Labor Market Dynamics:
The Role of Matching Efficiency and Quits

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Abstract

Separation rates fluctuate more over the business cycle in continental European countries than in Anglo-Saxon countries and thereby explain a larger fraction of unemployment dynamics. This paper shows in a search and matching model that these differences may either be driven by lower matching efficiency or lower quit rates in continental Europe. We use administrative data for Germany to test for the quantitative role of these two channels. We find no empirical evidence for the connection between matching efficiency and layoff rate dynamics. By contrast, our paper finds strong evidence for a negative connection between the quit rate and layoff rate dynamics and thereby provides an explanation for the more important role of separations in continental European countries.

\textit{JEL classification: E32, E24, J64.}

\textit{Keywords:} Separation Rate, Layoffs, Quits, Europe, United States.

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1 Motivation

Elsby et al. (2013) show that the job-finding rate is the major driver of unemployment fluctuations in Anglo-Saxon countries, while the separation rate is more important in continental European and in Nordic countries. Based on labor market flows for different OECD countries, they find a 85:15 split (job findings versus separations) for Anglo-Saxon countries and a 45:55 split for continental European and Nordic countries.\(^1\) Jung and Kuhn (2014) confirm these empirical findings in a detailed comparison of labor market dynamics between Germany and the United States. They show in a search and matching model that a lower matching efficiency in Germany can explain the higher separation rate volatility. Gartner et al. (2012) show in a simple partial equilibrium model that a lower quit rate in Germany may be the driver for a higher separation rate volatility.\(^2\)

We show in our paper that in a search and matching model both different matching efficiencies and different quit rates may be powerful drivers for the observed cross-country differences of job-finding rate and separation rate dynamics. However, the assumptions on wage bargaining are crucial for the importance of these channels. Against this background, we use rich administrative worker-level and establishment-level data for Germany to test for these channels (between sectors and at the establishment level). We find no evidence for a meaningful connection between matching efficiency and hiring/layoff dynamics at the sectoral level. By contrast, we find strong empirical evidence at the establishment level that higher quit rates are associated with lower layoff rate volatilities. In addition, we find evidence that layoffs are a less important driver for employment fluctuations at establishments with higher quit rates.

In the theoretical part of the paper, we show in a standard search and matching model that both, a lower matching efficiency and a lower quit rate, may lead to a larger separation rate volatility. However, the strength and existence of these two channels

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\(^1\) There is a large literature that examines worker flows in the U.S. (e.g., Fallick and Fleischman, 2004; Fujita and Ramey, 2009; Elsby et al., 2009), and in Europe (e.g., Petrongolo and Pissarides, 2008; Pissarides, 2009; Elsby et al., 2013; Jung and Kuhn, 2014). However, the reasons for the differences in labor market dynamics are still not fully understood.

\(^2\) Gartner et al. (2012) do not embed their argument into a search and matching model, they do discuss the role of different bargaining protocols and they do not show any evidence based on administrative data.
depends very much on the underlying bargaining mechanism. It is common practice in the search and matching literature to assume that the fall-back option under Nash bargaining is the outside option (i.e. unemployment). Under this assumption, market tightness shows up in the wage equation. Different matching efficiencies affect the dynamics of the wage and thereby the separation rate (see Jung and Kuhn (2014) for details). By contrast, if the fall-back option in the bargaining game is a waiting option (similar to Hall and Milgrom (2008)), wages do not depend on market tightness. We show analytically and numerically that matching efficiency does not matter for the dynamics of the separation rate in this case. We also show that a lower exogenous quit rate leads to larger fluctuations of the separation rate, as employment relationship last for longer and thereby the surplus fluctuates more than with a higher quit rate. This effect is stronger when market tightness does not show up in the bargaining.

To make statements on the relative role of job findings and separations for unemployment fluctuations, we also analyze the effect the effects of matching efficiency and quits on the job-finding rate. Matching efficiency has no effects on the volatility of the job-finding rate (independently of the assumed wage formation). By contrast, we show numerically that the volatility of the job-finding rate drops with higher quit rates.

Unfortunately, it is not possible to estimate the relative importance of matching efficiency and quits for labor market dynamics in a meaningful way based cross-country data because (i) on a cross-country level comparable time series are typically short, (ii) there are no comparable vacancy definitions to estimate matching efficiencies and (iii) separations are not split up into quits and layoffs.

Therefore, in the empirical part of the paper, we use German administrative labor market data to analyze the importance of these two channels. To test for the connection between matching efficiency and the layoff rate/job-finding rate volatility, we use a 2 percent worker sample, namely the Sample of Integrated Labour Market Biographies (SIAB), in combination with the Federal Employment Agency’s vacancy data. This allows us to estimate matching efficiencies at the industry level. We find no evidence for a negative connection between matching efficiency and layoff rate volatility (as in a
search and matching model with standard Nash bargaining). As shown in our theoretical part, these results may be rationalized by deviating from standard Nash bargaining with unemployment as fall-back option.

To test for the connection between the quit rate and the layoff rate/job-finding rate volatility, we start by running sector-level regressions based on the SIAB. We find some evidence for the negative connection between the quit rate and the volatility of layoffs and the job-finding rate.

Matching efficiency can only be measured at aggregated levels such as the sector. This leads to a small number of cross-sectional observations.\(^3\) However, as quit rate are available at the establishment level, we can test for the connection between quits rates and hiring/layoff volatility at this level. Therefore, we use the Administrative Wage and Labor Market Flow Panel (AWFP). The AWFP covers the universe of German establishments. In line with previous literature, we approximate quits by employment-to-employment transitions and layoffs by employment-to-unemployment or employment-to-non-employment transitions. In line with our theoretical results, we find a strong negative connection between the level of the quit rate and the volatility of the layoff rate. Interestingly, this connection is very robust and holds in each industry sector.

Overall, our paper finds a strong role for the quit rate on hiring and layoff dynamics and thereby the relative importance of these two channels for (un)employment fluctuations. Thereby, our empirical results open up the black box of labor market dynamics and thereby help to understand cross-country differences. This is an important prerequisite for understanding the effects of labor market institutions and aggregate shocks.

The rest of the paper is structured as follows. Section 2 derives a standard search and matching model with endogenous transitions between employment and unemployment. We show numerically and analytically how different matching efficiencies and quit rates affect the joint surplus of workers and firms and thereby the volatility of the job-finding rate and the layoff rate. Section 3 shows the empirical connection between matching efficiency and layoff rate as well as volatilities at the industry sectoral level for Germany.\(^4\)

\(^3\)Alternatively, matching efficiency may be measured at other aggregated levels such as occupation, which also leads to a small number of observations.
Section 4 shows the connection between quit rates and layoff rates as well as volatilities at the establishment level for Germany. Section 5 concludes.

2 Theory

We derive a standard search and matching model with endogenous separations (which serve as the theoretical counterpart to the empirically measured layoffs) and exogenous quits. Based on this model framework, we illustrate the effects of different matching efficiencies and quit rates on the volatility of separations.

2.1 Model Derivation

We use a standard search and matching model with endogenous separations. Matches, \( m_t \), follow a Cobb-Douglas constant returns matching function:

\[
m_t = \chi_t u_t^\alpha v_t^{1-\alpha},
\]

where \( \chi_t \) is the matching efficiency, \( u_t \) are unemployed workers and \( v_t \) are vacancies at time \( t \), and \( \alpha \) is the elasticity of the matching function with respect to unemployment.

The job-finding rate, \( p_t \), and the worker-finding rate, \( q_t \), are functions of market tightness (\( \theta_t = v_t/u_t-1 \)):

\[
p_t = \frac{m_t}{u_t} = \chi_t \theta_t^{1-\alpha},
\]

\[
q_t = \frac{m_t}{v_t} = \chi_t \theta_t^{-\alpha}.
\]
\[ V_t = -\kappa + q_t E_t \delta \left[ (1 - \sigma) (1 - \phi_{t+1}) J_{t+1}^I + (1 - (1 - \sigma) (1 - \phi_{t+1})) V_{t+1} \right] + (1 - q_t) E_t V_{t+1}. \] (4)

In equilibrium, due to the free-entry condition, the value of vacancies is driven to zero. Thus, vacancies are posted up to the point where hiring costs \((\kappa/q_t)\) are equal to the expected discounted profits from hiring

\[ \frac{\kappa}{q_t} = \delta E_t \left[ (1 - \sigma) (1 - \phi_{t+1}) J_{t+1}^I \right] \] (5)

Existing worker-firm pairs each period draw a realization, \(\varepsilon_{it}\), from an idiosyncratic match-specific shock distribution. The shock is drawn from a stable density function \(g(\varepsilon_{it})\). It is iid across workers and time. The value of an existing match with shock realization \(\varepsilon_{it}\) is

\[ J_t^I (\varepsilon_{it}) = a_t - w(\varepsilon_{it}) - \varepsilon_{it} + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J_{t+1}^I, \] (6)

where \(a\) is the productivity, \(w\) is the wage, \(\sigma\) is an exogenous quit probability, \(\phi\) is the endogenous separation rate, and \(J^I\) is the expected present value of an incumbent worker (which will be defined below). New and existing matches are hit by idiosyncratic cost-shocks and may therefore split up.

Note that we assume that workers that quit do not become unemployed. Instead, they exogenously transition to another firm.

Based on this shock realization, firms decide which workers they want to keep and which workers they want to fire. Firms are indifferent between firing and not firing at the cutoff point \(\tilde{\varepsilon}_{it}\), where \(J_t^I (\tilde{\varepsilon}_{it}) = 0:\)

\[ \tilde{\varepsilon}_{it} = a_t - w(\tilde{\varepsilon}_{it}) + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J_{t+1}^I. \] (7)

This allows us to define the endogenous firing rate \(\phi_t\), which is defined as:
\[
\phi_t = 1 - \int_{-\infty}^{\tilde{\varepsilon}_{it}} g(\varepsilon_{it}) \, d\varepsilon_{it}. \tag{8}
\]

Finally, firms calculate the expected ex-ante present value for a match (relying on the expected realization of the match-specific shock):

\[
J^I_t = a_t - \bar{w}_t - H(\tilde{\varepsilon}_{it}) + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J^I_{t+1}, \tag{9}
\]

where we define the average expected realization of the idiosyncratic shock and the expected wage as:

\[
H(\tilde{\varepsilon}_{it}) = \frac{\int_{-\infty}^{\tilde{\varepsilon}_{it}} \varepsilon_{it} g(\varepsilon_{it}) \, d\varepsilon_{it}}{1 - \phi_t}, \tag{10}
\]

\[
\bar{w}_t = \frac{\int_{-\infty}^{\tilde{\varepsilon}_{it}} w(\varepsilon_{it}) g(\varepsilon_{it}) \, d\varepsilon_{it}}{1 - \phi_t}. \tag{11}
\]

As usual in a search and matching model, wages are bilaterally efficient as long as they are in between workers’ and firms’ reservation wages. We will derive two wage formation mechanism in the next section that are in line with bilateral efficiency. However, the two wage formation mechanisms will differ in one important dimension.

