Estimating Labour Supply Functions Under Rationing

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Abstract

This paper demonstrates how data on individuals' non-market time allocations can be informative about their labour supply behaviour. A theoretical model of the allocation of time under rationing is presented. This shows that, in the presence of corner solutions in individuals' non-market time allocations, obtaining consistent estimates of the parameters of the labour supply function requires that the corner solutions be recognised. The model is estimated using data from the 2000 UK Time Use Survey. The results show that estimates of the wage elasticity of labour supply are indeed sensitive to the presence of corner solutions in the time allocated to non-market activities.

Key Words: Time use, Labour supply, Corner solutions, Simulation inference.

JEL Classification: C15, C34, J22.

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1 Introduction

There is an extensive literature dedicated to understanding individual labour supply.[1] The vast majority of the studies in this area use data from labour force surveys. However, recently, a small number of studies have used time use data to investigate individual labour supply (see, for example, Kooreman and Kapteyn, 1987, Klevmarken, 2005).

Time use surveys differ from labour force surveys in two important respects. Firstly, the method of data collection differs. Time use data are usually collected via diaries, where individuals are asked to detail their time use over a small number of days. In contrast, labour force surveys typically ask individuals to report their usual hours of work. Secondly, labour force surveys do not provide data on individuals' non-market time allocations. While the implications of the different methods of data collection have been studied extensively (see Carlin and Flood, 1997, Harvey, 1993, Juster and Stafford, 1991, Klevmarken, 2005, Stinson, 1999), little attention has been paid to how data on individuals' non-market time allocations could be used when estimating labour supply functions.

This paper investigates one key instance where data on individuals' non-market time allocations are informative about their labour supply behaviour. This is when some individuals face binding constraints, or rations, on their non-market time allocations. Specifically, this paper investigates the theoretical and empirical implications of corner solutions in individuals' non-market time allocations on their labour supply behaviour.

Theoretical models of choice under rationing include Neary and Roberts (1980), Lee and Pitt (1986) and Wales and Woodland (1983). A prevailing result from this work is that a constraint on one element of an individual's choice set affects the individual's demand for goods that are not subject to rationing. Translated into the context of labour supply, this implies that a constraint on some element of an individual's non-market time allocation affects their labour

[1] See the surveys by Blundell and MaCurdy (1999) and Killingsworth and Heckman (1986).

supply behaviour.

Empirically, corner solutions in the time allocated to non-market activities could have important implications for labour supply behaviour. Table 1 summarises time use data taken from the 2000 UK Time Use Survey.[2] Zero observations for the time allocated to non-market activities are very common. In particular, for both men and women, there are three non-market activities for which there are sizable proportions of zero observations; sports, volunteer work and social activities.[3]

Table 1 about here

In the absence of any corner solutions in the time allocated to non-market activities, it is valid to aggregate the time spent in all non-market activities into a single quantity. This is explained as follows. If an individual allocates positive time to all non-market activities, their value of time, at the margin, in each non-market activity is equal to their wage, if they work, or their reservation wage if they do not work. Thus, the relative prices of the individual's time in all non-market activities are fixed, and therefore Hick's (1936) composite commodity theorem can be applied. It follows that aggregation across non-market activities is valid, and, therefore, it is possible to obtain consistent estimates of the parameters of the labour supply

[2]See Section 4 for details on the 2000 UK Time Use Survey, and a description of the sample used in this application.

[3]Of course, there are alternative explanations for the zero observations. One such explanation is that the zero observations are due to the small number of sampled days (for a discussion of this issue, see Kalton, 1985). In this paper, the focus is on the corner solution explanation. This allows the zero observations to be explained within the framework of consumer theory, and consequently allows the application of established results from consumer theory regarding behaviour under rationing. The infrequency problem is minimised by using a combination of weekday and weekend day diaries and excluding diaries where the individual reports the day was unusual for some reason. Furthermore, many activities that occur infrequently are likely to be classified as other time use. By considering all activities that are classified as other time use as a single time use category, the infrequency problem is further reduced.

function without considering the allocation of time between non-market activities.

However, when corner solutions in the time allocated to non-market activities are present, an individual's value of time, or virtual price of time, in each of the constrained non-market activities will differ from their wage if they work, or reservation wage if they do not work. The relative prices of the individual's time in non-market activities are no longer fixed: They depend on the individual's wage and non-labour market income. Therefore, aggregation across non-market activities is not valid. In this case, correct estimation of the labour supply function requires explicit modelling of the corner solutions in the time allocated to non-market activities.

Below, a theoretical model of the allocation of time under rationing is presented. The model is implemented by assuming preferences take the Stone-Geary form, leading to a linear expenditure system for the demand functions. Two versions of the time allocation model are estimated. In the first model, corner solutions in the time allocated to non-market activities are ignored, and the time spent in all non-market activities is aggregated into a single quantity, leisure. This is referred to as the standard labour supply model. In the second model, a disaggregated classification of non-market time use is applied and, where appropriate, corner solutions in the time allocated to non-market activities are incorporated. This is referred to as the multivariate time allocation model. The first model is estimated by maximum likelihood. The second model is estimated using maximum simulated likelihood estimation, with the GHK simulator (see Börsh-Supan and Hajivassiliou, 1993, Hajivassiliou and McFadden, 1990 and Keane, 1994) used to evaluate the likelihood.

The theoretical analysis indicates that, in the case of Stone-Geary preferences, the wage elasticity of labour supply is biased downwards if corner solutions in the time allocated to non-market activities are ignored. The empirical results show that the magnitude of this bias is substantial.

This paper proceeds as follows. Section 2 introduces the theoretical model. Section 3 presents an empirical implementation of this model using the linear expenditure system, and gives formulas for the reservation wage, virtual prices and wage and non-labour market income elasticities. This section also contains a discussion of the empirical implications of corner solutions in individuals' non-market time allocations, with special attention paid to the case where preferences take the Stone-Geary form. Section 4 reviews the data. Section 5 presents the results and Section 6 concludes. The Appendix provides the econometric details behind the individual likelihood contributions.

2 Modelling the Allocation of Time Under Rationing

In this section an economically grounded model of the allocation of time between market work and several non-market activities is presented. The model is used to show the effects of constraints on individuals' non-market time allocations on their labour supply behaviour, and also serves as a starting point for the empirical implementation discussed in Section 3. The model is one of individual decision making. The complications posed by inter-relationships between individuals living in the same household are ignored.

The model is as follows. Each individual's non-market time is disaggregated into m possible uses, denoted by the vector $T_i = (T_{i1}, ..., T_{im})$ where T_{ij} is the time individual i spends in non-market activity j for $i = 1, ..., n$ and $j = 1, ..., m$. Each individual is assumed to have a well behaved utility function, $U(T_i, q_i)$, defined over the time spent in each of the m non-market activities and their consumption of the aggregate good, q_i.[4]

[4]One may interpret the time spent in non-market activities as contributing to the production of commodities that yield utility, as in Becker (1965) and Gronau (1977). In this case, $U(T_i, q_i)$ compounds preferences and technology. With sufficiently strong restrictions on preferences over commodities and on the production technology, the utility function is indeed well behaved (see Pollak and Wachter, 1975).

