Paying Gig Workers

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Preliminary version

November 17, 2016

Abstract

We study the compensation of gig workers in a natural field experiment. To derive testable predictions, this paper presents a formal model capturing a central feature of online freelance work: gig workers' ability to choose both how much to work and how big an effort to make. We analyse the set-up in a principal-agent model, showing that the optimal contract includes a sales-based commission and uses a gig-based piece rate to insure a risk-averse agent. This piece rate is increasing in her risk aversion, intrinsic motivation and ability. We then predict the effects of introducing a gig piece rate while reducing the commission rate. The effects on the agents' choices of quantity and quality are heterogeneous in their risk aversion, intrinsic motivation and ability.

Key Words: Incentives, Risk Aversion, Intrinsic Motivation, Sales Compensation, Multitasking, Field Experiment
JEL Classification: M52, J33, D23

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1 Introduction

The rise of the on-demand economy has seen a proliferation of firms that rely on a workforce composed of freelancers rather than regular employees.[1] Many of these firms operate virtual platforms on which freelancers are matched with customers. There is considerable variation between platforms in terms of the work parameters freelancers get to set themselves.[2] An aspect emphasised by many platforms, however, is that freelancers set their own schedules, deciding how much and when to work. Another characteristic shared by many platforms is that compensation is purely output-based. In other words, gig workers' compensation is often a direct function of the success of the gig. As a consequence, gig workers typically face substantially higher income uncertainty than regular employees while potentially enjoying greater flexibility in their work arrangements.

In this project we study compensation contracts for freelancers in the on-demand economy. To do so, we first describe a formal model that allows us to analyse different potential drivers of freelancers' quantity and quality choices. To empirically test the most important hypotheses derived from the theory, we will conduct a field experiment on an online platform in a second step. In the current version of this paper we present the analysis of the formal model.

The field experiment will be carried out in collaboration with an online platform run by a retail firm. That platform acts as an intermediary between clients[3] and gig workers, who provide remote shopping advice. Their service may result in the online sale of physical goods, which is handled by the platform. Gig workers decide how much they want to work: they set the quantity of slots they make available for client consultations. Their efforts also determine the quality achieved: the usefulness of their advice affects the sales to each client. At the outset, gig workers' compensation is commission-

[1] Whether or not contractually defining the status of on-demand workers as that of an independent contractor rather than an employee is legally valid (given the relevant jurisdiction's labour law) is being questioned by legal scholars - see, e.g., Prassl (2017).

[2] A recent article in The Economist discusses the example of Uber's drivers (The Economist Print Edition, November 5 2016)

[3] We will use the words "customer" and "client" interchangeably in this paper.

based, paying them a fraction of net sales to the clients they advised. This is the status quo against which we will test an intervention that changes the compensation scheme. The goal of the theoretical analysis presented here is to provide intuition about the mechanisms that might be at play, to inform the design of the optimal compensation policy for gig workers, and to generate testable predictions for the field experiment.

We first analyse a principal-agent model that captures essential features of the relationship between a platform and a gig worker. That is, we consider a principal designing a contract to motivate an agent who not only determines the quality of her work on all gigs but also how much labour input to provide (how many gigs to offer). We first characterise the optimal contract and show that optimal pay makes use of a sales-based commission but also insures a risk-averse agent through a gig-based piece rate (order bonus). That is, the principal optimally includes a payment in the contract that depends only on the number of gigs. This is in contrast to purely commission-based pay optimal for a risk-neutral agent. The key idea here is that a pure commission rate induces incentives to provide too little labour input when the agent is risk averse as the payoff from each gig is uncertain. An order bonus, however, provides stronger incentives to increase labour supply.

Second, we analyse the heterogeneity in agents' reactions to the contract with respect to risk aversion, ability and intrinsic motivation. When modelling intrinsic motivation we allow for both conscientiousness[4] and task enjoyment as potential drivers of a desire to work and to do a good job. We show that quality is increasing in an agent's intrinsic motivation and that an intrinsically motivated agent's quality choice responds less strongly to the commission rate.

Finally, we study a specific application of our model that allows us to derive predictions for the field experiment. We consider a principal that initially pays her agent a pure commission without an order bonus. The above result suggests that if the agent is somewhat risk averse the platform should

[4] We use conscientiousness in the sense of the Big Five personality trait, measuring the extent to which an agent is driven by a sense of duty when performing a task.

change its compensation policy and introduce an order bonus. We formally analyse a hypothetical experiment where - in expectation - the introduction of an order bonus is "paid for" by the reduction in the commission rate. That is, order bonus and commission are calibrated on a population of agents in such a way that the average agent's pay per order remains constant if agents do not adjust quality. We then show that such a move from a pure commission to a combination of a commission and an order bonus leads to an increase in average quantity which will be more pronounced for more risk averse agents and less pronounced for more able and more intrinsically motivated agents. The shift in compensation will reduce quality but this decrease will be less pronounced the higher the agent's intrinsic motivation. Finally, for a shift of any given size, profits will increase if and only if the agent is sufficiently risk averse.

2 The Model

Our framework builds on a multi-tasking model in the spirit of Holmström and Milgrom (1991). Consider an agent who works for a principal, providing a service to customers. The agent chooses the number of client orders to fulfil $n \in [0;\bar{n}]$ and the average service quality $q \in [0;\bar{q}]$. The agent has (potentially) imperfectly known ability $a \sim N\left(m, \sigma_a^2\right)$ with $m > 0$. She has convex costs of effort $c\left(q,n\right)$ where $c_{qq} > 0$, $c_{qqq} \geq 0$, $c_{nn} > 0$ and $c_{qn} > 0$ such that the marginal cost of q increases when n goes up - providing a given level of quality on more orders requires more effort. We also assume that the marginal average cost of quality per order is (weakly) decreasing in the number of orders:

$$\frac{\partial}{\partial q \partial n}\left(\frac{c\left(q,n\right)}{n}\right) \leq 0$$

which is equivalent to $c_{qn}\left(q,n\right)n \leq c_q\left(q,n\right)$. Moreover, we impose the following conditions that guarantee internal solutions: $c_q\left(0,n\right) \leq 0 \ \forall n$, $c_n\left(q,0\right) \leq 0 \ \forall q$, $\lim_{q\rightarrow\bar{q}} c_q\left(q,n\right) = \infty \ \forall n$, $\lim_{n\rightarrow\bar{n}} c_n\left(q,n\right) = \infty \ \forall q$ and

$\sqrt{c_{nn}c_{qq}} + c_{qn} > 1\ \forall q, n$.[5] Note that, in addition to the direct disutility of effort, the cost function can accommodate several behavioural components. For example, the agent may be intrinsically motivated for the task (and thus, for instance, may have negative marginal costs of effort up to a point or may have psychological costs of not providing an appropriate quality given the quantity she has chosen). We will later on consider a specific example to explore this possibility.