Workers who are not fired in the current period and those who are newly matched (and not immediately fired) are employed. Thus, the employment rate, \( n_t \), is defined as:

\[
n_{t+1} = (1 - \phi_t) (n_t + p_t u_t), \tag{12}
\]

with unemployment rate, \( u_t \):

\[
u_t = 1 - n_t. \tag{13}
\]

Note that the quit rate does not show up in the aggregate employment dynamics equation, as we assume that workers who quit their job exogenously transition to another firm.
2.2 Wage Formation

To close the model, we have to take a stance on wage formation. We present two possible versions.

2.2.1 Standard Nash: Fall-Back = Unemployment

Under standard Nash bargaining, workers and firms split whenever they do not agree. Thus, firms’ value under production is

$$J^I(\varepsilon_{it}) = a_t - w(\varepsilon_{it}) - \varepsilon_{it} + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J^I_{t+1},$$  \hspace{1cm} (14)

and the fallback-option is that the match is destroyed:

$$\bar{J}^I(\varepsilon_{it}) = 0.$$  \hspace{1cm} (15)

Workers’ value is

$$W(\varepsilon_{it}) = w(\varepsilon_{it}) + \delta E_t (1 - \sigma) (1 - \phi_{t+1}) W_{t+1} + \delta E_t (1 - (1 - \sigma) (1 - \phi_t)) U_{t+1}$$  \hspace{1cm} (16)

The fallback option is unemployment:

$$W(\varepsilon_{it}) = U_t = b + \delta E_{t+1} p_t (1 - \sigma) (1 - \phi_{t+1}) W_{t+1} + \delta E_t (1 - p_t (1 - \sigma) (1 - \phi_{t+1})) U_{t+1}.$$  \hspace{1cm} (17)

Thus, the Nash product is:

$$\Lambda(\varepsilon_{it}) = [J(\varepsilon_{it})]^{1-\gamma} [W(\varepsilon_{it}) - U_t]^{\gamma}$$  \hspace{1cm} (18)

where $\gamma$ is workers’ bargaining power.

Maximizing the Nash product with respect to wages yields (see Appendix for details):
\[ w(\varepsilon_{it}) = \gamma (a_t - \varepsilon_{it} + \kappa \theta_t) + (1 - \gamma) b, \]  

(19)

where \( \gamma \) is the bargaining power of workers.

### 2.2.2 Alternative Bargaining: Fall-Back = Waiting Period

As an alternative, we assume that the fallback option is equal to a waiting option. We assume that the fallback position is a waiting period (without negotiations) and to continue negotiations next period. This assumption is analogous to Hall and Milgrom (2008), Lechthaler et al. (2010), and Snower and Merkl (2006).

While the value of producing/working remains the same, the fallback option changes. Firms’ fallback option is

\[ \bar{J}^I_t (\varepsilon_{it}) = 0 + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J^I_{t+1}, \]  

(20)

and workers’ fallback option is:

\[ \bar{W}^I_t (\varepsilon_{it}) = b + \delta E_t (1 - \sigma) (1 - \phi_{t+1}) W_{t+1} + \delta E_t (1 - (1 - \sigma) (1 - \phi_{t+1})) U_{t+1}, \]  

(21)

where we assume for simplicity that the home production or government transfer is the same as under unemployment.

Thus, the Nash product is

\[ \Lambda (\varepsilon_{it}) = [a_t - w(\varepsilon_{it}) - \varepsilon_{it}]^{1-\gamma} [w(\varepsilon_{it}) - b]^\gamma. \]  

(22)

Maximizing the Nash product with respect to wages yields:

\[ w(\varepsilon_{it}) = \gamma (a_t - \varepsilon_{it}) + (1 - \gamma) b. \]  

(23)

Intuitively, under our alternative bargaining structure, both workers’ and firms’ outside option increases and thereby the reservation wage of workers increases and the reser-
vation wage of firms decreases. Comparing the two wage equations, two things stand out. First, the standard Nash bargaining wage is larger than the alternative wage formation. Second, the standard Nash bargained wage is responsive to the market tightness and thereby more responsive to the business cycle. This will turn out to be important when we analyze the impact of matching efficiency on labor market dynamics.

2.3 Analytical Results and Illustration

2.3.1 Analytical Results

Jung and Kuhn (2014) derive analytical results for the effect of the matching efficiency on the volatility of the job-finding rate (none) and the firing rate (negative). These analytical results are confirmed by our numerical exercise below.

It is important to keep in mind that Jung and Kuhn (2014) follow the standard convention in the literature that the fallback option under bargaining is the outside option. In different words, whenever workers and firm disagree in equilibrium, a match will be destroyed. In this case, market tightness shows up in the wage.

In the Appendix, we derive analytical results for the case where the outside option is a waiting period. In this case, market tightness does not show up in the bargaining. For this scenario, we derive the following analytical results:

\[
\frac{\partial^2 \ln \phi}{\partial \ln a \partial \chi} = 0 \quad (24)
\]

\[
\frac{\partial^2 \ln p}{\partial \ln a \partial \chi} = 0
\]

Both the elasticity of the job-finding rate and the firing rate are unaffected by the level of the matching efficiency under the alternative bargaining protocol.

We also show that the cross-derivative of the firing rate elasticity with respect to quits is positive:

\[
\frac{\partial^2 \ln \phi}{\partial \ln a \partial \sigma} > 0 \quad (25)
\]
As the elasticity of the firing rate with respect to productivity is negative, a positive cross-derivative means that a higher quit rate is associated with a lower volatility of the firing rate.

The cross-derivative of the job-finding rate elasticity with respect to quits is not clearly determined.

\[
\frac{\partial^2 \ln p}{\partial \ln a \partial \sigma} = ? \tag{26}
\]

We argue in the Appendix that a negative effect of quits on the job-finding rate elasticity is quite likely. This conjecture is confirmed by our numerical exercise below.

### 2.3.2 Comparative Statics: Illustration

What is the underlying intuition for the presented analytical results? Remember that the separation rate dynamics is driven by three equations, namely the cutoff point:

\[
\tilde{\epsilon}_{it} = a_t - w(\tilde{\epsilon}_{it}) + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J^I_{t+1}, \tag{27}
\]

the present value of an incumbent worker

\[
J^I_t = a_t - \bar{w}_t - H(\tilde{\epsilon}_{it}) + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J^I_{t+1}, \tag{28}
\]

and the separation rate:

\[
\phi_t = 1 - \int_{-\infty}^{\tilde{\epsilon}_{it}} g(\tilde{\epsilon}_{it}) d\tilde{\epsilon}_{it}. \tag{29}
\]

To illustrate the key model mechanisms, we write all equations in terms of the steady state and substitute (28) into (27):

\[
\tilde{\epsilon} = a - w(\tilde{\epsilon}) + (1 - \sigma) (1 - \phi) \delta \frac{a - \bar{w} - H}{1 - \delta (1 - \sigma) (1 - \phi)}. \tag{30}
\]

To gain intuition on this term, we assume \( \delta = 1 \) (i.e., no discounting), we omit \( \sigma \phi \approx 0 \), we ignore the future expected idiosyncratic shock realization, \( H \), and we use the
relationship that \( w(\tilde{\varepsilon}) = w - \alpha \varepsilon \), which holds in both bargaining regimes above. Thus, the simplified cutoff point is:

\[
\tilde{\varepsilon} = \frac{a - w}{(\sigma + \phi)(1 - \alpha)}.
\]

How do a lower quit rate and a higher matching efficiency affect the reaction of the joint surplus to productivity shocks? A lower quit rate \( \sigma \) increases the average employment span, thereby the expected present value and the cutoff point rise. In different words, lower quits are associated with lower firings. In addition, whenever the quit rate is low, the cutoff point can be expected to react more sensitively to productivity changes. To understand this point, assume the extreme case where quits are 100%, i.e., \( \sigma = 1 \). In this case, we can rewrite equation (31) as:

\[
\tilde{\varepsilon} = \frac{a - w}{(1 - \alpha)}.
\]

It is straightforward to see that changes in productivity have less of an effect on the cutoff in equation (31) than in equation (30) because \( (\sigma + \phi) < 1 \).

Interestingly, as can be seen in equation (31), there is no direct effect of matching efficiency on the cutoff point. However, there may be indirect effects via the wage, which we have ignored so far. While there is no such effect under our alternative bargaining structure (where market tightness does not show up in the wage), under standard Nash bargaining matching efficiency matter because market tightness is part of the wage. Remember that:

\[
w(\varepsilon) = \gamma (a - \varepsilon + \kappa \theta) + (1 - \gamma) b.
\]

Under Nash bargaining, the joint surplus of a match and the joint surplus under bargaining are the same (as the match is broken up when bargaining collapses). Thus, under Nash bargaining the cutoff point can be expressed in terms of the joint surplus (see Appendix A for derivations). Using the same simplifying assumptions as before, we can rewrite the cutoff point under standard Nash bargaining as:
\[ \hat{\varepsilon} = a - b + (1 - \sigma - \phi) \frac{a - w}{\sigma + \phi} + (1 - \sigma - \phi - p) \frac{w - b}{\sigma + \phi + p} \]  

(34)

Using the surplus splitting rule from Nash bargaining, we can simplify further:

\[ \hat{\varepsilon} = a - b + (1 - \sigma - \phi) \left( \frac{1}{\gamma} - p \right) \frac{w - b}{\sigma + \phi + p} \]  

(35)

The cutoff point under Nash bargaining in equation (35) also has the quit rate in the denominator. Thus, the same intuition as before applies for the effects of quits on the cutoff point. In addition, the job-finding rate \( p \) shows up on the right hand side of the equation. Technically, the job-finding rate plays a similar role as quits. Whenever the aggregate job-finding rate goes down, this will increase the joint surplus from a match because it is more difficult for a workers to find a new job.