Individual i faces the following optimisation problem

$$Max_{T_i,q_i} U(T_i, q_i) \tag{1}$$

subject to

$$q_i + w_i \sum_{j=1}^{m} T_{ij} \leqslant w_i T + a_i, \tag{2}$$

$$T_{ij} \geqslant 0, \text{ for } j = 1, ..., m, \tag{3}$$

$$T - \sum_{j=1}^{m} T_{ij} \geqslant 0. \tag{4}$$

In the above, T denotes total available time and $T_{iw} = T - \sum_{j=1}^{m} T_{ij}$ is the time individual i allocates to market work. w_i and a_i denote the individual's wage and non-labour market income respectively. (2) is the budget constraint while (3) and (4) are non-negativity constraints on the time spent in non-market activities and market work respectively.[5] The price of the aggregate good has been normalised to one. The Kuhn-Tucker conditions for this problem are as follows

$$U_{T_{ij}} - \lambda_i w_i + \mu_{ij} - \eta_i = 0, \text{ for } j = 1, ..., m, \tag{5}$$

$$U_{q_i} - \lambda_i = 0, \tag{6}$$

$$\lambda_i \geqslant 0, \tag{7}$$

$$\mu_{ij} \geqslant 0, \text{ for } j = 1, ..., m, \tag{8}$$

$$\eta_i \geqslant 0, \tag{9}$$

where λ_i is the multiplier on the budget constraint, μ_{ij} is the multiplier on the j^{th} non negativity constraint in (3) and η_i is the multiplier on the non-negativity constraint on market work, (4). Subscripts on the utility function denote partial derivatives.

Assuming local non-satiation, the budget constraint is strictly binding, implying $\lambda_i > 0$.

[5]The complete problem also includes the constraints $T_{ij} \leqslant T$ for $j = 1, ..., m$, and $T_{iw} \leqslant T$, however these constraints are not empirically important and therefore are ignored for what follows.

This allows the first order conditions given by (5) to be rearranged to produce

$$U_{T_{ij}} - \lambda_i(\underbrace{\underbrace{w_i + \frac{\eta_i}{\lambda_i}}_{w_i^*} - \frac{\mu_{ij}}{\lambda_i}}_{w_{ij}^*}) = 0, \text{ for } j = 1, ..., m. \tag{10}$$

Equation (10) implicitly defines w_i^*, individual i's reservation wage, and w_{ij}^*, individual i's virtual price of time in non-market activity j. For an individual who allocates zero time to market work $\eta_i > 0$, and therefore their reservation wage exceeds their market wage. $\mu_{ij} > 0$ if the individual allocates zero time to non-market activity j. In this case, the individual's virtual price of time in activity j is less than their market wage, if they work, or their reservation wage if they do not work. Intuitively, an individual's value of time is their wage, if they work, or their reservation wage if they do not work. Thus, the individual's decision to allocate time to any particular non-market activity depends on their value of time in the non-market activity, relative to their overall value of time.

Solving the Kuhn-Tucker conditions, given by equations (5)-(9), produces a system of constrained Marshallian demand functions. Using the definitions of the reservation wage and the virtual prices of time in the constrained non-market activities, the constrained demand functions can be expressed as the unconstrained demand functions evaluated at the reservation wage and the virtual prices of the constrained non-market activities (for a proof and further discussion see Neary and Roberts, 1980). Thus, the demand functions can be written as follows

$$T_j^{mc}(w_i, w_iT + a_i) = T_j^m(w_{i1}^*, ..., w_{im}^*, w_i^*, w_i^*T + a_i), \text{ for } j = 1, ..., m, \tag{11}$$

$$q^{mc}(w_i, w_iT + a_i) = q^m(w_{i1}^*, ..., w_{im}^*, w_i^*, w_i^*T + a_i), \tag{12}$$

where T_j^m and q^m are individual i's *unconstrained* Marshallian demand functions for time in non-market activity j and the aggregate good, and T_j^{mc} and q^{mc} are individual i's *constrained* Marshallian demand functions for time in non-market activity j and the aggregate good. The above imply a labour supply function of the form

$$T_w^{mc}(w_i, w_i T + a_i) = T_w^m(w_{i1}^*, ..., w_{im}^*, w_i^*, w_i^* T + a_i). \tag{13}$$

Expressing the demand functions in terms of virtual prices makes it clear that an individual's demand for time in unconstrained activities depends on the combination of binding and non-binding non-negativity constraints facing the individual. Specifically, an observation of zero time allocated to a non-market activity implies a value of time in that activity below the individual's value of time in the activities to which they allocate positive time. This effect, through the virtual price of time in the constrained activity, changes the individual's demand for time in the unconstrained activities, relative to the case where the demand for time in the constrained activity is positive.

The same is true for labour supply: Individuals facing different combinations of binding and non-binding non-negativity constraints on their non-market time allocations have different labour supply functions. The precise effect of a corner solution in the time allocated to a non-market activity on labour supply will depend on preferences. Suppose that time in the constrained non-market activity is a substitute for time in market work. Since the virtual price of time in the constrained activity is below the individual's wage, labour supply will be lower than in the absence of the constraint. Conversely, if time in the constrained non-market activity is a complement for time in market work, labour supply will be higher than in the absence of the constraint.

Thus, the functional form of the entire demand system, including the labour supply function, depends on the combination of corner solutions that is present. Ignoring any of the corner solutions leads to a misspecified model. Estimating such a model will produce inconsistent parameter estimates, and wage and income effects based on the misspecified model will be misleading. In order to obtain consistent parameter estimates, corner solutions must be incorporated.

3 Empirical Implementation

In this section it is shown that the linear expenditure system can be used to implement the above multivariate time allocation model, incorporating corner solutions in the time allocated to non-market activities and market work. The model takes the form of a multivariate Tobit with endogenous switching. The utility function and wage equation are specified to include observed and unobserved individual specific heterogeneity. Using the definitions of the reservation wage and virtual prices defined by equation (10), the likelihood can be derived. Additionally, closed form expressions can be found for the reservation wage, virtual prices and the wage and non-labour market income elasticities of labour supply and of time in non-market activities.

When specifying a functional form for preferences it is necessary to choose a utility function that permits corner solutions. Also, given that the wage is the only observed price, the demand functions must not involve cross price effects. For this application, preferences are assumed to be of the Stone-Geary form, Stone (1954). This leads to a linear expenditure system for the demand functions. The Stone-Geary utility function takes the following form

$$U(T_i, q_i; \varepsilon_i, Z_i) = \sum_{j=1}^{m} \alpha_{ij} \log(T_{ij} - \gamma_j) + \alpha_{iq} \log(q_i - \gamma_q). \tag{14}$$

The γ_js can be interpreted as minimum or subsistence quantities. Thus, a corner solution in the time allocated to non-market activity j is permitted if γ_j is negative.