When the agent fulfils n orders, she generates a level of sales

$$S = \sum_{i=1}^{n} (a + q + \varepsilon_i)$$

where $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$. The agent has an outside option that yields a reservation value $w_A > 0$ (with certainty). We allow for the possibility that the agent is risk averse with constant absolute risk aversion, where her Arrow-Pratt measure of absolute risk aversion is r.

Both total sales S and the number of orders worked are verifiable and we consider linear contracts that pay a wage

$$w = \alpha + \beta \cdot n + \gamma \cdot S,$$

where $\beta \geq 0$ is an order bonus, i.e., an order-based piece rate that does not depend on quality, and $\gamma \in [0,1]$ is a commission rate.

3 Analysis

3.1 Characterizing Optimal Contracts

The agent's objective function is

$$EU\left[\alpha + \beta n + \gamma\left(n(a+q) + \sum_{i=1}^{n}\varepsilon_i\right) - c(q,n)\right].$$

[5] The first four conditions are Inada-type conditions that also allow for negative marginal costs at small levels of effort. The last condition guarantees the concavity of the agent's objective function.

The variance of the agent's compensation is

$$V\left[\alpha+\beta n+\gamma\left(n\left(a+q\right)+\sum_{i=1}^{n}\varepsilon_i\right)\right]=\gamma^2\left(n^2\sigma_a^2+n\sigma_\varepsilon^2\right).$$

As the agent exhibits constant absolute risk aversion (see, for instance, Milgrom and Roberts (1992), Wolfstetter (2002) for details), she maximises

$$\max_{n,q}\alpha+\beta n+\gamma n\left(m+q\right)-c\left(q,n\right)-\frac{1}{2}r\gamma^2\left(n^2\sigma_a^2+n\sigma_\varepsilon^2\right)$$

with first order conditions[6]

$$\beta+\gamma\left(m+q\right)-c_n\left(q,n\right)-\frac{1}{2}r\gamma^2\left(2n\sigma_a^2+\sigma_\varepsilon^2\right)=0 \tag{IC1}$$

$$\text{and } \gamma n-c_q\left(q,n\right)=0. \tag{IC2}$$

To determine the optimal contract the principal maximises her expected profits

$$\max_{\alpha,\beta,\gamma,n,q}\left(1-\gamma\right)n\left(m+q\right)-\beta n-\alpha$$

subject to the incentive compatibility constraints (IC1) and (IC2) and the agent's participation constraint

$$\alpha+\beta n+\gamma n\left(m+q\right)-c\left(q,n\right)-\frac{1}{2}r\gamma^2\left(n^2\sigma_a^2+n\sigma_\varepsilon^2\right)\geq w_A.$$

As the agent's participation constraint must be binding (otherwise profits could be increased by reducing α without violating the incentive compatibility constraints) we can substitute α from the binding participation constraint. The principal thus maximises

$$n\left(m+q\right)-c\left(q,n\right)-\frac{1}{2}r\gamma^2\left(n^2\sigma_a^2+n\sigma_\varepsilon^2\right)$$

subject to the incentive compatibility constraints.

We first characterise the optimal contract under risk neutrality ($r=0$):

[6] See the Appendix for a proof of the concavity of the objective function.

Proposition 1 *If the agent is risk neutral ($r = 0$) the optimal contract never entails a strictly positive order bonus, i.e. $\beta = 0$, and the commission rate is $\gamma = 1$.*

Proof: See Appendix.

Intuitively, the order bonus β provides incentives only for quantity while the commission rate γ provides undistorted incentives for both quality and quantity: under a commission rate the agent's pay is a linear transformation of the principal's profits. Introducing an order bonus distorts the agent's decision favouring quantity (see, for instance, Feltham and Xie (1994) and Schnedler (2008) for analyses of performance measure 'congruence' in multitasking models).

However, this picture may change when the agent is risk averse. The reason is that a commission contract imposes income risk on the agent. An agent's risk aversion lowers her marginal return to quantity n as each additional order comes with an income risk (while risk aversion does not affect the returns to extra quality provision). In other words, risk aversion distorts the incentive effects of the commission, interfering with its ability to provide appropriate quantity incentives to risk averse agents. Here, an order bonus may become effective as it generates incentives to provide quantity without imposing risk on the agent. Indeed, we can show that under risk aversion the optimal contract entails both a (lower) commission rate and a strictly positive order bonus:

Proposition 2 *If the agent is risk averse, the optimal contract always includes an order bonus $\beta > 0$ and a commission rate $\gamma < 1$.*

Proof: See Appendix.

3.2 Intrinsic Motivation

We now impose more structure on the cost function in order to study comparative statics with respect to behavioural determinants of the agent's effort reaction. Consider the following specific cost function:

$$c(q,n) = n\left(\frac{\kappa}{2}q^2 - \frac{\eta}{2}\left(\tau - (q-q^*)^2\right)\right) + \frac{\nu}{2}n^2 \tag{1}$$

with $\eta, \kappa, \eta \in [0, \infty[$ and $\tau \in \left[0, q^{*2}\right]$. If $\tau = \eta = 0$, the agent is purely extrinsically motivated. In this case, the marginal cost of fulfilling another order (increasing n) as well the marginal cost of providing more quality per order (increasing q) is strictly increasing. Moreover, the cost of providing a quality level q on each order is linearly increasing in the number of order (i.e, for simplicity we ignore potential learning effects here).

If, however, $\eta > 0$, the cost function captures two behavioural motives for doing more and better work. To see the role the parameters τ and η play consider the intrinsic benefit from completing an order $\frac{\eta}{2}\left(\tau - (q-q^*)^2\right)$. The parameter η measures the agent's overall intrinsic motivation to complete orders, τ measures task enjoyment and q^* is a level of quality that is optimal from a welfare perspective.[7] An agent with a higher η has a stronger incentive to choose a quality level that is close to the normatively optimal level. For simplicity we assume that q^* is equal to the first-best quality.[8] If $\tau = 0$ then $\frac{\eta}{2}\left(\tau - (q-q^*)^2\right) < 0$; once an agent has decided to complete an order, the intrinsic motivation to provide higher quality puts a burden on the agent. Such a penalty for not doing a good job may be incurred by an individual that does not enjoy the task but is driven by a sense of duty, a feeling of obligation, or by conscientiousness. If, however, $\tau > 0$, her intrinsic motivation may give pleasure to the agent. In fact, if $\tau = q^{*2}$ then $\frac{\eta}{2}\left(\tau - (q-q^*)^2\right) > 0$ for all $q \in (0, 2q^*)$. In this case the agent enjoys working for on a client order (while of course still trading off the fun of working with the effort costs captured in the first term in (1)).

[7] Note that we have a setting in mind where the agent picks a set of items that is sent to customers via mail. The customers can then decide which items to keep and enjoy free returns of the items they do not want. As the firm incurs costs without earning anything on all returned items its ideal agent only selects items the customer wants to keep. In other words, in our setting the firm's objective function is closely aligned with the customer's interests.