The job-finding rate is directly affected by the matching efficiency. Thus, if matching efficiency is low, the joint surplus of a match is larger. Similar to quits, a lower matching efficiency leads to a larger sensitivity of the surplus with respect to productivity changes.

Summarizing, this section conveys the following insights. First, lower quits increase the length of the employment spell. Thereby, the level and the sensitivity of the cutoff point with respect to productivity increases. By contrast, there is no direct effect of matching efficiency on the cutoff point. However, there may be an indirect effect depending on the bargaining protocol.

Second, under Nash bargaining matching efficiency and quits affect the level and sensitivity of the joint surplus in the same qualitative way. In both cases, a less fluid labor market (with lower quits and lower matching efficiency) increases the surplus and increases the sensitivity of the surplus with respect to productivity. It has to be emphasized that we have derived these results under simplifying assumptions. In addition, we have only analyzed the reaction of the cutoff point and not the separation rate. The idiosyncratic distribution may have different curvature at different cutoff points. This may affect quantitative and even qualitative results. Therefore, we will show numerical illustrations for a realistic parameterization with a normal distribution in the next subsec-
tion. In the end, it is an empirical question whether the described effects are sufficiently strong to be found in the data, which we will analyze in the empirical part.

2.4 Parameterization of the Model

We parameterize our model on the quarterly frequency and choose a discount factor \( \delta = 0.99 \). The elasticity of the matching function is set to \( \alpha = 0.5 \). The value of benefits is set to \( b = 0.65 \). In addition, we assume standard Nash bargaining with an equal bargaining power for workers and firms (\( \gamma = 0.5 \)). We set the vacancy posting costs to \( \kappa = 0.1 \).

The remaining parameters are set based on administrative labor market flow data from Germany. We use employment-to-employment transitions in the data as a proxy for quits. The average quarterly flow rate from employment to employment is 2.6 percent (from 2000–2014).\(^4\) Therefore, we set \( \sigma = 0.026 \).

In addition, we have two targets from the administrative data. The average flow rate from employment to unemployment is 1.5 percent (measured as ratio of firms’ employment), which we interpret as endogenous separations. The average job finding rate is 18.7 percent per quarter (hirings as share of unemployment). We set the matching efficiency and the dispersion of the idiosyncratic productivity shock (assuming a normal distribution) to hit these two targets.\(^5\)

2.5 Numerical Exercises

2.5.1 Nash Bargaining

We start with standard Nash bargaining. To illustrate the effect of different quit rates and matching efficiencies for the dynamics of the job-finding rate and firing rate, we keep all parameters constant, except for the quit rate and the matching efficiency. First, we double the quit rate from 2.6 percent to 5.2 percent (which is closer to the level in the United States). When quits increase exogenously, the average duration of a match falls

\(^4\)In line with our estimations in the next sections, we choose the observation period from 2000 (first quarter) to 2014 (fourth quarter).

\(^5\)The mean of the idiosyncratic productivity shock is normalized to zero.
substantially. As a consequence, the present value of a match falls and firms choose a different cutoff point. Thus, the endogenous separation rate increases from 1.5 percent to 2 percent. Figure 1 shows the elasticity of the job-finding rate and the firing rate in these two economies with respect to aggregate productivity. In both cases, higher productivity leads to lower endogenous separations. The figure shows that the elasticity is larger for the firing rate and the job-finding rate with a lower quit rate. Note, however, that the quantitative effects of different quit rates on the volatility of the job-finding rate and the firing rate are relatively small with standard Nash bargaining.

![Figure 1: Elasticities of firing rate and job-finding rate separation rate under Nash bargaining: Change in quit rate](image)

It is not trivial to quantify how much larger the matching efficiency is in the United States relative to Germany (although economists would generally consider Germany to have a lower matching efficiency), as this is an unobservable variable. Therefore, we choose the new matching efficiency such that we obtain an increase of the endogenous separation rate to 2 percent (as in the previous exercise). When matching efficiency increases, the outside option for workers increases and thereby the value of a match falls. Thus, a larger fraction of matches does not have a positive joint surplus any more. Figure 2 shows that the elasticity of the firing rate with respect to productivity is smaller with a larger matching efficiency. By contrast, the elasticity of the job-finding rate is basically
unaffected by changes of the matching efficiency.

In a nutshell, both an increase of quits and matching efficiency leads a smaller elasticity of the separation rate with respect to aggregate productivity. Thus, both differences between continental European countries and Anglo-Saxon countries could be the driver for a larger volatility of the separation rate in the former countries.

2.5.2 Alternative Bargaining

We perform the same conceptional exercise under our alternative bargaining structure (i.e. without market tightness in the wage equation). This different bargaining protocol leads to two major changes.

First, the effects of quits on the elasticity of the job-finding rate and the firing rate is much larger. In our numerical exercise, a higher quit rate makes both the job-finding rate and the firing rate less responsive to productivity changes.

Second, as illustrated in the previous subsection, under this bargaining protocol matching efficiency leaves the response of the separation rate with respect to productivity shocks unaffected. By contrast, lower quit rate continue to amplify the effects of larger productivity shocks. The quantitative response is larger than under standard Nash
Figure 3: Elasticities of firing rate and job-finding rate under alternative bargaining: Change in quit rate

Figure 4: Elasticities of firing rate and job-finding rate under alternative bargaining: Change in matching efficiency
bargaining (see Fig. 3).

To the extent that market tightness has a limited (or no) influence on wages, there are smaller (or no) effects of matching efficiency on separation dynamics (see Fig. 4). In the end, it is an empirical question whether there is a meaningful connection between matching efficiency and endogenous separations, which we will test in the next section.

3 Matching Efficiency, Quits, and Flow Dynamics at the Sectoral Level

This section uses the cross-sectoral variation of matching efficiency in Germany to analyze the connection between matching efficiency and the volatility of the job-finding rate as well as the layoff rate. Matching efficiencies are estimated via a reduced form matching function applying a random effects panel estimation. For this purpose we assume the following: First, labor markets are separated. The natural choice for separated labor markets is either the industry level or the occupation level. We prefer the industry level because it has the advantage that we can perform a similar empirical exercise for establishments’ quit rates in the next section.6 Second, we abstract from differences in the dynamics of matching efficiency, i.e., the matching efficiency which we extract from the regression for every sector is constant across time. We think that this is sufficient as we are interested in the variation across industry sectors rather than the dynamics across time.7 Third, we assume that unemployed workers search in the industry they previously had a job in. We run the following regression in logs:

$$\log(p_{s,t}) = \sum_{s=1}^{S} \alpha_s \log(\chi_s) + \beta \cdot \log(\theta_{s,t}) + e_{s,t}$$

(36)

The sectoral job finding rate and the information on labor market tightness is ex-

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6Usually, establishments have several workers with different occupations. Thus, we cannot define establishment-specific dynamics. However, when we run our regressions on the connection between matching efficiency and layoff rate dynamics at the occupational level, the key message remains unaffected. Results are available on request.

7However, if we use time-varying matching efficiencies the qualitative implication stays the same. Results are available on request.
tracted from the SIAB, a 2 percent random sample drawn from the Integrated Employment Biographies (IEB) of the Institute for Employment Research (IAB) (see Ganzer et al. (2017) or Appendix C.1 for more details). The job finding rate is calculated as share of transitions between unemployment and employment over the one-period lagged stock of unemployment. The transitions are measured within one particular industry sector from one quarter to another between the first quarter in 2000 to the last quarter in 2014. The stock of unemployment in a particular industry sector is given by transcribing the information about the industry sector of the previous job onto a person’s spell of unemployment. Furthermore, we abstract from flows across different industry sectors. For that to be a reasonable assumption, we choose a medium scale industry sector classification which comprises 20 industry sectors.\(^8\) That, at least mostly, ensures that the main movement happens within the different industry sectors, which allows us to estimate the differences in matching efficiency accurately. For instance, in the largest industry sector (“industrial manufacturing”, measured in terms of employment), overall 90 percent of the observations are still in the same industry sector in the next spell. Related to job findings, it is still above 50 percent, i.e., given the workers is unemployed in the industrial manufacturing sector and finds a new job, with a 50% chance (across the period, on average) the job is still in the industrial manufacturing sector.\(^9\)

To generate labor market tightness, we merge sectoral vacancy data of the Federal Employment Agency. This data is just available after 2000, which naturally restricts our data to start in 2000. Our data ends in 2014 because of the AWFP, which provides data up to this year. After estimating the regression above, we extract the industry sector-specific fixed effects and exponentiate them, which delivers the matching efficiency for every industry sectors. Sectoral layoff rates are calculated in the same manner as job finding rates by taking transitions between employment and unemployment over last

\(^8\)The classification includes 21 industry sections (see Appendix C.2). We exclude section U (Activities of extraterritorial organisations and Bodies) and T (Activities of households as employers; differentiated goods- and services-producing activities of households for own use) as we have no or very limited vacancy information for it.

\(^9\)These numbers vary across industry sectors. However the smallest numbers are related to the smallest industry sectors. This is due to the fact, that the job finding rate is relatively low in these industry sectors, and just a few switches have a relatively high weight in this kind of exercise. All in all, the mode of switching strictly lies on the diagonal.
periods stock of employment. Then we seasonal adjust all panel times series using X-Arima-12. To gather insights in the relationship between matching efficiency and the layoff rate, we take averages across the observation period (2000q1–2014q4).

Table 1 shows how the coefficients of variation for different inflows and outflows are affected by different matching efficiency (me). The first row shows the effects of sector-specific me on the volatility of the job-finding rate (defined as matches from unemployment divided by unemployment). The second row shows the effects of me on the volatility of outflows into unemployment (outflows divided by employment). The third row shows the effects of me on inflows from non-employment (normalized by the employment stock, as the stock of non-employed is not properly defined). The fourth row shows the effects of me on outflows into non-employment.