Maximising (14) subject to the budget constraint, (2), and ignoring the non-negativity constraints produces the following system of Marshallian demand functions

$$T_{ij} = \gamma_j + \frac{\alpha_{ij}}{w_i} \left(w_i T + a_i - w_i \sum_{j=1}^{m} \gamma_j - \gamma_q \right), \text{ for } j = 1, ..., m, \tag{15}$$

$$q_i = \gamma_q + \alpha_{iq} \left(w_i T + a_i - w_i \sum_{j=1}^{m} \gamma_j - \gamma_q \right). \tag{16}$$

Consequently, the labour supply function is given by

$$T_{iw} = \frac{\gamma_q - a}{w_i} + \frac{\alpha_{iq}}{w_i} \left(w_i T + a_i - w_i \sum_{j=1}^{m} \gamma_j - \gamma_q \right). \tag{17}$$

Inspecting the above reveals an absence of cross price effects, as required.

Both observed and unobserved individual specific heterogeneity are incorporated into the utility function through the α_is in the following way

$$\alpha_{ij} = \frac{\exp(\varepsilon_{ij} + Z_i'\beta_j)}{\sum_{j=1}^{m} \exp(\varepsilon_{ij} + Z_i'\beta_j) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}, \text{ for } j = 1, ..., m-1, \tag{18}$$

$$\alpha_{im} = \frac{1}{\sum_{j=1}^{m} \exp(\varepsilon_{ij} + Z_i'\beta_j) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}, \tag{19}$$

$$\alpha_{iq} = \frac{\exp(\varepsilon_{iq} + Z_i'\beta_q)}{\sum_{j=1}^{m} \exp(\varepsilon_{ij} + Z_i'\beta_j) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}. \tag{20}$$

Here, Z_i is a vector of observed individual characteristics, and $\varepsilon_i = (\varepsilon_{i1,...,}\varepsilon_{im-1}, \varepsilon_{iq})$ is an m dimensional vector representing the unobserved component of preferences. The identifying normalisations $\varepsilon_{im} = 0$ for all i and $\beta_m = 0$ have been made. Therefore ε_{ij} represents the unobserved component of individual i's preference for time in non-market activity j relative to time in the m^{th} non-market activity. Likewise, $Z_i'\beta_j$ represents the observed component of individual i's preference for time in non-market activity j relative to time in the m^{th} non-market activity. It is assumed that ε_i is known to the individual when they make their time allocation decision, however ε_i is not observed by the econometrician. Furthermore ε_i is assumed to be independent of Z_i for $i = 1, ..., n$ and independent across individuals.[6]

The properties of the above specification of the linear expenditure system are now discussed. The specification of the α_is given in equations (18)-(20) ensures $0 < \alpha_{ij} < 1$ for $j = 1, ..., m$, $0 < \alpha_{iq} < 1$ and $\sum_{j=1}^{m} \alpha_{ij} + \alpha_{iq} = 1$. The first two conditions are necessary and sufficient for global concavity of the cost function, and therefore ensure negativity of the demand system. The third condition is necessary and sufficient for the demand functions to satisfy adding up and to be homogeneous of degree zero in prices and income.

[6] In this specification of the linear expenditure system the γs are assumed to be constant across individuals. Obviously this is not entirely realistic, for example, one might expect that the minimum quantity of goods, γ_q, varies with the number of children in the household. However, given the already complex nature of the model, incorporating demographic variables in the γs is not attempted.

Since the model consists of a system of censored demand functions it is important to ensure that the model is coherent (see Gourieroux *et al.*, 1980, Ransom, 1987, van Soest *et al.*, 1993). For the model in hand, coherency requires that each realisation of the random vector ε_i corresponds to a unique vector of endogenous variables (T_i, q_i), and for every observed (T_i, q_i) there must exist some ε_i that can generate this outcome. Global concavity of the cost function is sufficient, although not necessary, to ensure the system of censored demand functions is coherent. The above stochastic specification ensures that the cost function is globally concave, thus the system of censored demand functions is indeed coherent. This allows the model to be estimated without needing to further restrict the parameter space to ensure coherency.

Wages are only observed for individuals who work in the market. However, wages are also required for non-working individuals. Thus, a wage equation is specified, and is estimated jointly with the demand system, as in Heckman (1974). Log wages as assumed to be linear in a vector of observed individual characteristics, X_i, with an additive error term, ε_{iw}

$$\log(w_i) = X_i'\delta + \varepsilon_{iw}. \tag{21}$$

Let $e_i = (\varepsilon_{iw}, \varepsilon_{i1}, ..., \varepsilon_{im-1})$. e_i is assumed to be independently and identically normally distributed with zero mean and an unrestricted covariance matrix.

Expressions for each term in the likelihood can now be derived. All individuals falls into one of three cases, depending on the combination of binding and non-binding constraints they are facing. In case (i) all non-negativity constraints are non-binding, in case (ii) there are binding non-negativity constraints on the time allocated to the first l non-market activities, and in case (iii) there are binding non-negativity constraints on the time allocated to the first l non-market activities and also on the time spent in market work. The Appendix contains derivations of the likelihood contributions for each of the three cases.

When there are individuals facing multiple binding non-negativity constraints the likelihood contains high dimensional integrals. The dimension of the integral an individual contributes to the likelihood is equal to the number of binding non-negativity constraints facing the individual. Except in special cases, it is computationally difficult to numerically evaluate multivariate normal distribution functions with more than three dimensions. The solution proposed here is to use the GHK simulator due to Börsh-Supan and Hajivassiliou (1993), Hajivassiliou and McFadden (1990) and Keane (1994) to evaluate the probability each individual contributes to the likelihood.

In the standard labour supply model, corner solutions in the time allocated to non-market activities are ignored, but corner solution for labour supply are still permitted. The time spent in all non-market activities is aggregated into a single quantity, labelled leisure. Preferences are assumed to be defined over leisure and the aggregate good. Here, preferences are again assumed to be of the Stone-Geary form. Thus, the demand functions for leisure and goods, and the labour supply function, take the same form as above. The likelihood for the standard labour supply model can be derived in the same way as for the multivariate time allocation model. The standard labour supply model is estimated by maximum likelihood.