[8] We thus have that $(n^*, q^*) = \arg\max_{n,q} n(m+q) - \left(n\left(\frac{\kappa}{2}q^2 - \frac{\eta}{2}\left(\tau - (q-q^*)^2\right)\right) + \frac{\nu}{2}n^2\right)$ which yields quality level $q^* = \frac{1}{\kappa}$.

We first characterise an agent's reaction to a contract with a commission rate $\gamma \in [0,1]$ and an order bonus $\beta \geq 0$. We substitute the marginal costs into the incentive compatibility conditions (IC1) and (IC2) to obtain the following result:

Proposition 3 *The agent chooses quality level*

$$q = \frac{\gamma + \eta q^*}{\kappa + \eta}$$

and quantity

$$n = \frac{1}{\nu + r\gamma^2 \sigma_a^2} \left(\beta + \gamma m + \frac{(\gamma + \eta q^*)^2}{2(\kappa + \eta)} - \eta \frac{q^{*2} - \tau}{2} - \frac{1}{2} r \gamma^2 \sigma_\varepsilon^2 \right).$$

The agent's choice of quality is increasing in her intrinsic motivation η while the effect of the commission rate γ on her choice of quality q is decreasing in η. The agent's choice of quantity n is decreasing in her risk aversion r and increasing in mean ability m. Her choice of n is increasing in η if and only if task enjoyment τ is sufficiently strong.

Proof: See Appendix.

If the agent is intrinsically motivated she makes a greater effort to provide quality; at the same time her quality provision is less responsive to the commission rate. The intrinsic desire to do a good job may lead to a reduction in quantity if task enjoyment τ is small, i.e., if the agent does not much enjoy the task per se but is nevertheless intrinsically compelled to provide quality, (e.g., by her conscientiousness). Such an agent anticipates that she will invest more effort every time she completes an order, receiving lower utility than a purely selfish agent. She rationally fulfils fewer orders, working harder on each individual order. Instead an agent who is intrinsically motivated and enjoys the task (sufficiently high τ) both completes a higher number of orders and provides higher quality on them.

Finally, when we further simplify the model by assuming that there is no uncertainty about the agent's talent ($\sigma_a^2 = 0$) we can derive closed-form solutions for the optimal contract parameters:

Proposition 4 *When $\sigma_a^2 = 0$ the optimal commission rate is*

$$\gamma = \frac{1}{1 + r\sigma_\varepsilon^2 (\kappa + \eta)},$$

and the optimal order bonus is

$$\beta = r\sigma_\varepsilon^2 \left(\frac{cm + \eta (m + q^*)}{1 + r\sigma_\varepsilon^2 (\kappa + \eta)} + \frac{1}{2 (1 + r\sigma_\varepsilon^2 (\kappa + \eta))^2} \right)$$

In the optimal contract the commission rate is strictly decreasing in the agent's risk aversion r and intrinsic motivation η. The optimal order bonus is strictly increasing in the agent's risk aversion r, intrinsic motivation η and mean ability m.

Proof: See Appendix.

3.3 Application: Deriving Experimental Predictions

We now turn to an application designed to yield testable predictions for the field experiment we will conduct. Consider a principal that initially pays her agent a pure commission without an order bonus. Our results from 3.1 suggest that if the agent is somewhat risk averse the principal should reduce the commission rate while introducing an order bonus. We analyse a particular change in contract the principal may experiment with: introducing an order bonus that is paid for by the simultaneous reduction of the commission rate.

To this end, consider a shift from a pure commission rate $\gamma_0 \in]0, 1]$ to a lower commission rate $\gamma_1 < \gamma_0$ combined with an order bonus $\beta > 0$; the relative size of $\gamma_0 - \gamma_1$ and β is calibrated on a population of agents in such a way that the average agent's pay per order remains constant if agents do not adjust quality. We now analyse the (heterogeneous) effects of such an intervention on expected quantity, quality and profits. For this purpose we assume that agents know their ability a when choosing their efforts and thus a person i is characterized by a vector $(a_i, r_i, \eta_i, \tau_i)$. Moreover, we assume that the personality traits are uncorrelated.[9]

First note that a shift that keeps the payment per order constant (at prior quality) in the population of agents will imply that

$$E\left[\gamma_0\left(a_i + q_{i0}\right)\right] = \beta + \gamma_1\left(m + E\left[q_{i0}\right]\right) \Leftrightarrow$$

$$\beta = \left(\gamma_0 - \gamma_1\right)\left(m + E\left[\frac{\gamma_0 + \eta q^*}{\kappa + \eta_i}\right]\right).$$

As

$$E\left[\Delta q_i\right] = E\left[\frac{\gamma_1 + \eta_i q^*}{\kappa + \eta_i} - \frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i}\right] = E\left[\frac{\gamma_1 - \gamma_0}{\kappa + \eta_i}\right]$$

it is clear that there will be a loss in quality and

$$\frac{\partial E\left[\frac{\gamma_1 - \gamma_0}{\kappa + \eta_i} |\eta_i\right]}{\partial \eta_i} = -\frac{\gamma_1 - \gamma_0}{\left(\kappa + \eta_i\right)^2} > 0$$

such that the loss in quality is the smaller, the more intrinsically motivated an agent is (higher η_i). We can moreover show that quantity increases (heterogeneously) and that profits increase for a certain type of agent:

Proposition 5 *Consider a shift from a pure commission rate $\gamma_0 \in]0, 1]$ to a lower commission rate $\gamma_1 < \gamma_0$ combined with an order bonus $\beta > 0$.*
(i) Such a shift reduces expected quality $E\left[\Delta q_i\right] < 0$. The effect is the smaller, the more intrinsically motivated an agent is (i.e., $\frac{\partial E[\Delta q_i|\eta_i]}{\partial \eta_i} > 0$).
(ii) The shift increases expected quantity $E\left[\Delta n_i\right] > 0$; the effect is the larger, the more risk averse the agent (i.e., $\frac{\partial E[\Delta n_i|r_i]}{\partial r_i} > 0$), the less able the agent (i.e., $\frac{\partial E[\Delta n_i|a_i]}{\partial a_i} < 0$), and the less intrinsically motivated the agent is (i.e., $\frac{\partial E[\Delta n_i|\eta_i]}{\partial \eta_i} < 0$).

[9] Note that only the proof of claim (iii) in Proposition 5 will hinge on the assumption that the traits are uncorrelated. The predictions for the effects on quality and quantity also arise when the traits are correlated.

(iii) The shift increases expected profits if and only if the agent is sufficiently risk averse.

Proof: See Appendix.

4 Conclusion

An important feature of freelance work is the freedom to set one's own schedule: a freelancer in our setting, for example, can change the number of jobs from one day to the next. In this paper we investigate the consequences of such worker flexibility for a firm trying to determine the optimal pay structure for its workforce. We formally analyse a principal-agent model that incorporates the agent's choice of quantity worked and accommodates heterogeneity in risk aversion, ability and intrinsic motivation. We show that the optimal contract for a risk-averse agent in this setting combines a sales-based commission rate with an order-based piece rate (order bonus). Moreover, the optimal order bonus is increasing in the agent's risk aversion, ability and intrinsic motivation.