Table 1: Regression Results: Effects of Matching Efficiency on Flow Rate Volatilities

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>matching efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient of variation job-finding rate (ue)</td>
<td>-0.0091</td>
</tr>
<tr>
<td></td>
<td>(0.935)</td>
</tr>
<tr>
<td>coefficient of variation of separation rate (eu)</td>
<td>0.0184</td>
</tr>
<tr>
<td></td>
<td>(0.492)</td>
</tr>
<tr>
<td>coefficient of variation of inflow rate (non-e-e)</td>
<td>0.0352</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
</tr>
<tr>
<td>coefficient of variation of outflow rate (e-non-e)</td>
<td>-0.0080</td>
</tr>
<tr>
<td></td>
<td>(0.447)</td>
</tr>
</tbody>
</table>

Notes: P-values in parenthesis. Regression with robust (Huber/White) standard errors. Own calculations.

In all four cases, we find no statistically significant effect (at conventional levels) of matching efficiency on labor market flow volatilities. In different words, our regressions provide no evidence that me matters for labor market volatilities.

The estimated matching efficiency may suffer from problems (e.g. if the Cobb-Douglas constant-returns matching function is a misspecification). Therefore, as a robustness check, we use the average job-finding rate in each sector, which is an alternative proxy how easy it is to find a new job in a specific industry sector. Table 2 shows the estimated coefficients for this scenario. As none of the estimated coefficients is significant at con-
ventional levels, this confirms the results from Table 1 that we cannot find a meaningful connection between the fluidity of a labor market (in terms of the ability to find a new job) and the volatility of job findings and separations.

Table 2: Regression Results: Effects of Job-Finding Rate on Flow Rate Volatilities

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>job-finding rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient of variation job-finding rate (ue)</td>
<td>-0.3164 (0.838)</td>
</tr>
<tr>
<td>coefficient of variation of separation rate (eu)</td>
<td>0.1210 (0.775)</td>
</tr>
<tr>
<td>coefficient of variation of inflow rate (non-e-e)</td>
<td>0.4552 (0.111)</td>
</tr>
<tr>
<td>coefficient of variation of outflow rate (e-non-e)</td>
<td>-0.1556 (0.226)</td>
</tr>
</tbody>
</table>

Notes: P-values in parenthesis. Regression with robust (Huber/White) standard errors. Own calculations.

Note that the zero effect of matching efficiencies and the job-finding rate on job-finding rate volatilities is in line with our theoretical model (independently of wage formation). By contrast, the zero effect of matching efficiencies and job finding rate levels on the layoff rate volatility would only be in line with a model where market tightness does not show up in bargaining (i.e. our alternative wage formation mechanism).

Next, we estimate the connection between the quit rate and labor market volatilities at the sectoral level. Table 3 shows the estimated effects of different sector-specific quit rates (approximated as employment-to-employment transitions) on the coefficient of variation for inflows and outflows. The inflow and outflow rates are defined analogously to Table 1.

The estimated coefficients are negative in all four cases. However, they are only statistically significant at conventional levels for the separation rates. In different words, we find that a lower quit rate is associated with a higher volatility of the layoff rate. This empirical result is in line with the theoretical model.

To sum up: The estimation results at the sectoral level are in line with the predictions from the search and matching model with non-conventional bargaining (i.e. without
Table 3: Regression for quit rates and volatilities

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>quit rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient of variation job-finding rate (ue)</td>
<td>-3.078</td>
</tr>
<tr>
<td></td>
<td>(0.168)</td>
</tr>
<tr>
<td>coefficient of variation of separation rate (eu)</td>
<td>-4.2124</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
</tr>
<tr>
<td>coefficient of variation of inflow rate (e-non-e)</td>
<td>-0.1698</td>
</tr>
<tr>
<td></td>
<td>(0.757)</td>
</tr>
<tr>
<td>coefficient of variation of outflow rate (non-e-e)</td>
<td>-0.9461</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
</tr>
</tbody>
</table>

Notes: P-values in parenthesis. Regression with robust (Huber/White) standard errors. Own calculations.

tightness in the wage equation). Although these regressions at the sectoral level allow us to estimate the effects of matching efficiency on volatilities, sector-level regressions suffer from shortcomings. In the cross section we only have a small number of observations. In addition, sector-level business cycles may distort our results. To circumvent these shortcomings, we estimate the effect of quits on labor market volatilities at the establishment level in the next section. Note that this is not possible for matching efficiency, as me is considered to be market specific (i.e. we do not have an establishment-specific me).

4 Quits and Flow Dynamics at the Establishment Level

4.1 Effect of Quit Rate on Volatilities

In this section we analyze the connection between quit rates and the dynamics of the layoff rate. As our data set (see below) does not contain any information on quits and layoffs, we use —similar to other papers (e.g., Dustmann and U. Schönberg, 2009)— employment-to-employment-flows (ee-flows) as a proxy for quits and employment-to-unemployment-flows (eu-flows, ue-flows) and alternatively employment-to-non-employment-flows (e-non-e-flows and non-e-e-flows) as a proxies for layoffs and hirings. Given that quits and layoffs happen at the establishment level, the natural unit of observation is the establishment.
For our analysis we use the Administrative Wage and Labor Market Flow Panel (AWFP, see Stüber and Seth (2018) and Appendix C.3). The AWFP provides employment, labor flow, and earnings data for the universe of German establishments (Betriebe) for the years 1975–2014. Its main data source is the Employment History (Beschäftigten Historik, BeH) of the IAB. It is an individual-level data set covering all workers in Germany subject to social security contributions. From the BeH, the AWFP aggregates the worker and job flow information to the establishment level.\(^{10}\) To ensure consistency over time, most variables in the AWFP — and all variables used in this paper — are calculated on a full-time worker basis.\(^{11}\)

For our analysis, we use the AWFP at the quarterly frequency for the years 2000–2014.\(^{12}\) We are interested in the connection between the quit rate (ee-flows) and the dynamics of the layoff rate (eu-flows and e-non-e-flows) and the hiring rate (ue-flows and non-e-u-flows). Therefore, we calculate, on the establishment level, the coefficient of variation (cv) for the layoff rate and hiring rate (hr). In case one of these rates is zero, we get a missing value for its cv. These missings are set to zero (if the sd is non-missing).

Then we run the following regressions:

\[
.cv(\phi_e) = \beta_0 + \beta_1 \sigma_e + \beta_2 X_e + \epsilon_{e,t} \tag{37}
\]

\[
.cv(hr_e) = \beta_0 + \beta_1 \sigma_e + \beta_2 X_e + \epsilon_{e,t} \tag{38}
\]

As we want to ensure that our results are not driven by differences across industry sectors, we run this regression for all establishments within a given industry sector. The vector \(X_e\) contains establishment characteristics that may have an effect of hiring and firing volatilities beyond the quit rate, namely: mean number of full-time workers, mean share of male workers and the means of the mean age and mean tenure of the workers.\(^{13}\)

---

\(^{10}\)Before this aggregation, the data on individuals undergo numerous validation procedures. Details on the validation methods are described in Stüber and Seth (2018).

\(^{11}\)To be more precise, variables are calculated on a ‘regular worker’ basis, see Appendix C.3.

\(^{12}\)This is the time period for which we can analyze both, the matching efficiency and layoff rate dynamics (see Section 3) and the quits and layoff rate dynamics.

\(^{13}\)We do no show the estimated coefficients for these control variables. Results are available on request.
Figures 5 and 6 show the point estimates $\beta_1$ of Equations (37) and (38) for every single industry sector. There is a clear-cut negative connection between the quit rate and the coefficient of variation of the layoff rate. The estimated coefficient is negative and statistically significant at the 5 percent level in each of the 19 industry sector. This is both the case when we use employment to unemployment flows and employment to non-employment flows as proxy for layoffs. Thus, our estimations show a remarkably stable effect of quits on the volatility of layoffs.

The estimated effect effect of quits on the volatility of the hiring rate is negative in most cases (in 14 out of 19 cases in Figure 5 and in 15 out of 19 cases in Figure 6).

![Figure 5: Estimated coefficients for the effect of quit rates on layoff rate and hiring rate volatility (approximated as flows into unemployment)](image)

The negative effect of the quit rate on labor market volatility is in line with a search and matching model with non-standard bargaining. Interestingly, in most sectors the effects of the quit rate on the layoff rate is larger (in absolute terms) than on the hiring rate. This can be seen by comparing the blue bars (layoff rate volatility) and the red bars (hiring rate volatility) in Figures 5 and 6.
Thus, Figures 5 and 6 suggest that quits may have a larger effect on the firing rate volatility than on the hiring rate volatility. This brings us back to the role of quits for (un)employment dynamics. We will address this question in the next subsection where we do a variance decomposition at establishment level.

4.2 Effect of Quits on Employment Variance Decomposition

Our variance decomposition follows the idea of Fujita and Ramey (2009) and is constructed as follows: First we approximate employment given the inflow and outflow rates for each establishment and calculate the relative first difference.

\[ n_{e,t} = irr_{e,t} * n_{e,t-1} + (1 -orr_{e,t}) * n_{e,t-1} \]  
\[ (39) \]

According to Fujita and Ramey (2009), this allows us to derive the different betas:

\[ \beta_{e}^{irr} = \frac{Cov(dn_e, irr_e)}{Var(dn_e)} \]  
\[ (40) \]

\[ \beta_{e}^{orr} = \frac{Cov(dn_e, orr_e)}{Var(dn_e)} \]  
\[ (41) \]
where $dn$ is the relative first difference of the approximated employment stock. We proceed by selecting establishments that have at least 10 valid observation in the approximated employment stock.

Once we have calculated the relative contribution of the inflow rate for each establishment, we estimate the effect of the quit rate on the calculated beta-coefficient. We estimate the effect of different quit rate for the role of separations at the establishment level, using the following equation:

$$
\beta_{e}^{corr} = \beta_{0} + \beta_{1}\sigma_{e} + \beta_{2}X_{e} + \epsilon_{e,t}
$$

(42)

Figures 8 and 7 show the a higher quit rate is associated with a less important role for layoffs in terms of the contribution to employment dynamics in most sectors. In different words, we show that establishments with a larger quit rate adjust their workforce more along the hiring margin than along the layoff margin.
Our empirical results is in line with our analytical results, where we show that a higher quit rate leads to more volatile layoffs, while the effect on the hiring rate is analytically not clear.