3.1 Elasticities and Virtual Prices

Given the functional form of the linear expenditure system, it is possible to find closed form expressions for the reservation wage and the virtual prices of time in the constrained non-market activities.[7] An individual who works in the market and allocates zero time to non-market activities $j = 1, ..., l$ has virtual prices of time in the first l non-market activities given by

$$w_{ij}^{*} = -\frac{\alpha_{ij}(w_i T + a_i - w_i \sum_{j=l+1}^{m} \gamma_j - \gamma_q)}{\gamma_j (1 - \sum_{j=1}^{l} \alpha_{ij})}, \text{ for } j = 1, ..., l. \qquad (22)$$

An individual who does not work in the market and allocates zero time to non-market activities $j = 1, ..., l$ and positive time to all other non-market activities has a reservation wage and virtual

[7] The expressions in this section refer to the multivariate time allocation model. The corresponding quantities for the standard labour supply model, where appropriate, are obvious special cases.

prices of time in the first l non-market activities given by

$$w_i^* = \frac{(1-\alpha_{iq})(a_i - \gamma_{iq} - \sum_{j=1}^{l} w_{ij}^* \gamma_j)}{\left(T - \sum_{j=l+1}^{m} \gamma_j\right) \alpha_{iq}}, \quad (23)$$

$$w_{ij}^* = -\frac{\alpha_{ij}(a_i - \gamma_q)}{\gamma_j \alpha_{iq}}, \text{ for } j = 1..., l. \quad (24)$$

The wage and non-labour market income elasticities of labour supply and of time in non-market activities can be found by combining the demand functions given in equations (15)-(17) with the formulas for the reservation wage and virtual prices given in equations (22)-(24). Below, the formulas for the wage elasticities of labour supply and of time in non-market activities are presented, for the case where there are binding constraints on the time spent in the first l non-market activities.

$$\epsilon_{w,T_w} = \frac{(1-\alpha_{iq} - \sum_{j=1}^{l} \alpha_{ij})(a_i - \gamma_q)}{T_{iw} w_i (1 - \sum_{j=1}^{l} \alpha_{ij})}, \quad (25)$$

$$\epsilon_{w,T_j} = -\frac{\alpha_{ij}(a_i - \gamma_q)}{T_{ij} w_i (1 - \sum_{j=1}^{l} \alpha_{ij})}, \text{ for } j = l+1, ..., m. \quad (26)$$

Similarly, the non-labour market income elasticities of labour supply and of time in non-market activities for the case where there are binding constraints on the time spent in the first l non-market activities are given by

$$\epsilon_{a,T_a} = -\frac{(1-\alpha_{iq} - \sum_{j=1}^{l} \alpha_{ij}) a_i}{T_{iw} w_i (1 - \sum_{j=1}^{l} \alpha_{ij})}, \quad (27)$$

$$\epsilon_{a,T_j} = \frac{\alpha_{ij} a_i}{T_{ij} w_i (1 - \sum_{j=1}^{l} \alpha_{ij})}, \text{ for } j = l+1, ..., m. \quad (28)$$

The wage elasticities of labour supply and of time in non-market activities are zero for individuals who do not work in the market. The non-labour market income elasticity of labour supply is also zero for individuals who do not work in the market.

Inspecting the above formulas reveals that time in each non-market activity is a substitute for time in market work. This is a consequence of assuming Stone-Geary preferences. The above formulas also show that the wage elasticities depend on the parameters γ_q, α_{ij} for $j = 1, ..., m$ and α_{iq}, whereas the non-labour market income elasticities depend only on α_{ij} for

$j = 1, ..., m$ and α_{iq}. Thus, given the restriction $\sum_{j=1}^{m} \alpha_{ij} + \alpha_{iq} = 1$, the total effects of demographic variables and non-labour market income on the demand functions and elasticities are restricted. This is important when interpreting the results.

3.2 Empirical Implications of Ignoring Corner Solutions

Thus far, it has been established that ignoring corner solutions in the time allocated to non-market activities leads to inconsistent estimates of the parameters of the labour supply function. In this section, the effects of ignoring corner solutions on the estimated demand system, and in particular on the estimated labour supply function, are explored in more detail. This is first discussed for the general case then specialised to the case of the linear expenditure system.

Note that a corner solution in the time allocated to a non-market activity implies a negative unconstrained demand for time in the activity.[8] Ignoring corner solutions in the time allocated to non-market activities leads to estimated unconstrained demands for time in the constrained activities that display less variation than the true unconstrained demands. The lesser variation in the estimated unconstrained demands corresponds to some combination of smaller wage effects, smaller non-labour market income effects and smaller effects of demographic variables. The relative contributions of the wage, non-labour market income and demographic variables to the lower variation in the unconstrained demands depends on the functional form of the demand system. Consequently, the effect of corner solutions in the time allocated to non-market activities on labour supply behaviour depends on preferences. Thus, in general, ignoring corner solutions in the time allocated to non-market activities can lead to upwards or downwards bias in the estimated wage elasticity of labour supply.

The implications of ignoring corner solutions in the time allocated to non-market activities when preferences take the Stone-Geary form are now explored. Consider the unconstrained demand functions for time in non-market activities when corner solutions are present. The

[8]For this discussion, unconstrained demands are taken to be evaluated at the market wage.

effects of non-labour market income and demographic variables on these demand functions are limited by the restriction $\sum_{j=1}^{m} \alpha_{ij} + \alpha_{iq} = 1$. Therefore, variations in non-labour market income or demographic variables is unlikely to cause the variation in the unconstrained demands required to produce the observed corner solutions. Instead, the required variation in the unconstrained demands is likely to be due to a large wage effect. Furthermore, in the linear expenditure system, time in each non-market activity is a substitute for time in market work. Therefore, if the time allocated to some non-market activities is highly wage sensitive, the time allocated to market work will necessarily be highly wage sensitive.

Suppose a linear expenditure system is estimated ignoring the corner solutions in the time allocated to non-market activities. The estimated unconstrained demand functions for time in non-market activities will display less variation than the true demand functions. The lower variation is likely to be due to a smaller effect of the wage on the time allocated to non-market activities. This implies a smaller effect of the wage on labour supply. Thus, in the case of the linear expenditure system, ignoring corner solutions in the time allocated to non-market activities is likely to cause the estimated wage elasticity of labour supply to be biased downwards.

4 An Overview of the Data

The data are taken from the 2000 UK Time Use Survey. The main aim of this survey was to measure how individuals allocate their time between various activities. The time use data were collected via two 24 hour time use diaries, one for a weekday and one for a weekend day. For every 10 minute interval in each 24 period, individuals were asked to record their primary and secondary activities as well as information on their location and who they were with. Household and individual questionnaires were used gather background information and demographics.

Attention is restricted to individuals aged 18 years or over. Retired individuals and students have been excluded. Any diaries where the individual reported the day was unusual for some reason were also excluded. This sample consists of 1832 females and 1433 males. Since the time allocation patterns for males and females differ substantially (see Table 1), separate models are estimated for males and females. As in common when using time use data, an equivalent week has been constructed for each individual. The time an individual spends on each activity during an equivalent week is defined as five times the weekday diary observation plus two times the weekend observation. For the purpose of estimating the demand system, these equivalent times are taken to represent individuals' demands for time in various activities over a typical week. When estimating the multivariate time allocation model, eight different time uses are distinguished. The definition of each is given below.

Market work Working time in main job, coffee breaks and other breaks in main job, working time in secondary job, coffee breaks or other breaks in secondary job, other activities relating to employment, excluding activities relating to job search.