Based on this model we derive predictions that we can test in a natural field experiment we will conduct in collaboration with an online platform. For this purpose we study the effects of a move from a pure commission rate to a combination of an order bonus and a lower commission rate (set in such a way that at prior quality levels expected payments per gig remain constant). The model predictions that we will test are the following: first, the intervention leads to an increase in average quantity and this increase is more pronounced for more risk averse agents. Second, the shift in compensation reduces quality but to a lesser extent the higher the agent's intrinsic motivation. Finally, for a shift of any given size, profits will increase if and only if the agent is sufficiently risk averse.

Extensions of our model provide a framework for studying other questions beyond the predictions we test in the field experiment. These include the dynamics of employee and employer learning (i.e. by studying the effecs of changes in σ_a^2 - the uncertainty about an agent's ability) as well as the selection and sorting of workers into and out of freelance jobs depending on individual characteristics and on the (menu of) contracts offered.

5 Appendix

Concavity of the agent's objective function
The objective function is strictly concave if

$$\begin{aligned} -c_{nn}(q,n) - r\gamma^2 2\sigma_a^2 &< 0 \\ -c_{qq}(q,n) &< 0 \\ \left(-c_{nn}(q,n) - r\gamma^2 2\sigma_a^2\right)\left(-c_{qq}(q,n)\right) - \left(\gamma - c_{qn}(q,n)\right)^2 &\geq 0 \end{aligned}$$

the latter is equivalent to

$$\left(c_{nn}(q,n) + r\gamma^2 2\sigma_a^2\right)\left(c_{qq}(q,n)\right) - \left(\gamma - c_{qn}(q,n)\right)^2 \geq 0$$

which always holds if

$$c_{nn}(q,n)\, c_{qq}(q,n) \geq \left(\gamma - c_{qn}(q,n)\right)^2$$

for all γ, q, n

(i) if $\gamma - c_{qn}(q,n) \geq 0$

$$\Leftrightarrow \sqrt{c_{nn}(q,n)\, c_{qq}(q,n)} \geq \left(\gamma - c_{qn}(q,n)\right)$$

which always holds (when $\gamma \leq 1$) if $\sqrt{c_{nn}(q,n)\, c_{qq}(q,n)} + c_{qn}(q,n) \geq 1$

(ii) if $\gamma - c_{qn}(q,n) < 0$

$$\Leftrightarrow \sqrt{c_{nn}(q,n)\, c_{qq}(q,n)} \geq -\left(\gamma - c_{qn}(q,n)\right)$$

$$\Leftrightarrow \gamma \geq c_{qn}(q,n) - \sqrt{c_{nn}(q,n)\, c_{qq}(q,n)}$$

which always holds because of the convexity of the cost function.

Proof of Proposition 1: The Lagrangean becomes

$$\mathcal{L} = n(m+q) - c(q,n) - \lambda_1\left(\beta + \gamma(m+q) - c_n(q,n)\right) - \lambda_2\left(\gamma n - c_q(q,n)\right)$$

and

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \beta} &= -\lambda_1 = 0 \\
\frac{\partial \mathcal{L}}{\partial \gamma} &= -\lambda_1 (m+q) - \lambda_2 n = 0 \\
\frac{\partial \mathcal{L}}{\partial n} &= m + q - c_n (q,n) + \lambda_1 c_{nn} (q,n) - \lambda_2 (\gamma - c_{qn} (q,n)) = 0 \\
\frac{\partial \mathcal{L}}{\partial q} &= n - c_q (q,n) - \lambda_1 (\gamma - c_{qn} (q,n)) + \lambda_2 c_{qq} (q,n) = 0.
\end{aligned}
$$

Thus $\lambda_1 = 0$ and, in turn, $\lambda_2 = 0$ such that

$$
\begin{aligned}
m + q - c_n (q,n) &= 0 \\
n - c_q (q,n) &= 0
\end{aligned}
$$

From the incentive compatibility constraints we must thus have that $\gamma n - c_q (q,n) = 0$ which implies $\gamma = 1$. And

$$\beta + \gamma (m+q) - c_n (q,n) = 0$$

which implies that $\beta = 0$. ■

Proof of Proposition 2: From the Lagrangean

$$
\begin{aligned}
\mathcal{L} &= n (m+q) - c(q,n) - \frac{1}{2} r \gamma^2 \left(n^2 \sigma_a^2 + n \sigma_\varepsilon^2\right) \\
&\quad -\lambda_1 \left(\beta + \gamma (m+q) - c_n (q,n) - \frac{1}{2} r \gamma^2 \left(2n \sigma_a^2 + \sigma_\varepsilon^2\right)\right) \\
&\quad -\lambda_2 (\gamma n - c_q (q,n))
\end{aligned}
$$

we obtain

$$\frac{\partial \mathcal{L}}{\partial \beta} = -\lambda_1 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = -r\gamma\left(n^2\sigma_a^2 + n\sigma_\varepsilon^2\right) - \lambda_1\left(m + q - r\gamma\left(2n\sigma_a^2 + \sigma_\varepsilon^2\right)\right) - \lambda_2 n \tag{3}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n} &= m + q - c_n(q,n) - r\gamma^2\left(2n\sigma_a^2 + \sigma_\varepsilon^2\right) \\ &\quad +\lambda_1\left(c_{nn}(q,n) + r\gamma^2\sigma_a^2\right) - \lambda_2\left(\gamma - c_{qn}(q,n)\right)\end{aligned} \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial q} = n - c_q(q,n) - \lambda_1\left(\gamma - c_{qn}(q,n)\right) + \lambda_2 c_{qq}(q,n). \tag{5}$$

Setting (2) through (5) equal to zero, we have $\lambda_1 = 0$ from (2) and consequently $\lambda_2 = -r\gamma\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)$ from (3). Substituting these and simplifying, the remaining two conditions become

$$m + q - c_n(q,n) - r\gamma^2 n\sigma_a^2 - r\gamma\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)c_{qn}(q,n) = 0, \tag{6}$$

$$n - c_q(q,n) - r\gamma\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)c_{qq}(q,n) = 0. \tag{7}$$

