:

$$Out(a) = E_t[Out(a)] + \Bigg[Out(a) - E_t[Out(a)]\Bigg], \tag{2}$$

where $E_t[Out(a)]$ is the expected outside option of worker with skill level a. With standard algebraic manipulations one obtains:

$$Out(a) = E_t[Out(a)]\Bigg[1 + \frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]}\Bigg], \tag{3}$$

and considering a first order Taylor approximation, we have:

$$ln[Out(a)] = ln\Bigg[E_t[Out(a)]\Bigg] + \frac{Out(a) - E_t[(Out(a)]}{E_t[Out(a)]}. \tag{4}$$

Inside the expected value of the outside option. We have that the expected value of the outside option of the worker is given by:

$$ln\Bigg[E_t(Out(a))\Bigg] = ln\Bigg\{E_t \sum_{j=1}^{J} \xi_j^u(a)\Bigg\{b + \theta_j q(\theta_j)\Bigg(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Bigg) - c_l\Bigg\}\Bigg\}. \tag{5}$$

At this point, we consider a first order Taylor approximation around the initial value of a for each worker. Thus:

$$E_t\Bigg(Out(a)\Bigg) = E_t\Bigg(Out(a_{\tau 0(i)})\Bigg) + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Bigg|_{\tilde{a}\in[a_{\tau 0(i)};a]}\Bigg(a - a_{\tau 0(i)}\Bigg), \tag{6}$$

where $a_{\tau 0(i)}$ represents the skill value of the worker in the moment he enters the labour market.

Moreover, let us consider the expected value of skill a worker with a in the current period t should have had in the first period of her current contract - $E_t[a_{\tau 0(i)}|a]$.

Consequently, equation 6 becomes:

$$E_t\Big(Out(a)\Big) = E_t\Big(Out(a_{\tau_0(i)})\Big) + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i);a}]}\Big(a - E_t[a_{\tau_0(i)}|a]\Big) + \\ + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i);a}]}\Big(E_t[a_{\tau_0(i)}|a] - E_t[a_{\tau_0(i)}]\Big), \tag{7}$$

and after a first order Taylor approximation around the logarithm of expected value of outside option, one obtains:

$$ln\Big[E_t\Big(Out(a)\Big)\Big] \approx ln\Big[E_t\Big(Out(a_{\tau_0(i)})\Big)\Big] + \\ + \frac{1}{E_t[Out(a_{\tau_0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i);a}]}\Big(a - E_t[a_{\tau_0(i)}|a]\Big) + \\ + \frac{1}{E_t[Out(a_{\tau_0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i);a}]}\Big(E_t[a_{\tau_0(i)}|a] - E_t[a_{\tau_0(i)}]\Big). \tag{8}$$

Functional form of empirical model for outside options. Combining equations (4) and (8), we have:

$$ln[Out(a)] = \underbrace{\frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]}}_{\text{Task-market-time effect - }\lambda(j,t)} \\ \underbrace{+\, ln\Big[E_t\Big(Out[a_{\tau_0(i)}]\Big)\Big] + \frac{1}{E_t[Out(a_{\tau_0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i);a}]}\Big(a - E_t[a_{\tau_0(i)}|a]\Big) +}_{\substack{\text{expected contract profile -}\\ \psi(t-\tau_0(i),rank(i,t),female_i)}} \\ \underbrace{+\, \frac{1}{E_t[Out(a_{\tau_0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i,f);a}]}\Big(E_t[a_{\tau_0(i)}|a] - E_t[a_{\tau_0(i)}]\Big)}_{\text{disturbance - }v^S_{i,t}}. \tag{9}$$

where we explicitly introduce a parallel trend assumption, namely:

$$\frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]} = \frac{Out - E_t[Out]}{E_t[Out]}(j,t) = \lambda(j,t). \tag{10}$$

Intuitively, one is assuming that the evolution of outside option value of every worker of a given type a within a task market j is identical.