Sports Sports and outdoor activities, physical exercise, productive exercise, hobbies and games, computing, collecting, correspondence, solo games, games played with others, computer games and gambling.

Volunteer work Volunteer work and meetings, work for an organisation, volunteer work through an organisation, other organisational work, informal help to other households, participatory voluntary activities including meetings and religious activities.

Social activities Social life and entertainment, socialising with household members, visiting and receiving visitors, feasts, telephone conversations, cinema, theatre and concerts, visiting art exhibitions, museums and libraries, sports events, resting, other entertainment and cultural activities.

Home production Food management and preparation, cleaning dwelling, cleaning yard, making and care of textiles, gardening and pet care, house construction and renovation, shopping, commercial or administrative services, personal services, care of another household member, including childcare.

Media activities Reading, watching television, listening to the radio, music or recordings, other mass media activities.

Other time use Classes and lectures, homework, other activities relating to school or university, free time study, travel related to work, activities relating to job search, unspecified time use.

Sleep Sleep, eating, washing and dressing, other activities relating to personal care.

When estimating the multivariate time allocation model, corner solutions in market work, sports, volunteer work and social activities are incorporated. In total, there are sixteen different combinations of binding and non-binding non-negativity constraints. The numbers of individuals facing each combination of binding and non-binding non-negativity constraints are shown in Table 2.

Tables 2 and 3 about here

Table 3 summarises the demographic and wage data for males and females. AGE is age is years, EDUCATION is an indicator variable taking the value one if the individual has an educational level of A-Levels or above and zero otherwise and CHILDREN is the number of children under 16 years of age present in the household. WAGE is the hourly wage in pounds. Wage data for employed and self employed individuals were collected via the individual questionnaire. Employed individuals were asked to report their last take home pay after deductions and the period covered by their last take home pay. Individuals refusing to answer this question were asked to report their monthly take home pay. Self employed individuals were asked to report their monthly take home pay. All working individuals were asked to report the hours worked in a typical week. Using this information, an hourly wage was constructed for all individuals in employment.

Non-labour market income is defined as weekly household income less the weekly labour market income of all household members, divided by the number of household members. Consumption of the aggregate good is defined as weekly non-labour market income, plus the wage times hours of market work during the equivalent week. Thus, income is assumed to be equal to consumption, and the possibility of consumption smoothing has been excluded.[9]

The regressors used in the wage equation are AGE, AGE^2, EDUCATION and an intercept. Z_i consists of AGE, AGE^2, EDUCATION, CHILDREN and an intercept.[10]

5 Results

In this section the results for the standard labour supply model and the multivariate time allocation model are presented. The estimated wage elasticities of labour supply according to the two models are compared.

The parameter estimates for the standard labour supply model are shown in Table 4, and parameter estimates for the multivariate time allocation model are shown in Tables 5-8.[11,12]

Tables 4 - 8 about here.

[9] The data do not allow one to distinguish between income and consumption.

[10] Excluding the variable CHILDREN from the wage equation ensures that identification of the wage equation is not solely reliant on the non-linearity of the functional form.

[11] When estimating the multivariate time allocation model, 20 replications were used when simulating the likelihood. The results appear to be robust to the number of replications.

[12] All numerical calculations were performed using MATLAB.

Both models produce reasonable estimates of the parameters of the wage equations. The standard labour supply model suggests a minimum quantity of leisure of 76.65 hours per week for females and 48.84 hours per week for males. The results for the multivariate time allocation model show that for males the γ_js are negative for all non-market activities and for females they are negative for all non-market activities except sleep. The specification implemented here constrains γ_{sports}, $\gamma_{volunteer\ work}$ and $\gamma_{social\ activities}$ to be negative but places no restrictions on the other γ_js. However, finding negative γ_js for the other non-market activities is not inconsistent with the framework presented above; it is possible for an activity to have a negative γ, and yet no individual be observed at a corner solution with respect to this activity.

In the standard labour supply model the γ_qs are far lower than the values of $\gamma_{leisure}$. Similarly, in the multivariate time allocation model, the γ_qs are lower than any of the γ_js. Given the way in which γ_q enters the formulas for the wage elasticities, this finding suggests that the variation in unconstrained demands implied by the observed corner solutions is mostly due to a large wage effect, as opposed to the effects of either demographic variables or non-labour market income.

In the standard labour supply model, for both males and females, the error in the wage equation in negatively correlated with the error in the demand function for leisure. This implies individuals who have a higher unobserved components to their wages tend to have a higher unobserved preference for leisure relative to goods. Tables 7 and 8 give the estimated covariance matrices for males and females, according to the multivariate time allocation model. Examining these tables shows females' unobserved preference for time in volunteer work is negative correlated with the error in the wage equation. For males, unobserved preferences for time in both volunteer work and social activities are negatively correlated with the error in the wage equation. For both males and females all other correlations of unobserved preferences with the error in the wage equation are positive. It is interesting to note that the correlation of the unobserved preferences for time in social activities and media activities is negative for both males and females. This means that individuals who have a high unobserved taste for time in media activities are ceteris paribus likely to have a relatively low preference for time in social activities, and vice versa. A similar interpretation can be given to the other correlations in these tables.

The demand functions given by equations (15)-(17) and the wage elasticities given by equations (25) and (26) depend on both the observed individual specific heterogeneity, Z_i, and the unobserved heterogeneity, ε_i. Since ε_i is unobserved, the estimated elasticities and demand functions are evaluated by simulation. To assess the fit of the two models and obtain wage elasticities for the individuals in the sample, the observed Z_i and non-labour market income for each individual are used and 100 values of ε_i are drawn for each individual.

Table 9 shows the predicted proportion of individuals with positive labour supply, the mean predicted labour supply and median predicated wage elasticity according to the standard labour supply model and the multivariate time allocation model. Both models predict the proportion of non-zero observations and the mean hours of labour supply reasonably well. There are, however, substantial differences in the estimated wage elasticity of labour supply according to the two models. Specifically, the standard labour supply model predicts a median wage elasticity of labour supply of 1.04 for females and 1.67 for males, where the lower elasticity for females is due to the lower proportion of females with positive labour supply. In contrast, the multivariate time allocation model predicts a median wage elasticity of labour supply of 3.97 for females and 3.72 for males. These results support the discussion in Section 3.2: In the case of the linear expenditure system, ignoring the corner solutions in the time allocated to non-market activities leads to a downwards bias in the estimated wage elasticity of labour supply.

Table 9 about here

Finally, Table 10 shows the predicted allocation of time to non-market activities, together with the predicted wage elasticities for time in non-market activities. The multivariate time allocation model predicts women allocate more time than men to social activities, home production and sleep, whereas men spend longer than women in market work, sports and media activities. The mean predicted time allocated to volunteer work and other time use is similar for men and women. This is entirely consistent with the observed time allocations summarised in Table 1. The model also gives reasonable predictions of the proportions of participants for sports, volunteer work and social activities. This suggests that the multivariate time allocation model provides an accurate description of individuals' non-market time allocation behaviour.