Furthermore, our empirical result provides an explanation for cross-country differences. Elsby et al. (2013) show that separations contribute more to unemployment dynamics in Anglo-Saxon countries than in continental European countries. At the same time, labor market flows and also employment-to-employment transitions are much lower. Our model provides an explanation for these differences. Our empirical results (both at the sectoral and establishment level) show that there is an important role for the quit rate in terms of (un)employment dynamics.

5 Conclusion

In a standard search and matching model, both a lower quit rate and a lower matching efficiency may generate a larger volatility of the layoff rate. Our paper tests this connection based on German microeconomic data. While we find no evidence for a negative connection between matching efficiency and layoff rate volatility, there is a stable negative connection between the average quit rate and layoff rate volatility. Thus, lower quit rates
in continental European countries appear to be a key driver for the large fluctuation of the layoff rate over the business cycle.

At the establishment level, we find that lower quits are associated with a more important role for the layoff rate (relative to the hiring rate) in terms of employment dynamics. This is in line with lower transition rates in European countries and a more important role for separations in terms of unemployment dynamics.
References


A Theoretical Equations and Derivations

A.1 Derivation of Standard Nash Bargaining

Starting point for deriving the Nash wage are the net present values for firms and workers:

\[ J^I (\varepsilon_{it}) = a_t - w (\varepsilon_{it}) - \varepsilon_{it} + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J^I_{t+1}, \quad (A.1) \]

\[ W (\varepsilon_{it}) = w (\varepsilon_{it}) + \delta E_t (1 - \sigma) (1 - \phi_{t+1}) W_{t+1} + \delta E_t (1 - (1 - \sigma) (1 - \phi_{t+1})) U_{t+1} \quad (A.2) \]

\[ U_t = b + \delta E_{t+1} p_t (1 - \sigma) (1 - \phi_{t+1}) W_{t+1} + \delta E_t (1 - p_t (1 - \sigma) (1 - \phi_{t+1})) U_{t+1}. \quad (A.3) \]

Thus:

\[ W (\varepsilon_{it}) - U_t \]
\[ = w (\varepsilon_{it}) - b + \delta E_t (1 - \sigma) (1 - \phi_{t+1}) (1 - p_t) (W_{t+1} - U_{t+1}) \quad (A.4) \]
\[ \Lambda (\varepsilon_{it}) = [J (\varepsilon_{it})]^{1-\gamma} [W (\varepsilon_{it}) - U_{t+1}]^\gamma \quad (A.6) \]

Maximization with respect to the wage yields:

\[ \frac{\partial \Lambda (\varepsilon_{it})}{\partial w (\varepsilon_{it})} = (1 - \gamma) [J (\varepsilon_{it})]^{-\gamma} [W (\varepsilon_{it}) - U_{t+1}]^{\gamma-1} \frac{\partial J (\varepsilon_{it})}{\partial w (\varepsilon_{it})} + \gamma [J (\varepsilon_{it})]^{1-\gamma} [W (\varepsilon_{it}) - U_{t+1}]^{\gamma-1} = 0 \quad (A.7) \]
\[(1 - \gamma) [J(\varepsilon_{it})]^{-\gamma} [W(\varepsilon_{it}) - U_{t+1}]^\gamma = \gamma [J(\varepsilon_{it})]^{1-\gamma} [W(\varepsilon_{it}) - U_{t+1}]^{\gamma-1} \quad (A.8)\]

\[(1 - \gamma) [W(\varepsilon_{it}) - U_{t+1}] = \gamma [J(\varepsilon_{it})] \quad (A.9)\]

\[(1 - \gamma) [w(\varepsilon_{it}) - b + \delta E_t [(1 - \sigma) (1 - \phi_{t+1}) (1 - p_t)] [W_{t+1} - U_{t+1}]]^\gamma = \gamma [a_t - w(\varepsilon_{it}) - \varepsilon_{it} + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J_{t+1}^I]^{1-\gamma} \quad (A.10)\]

Using the relationship from equation (A.9) in period t+1 for incumbent workers, we obtain:

\[(1 - \gamma) [W_{t+1}^I - U_{t+1}] = \gamma [J_{t+1}^I] \quad (A.12)\]

\[(1 - \gamma) [w(\varepsilon_{it}) - b + \delta E_t [(1 - \sigma) (1 - \phi_{t+1}) (1 - p_t)] \left(\frac{\gamma}{(1 - \gamma)} J_{t+1}^I\right)]^\gamma = \gamma [a_t - w(\varepsilon_{it}) - \varepsilon_{it} + E_t (1 - \sigma) (1 - \phi_{t+1}) \delta J_{t+1}^I]^{1-\gamma} \quad (A.13)\]

Thus:

\[w(\varepsilon_{it}) = \gamma [a_t - \varepsilon_{it} + E_t p_t \delta (1 - \sigma) (1 - \phi_{t+1}) J_{t+1}^I] + (1 - \gamma) b \quad (A.15)\]

We substitute the job-creation into the previous equation:

\[\frac{\kappa}{q_t} = \delta E_t [(1 - \sigma) (1 - \phi_{t+1}) J_{t+1}^I] \quad (A.16)\]

and obtain the following bargained wage:
\[w(\varepsilon_t) = \gamma [a_t - \varepsilon_t + \mu_t \frac{\kappa}{q_t}] + (1 - \gamma) b \quad \text{(A.17)}\]
\[= \gamma [a_t - \varepsilon_t + \kappa \theta_t] + (1 - \gamma) b \quad \text{(A.18)}\]

A.2 Derivation of the Joint Surplus

The steady state value of match at the cutoff point is:

\[J(\bar{\varepsilon}) = a - w(\bar{\varepsilon}) - \bar{\varepsilon} + (1 - \sigma) (1 - \phi) \delta J^I. \quad \text{(A.19)}\]

The steady state value of an incumbent worker is

\[J^I = \frac{a - \bar{w} - H}{1 - \delta (1 - \sigma) (1 - \phi)}. \quad \text{(A.20)}\]

A worker’s value of employment at the cutoff point is:

\[W(\bar{\varepsilon}) = w(\bar{\varepsilon}) + \delta (1 - \sigma) (1 - \phi) W^I + \delta (1 - (1 - \sigma) (1 - \phi)) U. \quad \text{(A.21)}\]

A worker’s value of unemployment is:

\[U = b + \delta p (1 - \sigma) (1 - \phi) W^I + \delta (1 - p (1 - \sigma) (1 - \phi)) U. \quad \text{(A.22)}\]

The surplus of a worker at the cutoff point is

\[W(\bar{\varepsilon}) - U = w(\bar{\varepsilon}) - b + \delta ((1 - \sigma) (1 - \phi) (1 - p)) (W^I - U). \quad \text{(A.23)}\]

Average surplus:

\[W^I - U = w - b + \delta (1 - \sigma) (1 - \phi) (1 - p) (W^I - U). \quad \text{(A.24)}\]

\[W^I - U = \frac{w - b}{1 - \delta (1 - \sigma) (1 - \phi) (1 - p)}. \quad \text{(A.25)}\]
Thus, we can define the match value in terms of the joint surplus:

\[
S(\tilde{\varepsilon}) = J(\tilde{\varepsilon}) + W(\tilde{\varepsilon}) - U + a - b - \tilde{\varepsilon} + \delta (1 - \sigma) (1 - \phi) J I + \delta ((1 - \sigma) (1 - \phi) (1 - p)) (W I - U) \quad (A.26)
\]

Setting \( S(\tilde{\varepsilon}) = 0 \), we obtain:

\[
\tilde{\varepsilon} = a - b + (1 - \sigma) (1 - \phi) \frac{a - w - H}{1 - \delta (1 - \sigma) (1 - \phi)} + (1 - \sigma) (1 - \phi) (1 - p) \frac{w - b}{1 - \delta ((1 - \sigma) (1 - \phi) (1 - p))}. \quad (A.27)
\]

Using the simplifying assumptions described in the main part, we obtain:

\[
\tilde{\varepsilon} = a - b + (1 - \sigma - \phi) \frac{a - w}{\sigma + \phi} + (1 - \sigma - \phi - p) \frac{w - b}{\sigma + \phi + p}. \quad (A.28)
\]

Alternative way of expressing this equation, using the relation from Nash bargaining:

\[
J I = \frac{(1 - \gamma)}{\gamma} (W I - U) \quad (A.29)
\]

and substituting this into the surplus of the marginal worker:

\[
\tilde{\varepsilon} = a - b + \delta (1 - \sigma) (1 - \phi) \frac{(1 - \gamma)}{\gamma} (W I - U) + \delta ((1 - \sigma) (1 - \phi) (1 - p)) (W I (A.80))
\]

\[
= a - b + \delta (1 - \sigma) (1 - \phi) \left( \frac{1 - \gamma}{\gamma} + 1 - p \right) (W I - U) \quad (A.31)
\]

\[
= a - b + \delta (1 - \sigma) (1 - \phi) \left( \frac{1}{\gamma} - p \right) \frac{w - b}{1 - \delta (1 - \sigma) (1 - \phi) (1 - p)} \quad (A.32)
\]

With our simplifying assumptions, we obtain:

\[
\tilde{\varepsilon} = a - b + (1 - \sigma - \phi) \left( \frac{1 - \gamma}{\gamma} + 1 - p \right) \frac{w - b}{\sigma + \phi + p} \quad (A.33)
\]
A.3 Model Equations for Simulation

Firing rate:

\[
\phi_t = 1 - \int_{-\infty}^{\tilde{\varepsilon}_t} g(\varepsilon_{it}) \, d\varepsilon_{it} \quad (A.34)
\]

Cutoff point:

\[
\tilde{\varepsilon}_{it} = a_t - w(\tilde{\varepsilon}_{it}) + (1 - \sigma) (1 - \phi_{t+1}) \delta J_{t+1}^I \quad (A.35)
\]

Ex-ante profits of a hired worker:

\[
J^I_t = a_t - \bar{w}_t - H(\tilde{\varepsilon}_{it}) + (1 - \sigma) (1 - \phi_{t+1}) E_t \delta J_{t+1}^I \quad (A.36)
\]