The reduced form of Outside Options and measurement error. Altogether, we therefore have that the reduced form representation, as presented in equation (28) of the text, is given by:

$$ln[w^{\star}_{bargain}] = \underbrace{\lambda(j,t) + \psi(t-\tau_0(i), rank(i,t), female_i)}_{\widehat{Out(a)}} + v^S_{i,t}. \tag{11}$$

Moreover, following the insight of Pei et al. (2018), notice that in the case of existence of a classical measurement error in $w_{bargain}$, so that:

$$w_{bargain} = w^{\star}_{bargain} + \eta(i,t), \tag{12}$$

where $\eta(i,t)$ corresponds to the classical measurement error, with the following properties:

1. $E[\eta(i,t)] = 0$;
2. $E[\lambda(j,t)\eta(i,t)] = 0$;
3. $E[\psi(t-\tau_0(i), rank(i,t))\eta(i,t)] = 0$;
4. $E[v^S_{i,t}\eta(i,t)] = 0$.

Consequently:

$$ln[w_{bargain}] = \underbrace{\lambda(j,t) + \psi(t-\tau_0(i), rank(i,t), female_i)}_{\widehat{Out(a)}} + \underbrace{v^S_{i,t} - \eta(i,t)}_{v_{i,t}}, \tag{13}$$

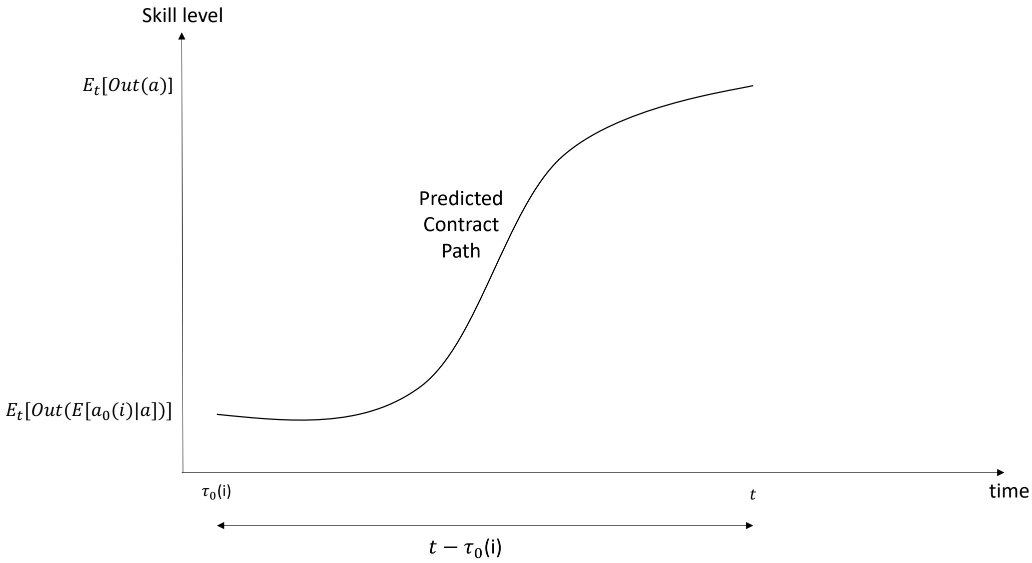
with $v_{i,t}$ corresponding to the composite error term.

In a nutshell, the first stage of our empirical implementation, beyond providing empirical structure to our estimation, also provides relevant answer to the existence of measurement error, particularly given $w_{bargain}$ corresponds to a proxy. As long as the measurement error is classical, it only has efficiency impacts, and not on the consistency of the estimates. Given the high dimensionality of our data, naturally efficiency of the estimator does not lie in the top of our priorities.