6 Conclusion

The implications of corner solutions in individuals' non-market time allocations on their labour supply behaviour have been explored. Two models have been estimated: The standard labour supply model, which assumes an absence of corner solutions in the time allocated to non-market activities, and the multivariate time allocation model, which uses a disaggregated classification of non-market time use and, where appropriate, incorporates corner solutions in the time allocated to non-market activities. In both cases, preferences have been assumed to be of the Stone-Geary form.

As expected, the estimated wage elasticity of labour supply is biased downwards when the corner solutions in the time allocated to non-market activities are ignored. Of course, this finding is somewhat driven by the functional form of the linear expenditure system. With an alternative functional form, ignoring corner solutions will have a different effect on the estimated labour supply function.

Indeed, one might view the above estimated wage elasticities of labour supply according to the multivariate time allocation model as being implausibly high. If so, this is principally an objection to the functional form employed here: Irrespective of ones preferred specification of preferences, the results presented in this paper suggest that it may be important to consider constraints on individuals' non-market time allocations when estimating labour supply functions.

Appendix

Consider the first order conditions for case (i), where all non-negativity constraints are non-binding

$$\frac{\alpha_{ij}}{T_{ij}-\gamma_j} - \lambda_i w_i = 0, \text{ for } j = 1, ..., m, \tag{29}$$

$$\frac{\alpha_{iq}}{q_i-\gamma_q} - \lambda_i = 0. \tag{30}$$

Assume the m^{th} good is always consumed. Dividing the above equations by the m^{th} first order condition and taking logs gives

$$\varepsilon_{ij} = \log(T_{ij}-\gamma_j) - \log(T_{im}-\gamma_m) - Z_i'\beta_j, \text{ for } j = 1, ..., m-1, \tag{31}$$

$$\varepsilon_{iq} = \log(q_i-\gamma_q) - \log(T_{im}-\gamma_m) - \log(w_i) - Z_i'\beta_q. \tag{32}$$

Thus, the contribution to the likelihood of an individual who falls into case (i) is given by

$$L_{i1} = f_1(w_i, T_{i1,....,}T_{im-1}, q_i|X_i, Z_i) \tag{33}$$

$$= f_{1a}(\varepsilon_{iw}, \varepsilon_{i1}, ..., \varepsilon_{im-1}, \varepsilon_{iq}|X_i, Z_i)\left|\frac{\partial\bar{\varepsilon}_i}{\partial\bar{T}_i}\right|, \tag{34}$$

where f_1 is the joint density of $\bar{T}_i = (w_i, T_{i1,....,}T_{im-1}, q_i)$ conditional on the observed regressors X_i and Z_i, f_{1a} is the multivariate normal density function of $\bar{\varepsilon}_i = (\varepsilon_{iw}, \varepsilon_{i1}, ..., \varepsilon_{im-1}, \varepsilon_{iq})$ and $\left|\frac{\partial\bar{\varepsilon}_i}{\partial\bar{T}_i}\right|$ is the absolute Jacobian from $\bar{T}_i$ to $\bar{\varepsilon}_i$. Using the budget constraint (2) and the utility function (14) gives

$$\left|\frac{\partial \bar{\varepsilon}_i}{\partial \bar{T}_i}\right| = \begin{vmatrix} \frac{1}{T_{i1}-\gamma_1} + \frac{1}{w_i(T_{im}-\gamma_m)} & . & . & \frac{1}{w_i(T_{im}-\gamma_m)} \\ . & . & & . \\ . & & \frac{1}{T_{im-1}-\gamma_{m-1}} + \frac{1}{w_i(T_{im}-\gamma_m)} & . \\ \frac{1}{w_i(T_{im}-\gamma_m)} & . & . & \frac{1}{q_i-\gamma_q} + \frac{1}{w_i(T_{im}-\gamma_m)} \end{vmatrix}. \tag{35}$$

Moving to case (ii), where the non-negativity constraints on the time spent in the first l non-market activities are binding, the first order conditions are given by

$$\frac{\alpha_{ij}}{-\gamma_j} - \lambda_i w_{ij}^* = 0, \text{ for } j = 1, ..., l, \tag{36}$$

$$\frac{\alpha_{ij}}{T_{ij}-\gamma_j} - \lambda_i w_i = 0, \text{ for } j = l+1, ..., m, \tag{37}$$

$$\frac{\alpha_{iq}}{q_i-\gamma_q} - \lambda_i = 0. \tag{38}$$

Again, dividing by the m^{th} first order condition and taking logs gives

$$P(w_{ij}^* \leqslant w_i|Z_i) = P(\varepsilon_{ij} \leqslant \log(-\gamma_j) - \log(T_{im}-\gamma_m) - Z_i'\beta_j), \text{ for } j = 1, ..., l, \tag{39}$$

$$\varepsilon_{ij} = \log(T_{ij}-\gamma_j) - \log(T_{im}-\gamma_m) - Z_i'\beta_j, \text{ for } j = l+1, ..., m-1, \tag{40}$$

$$\varepsilon_{iq} = \log(q_i-\gamma_q) - \log(w_i) - \log(T_{im}-\gamma_m) - Z_i'\beta_q. \tag{41}$$

Thus, the contribution to the likelihood of an individual who falls into case (ii) is given by

$$L_{i2} = P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, T_{il+1}, ..., T_{im-1}, q_i, w_i|X_i, Z_i) \tag{42}$$

$$= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i|T_{il+1}, ..., T_{im-1}, q_i, w_i, X_i, Z_i) f_2(T_{il+1}, ..., T_{im-1}, q_i, w_i|X_i, Z_i) \tag{43}$$

$$= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i|T_{il+1}, ..., T_{im-1}, q_i, w_i, X_i, Z_i) f_{2a}(\varepsilon_{iw}, \varepsilon_{il+1}, ..., \varepsilon_{im-1}, \varepsilon_{iq}|X_i, Z_i) \left|\frac{\partial \breve{\varepsilon}_i}{\partial \breve{T}_i}\right| \tag{44}$$

where $P(.)$ is a l variate normal distribution function, f_2 is the joint density of $\breve{T}_i = (w_i, T_{il+1,....,}T_{im-1}, q_i)$ conditional on the observed regressors X_i and Z_i, f_{2a} is the multivariate normal density function of $\breve{\varepsilon}_i = (\varepsilon_{iw}, \varepsilon_{il+1}, ..., \varepsilon_{im-1}, \varepsilon_{iq})$ and $\left|\frac{\partial \breve{\varepsilon}_i}{\partial \breve{T}_i}\right|$ is the absolute Jacobian from $\breve{T}_i$ to $\breve{\varepsilon}_i$. $\left|\frac{\partial \breve{\varepsilon}_i}{\partial \breve{T}_i}\right|$ has a similar structure to (35).