Conditional expectations for idiosyncratic shock realization:

\[
H(\tilde{\varepsilon}_{it}) = \int_{-\infty}^{\tilde{\varepsilon}_{it+1}} g(\varepsilon_{it}) \, d\varepsilon_{it} \quad (A.37)
\]

Nash bargained wage for workers with idiosyncratic realization \( \varepsilon_{it} \):

\[
w(\varepsilon_{it}) = \gamma (a_t - \varepsilon_{it} + \kappa \theta_t) + (1 - \gamma) b, \quad (A.38)
\]

Conditional expected wage for incumbent workers:

\[
\bar{w}_t = \frac{\int_{-\infty}^{\tilde{\varepsilon}_{it+1}} w(\varepsilon_{it}) \, g(\varepsilon_{it}) \, d\varepsilon_{it}}{1 - \phi_t} \quad (A.39)
\]

Wage for a newly matched worker, standard Nash bargaining:

\[
w_t = \gamma (a_t + \kappa \theta_t) + (1 - \gamma) b, \quad (A.40)
\]

Alternatively

\[
w_t = \gamma a_t + (1 - \gamma) b, \quad (A.41)
\]
Job-finding rate:

\[ p_t = \chi_t \theta_t^{1-\alpha} \]  
(A.42)

Worker-finding rate:

\[ q_t = \chi_t \theta_t^{-\alpha}. \]  
(A.43)

Vacancy posting condition:

\[ \frac{\kappa}{q_t} = \delta E_t \left( (1 - \sigma) (1 - \phi_{t+1}) J_{t+1}^I \right) \]  
(A.44)

Employment dynamics equation:

\[ n_{t+1} = (1 - \phi_t) (1 - \sigma) (n_t + p_t u_t), \]  
(A.45)

Definition of unemployment:

\[ u_t = 1 - n_t \]  
(A.46)

Endogenous variables: \( \phi_t, \tilde{e}_{it}, w(\varepsilon_{it}), J_t^I, H(\tilde{e}_{it}), \bar{w}_{it}, w_t, p_t, q_t, \theta_t, n_t, u_t \)
B Analytical Steady State Results

B.1 Model Equations

We rewrite all our equations in terms of the steady state. The expected average value of a match is:

\[ J^I = a - \bar{w} - H(\tilde{\varepsilon}) + \delta (1 - \phi) (1 - \sigma) J^I, \]  
\[ (B.47) \]

\[ J^I = \frac{a - \bar{w} - H(\tilde{\varepsilon})}{1 - \delta (1 - \phi) (1 - \sigma)}. \]  
\[ (B.48) \]

The cutoff point is:

\[ \tilde{\varepsilon} = a - w(\varepsilon) + \delta (1 - \phi) (1 - \sigma) J^I. \]  
\[ (B.49) \]

Substituting (B.48) into (B.49):

\[ \tilde{\varepsilon} = a - w(\varepsilon) + \delta (1 - \phi) (1 - \sigma) \frac{a - \bar{w} - H(\tilde{\varepsilon})}{1 - \delta (1 - \phi) (1 - \sigma)} \]  
\[ (B.50) \]

In order to derive analytical results, we use the following simplifying assumptions: i) the wage is assumed to be a fixed proportion of productivity, i.e. \( w(\varepsilon) = \gamma (a - \varepsilon) \). This assumption is a simplified version of our alternative bargaining model. ii) We assume that the average idiosyncratic realization of the shock (conditional on being hired) is equal to zero \( H(\tilde{\varepsilon}) = 0 \). iii) We assume a uniform distribution for the idiosyncratic shock with lower support \(-z\) and upper support \(z\). iv) We assume that \( \delta = 1 \).

Based on these assumptions, we obtain the following approximative equation (as \( \phi \sigma \approx 0 \)):

\[ \tilde{\varepsilon} = a (1 - \gamma) - \gamma \varepsilon + (1 - \phi) (1 - \sigma) \frac{a (1 - \gamma)}{\phi + \sigma}, \]  
\[ (B.51) \]

\[ \tilde{\varepsilon} = \frac{a}{\phi + \sigma}. \]  
\[ (B.52) \]
The firing rate is:
\[
\phi = 1 - \int_{-\infty}^{\hat{\varepsilon}} g(\varepsilon) d\varepsilon. \tag{B.53}
\]

Based on the assumed uniform distribution, we obtain a closed-form solution for the firing rate:
\[
\phi = 1 - \hat{\varepsilon} + z = 1 - \frac{a}{\phi + \sigma} + \frac{\sigma + \rho}{2z}. \tag{B.54}
\]

To derive an analytical expression for the job-finding rate, we start with the vacancy posting condition:
\[
\frac{\kappa}{q(\theta)} = \delta (1 - \sigma) (1 - \phi) J^I. \tag{B.55}
\]

Using equation (B.48) and our simplifying assumptions, we obtain:
\[
\frac{\kappa}{q(\theta)} = (1 - \sigma) (1 - \phi) \frac{a (1 - \gamma)}{\phi + \sigma}. \tag{B.56}
\]

For notational convenience, we abbreviate the right-hand side with: \( \pi = (1 - \sigma) (1 - \phi) \frac{a (1 - \gamma)}{\phi + \sigma} \).

Using the definition for the worker-finding rate, we obtain:
\[
\theta = \left( \frac{\pi \chi}{\kappa} \right)^{\frac{1}{\alpha}}. \tag{B.57}
\]

Thus, the job-finding rate is:
\[
p = \chi \theta^{1-\alpha} = \chi \left( \frac{\pi \chi}{\kappa} \right)^{\frac{1-alpha}{\alpha}} = \chi^{\frac{1}{\alpha}} \left( \frac{\pi}{\kappa} \right)^{\frac{1-alpha}{\alpha}}. \tag{B.58}
\]

### B.2 Firing Rate Volatility and Quits

We start by deriving the response of the cutoff point to aggregate productivity changes. In order to calculate \( \frac{\partial \phi}{\partial \alpha} \), we substitute equation (B.54) into equation (B.52):
\[ \tilde{\varepsilon} = \frac{a}{\phi + \sigma} = \frac{a}{1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma}. \]  

(B.59)

Using the implicit functions theorem, we obtain:

\[ \frac{\partial \tilde{\varepsilon}}{\partial a} = \frac{(1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma) - \frac{a}{2z} \frac{\partial \tilde{\varepsilon}}{\partial a}}{(1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2}. \]  

(B.60)

We rewrite this expression:

\[ \frac{\partial \tilde{\varepsilon}}{\partial a} \left( (1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2 - \frac{a}{2z} \right) = \frac{1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma}{(1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2} \]  

(B.61)

\[ \frac{\partial \tilde{\varepsilon}}{\partial a} = \frac{1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma}{(1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2 - \frac{a}{2z}} \]  

(B.62)

Finally, we want to know how the elasticity of the cutoff point if affected by the level of the quit rate?

\[ \frac{\partial^2 \tilde{\varepsilon}}{\partial a \partial \sigma} = \frac{\left( (1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2 - \frac{a}{2z} \right)(1 - \frac{\partial \varepsilon}{\partial \sigma} \frac{1}{2z})}{(1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2 - \frac{a}{2z}} \]  

(B.63)

\[ = -\left( (1 - \frac{\tilde{\varepsilon} + z}{2z} + \sigma)^2 + \frac{a}{2z} \right)(1 - \frac{\partial \varepsilon}{\partial \sigma} \frac{1}{2z}) < 0 \]  

(B.64)

As the denominator is positive and the numerator is negative (given that \( \frac{\partial \varepsilon}{\partial \sigma} < 0 \)), 

\[ \frac{\partial^2 \tilde{\varepsilon}}{\partial a \partial \sigma} < 0. \]  

This expression shows that a higher level of quits leads to a smaller (absolute) sensitivity of the cutoff point in response to productivity shocks.

Using equation (B.54), we can calculate:

\[ \frac{\partial \phi}{\partial a} = \frac{\partial \phi}{\partial \tilde{\varepsilon}} \frac{\partial \tilde{\varepsilon}}{\partial a} = -\frac{1}{2z} \frac{\partial \tilde{\varepsilon}}{\partial a}. \]  

(B.65)

This allows us to calculate the cross-derivative of the firing rate with respect to quits:
\[
\frac{\partial^2 \phi}{\partial a \partial \sigma} = -\frac{1}{2z} \frac{\partial \tilde{\varepsilon}}{\partial a \partial \sigma} > 0.
\] (B.66)

Remember that the sensitivity of the firing rate with respect to quits is negative. A positive cross-derivative \(\frac{\partial^2 \phi}{\partial a \partial \sigma} > 0\) means that this negative reaction is getting larger with larger quits (i.e. it is closer to zero).

In order to see what this means for the responsiveness of the elasticity of the firing rate with respect to a different quit rate level, we first calculate the elasticity:

\[
\frac{\partial \ln \phi}{\partial \ln a} = \frac{\partial \phi}{\partial a} a \phi.
\] (B.67)

What is the responsiveness of the elasticity of the firing rate with respect to different quit rate levels?

\[
\frac{\partial^2 \ln \phi}{\partial \ln a \partial \sigma} = \frac{\partial}{\partial \sigma} \left( \frac{\partial \phi}{\partial a} \right) a \phi = \frac{\phi a \frac{\partial^2 \phi}{\partial a \partial \sigma} - \frac{\partial \phi}{\partial a} a \frac{\partial \phi}{\partial \sigma}}{\phi^2} > 0
\] (B.68)

We know that \(\frac{\partial^2 \phi}{\partial a \partial \sigma} > 0\). Further, \(\frac{\partial \phi}{\partial a} < 0\) and \(\frac{\partial \phi}{\partial \sigma} > 0\). Therefore, \(\frac{\partial^2 \ln \phi}{\partial \ln a \partial \sigma} > 0\).

Keep in mind that the elasticity of the firing rate with respect to productivity is negative. Thus, a positive second derivative with respect to quits means that this negative elasticity increases in case of a higher quit rate. In different words, in absolute terms the elasticity of the firing rate falls with a higher \(\sigma\). Based on these derivations, we expect that the volatility of firings (which is an absolute concept) falls with lower quits.