Intuition on the expected contract profile. We take advantage of the knowledge of: (i) the actual rank of the worker, which is linked with $E_t[Out(a)]$, apart from the trend behaviour; (ii) the experience of the worker, given by $t - \tau_0(i)$, so that we are capable to estimate the predicted contract path of each worker; and (iii) we allow for heterogeneous contract profiles by gender.

The identification of the predicted contract path enables the estimation of a time-task market effect, so that it controls for any time trend. Altogether, we are bunching the information of the workers sharing the same contract at collective agreement level (i.e. experience, actual rank and gender), and thus we improve our position to better value the individual-task market effect. Accordingly our identification follows the intuition of figure C1.

Figure C1: Structure of the Model in each moment t.



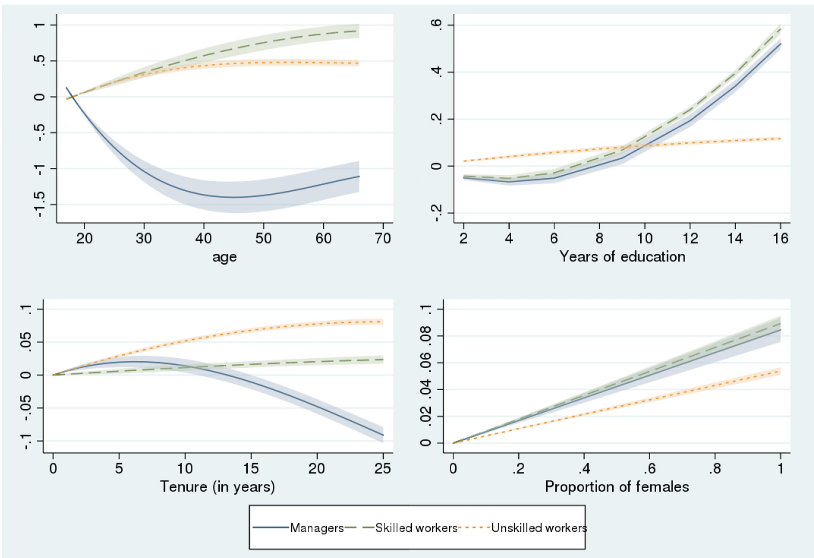
D - Empirical Results on Operational Costs

Regarding the estimates of the operating costs of the prime empirical wage setting equation, figure D1 displays the total effect of education, tenure, age and female proportions at the rank on the profile of real productivity of the match per hour, as well as some of its distributional features on those dimensions. As a general remark, the estimated behaviour of the productivity is consistent with the broad literature on wage determination, and as well with Postel-Vinay and Robin (2002), where the contribution of the individual characteristics become more relevant for higher skill level task markets.

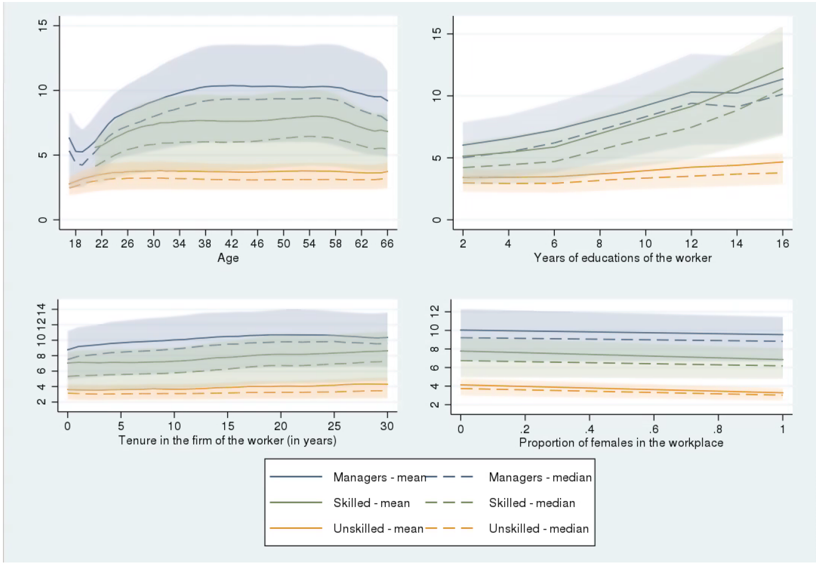
In broad brushstrokes, education has little effect on the unskilled workers, while it increases exponentially the productivity of the remainder workforce, confirming in our model its role as primary choice-based productivity enhancer when the job can benefit from the proceeds of schoolwork. Also, it conforms with the idea of potential over-education when the worker doesn't land on a suitable rank position given her education (see Hartog (2000) for further details on over-education for Portugal).