Using (IC2) we can substitute $c_q(q,n) = \gamma n$ into (7) to obtain

$$\begin{aligned}&n - \gamma n - r\gamma\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)c_{qq}(q,n) = 0 \\ &\Leftrightarrow \gamma = \frac{n}{n + r\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)c_{qq}(q,n)} < 1 \\ &\Leftrightarrow \gamma = \frac{1}{1 + r\left(\sigma_a^2 + \frac{\sigma_\varepsilon^2}{n}\right)c_{qq}(q,n)}\end{aligned} \tag{8}$$

when $r > 0$. Moreover, from (IC1) and (6) we have that

$$m+q-c_n(q,n)-r\gamma^2 n\sigma_a^2-r\gamma\left(n\sigma_a^2+\sigma_\varepsilon^2\right)c_{qn}(q,n) = \beta+\gamma(m+q)-c_n(q,n)-\frac{1}{2}r\gamma^2\left(2n\sigma_a^2+\sigma_\varepsilon^2\right)$$

$$m + q + r\gamma^2\left(\frac{1}{2}\sigma_\varepsilon^2\right) - r\gamma\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)c_{qn}(q,n) = \beta + \gamma(m+q)$$

$$\Leftrightarrow \beta = (1-\gamma)(m+q) - r\gamma\left(n\sigma_a^2 + \sigma_\varepsilon^2\right)c_{qn}(q,n) + r\gamma^2\frac{1}{2}\sigma_\varepsilon^2$$

If we substitute $\gamma = \frac{1}{1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}(q,n)}$

$$\begin{aligned}\Leftrightarrow \beta = \quad & \left(1 - \frac{1}{1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}\left(q,n\right)}\right)(m+q) \\ & -r\frac{1}{1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}\left(q,n\right)}\left(n\sigma_a^2+\sigma_\varepsilon^2\right)c_{qn}\left(q,n\right) \\ & +r\left(\frac{1}{1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}\left(q,n\right)}\right)^2\frac{1}{2}\sigma_\varepsilon^2\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad \beta = & \, r\left(n\sigma_a^2+\sigma_\varepsilon^2\right)\frac{\frac{1}{n}c_{qq}\left(q,n\right)(m+q)-c_{qn}\left(q,n\right)}{1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}\left(q,n\right)} \\ & +\frac{r\sigma_\varepsilon^2}{2\left(1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}\left(q,n\right)\right)^2}\end{aligned} \tag{9}$$

This will be strictly positive if $r > 0$ and

$$2\left(n\sigma_a^2+\sigma_\varepsilon^2\right)\left(\frac{1}{n}c_{qq}\left(q,n\right)(m+q)-c_{qn}\left(q,n\right)\right)\left(1+r\left(\sigma_a^2+\frac{\sigma_\varepsilon^2}{n}\right)c_{qq}\left(q,n\right)\right)+\sigma_\varepsilon^2>0$$

will always be the case for all $m > 0$ if

$$c_{qq}\left(q,n\right)q \geq c_{qn}\left(q,n\right)n.$$

Because of the assumption that $c_{qn}\left(q,n\right)n \leq c_q\left(q,n\right)$ this condition holds if

$$c_{qq}\left(q,n\right)q \geq c_q\left(q,n\right). \tag{10}$$

Note that due to $c_{qqq} \geq 0$ the marginal costs of quality are (weakly) convex and thus

$$c_q\left(n,0\right) \geq c_q\left(n,q\right)+c_{qq}\left(n,q\right)(0-q) \Leftrightarrow$$

$$c_{qq}(n,q)\,q \geq c_q(n,q) - \underbrace{c_q(n,0)}_{\leq 0} \quad \forall q,n$$

which implies condition (10). ■

Proof of Proposition 3: Conditions (IC1) and (IC2) become

$$\beta + \gamma(m+q) - \left(\frac{\kappa}{2}q^2 - \frac{\eta}{2}\left(\tau - (q-q^*)^2\right)\right) - \nu n - \frac{1}{2}r\gamma^2\left(2n\sigma_a^2 + \sigma_\varepsilon^2\right) = 0$$
$$\text{and } \gamma n - n\left(\kappa q + \eta(q-q^*)\right) = 0.$$

such that from (IC2)

$$\gamma n - n\left(\kappa q + \eta(q-q^*)\right) = 0$$

$$\Leftrightarrow q = \frac{\gamma + \eta q^*}{\kappa + \eta}$$

with

$$\frac{\partial q}{\partial \gamma} = \frac{1}{\kappa+\eta} \text{ and}$$

$$\begin{aligned}\frac{\partial q}{\partial \eta} &= \frac{q^*(\kappa+\eta) - (\gamma + \eta q^*)}{(\kappa+\eta)^2} \\ &= \frac{\kappa q^* - \gamma}{(\kappa+\eta)^2} = \frac{1-\gamma}{(\kappa+\eta)^2} > 0\end{aligned}$$

We compute n by rearranging (IC1) and simplifying to obtain

$$\beta + \gamma(m+q) - \left(\frac{\kappa}{2}q^2 - \frac{\eta}{2}\left(\tau - (q-q^*)^2\right)\right) - \nu n - \frac{1}{2}r\gamma^2\left(2n\sigma_a^2 + \sigma_\varepsilon^2\right) = 0$$

$$\beta + \gamma(m+q) - \left(\frac{\kappa}{2}q^2 - \frac{\eta}{2}\left(\tau - (q-q^*)^2\right)\right) - \nu n - r\gamma^2 n\sigma_a^2 - \frac{1}{2}r\gamma^2\sigma_\varepsilon^2 = 0$$

$$
\begin{aligned}
n &= \frac{\beta+\gamma\left(m+q\right)-\left(\frac{\kappa}{2}q^{2}-\frac{\eta}{2}\left(\tau-\left(q-q^{*}\right)^{2}\right)\right)-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}\\
&= \frac{\beta+\gamma\left(m+\frac{\gamma+\eta q^{*}}{\kappa+\eta}\right)-\left(\frac{\kappa}{2}\left(\frac{\gamma+\eta q^{*}}{\kappa+\eta}\right)^{2}-\frac{\eta}{2}\left(\tau-\left(\frac{\gamma+\eta q^{*}}{\kappa+\eta}-q^{*}\right)^{2}\right)\right)-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}\\
&= \frac{\beta+\gamma m+\gamma\frac{\gamma+\eta q^{*}}{\kappa+\eta}-\frac{\kappa}{2}\frac{\left(\gamma+\eta q^{*}\right)^{2}}{\left(\kappa+\eta\right)^{2}}+\frac{\eta}{2}\tau-\frac{\eta}{2}\left(\left(\frac{\gamma+\eta q^{*}}{\kappa+\eta}\right)^{2}-2\left(\frac{\gamma+\eta q^{*}}{\kappa+\eta}\right)q^{*}+q^{*2}\right)-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}\\
&= \frac{\beta+\gamma m+\gamma\frac{\gamma+\eta q^{*}}{\kappa+\eta}-\frac{\kappa}{2}\frac{\left(\gamma+\eta q^{*}\right)^{2}}{\left(\kappa+\eta\right)^{2}}+\frac{\eta}{2}\tau-\frac{\eta}{2}\left(\frac{\gamma+\eta q^{*}}{\kappa+\eta}\right)^{2}+\eta\left(\frac{\gamma+\eta q^{*}}{\kappa+\eta}\right)q^{*}-\frac{\eta}{2}q^{*2}-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}\\
&= \frac{\beta+\gamma m+\frac{\left(\gamma+\eta q^{*}\right)^{2}}{\kappa+\eta}-\frac{1}{2}\frac{\left(\gamma+\eta q^{*}\right)^{2}}{\left(\kappa+\eta\right)}-\eta\frac{q^{*2}-\tau}{2}-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}\\
&= \frac{\beta+\gamma m+\frac{\left(\gamma+\eta q^{*}\right)^{2}}{2\left(\kappa+\eta\right)}-\eta\frac{q^{*2}-\tau}{2}-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}.
\end{aligned}
$$