### B.3 Firing Rate Rate Volatility and Matching Efficiency

Under our alternative wage formation where market tightness does not show up in the bargaining, matching efficiency does not affect wages and thereby firings are unaffected by matching efficiency. Given that

\[
\phi = 1 - \frac{\tilde{\varepsilon} + z}{2z} = 1 - \frac{a}{\phi + \sigma} + \frac{z}{2z}.
\] (B.69)
\[
\frac{\partial^2 \ln \phi}{\partial \ln a \partial \chi} = 0 \tag{B.70}
\]

Thus, without market tightness in the bargaining equation, we expect no effect of matching efficiency on the volatility of the firing rate.

### B.4 Job-Finding Rate Volatility and Quits

What is the elasticity of the job-finding rate with respect to productivity? We use equation (B.58) to calculate:

\[
\frac{\partial \ln p}{\partial \ln a} = \frac{\partial p}{\partial a} \frac{a}{p} = \chi^{\frac{1}{\alpha}} \left( \frac{\pi}{\kappa} \right)^{\frac{1-\alpha}{\alpha}} \frac{\partial \pi}{\partial a} \frac{1}{p} \tag{B.71}
\]

Using the expression for \( \pi = \frac{(1-\sigma)(1-\phi) a (1-\gamma)}{\phi + \sigma} \).

\[
\frac{\partial \pi}{\partial a} = (\phi + \sigma) \left[ (1-\sigma)(1-\phi)(1-\gamma) - \frac{\partial \phi}{\partial a} (1-\sigma) a (1-\gamma) \right] - (1-\sigma)(1-\phi) a (1-\gamma) \frac{\partial \phi}{\partial a} \frac{1}{(\phi + \sigma)^2} \tag{B.72}
\]

Let’s simplify this expression:

\[
\frac{\partial \pi}{\partial a} = \frac{(\phi + \sigma)(1-\sigma)(1-\phi)(1-\gamma) - \frac{\partial \phi}{\partial a} (1-\sigma) a (1-\gamma)(1+\sigma)}{(\phi + \sigma)^2} \tag{B.73}
\]

Combining equations (B.71) and (B.72), yields a relatively complicated expression. However, to illustrate the mechanism, it is useful to consider exogenous separations. In this case: \( \frac{\partial \phi}{\partial a} = 0 \). And the expression would simplify to:

\[
\frac{\partial \ln p}{\partial \ln a} = \chi^{\frac{1}{\alpha}} \frac{1-\alpha}{\alpha} \left( \frac{\pi}{\kappa} \right)^{\frac{1-\alpha}{\alpha}} \frac{(1-\sigma)(1-\phi)(1-\gamma) a}{(\phi + \sigma) \kappa} \frac{1}{p} \tag{B.74}
\]

Using the definition of the job-finding rate in equation (B.58), we obtain:

\[
\frac{\partial \ln p}{\partial \ln a} = \frac{1-\alpha}{\alpha} \tag{B.75}
\]
Under endogenous separations, this expression changes to:

\[
\frac{\partial \ln p}{\partial \ln a} = \chi \frac{1 - 1}{\alpha} \left( \frac{\pi}{\kappa} \right)^{\frac{1-\alpha}{\alpha}} \frac{(\phi + \sigma)(1 - \sigma)(1 - \phi)(1 - \gamma) - \frac{\partial \phi}{\partial a} (1 - \gamma) a (1 - \phi)(1 + \sigma)}{\frac{\partial \phi}{\partial a} (1 - \phi)(1 - \gamma) a (1 + \sigma)}
\]

It is straightforward to see that the elasticity of the job-finding rate is larger under endogenous separations, as \( \frac{\partial \phi}{\partial a} < 0 \).

What is the sensitivity of this elasticity with respect to quits? To illustrate this, let’s do the following substitution \( f = \frac{a(1 + \sigma)}{(\phi + \sigma)(1 - \phi)} \).

\[
\frac{\partial^2 \ln p}{\partial \ln a \partial \sigma} = -\frac{1 - 1}{\alpha} \left( \frac{\partial^2 \phi \partial a \partial \sigma + \partial f \partial \phi}{\partial a \partial \sigma} \right)
\]

We know that \( \frac{\partial^2 \phi}{\partial a \partial \sigma} > 0 \), \( \frac{\partial \phi}{\partial a} < 0 \).

What is the sign of \( \frac{\partial f}{\partial \sigma} \)?

\[
\frac{\partial f}{\partial \sigma} = \frac{(\phi + \sigma)(1 - \phi) a - a (1 + \sigma) (\frac{\partial \phi}{\partial \sigma} - 2 \phi \frac{\partial \phi}{\partial a} + 1 - \phi - \sigma \frac{\partial \phi}{\partial a})}{[(\phi + \sigma)(1 - \phi)]^2}
\]

We know that \( \frac{\partial \phi}{\partial \sigma} > 0 \). For the general case, we cannot make any statement on the sign.

However, if we switch off the second-order effect that quits affect the level of firings (\( \frac{\partial \phi}{\partial \sigma} = 0 \)), the expression collapses to:
\[ \frac{\partial f}{\partial \sigma} = \frac{(\phi + \sigma)(1 - \phi) a - a(1 + \sigma)(1 - \phi)}{[(\phi + \sigma)(1 - \phi)]^2} \]

\[ = -\frac{(1 - \phi)(1 - \phi) a}{[(\phi + \sigma)(1 - \phi)]^2} > 0 \]

In this case:

\[ \frac{\partial^2 \ln p}{\partial \ln a \partial \sigma} = -\frac{1 - \alpha}{\alpha} \left( \frac{\partial^2 \phi}{\partial a \partial \sigma} f + \frac{\partial f}{\partial \sigma} \frac{\partial \phi}{\partial a} \right) \]

\[ \frac{\partial^2 \phi}{\partial a \partial \sigma} > 0, \frac{\partial \phi}{\partial a} < 0, \frac{\partial f}{\partial \sigma} < 0. \] Thus, \[ \frac{\partial^2 \ln p}{\partial \ln a \partial \sigma} < 0. \]

In different word, although we cannot make a general statement. If second-order effects are sufficiently small, higher quits lead to a lower volatility of the job-finding rate.

### B.5 Job-Finding Rate Volatility and Matching Efficiency

\[ \frac{\partial \ln p}{\partial \ln a} = 1 - \frac{\alpha}{\alpha} \left( 1 - \frac{\partial \phi}{\partial a} \left( \frac{\phi a (2 - \gamma)}{(\phi + \sigma)(1 - \phi)(1 - \gamma)} \right) \right) \]

As matching efficiency does not show up in the elasticity of the job-finding rate, the following result holds:

\[ \frac{\partial^2 \ln p}{\partial \ln a \partial \chi} = 0 \]

### C Datasets and Definitions

#### C.1 Sample of Integrated Labour Market Biographies

The Sample of Integrated Labour Market Biographies 1975–2014 (SIAB, see Antoni et al., 2016) covers the employment histories of nearly 1.8 million individuals working in Germany.\footnote{The SIAB is produced at the Research Data Centre (FDZ) of the Federal Employment Agency (BA) at the IAB. The data in the weakly anonymous version may only be analysed in the context of a research visit at the FDZ and subsequent remote data access (see Antoni et al., 2016).}

The SIAB is a 2% sample of the population of the Integrated Employment Biographies (IEB) of the Institute for Employment Research (IAB). It is a rich data set that tracks
a persons’ (un-) employment episodes over time, and a set of individual characteristics such as age, gender, occupation, wages etc. and their changes over time. Furthermore establishment information can be merged, which allows to track the industry sector of the establishment a persons works or has worked at. The IEB comprises, inter alia, all individuals who showed one of the following statuses at least once since 1975:

- employment subject to social security,
- marginal part-time employment (since 1999),
- receipt of benefits in accordance with Social Code Book III,
- receipt of benefits in accordance with Social Code Book II (since 2005).

C.2 Industry Classification

For our analysis we aggregate the 5-digit industry classifications to the 21 sections (A to U) according to the 2008 “Structure of the Classification of Economic Activities” of the Federal Statistical Office (Statistisches Bundesamt, see Table C.1). For the analysis we do not consider the section U (Activities of extraterritorial organisations and Bodies) as we have no vacancy information for it.
Table C.1: Sections of the 2008 “Structure of the Classification of Economic Activities”

A Agriculture, forestry and fishing  
B Mining and quarrying  
C Manufacturing  
D Electricity, gas, steam and air conditioning supply  
E Water supply; sewerage, waste management and remediation activities  
F Construction  
G Wholesale and retail trade; repair of motor vehicles and motorcycles  
H Transportation and storage  
I Accommodation and food service activities  
J Information and communication  
K Financial and insurance activities  
L Real estate activities  
M Professional, scientific and technical activities  
N Administrative and support service activities  
O Public administration and defence; compulsory social security  
P Education  
Q Human health and social work activities  
R Arts, entertainment and recreation  
S Other service activities  
T Activities of households as employers; differentiated goods- and services-producing activities of households for own use  
U Activities of extraterritorial organisations and bodies

C.3 The Administrative Wage and Labor Market Flow Panel

The Administrative Wage and Labor Market Flow Panel (AWFP, see Stüber and Seth, 2018) provides employment, labor flow, and earnings data for the universe of German establishments (Betriebe) for the years 1975–2014.\(^{15}\) It is available at an annual and a quarterly frequency. The AWFPs main data source is the Employment History (Beschäftigten Historik, BeH) of the IAB. The BeH is an individual-level data set covering all workers in Germany subject to social security. Marginal part-time workers (\textit{geringfügig Beschäftigte}) have been covered since 1999.\(^{16}\) The information in the BeH stems from the notification procedure for social security. This procedure requires employers to keep the social se-

\(^{15}\)The AFP was generated within the German Science Foundations’ (DFG) priority program “The German Labor Market in a Globalized World” (SPP 1764). A 50% sample of the AWFP may be analysed in the context of a research visit at the FDZ and subsequent remote data access (see Stüber and Seth, 2019).

\(^{16}\)The main types of employees not covered by the BeH are civil servants (Beamte), military personnel, and the self-employed.
curity agencies informed about their employees by reporting any start and end date of employment and by annually confirming existing employment relationships.