Differently, the tenure on the firm albeit displaying lower effects when compared with education, present a reversed relationship. The lower fragments of the wage tables, primarily unskilled workers, but also at a lower magnitude skilled ones, benefit the most from the permanence at the firm, with a stable monotonic relationship, even for very long tenancies. In a different archetype, the productivity gains for managers while increasing more sizably at early stages of the worker-firm match fade after the first five years, becoming increasingly detrimental for very long tenancies. Overall, such profiles could be consistent with Acemoglu and Pischke (1999), where increases in minimum wages, more likely to affect the bottom of the wage tables, induce higher levels of on-job training sponsored by firms.

Figure D1: The operating cost estimates.
Panel A: Estimated total effect on real productivity per hour.



Panel B: Distributional features of the real productivity per hour.



Notes: On panel A, the fading grey shades corresponds to the $95^{th} - 5^{th}$ confidence interval. On panel B, the fading shades corresponds to the $75^{th} - 25^{th}$ percentile range of the implied distribution. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

Regarding the age profiles, the gains for skilled and unskilled workers are increasing with the wage tables, though ceasing to further accrue earlier in the career the lower the position of the worker on those wage tables. Relatively to managers, increases in experience translates in consistent real productivity gains at a latter stage in their careers, broadly after around the 40 years of age, which however coincides with the most sizable mass of existing managers.[73] Before the cited threshold, the younger the manager the higher his real productivity, ceteris paribus. Tentatively, their appointment encloses relevant traits that make them particularly productive, as it is likely to be linked with an entrepreneurial/family-owned business perspective - the ideal, and perhaps unique green-way to land on a managerial position so early in their careers (see Blanchflower and Oswald (2007) for an analysis on the traits of young entrepreneurs).

E - A Toy Model

In this appendix we present a toy version of the model presented in the text. This toy version abstracts from several key ingredients of the main model, but preserves the link between the model and the identification strategy implemented.

Labour market structure. The labour market is frictional, workers search for jobs and get randomly matched with vacancies posted by firms.

[73] In the period of analysis, 90 percent of the managers are older than 28 years, and 75 percent are older than 33.

Consider a standard constant returns to scale matching function as M(u,v), and define the labour market tightness as $\theta = \frac{v}{u}$, where v is the number of vacancies in the market posted by firms, and u is the number of workers unemployed and searching for a job. Accordingly, the poisson rate at which workers meet a vacancy is given by $\theta q(\theta)$, and the poisson rate at which a vacancy is filled corresponds to $q(\theta)$.

Worker and firm heterogeneity. Further assume the economy is populated by exogenously capital heterogeneous firms, and exogenously skill heterogeneous workers, with $K \in [0,1]$ and $A \in [0,1]$, respectively. The production function of a job filled in a type-K firm by a worker of type-A, is given by:

$$F(K, A) = A + f(K). \tag{1}$$

Value Functions. Considering the setting, the value function of a vacancy is given by:

$$rV(K) = -\gamma + q(\theta)\Big[E_A[J(A,K)] - V(K)\Big], \tag{2}$$

where r is the discount rate, γ the cost of posting a vacancy and $E_A[J(A,K)]$ the expected value of a vacancy filled in a type-K firm.