We then have

$$\frac{\partial n}{\partial\tau}=\frac{\frac{1}{2}\eta}{\nu+r\gamma^{2}\sigma_{a}^{2}}>0,$$

$$
\begin{aligned}
\frac{\partial n}{\partial r} &= \frac{\partial}{\partial r}\left\{\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)^{-1}\left(\beta+\gamma m+\frac{\left(\gamma+\eta q^{*}\right)^{2}}{2\left(\kappa+\eta\right)}-\eta\frac{q^{*2}-\tau}{2}-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}\right)\right\}\\
&= -\frac{\gamma^{2}\sigma_{a}^{2}}{\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)^{2}}\left(\beta+\gamma m+\frac{\left(\gamma+\eta q^{*}\right)^{2}}{2\left(\kappa+\eta\right)}-\eta\frac{q^{*2}-\tau}{2}-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}\right)-\frac{\gamma^{2}\sigma_{\varepsilon}^{2}}{2\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)}\\
&= -\frac{\gamma^{2}\sigma_{a}^{2}}{\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)}\frac{\beta+\gamma m+\frac{\left(\gamma+\eta q^{*}\right)^{2}}{2\left(\kappa+\eta\right)}-\eta\frac{q^{*2}-\tau}{2}-\frac{1}{2}r\gamma^{2}\sigma_{\varepsilon}^{2}}{\nu+r\gamma^{2}\sigma_{a}^{2}}-\frac{\gamma^{2}\sigma_{\varepsilon}^{2}}{2\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)}\\
&= -\frac{\gamma^{2}\sigma_{a}^{2}}{\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)}n-\frac{\gamma^{2}\sigma_{\varepsilon}^{2}}{2\left(\nu+r\gamma^{2}\sigma_{a}^{2}\right)}<0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial n}{\partial \eta} &= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{2\left(\gamma+\eta q^*\right)q^*2\left(\kappa+\eta\right)-\left(\gamma+\eta q^*\right)^2 2}{4\left(\kappa+\eta\right)^2}-\frac{q^{*2}-\tau}{2}\right) \\
&= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\left(\gamma+\eta q^*\right)\frac{\left(2c+\eta\right)q^*-\gamma}{2\left(\kappa+\eta\right)^2}-\frac{q^{*2}-\tau}{2}\right) \\
&= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{\left(\gamma+\eta q^*\right)\left(2c+\eta\right)q^*-\left(\gamma+\eta q^*\right)\gamma-q^{*2}\left(\kappa+\eta\right)^2}{2\left(\kappa+\eta\right)^2}+\frac{\tau}{2}\right) \\
&= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{\left(\gamma+\eta q^*\right)2cq^*+\left(\gamma+\eta q^*\right)\eta q^*-\left(\gamma+\eta q^*\right)\gamma-q^{*2}\left(\kappa^2+2c\eta+\eta^2\right)}{2\left(\kappa+\eta\right)^2}+\frac{\tau}{2}\right) \\
&= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{2cq^*\gamma+2c\eta q^{*2}+\gamma\eta q^*+\eta^2q^{*2}-\gamma^2-\eta q^*\gamma-\kappa^2q^{*2}-2c\eta q^{*2}-\eta^2q^{*2}}{2\left(\kappa+\eta\right)^2}+\frac{\tau}{2}\right) \\
&= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{2cq^*\gamma-\gamma^2-\kappa^2q^{*2}}{2\left(\kappa+\eta\right)^2}+\frac{\tau}{2}\right) \\
&= \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{\tau}{2}-\frac{\left(\gamma-cq^*\right)^2}{2\left(\kappa+\eta\right)^2}\right).
\end{aligned}
$$

$\frac{\partial n}{\partial \eta}$ is strictly negative if $\tau = 0$ and for $\tau = q^{*2}$ it is equal to

$$
\begin{aligned}
& \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{q^{*2}\left(\kappa+\eta\right)^2-\left(\gamma-cq^*\right)^2}{2\left(\kappa+\eta\right)^2}\right) \\
=\; & \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{\kappa^2q^{*2}+2c\eta q^{*2}+\eta^2q^{*2}-\gamma^2+2cq^*\gamma-\kappa^2q^{*2}}{2\left(\kappa+\eta\right)^2}\right) \\
=\; & \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{2c\eta q^{*2}+\eta^2q^{*2}+\left(2cq^*-\gamma\right)\gamma}{2\left(\kappa+\eta\right)^2}\right) \\
=\; & \frac{1}{\nu + r\gamma^2\sigma_a^2}\left(\frac{2\frac{\eta}{\kappa}+\frac{\eta^2}{\kappa^2}+\gamma\left(2-\gamma\right)}{2\left(\kappa+\eta\right)^2}\right) > 0
\end{aligned}
$$

Moreover, n is strictly increasing in τ which completes the proof. ■

Proof of Proposition 4: The optimal values of γ and β are obtained by substituting c_{qq} and c_{qn} into expressions (8) and (9):

$$\gamma = \frac{1}{1 + r\left(\sigma_a^2 + \frac{\sigma_\varepsilon^2}{n}\right) c_{qq}(q,n)}$$

$$\beta = r\left(n\sigma_a^2 + \sigma_\varepsilon^2\right) \frac{\frac{1}{n} c_{qq}(q,n)(m+q) - c_{qn}(q,n)}{1 + r\left(\sigma_a^2 + \frac{\sigma_\varepsilon^2}{n}\right) c_{qq}(q,n)} + \frac{r\sigma_\varepsilon^2}{2\left(1 + r\left(\sigma_a^2 + \frac{\sigma_\varepsilon^2}{n}\right) c_{qq}(q,n)\right)^2}$$

The cost function is given by

$$c(q,n) = n\left(\frac{\kappa}{2}q^2 - \frac{\eta}{2}\left(\tau - (q-q^*)^2\right)\right) + \frac{\nu}{2}n^2.$$

$$c_q = n\left(\kappa q + \eta\left(q - q^*\right)\right)$$

$$c_{qq} = n\left(\kappa + \eta\right)$$

$$c_{qn} = \left(\kappa q + \eta\left(q - q^*\right)\right)$$

Substituting and setting $\sigma_a^2 = 0$,

$$\gamma = \frac{1}{1 + r\sigma_\varepsilon^2(\kappa + \eta)}$$

which is strictly decreasing in r and in η.