In case (iii) the individual faces binding non-negativity constraints on the time spent in each of the first l non-market activities and also on the time spent in market work. In this case the first order conditions are given by

$$\frac{\alpha_{ij}}{-\gamma_j} - \lambda_i w_{ij}^* = 0, \text{ for } j = 1, ..., l, \tag{45}$$

$$\frac{\alpha_{ij}}{T_{ij} - \gamma_j} - \lambda_i w_i^* = 0, \text{ for } j = l+1, ..., m, \tag{46}$$

$$\frac{\alpha_{iq}}{a_i - \gamma_q} - \lambda_i = 0. \tag{47}$$

Note that in equation (46) the market wage, w_i, has been replaced by the reservation wage, w_i^*. Dividing by the m^{th} first order condition and taking logs gives

$$P(w_{ij}^* \leqslant w_i^*) = P(\varepsilon_{ij} \leqslant \log(-\gamma_j) - \log(T_{im} - \gamma_m) - Z_i'\beta_j), \text{ for } \ j = 1, ..., l, \tag{48}$$

$$\varepsilon_{ij} = \log(T_{ij} - \gamma_j) - \log(T_{im} - \gamma_m) - Z_i'\beta_j, \text{ for } j = l+1, ..., m-1, \tag{49}$$

$$P(w_i^* \geqslant w_i) = P(\varepsilon_{iq} \leqslant \log(q_i - \gamma_q) - \log(T_{im} - \gamma_m) - \log(w_i) - Z_i'\beta_q). \tag{50}$$

Thus, the contribution to the likelihood of an individual who falls into case (iii) is given by

$$L_{i3} = P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, w_i^* \geqslant w_i, T_{il+1}, ..., T_{im-1}|X_i, Z_i) \tag{51}$$

$$= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, w_i^* \geqslant w_i|T_{il+1}, ..., T_{im-1}, X_i, Z_i) f_3(T_{il+1}, ..., T_{im-1}|X_i, Z_i) \tag{52}$$

$$= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, w_i^* \geqslant w_i|T_{il+1}, ..., T_{im-1}, X_i, Z_i) f_{3a}(\varepsilon_{il+1}, ..., \varepsilon_{im-1}|X_i, Z_i) \left|\frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}\, i}\right| \tag{53}$$

where $P(.)$ is a $l+1$ variate normal distribution function, f_3 is the joint density of $\tilde{T}_i = (T_{l+1i,....,}T_{m-1i})$ conditional on the observed regressors X_i and Z_i, f_{3a} is the multivariate normal density function of $\tilde{\varepsilon}_i = (\varepsilon_{il+1}, ..., \varepsilon_{im-1})$ and $\left|\frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}_i}\right|$ is the absolute Jacobian from $\tilde{T}_i$ to $\tilde{\varepsilon}_i$. Again $\left|\frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}_i}\right|$ has a similar structure to (35). The likelihood can be formed by combining the probabilities given by (33), (44) and (53).

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	Females (1832 observations)			Males (1433 observations)		
Activity	% non-zero	Mean of positive observations	Mean of all observations	% non-zero	Mean of positive observations	Mean of all observations
Market work	62.99	33.40	21.04	85.20	45.08	38.41
Sports	44.87	6.12	2.75	53.87	8.45	4.55
Volunteer work	26.80	7.03	1.89	20.51	6.02	1.24
Social activities	91.43	10.38	9.49	86.39	8.67	7.49
Home production	99.23	30.24	30.01	96.65	15.64	15.12
Media activities	97.21	16.55	16.09	98.32	19.00	18.68
Other time use	99.07	12.70	12.58	99.51	12.49	12.43
Sleep	1	74.15	74.15	1	70.09	70.09

Table 1: Summary of Time Use data: Hours per equivalent week.

Category	Females	Males	Category	Females	Males
W, Sp, V, Sc	118	112	$\underline{W}, \underline{Sp}, V, Sc$	81	20
$W, \underline{Sp}, V, Sc$	158	98	$\underline{W}, Sp, \underline{V}, Sc$	216	84
$W, Sp, \underline{V}, Sc$	336	438	$\underline{W}, Sp, V, \underline{Sc}$	4	1
$W, Sp, V, \underline{Sc}$	6	5	$W, \underline{Sp}, \underline{V}, \underline{Sc}$	65	84
$\underline{W}, Sp, V, Sc$	112	45	$\underline{W}, Sp, \underline{V}, \underline{Sc}$	206	47
$W, \underline{Sp}, \underline{V}, Sc$	448	394	$\underline{W}, \underline{Sp}, V, \underline{Sc}$	4	0
$W, \underline{Sp}, V, \underline{Sc}$	8	13	$\underline{W}, \underline{Sp}, \underline{V}, Sc$	15	10
$W, Sp, \underline{V}, \underline{Sc}$	15	77	$\underline{W}, \underline{Sp}, \underline{V}, \underline{Sc}$	40	5

Table 2: Number of individuals falling into each combination of binding and non-binding non-negativity constraints: W, Sp, V and Sc denote Market work, Sports, Volunteer work and Social activities respectively. An underscore denotes a zero observation for the corresponding activity.

	Females		Males	
	Mean	Standard deviation	Mean	Standard deviation
AGE	39.84	11.01	40.50	11.62
EDUCATION	0.28	0.45	0.29	0.46
CHILDREN	1.22	1.25	1.05	1.16
WAGE	6.64	5.05	8.43	5.54

Table 3: Summary of demographic and wage data.

	Females	Males
Wage equation		
INTERCEPT	1.05 (0.209)	0.87 (0.011)
AGE	0.03 (0.011)	0.05 (0.019)
AGE^2	−0.0003 (0.00013)	−0.0005 (0.00016)
EDUCATION	0.44 (0.025)	0.34 (0.030)
β_q		
INTERCEPT	3.00 (0.202)	9.30 (0.060)
AGE	0.03 (0.012)	−0.02 (0.004)
AGE^2	−0.0005 (0.00015)	0.0001 (0.00005)
EDUCATION	−0.33 (0.035)	−0.36 (0.037)
CHILDREN	−0.12 (0.010)	−0.02 (0.008)
$\gamma_{leisure}$	76.65 (0.86)	−4.19 (48.84)
γ_q	−10032.71 (361.72)	−3552211.26 (91648.80)
σ_w^2	0.15 (0.001)	0.23 (0.002)
σ_{wq}	−0.15 (0.021)	−0.25 (0.026)
σ_q^2	0.35 (0.002)	0.37 (0.003)

Table 4: Parameter estimates for the standard labour supply model: Standard errors in parenthesis.