From the BeH, the AWFP aggregates the worker and job flow information to the establishment level, rendering an establishment the observational unit. Before this aggregation, the data on individuals undergo numerous validation procedures. Further details on the data set are described in Stüber and Seth (2018).

To ensure consistency over time, most variables in the AWFP—and all variables used in this paper—are calculated on a ‘regular worker’ basis. In the AWFP, a person is defined as a ‘regular worker’ when she is employed full-time and belongs to one of the following person groups: ‘employees subject to social security without special features’, ‘seamen’ or ‘maritime pilots.’ Therefore (marginal) part-time employees, employees in partial retirement, interns, etc., are not counted as regular workers.

For our analysis, we use the AWFP at the quarterly frequency for the years 2000–2014.\textsuperscript{17} We drop all establishments in the AWFP that change the industry sector and/or the federal state. Some small establishments have data ”gaps” for certain quarters. These gaps appear if the establishment does not employ any full-time worker in a quarter. We fill these gaps by setting the stock of full-time workers to zero and imputing time-consistent variables from the previous or subsequent quarter. When we calculate means of variables for establishments, e.g., the mean share of male workers, we only use original AWFP observations. As already described in Section 3, we aggregate the data to a medium scale industry sector classification which comprises 20 industry sectors (see Appendix C.2).

Wages in the AWFP are defined as the mean real daily wages of all employed full-time workers in a particular establishment. The daily wages include the base salary, all bonuses and special payments (such as performance bonuses, holiday pay, or Christmas allowance), fringe benefits, and other monetary compensations received throughout the year (or the duration of the employment spell). Therefore, the daily wages correspond more to a measure of total compensation than to a daily base wage. Workers’ daily wages above the contribution assessment ceiling are imputed following Card et al. (2015) before

\textsuperscript{17}This is the time period for which we can analyze both, the matching efficiency and layoff rate dynamics (see Section 3) and the quits and layoff rate dynamics.
aggregating the data to the establishment level.\textsuperscript{18}

Stocks and flows are calculated using an “end-of-period” definition:

- The stock of employees of an establishment in year $t$ equals the number of full-time workers on the last day of year $t$.

- Inflows of employees into an establishment for year $t$ equal the number of full-time workers who were regularly employed on the last day of year $t$ but not so on the last day of the preceding year, $t-1$.

- Outflows of employees from an establishment for year $t$ equal the number of full-time workers who were regularly employed on the last day of the preceding year ($t-1$) but not so on the last day of year $t$.

\textsuperscript{18}For details see Appendix 8.2 of Schmucker et al. (2016).
D Insights within and across sectors

D.1 Decomposition exercise

To gather some insights on the contributions of separations and job findings to the dynamics of unemployment and employment, we use the decomposition method of Fujita and Ramey (2009). Because it is only possible to decompose variations in unemployment in the SIAB, we then take another step by decomposing employment in the AWFP. Here we use two different modes of calculations. In one mode we take averages across establishments, that operate in a certain sector, and run our decomposition. In the second mode, we run the decomposition for each establishment separately and average afterwards.

With this decomposition exercise we exploit two things: First, how the contributions to fluctuations differ across sectors, for unemployment and employment. Second, whether differences across sector differ using different levels of aggregation.

Table D.2: Decomposing fluctuations in unemployment across sectors

<table>
<thead>
<tr>
<th>sector</th>
<th>$\beta_{jfr}$</th>
<th>$\beta_{sep}$</th>
<th>$\beta_{rest}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.6269</td>
<td>0.3830</td>
<td>-0.0099</td>
</tr>
<tr>
<td>B</td>
<td>0.6758</td>
<td>0.3313</td>
<td>-0.0070</td>
</tr>
<tr>
<td>C</td>
<td>0.4564</td>
<td>0.5406</td>
<td>0.0030</td>
</tr>
<tr>
<td>D</td>
<td>0.3458</td>
<td>0.6541</td>
<td>0.0001</td>
</tr>
<tr>
<td>E</td>
<td>0.7645</td>
<td>0.2076</td>
<td>0.0279</td>
</tr>
<tr>
<td>F</td>
<td>0.5150</td>
<td>0.5004</td>
<td>-0.0154</td>
</tr>
<tr>
<td>G</td>
<td>0.5360</td>
<td>0.4643</td>
<td>-0.0003</td>
</tr>
<tr>
<td>H</td>
<td>0.5959</td>
<td>0.3873</td>
<td>0.0168</td>
</tr>
<tr>
<td>I</td>
<td>0.6749</td>
<td>0.3181</td>
<td>0.0070</td>
</tr>
<tr>
<td>J</td>
<td>0.6632</td>
<td>0.3274</td>
<td>0.0093</td>
</tr>
<tr>
<td>K</td>
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<td>0.2666</td>
<td>-0.0007</td>
</tr>
<tr>
<td>L</td>
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<td>0.1238</td>
<td>0.0045</td>
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<tr>
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<td>0.0030</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>P</td>
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<td>0.3029</td>
<td>0.0063</td>
</tr>
<tr>
<td>Q</td>
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<td>0.4590</td>
<td>0.0011</td>
</tr>
<tr>
<td>R</td>
<td>0.7087</td>
<td>0.2792</td>
<td>0.0121</td>
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<tr>
<td>S</td>
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<td>0.3054</td>
<td>-0.0018</td>
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<tr>
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<td>0.7478</td>
<td>0.0008</td>
</tr>
<tr>
<td>U</td>
<td>0.0116</td>
<td>0.9826</td>
<td>0.0058</td>
</tr>
</tbody>
</table>

On average 0.5923 0.4042 0.0034
Table D.2 shows the decomposition for unemployment fluctuations. We first calculate a steady state unemployment rate, $u_{t}^{ss} = s_{t}/(s_{t} + f_{t})$ where $s$ and $f$ denote the separation rates and $f$ the job-finding rates. Then we extract the proportions of variation of steady state unemployment rate and the separation and job-finding rates by calculating the betas according to Fujita and Ramey (2009). For this exercise we log the steady state unemployment rates, the job-finding rates and the separation rates in each sector. Then we calculate first differences for each factor and normalize the job finding and separation rates. The betas are then given by the covariance of the job-finding rates (and separations rates, respectively) and the steady state unemployment rate, relative to the variance of the steady state unemployment rates.

### D.2 Fujita and Ramey Method

#### D.2.1 Unemployment

Here is how I understand Fujita and Ramey (2009). They approximate the unemployment rate by steady state unemployment:

$$u_{t} \approx \frac{s_{t}}{s_{t} + f_{t}} = u_{t}^{ss}$$  \hspace{1cm} (D.76)

The same approximation is applied to the trend (denoted with a bar):

$$\bar{u}_{t} \approx \frac{\bar{s}_{t}}{\bar{s}_{t} + \bar{f}_{t}} = \bar{u}_{t}^{ss}$$  \hspace{1cm} (D.77)

Via log-linearization, the authors obtain:

$$\ln \left( \frac{u_{t}^{ss}}{u_{t-1}^{ss}} \right) = (1 - \bar{u}_{t}^{ss}) \ln \left( \frac{s_{t}}{\bar{s}_{t}} \right) - (1 - \bar{u}_{t}^{ss}) \ln \left( \frac{f_{t}}{\bar{f}_{t}} \right) + \varepsilon_{t}$$  \hspace{1cm} (D.78)

When using first differences as trend ($u_{t}^{ss} = u_{t-1}^{ss}$, etc.). Thus:

$$\Delta \ln \left( u_{t}^{ss} \right) = (1 - \bar{u}_{t}^{ss}) \Delta \ln \left( s_{t} \right) - (1 - \bar{u}_{t}^{ss}) \Delta \ln \left( f_{t} \right) + \varepsilon_{t}$$  \hspace{1cm} (D.79)

The authors summarize the previous two equations more compactly:
\[ du_t^{ss} = du_t^{sr} + du_t^{jr} + \varepsilon_t, \]  
(D.80)

where \( du_t^{sr} = (1 - \bar{u}_t^{ss}) \Delta \ln (s_t) \) in case of first differences.

They derive the variance in terms the covariance:

\[ \text{Var} (du_t^{ss}) = \text{Cov} (du_t^{ss}, du_t^{sr}) + \text{cov} \left( du_t^{ss}, du_t^{jr} \right) + \text{cov} (du_t^{ss}, \varepsilon_t) \]  
(D.81)

By dividing by \( \text{Var} (du_t^{ss}) \), they define:

\[ \beta^{jr} = \frac{\text{Cov} (du_t^{ss}, du_t^{sr})}{\text{Var} (du_t^{ss})} \]  
(D.82)

with

\[ 1 = \beta^{sr} + \beta^{jr} + \beta^\varepsilon \]  
(D.83)

### D.2.2 Decomposition for Employment

In case of employment:

\[ \Delta n_t = irr_t n_{t-1} - orr_t n_{t-1} + \varepsilon_t \]  
(D.84)

Note that I define the inflow rate as \( irr_t \) and the outflow rate as \( orr_t \) (in line with the notation in the empirical part). Also important: on the left-hand side, we now have the absolute difference of employment. Although equation (D.84) is not an approximation, we have a rest term (as workers may be joining and leaving the firm via on-the-job search).

To obtain the relative difference, I divide both sides by \( n_t \).

\[ \frac{\Delta n_t}{n_{t-1}} = irr_t - orr_t + \frac{\varepsilon_t}{n_{t-1}} \]  
(D.85)

We can now apply the same decomposition exercise as before, by defining: \( dn = \frac{\Delta n_t}{n_t} \), \( cirr = irr_t \), where I use a \( c \) (for contribution) instead of a \( d \) because it is not a first difference, \( corr = orr_t \), and \( c\varepsilon = \frac{\varepsilon_t}{n_t} \).
\[ \text{Var}(dn) = \text{Cov}(dn, \text{cirr}) + \text{cov}(dn, \text{corr}) + \text{cov}(dn, \varepsilon) \quad (D.86) \]

This allows us to derive the different betas:

\[ \beta_{\text{irr}} = \frac{\text{Cov}(dn, \text{cirr})}{\text{Var}(dn)} \quad (D.87) \]

\[ \beta_{\text{orr}} = \frac{\text{Cov}(dn, \text{corr})}{\text{Var}(dn)} \quad (D.88) \]