Substituting c_{qq} and c_{qn} into β and setting $\sigma_a^2 = 0$ we obtain

$$\begin{aligned}
\beta &= r\sigma_\varepsilon^2 \frac{(\kappa+\eta)(m+q) - (cq + \eta(q-q^*))}{1 + r\sigma_\varepsilon^2(\kappa+\eta)} + \frac{r\sigma_\varepsilon^2}{2\left(1 + r\sigma_\varepsilon^2(\kappa+\eta)\right)^2} \\
&= r\sigma_\varepsilon^2 \frac{cm + \eta(m+q^*)}{1 + r\sigma_\varepsilon^2(\kappa+\eta)} + \frac{r\sigma_\varepsilon^2}{2\left(1 + r\sigma_\varepsilon^2(\kappa+\eta)\right)^2} \\
&= r\sigma_\varepsilon^2 \left(\frac{cm + \eta(m+q^*)}{1 + r\sigma_\varepsilon^2(\kappa+\eta)} + \frac{1}{2\left(1 + r\sigma_\varepsilon^2(\kappa+\eta)\right)^2}\right).
\end{aligned}$$

For the comparative statics note that β can be rearranged to obtain

$$\beta = \sigma_\varepsilon^2 \left(\frac{cm + \eta(m+q^*)}{\frac{1}{r} + \sigma_\varepsilon^2(\kappa+\eta)} + \frac{1}{2\left(\frac{1}{r} + \sigma_\varepsilon^2(\kappa+\eta)\right)^2}\right),$$

which is strictly increasing in r and in m. Finally,

$$\begin{aligned}\frac{\partial\beta}{\partial\eta} &= r\sigma_\varepsilon^2\left(\frac{(m+q^*)\left(1+r\sigma_\varepsilon^2(\kappa+\eta)\right)-(cm+\eta(m+q^*))\,r\sigma_\varepsilon^2}{\left(1+r\sigma_\varepsilon^2(\kappa+\eta)\right)^2}-\frac{r\sigma_\varepsilon^2}{\left(1+r\sigma_\varepsilon^2(\kappa+\eta)\right)^3}\right)\\ &= r\sigma_\varepsilon^2\left(\frac{m+q^*+r\sigma_\varepsilon^2(\kappa+\eta)(m+q^*)-r\sigma_\varepsilon^2cm-r\sigma_\varepsilon^2\eta(m+q^*)}{\left(1+r\sigma_\varepsilon^2(\kappa+\eta)\right)^2}-\frac{r\sigma_\varepsilon^2}{\left(1+r\sigma_\varepsilon^2(\kappa+\eta)\right)^3}\right)\\ &= \frac{r\sigma_\varepsilon^2}{\left(1+r\sigma_\varepsilon^2(\kappa+\eta)\right)^2}\left(m+q^*+r\sigma_\varepsilon^2\left(\frac{r\sigma_\varepsilon^2(\kappa+\eta)}{1+r\sigma_\varepsilon^2(\kappa+\eta)}\right)\right)>0.\end{aligned}$$

■

Proof of Proposition 5:

Claim (i) directly follows from the considerations in the text.

Claim (ii): Consider

$$\begin{aligned}\Delta n_i &= \frac{1}{\nu}\left(\beta+\gamma_1 a_i+\frac{(\gamma_1+\eta_i q^*)^2}{2(\kappa+\eta_i)}-\eta_i\frac{q^{*2}-\tau}{2}-\frac{1}{2}r_i\gamma_1^2\sigma_\varepsilon^2\right.\\ &\qquad\left.-\left(\gamma_0 a_i+\frac{(\gamma_0+\eta_i q^*)^2}{2(\kappa+\eta_i)}-\eta_i\frac{q^{*2}-\tau}{2}-\frac{1}{2}r_i\gamma_0^2\sigma_\varepsilon^2\right)\right)\end{aligned}$$

$$=\frac{1}{\nu}\left(\beta+(\gamma_1-\gamma_0)\,a_i+\frac{(\gamma_1+\eta_i q^*)^2}{2(\kappa+\eta_i)}-\frac{(\gamma_0+\eta_i q^*)^2}{2(\kappa+\eta_i)}+\frac{1}{2}r_i\left(\gamma_0^2-\gamma_1^2\right)\sigma_\varepsilon^2\right)$$

$$=\frac{1}{\nu}\left(\beta+(\gamma_1-\gamma_0)\,a_i+\frac{\gamma_1^2-\gamma_0^2+2(\gamma_1-\gamma_0)\,\eta_i q^*}{2(\kappa+\eta_i)}+\frac{1}{2}r_i\left(\gamma_0^2-\gamma_1^2\right)\sigma_\varepsilon^2\right)$$

$$=\frac{1}{\nu}\left(\beta+(\gamma_1-\gamma_0)\,a_i+\frac{(\gamma_1-\gamma_0)(\gamma_1+\gamma_0)+2(\gamma_1-\gamma_0)\,\eta_i q^*}{2(\kappa+\eta_i)}+\frac{1}{2}r_i\left(\gamma_0^2-\gamma_1^2\right)\sigma_\varepsilon^2\right)$$

$$=\frac{1}{\nu}\left(\beta-(\gamma_0-\gamma_1)\left(a_i+\frac{\gamma_1+\gamma_0+2\eta_i q^*}{2(\kappa+\eta_i)}\right)+\frac{1}{2}r_i\left(\gamma_0^2-\gamma_1^2\right)\sigma_\varepsilon^2\right)$$

with $\beta = (\gamma_0 - \gamma_1)\left(m + E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)$

$$\Delta n_i = \frac{1}{\nu}\left((\gamma_0 - \gamma_1)\left(m + E\left[\frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i}\right]\right) - (\gamma_0 - \gamma_1)\left(a_i + \frac{\gamma_1 + \gamma_0 + 2\eta_i q^*}{2(\kappa + \eta_i)}\right) + \frac{1}{2} r_i \left(\gamma_0^2 - \gamma_1^2\right)\sigma_\varepsilon^2\right)$$

$$= \frac{1}{\nu}\left((\gamma_0 - \gamma_1)\left(m - a_i + E\left[\frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i}\right] - \frac{\gamma_1 + \gamma_0 + 2\eta_i q^*}{2(\kappa + \eta_i)}\right) + \frac{1}{2} r_i \left(\gamma_0^2 - \gamma_1^2\right)\sigma_\varepsilon^2\right)$$

Now consider the effect of the treatment on quantity in the population, which is given by

$$E[\Delta n_i] = \frac{1}{\nu}\left((\gamma_0 - \gamma_1)\left(E\left[\frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i} - \frac{\gamma_1 + \gamma_0 + 2\eta_i q^*}{2(\kappa + \eta_i)}\right]\right) + \frac{1}{2} E[r_i]\left(\gamma_0^2 - \gamma_1^2\right)\sigma_\varepsilon^2\right)$$

$$= \frac{1}{\nu}\left((\gamma_0 - \gamma_1)\left(E\left[\frac{\gamma_0 - \gamma_1}{2(\kappa + \eta_i)}\right]\right) + \frac{1}{2} E[r_i]\left(\gamma_0^2 - \gamma_1^2\right)\sigma_\varepsilon^2\right) > 0$$

$$= \frac{1}{\nu}\left(\frac{(\gamma_0 - \gamma_1)^2}{2} E\left[\frac{1}{\kappa + \eta_i}\right] + \frac{1}{2} E[r_i]\left(\gamma_0^2 - \gamma_1^2\right)\sigma_\varepsilon^2\right) > 0$$