The value function of a filled vacancy constituting a match between a type-A worker and a type-K firm corresponds to:

$$rJ(A,K) = A + f(K) - (r-\delta)K - w(A,K) - s(A,K)\Big[J(A,K) - V(K)\Big], \tag{3}$$

where δ is the depreciation rate, s is the probability of termination of the match and $w(K,A)$ is the wage earned by the type-A worker in such filled vacancy. Moreover, the value function of a type-A employed worker in a type-K firm is:

$$r\Xi(A,K) = w(A,K) + s(A,K)\Big[U(A) - \Xi(A,K)\Big], \tag{4}$$

with $U(A)$ corresponding to the value function of a type-A worker which is unemployed. Such value function is in detailed given by:

$$rU(A) = b + \theta q(\theta)\Big[E_A[\Xi(A,K)|K \in \Theta] - U(A)\Big], \tag{5}$$

with b corresponding to the flow of unemployment benefits, Θ consisting in the set of type-K firms which are expected to post vacancies, and $E_A[J(A,K)|K \in \Theta]$ representing the expected value of filled vacancies by a type-A worker, given the type-K firms posting vacancies.

Free Entry. It is assumed that there is free entry for posting vacancies. Accordingly, it holds that $V(K) = 0$ for all type-K firms, and consequently:

$$E_A[J(A, K)] = \frac{\gamma}{q(\theta)}. \tag{6}$$

Termination rate. It is assumed that a filled vacancy can be terminated either due to exogenous reasons happening at rate s, or due to economic reasons. Accordingly, it holds that:

$$s(A, K) = \begin{cases} 1 \text{ if } A + f(K) - (r - \delta)K - rU(A) \geq 0 \\ s \text{ if } A + f(K) - (r - \delta)K - rU(A) < 0. \end{cases} \tag{7}$$

Wage bargaining and equilibrium wages. The wage is set through a 2 stage bargaining. In detail:

1. The first takes place at national level between a fully representative right-to-manage union and a fully representative employers' association;
2. The second stage happens subsequently at firm level, between the referred union and the firm.

At national level, the union and the firm association bargain the minima wage requirement for each type-A worker, with the union committing to abstain from a full national lockdown. Moreover, the sides agree the minima wage requirement by bargaining an average national wage for type-A worker and then consider the least minimum viable match, namely the wage that would arise in the absence of an average positive quasi-rent at vacancy-filled level - $E_K[QR(A, K)]$. Notice that the quasi-rent corresponds to:

$$QR(A, K) = A + f(K) - (r - \delta)K - rU(A). \tag{8}$$

Therefore, the bargaining at national level solves:

$$w_{MIN}(A) = argmax_{E_K[w(A,K)]} \Big[E_K[\Xi(A,K)] - U(A)\Big]^{\beta} \Big[E_K[J(A,K)] - V(K)\Big]^{1-\beta}$$

subject to

$$A + E_K[f(K)] - (r-\delta)E(K) - rU(A) = 0 \quad \text{(Quasi-Rent Constraint)}, \tag{9}$$

where β corresponds to the union's bargaining power. Altogether, the solution of optimization problem in equation (9) corresponds to:

$$w_{MIN}(A) = rU(A). \tag{10}$$

At firm level, the union and a single firm bargain over wages, with the union committing to abstain from a full lockdown, and considering the minimum wage requirement set. So it holds:

$$w(A,K) = argmax_{w(A,K)} \Big[\Xi(A,K) - U(A)\Big]^{\beta} \Big[J(A,K) - V(K)\Big]^{1-\beta}$$

subject to

$$w(A,K) \geq w_{MIN}(A) \quad \text{(National bargaining constraint)}. \tag{11}$$

Accordingly, the solution corresponds to:

$$w(A,K) = \begin{cases} (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big] & \text{if } A + f(K) - (r-\delta)K - rU(A) \geq 0 \\ w_{MIN}(A) & \text{if } A + f(K) - (r-\delta)K - rU(A) < 0, \end{cases} \tag{12}$$

which given the termination rate, translates to:

$$w(A,K) = (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big]. \tag{13}$$