	Females	Males
γ_{sports}	−13.25 (0.58)	−10.13 (0.65)
$\gamma_{volunteer\ work}$	−53.04 (1.38)	−52.19 (1.87)
$\gamma_{social\ activiites}$	−7.22 (0.42)	−5.28 (0.36)
$\gamma_{home\ production}$	−13.57 (0.77)	−3.62 (0.39)
$\gamma_{media\ activities}$	−7.71 (0.48)	−12.96 (0.63)
$\gamma_{other\ time\ use}$	−4.78 (0.40)	−3.22 (0.37)
γ_{sleep}	4.50 (0.028)	−15.07 (0.83)
γ_q	−38327.45 (886.50)	−43162.57 (909.48)

Table 5: Parameter estimates for the multivariate time allocation model: Estimates of γ_j for $j = 1, ..., 7$ and γ_q for females and males. Standard errors in parenthesis.

	Females	Males		Females	Males
β_{sports}			$\beta_{media\ activities}$		
INTERCEPT	−1.94 (0.18)	−2.04 (0.21)	INTERCEPT	−1.34 (0.12)	−1.14 (0.10)
AGE	0.0085 (0.0097)	−0.0031 (0.011)	AGE	0.0083 (0.0064)	0.0024 (0.0054)
AGE^2	−0.000072 (0.00011)	0.000010 (0.00013)	AGE^2	−0.000052 (0.000076)	0.000013 (0.0063)
EDUCATION	0.13 (0.027)	0.22 (0.039)	EDUCATION	−0.13 (0.026)	−0.079 (0.022)
CHILDREN	−0.019 (0.011)	0.033 (0.017)	CHILDREN	−0.019 (0.020)	−0.0044 (0.0093)
$\beta_{volunteer\ work}$			$\beta_{other\ time\ use}$		
INTERCEPT	−0.78 (0.086)	−0.98 (0.076)	INTERCEPT	−1.53 (0.16)	−1.83 (0.17)
AGE	0.014 (0.0040)	0.011 (0.0041)	AGE	0.0046 (0.0085)	−0.0010 (0.0093)
AGE^2	−0.011 (0.000047)	−0.000091 (0.000048)	AGE^2	−0.00011 (0.000097)	−0.00001 (0.011)
EDUCATION	0.000001 (0.016)	0.000006 (0.018)	EDUCATION	0.066 (0.026)	0.15 (0.031)
CHILDREN	0.013 (0.0063)	0.019 (0.0076)	CHILDREN	0.019 (0.010)	0.028 (0.013)
$\beta_{social\ activities}$			β_q		
INTERCEPT	−0.94 (0.16)	−1.55 (0.18)	INTERCEPT	4.73 (0.054)	4.55 (0.15)
AGE	−0.033 (0.0077)	−0.030 (0.0090)	AGE	−0.0004 (0.0043)	−0.0064 (0.0081)
AGE^2	0.00041 (0.000092)	0.00038 (0.00011)	AGE^2	−0.000027 (0.0061)	0.000030 (0.000057)
EDUCATION	0.023 (0.028)	−0.0018 (0.038)	EDUCATION	−0.42 (0.026)	−0.32 (0.030)
CHILDREN	0.019 (0.011)	0.0013 (0.016)	CHILDREN	−0.012 (0.0038)	0.0052 (0.0035)
$\beta_{home\ production}$			Wage equation		
INTERCEPT	−1.02 (0.11)	−2.4616 (0.23)	INTERCEPT	1.16 (0.088)	1.34 (0.15)
AGE	0.0079 (0.0055)	0.0148 (0.013)	AGE	0.021 (0.0048)	0.024 (0.0069)
AGE^2	0.000004 (0.000066)	0.000009 (0.014)	AGE^2	−0.00022 (0.000066)	−0.00023 (0.000083)
EDUCATION	−0.019 (0.022)	0.0135 (0.039)	EDUCATION	0.44 (0.025)	0.33 (0.029)
CHILDREN	0.13 (0.0090)	0.1165 (0.017)			

Table 6: Parameter estimates for multivariate time allocation model: Estimates of β_j for $j = 1, ..., 6$, β_q and the parameters of the wage equations for males and females. Standard errors in parenthesis.

	ε_w	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_q
ε_w	0.15	0.0024	−0.0016	0.025	0.0083	0.018	0.016	0.0024
ε_1	.	0.22	0.028	0.025	0.0062	0.017	0.047	0.020
ε_2	.	.	0.066	0.039	0.018	0.016	0.045	0.026
ε_3	.	.	.	0.28	0.016	−0.019	0.045	0.025
ε_4	.	.	.	.	0.17	0.018	0.0016	0.016
ε_5	.	.	.	.	.	0.23	−0.017	0.021
ε_6	.	.	.	.	.	.	0.26	0.040
ε_q	.	.	.	.	.	.	.	0.029

Table 7: Females: Estimated covariance matrix.

	ε_w	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_q
ε_w	0.23	0.011	−0.011	−0.0032	0.0096	0.0058	0.030	−0.0028
ε_1	.	0.38	0.014	0.00081	−0.038	0.027	0.0080	0.012
ε_2	.	.	0.057	0.033	0.019	0.0044	0.032	0.017
ε_3	.	.	.	0.41	0.046	−0.026	0.071	0.023
ε_4	.	.	.	.	0.43	0.014	−0.0041	0.018
ε_5	.	.	.	.	.	0.14	−0.0071	0.015
ε_6	.	.	.	.	.	.	0.27	0.025
ε_q	.	.	.	.	.	.	.	0.020

Table 8: Males: Estimated covariance matrix.

Females					Males				
Labour supply (hours per week)			ϵ		Labour supply (hours per week)			ϵ	
% non-zero	Positive	All	Positive	All	% non-zero	Positive	All	Positive	All
Standard labour supply model									
66.49	30.59	20.34	1.94	1.04	90.40	41.59	37.60	1.85	1.67
Multivariate time allocation model									
68.5	29.67	20.33	6.45	3.97	90.72	42.20	38.29	4.06	3.72

Table 9: Comparison of the standard labour supply model and the multivariate time allocation model: ϵ is the median predicted wage elasticity of labour supply. Positive refers to individuals whose predicted labour supply is greater than zero, and All refers to all individuals.

	Females					Males				
	Mean time (hours per week)			ϵ		Mean time (hours per week)			ϵ	
Activity	% non-zero	Positive	All	Positive	All	% non-zero	Positive	All	Positive	All
Sports	0.30	6.14	2.79	-4.08	0	0.54	8.31	4.51	-2.76	-0.14
Volunteer work	0.17	7.05	1.88	-11.04	0	0.20	6.56	1.31	-11.40	0
Social activities	0.62	10.48	9.67	-1.87	-1.44	0.86	8.66	7.45	-1.79	-1.55
Home production	1	30.11		-1.37		1	15.31		-1.23	
Media activities	1	16.30		-1.34		1	18.63		-1.65	
Other time use	1	12.58		-1.25		1	12.35		-1.23	
Sleep	1	74.34		-0.89		1	70.14		-1.17	

Table 10: Mean predicted times and median predicted wage elasticities: ϵ is the median predicted wage elasticity. Positive refers to individuals whose predicted time in the activity is greater than zero, and All refers to all individuals.