Now we can consider the partial derivatives

$$\begin{aligned}
\frac{\partial E[\Delta n_i | r_i]}{\partial r_i} &= \frac{\left(\gamma_0^2 - \gamma_1^2\right)\sigma_\varepsilon^2}{2c_n} > 0, \\
\frac{\partial E[\Delta n_i | a_i]}{\partial a_i} &= -\frac{\gamma_0 - \gamma_1}{\nu} < 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial E[\Delta n_i | \eta_i]}{\partial \eta_i} &= -\frac{(\gamma_0 - \gamma_1)}{\nu}\frac{4q^*(\kappa + \eta_i) - 2(\gamma_1 + \gamma_0 + 2\eta_i q^*)}{4(\kappa + \eta_i)^2} \\
&= -\frac{(\gamma_0 - \gamma_1)}{\nu}\frac{2cq^* - \gamma_1 - \gamma_0}{2(\kappa + \eta_i)^2} \\
&= -\frac{(\gamma_0 - \gamma_1)}{\nu}\frac{2 - \gamma_1 - \gamma_0}{2(\kappa + \eta_i)^2} < 0.
\end{aligned}$$

Claim (iii): Compare profits generated by an agent i before and after the shift. Initial profits are

$$\Pi_{i0} = (1-\gamma_0)n_{i0}(a_i + q_{i0})$$

Profits after the shift are

$$\Pi_{i1} = (1-\gamma_1)n_{i1}(a_i + q_{i1}) - \beta n_{i1}$$

such that the change in profits is

$$\Delta\Pi_i = \left((1-\gamma_1)(a_i + q_{i1}) - \beta\right) n_{i1} - (1-\gamma_0)\, n_{i0}\,(a_i + q_{i0})$$

substituting $n_{it} = \frac{1}{\nu}\left(\beta \cdot t + \gamma_t a_i + \frac{(\gamma_t + \eta_i q^*)^2}{2(\kappa+\eta_i)} - \eta_i \frac{q^{*2} - \tau_i}{2} - \frac{1}{2} r_i \gamma_t^2 \sigma_\varepsilon^2\right), t \in \{0,1\}$,

$$\begin{aligned}
\Delta\Pi_i \;&=\; \left((1-\gamma_1)\left(a_i + \frac{\gamma_1 + \eta_i q^*}{\kappa + \eta_i}\right) - \beta\right)\frac{1}{\nu}\left(\beta + \gamma_1 a_i + \frac{(\gamma_1 + \eta_i q^*)^2}{2(\kappa + \eta_i)} - \eta_i \frac{q^{*2} - \tau_i}{2} - \frac{1}{2} r_i \gamma_1^2 \sigma_\varepsilon^2\right) \\
&\quad -(1-\gamma_0)\frac{1}{\nu}\left(\gamma_0 a_i + \frac{(\gamma_0 + \eta_i q^*)^2}{2(\kappa+\eta_i)} - \eta_i \frac{q^{*2} - \tau_i}{2} - \frac{1}{2} r_i \gamma_0^2 \sigma_\varepsilon^2\right)\left(a_i + \frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i}\right)
\end{aligned}$$

First, note that $\Delta\Pi_i$ is a linear function of r_i. Taking the first derivative with respect to r_i we obtain

$$\begin{aligned}
\frac{\partial \Delta\Pi_i}{\partial r_i} \;&=\; \left((1-\gamma_1)\left(a_i + \frac{\gamma_1 + \eta_i q^*}{\kappa+\eta_i}\right) - \beta\right)\frac{1}{\nu}\left(-\frac{1}{2}\gamma_1^2\sigma_\varepsilon^2\right) - (1-\gamma_0)\frac{1}{\nu}\left(-\frac{1}{2}\gamma_0^2\sigma_\varepsilon^2\right)\left(a_i + \frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i}\right) \\
&=\; \frac{\sigma_\varepsilon^2}{2c_n}\left[\gamma_0^2(1-\gamma_0)\left(a_i + \frac{\gamma_0 + \eta_i q^*}{\kappa+\eta_i}\right) - \gamma_1^2(1-\gamma_1)\left(a_i + \frac{\gamma_1 + \eta_i q^*}{\kappa + \eta_i}\right) + \beta\gamma_1^2\right].
\end{aligned}$$

Substituting $\beta = (\gamma_0 - \gamma_1)\left(m + E\left[\frac{\gamma_0 + \eta_i q^*}{\kappa + \eta_i}\right]\right)$

$$\begin{aligned}
\frac{\partial \Delta\Pi_i}{\partial r_i} \;&=\; \frac{\sigma_\varepsilon^2}{2c_n}\Bigg[\gamma_0^2(1-\gamma_0)\left(a_i + \frac{\gamma_0 + \eta_i q^*}{\kappa+\eta_i}\right) - \gamma_1^2(1-\gamma_1)\left(a_i + \frac{\gamma_1 + \eta_i q^*}{\kappa + \eta_i}\right) \\
&\quad + \gamma_1^2(\gamma_0 - \gamma_1)\left(m + E\left[\frac{\gamma_0 + \eta_i q^*}{\kappa+\eta_i}\right]\right)\Bigg]
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial E\left[\Delta\Pi_i|r_i\right]}{\partial r_i} &= \frac{\sigma_\varepsilon^2}{2c_n}\left[\gamma_0^2\left(1-\gamma_0\right)\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)-\gamma_1^2(1-\gamma_1)\left(m+E\left[\frac{\gamma_1+\eta_i q^*}{\kappa+\eta_i}\right]\right)\right. \\
&\quad \left.+\gamma_1^2\left(\gamma_0-\gamma_1\right)\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)\right] \\
&> \frac{\sigma_\varepsilon^2}{2c_n}\left[\gamma_0^2\left(1-\gamma_0\right)\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)-\gamma_1^2(1-\gamma_1)\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)\right. \\
&\quad \left.+\gamma_1^2\left(\gamma_0-\gamma_1\right)\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)\right] \\
&= \frac{\sigma_\varepsilon^2\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)}{2c_n}\left[\gamma_0^2\left(1-\gamma_0\right)-\gamma_1^2(1-\gamma_1)+\gamma_1^2\left(\gamma_0-\gamma_1\right)\right] \\
&= \frac{\sigma_\varepsilon^2\left(m+E\left[\frac{\gamma_0+\eta_i q^*}{\kappa+\eta_i}\right]\right)}{2c_n}\left(\gamma_0^2-\gamma_1^2\right)\left(1-\gamma_0\right)>0,
\end{aligned}$$

as required. ■

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