Empirical Identification. Consider the difference between the equilibrium wage defined in equation (13), and the average wage of type-K firm:

$$w(K,A) - E_A[w(K,A)|K] = (1-\beta)\Big[rU(A) - rE_A[U(A)|K]\Big] + \beta\Big[A - E[A|K]\Big], \tag{14}$$

which can be represented as:

$$\ln w(A,K) = \ln \Bigg[E_A[w(K,A)|K] + (1-\beta)\Big[rU(A) - rE_A[U(A)|K]\Big] + \beta\Big[A - E[A|K]\Big]\Bigg]. \tag{15}$$

Given the solution of national wide bargain, equation (15) can be translated to:

$$\ln w(A,K) = \ln \Bigg[E_A[w(K,A)|K] + (1-\beta)\Big[w_{MIN}(A) - E_A[w_{MIN}(A)|K]\Big] + \beta\Big[A - E[A|K]\Big]\Bigg]. \tag{16}$$

Equation (14) corresponds to the empirical model to be implemented resorting to a Non-linear Least Squares algorithm.

Derivation of the set of minimum wage requirement. Given the optimization problem presented in equation (9), the value functions in equations (2)-(5), the free entry condition and the termination rate, we have that the minimum wage requirement set for viable filled vacancies solves the following system:

$$\begin{cases} (1-\beta)\Big[E_K[\Xi(A,K)] - U(A)\Big] = \beta\Big[E_K[J(A,K)]\Big] \\ A + E_K[f(K)] - (r-\delta)E(K) - rU(A) = 0 \\ rE_K[\Xi(A,K)] = E_K[w(A,K)] + s\Big[U(A) - E_K[\Xi(A,K)]\Big] \\ rE_K[J(A,K)] = A + E_K[f(K)] - (r-\delta)E[K] - E_K[w(A,K)] - s\Big[E_K[J(A,K)] - E_K[V(K)]\Big]. \end{cases}$$

Consequently, we have that the system can be reduced to:

$$\begin{cases} (1-\beta)\Big[E_K[w(A,K)] - rU(A)\Big] = \beta\Big[A + E_K[f(K)] - (r-\delta)E[K] - E_K[w(A,K)]\Big] \\ A + E_K[f(K)] - (r-\delta)E(K) - rU(A) = 0, \end{cases}$$

and the set of wage minimum is given by:

$$w_{MIN}(A) = E_K[w(A,K)] = (1-\beta)rU(A) + \beta\Big[A + E_K[f(K)] - (r-\delta)E(K)\Big]$$

which can be reduced to:

$$w_{MIN}(A) = rU(A).$$

Derivation of the equilibrium wage. Given the optimization problem presented in equation (11), the value functions in equations (2)-(5), the free entry condition and the termination rate, the system that solves the equilibrium wages is given by:

$$\begin{cases} (1-\beta)\Big[\Xi(A,K) - U(A)\Big] = \beta\Big[J(A,K)\Big] \\ r\Xi(A,K) = w(A,K) + s\Big[U(A) - \Xi(A,K)\Big] \\ rJ(A,K) = A + f(K) - (r-\delta)K - w(A,K) - s\Big[J(A,K) - V(K)]\Big]. \end{cases}$$

The referred system can be reduced to:

$$w(A,K) = (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big],$$

which corresponds to the equilibrium wage equation in the interior solution. Thus:

$$w(A,K) = \begin{cases} (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big] & \text{if } A + f(K) - (r-\delta)K - rU(A) \geq 0 \\ w_{MIN}(A) & \text{if } A + f(K) - (r-\delta)K - rU(A) < 0. \end{cases}$$

However, notice given the termination rate, the equilibrium wage holds in the interior solution.